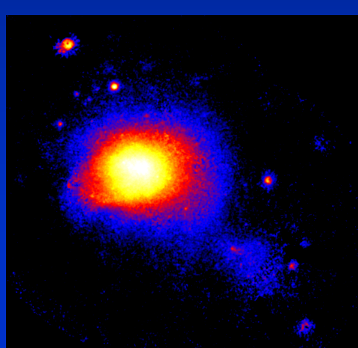
MHD Turbulence and the problem of Viscosity.

Alex Schekochihin, Cambridge.
Steve Cowley, UCLA & Imperial.
Russell Kulsrud, Princeton.
Greg Hammett, Princeton.
Prateek Sharma, Princeton.
Eliot Quataert, Berkeley.
Bill Dorland, Maryland.
Ben Chandran, Iowa.
Jim McWilliams, UCLA.
Jason Maron, AMNH.

Cluster Parameters



- Size l_0~1-3Mpc.
- Turbulent Flow $u_0 \le C_s$. Perhaps from mergers.
- T~1-10 keV.
- n~10^{-2}-10^{-3}cm^{-3}.
- l_0/u_0 ~ 10^9years.
- Reynolds #, Re ~ 10^2 - 10^3.
- Magnetic Reynolds #, Rm ~ 10^{29}.
- **B** ~ 1 - 10μG.

LABORATORY FOR MHD TUBULENCE?

Some Questions.

1. Can magnetic fields be amplified by the Turbulence in clusters to the size and spectrum seen in observations?

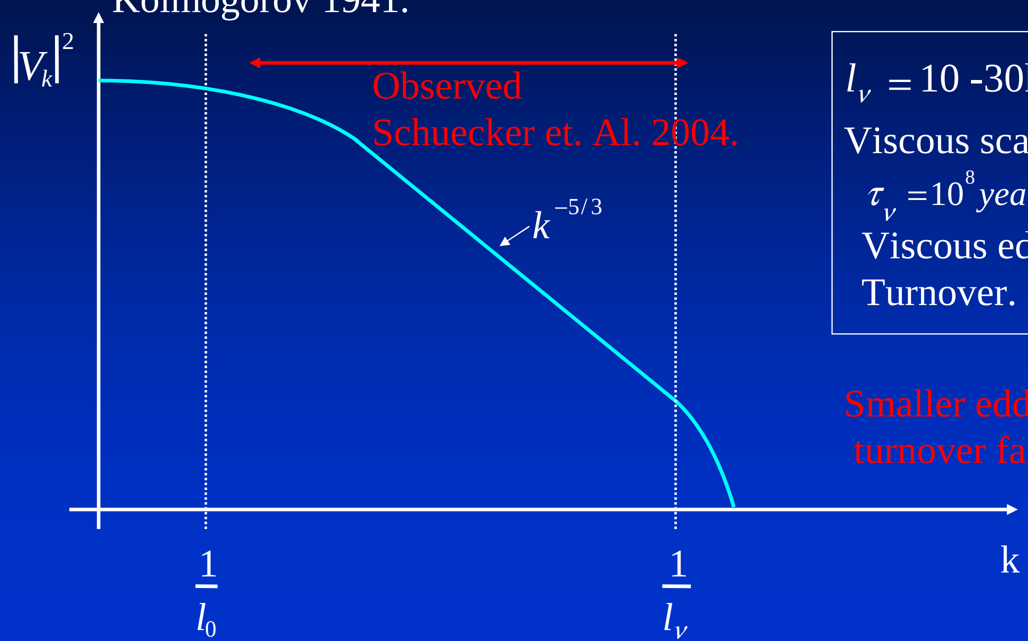
2. What is the small scale structure of the Magnetic field and is this universal in MHD?

3. How do the collisionless scales affect the Dynamics?

4. When the turbulence is created by MRI in discs are the small scales different? Where does the energy go?

Velocity Spectrum

Kolmogorov 1941.



