Motivating Holographic quantum error correcting codes

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Joint work with: Beni Yoshida, Daniel Harlow & John Preskill

Outline

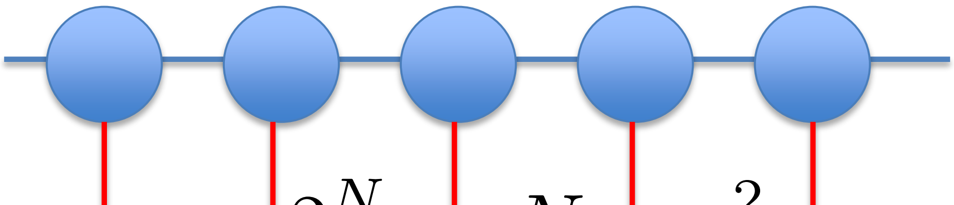
- History of tensor networks in holography
- Bulk boundary locality and QECC
- Perfect tensors
- Holographic codes (local bulk reconstruction)
- Holographic states (RT saturation)
- Conclusions + Open problems

Tensor network states in condensed matter

Compression based on **entanglement area laws**

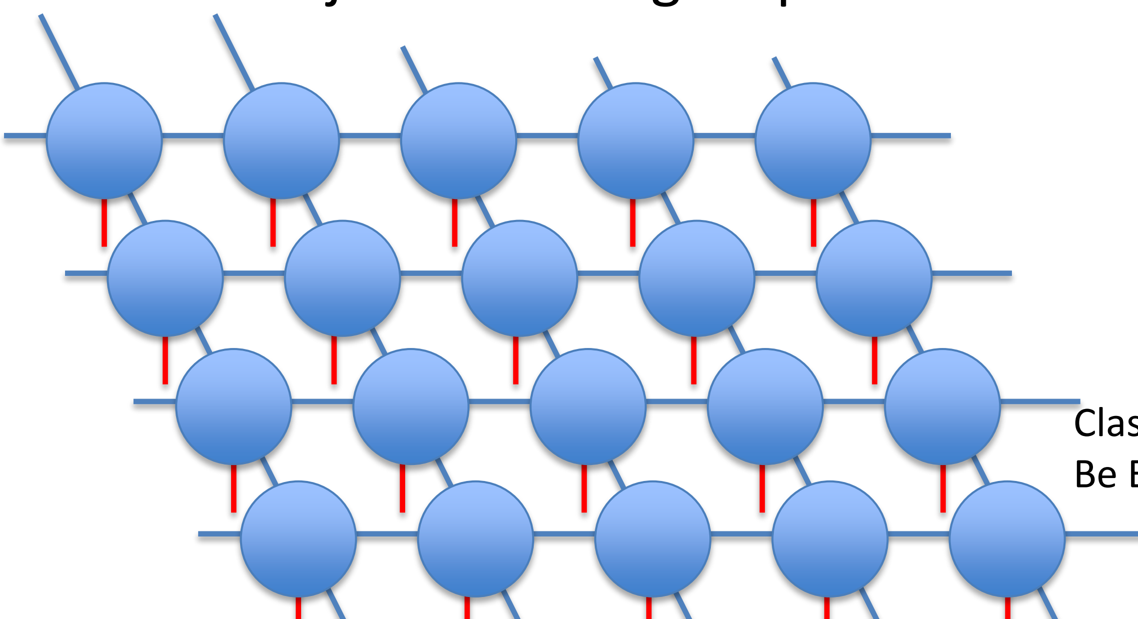
MPS = Matrix product state representations

Perez-Garcia, D.; Verstraete, F.; Wolf, M.M. (2008).

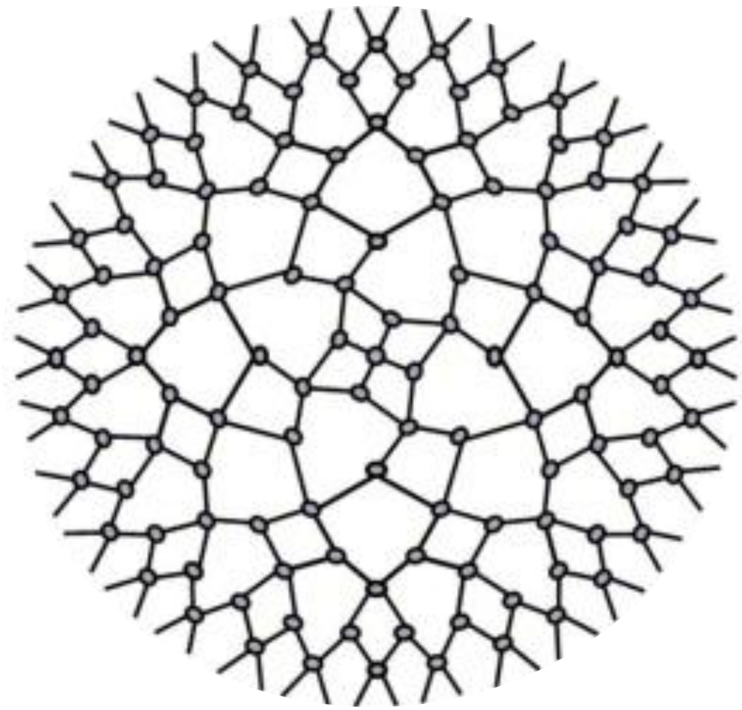


$$2^N \to N \times \chi^2$$

PEPS = Projected entangled pair states

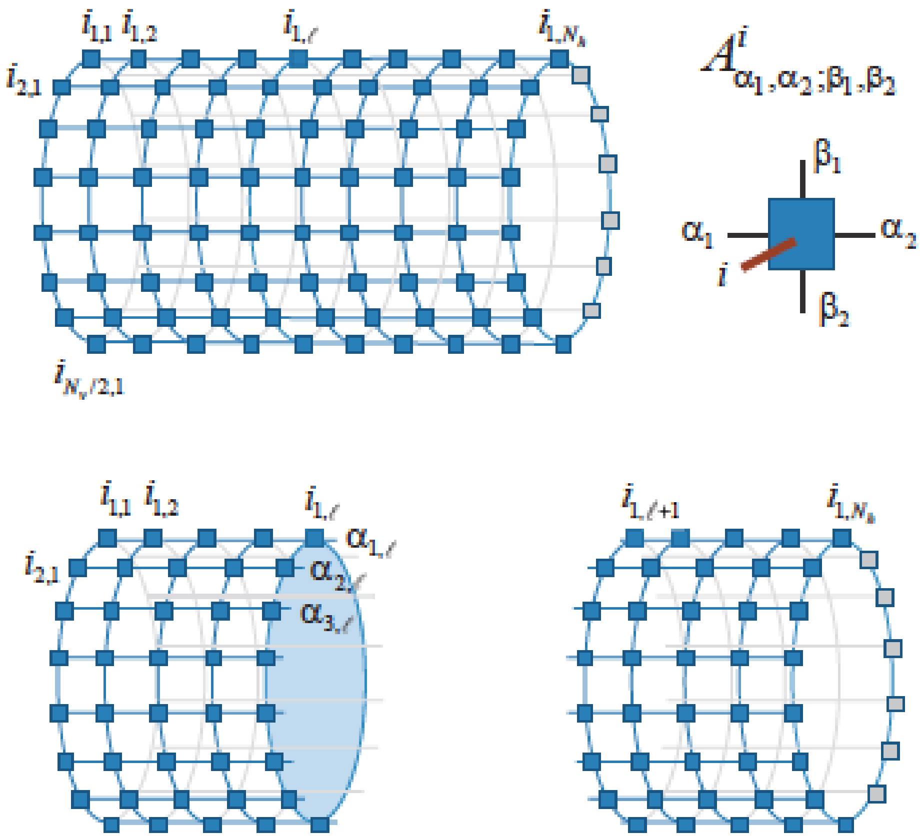


MERA



Class of Quantum Many-Body States That Can Be Efficiently Simulated Guifre Vidal (2008)

Entanglement spectrum & boundary theories in PEPS



- Entanglement spectrum as thermal state of "Boundary Hamiltonian"
- Quantum criticality => Non-local Hamiltonian.

Entanglement Hamiltonian = Modular Hamiltonian

Entanglement spectrum and boundary theories with projected entangled-pair states. Cirac, J. I., Poilblanc, D., Schuch, N., & Verstraete, F. (2011).

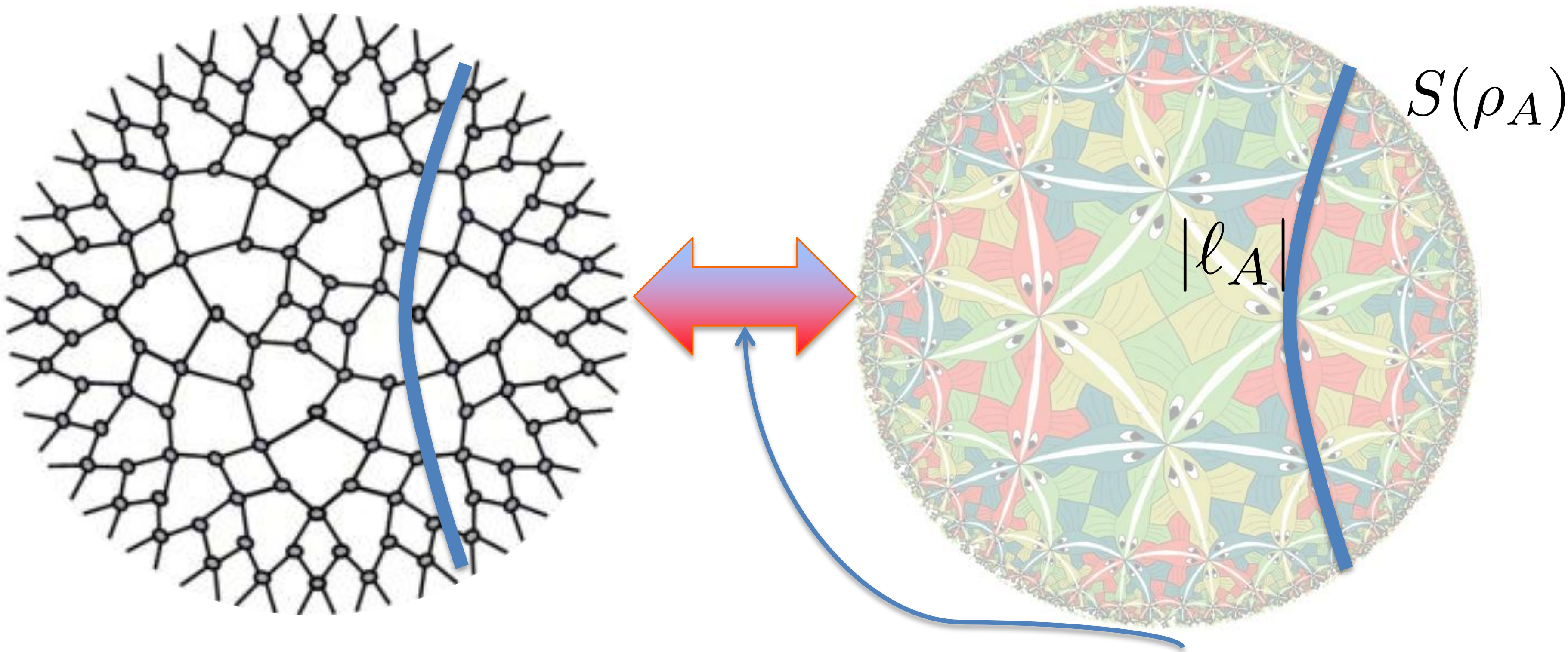
Ryu-Takayanagi

Holographic Derivation of Entanglement Entropy from AdS/CFT (2006)

