Solution of the Dissipative, Quantum XY Model*

Application to Local Quantum-Criticality in Cuprates and in a Class of Metallic Anti-ferromagnets and 2 D-superconductor-insulator QCP's.

Based on Analytic work:
Vivek Aji and CMV - PRL 2007, PRB 2009, 2010.

Quantum Monte-Carlo:
Spaersted, Stensted, Sudbo - PRB 2012,
Lijun Zhu and CMV (Unpublished)

Quantum-criticality in 2D itinerant AFM:
CMV (Unpublished)

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*Chakravarty, Kivelson, Ingold, Luther; MPA Fisher (1986)

Plan of this Lecture:

1. What are local QCF's? Need for introducing them in Cuprates, AFM criticality in Heavy fermions, and possibly the Fe-based Superconductors.

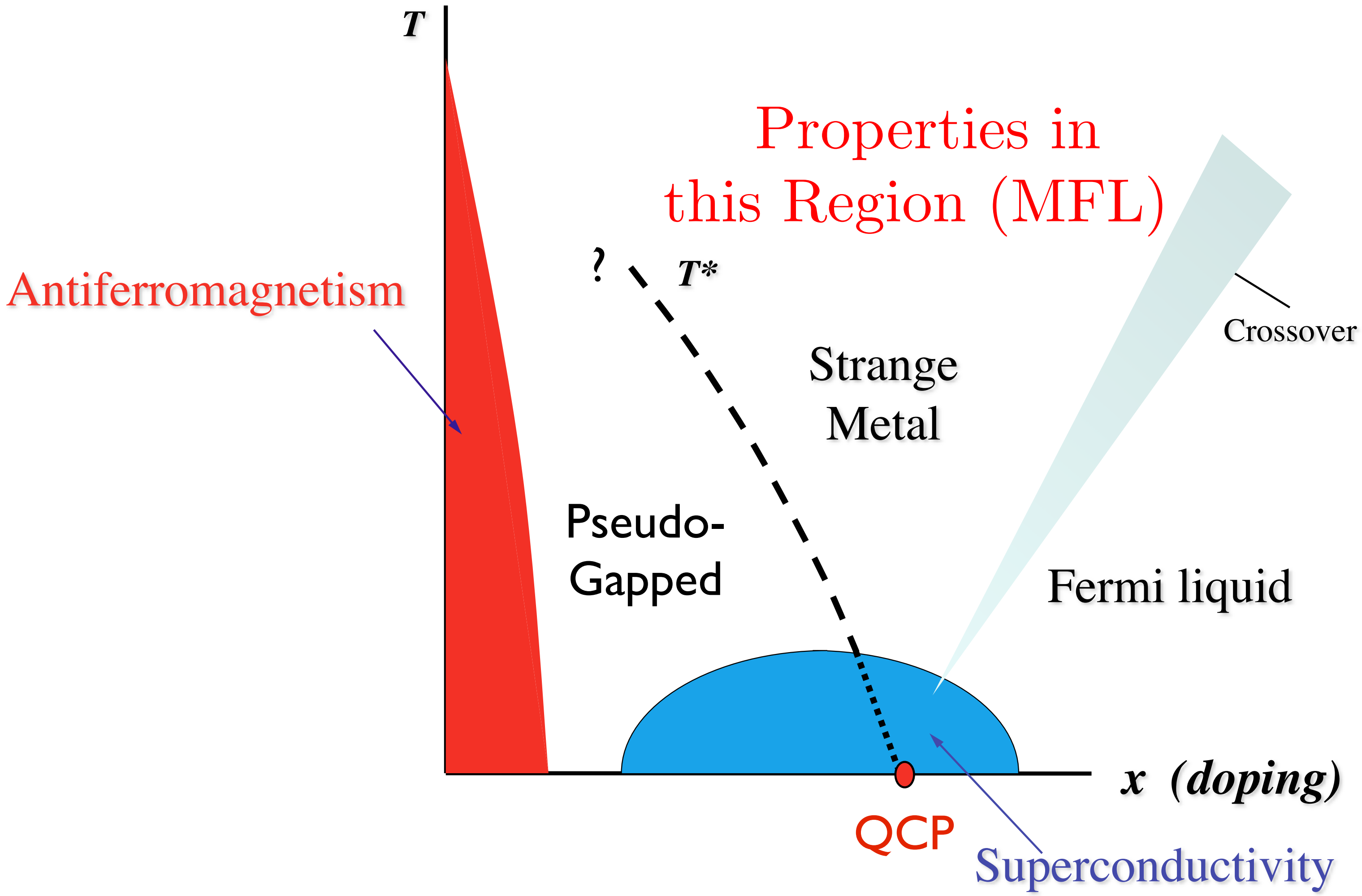
2. A simple model for itinerant AFM - Mapped to a decorated Diss. Qtm. XY model.

3. Solution of the Model.

4. Test of the solution by Qtm. Monte-Carlo calcs.

Need for Local QC in Cuprates.

Universal Phase Diagram



Unusual Properties in the "Strange Metal Phase"

Resistivity, $R = a + bT$

Optical conductivity $\sigma(\omega) \propto \omega^{-1} \ln(\omega)$

Raman Scattering, $I(\omega) \propto constant, \omega \leq \omega_c$

Tunneling, $G(V) \propto |V|$

Nuclear relaxation rate on Cu, $T_1^{-1} = A + BT$

Landau Fermi-Liquid Theory and Quasi-particle concepts do not work.

Point of view:

The diverse anomalies must all come from one and the same basic physics and finding it is the central problem.

Contrast, for example:

Raman Scattering (q $\rightarrow$ 0, as a fn. of)

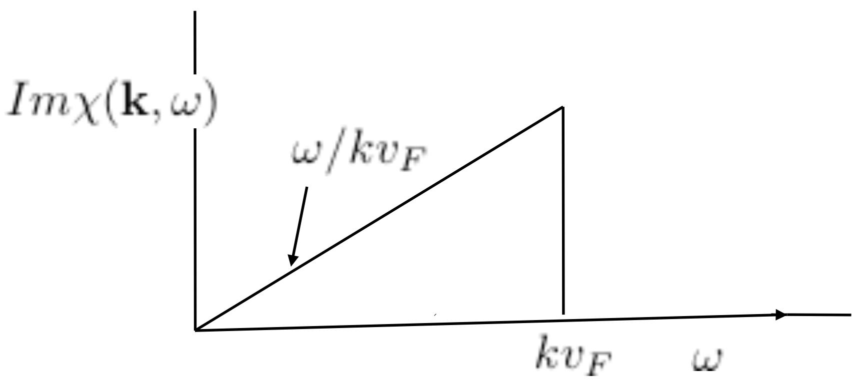
$$S(\omega) \approx constant, \omega \lesssim \omega_c, \quad \omega_c \approx 0.4eV$$

Nuclear relaxation rate: $(\omega \to 0, \text{ integrated over } \mathbf{q})$

On Cu, $T_1^{-1} \approx A + BT$

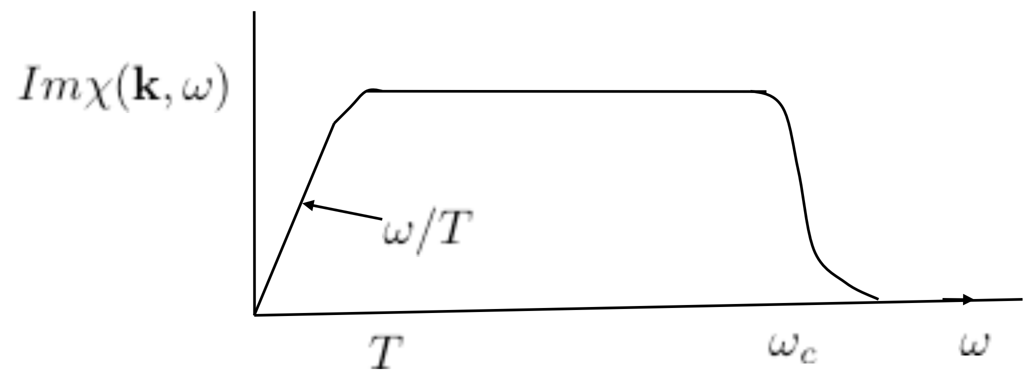
What does not work and what works.

Landau Fermi-liquid



Low Density of Low-energy Excitations.
Scattering rate of electrons $\propto T^2$

Marginal Fermi Liquid



High Density of Low-energy Excitations controlled only by the temperature of measurement and independent of momentum.

Scattering rate of electrons predicted $\propto (T, |\omega|)$, ind. of momentum.

Scale-Invariant Spectrum- implies a phase transition at $T \rightarrow 0$.

Sp. Heat ~T ln T.

A New Paradigm for Criticality

$$Im\ \chi(\omega, \mathbf{q}) \propto -\omega/T \quad \text{for } \omega/T << 1,$$
$$\propto -sgn(\omega) \quad \text{for } \omega_c >> \omega >> T.$$

$$Re\ \chi(\omega, \mathbf{q}) \propto ln\ (\omega_c/x)\ , x \approx max(\omega, T).$$

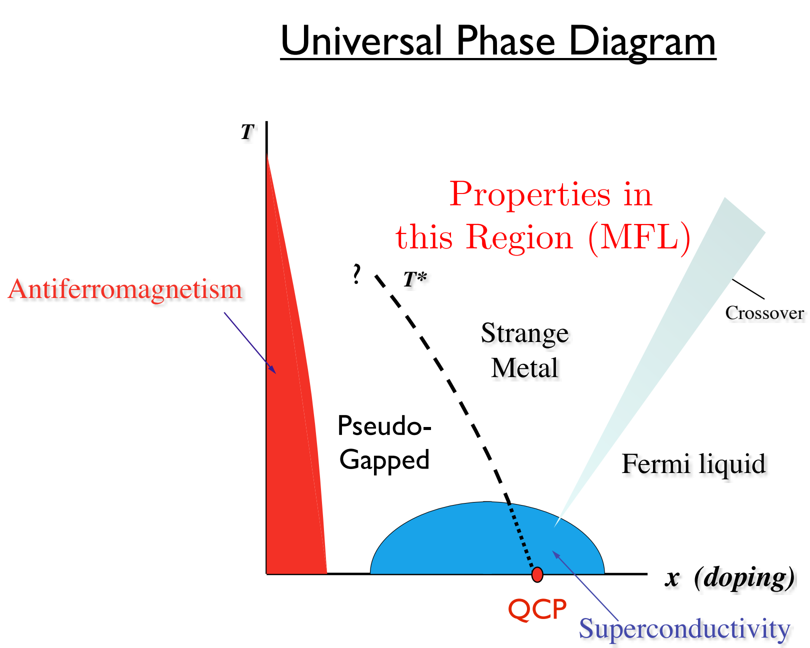
(Slightly different forms are also allowed.)

