Physics of Astrophysical Outflows and Accretion Disks (Santa Barbara) Jun 2 2005

Local Behavior of the Magnetorotational Instability (MRI) in Accretion Disks

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Motivation

- Angular Momentum Transport by MRI in Accretion Disks
  - Protoplanetary Disks
  - Planet Formation Theory
- Questions:

  " How does the magnetic dissipation affect the MRI? "

  " What determines the saturation level of the stress? "

OUTLINE

- Effect of the Ohmic Dissipation
  - Linear Analysis
  - Application to Protoplanetary disks
  - Nonlinear Simulations
- Saturation Mechanism of the MRI
  - Dependence of Saturation Level on Physical Quantities

Linear Analysis

- Ideal MHD

$$\omega_{\max} = \frac{3}{4}\Omega$$

$$\lambda = \frac{v_A}{\Omega}$$

- Ohmic Dissipation

"Magnetic Reynolds Number"

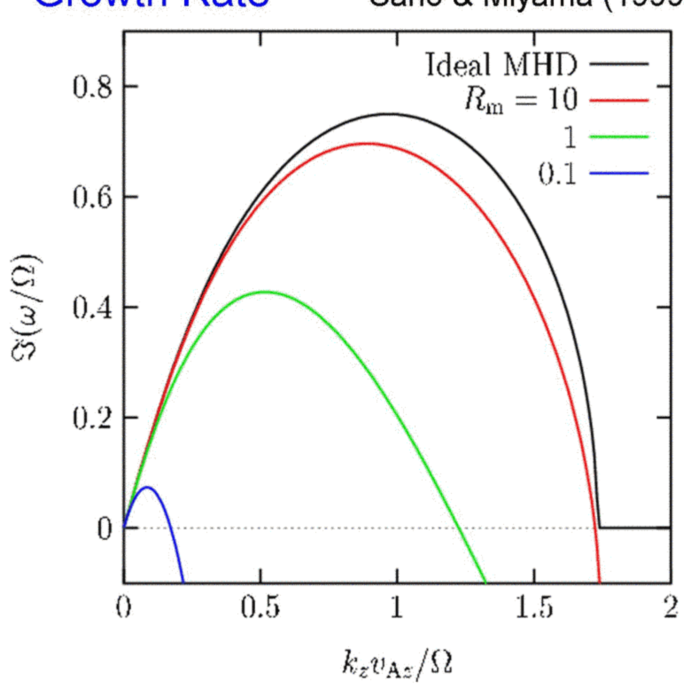
$$Re_M = \frac{VL}{\eta} = \frac{v_A^2}{\Omega \eta}$$

$$V = v_A \quad L = \frac{v_A}{\Omega}$$

Jin (1996)

Growth Rate

Sano & Miyama (1999)



Wave Number

Resistive Regime

Magnetic Reynolds Number << 1

- Most Unstable Wavelength

$$\lambda \approx \frac{v_A}{\Omega} \longrightarrow \lambda \approx \frac{\eta}{v_A}$$

Ideal MHD

$$(kv_A)^{-1} \sim (k^2\eta)^{-1}$$

(Alfven Time) = (Dissipation Time)

- Maximum Growth Rate

$$\omega_{\max} \approx \Omega \longrightarrow \omega_{\max} \approx \frac{v_A^2}{\eta}$$