Minimal surface = entanglement entropy

MERA

AdS metric

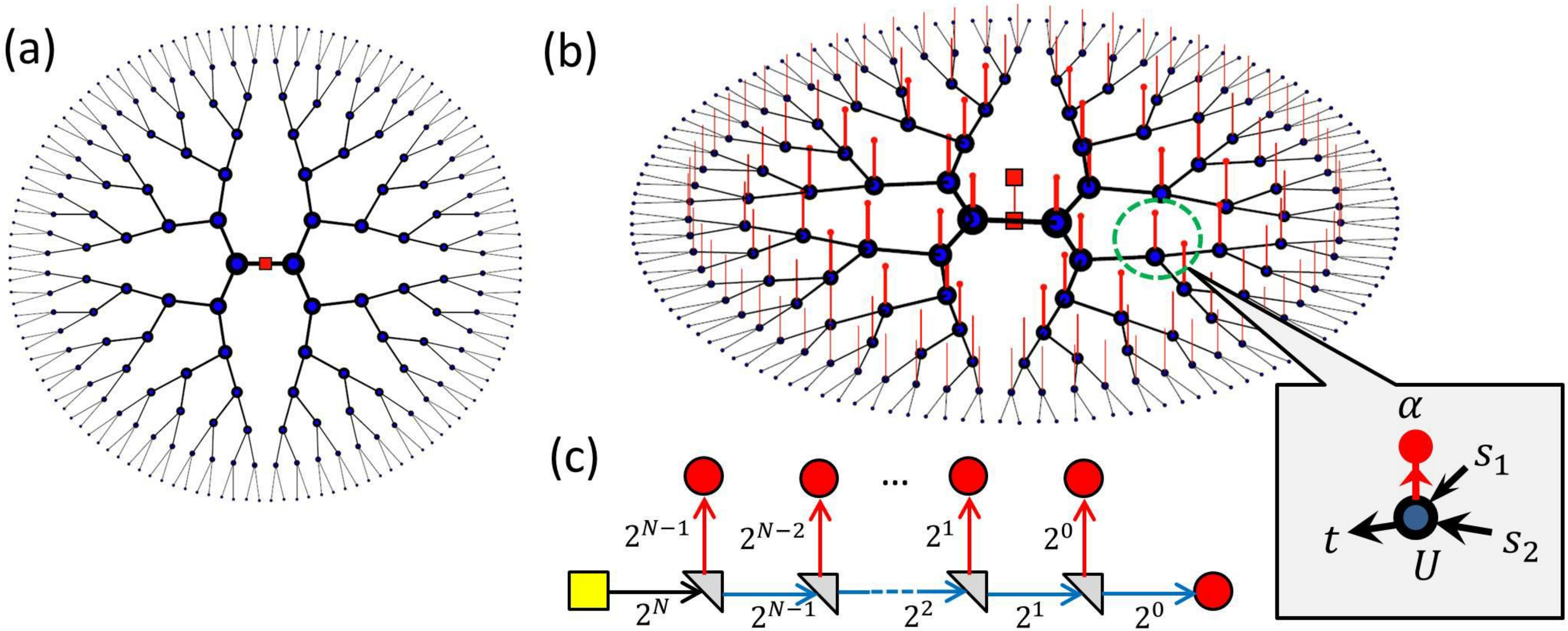


Class of Quantum Many-Body States That Can Be Efficiently Simulated Guifre Vidal (2008)

Entanglement renormalization and holography Brian Swingle (2012)

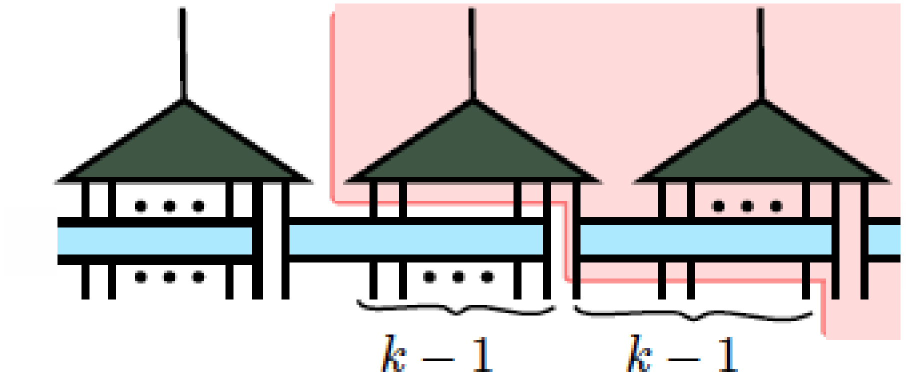
Holography from tensor networks
(network as the Isomorphism)

Bulk boundary Isomorphism $\mathcal{H}_{Blk} \equiv \mathcal{H}_{Bdy}$

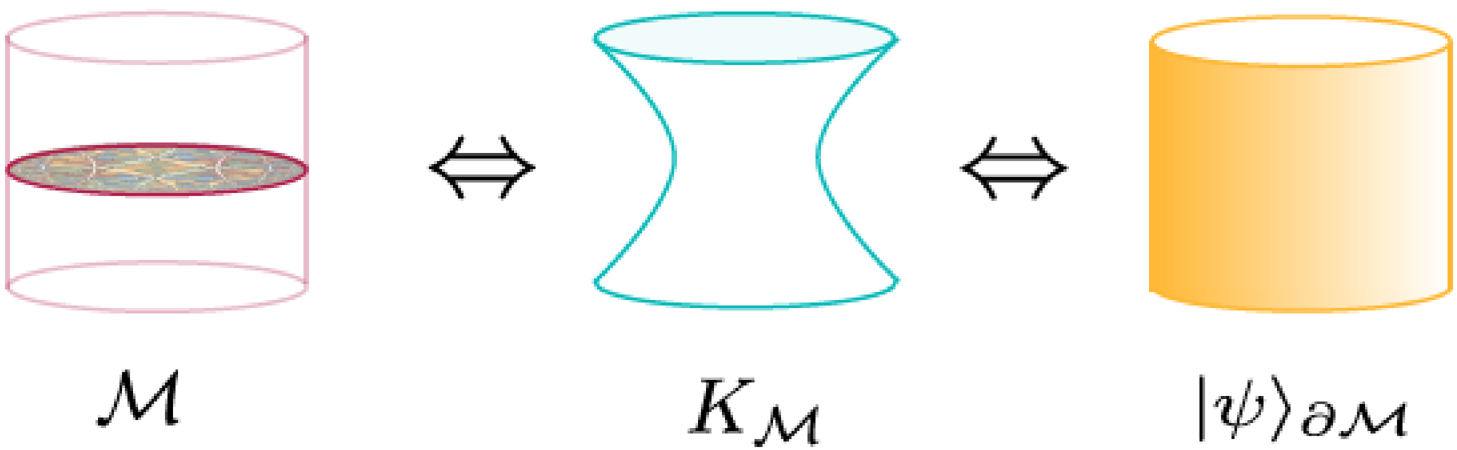


Exact Holographic Mapping Xiao-Liang Qi (2013)

AdS/MERA Critics

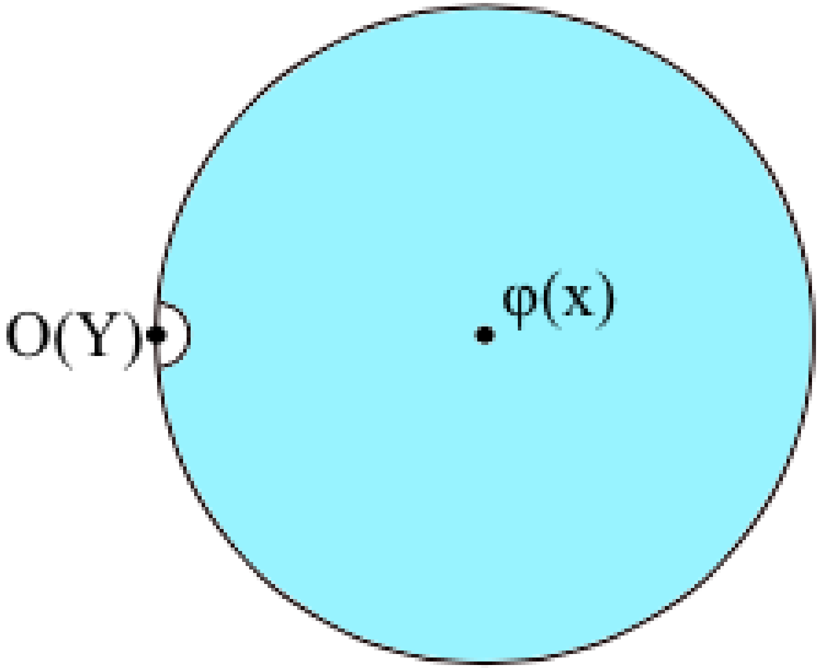


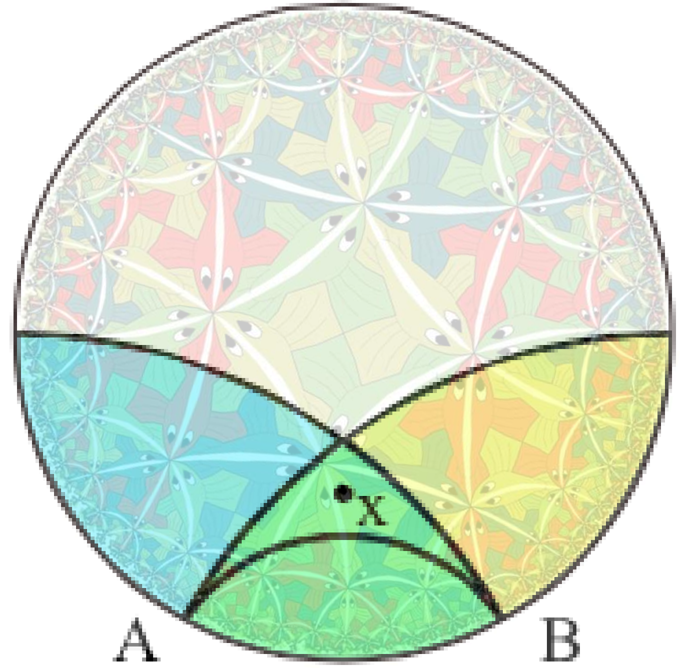
Consistency conditions for an AdS/MERA correspondence. Bao, N., et al. (2015).



Integral Geometry and Holography Czech, B., et al. (2015)

Bulk/Boundary locality and QECC





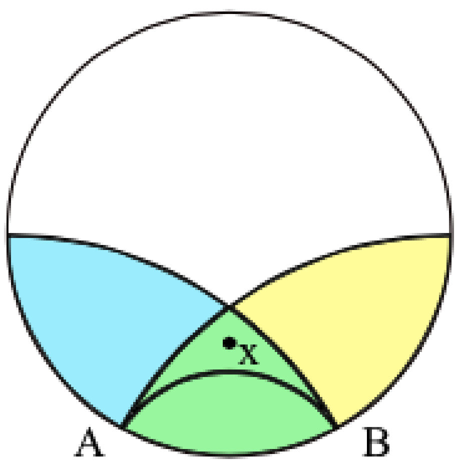
$$\phi(x)|\omega\rangle \to \Phi_A|\Omega\rangle = \Phi_B|\Omega\rangle$$

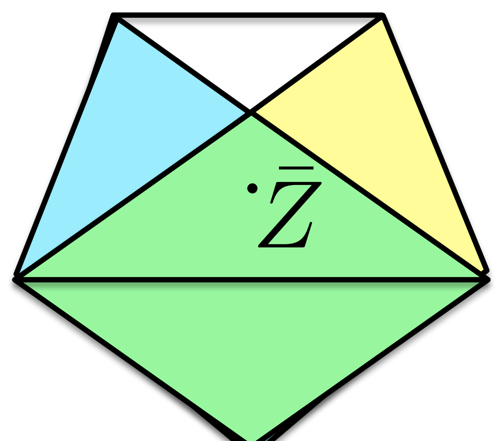
Multiple AdS-Rindler reconstructions on CFT

Bulk locality and quantum error correction in AdS/CFT. Almheiri, A., Dong, X., & Harlow, D. (2015)

2) QECC: Encoder = Isometry

AdS/CFT	QECC
Bulk (AdS) operators	Logical operators
CFT operator(s)	Physical operator(s)
AdS-Rindler reconstruction	Systematic sub-region physical realization of logical operators

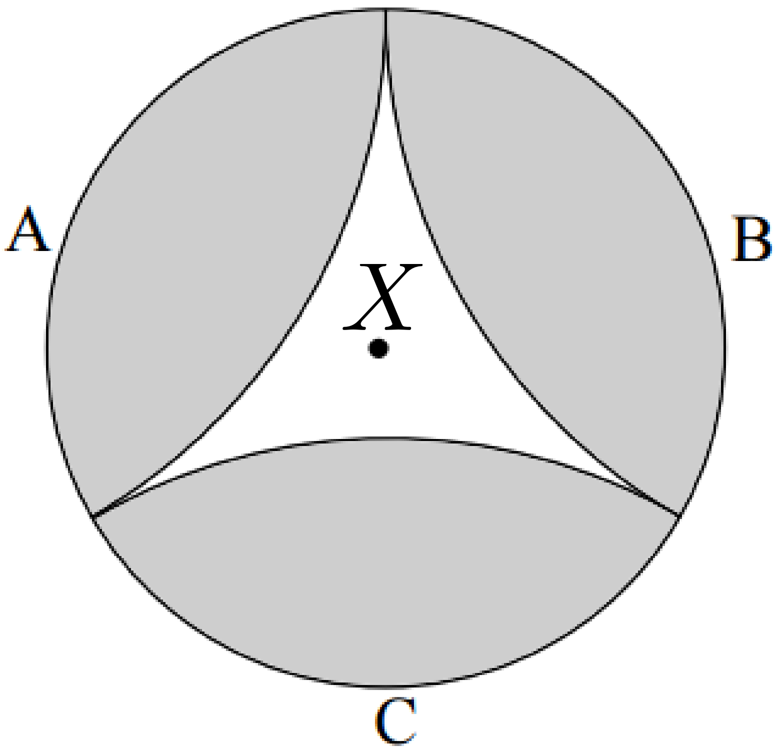




$$\tilde{Z} = X \otimes Z \otimes Z \otimes I \otimes I$$

$$\tilde{Z} = I \otimes X \otimes Z \otimes Z \otimes I$$

Sharpening the paradox



$$A \cup B \cup C = Boundary$$

$$\omega \qquad X \omega X^\dagger$$

$$\rho \neq \tilde{X} \rho \tilde{X}^\dagger = \sigma$$

$$\rho_A = \sigma_A \qquad \rho_B = \sigma_B \qquad \rho_C = \sigma_C$$

$$\rho_{BC} \neq \sigma_{BC} \qquad \rho_{CA} \neq \sigma_{CA} \qquad \rho_{AB} \neq \sigma_{AB}$$

Information in non-local correlations like QECC.