$l_\nu = 10$ -30kpc

Viscous scale

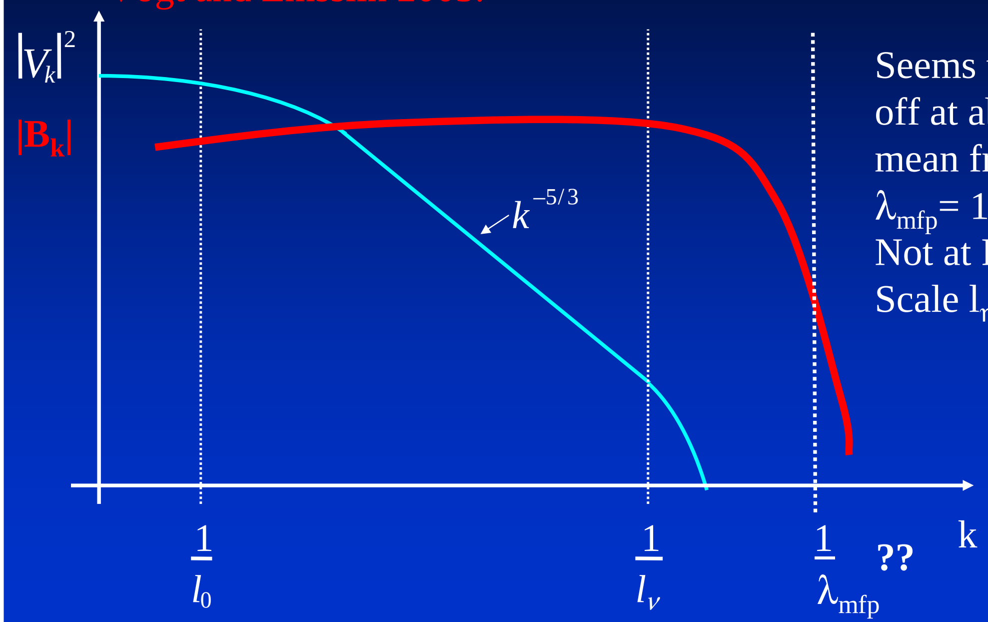
$\tau_\nu = 10^8$ *years*

Viscous eddy Turnover.

Smaller eddies turnover faster.

Oberved Magnetic Spectrum

Vogt and Emsslin 2003.



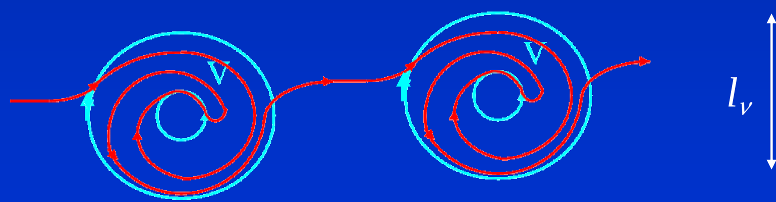
Seems to cut off at about the mean free path - λ_{mfp}= 1-10kpc. Not at Resistive Scale $l_\eta = 10^7$cm.

Homogeneous Dynamo -- History

- MHD invented ~ WWII (*Alfven, etc.*).
- First turbulent dynamo. *Batchelor 1950, Proposed that Homogeneous turbulence will amplify magnetic field.*
- MHD turbulence should (?) give equipartition, $|B_k|^2 = |V_k|^2$ *Bierman and Schluter 1951*. Alfven waves at small scale *Kraichnan, Goldreich, Montgomery, Mattheus etc.*
- By late 1950s helical dynamos proposed *Parker, Zeldovich, Krauss, Radler ……*
- Homogeneous dynamo does not exist? *Moffatt, 1963, 1978…….*
- Delta correlated dynamo, *Kazantsev 1967, Vainstein 1970, Kulsrud and Anderson 1990.* Is it relevant?
- Numerical homogeneous dynamo does exist, *Meneguzzi and Poquet 1982 (Pr =1 and 64^3) Kinney, Chandran, Maron, Schekochihin, Cowley, McWiiliams, (1999 -- …. Mostly Pr>> 1.)*
- Homogeneous Dynamo depends crucially on magnetic Prandtl number, *Pr = υ/η.*

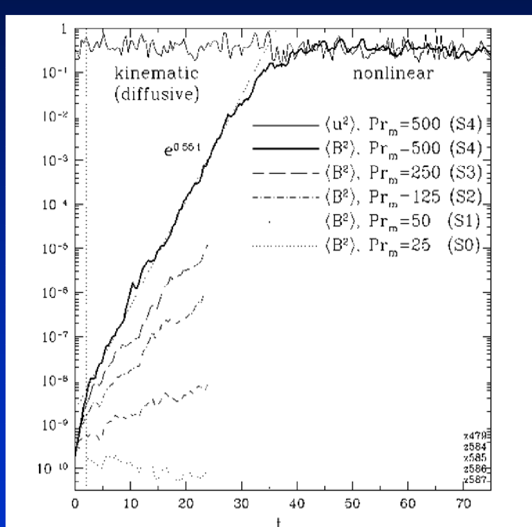
The Large Prandtl Number Case: Galaxies, Clusters, Hot Discs.