Ideal MHD

$$\tau_{dis}^{-1} \sim \eta/\lambda^2 \sim v_A^2/\eta$$

- Magnetic Reynolds Number

$$\frac{v_A^2}{\eta\Omega} \approx 1 \qquad \tau_{dis} \sim \Omega^{-1}$$

Diffusivity

$$\eta = \frac{c^2}{4\pi\sigma_c}$$

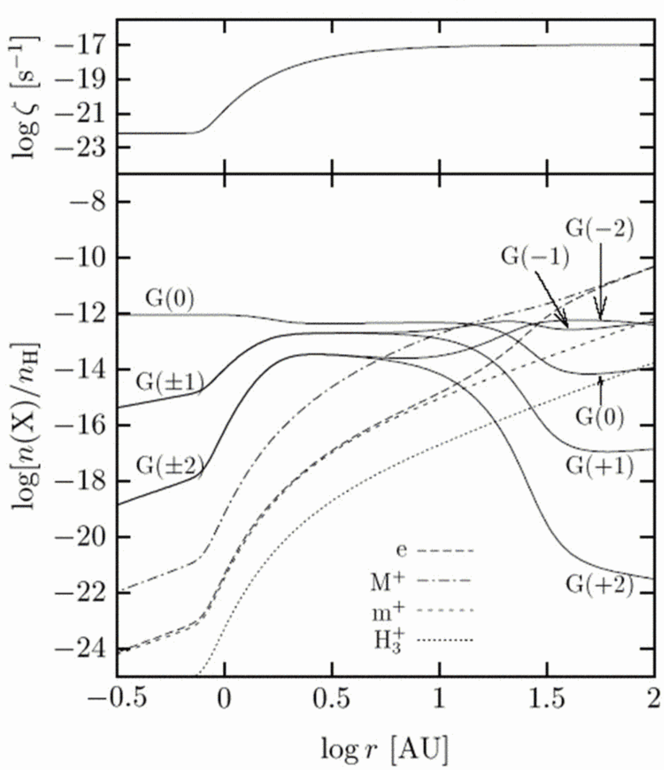
- Conductivity

$$\sigma_c = \sum_\nu \frac{(eq_\nu)^2 \tau_\nu n_\nu}{m_\nu}$$

- Number Density of Charged Particles
- Recombination on Grain Surface

Sano et al. (2000)