1. A singularity at $T \to 0$, i.e. a Quantum Critical Point.
2. Scale-invariant in frequency but spatially local!

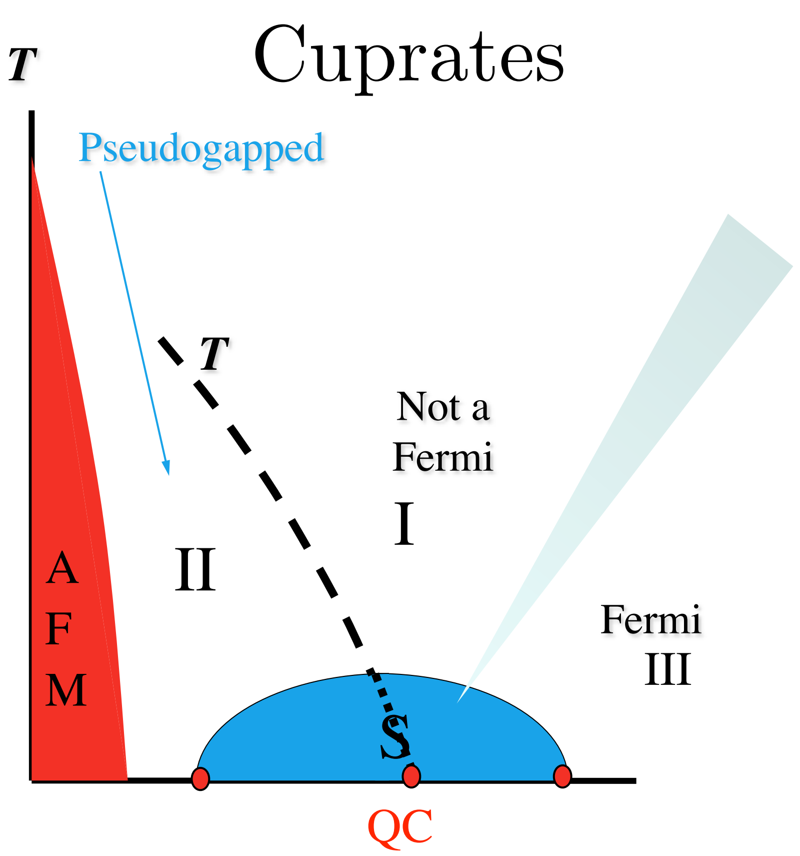
Crazy but important predictions from it worked.

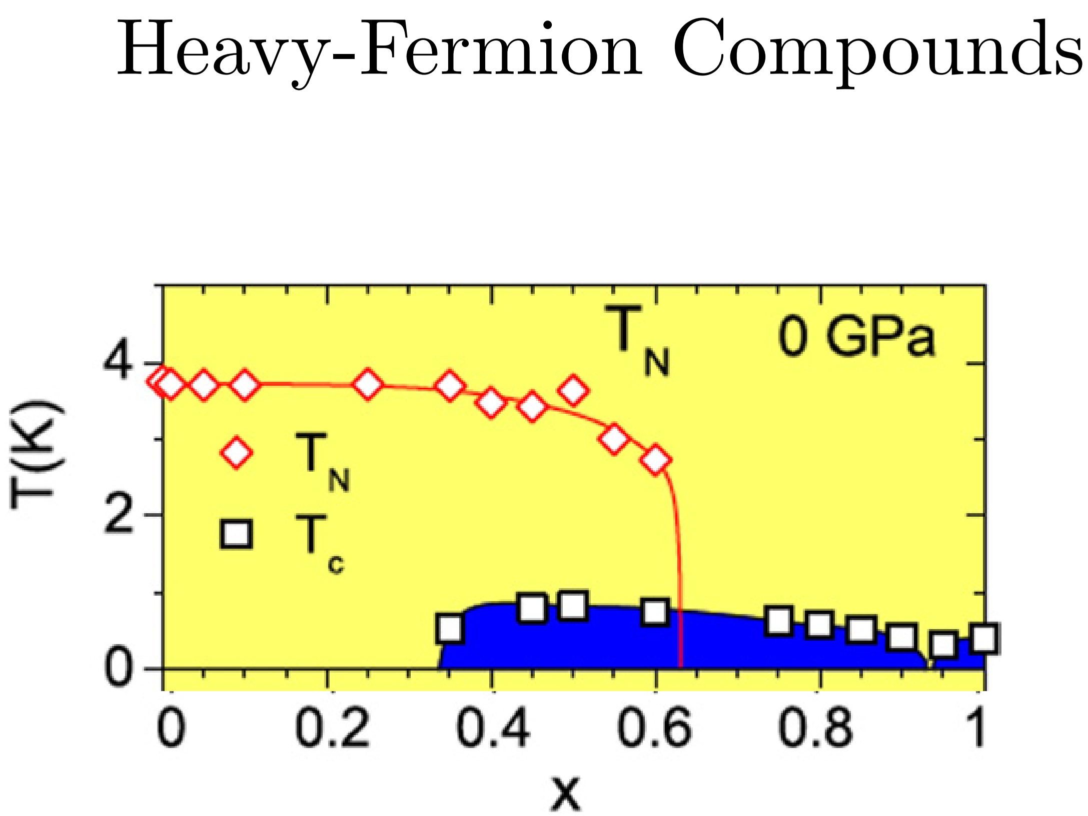
Question for Cuprates:

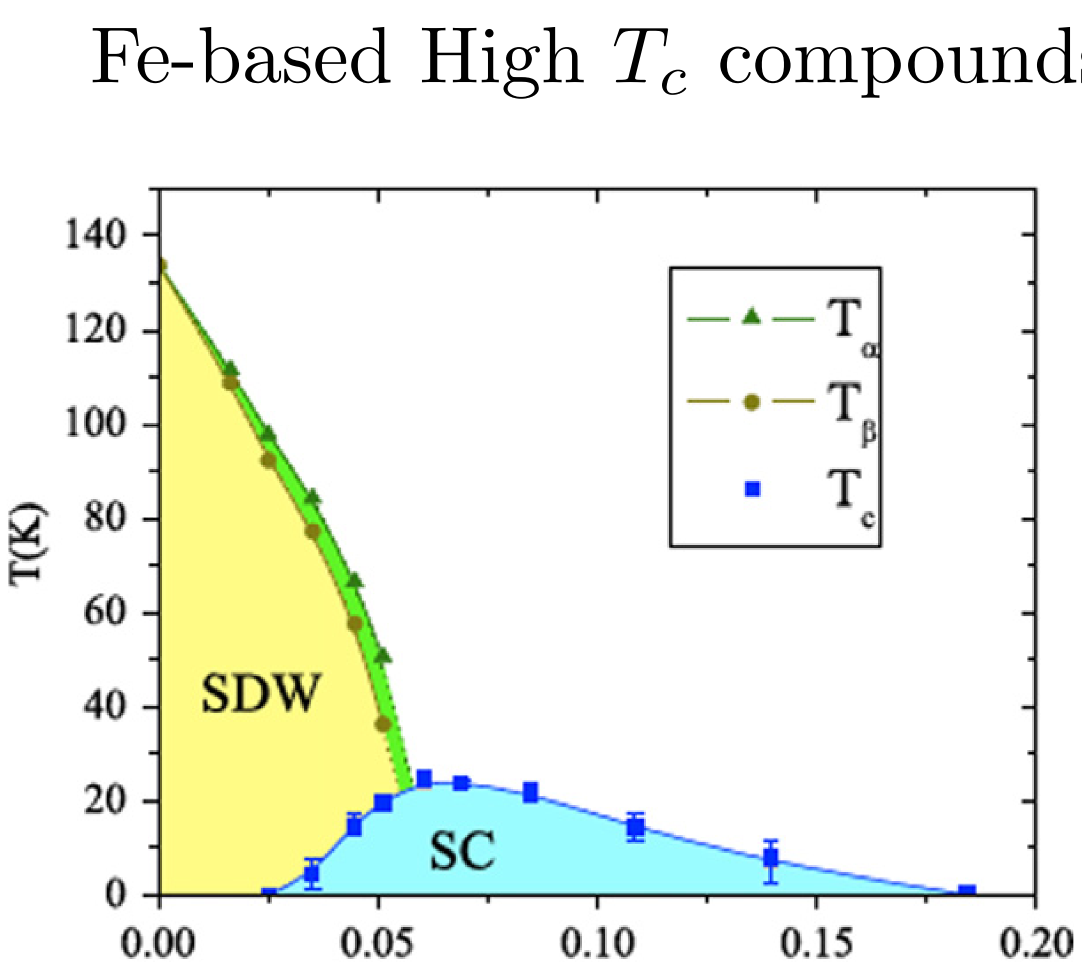
What is this a fluctuation of?



