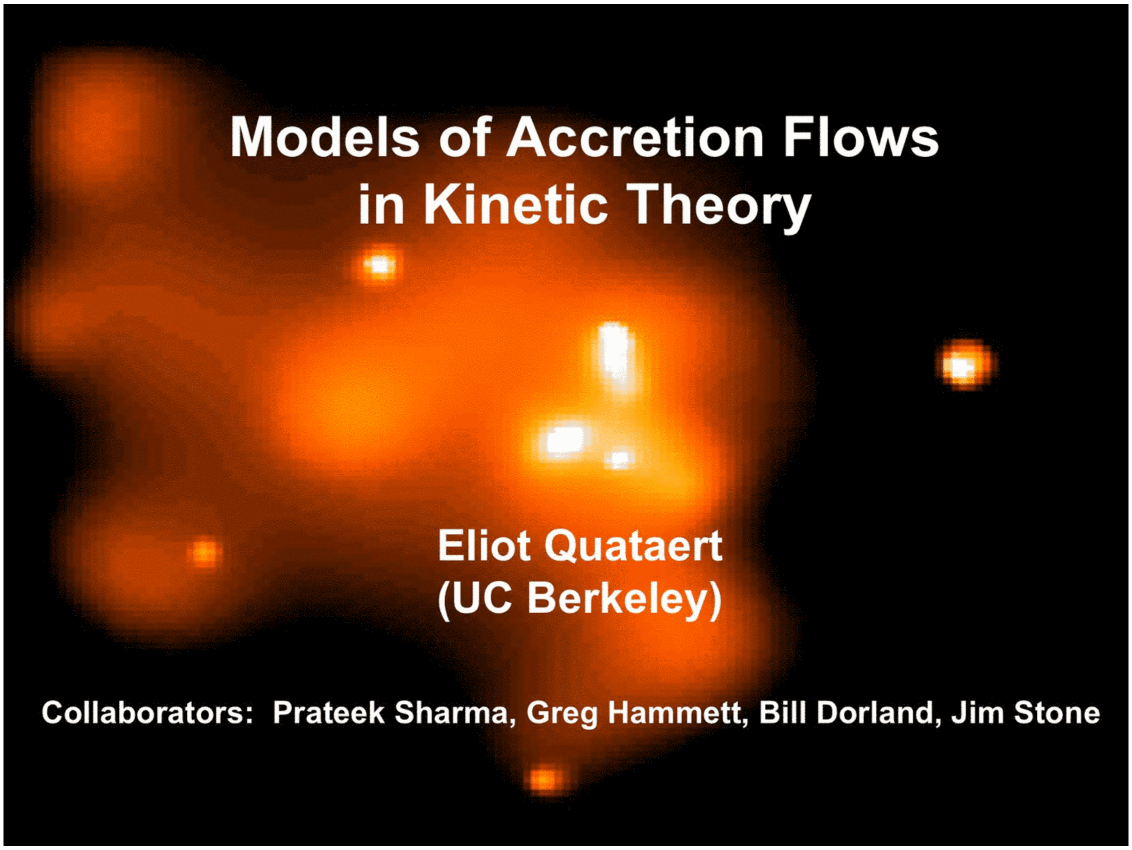

Models of Accretion Flows in Kinetic Theory

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(UC Berkeley)

Collaborators: Prateek Sharma, Greg Hammett, Bill Dorland, Jim Stone

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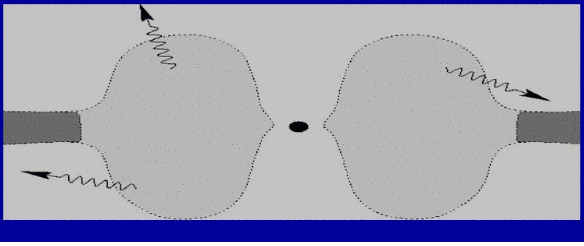
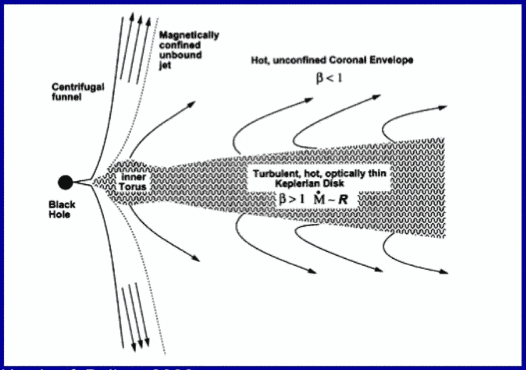
Radiatively Inefficient Accretion

$$\boxed{L << \dot{M}c^2 \quad \dot{\mathrm{M}} < \alpha^2 \dot{\mathrm{M}}_{\mathrm{EDD}}}$$

- Cooling Time >> Inflow Time
- $T_p \sim 10^{11}\text{-}10^{12}$ K > $T_e \sim 10^9\text{-}10^{11}$ K
- Collisionless plasma w/ e-p collision time >> inflow time
- Low-luminosity XRBs & AGN
  - most accreting compact objects, most of the time

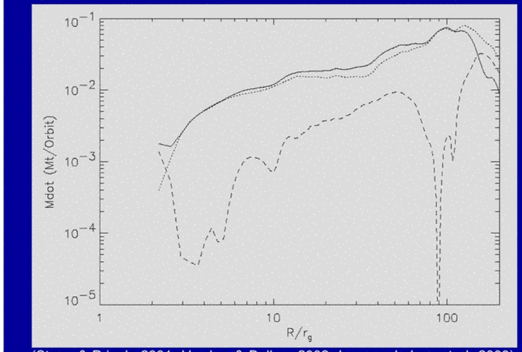


Numerical Simulations



Hawley & Balbus 2002

Accretion Rate



(Stone & Pringle 2001; Hawley & Balbus 2002; Igumenshchev et al. 2003)

