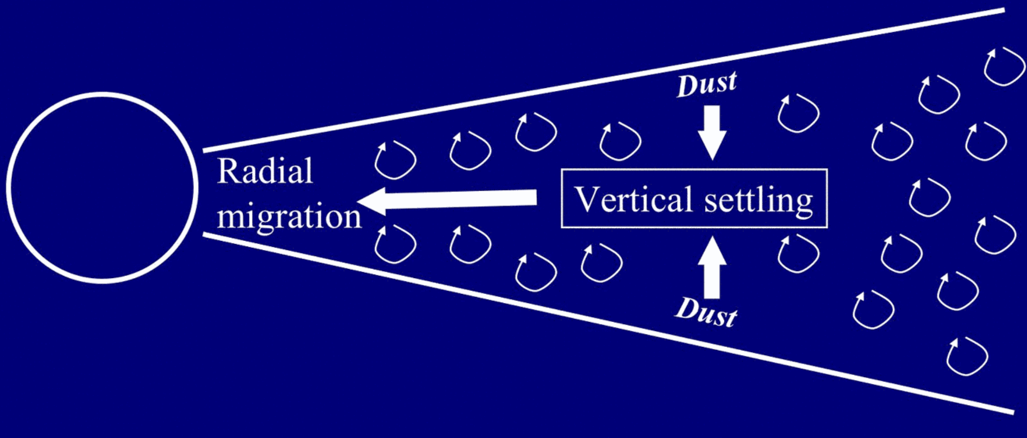
# Dust dynamics in turbulent protoplanetary disks

S.Fromang (QMUL)
J.Papaloizou (DAMTP, Cambridge)
R.Nelson (QMUL)

---

# Overview



*Dust*

Radial migration

Vertical settling

*Dust*

- How does MHD turbulence affect dust settling?
- How does MHD turbulence affect dust radial migration?

# Drag forces

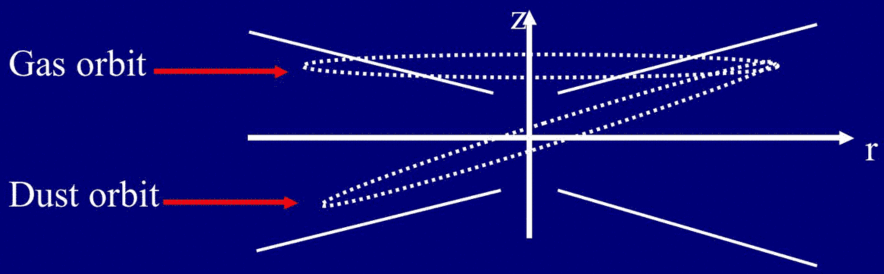
Force on the dust in the Epstein regime: $F_{drag} = m_p \frac{\rho c_s}{\rho_{dust} a}(v_g - v_{dust})$

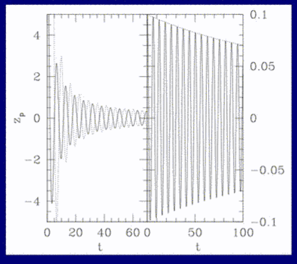
Stopping time scale: $\tau_s = \frac{\rho_{dust} a}{\rho c_s}$

- **If $\Omega\tau_s<1$ ⇒ good coupling, 2 fluid description (Garaud et al, 2004)**
- **If $\Omega\tau_s$~1 ⇒ weak coupling, N-body approach better**

---

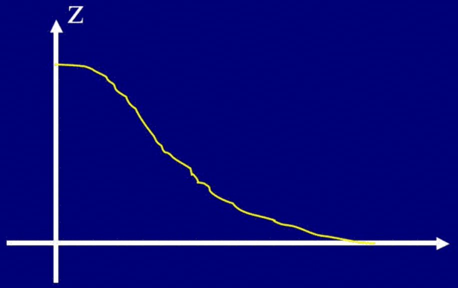
# Vertical settling





Weak coupling limit

*(Garaud et al. 2004)*



Strong coupling limit

# Radial migration

**Weak coupling limit:**

- gas orbits at <u>sub-Keplerian</u> velocity
- dust orbits at <u>Keplerian</u> velocity
  - ⇒ dust feels an head-wind
  - ⇒ **migrate inward**

**Strong coupling limit:**

- gas & dust orbits at sub-Keplerian velocity
- dust does **NOT** feel radial pressure gradient
  - ⇒ **migrate inward**

For 1 meter particles: $\tau_{mig} \sim 10^{2\text{-}3}$ years

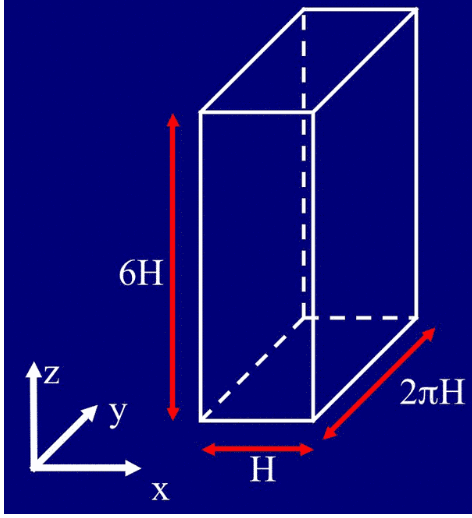
(Weidenschilling 1977)