Error correction condition

$$\mathcal{E}nc : \mathcal{H}_{logical} \to \mathcal{H}_{physical}$$

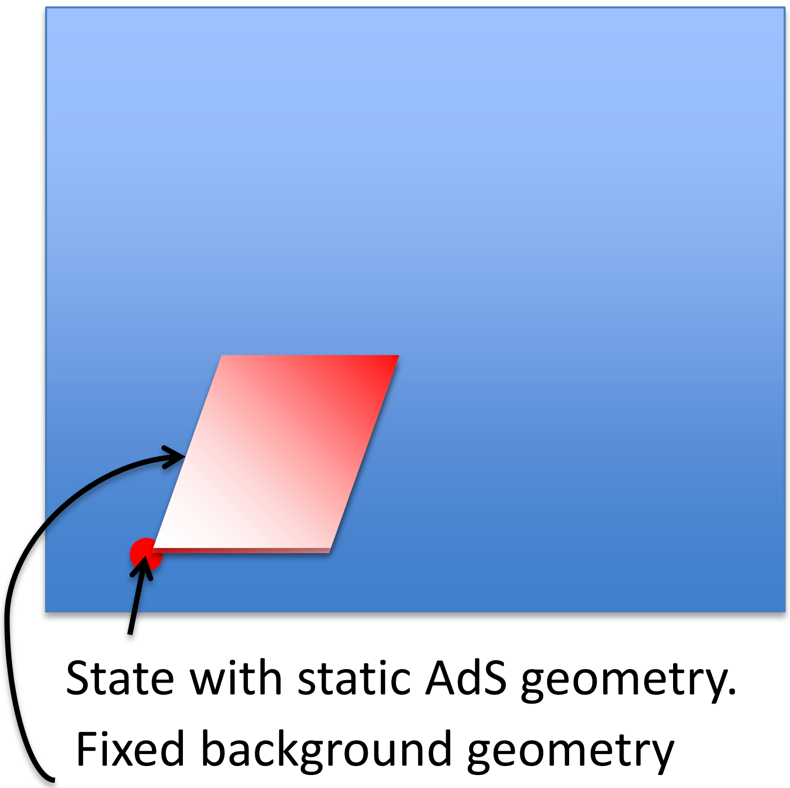
$$\mathcal{N}oise : \mathcal{H}_{physical} \to \mathcal{H}_{physical}$$

$$\mathcal{D}ec : \mathcal{H}_{physical} \to \mathcal{H}_{logical}$$

$$\mathcal{D}ec \circ \mathcal{N}oise \circ \mathcal{E}nc(|\psi\rangle\langle\psi|) = |\psi\rangle\langle\psi|$$

Where is the full Hilbert space?

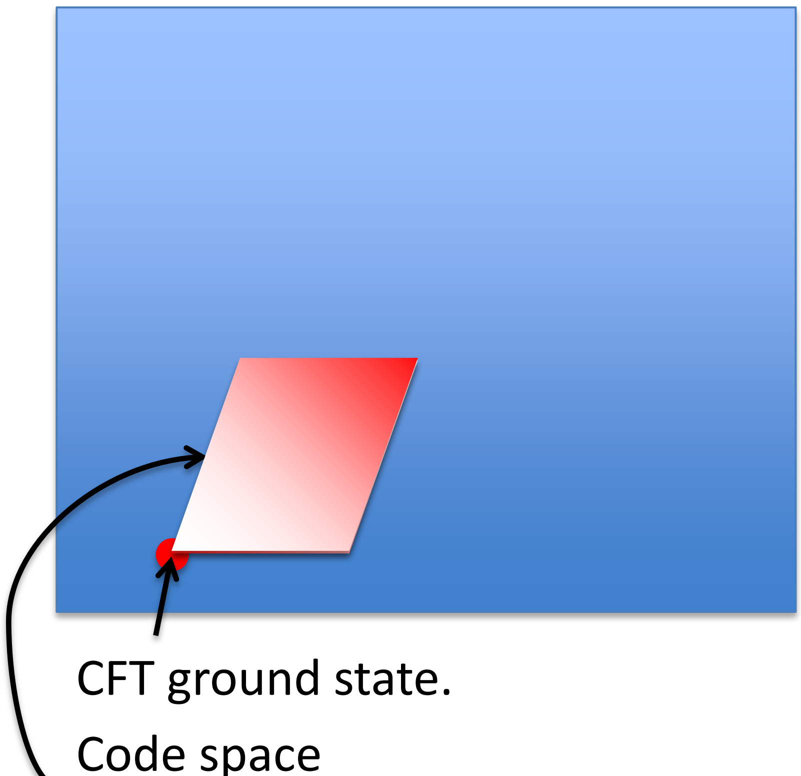
Bulk Hilbert space



State with static AdS geometry.

Fixed background geometry
"low-energy" subspace

Boundary Hilbert space

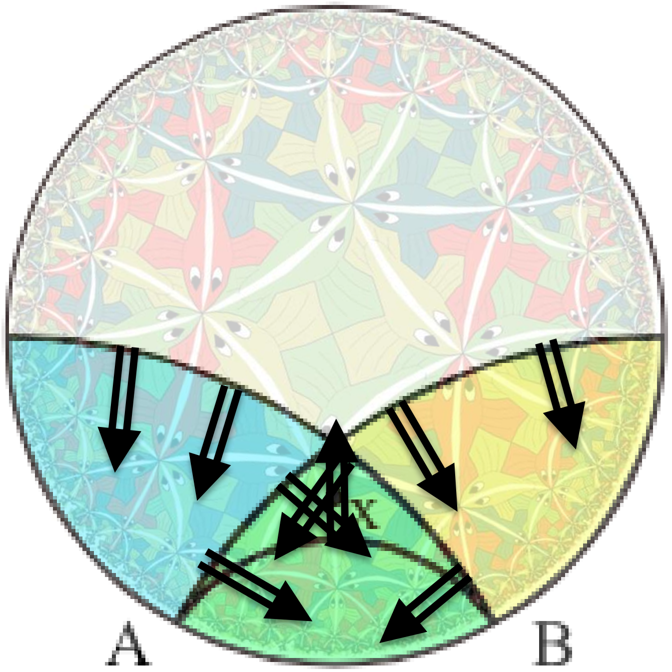


CFT ground state.

Code space
subspace within CFT HS

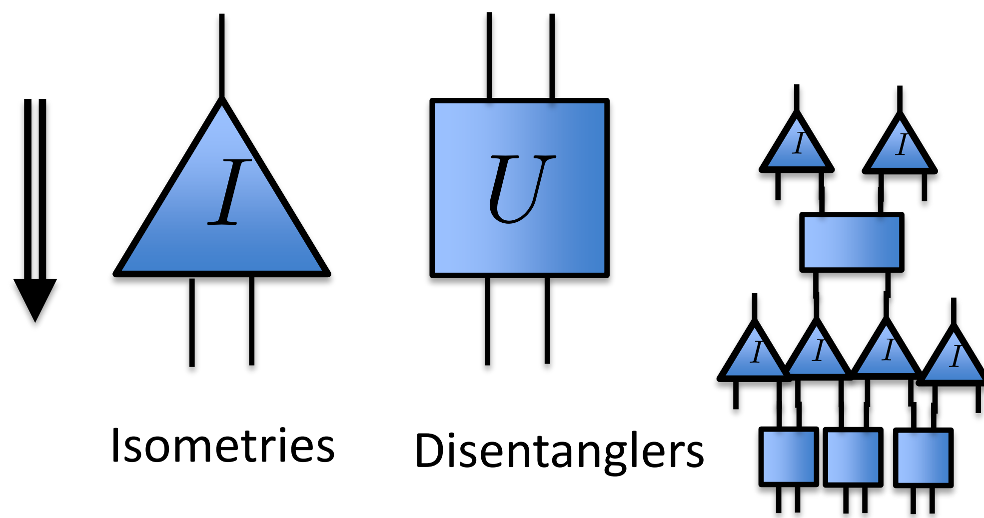
1) Manifest need for Bulk "Isotropy"

Isotropy: In bulk, all directions are created equal.