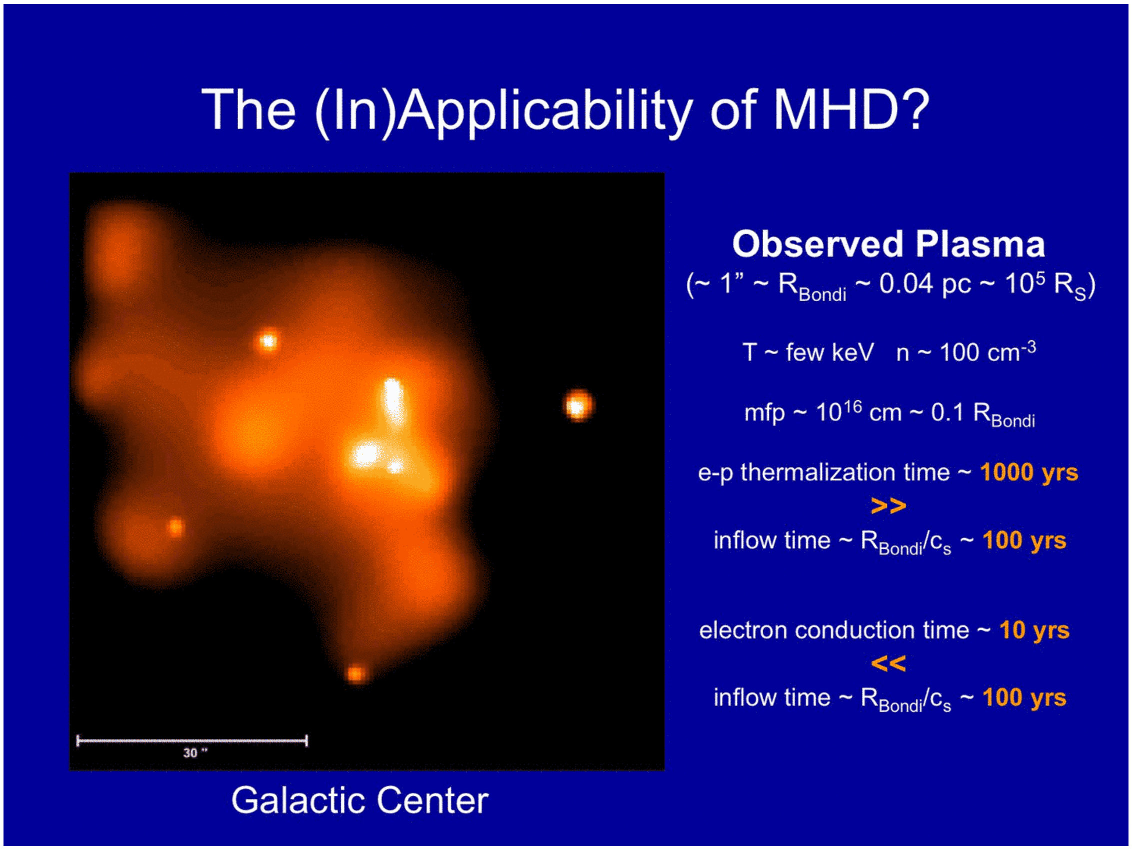
Simulations indicate that very little of the available mass accretes onto the central object (both MHD & α models)

Top 2 List

1. Do MHD Calculations Qualitatively Capture Global Dynamics or will a Proper Kinetic Description yield Surprises?

2. Electron vs. Ion Energetics (electrons radiate, not ions)

    - Heating: MHD turbulence, reconnection, shocks, ….
    - Energy Transfer: Electron-ion coupling, heat conduction, ….



The (In)Applicability of MHD?

Galactic Center

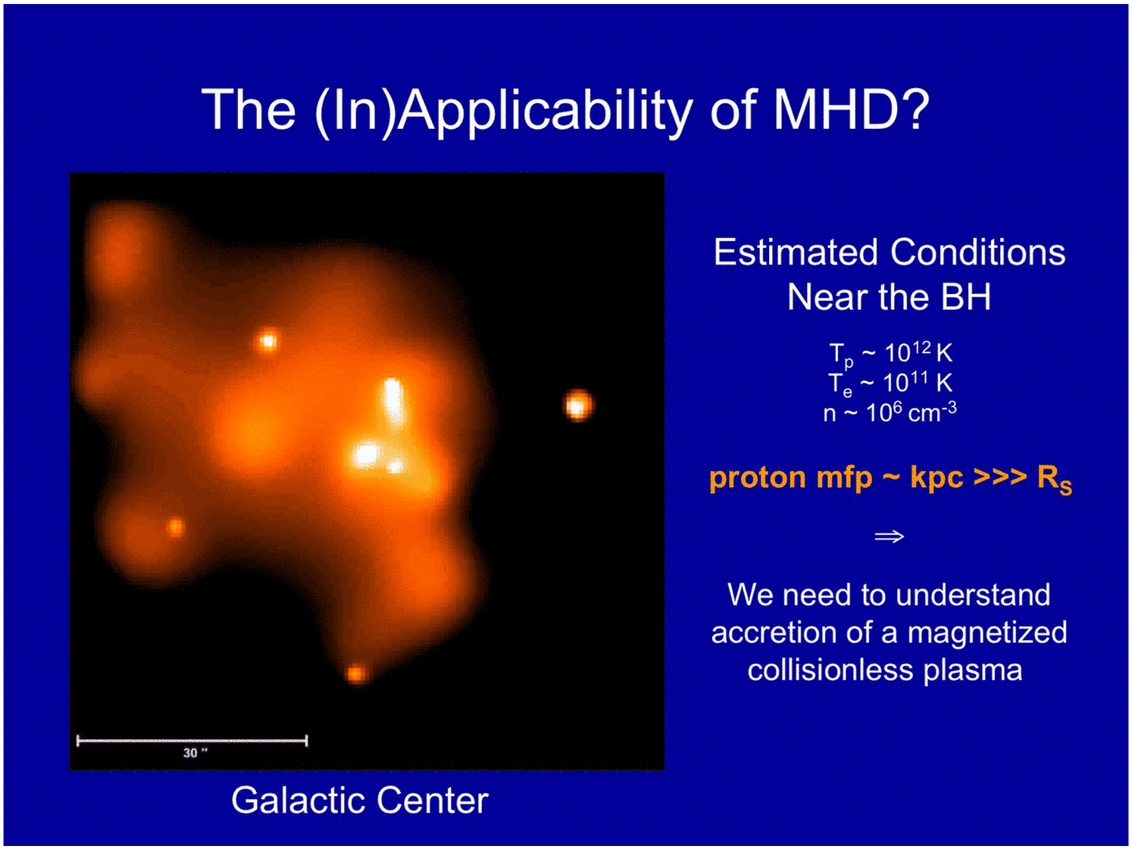
Observed Plasma
(~ 1" ~ R_{Bondi} ~ 0.04 pc ~ 10^5 R_S)

T ~ few keV   n ~ 100 cm^{-3}

mfp ~ 10^{16} cm ~ 0.1 R_{Bondi}

e-p thermalization time ~ **1000 yrs**
>>
inflow time ~ R_{Bondi}/c_s ~ **100 yrs**

electron conduction time ~ **10 yrs**
<<
inflow time ~ R_{Bondi}/c_s ~ **100 yrs**



The (In)Applicability of MHD?