---

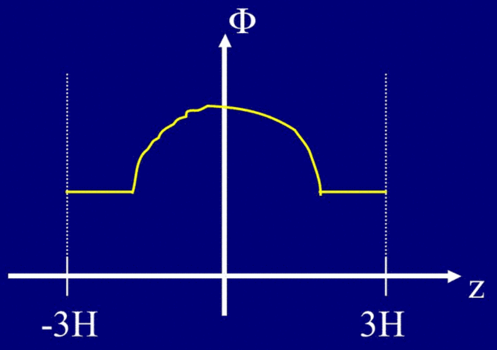
# Dust vertical settling

# Disk model: setup

- Stratified shearing box (Stone et al. 1996) with ZEUS
- Resolution: $(N_r,N_\phi,N_z)=(32,100,192)$

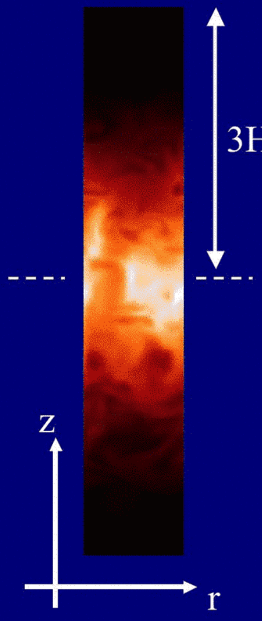


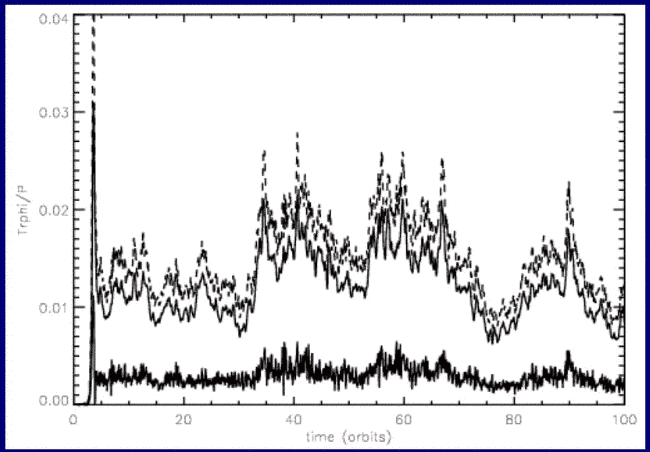
- Boundary conditions periodic in z



---

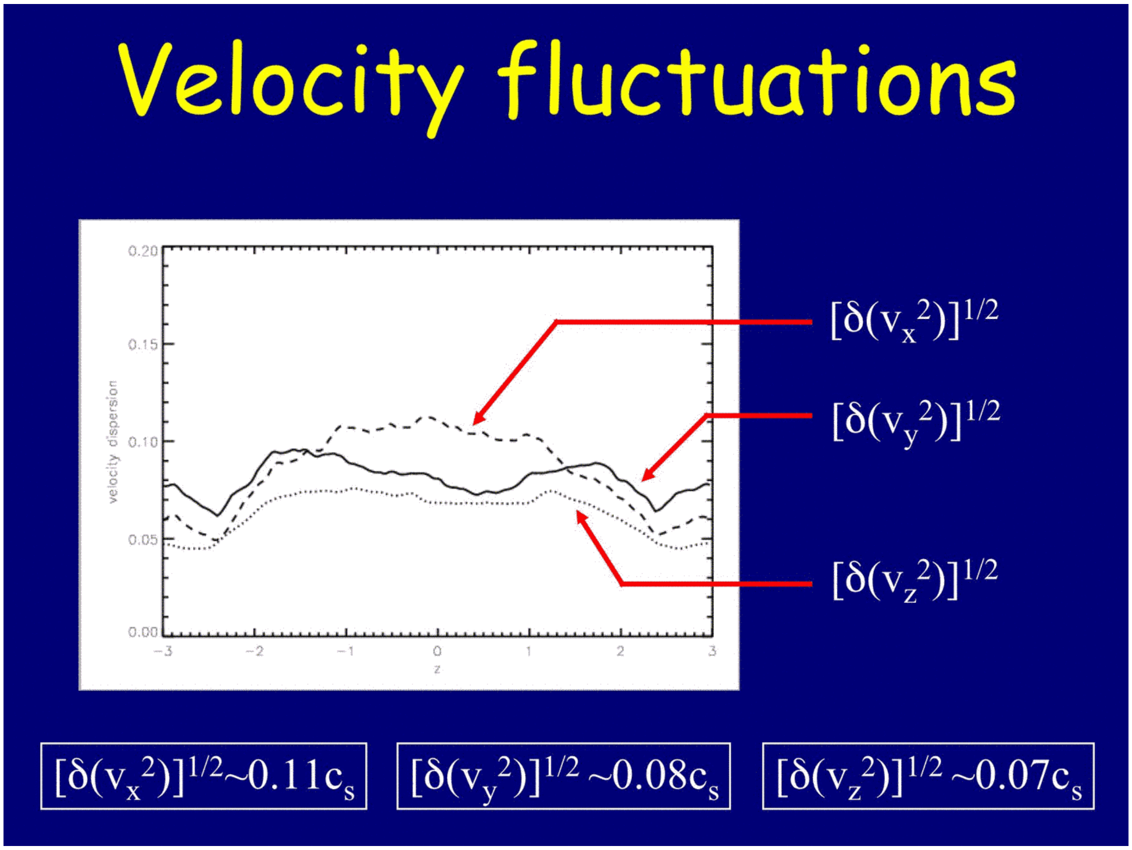
# Disk model: properties





Time history for the Maxwell and Reynolds stress tensor

*Similar to Stone et al. (1996)*

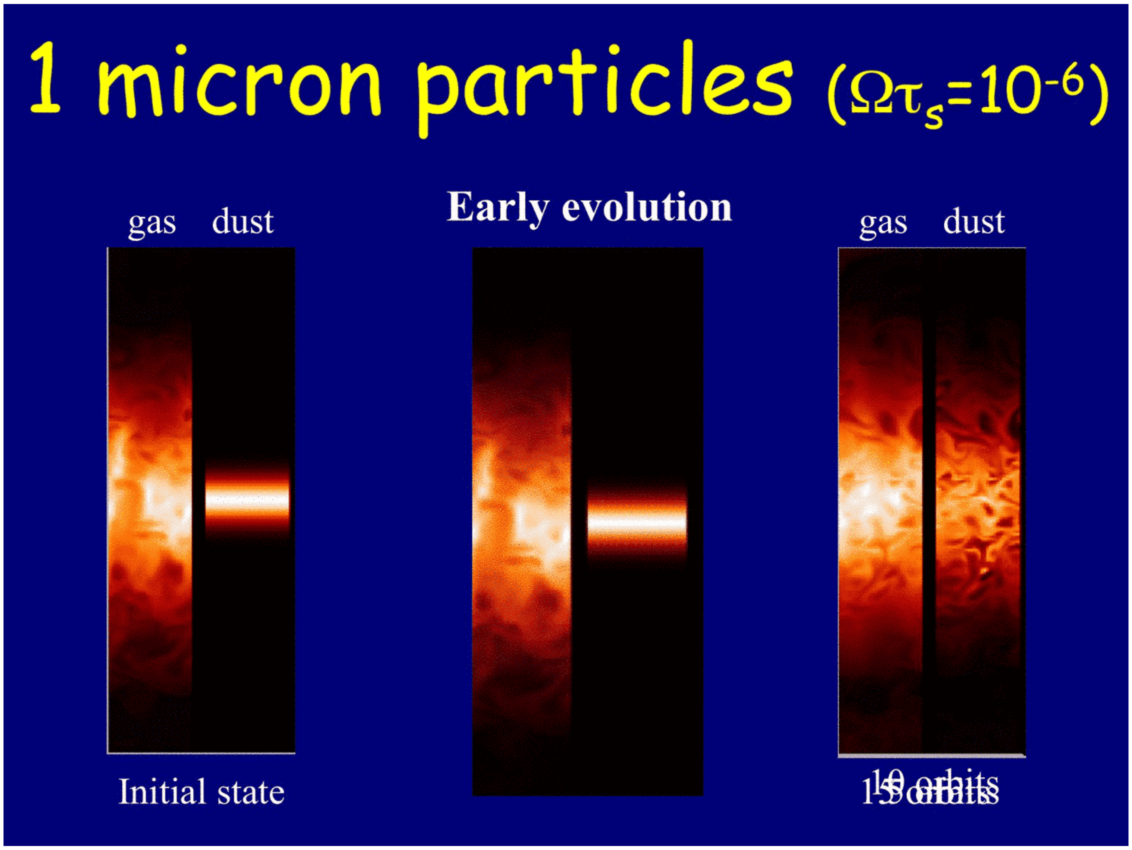


# Velocity fluctuations

$[\delta(v_x^2)]^{1/2}\sim 0.11c_s$

$[\delta(v_y^2)]^{1/2}\sim 0.08c_s$

$[\delta(v_z^2)]^{1/2}\sim 0.07c_s$