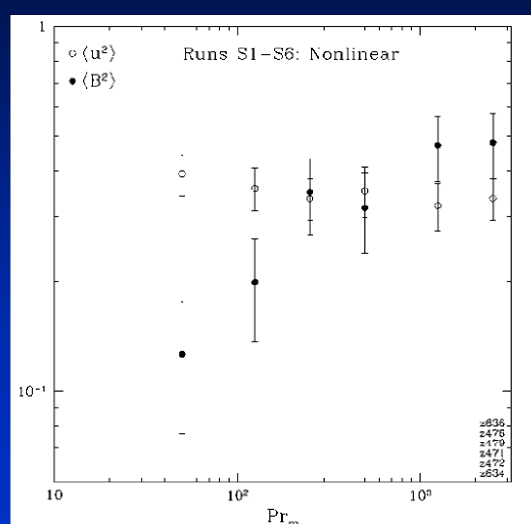
- Magnetic Prandtl number = $Pr = \upsilon/\eta$.
- On the turnover time of the viscous eddies the "seed field" grows. The field develops structure below the viscous scale down to the resistive scale $l_\eta = Pr^{-1/2}\, l_\nu$



Large Pr Dynamo: Numerical.

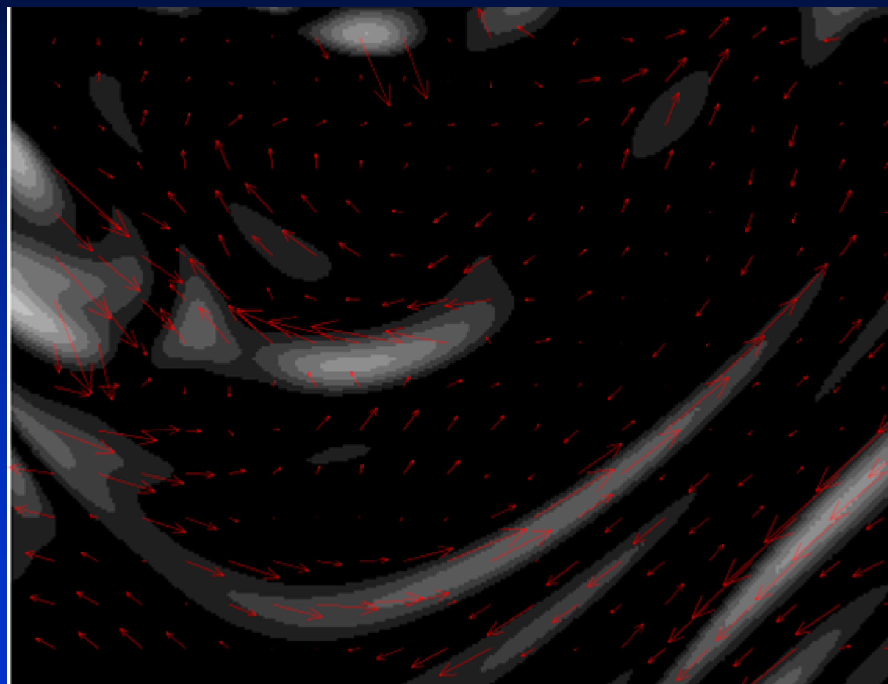
Schekochihin, SCC, McWilliams, Maron Ap. J. 2004.



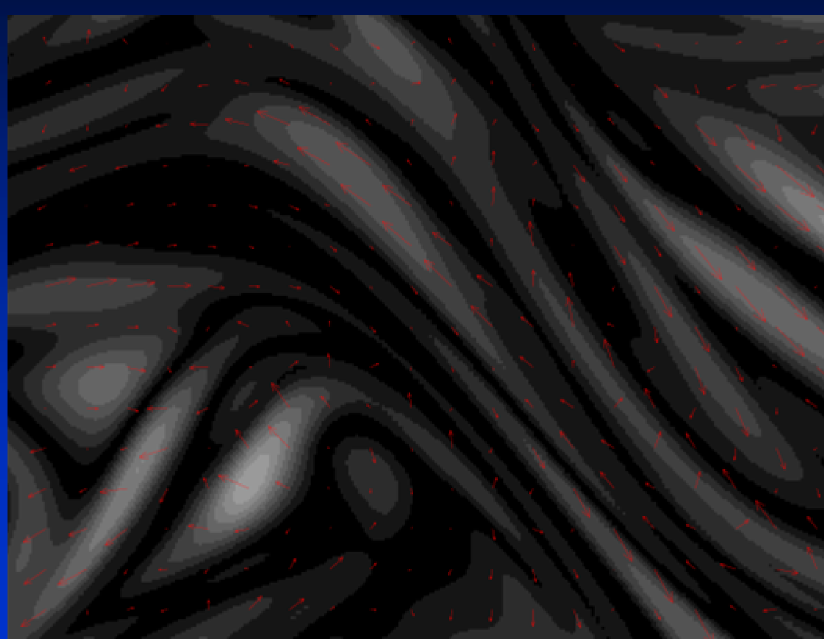


Kinematic Stage - Intermittent Folded Structure.