Galactic Center

Estimated Conditions
Near the BH

T_p ~ 10^{12} K
T_e ~ 10^{11} K
n ~ 10^6 cm^{-3}

proton mfp ~ kpc >>> R_S

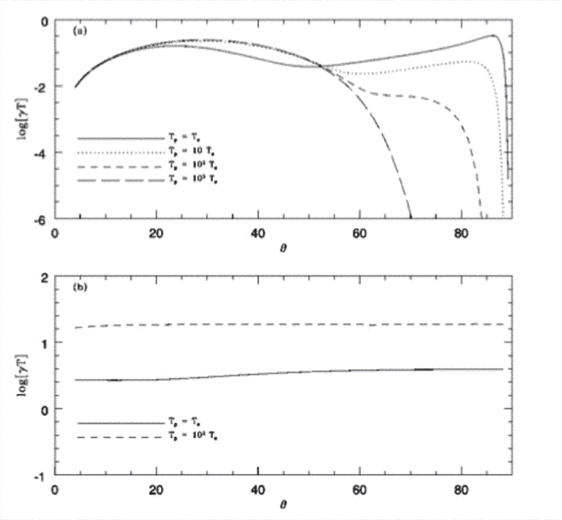
⇒

We need to understand
accretion of a magnetized
collisionless plasma

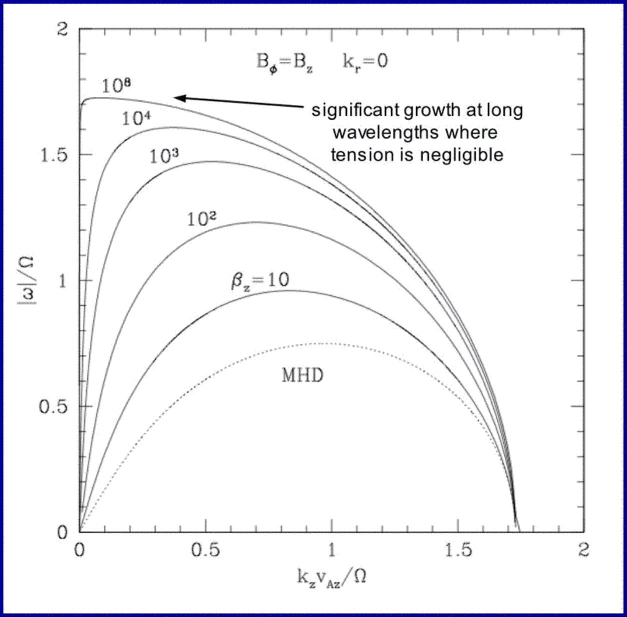
Kinetic Physics

- Collisionless Damping (all k)
  - Alfven wave also damped bec. coupled to slow wave via rotation
- Free Streaming Along Field Lines
  - Heat & Momentum Transport
- Anisotropic Pressure Tensor
- Mean Free Path set by B-field & Wave-Particle Interactions

Collisionless Damping in β ~ 1 Plasma



The MRI in a Collisionless Plasma



Quataert, Dorland, Hammett 2002; also Sharma et al. 2003; Balbus 2004

B-field channels particle transport along field lines

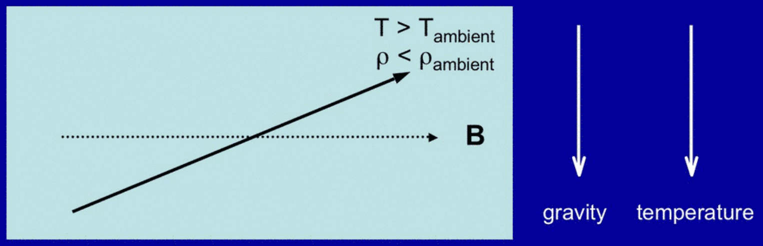
angular momentum transport via anisotropic pressure (viscosity!) in addition to magnetic stresses

$$F_\varphi \propto \left(\frac{B_z B_\varphi}{B^2}\right)\left(\delta p_{\parallel} - \delta p_{\perp}\right)$$

Collisionless Convection

(Balbus 2000; Karin Sandstrom & EQ 200N)

- Convection (Buoyancy) May be Dynamically Impt in Hot, Thick Disks
- Schwarzschild Criterion for Instability in Hydro & MHD (β >> 1): **$ds/dr < 0$**



$t_{conduction} < t_{buoyancy} \Rightarrow \nabla_{\parallel} T \approx 0$ (temp constant along field lines)

"Magneto-Thermal Instability" (MTI) if **$dT/dr < 0$**

$$\gamma^2 \approx -g \frac{d \ln T}{dr}$$

Nonlinear Evolution Simulated Using Kinetic-MHD

- Large-scale Dynamics of collisionless plasmas: expand Vlasov equation retaining "slow timescale" & "large lengthscale" assumptions of MHD (e.g., Kulsrud 1983)
- Particles efficiently transport heat and momentum along field-lines

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,$$
$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho (\mathbf{V} \cdot \nabla) \mathbf{V} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \nabla \cdot \mathbf{P} + \mathbf{F}_{\mathbf{g}},$$
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}),$$
$$\mathbf{P} = p_{\perp} \mathbf{I} + (p_{\parallel} - p_{\perp}) \hat{\mathbf{b}}\hat{\mathbf{b}},$$