# 1 micron particles ($\Omega\tau_s=10^{-6}$)

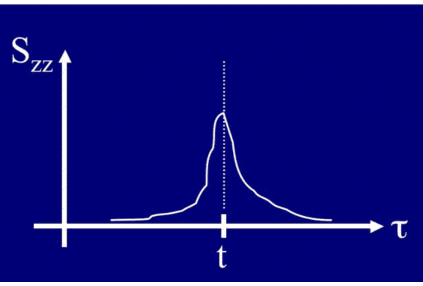
# A simple model

- 3D, isotropic, steady state turbulence

$z(T)=\int_0^T v_z(t)dt$ gives <z(T)>=0

$$<z^2(T)>=\int_0^T dt\int_0^T <v_z(t)v_z(\tau)>d\tau$$

$$S_{zz}(t,\tau)=<v_z(t)v_z(\tau)>=S_{zz}(t-\tau)$$



For large enough T,

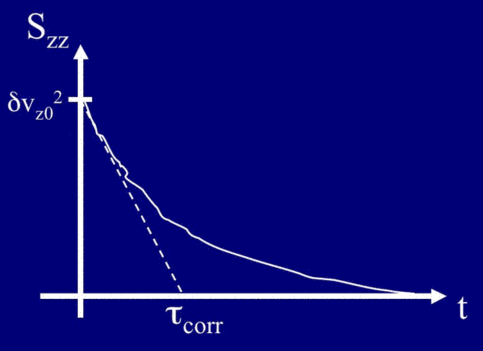
$\int_0^T S_{zz}(t-\tau)d\tau$

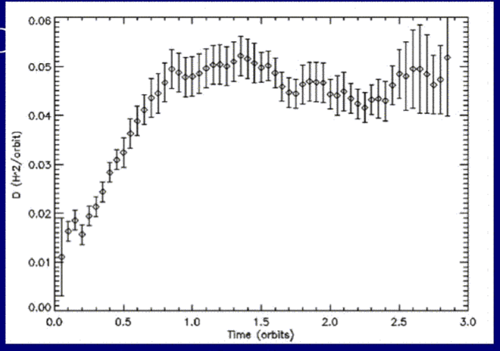
becomes independant of t

# Diffusion coefficient

Then: $<z^2(T)>=2D_{turb}T$ with

$$D_{turb}=\int_0^T S_{zz}(\tau)d\tau=\int_0^T <v_z(0)v_z(\tau)>d\tau$$

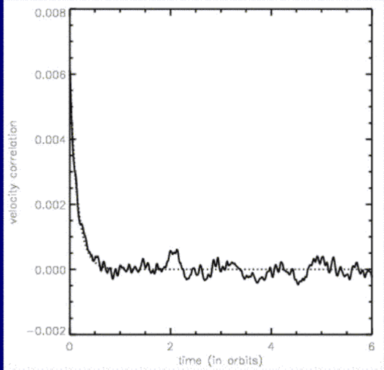




(Carballido, Stone & Pringle 2005)