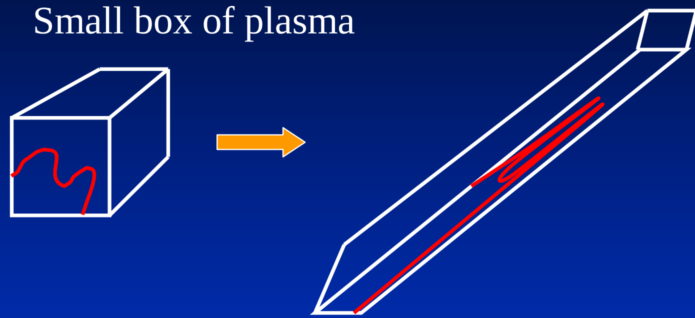
Grayscale is |B|.



Less intermittent, but still Folded saturated state.



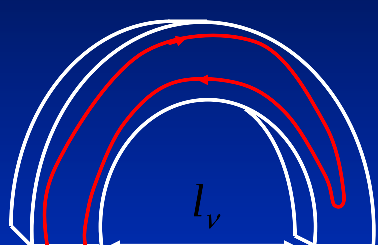
Small box of plasma



We can think of a little bit of the field line being stretched by the flow. To preserve volume some other direction must compress making field lines get closer together. Lines align with stretching direction.

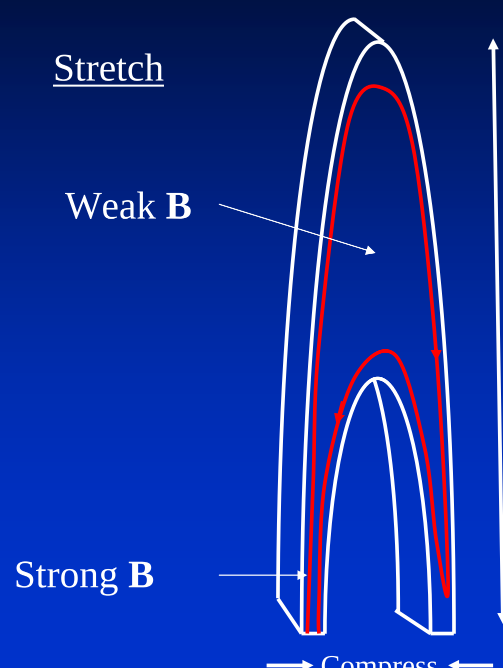
Bending and Stretching.

<u>Bend</u>



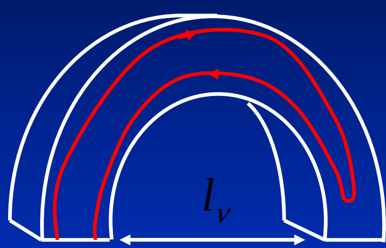
Curvature and |B| anti-correlated.

<u>Stretch</u>



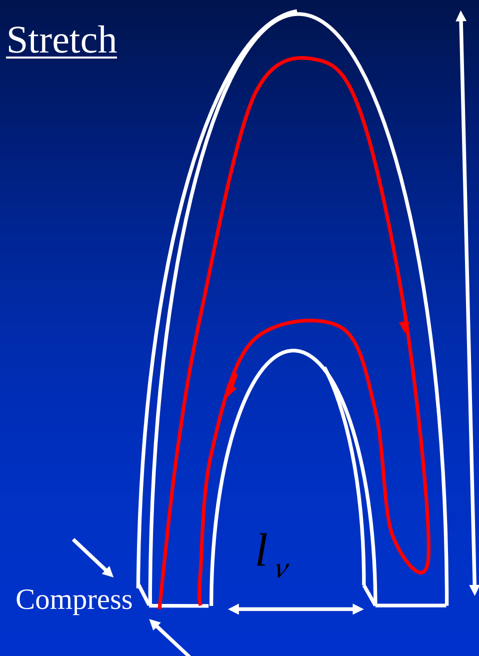
Amplification without generating smaller scales.

Bend



Compress in the direction along which **B** doesn't change. Only some of the random motions do this.