Evolution of the Pressure Tensor

$$\frac{\partial f}{\partial t} + \left(v_\parallel \hat{\mathbf{b}} + \mathbf{v}_E\right) \cdot \nabla f + \left(-\hat{\mathbf{b}} \cdot \frac{D\mathbf{v}_E}{Dt} - \mu \hat{\mathbf{b}} \cdot \nabla B + \frac{e}{m}(E_\parallel + F_{g\parallel}/e)\right) \frac{\partial f}{\partial v_\parallel} = C(f),$$

$$\rho B \frac{d}{dt}\left(\frac{p_\perp}{\rho B}\right) = -\nabla \cdot (\hat{\mathbf{b}} q_\perp) - q_\perp \nabla \cdot \hat{\mathbf{b}}$$

$$\frac{\rho^3}{B^2} \frac{d}{dt}\left(\frac{p_\parallel B^2}{\rho^3}\right) = -\nabla \cdot (\hat{\mathbf{b}} q_\parallel) + 2 q_\perp \nabla \cdot \hat{\mathbf{b}},$$

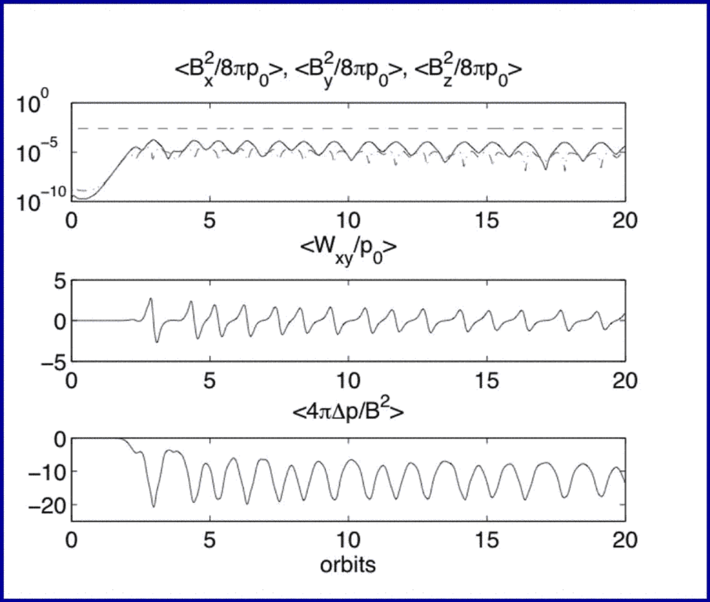
adiabatic invariance of $\mu \sim mv^2_\perp/B \sim T_\perp/B$

$q = 0$ CGL or Double Adiabatic Theory

$$q \approx \frac{n v_{th}}{|k_\parallel|} \nabla_\parallel T$$

Closure Models for Heat Flux (temp gradients wiped out on ~ a crossing time)

Local Sims of the Kinetic MRI



Saturates at Linear Amplitude!

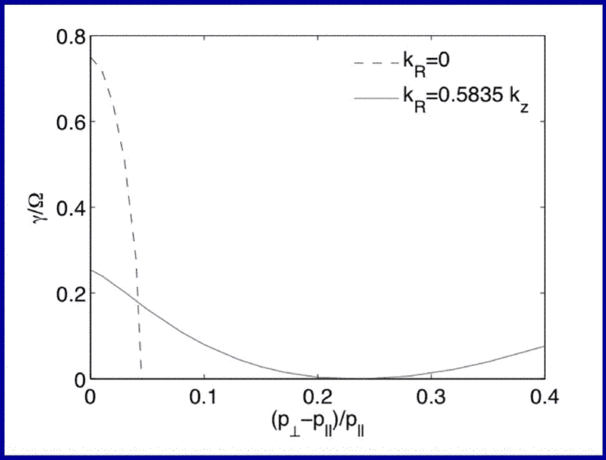
$\delta B/B \sim 1/\beta$

Box filled with stable anisotropic Alfven Waves

Pressure Anisotropy

- $\mu \propto T_\perp / B = \text{constant} \;\Rightarrow\; T_\perp > T_\parallel$ as B ↑

MRI Growth Rate vs. Pressure Anisotropy ($\beta \sim 100$)



Uniform Anisotropic Plasma

$$\omega^2 = k_\parallel^2\left[v_A^2 + \left(\frac{p_\perp - p_\parallel}{\rho}\right)\right]$$

$\Delta p \sim B^2$ Dynamically Important

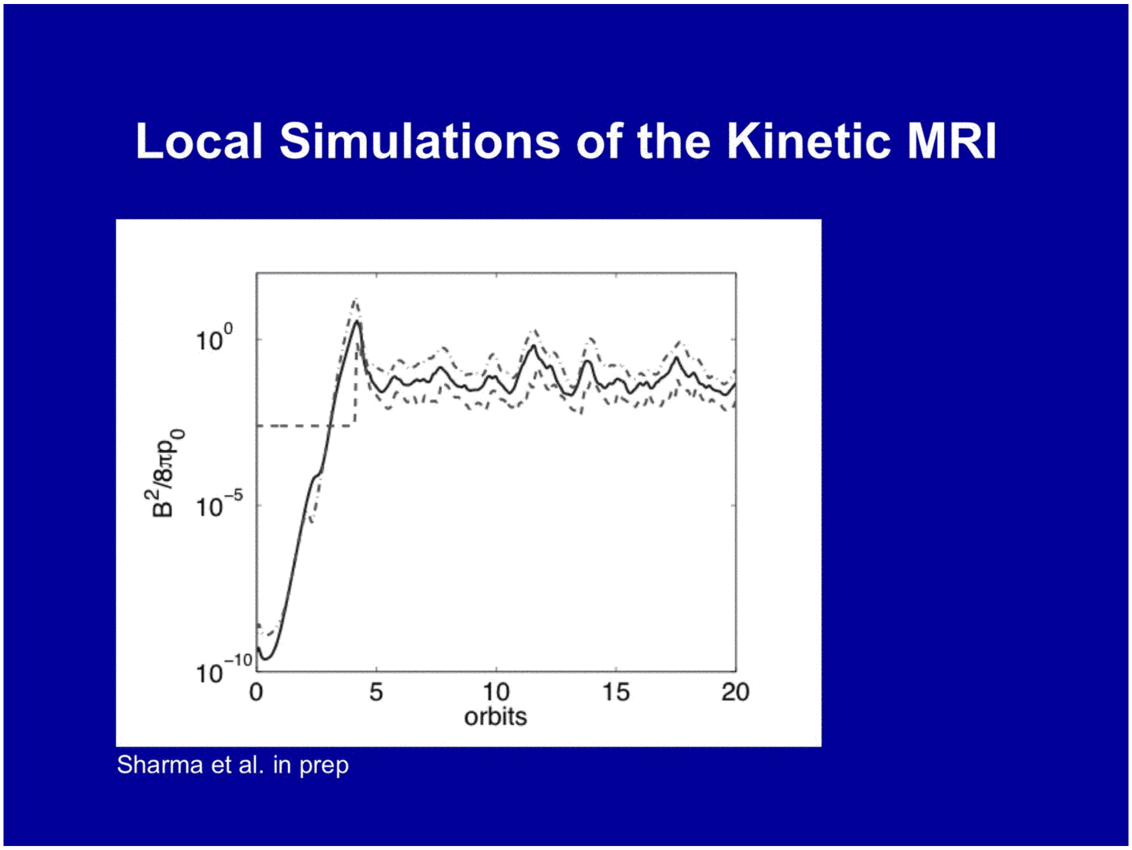
($\Delta p/p \sim \beta^{-1} << 1$)