# Measured $D_{dust}$

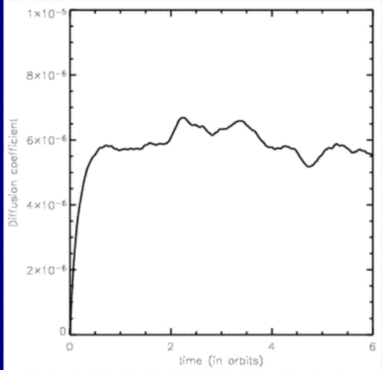
- Restart turbulent model at times t=40, 50, 60 70, 80 orbits
- Calculate $S_{zz}$, $D_{turb}$ (space averaging for $|z|<H$)
- Average between the models



Velocity correlation

$S_{zz}=v_{z0}^2 e^{-t/tcorr}$

with tcorr=0.15 orbits



Diffusion coefficient

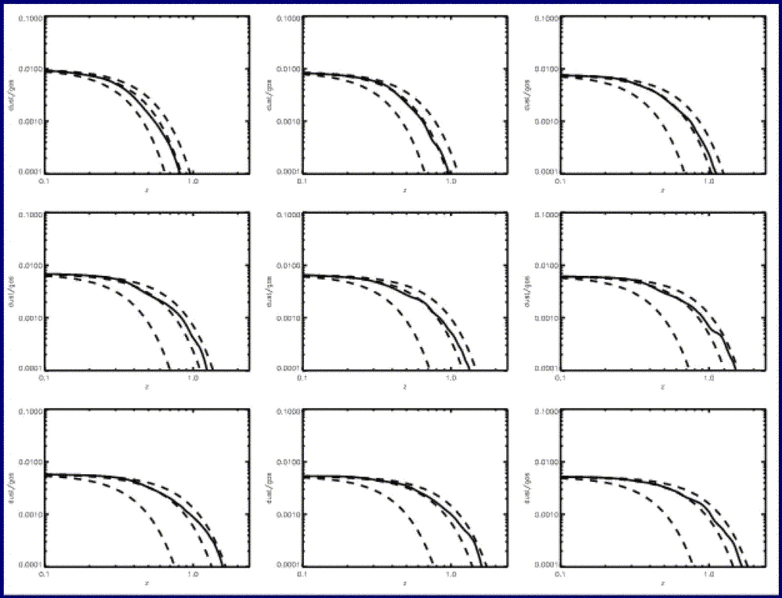
$<D_{dust}>=6.10^{-6}$

# Diffusive evolution

t=0.15 orbits
0.45 orbits
0.75 orbits

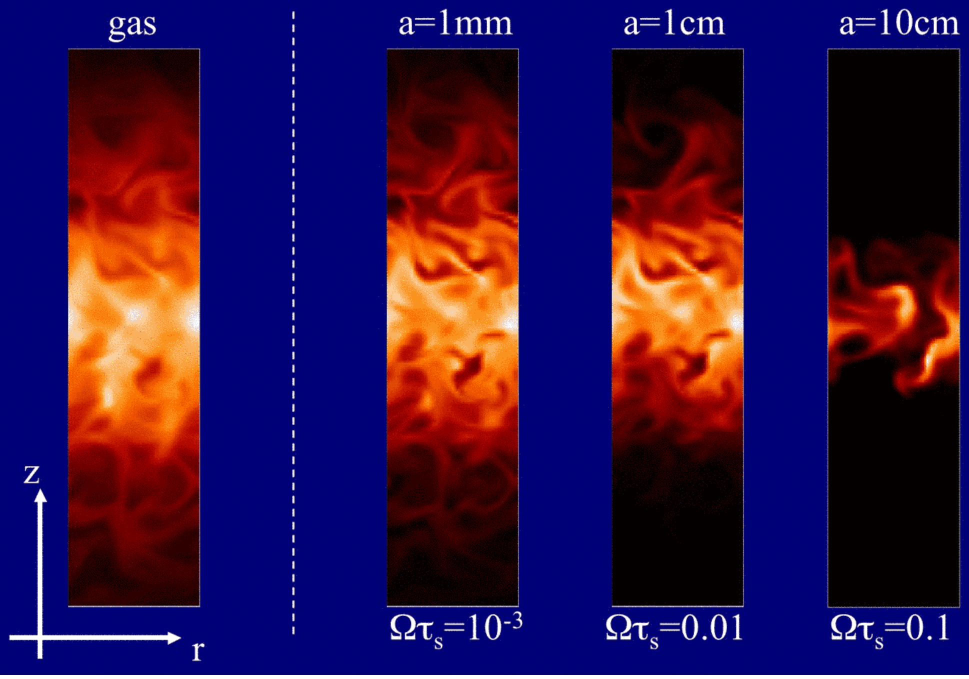