MERA tensors

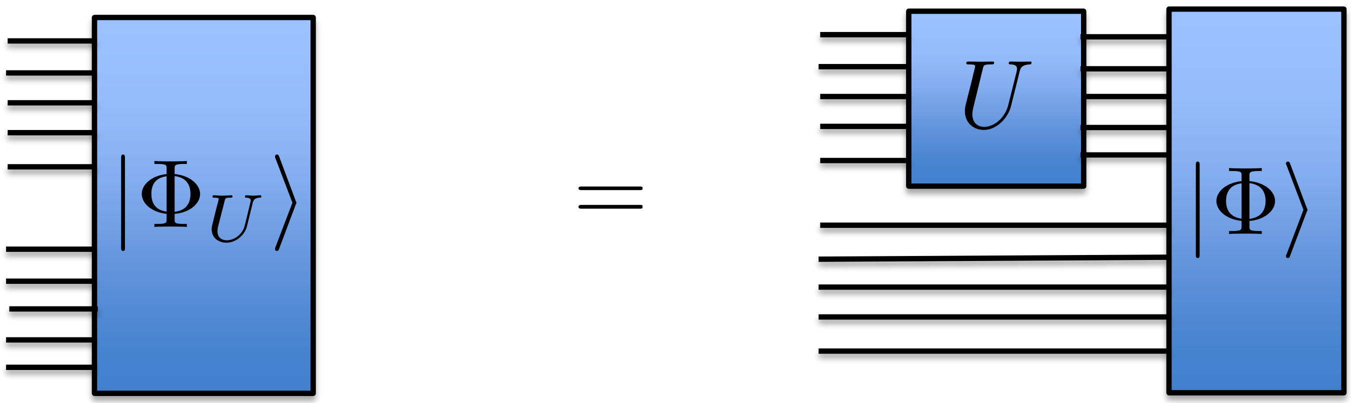
preferred renormalization direction



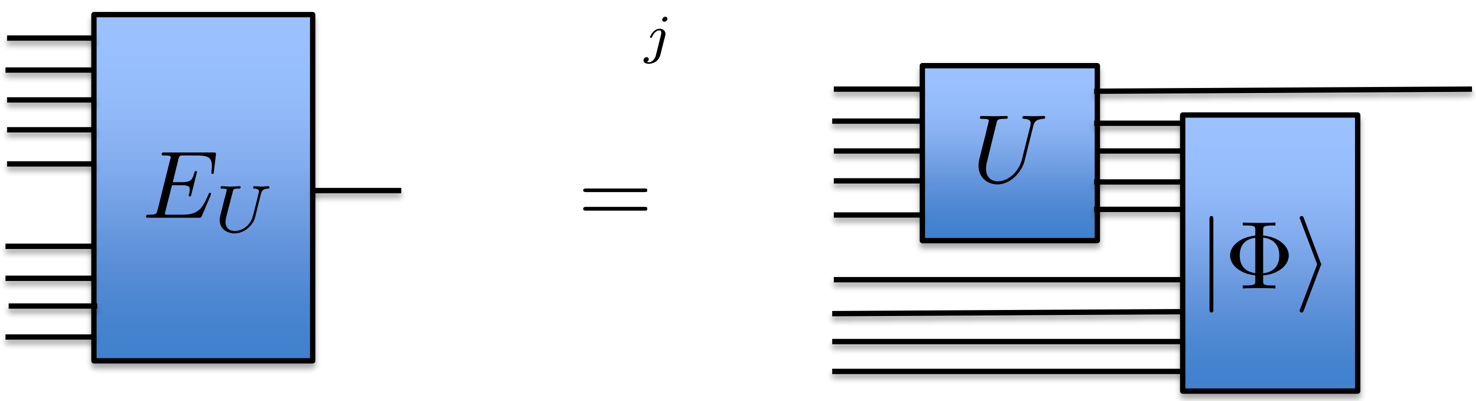
Isometry: Preserves inner product

$$\langle \psi | \phi \rangle = \langle \psi | I^{\dagger} I | \phi \rangle$$

Maximally entangled = Isometry



$$|\Phi\rangle = \sum_j |j, j\rangle$$



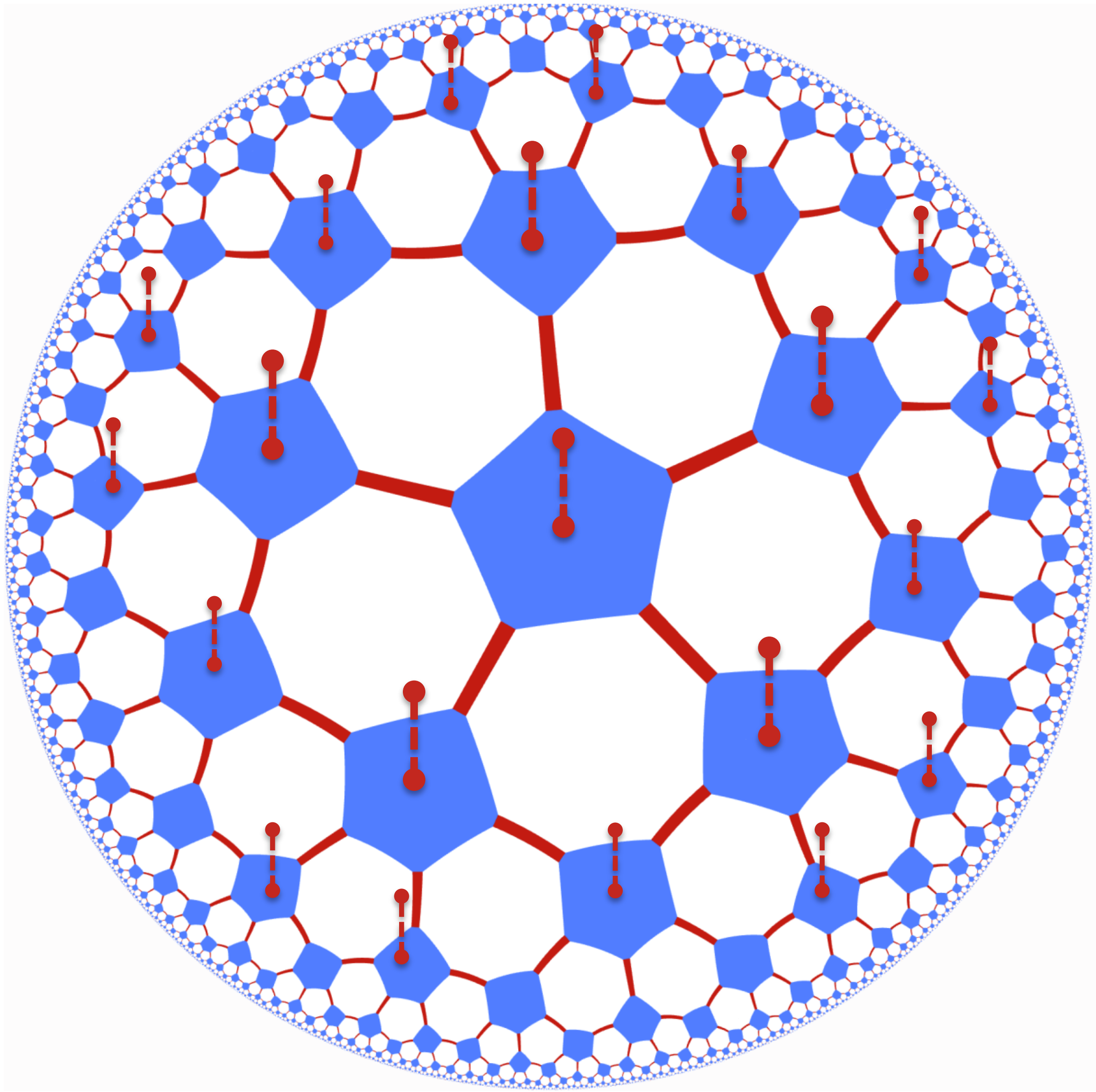
Choi-Jamiokowski Isomorphism

Perfect tensors = absolutely maximally entangled (AME)

- Maximally entangled along **all possible** cuts
- Always proportional to unitary or **isometry**

Holographic Code

(example lattice choice)

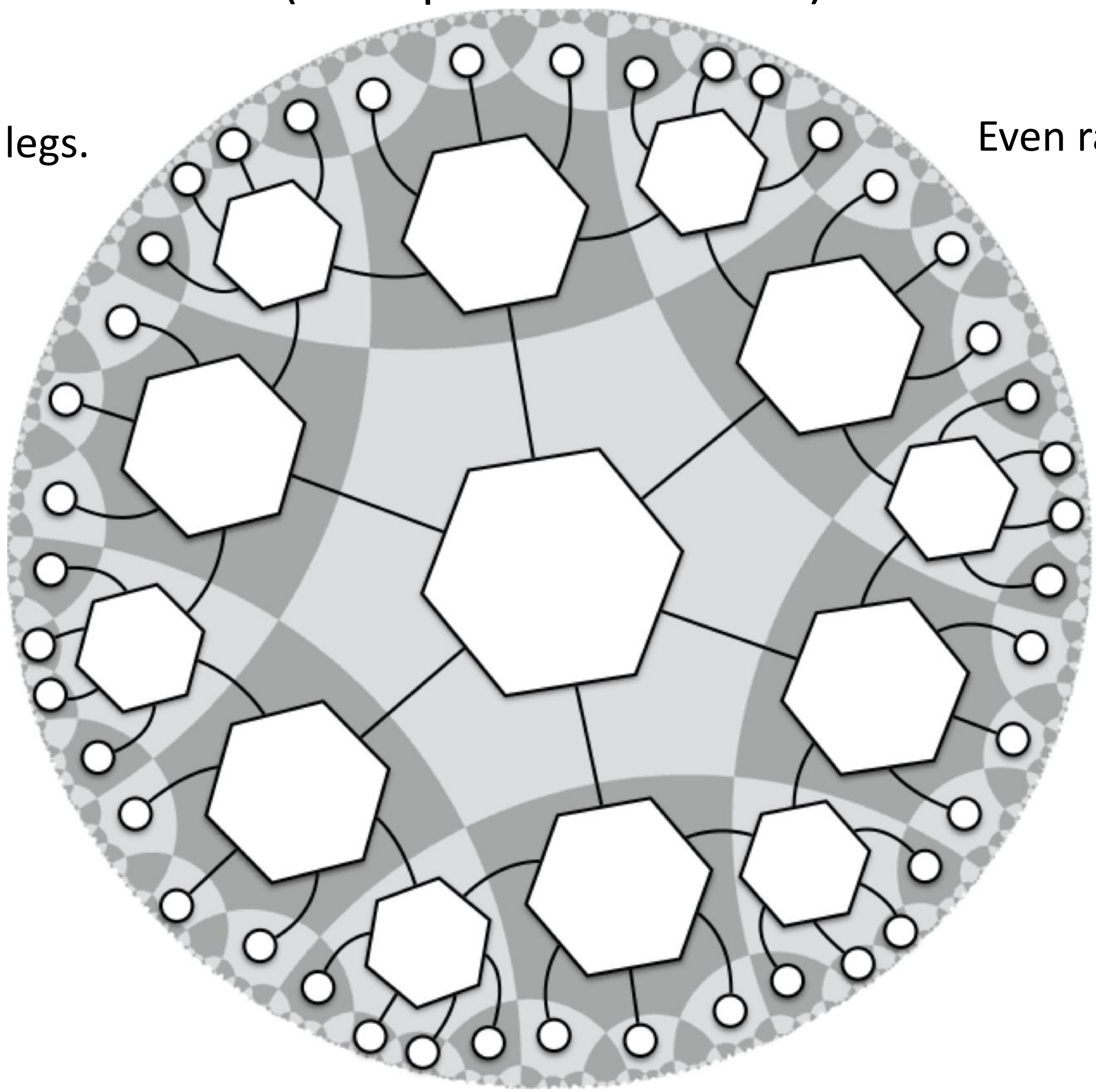


Holographic state

(example lattice choice)

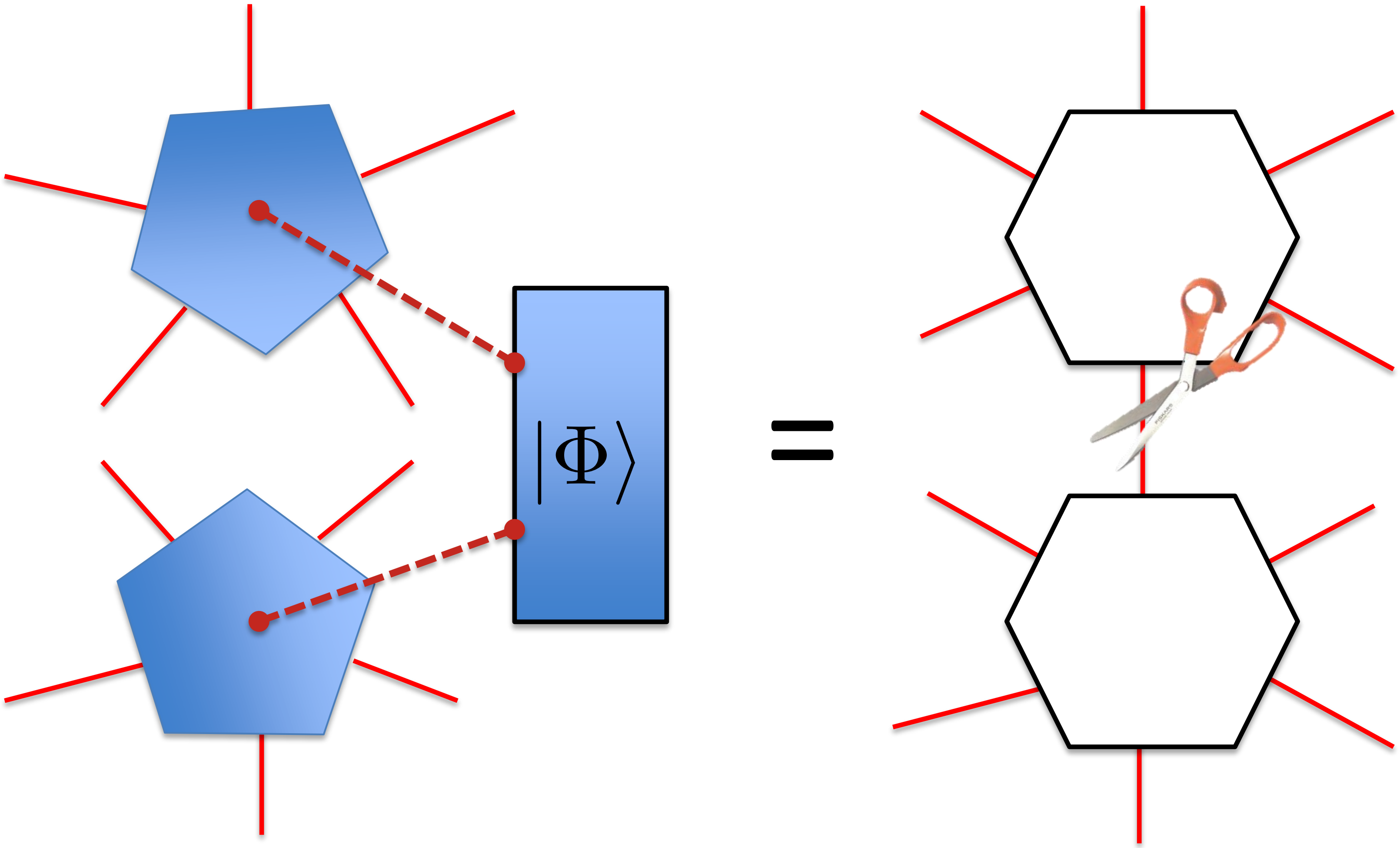
No bulk/logical legs.