Common Features of Quant. Criticality at different transitions







The itinerant AFM Q.C. Problem

There exists a canonical Transformation between a repulsive Hubbard model at arbitrary filling and a decorated attractive Hubbard model at finite Zeeman field.

$$H = \sum_{<ij>,\sigma=\uparrow,\downarrow} t_{ij} a^{\dagger}_{i,\sigma} a_{j,\sigma} + H.C. + U \sum_i (n_{i\uparrow} - 1/2)(n_{i\downarrow} - 1/2)$$
$$+ I_z \sum_i (S^z_i)^2 - \mu \sum_i n_i.$$

This model at filling away from 1/2 has an income. AFM transition at a vector $\mathbf{Q}_0 + \mathbf{q}_0, \quad \mathbf{Q_0} = \pi/\mathbf{R}_0$

The canonical transformation:

$$a_{i,\uparrow} \rightarrow e^{i\phi_i} \tilde{a}_{i,\uparrow}; \; a^{\dagger}_{i,\uparrow} \rightarrow e^{-i\phi_i} \tilde{a}^{\dagger}_{i,\uparrow};$$
$$a_{i,\downarrow} \rightarrow \tilde{a}^{\dagger}_{i,\downarrow} e^{iQ_0.R_i + i\phi_i}; \; a^{\dagger}_{i,\downarrow} \rightarrow \tilde{a}_{i,\downarrow} e^{-iQ_0.R_i - i\phi_i}.$$

with

$$\phi_i = -\frac{1}{2} \mathbf{q}_0 \cdot \mathbf{R}_i,$$

leads to:

$$\tilde{H} = \sum_{<ij>,\sigma} \tilde{t}_{ij} e^{-i\sigma(\phi_i - \phi_j)} \tilde{a}^{\dagger}_{i,\sigma} \tilde{a}_{j,\sigma} + H.C. - \tilde{U} \sum (\tilde{n}_{i\uparrow} - 1/2)(\tilde{n}_{i\downarrow} - 1/2) - \tilde{h} \sum_i \tilde{S}^z_i$$

spin-orbit coupling. Attractive Interaction. Zeeman-Fld.

$$\tilde{t} \; = \; t; \quad \tilde{U} = U - 2I_z, \tilde{h} = \mu, \tilde{\mu} = h.$$

With the canon. transformation,

$$S_i^+ \rightarrow e^{i(\mathbf{Q}_0 + \mathbf{q}_0).\mathbf{R}_i} \tilde{a}^+_{i\uparrow} \tilde{a}^+_{i\downarrow}, \quad S_i^- \rightarrow e^{-i(\mathbf{Q}_0 + \mathbf{q}_0).\mathbf{R}_i} \tilde{a}_{i\downarrow} \tilde{a}_{i\uparrow}$$

$$\chi^H_{(S^+S^-)}(\mathbf{Q} + \mathbf{q}, \omega) = \chi^{\tilde{H}}_{(\Delta^+\Delta^-)}(\mathbf{q}, \omega).$$

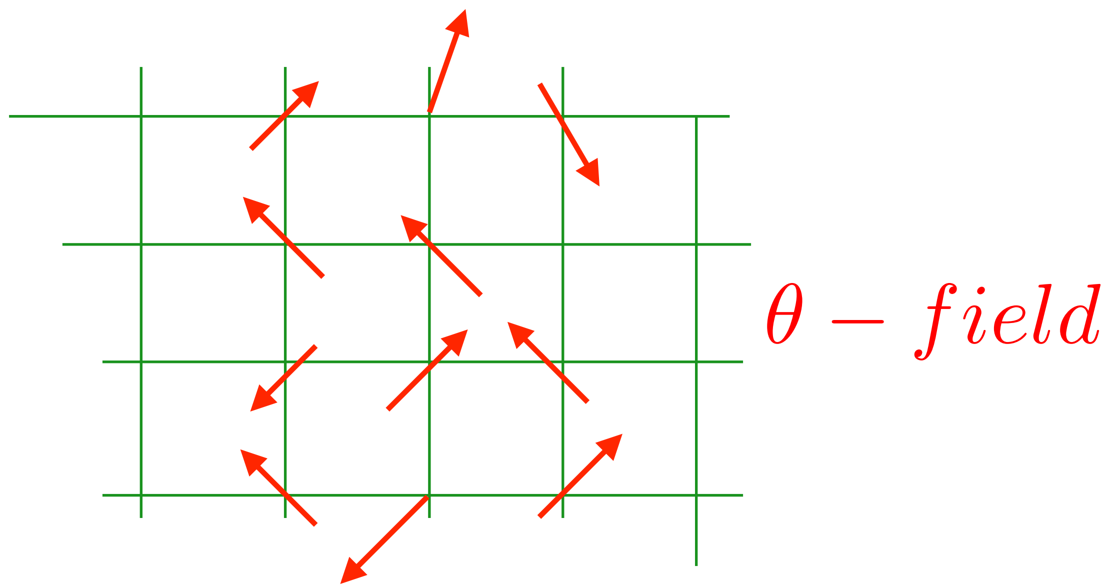
In 2D, the the latter is calculable from the 2D-XY model with Dissipation.

The usual Dissipation used for the AFM model transforms to the Caldeira-Legett Dissipation used for the XY Model.

Quantum-Critical Fluctuations of the Model

(Vivek Aji, CMV - PRL 2007, PRB-2009, 2010)

Classical Model: XY model with 4-fold Anisotropy



$$\mathcal{L} = \sum_{<ij>} K \cos(\theta_i - \theta_j) + K_4 \cos 2(\theta_i - \theta_j) + h_4 \cos(4\theta_i)$$

Anisotropy: Marginally Irrelevant in the Fluctuation region,
Highly relevant in the ordered region.

Topological Phase Transition (Kosterlitz-Thouless, Berezinsky)
Ordering by Binding of vortices of opposite circulation.

Quantum Model:

Add Kinetic Energy of Rotors:

$$K_\tau \sum_i \left(\frac{\partial \theta_i}{\partial t}\right)^2$$

And

Dissipation by Decay into FErmions,

Caldeira-Leggett form, in Fourier space:

$$\alpha \sum_{\mathbf{k},\omega} |\omega| \; k^2 \; |\theta(\mathbf{k},\omega)|^2$$

Model has a phase transition as a function of α.

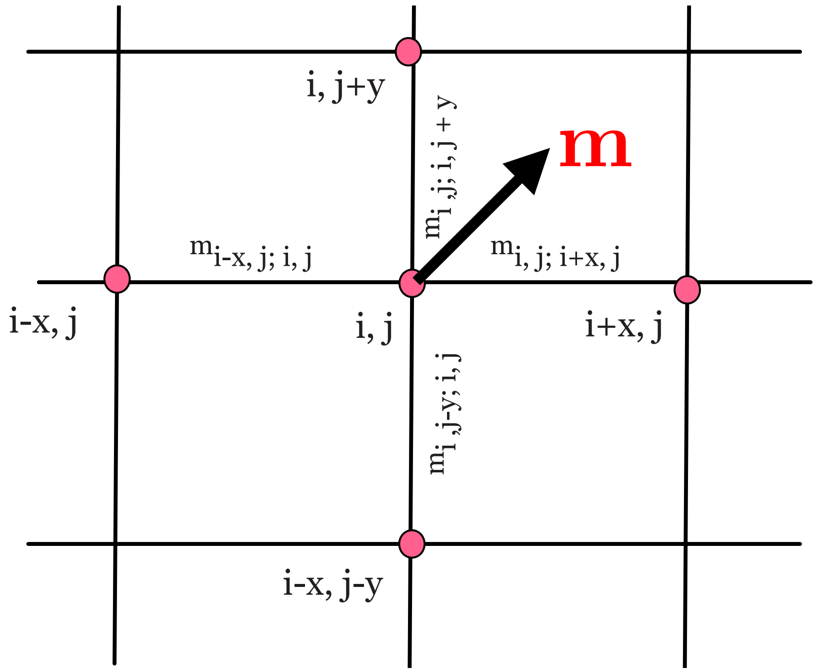
Surprisingly, the Quantum model is as solvable as the Classical XY Model, with remarkable Properties.

By Finding the Right Variables.

Right Variables.

Define a link variable m, which is the difference of θ's at adjacent sites.

m is a discrete integer field which lives on links.



The $\mathbf{m}$ field can be divided into solenoidal and irrotational:

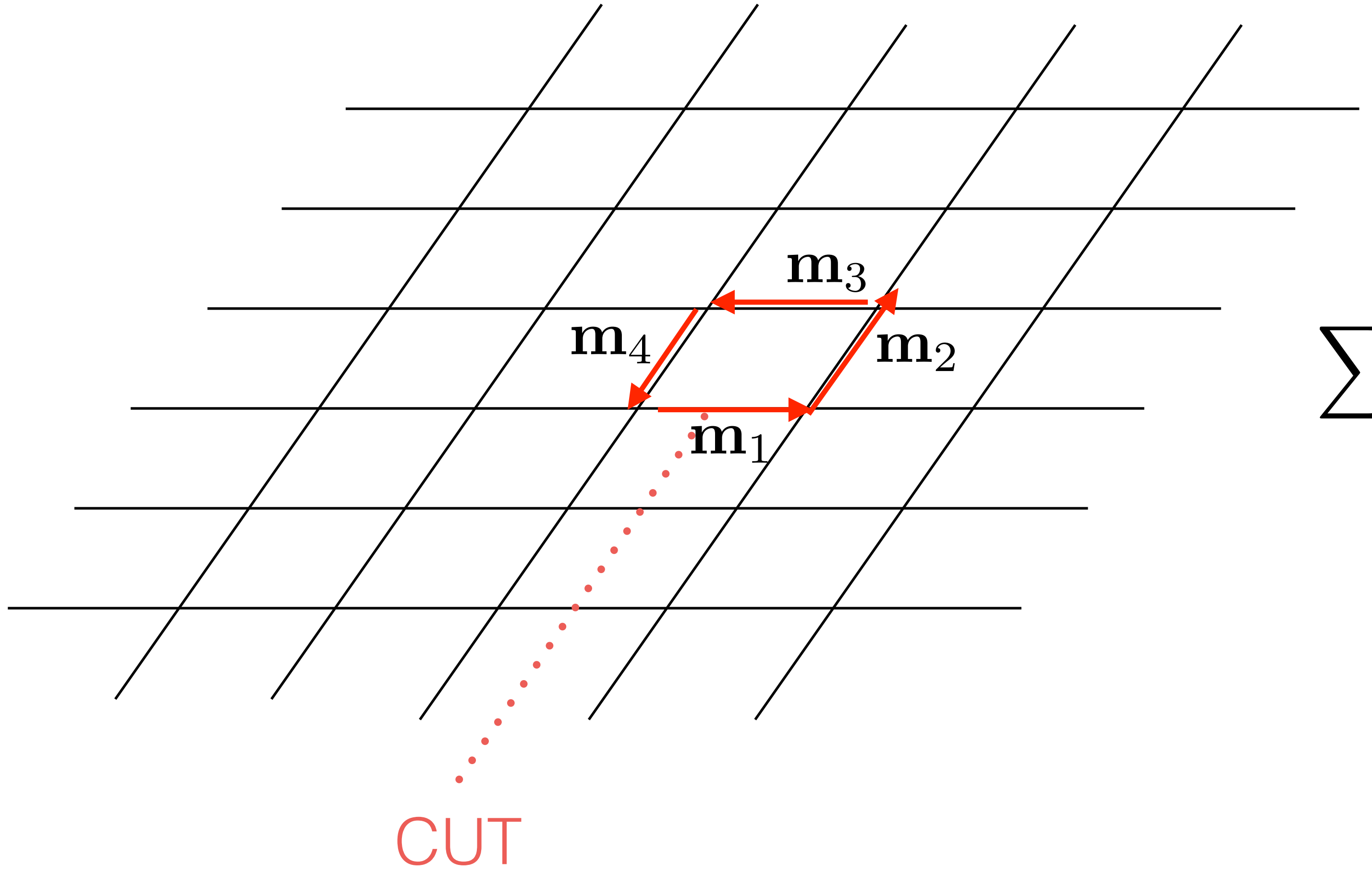
$$\mathbf{m} = \mathbf{m}_\ell + \mathbf{m}_t$$

Define

$$\nabla \times \mathbf{m}_t = \rho_v \hat{\mathbf{z}} \quad : \text{Vortex}$$

$$\frac{\partial \nabla \cdot \mathbf{m}_\ell}{\partial t} = \rho_w \quad : \text{Warp}$$

Vortex

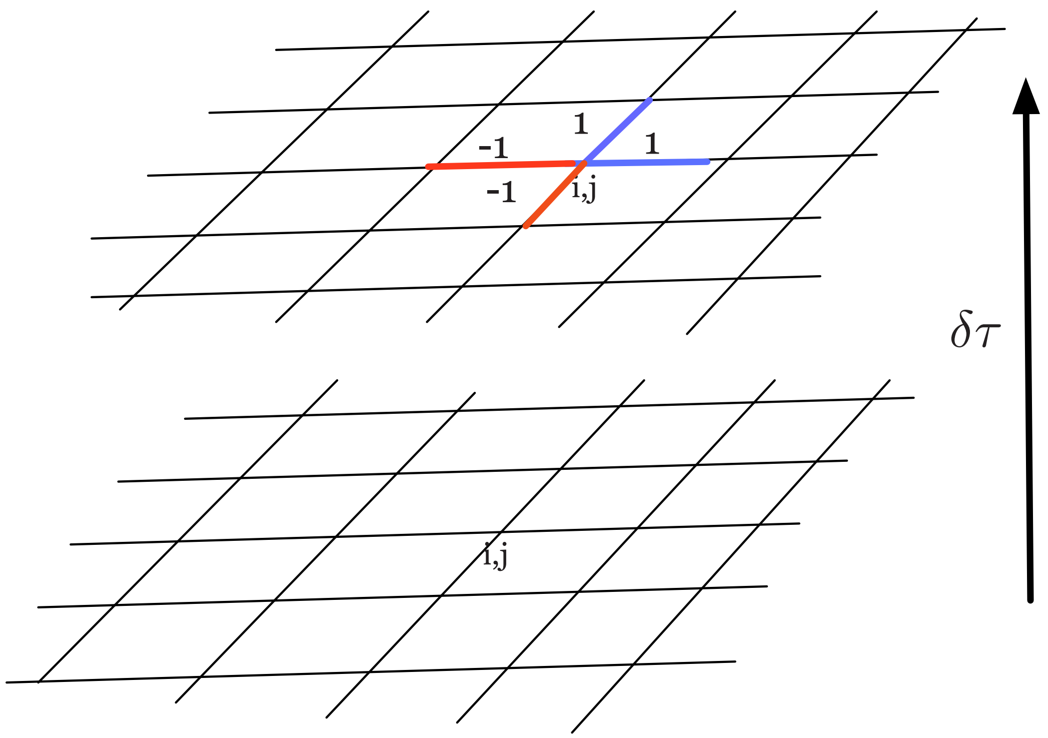


$$\sum_i \mathbf{m}_i = 2\pi$$

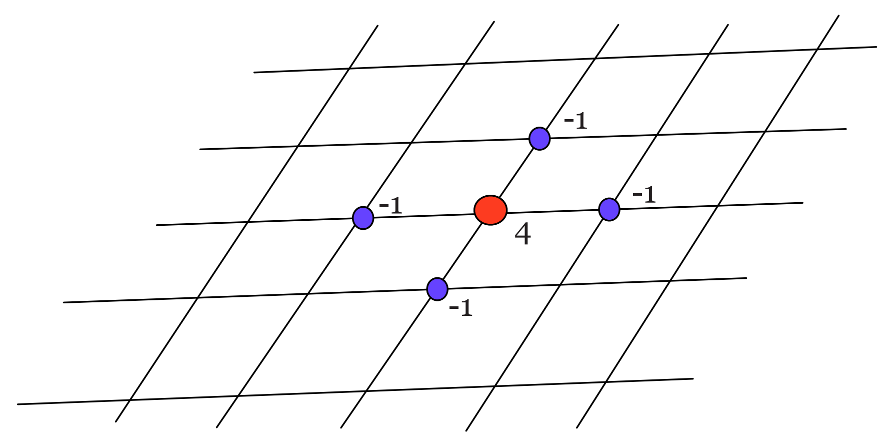
What is a warp?

Jump in Phase by 2π at a point in space between two time-slices,

Change in $\mathbf{m}$:



Change in $\nabla \cdot \mathbf{m}$:



Creates a monopole of charge 4 surrounded by 4 monopoles of charge -1.

Then one finds that the singular part of the action decouples as:

$$\mathrm{S} = \int d\tau d\mathbf{r} \mathbf{dr'} \mathbf{J} \rho_{\mathbf{v}}(\mathbf{r},\tau)\rho_{\mathbf{v}}(\mathbf{r'},\mathbf{t}) \mathbf{ln}\,|\mathbf{r}-\mathbf{r'}| + \int \mathbf{drd}\tau\mathbf{d}\tau' \alpha \rho_{\mathbf{w}}(\mathbf{r},\tau)\rho_{\mathbf{w}}(\mathbf{r},\tau') \mathbf{ln}\,|\tau-\tau'|$$

$$+ \int d\mathbf{r} d\mathbf{r}' d\tau d\tau' \rho_w(\mathbf{r},\tau)\rho_w(\mathbf{r}',\tau') \frac{1}{\sqrt{(|\mathbf{r}-\mathbf{r}'|^2 + v^2(\tau-\tau')^2}}.$$

The last term is Coulomb interaction in 3 D, which by itself does not lead to any transition.
The first two terms are orthogonal.
So, at critical points, the problem is soluble.
The last term determines cross-overs between the critical lines in parameter space.

Solution:

The Action can be transformed
(after integration of small amplitude fluctuations),
to a model for orthogonal topological excitations,
warps and vortices.

When warps dominate, the correlation function
of the order parameter in fluctuation regime,

$$\langle e^{i\theta(\mathbf{r},\tau)} e^{-i\theta(\mathbf{r}',\tau')} \rangle \propto \delta(\mathbf{r}-\mathbf{r}') \frac{1}{\tau-\tau'} \equiv G_\theta(\mathbf{r}-\mathbf{r}', \tau-\tau')$$

Fourier transform of this is
$-\tanh(\frac{\omega}{2T})$,
with a cut-off $\omega_c = \sqrt{K_\tau K}$

Local Criticality with ω/T - scaling.

(Cross-over functions calculated, detailed q dependence ?)

Phase Diagram and Correlation functions by Qtm. Monte-Carlo Calcs.