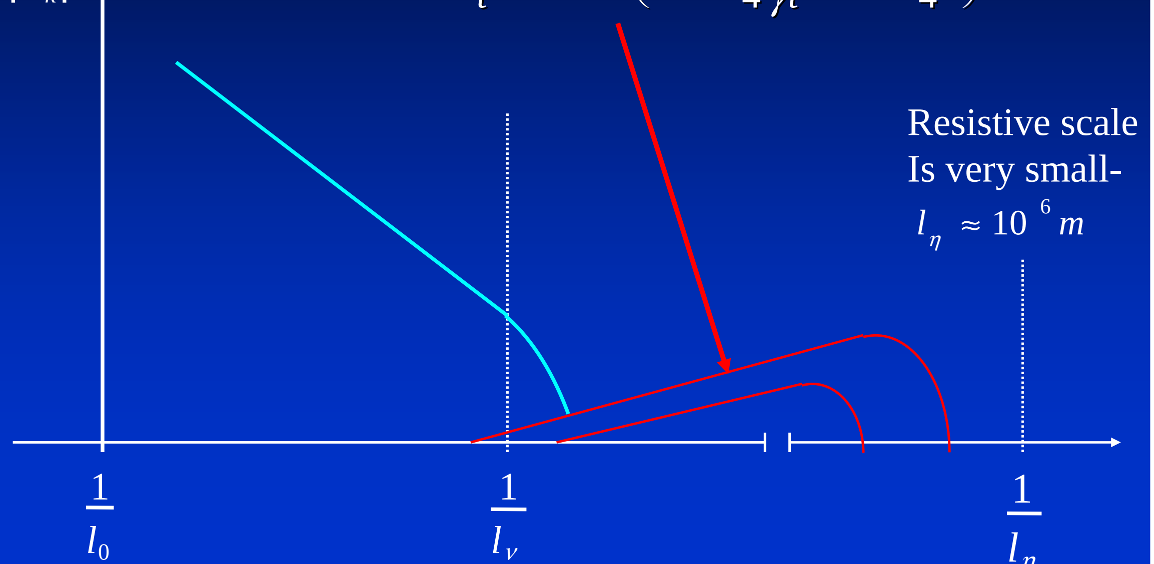
Stretch



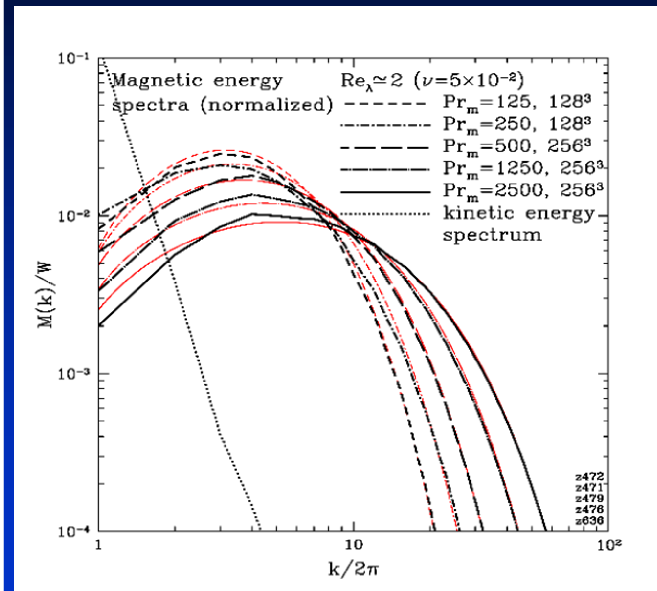
Kinematic Large Pr Dynamo

$|V_k|^2$

$$|B(k,t)|^2 = \frac{k^{3/2}}{t^{1/2}} \exp\left(-\frac{5(\ln k)^2}{4\gamma t} + \frac{3\gamma t}{4}\right)$$