1.05 orbits
1.45 orbits
1.75 orbits

2.05 orbits
2.35 orbits
2.65 orbits



$D_{dust}=10^{-6},6.10^{-6},10^{-5}$

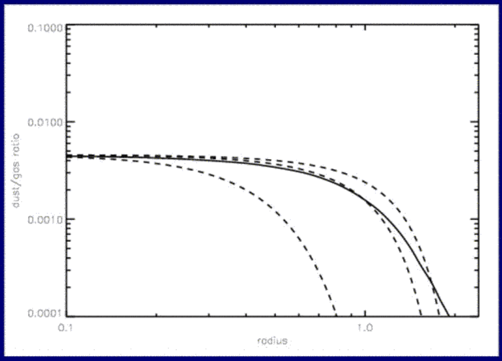
# Larger particles



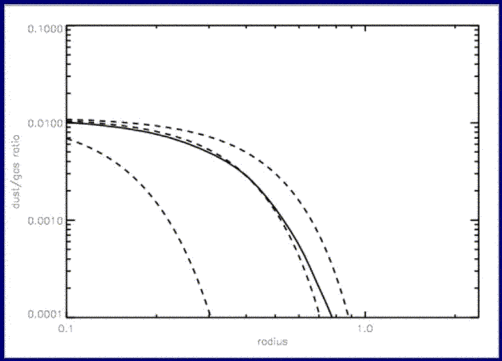
# Steady state

$$\frac{\partial \rho_d}{\partial t} - \frac{\partial}{\partial z}(z\Omega^2\tau_f\rho_d) = \frac{\partial}{\partial z}\left[D_{dust}\frac{\partial \rho_d}{\partial z}\right]$$

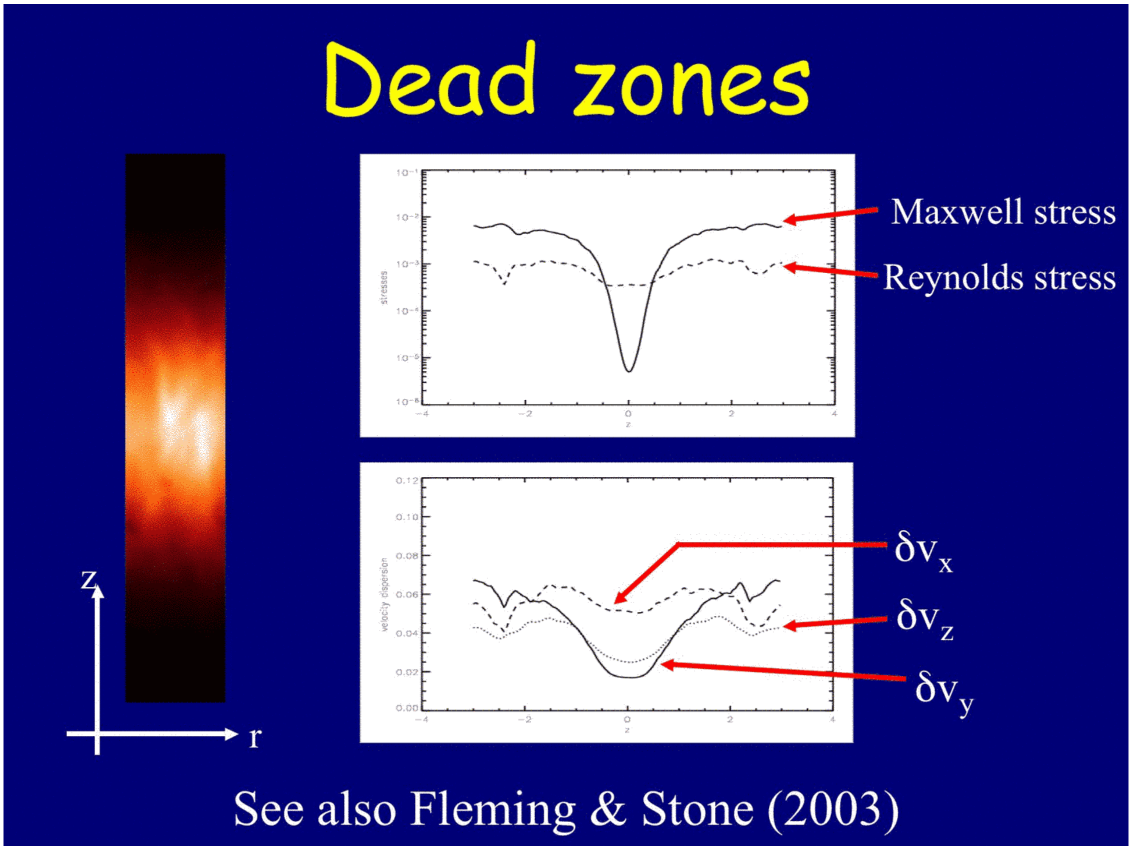
with $D_{dust}=10^{-6}, 6.10^{-6}, 10^{-5}$