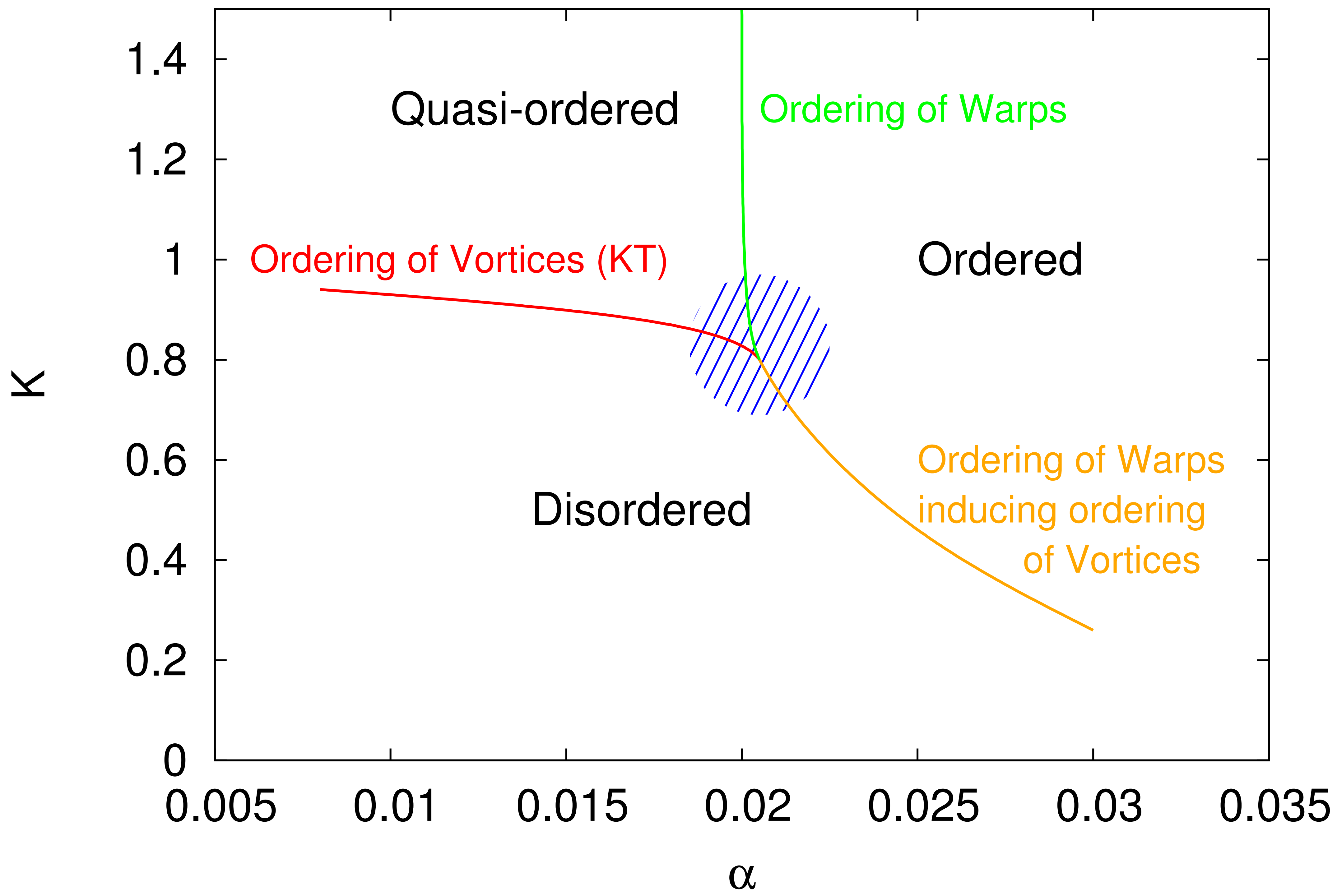
Three parameters:
Moment of Inertia K_τ, Coupling K, Dissipation α

In a large region of parameters,
Transition to "ordered" state driven by α.

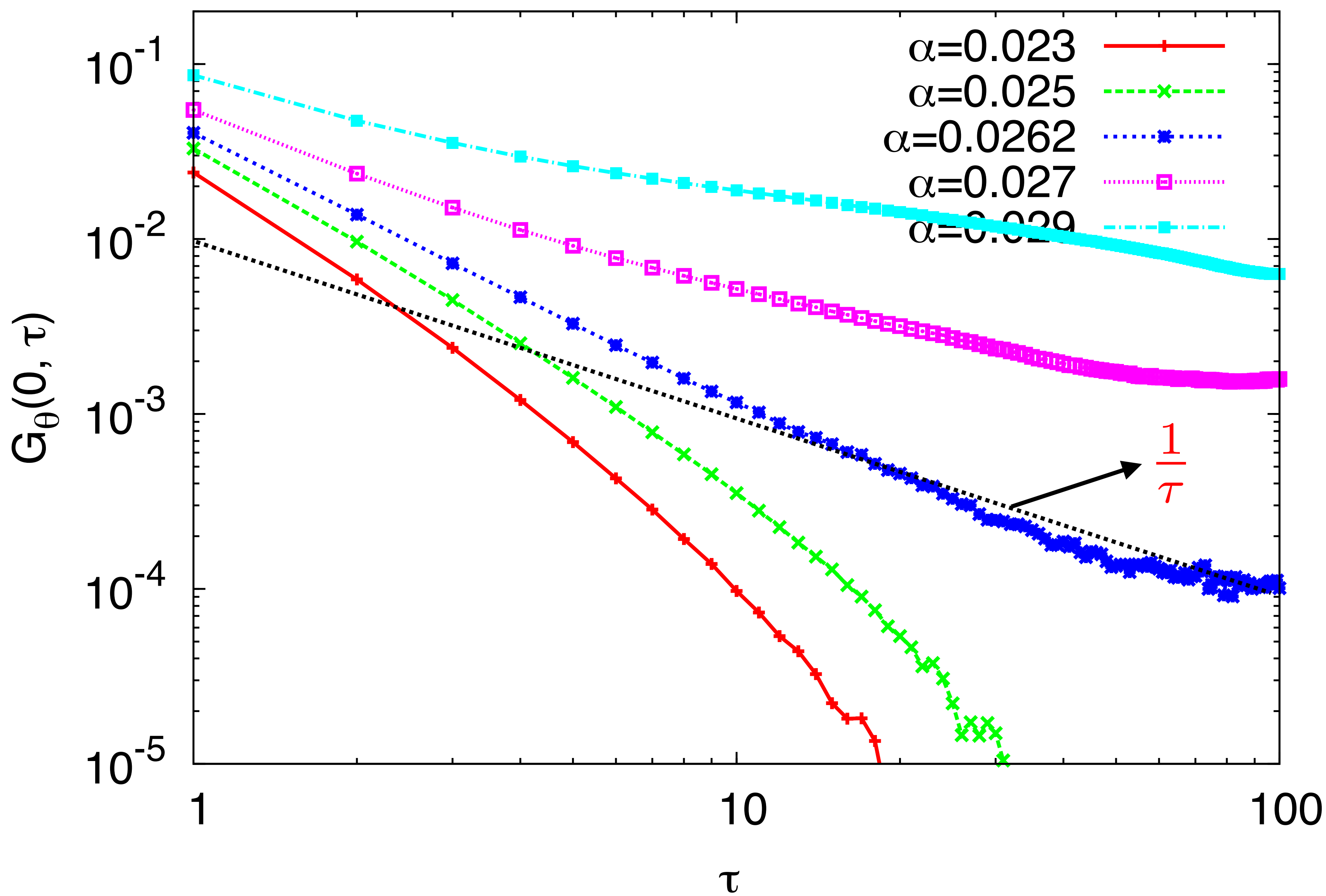
At this transition, change in the density of 'warps'
as a function of $(\alpha - \alpha_c)$.
Different for $(\alpha - \alpha_c) > 0$, and < 0.

Correlation functions:
$< e^{-i\theta_{i,\tau}} e^{i\theta_{j,\tau'}} >$ calculated.

Phase Diagram for a fixed K_τ.



Correlation in time.

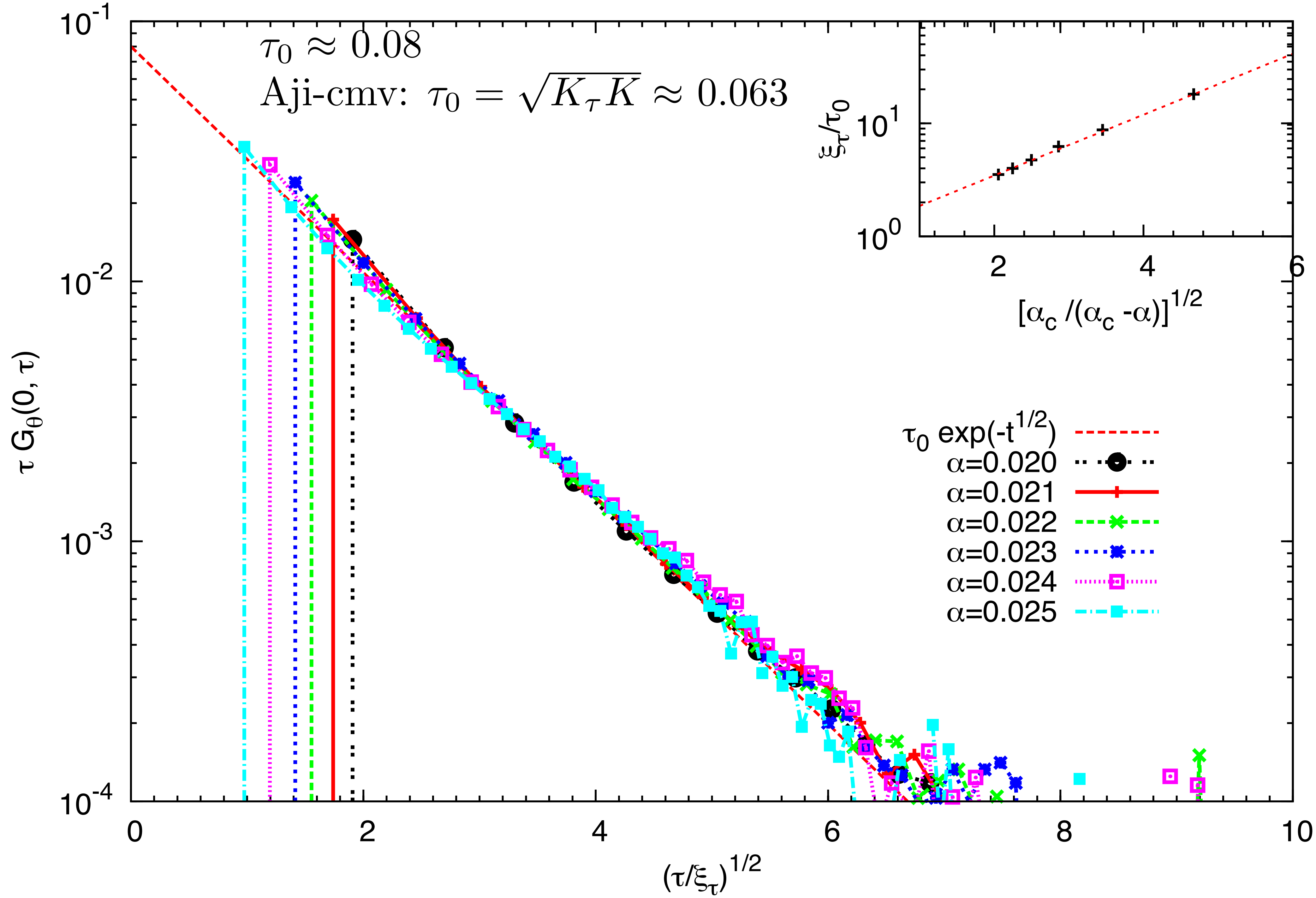


Scaled Correlation in time for $\alpha < \alpha_c$.