Pressure Anisotropy Unstable to Kinetic Microinstabilities

- $p_\perp \neq p_\parallel$ unstable to small-scale (~ Larmor radius) modes that act to isotropize the pressure tensor (velocity space anisotropy)
  - mirror, firehose, ion cyclotron, whistler instabilities
- waves w/ frequencies ~ Ω_{cyc} violate μ invariance & pitch-angle scatter
  - provide effective collisions & set effective mean free path in collisionless plasma
  - $|\Delta p| > \# B^2$ required for growth rate ~ disk period
- Use “subgrid” scattering model in disk simulations

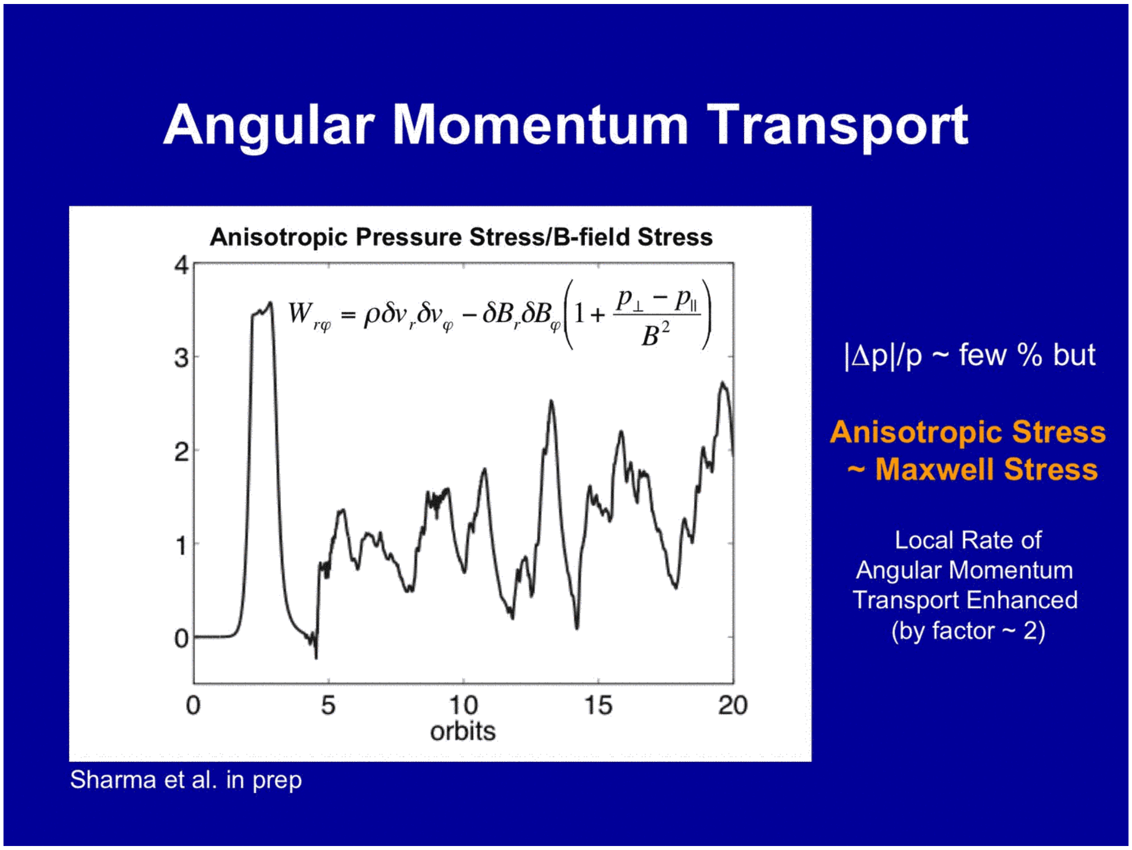
$$\frac{\partial p_\perp}{\partial t} = \ldots - \nu(p_\perp, p_\parallel, \beta)\left[p_\perp - p_\parallel\right]$$

$$\frac{\partial p_\parallel}{\partial t} = \ldots - \nu(p_\perp, p_\parallel, \beta)\left[p_\parallel - p_\perp\right]$$