a=1cm

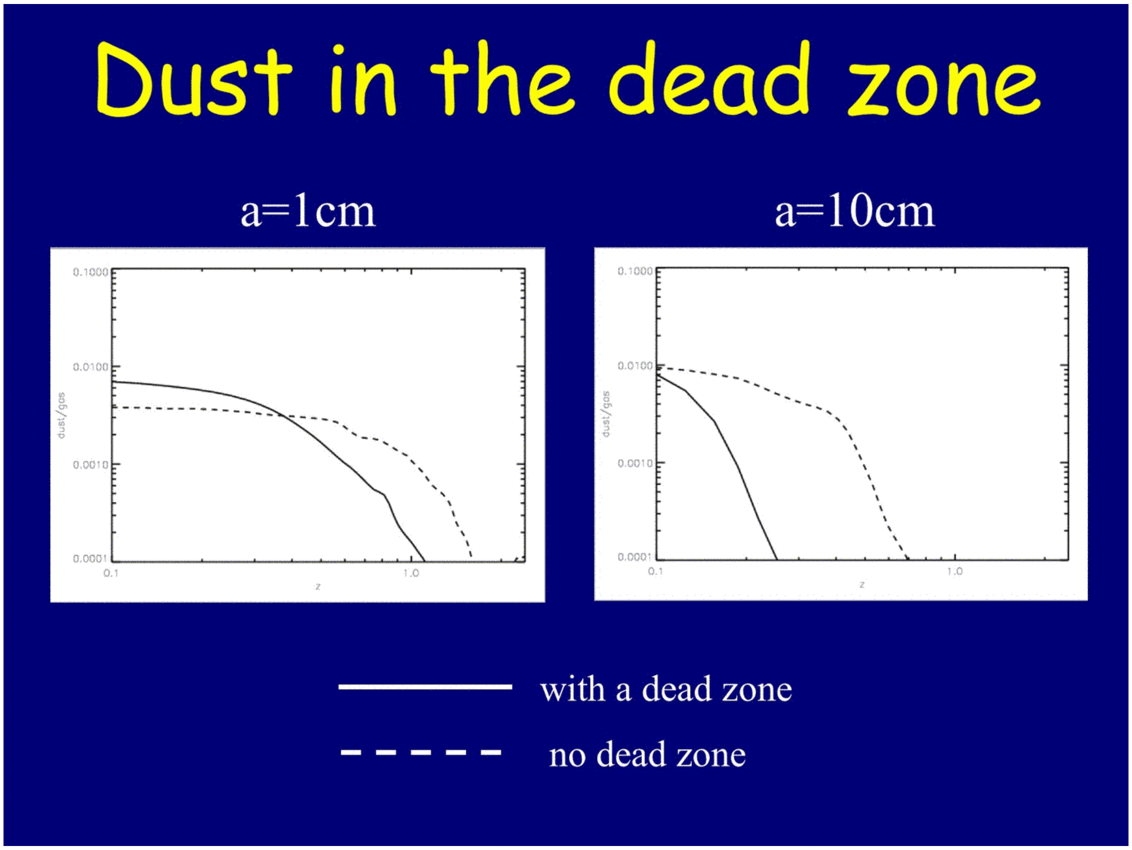


a=10cm



# Dead zones

See also Fleming & Stone (2003)



# Dust in the dead zone

with a dead zone

no dead zone

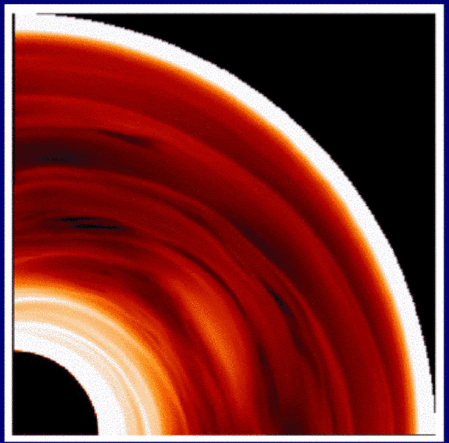
# Solid bodies radial migration

---

# The disk model

- Cylindrical disk model, based on Steinacker & Papaloizou (2002), Papaloizou & Nelson (2003, 2004), Nelson (2005)

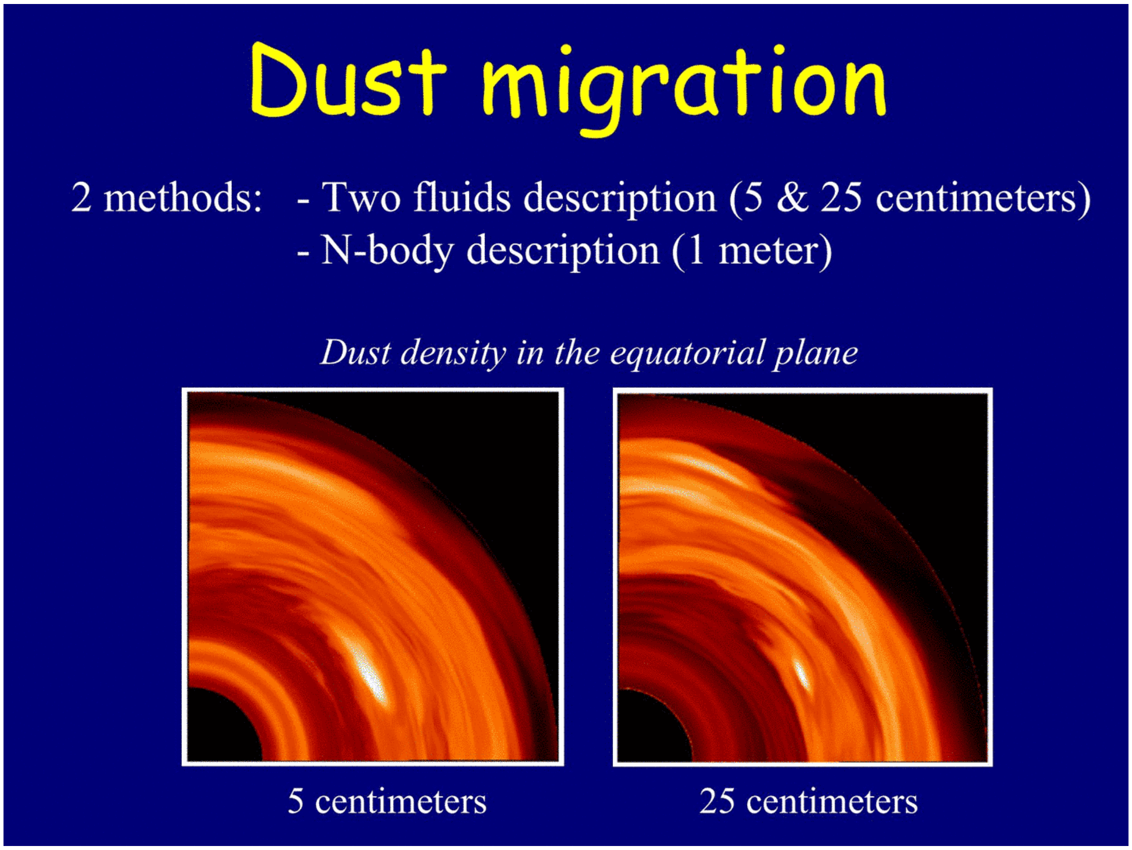
-Models computed with GLOBAL (Hawley & Stone 1995) and NIRVANA (Ziegler & Yorke 1997)