$$G_\theta(0,\tau) = \frac{1}{\tau}\; e^{(-\tau\xi_\tau)^{1/2}}$$

$$\xi_\tau = \tau_0 e^{[\alpha_c/(\alpha_c-\alpha)]^{1/2}]}$$

$\rightarrow$ Criticality with ω/T-scaling, with crossover.

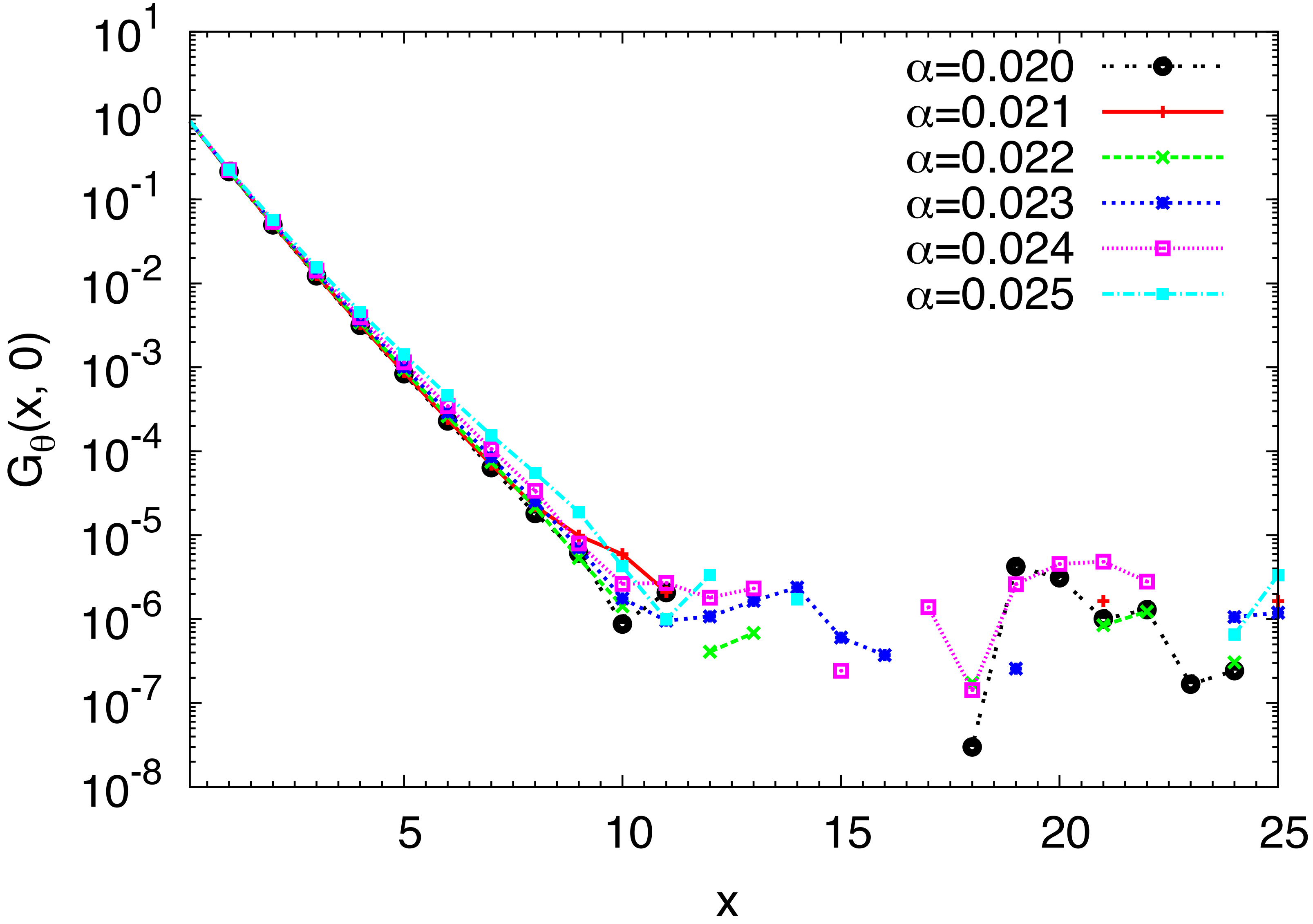


Correlation as a function of space.