Local Simulations of the Kinetic MRI

Sharma et al. in prep



Angular Momentum Transport

Anisotropic Pressure Stress/B-field Stress

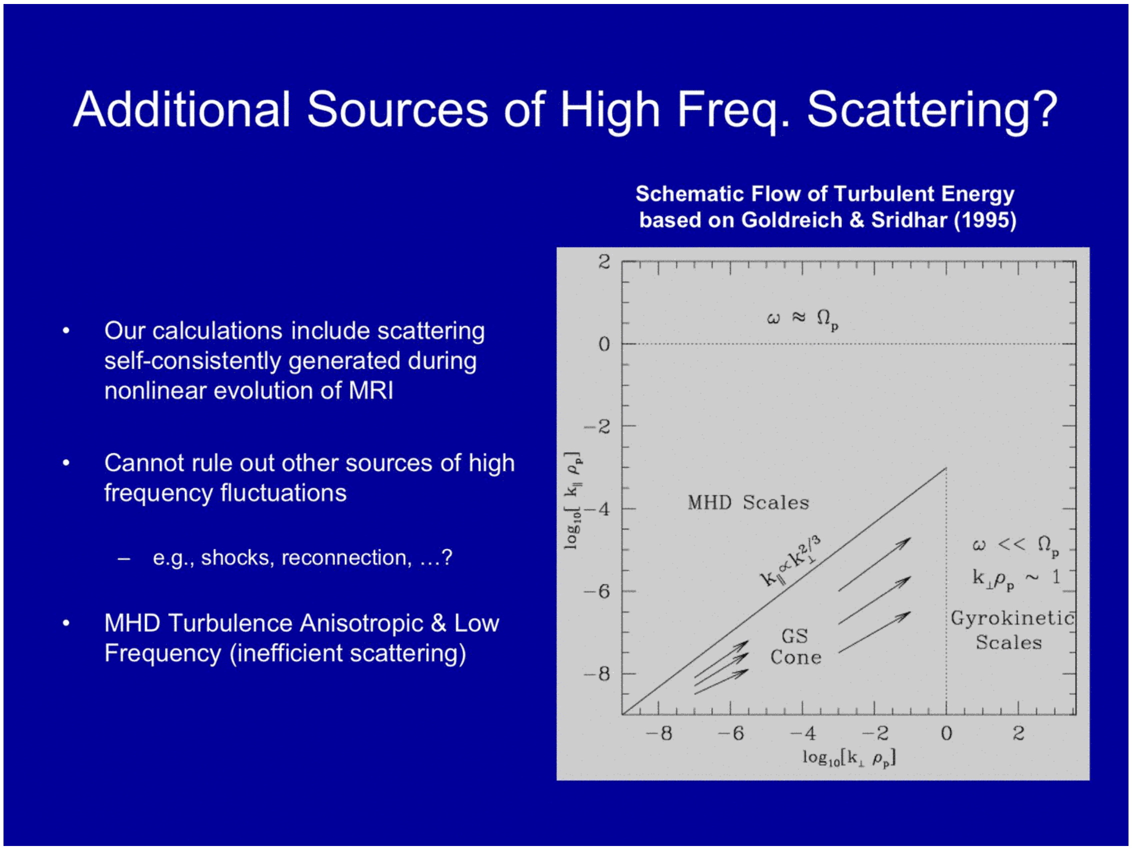
$$W_{r\varphi} = \rho\delta v_r \delta v_\varphi - \delta B_r \delta B_\varphi \left(1 + \frac{p_\perp - p_\parallel}{B^2}\right)$$

Sharma et al. in prep

|Δp|/p ~ few % but

Anisotropic Stress ~ Maxwell Stress

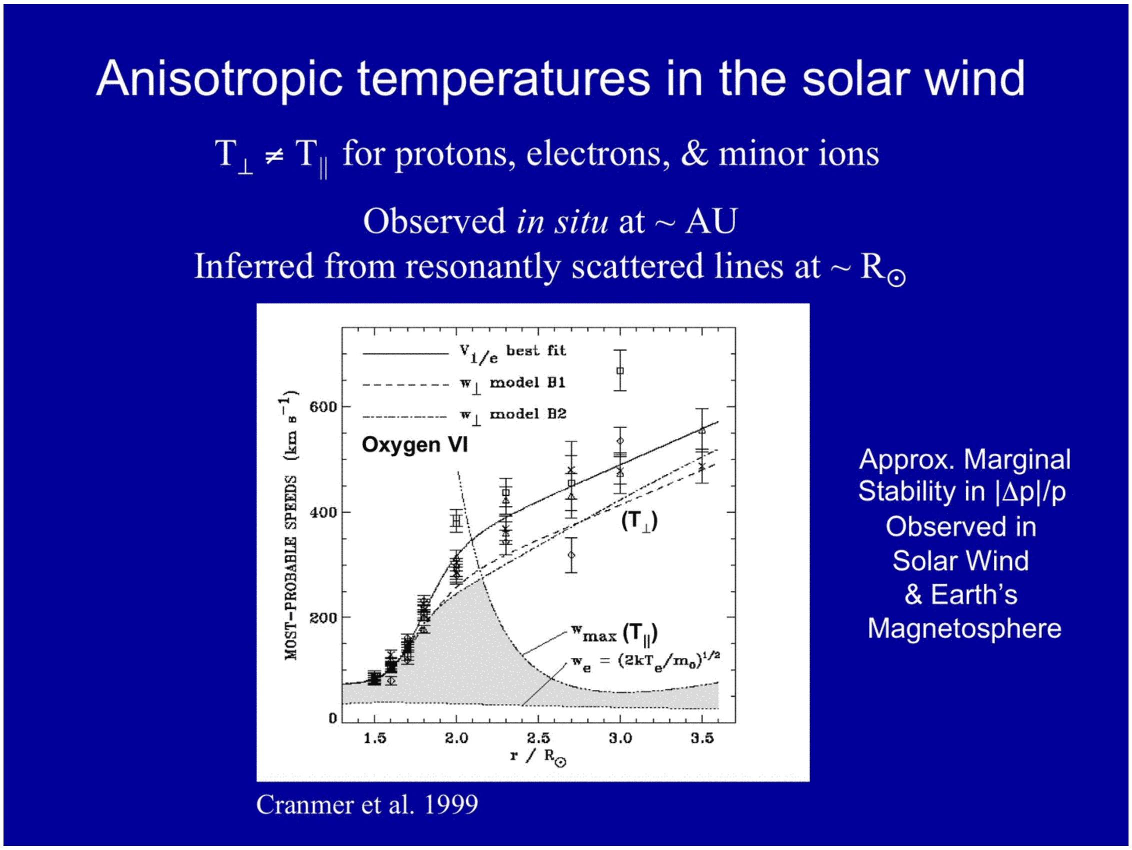
Local Rate of
Angular Momentum
Transport Enhanced
(by factor ~ 2)



Additional Sources of High Freq. Scattering?