Abundances of Charged Particle

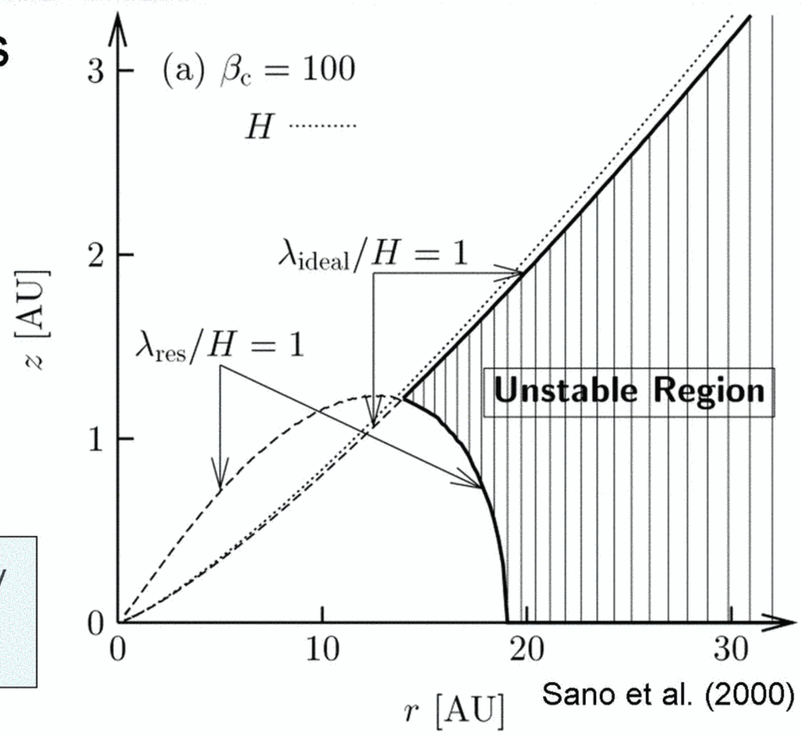


Unstable Region

- Outer Part of the Disk
- Critical Radius
  ▯ 10AU

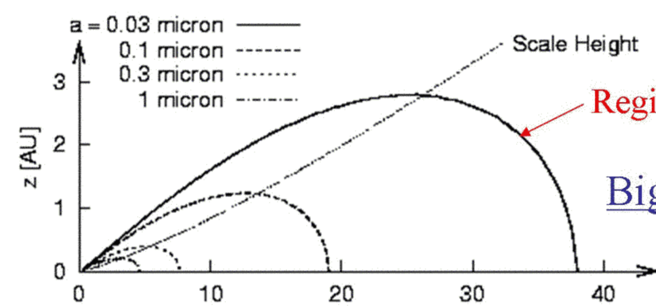
- Inner Part
- Dead Zone

Condition for Instability
$\lambda_{MRI} < H$



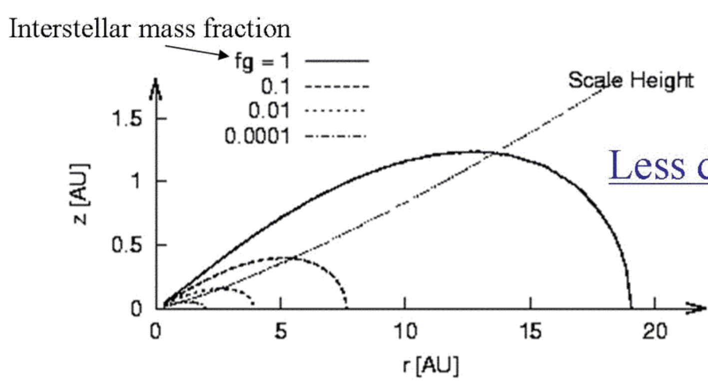
Sano et al. (2000)

An important caveat: Dust can have a large effect on the ionization fraction (Wardle & Ng 1999; Sano et al. 2000)



Region inside which MRI is suppressed

Big grains have less effect



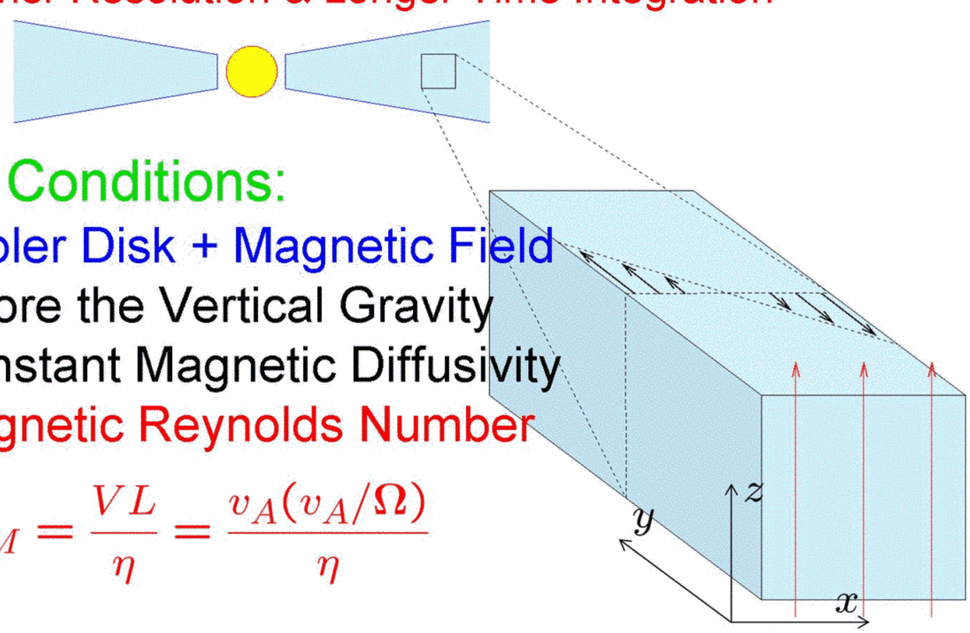
Less dust (settling) reduces effect

MRI in Protoplanetary Disks

- Ohmic dissipation is quite important.
- Inner Dead Zone + Outer Active Region
- Mass Accretion in Dead Zone
  - Gravitational Instability?
  - Layered Accretion?
- Characteristics of dust grains affect the size of magnetorotationally unstable region.
  - MHD evolution ←→ Dust Evolution

Nonlinear Evolution of MRI

- Local Shearing-Box Calculations
  - Advantage:
    Higher Resolution & Longer Time Integration



- Initial Conditions:
  - Kepler Disk + Magnetic Field
  - Ignore the Vertical Gravity
  - Constant Magnetic Diffusivity
  - Magnetic Reynolds Number

$$Re_M = \frac{VL}{\eta} = \frac{v_A(v_A/\Omega)}{\eta}$$

Basic Equations

$$\frac{\partial \rho}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \rho = -\rho \boldsymbol{\nabla} \cdot \boldsymbol{v}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \boldsymbol{v} = -\frac{\boldsymbol{\nabla} P}{\rho} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{c\rho} - 2\boldsymbol{\Omega} \times \boldsymbol{v} + 2q\Omega^2 x \hat{\boldsymbol{x}}$$