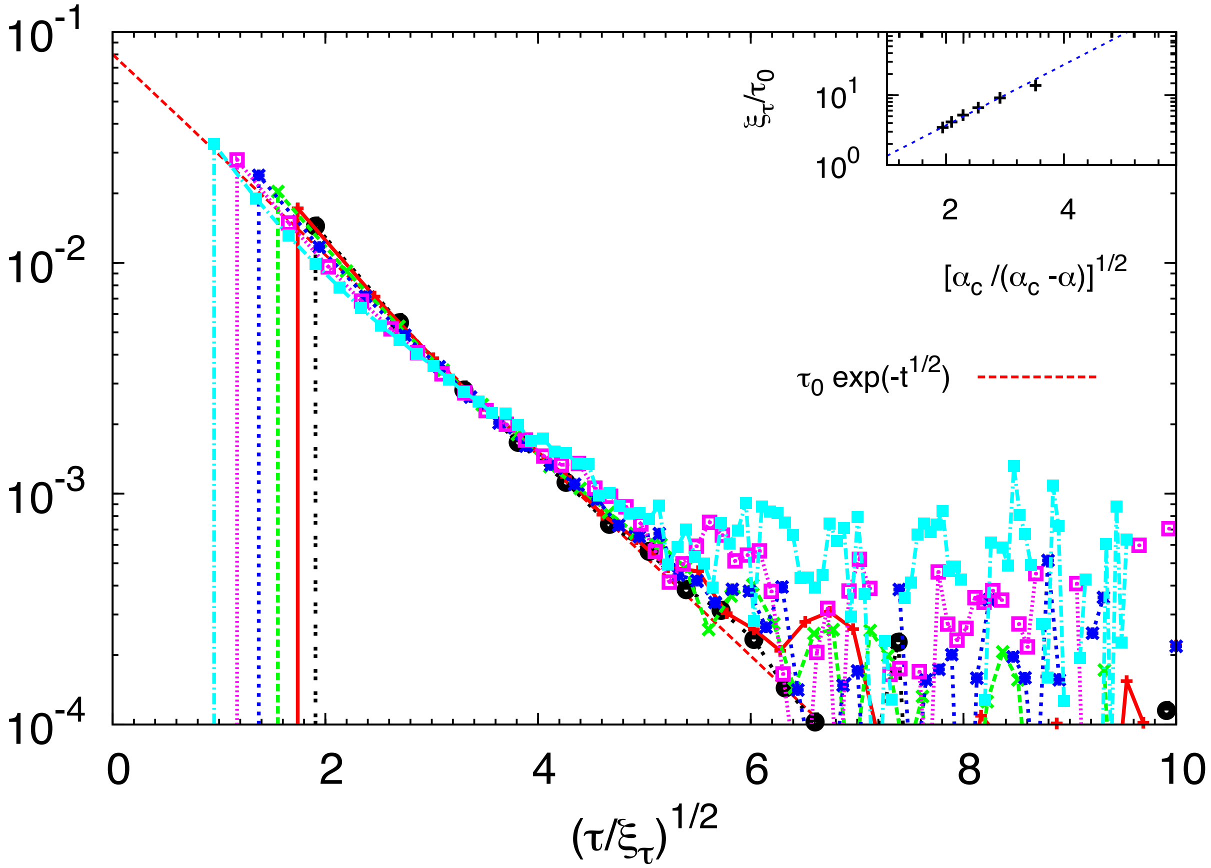
$$G_\theta(x,0) = \exp(-x/\xi_0),$$

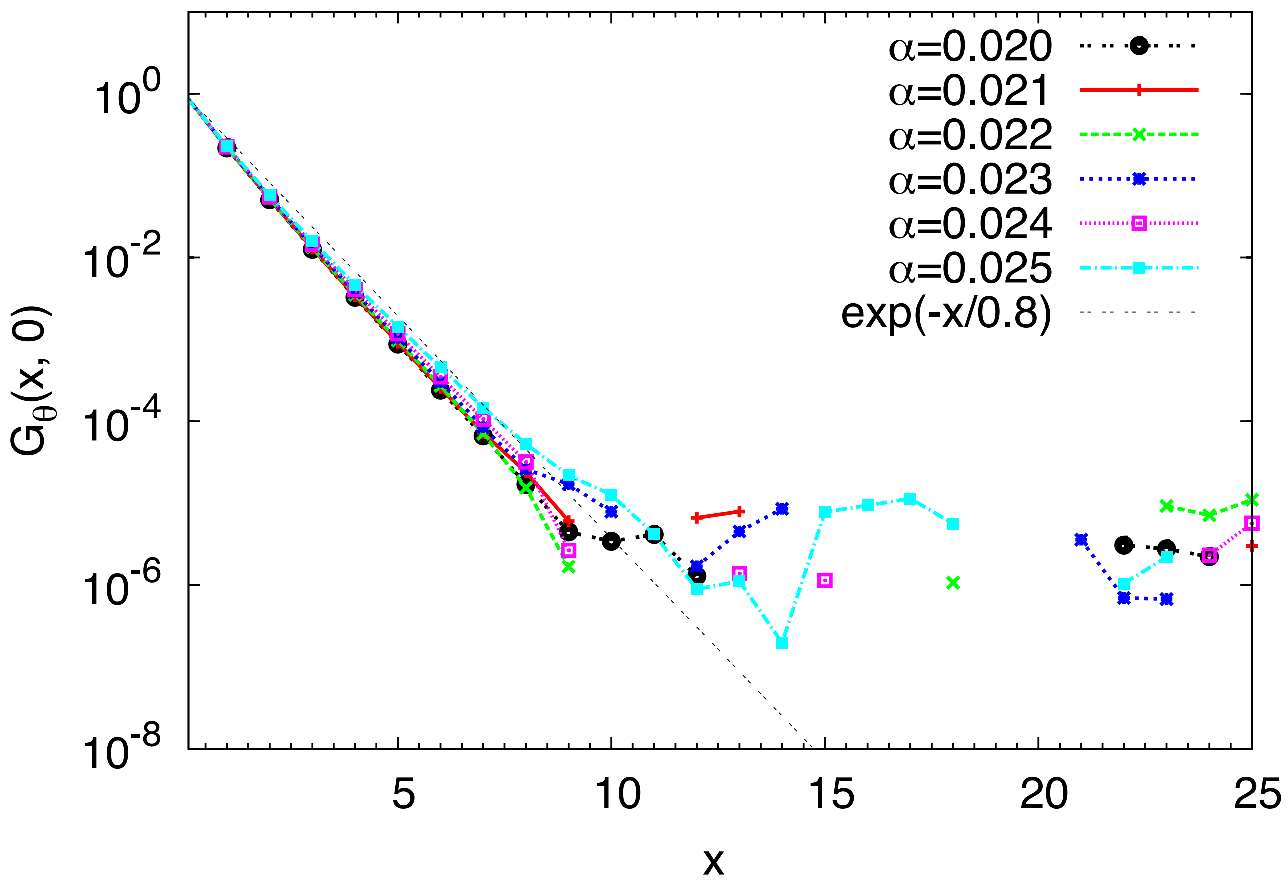
$$\xi_0 \approx 1, \text{ ind. of } \alpha$$

→ Spatially Local Criticality.



Corr. fns. With four-fold anisotropy parameter h4 = 5





Summary:

1. Quantum-Criticality of some models in 2D, with appropriate dissipation, can only be described by topological excitatations.

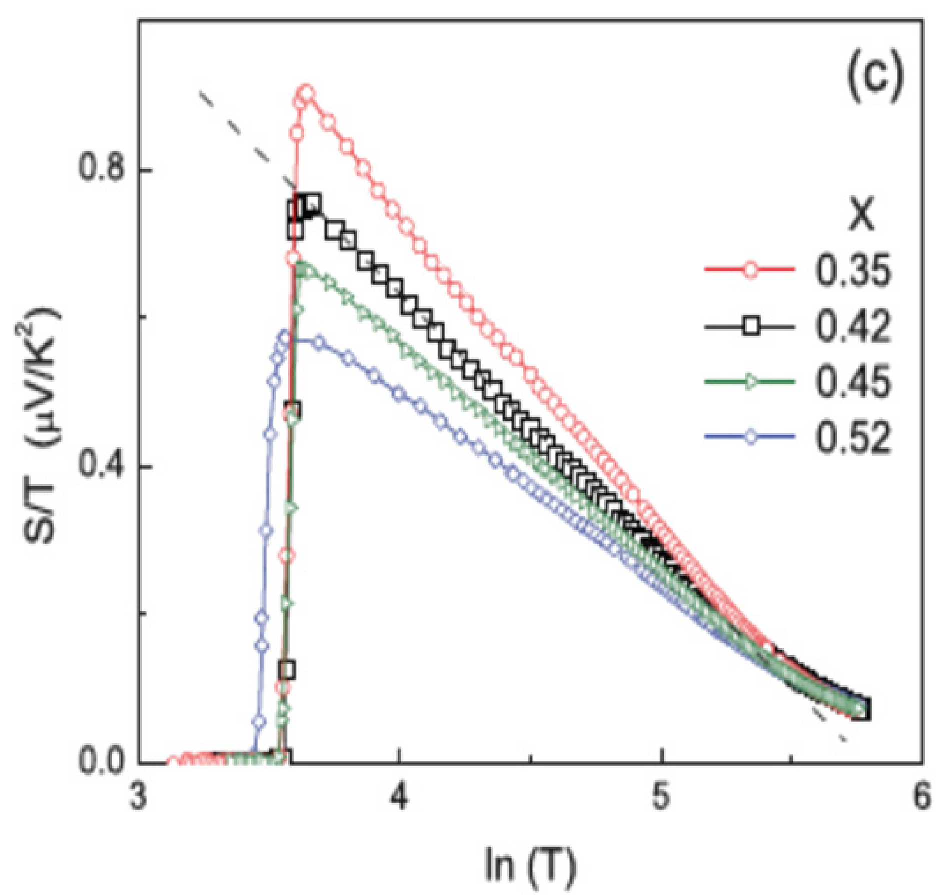
2. In XY Models with 4-fold or higher anisotropy, proliferation of a new class of topological excs.- 'warps' leads to spatial locality and ω/T scaling.

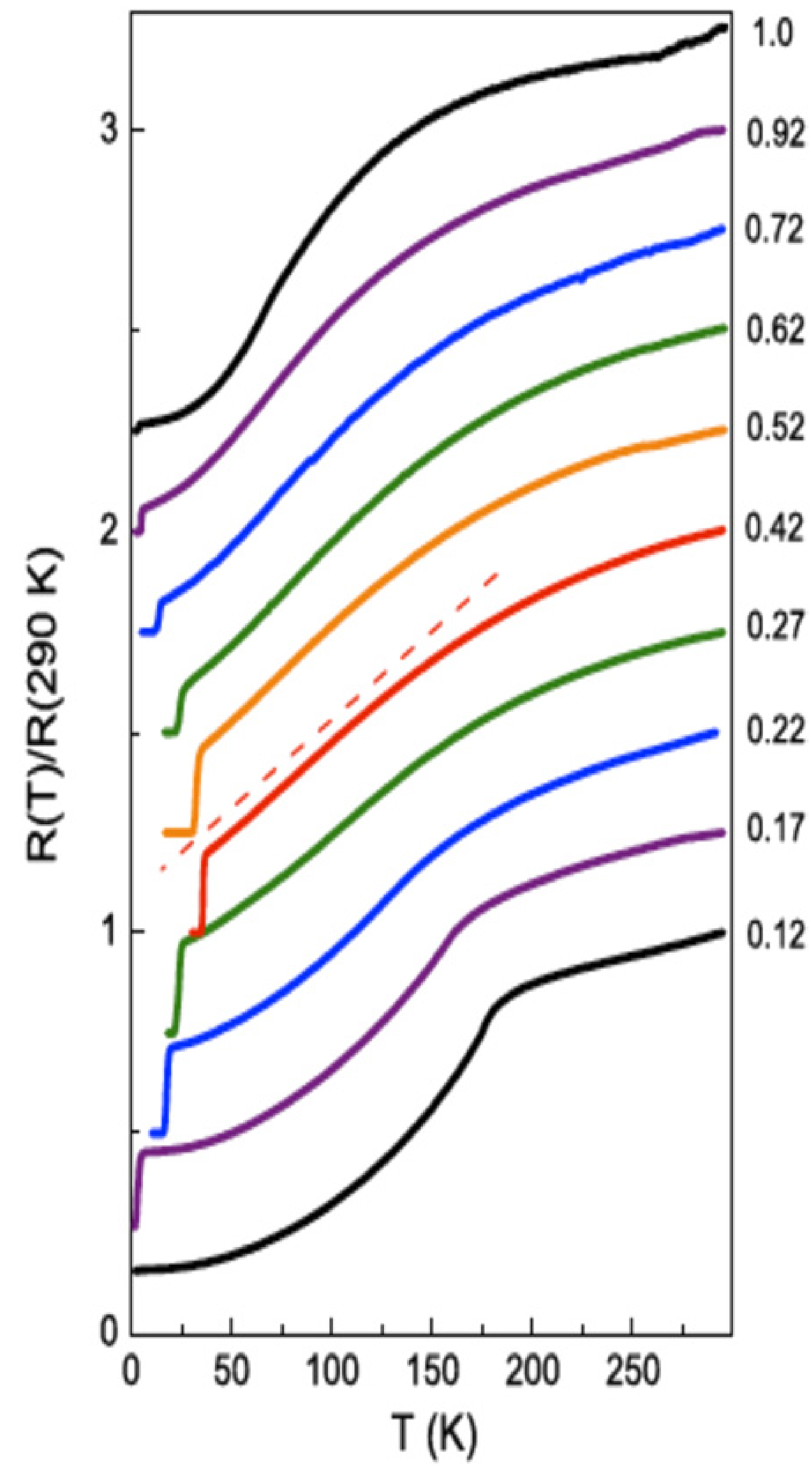
3. Model appears soluble in a controlled way.