Even rank tensors.



Ryu-Takayanagi -> Entanglement entropy = length of bulk geodesics.

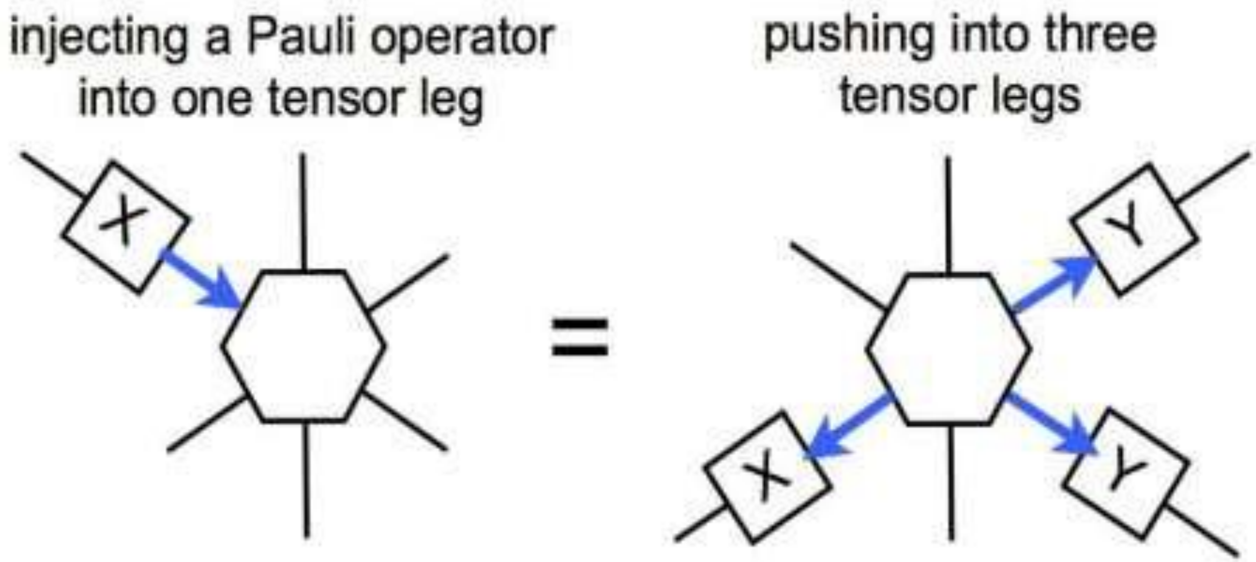
Connection between state & code



Holographic codes

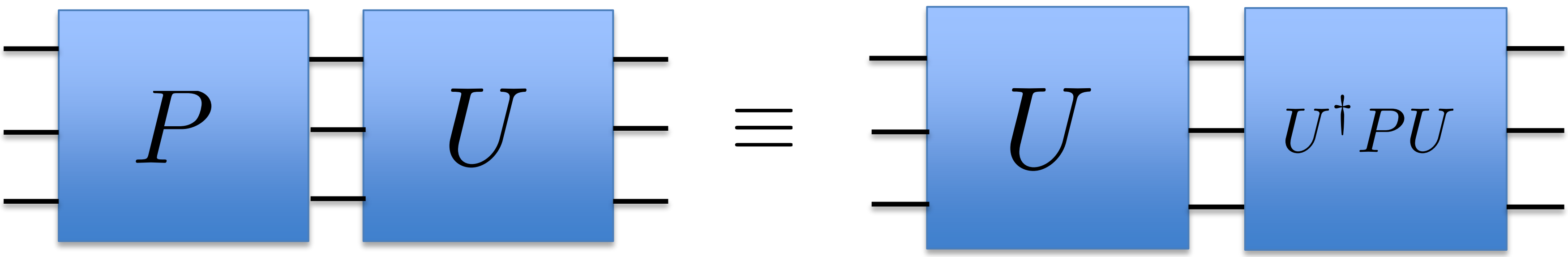
Bulk reconstruction & the greedy wedge

Pauli pushing on Holographic Stabilizer tensors



- Clifford case: Map Paulis to tensor product Paulis
- General case: Map single site operators to entangling operators.

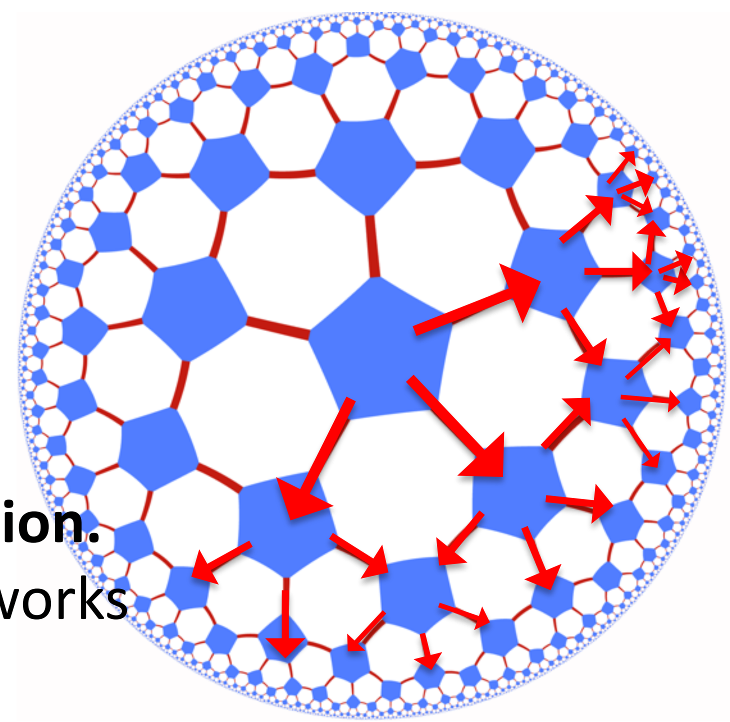
Holographic QECC
Operator pushing



More out than in.

=>

Hyperbolic lattice

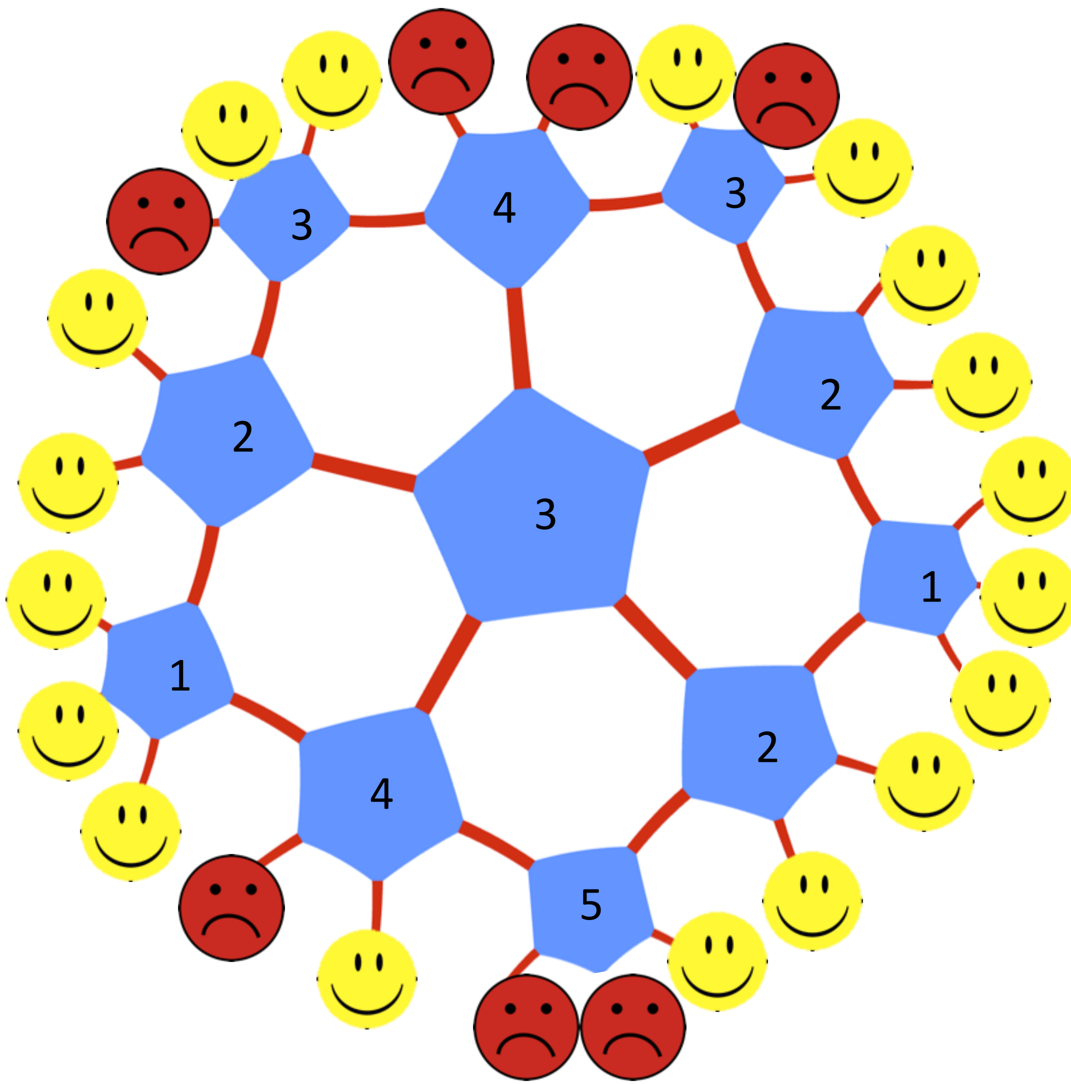


U guarantees **Hermiticity & Norm preservation.**

Otherwise, any pseudo-invertible operator works

Greedy erasure recovery

(Dimension independent)



Give me the stabilizers!

Holographic code: has bulk/logical legs.

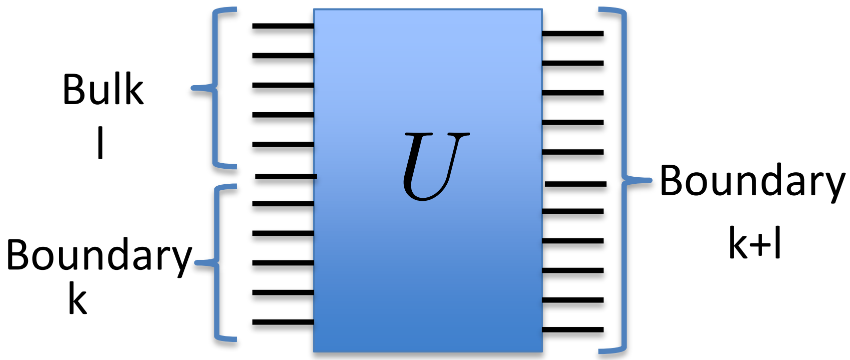
Is the encoder normalized? 