- Our calculations include scattering self-consistently generated during nonlinear evolution of MRI
- Cannot rule out other sources of high frequency fluctuations
  - e.g., shocks, reconnection, …?
- MHD Turbulence Anisotropic & Low Frequency (inefficient scattering)

Schematic Flow of Turbulent Energy based on Goldreich & Sridhar (1995)



Anisotropic temperatures in the solar wind

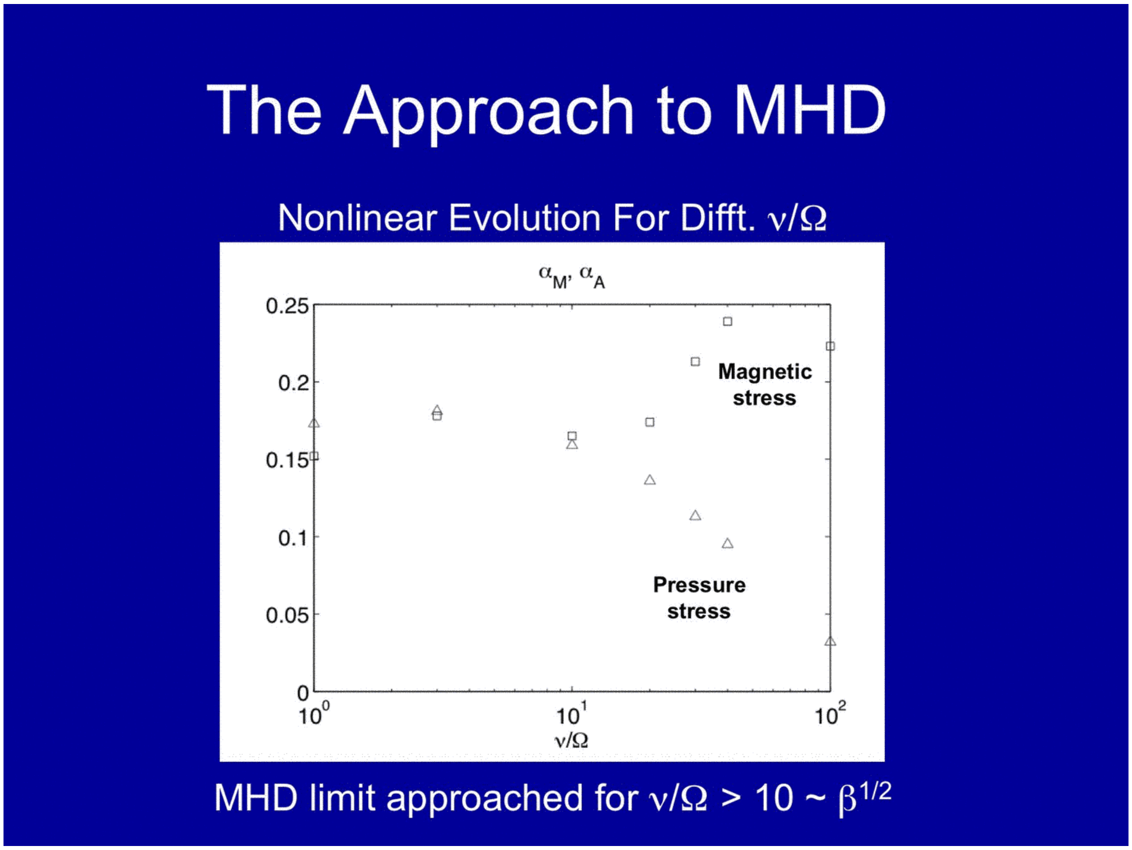
$T_\perp \neq T_\parallel$ for protons, electrons, & minor ions

Observed *in situ* at ~ AU

Inferred from resonantly scattered lines at ~ $R_\odot$

Cranmer et al. 1999

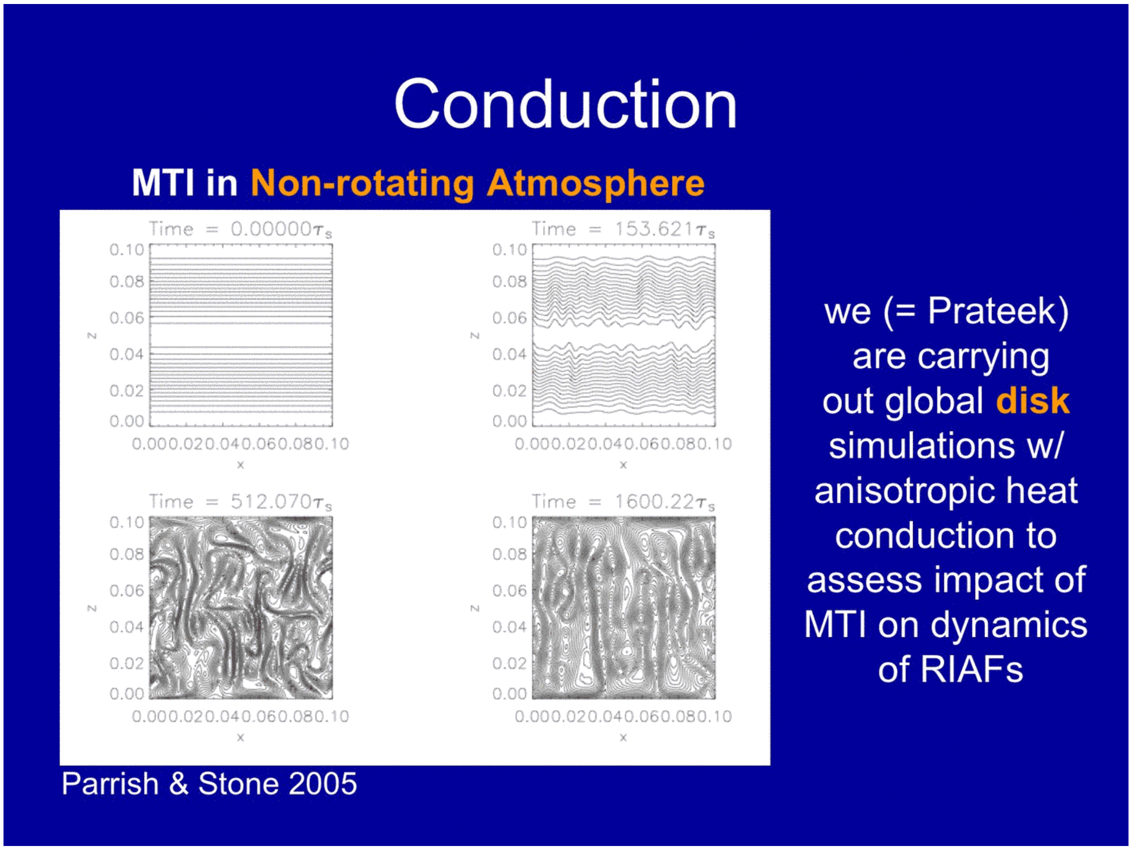
Approx. Marginal Stability in |Δp|/p Observed in Solar Wind & Earth's Magnetosphere