Saturated Energy Spectra: Simulation and Theory

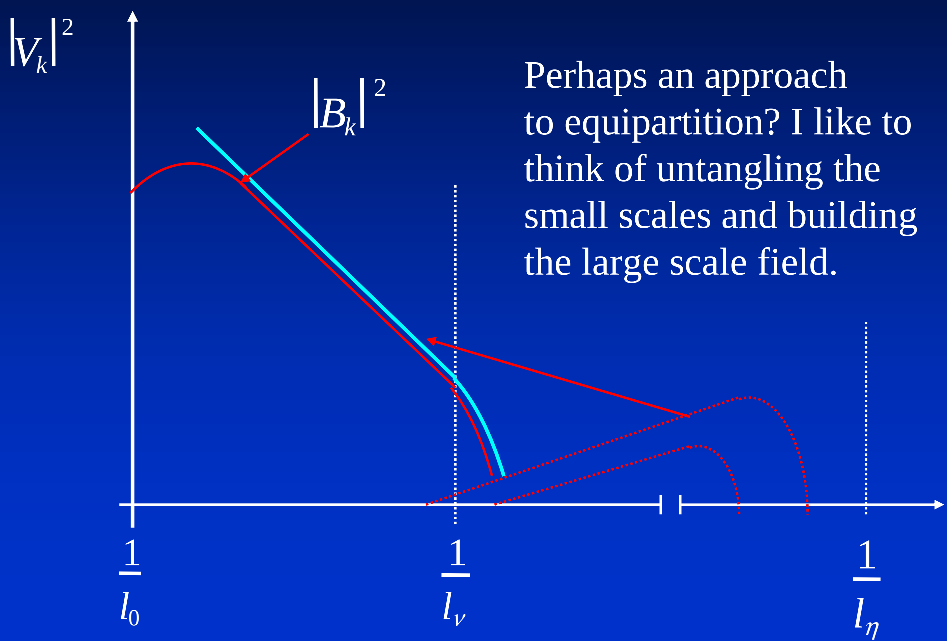


- Fit using same parameters for all *Pr*.
- If *Pr* becomes very large $M(k) \sim k^{0.23}$.
- Magnetic field small scale dominated.

Re >> 1, Pr >> 1 the nonlinear dynamo.

- At Re >> 1 the first sign of nonlinearity is when $B^2 \sim V^2_\nu$ the energy of the viscous eddies.
- Saturation does not occur at this stage magnetic energy grows to be of order the total kinetic energy.

The Inverse Cascade?

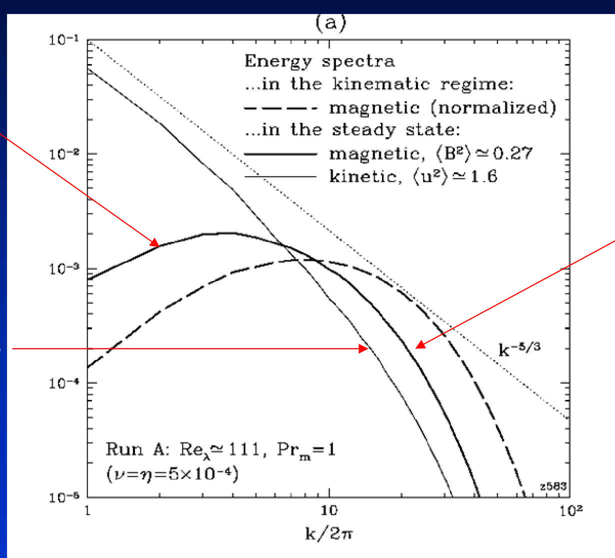


Perhaps an approach to equipartition? I like to think of untangling the small scales and building the large scale field.

No Numerical evidence for equipartition.

Stirring scale still folding Field lines.

Alfven waves on folded field lines?

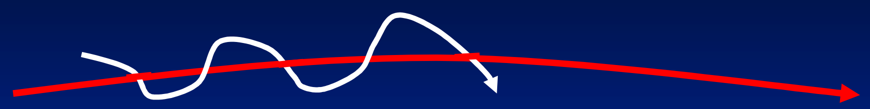


Folded field lines - small scale energy.

QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

Mean Field, Alfven waves
Maron and Goldreich.

Magnetized Viscosity.

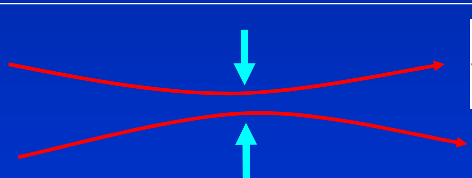


$$\mu = \frac{v_\perp^2}{B} = constant$$

Collisionless particle motion restricted to being close to field line and conserving μ.

$$\mathbf{P} = P_\perp(\mathbf{I} - \mathbf{bb}) + P_\parallel \mathbf{bb} \quad P_\perp =< \frac{1}{2} m_i v_\perp^2 > \quad P_\parallel =< m_i v_\parallel^2 >$$

Anisotropic pressure tensor.



Compressing Field

$$\frac{1}{B}\frac{dB}{dt} \sim \frac{1}{< v_\perp^2 >}\frac{d < v_\perp^2 >}{dt}$$

Collisionless. Relaxed by Collisions.

$$P_\perp - P_\parallel \sim \frac{1}{\nu B}\frac{dB}{dt}$$

Incompressible Braginski MHD.