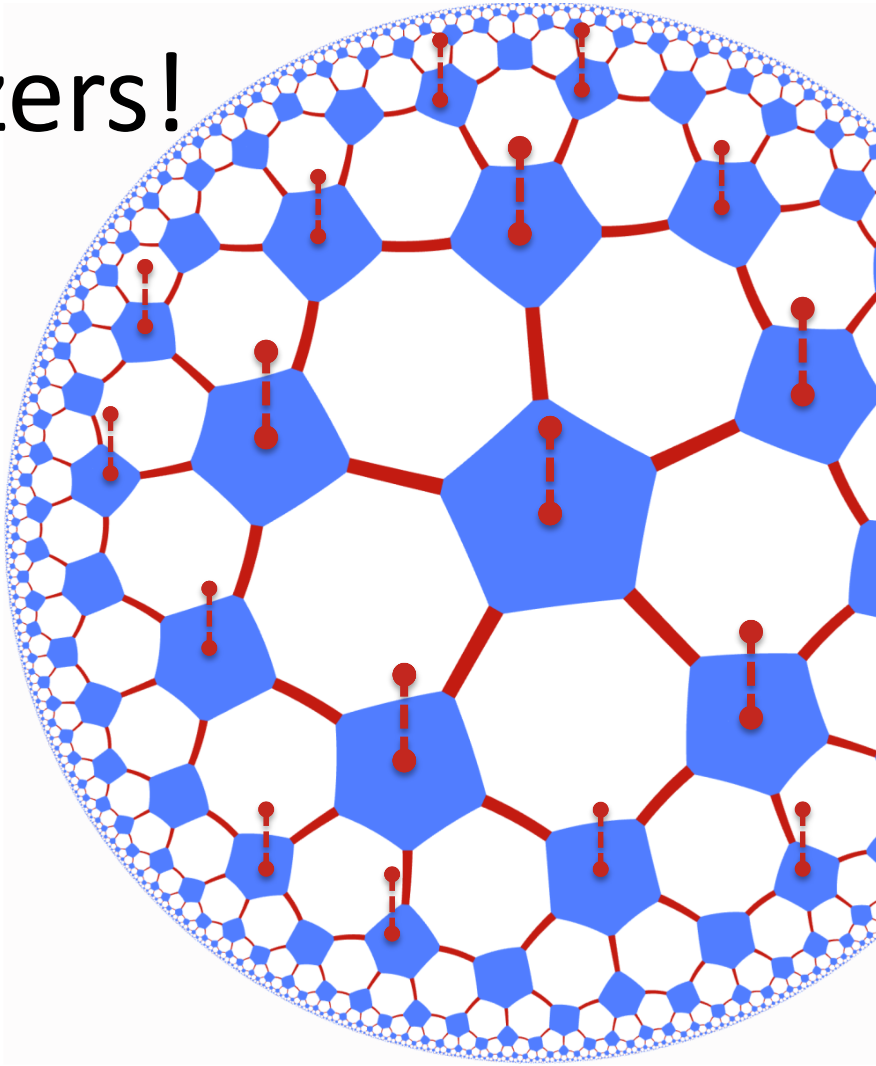
If we can build a DAG-Flux
with all bulk as input.

$$T = U$$

Iff greedy algorithm succeeds.

|Boundary|=2k+l |Bulk|=l

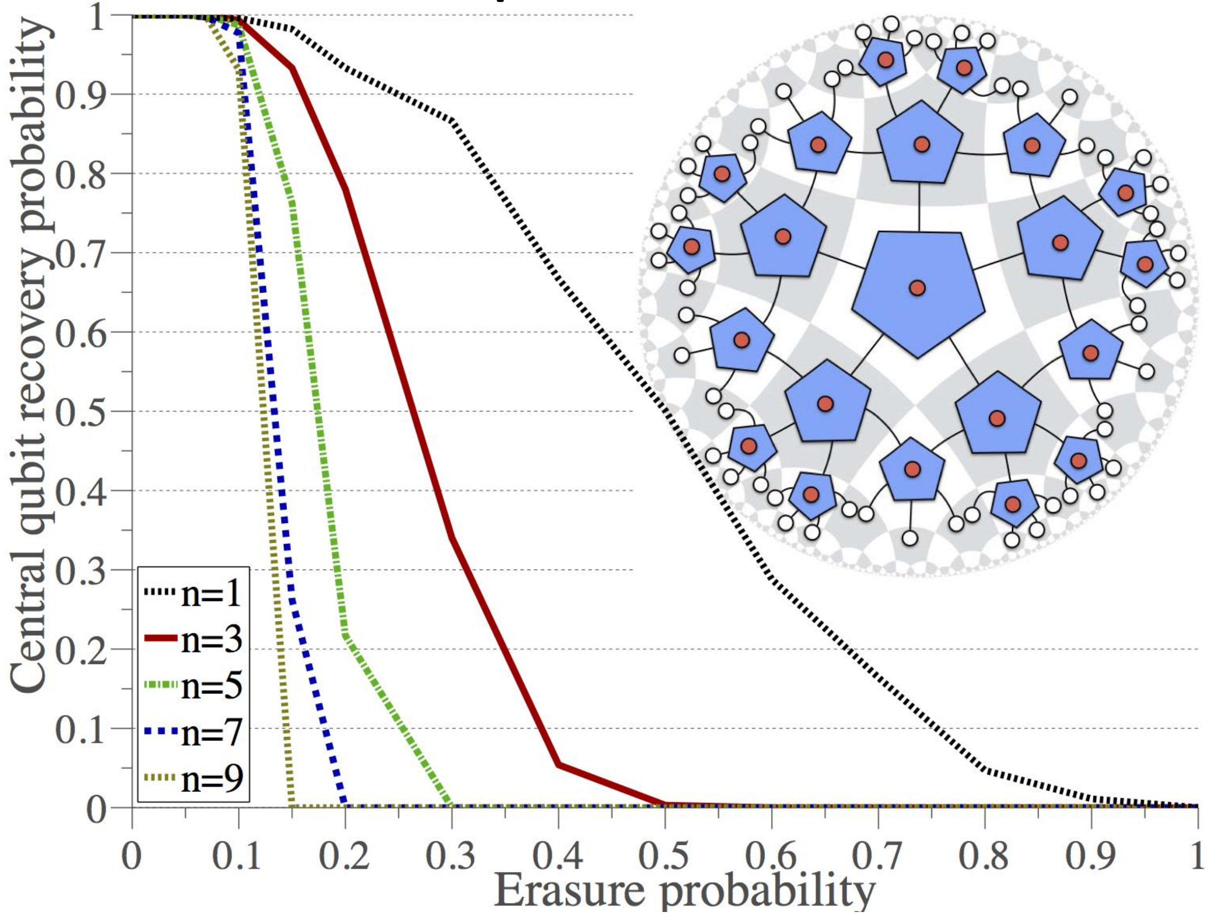




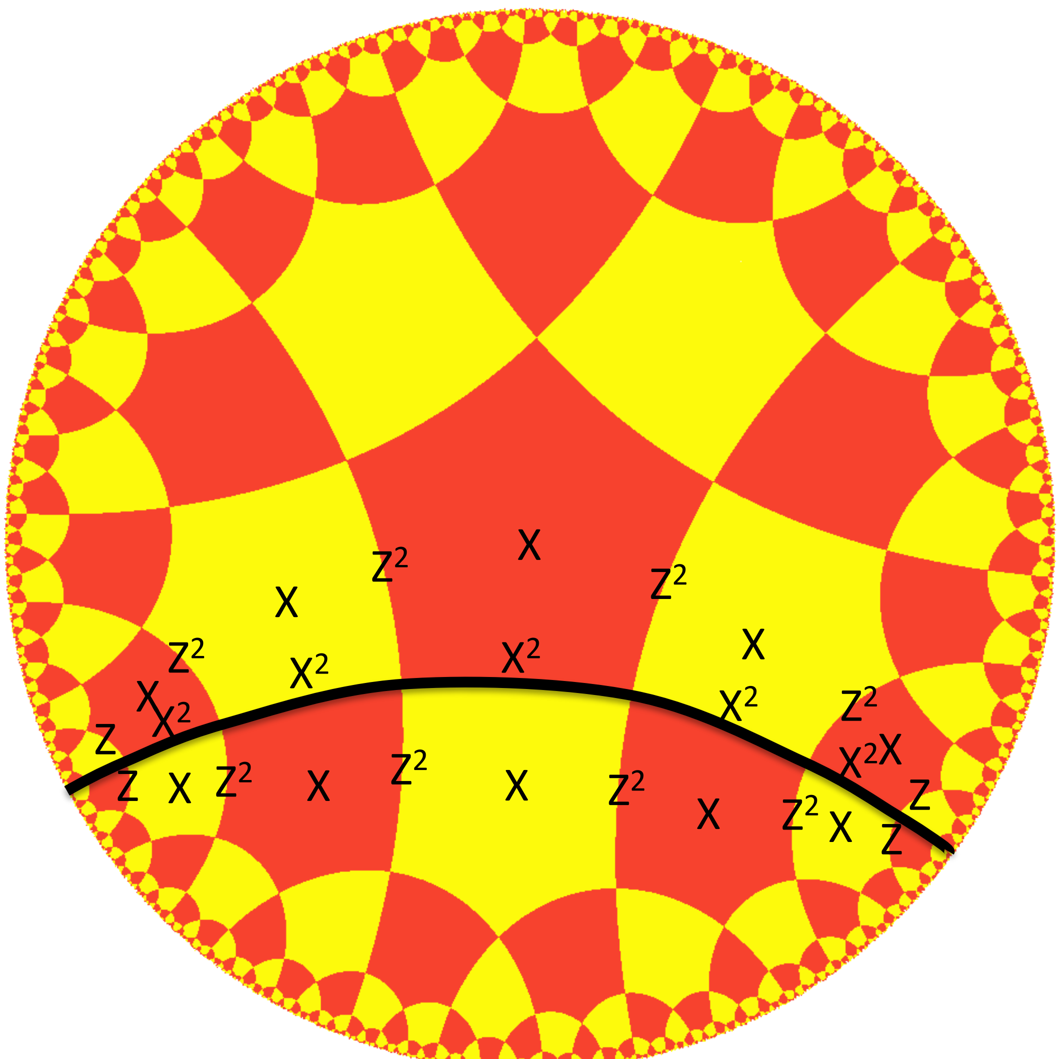
$$\mathcal{S} = \langle X_j \otimes U^\dagger X_j^\dagger U, Z_j \otimes U^\dagger Z_j^\dagger U \rangle$$

Code property checklist

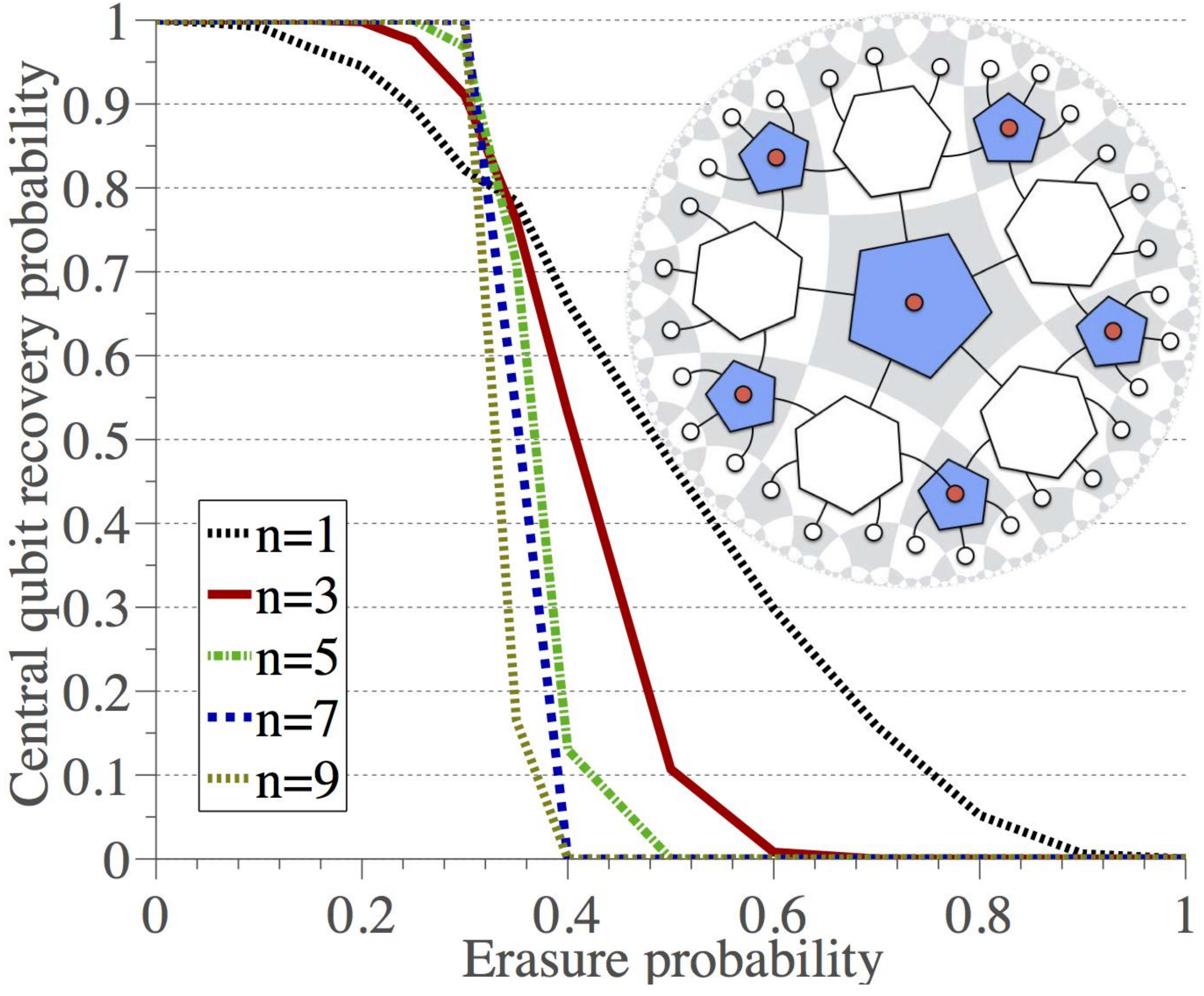
- Code distance = (3 for suburban logicals)
- Does the central qubit have a threshold?