*Density in the equatorial planet*

- Resolution: $(N_r,N_\phi,N_z)=(260,152,44)$
- r in [1,5]
- $\phi$ in $[0, \pi/2]$
- Toroidal field initially

$$\frac{T_{r\phi}^{Max}+T_{r\phi}^{Rey}}{P}\sim 10^{-2}$$



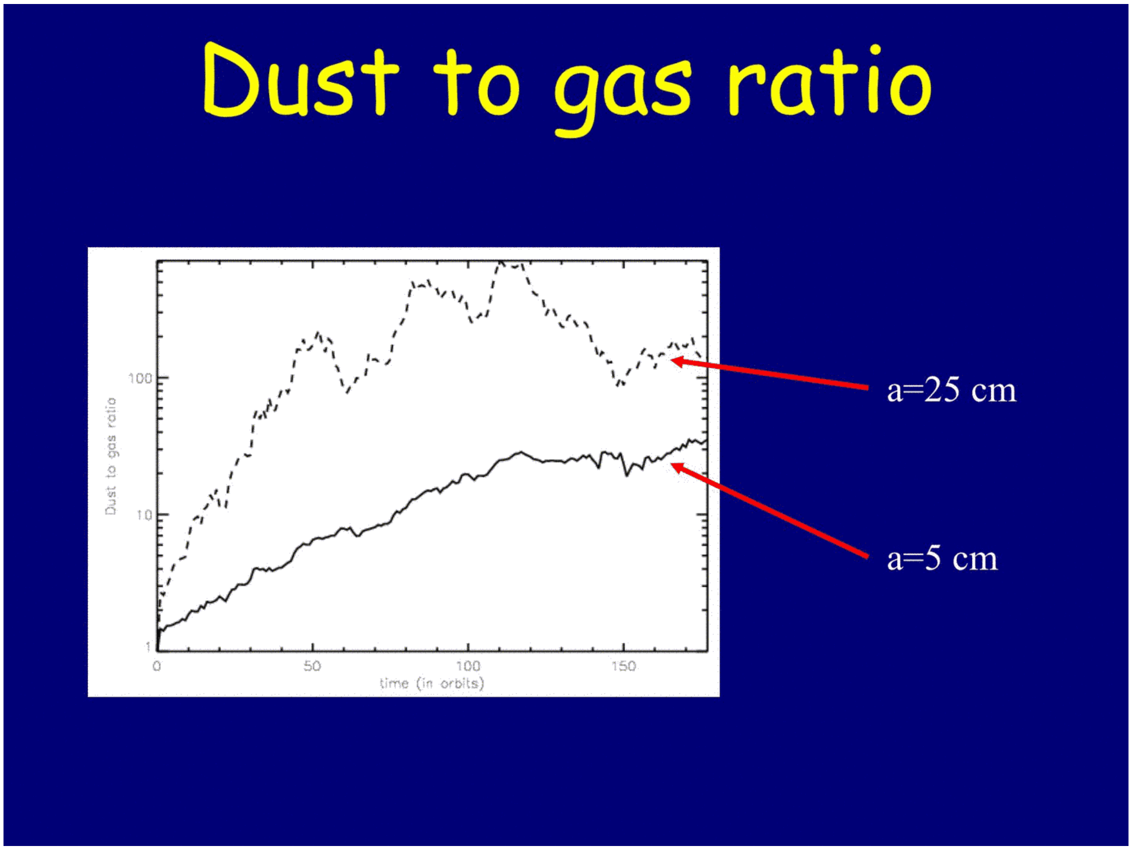
# Dust migration

2 methods:
- Two fluids description (5 & 25 centimeters)
- N-body description (1 meter)

*Dust density in the equatorial plane*

5 centimeters

25 centimeters



# Dust to gas ratio

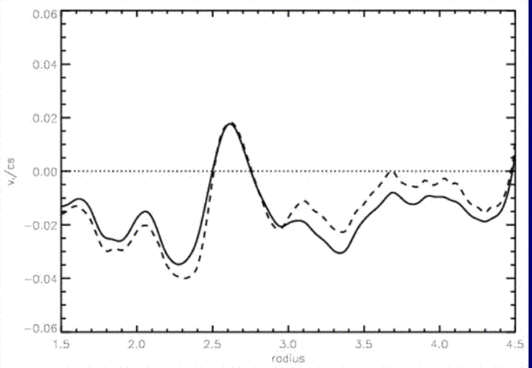
a=25 cm

a=5 cm

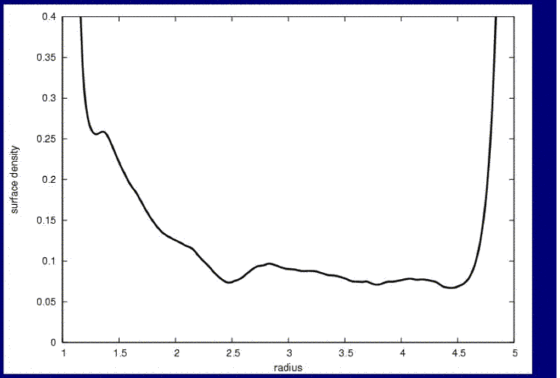
# Drift velocity

(1) $$v_r^{dust} = \frac{\tau_s}{\rho}\frac{\partial P}{\partial r}$$ (Weidenschilling 1977)