4. Solution directly applicable to Cuprate and planar AFM Quantum-criticality.

5. Detailed applications to Heavy Fermions and Fe-compounds?

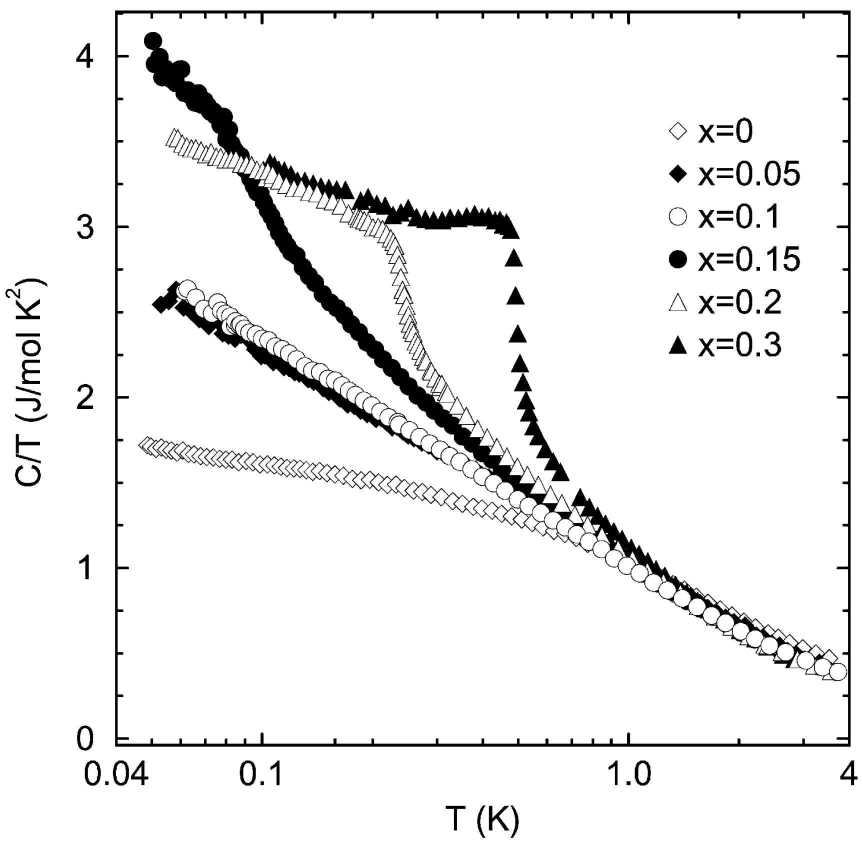
ThermoPower and Resistivity of $Fe_{1-x}Co_xAs_2$

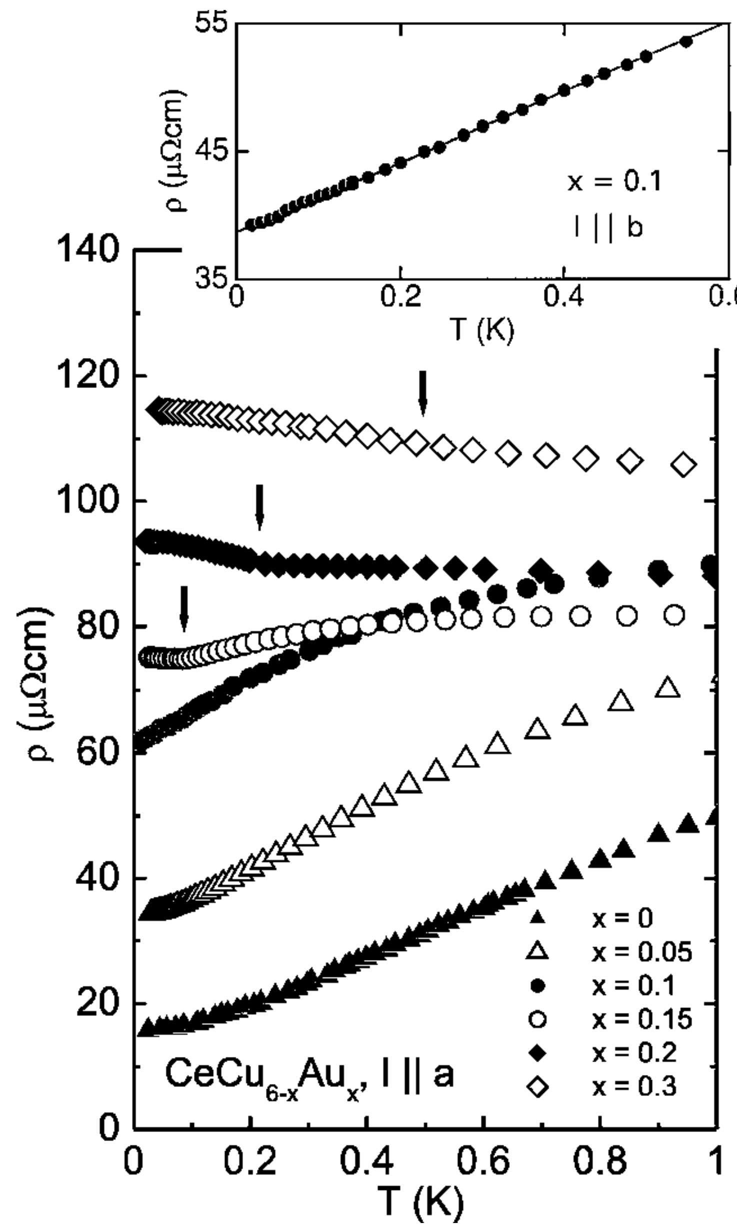


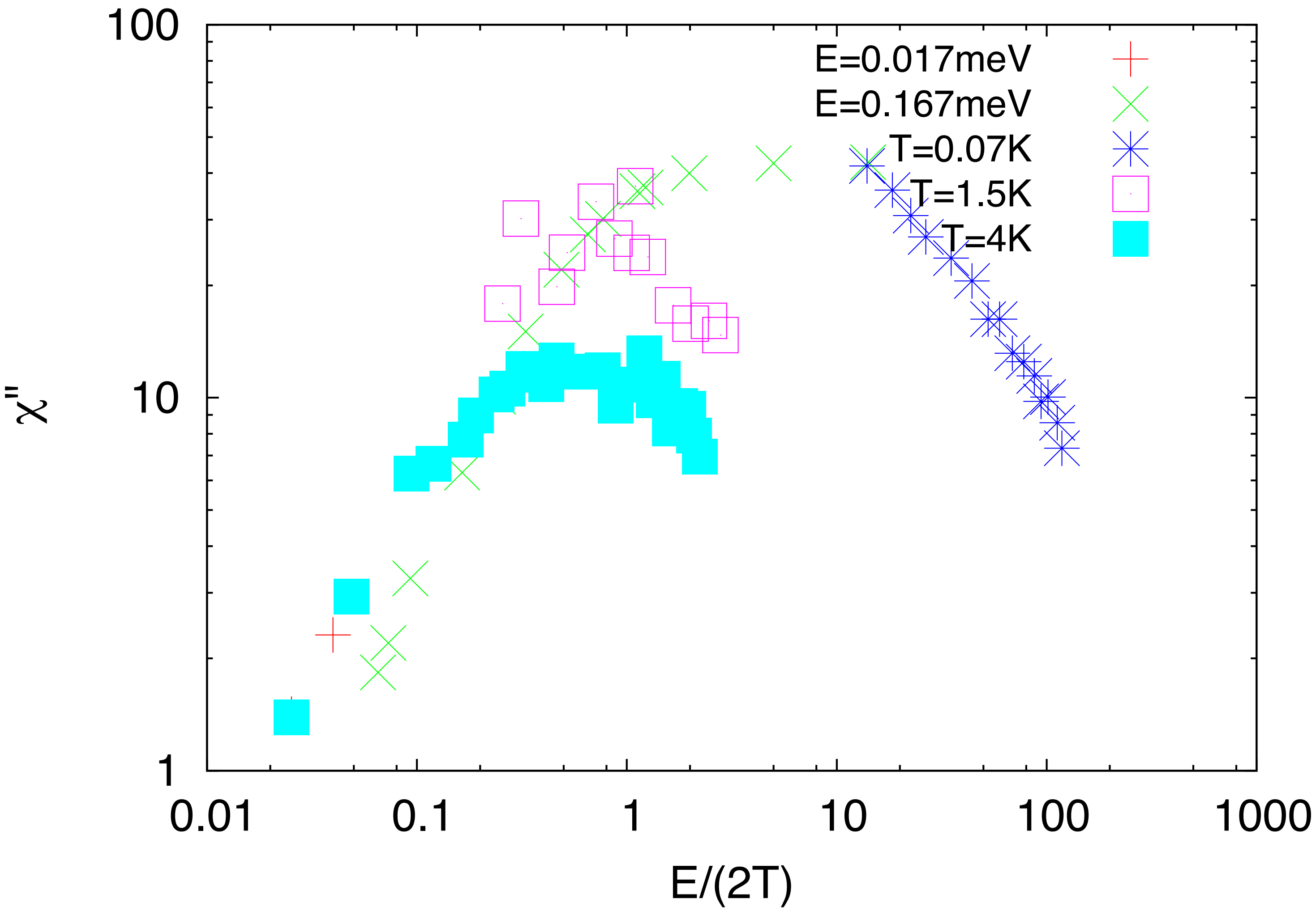


Also (|B|/T) scaling of the resistivity (Analytis 2014)

Heavy-Fermions: CeCu(6-x) Au(x)







100
10
1
0.01
0.1
1
10
100
1000
E/(2T)
χ''
E=0.017meV
E=0.167meV
T=0.07K
T=1.5K
T=4K

Slope ≃ 1, gives linear in T resistivity, T ln T sp. ht.