Weight 4 logical ops. affecting downtown logicals



Make the code less dense



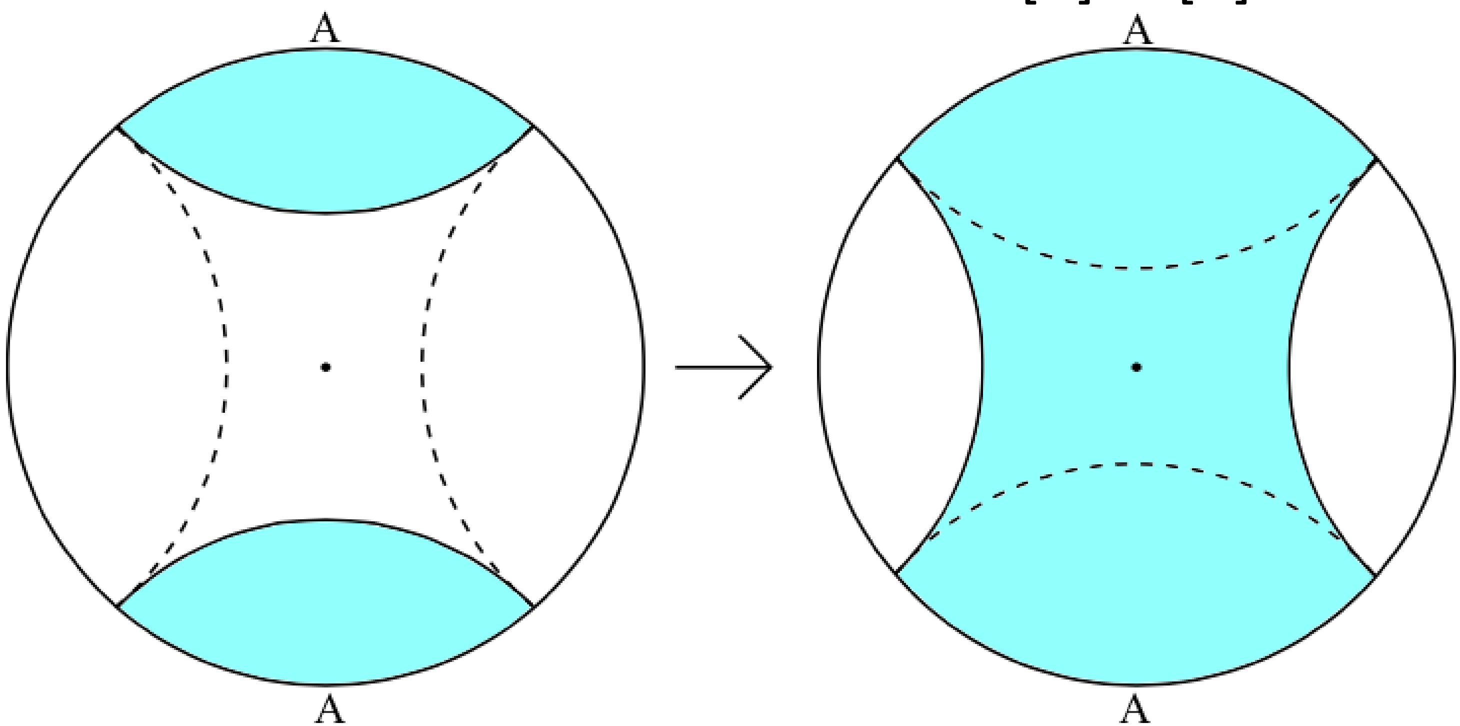
Pentagons and hexagons (4 polygons per vertex)