*Radial velocity*



*Gas surface density*

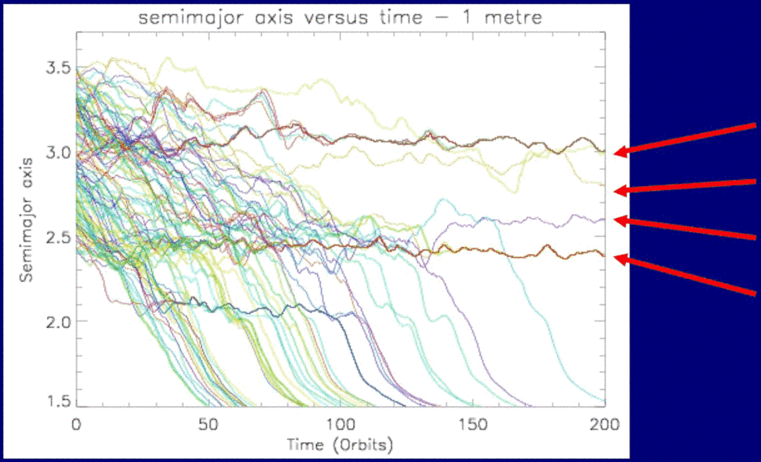


——— Numerical simulations

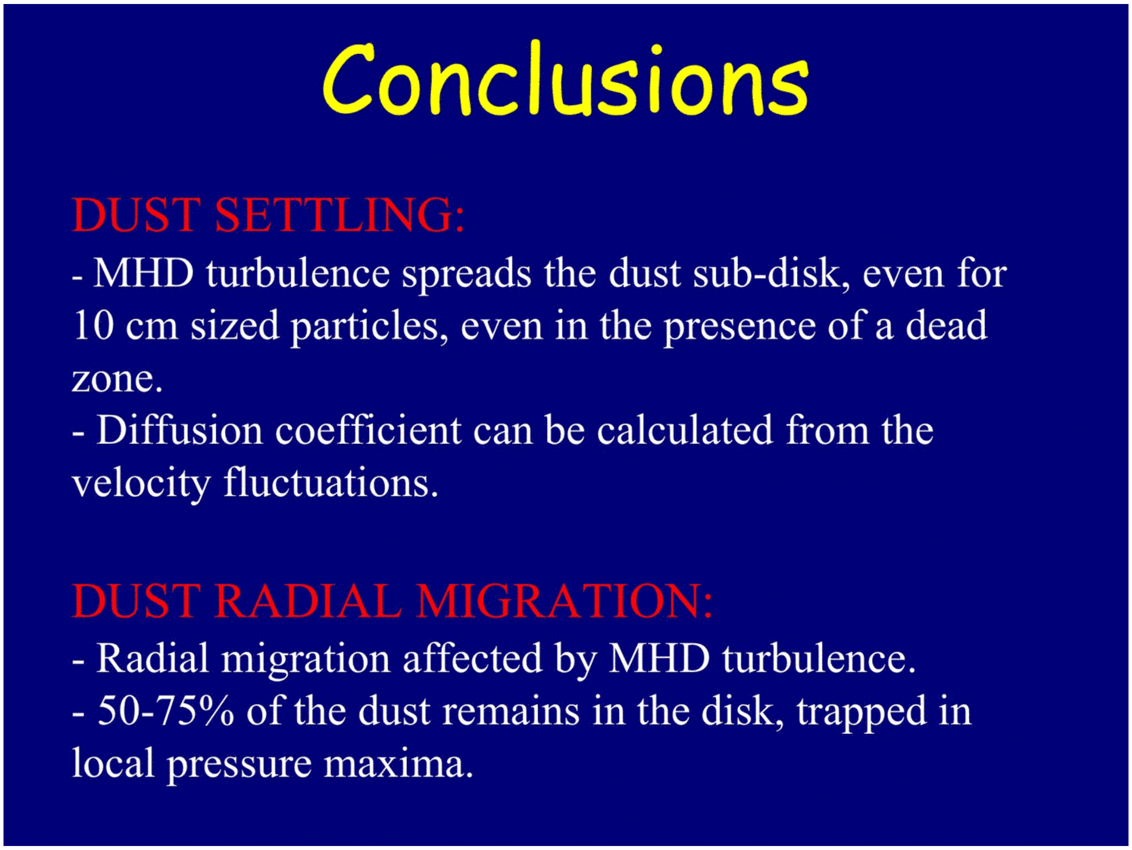
– – – Using equation (1)

# N-body approach

- Gas simulations + 3000 particles (a=1 meter) using NIRVANA



Solid body
accumulation
at a few radii



# Conclusions

DUST SETTLING:

- MHD turbulence spreads the dust sub-disk, even for 10 cm sized particles, even in the presence of a dead zone.
- Diffusion coefficient can be calculated from the velocity fluctuations.

DUST RADIAL MIGRATION:

- Radial migration affected by MHD turbulence.
- 50-75% of the dust remains in the disk, trapped in local pressure maxima.