The Approach to MHD

Nonlinear Evolution For Difft. ν/Ω

MHD limit approached for $\nu/\Omega > 10 \sim \beta^{1/2}$



Conduction

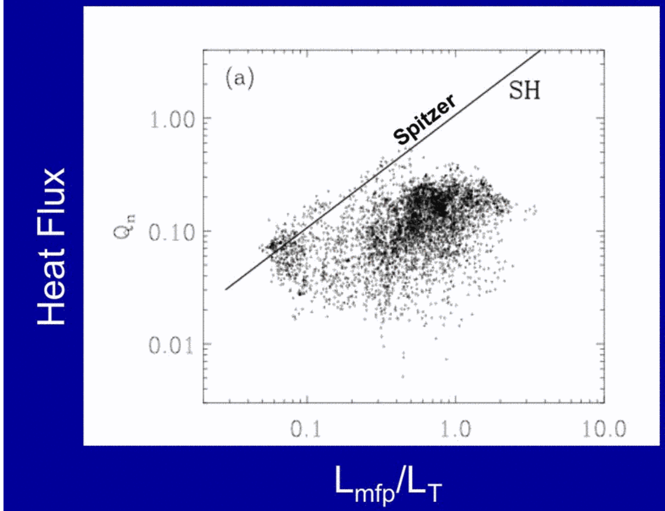
MTI in Non-rotating Atmosphere

Parrish & Stone 2005

we (= Prateek) are carrying out global **disk** simulations w/ anisotropic heat conduction to assess impact of MTI on dynamics of RIAFs

Electron Heat Flux in the Solar Wind

In Situ Measurements at ~ 1 AU (Wind)



Salem et al. 2003

At ~ 1 AU, solar wind has $\beta \sim 1$ & is fully MHD turbulent

Electron Heat Flux is ~ 0.1-1 Spitzer

At **$L_{mfp}/L_T \sim 1$** measured heat flux is **$Q \sim 0.3\ n\ m_e\ v_e^3$** (nearly identical to saturated flux from Cowie & McKee1977)

Effects of Heat Conduction

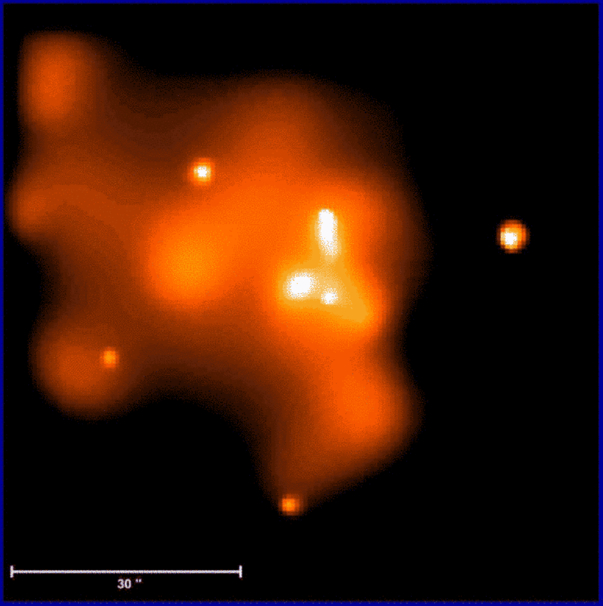
$$\underbrace{nT_e v_r \frac{ds_e}{dr}}_{\sim\, nm_e v_r v_e^2/r} = q_+ - \underbrace{\nabla \bullet Q}_{\sim\, nm_e v_e^3/r} - q_{rad}$$

Outward heat flux dominates inward advection by ~ v_e/v_r ~ 100 (for ions, the terms are comparable so harder to assess)

Conjecture ($t_{cond} < t_{cool}$): $4\pi R^2 Q_r \sim \text{constant} \sim \varepsilon \dot{M} c^2$

Implications for global disk dynamics, electron temperature, etc. being explored (may depend on T_e/T_i & v_e/c and thus on accretion rate, radius, …)

Electron Heat Flux



Observed Plasma

(~ 1" ~ R_{Bondi} ~ 0.04 pc ~ 10^5 R_S)

T ~ 2 keV   n ~ 100 cm^{-3}

mfp ~ 10^{16} cm ~ 0.1 R_{Bondi}

$$L_c \approx 10^{38} R_{Bondi} \left(\frac{T_e}{2\ \mathrm{keV}}\right)^{7/2} \mathrm{ergs/s}$$

$$\approx 100\ L_{rad} \approx 10^{-9}\ M_{\odot} yr^{-1} c^2 \sim 0.1 \dot{M} c^2$$

L_c ~ const $\Rightarrow T_e \propto r^{-2/7}$

(consistent w/ factor ~ 2
dec. in T_e from ~ 0.1-1 pc)

Electron Heat Flux is a Significant Energy Sink For the Accretion Flow

Summary

- RIAFs: Accretion via a Collisionless Plasma (L_{mfp}/R_s ~ 10^9 for Sgr A*)
  - Relevant to low-luminosity XRBs & AGN (most accreting compact objects, most of the time)
- Instabilities that Determine Accretion Dynamics Qualitatively Different in Collisionless Plasmas (MRI, Convection)
- Local Simulations of Kinetic MRI Similar to MHD with Enhanced Transport due to Anisotropic Pressure Stresses ($p_\perp \neq p_{||}$)
- Conductive Heat Flux May be a Dynamically Impt. Energy Sink