Causal wedge   Entanglement wedge

$C[A] = C[A_1] \cup C[A_2]$

$E^c[A] = E[A^c]$

$E[A] \geq C[A]$



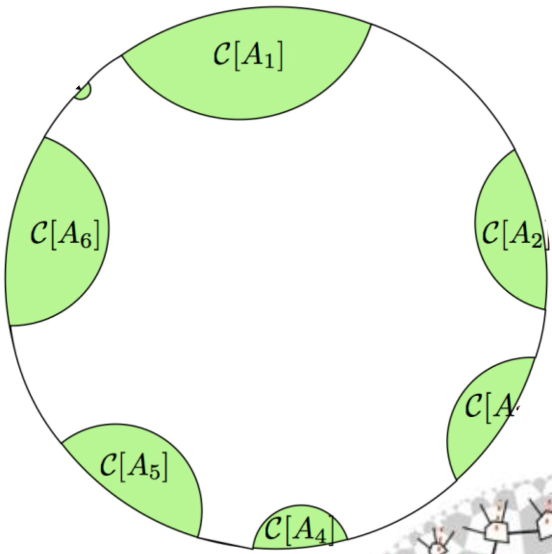
$E[A] = C[A]$

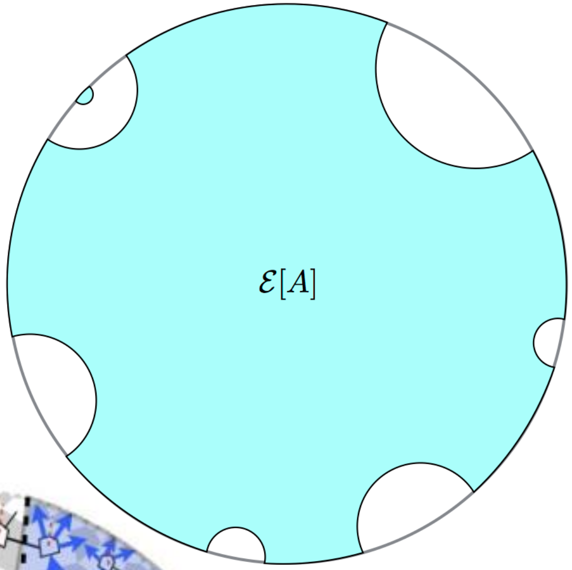
$E[A] > C[A]$

$|E[A]| = |C[A]| + 2$

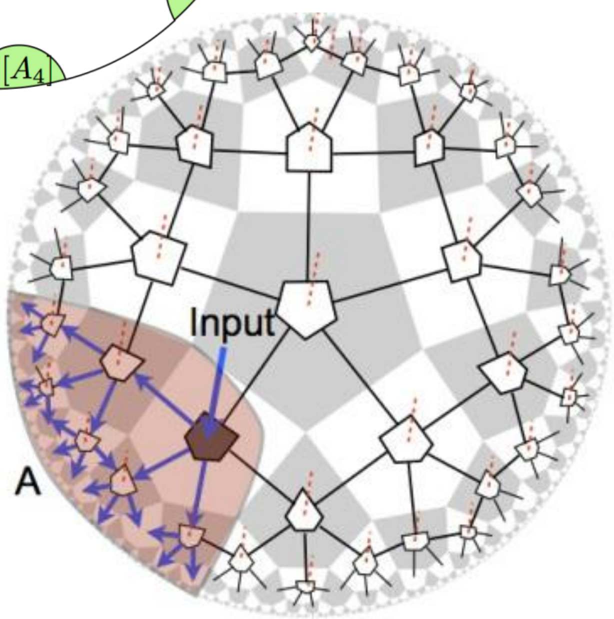
Causal wedge

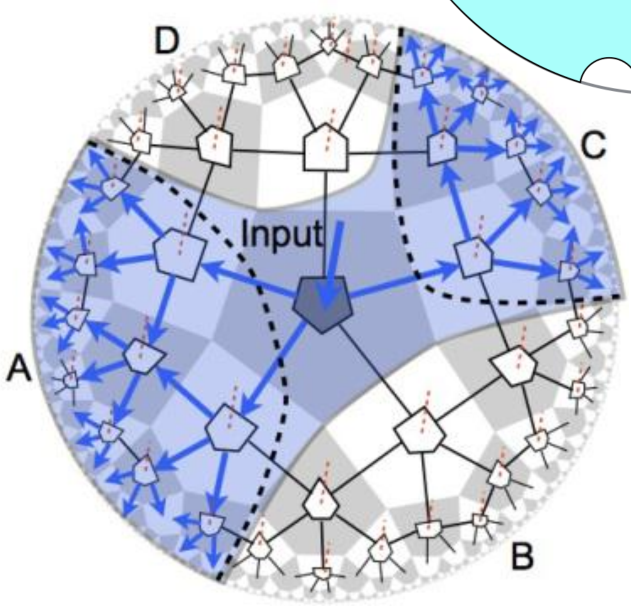
Entanglement wedge





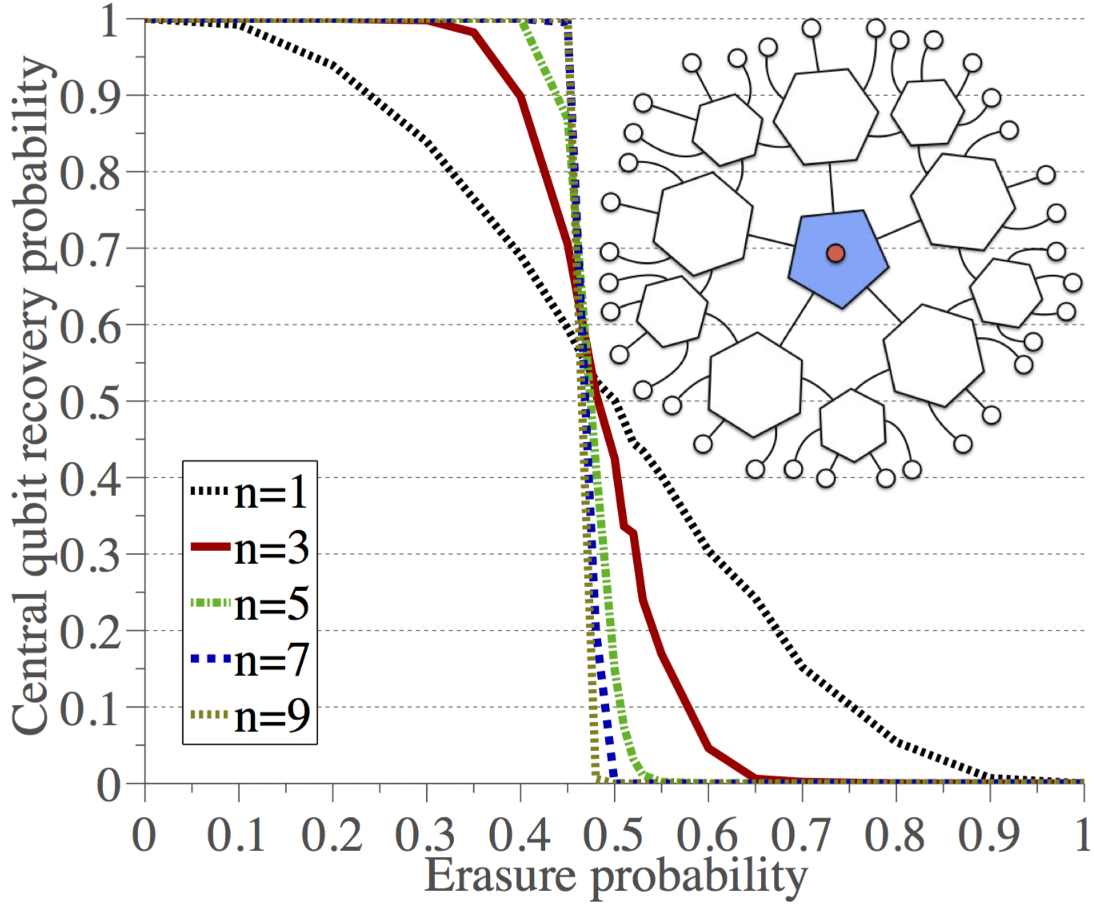
Reconstruction Isometries

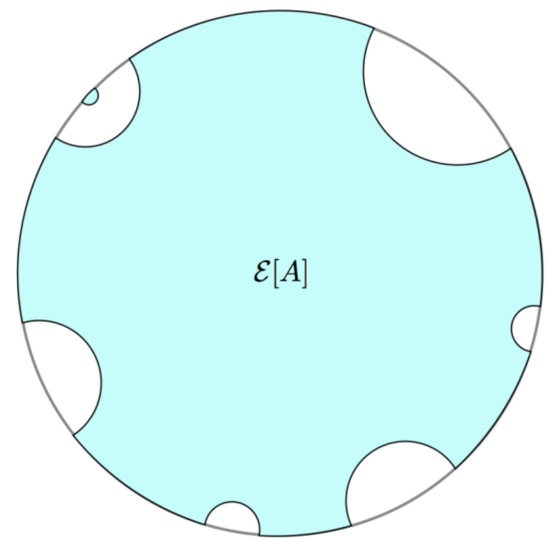




Deep beyond the causal wedge

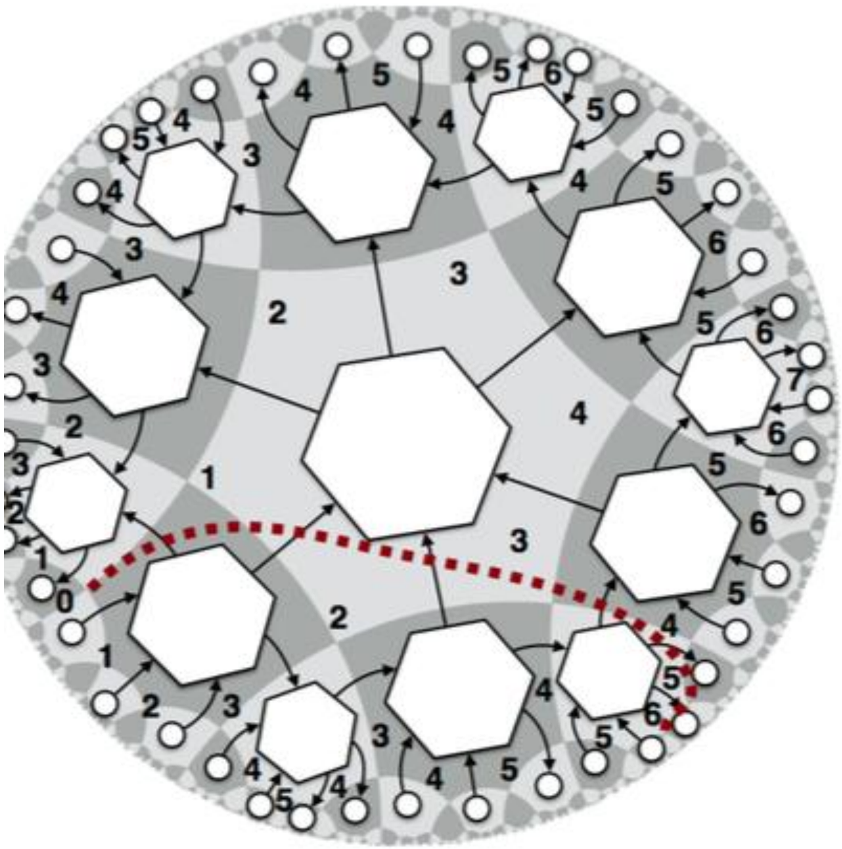
- Numerical greedy recovery threshold ~0.52.
- Actual threshold of 0.5?



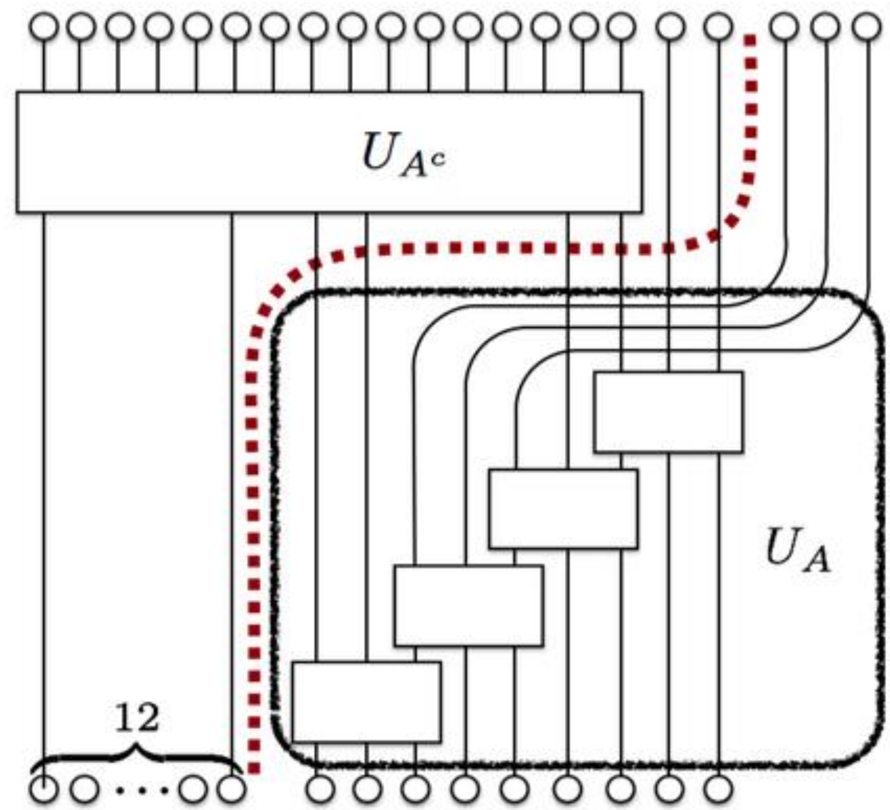


Holographic states
and
Exact Ryu-Takayanagi saturation

Ryu-Takayanagi is saturated

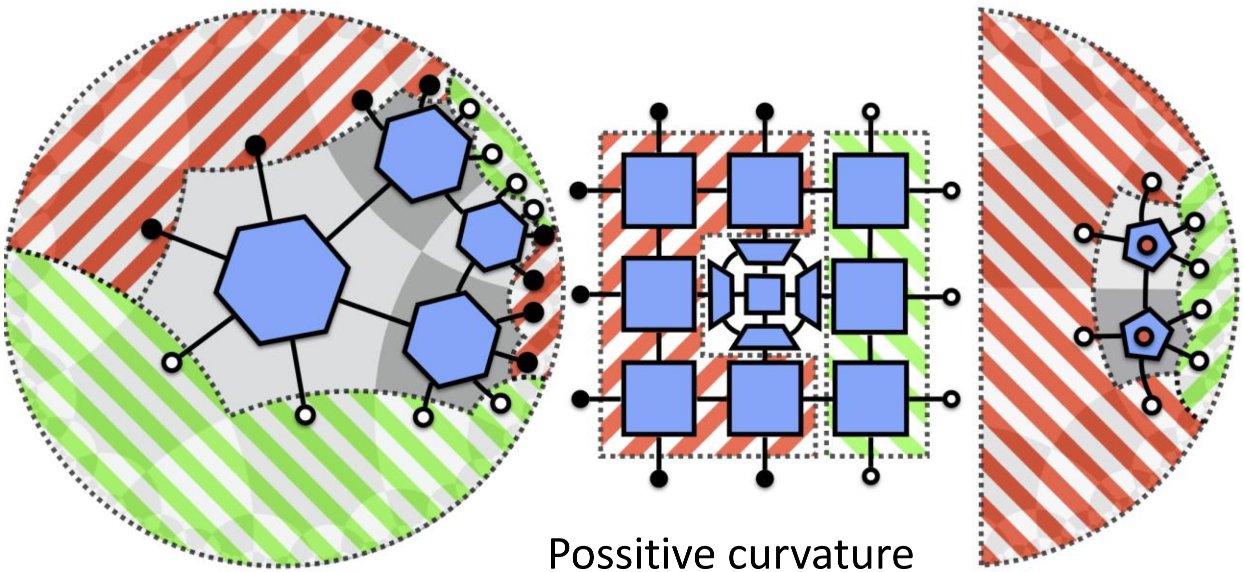


(a) Circuit interpretation construction



(b) Bulk operator reconstruction circuit

Assumptions for RT saturation.



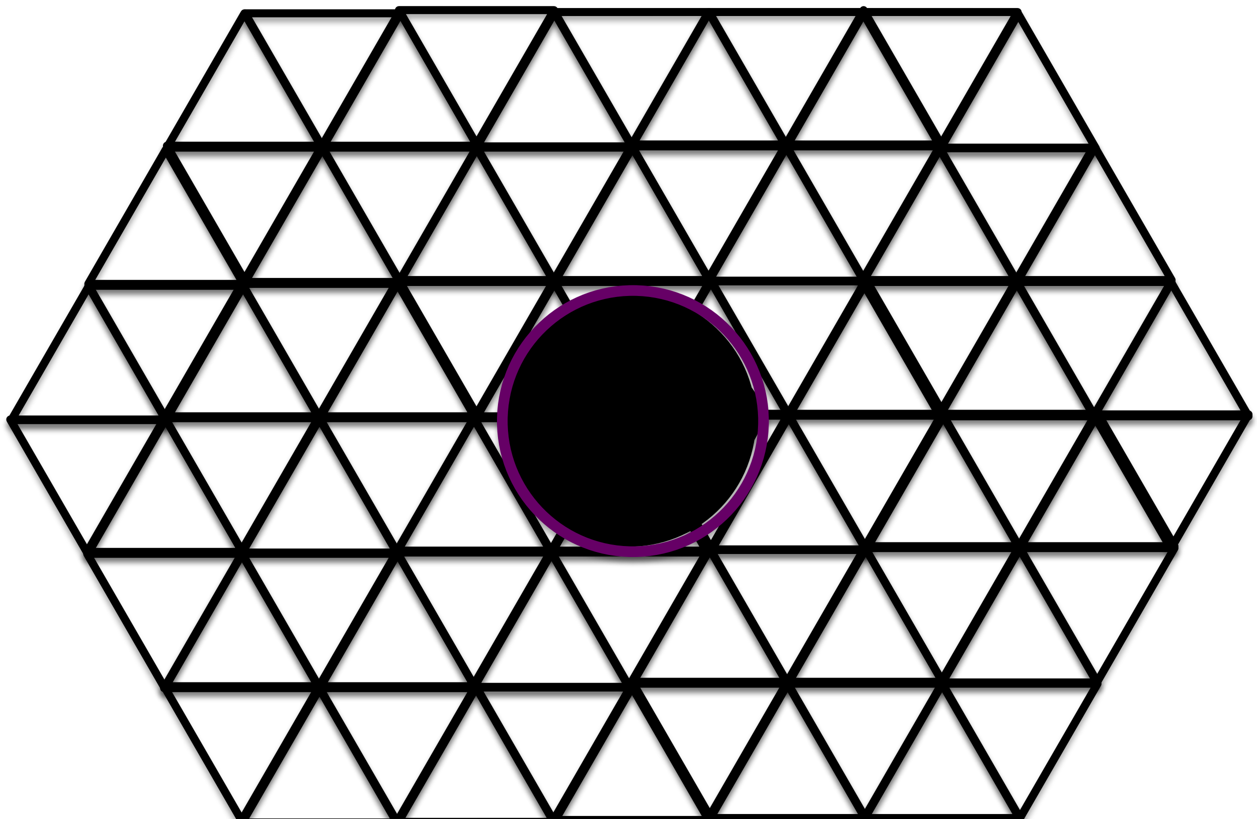
Possitive curvature

Multiple boundary components

Holographic **code**

- Simple 2D graph structure
- Single connected boundary component.
- Non-positive graph theoretic curvature. (Flat space allowed)
- Holographic state (not code with bulk inputs)

Black holes and Bekenstein-Hawking



Conclusions

- Illustrated power of perfect tensors
  - For constructing QECC
  - For providing toy connection of entanglement and geometry
- Constructed a family of holographic codes
  - Bulk locality
  - Erasure recovery possible (beyond causal wedge)
- Constructed holographic “vacuum” states
  - Proved exact Ryu-Takayanagi entanglement entropy