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right] = -\nabla p + \nabla\cdot\left[(P_\perp - P_\parallel)\mathbf{bb}\right] + \mathbf{B}\cdot\nabla\mathbf{B}$$

$$\nabla\cdot\mathbf{v} = 0$$

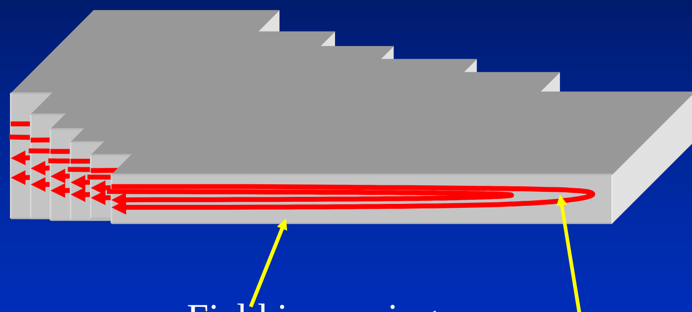
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B})$$

$$P_\perp - P_\parallel = \frac{p}{\nu}\mathbf{b}\cdot\nabla\mathbf{v}\cdot\mathbf{b} = \frac{p}{\nu}\frac{1}{B}\frac{dB}{dt}$$

This viscosity does not damp alfven waves and therefore allows velocities below the viscous cutoff. Can these velocities unwind the small scale field?
Kulsrud, Cowley, Gruzinov and Sudan, 1996. Malyshkin and Kulsrud 2002.

Stretching and compressing

Stretched at the turnover rate of the viscous eddies.



Using Braginskii's Expression we get $P_\parallel - P_\perp \sim Re^{-1/2} P$

Field increasing
$P_\parallel < P_\perp$ Mirror mode
Unstable.

Field decreasing
$P_\parallel > P_\perp$ Firehose
Unstable.

Firehose Instability.

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p - \nabla\cdot[(P_\parallel - P_\perp)\mathbf{bb}] + \mathbf{J}\times\mathbf{B}$$

Look at instabilities that are smaller scale than the field and growing faster than the stretching rate.. We take a constant unperturbed stretching and $\mathbf{B_0}$.

$$\rho\left(\frac{\partial \delta\mathbf{v}}{\partial t}\right) = -\mathbf{k}(\delta p + \delta\mathbf{B}\cdot\mathbf{B}) - \mathbf{k}\cdot[(\delta P_\parallel - \delta P_\perp)\mathbf{b_0 b_0}] + [B_0^2 - (P_\parallel - P_\perp)]\delta(\mathbf{b}\cdot\nabla\mathbf{b}) + (\mathbf{B_0}\cdot\nabla\delta B_\parallel)\mathbf{b_0}$$

Field still frozen to the plasma. $\delta\mathbf{b} = \mathbf{b_0}\cdot\nabla\xi_\perp$

For Alfven wave Polarization. $\xi_\perp \propto \mathbf{k}\times\mathbf{b_0}$

$$\gamma^2 = -k_\parallel^2\left[\frac{B_0^2}{\rho} - \frac{(P_\parallel - P_\perp)}{\rho}\right]$$

Unstable if: $(P_\parallel - P_\perp) > B_0^2$ $\quad \beta > Re^{-1/2}$

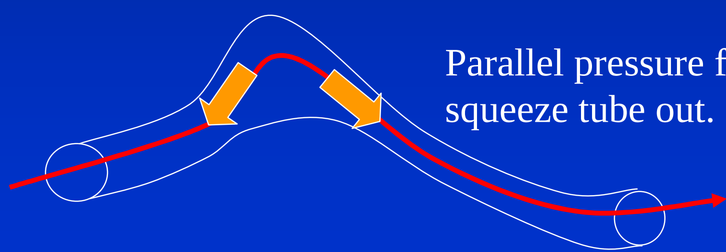
More Firehose.

Unstable when $B^2 <$ energy in viscous scale eddies (roughly for $B < 5\mu G$). Obviously in early stages of dynamo. Since

$$\gamma \sim k_{||} C_{sound} (Re)^{-1/4}$$

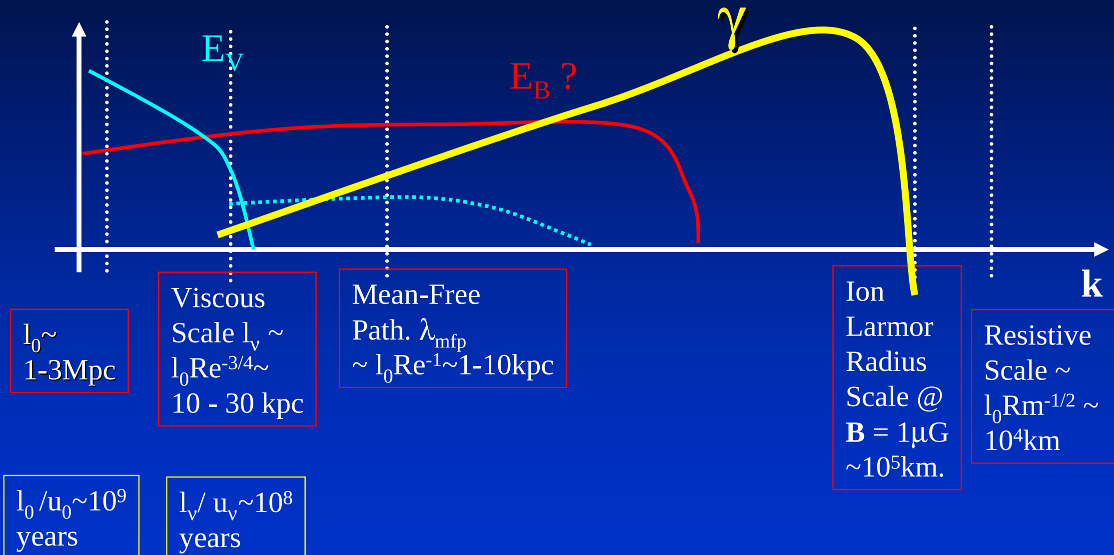
Growth at small scales is very fast. Collisionless theory gives same growth formula down to $k_{||}\rho_i \sim (Re)^{-1/4}$ where:

$$\gamma_{max} \sim \Omega_{ci} (Re)^{-1/4}$$



Parallel pressure forces squeeze tube out.

Scales



Enhanced Scattering.

The effect of the firehose and mirror instabilities may be to scatter the particles making an effective mean free path of order ρ_I

Viscosity decreased $v_{thi}\,\lambda_{mfp} \rightarrow v_{thi}\,\rho_{ii}$ Large Re.

Has the interesting effect of increasing the initial field growth because it increases the eddy turnover rate of the viscous eddies. A key question is: how does it affect the plasma transport processes? Does it make MHD a better approximation?

What was this talk about? Conclusions?

- HOMOGENEOUS DYNAMO - NO MEAN FIELD.
- Structure of the homogeneous dynamo, *Pr >> 1*. Folding, intermittency.
- Sustained MHD turbulence -- no equipartition. Folding persists, not alfven waves on large scale field.
- Alfven waves and folds. Maybe on the folds.
- Model of saturated state. Gives good spectrum shape with assumption of suppression of bending motions.
- Unstable to growth of Firehose instabilities.
- Enhanced collisionality from instabilities.

Kazantsev Model, 1967.

See also Kulsrud and Anderson 1990

- Delta correlated velocity -- lots of jitters.

$$< \mathbf{u}(t,\mathbf{r})\mathbf{u}(t',\mathbf{r}') >= \delta(t-t')\kappa(\mathbf{r}-\mathbf{r}')$$

in k space

$$< \mathbf{u_k}(t)\mathbf{u}(t')_{\mathbf{k}'} >= \delta(t-t')\delta_{\mathbf{k},\mathbf{k}'}\ \kappa(k)(\mathbf{I}-\hat{\mathbf{k}}\hat{\mathbf{k}})$$

note that dimensionally

$$\kappa(k) \sim \tau_c u_k^2$$

where τ_c is the "correlation time".

Nonlinear saturation model.

We modified the delta correlated model to make velocity correlate with magnetic field direction. We then partially suppressed bending motions but not interchange motions as B^2 approaches V^2 and find steady state.

$$< \mathbf{u_k}(t)\mathbf{u}(t')_{\mathbf{k}'} >= \delta(t-t')\delta_{\mathbf{k},\mathbf{k}'}\ [\kappa^1(k,\mu)(\mathbf{I}-\hat{\mathbf{k}}\hat{\mathbf{k}})+ \kappa^2(k,\mu)(\mathbf{bb}+\mu^2\hat{\mathbf{k}}\hat{\mathbf{k}}-\mu\hat{\mathbf{k}}\mathbf{b}-\mu\mathbf{b}\hat{\mathbf{k}})$$

with $\mu = \hat{\mathbf{k}}\cdot\mathbf{b}$.

Braginskii's Viscosity.

$$\nabla \cdot \mathbf{P} = \nabla p + \nabla \cdot \left[\left(P_\perp - P_\parallel \right) \mathbf{b}\mathbf{b} \right]$$

$$\left(P_\perp - P_\parallel \right) \sim \frac{nT}{\nu B}\frac{dB}{dt} \sim \frac{nT}{\nu}\mathbf{b} \cdot \nabla \mathbf{v} \cdot \mathbf{b}$$

This viscosity does not damp alfven waves and therefore allows velocities below the viscous cutoff. Can these velocities unwind the small scale field? Kulsrud, Cowley, Gruzinov and Sudan, 1996. Malyshkin and Kulsrud 2002.

With this kind of viscosity the plasma is unstable to rapidly growing instabilities at scales from the viscous to the ion larmor radius scale. Formally MHD with the correct viscosity for a fully ionized (and Magnetized) plasma is Ill Posed unless $\beta < \beta_c$ everywhere.