Coriolis Force & Effective Potential

$$\frac{\partial \epsilon}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \epsilon = -\frac{P \boldsymbol{\nabla} \cdot \boldsymbol{v}}{\rho} + \frac{4\pi\eta \boldsymbol{J}^2}{c^2 \rho}$$

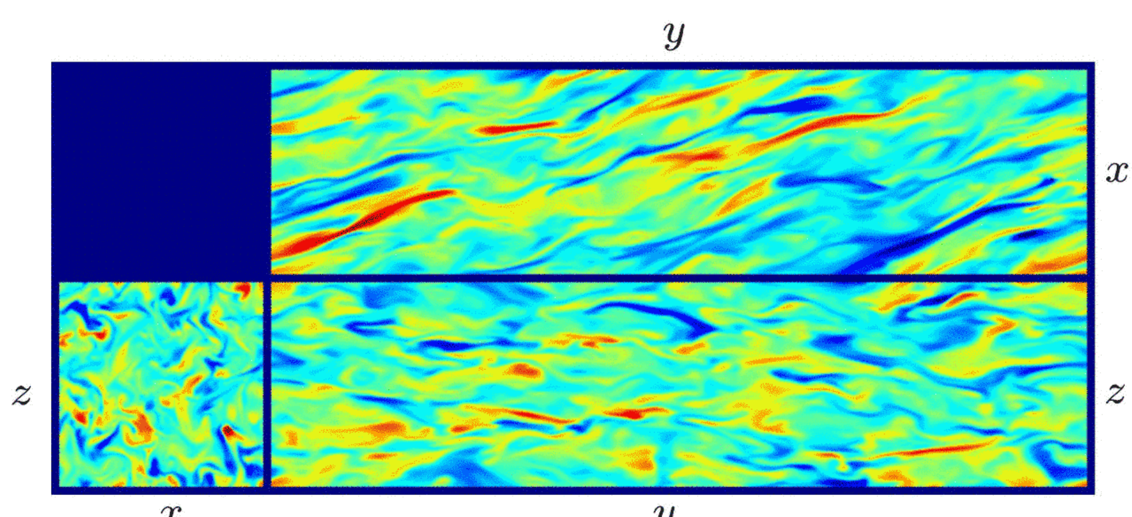
$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \left(\boldsymbol{v} \times \boldsymbol{B} - \frac{4\pi\eta \boldsymbol{J}}{c} \right)$$

Ohmic Dissipation

$$\boldsymbol{J} = \frac{c}{4\pi} (\boldsymbol{\nabla} \times \boldsymbol{B})$$

MHD Turbulence

- Unisotropic Turbulence
- Zero-Net Flux Cases
  → Small Scale Fluctuations



Color: Toroidal Field Strength

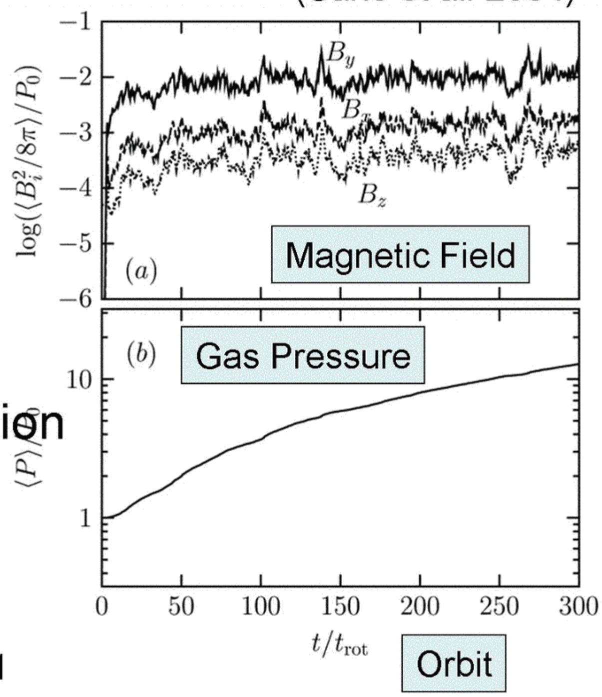
Nonlinear Evolution

- Time Evolution of Magnetic Fields
  - Time-Averaged Maxwell Stress

(Sano et al. 2004)

$$\langle\langle w_M \rangle\rangle = \left\langle\left\langle -\frac{B_R B_\phi}{4\pi} \right\rangle\right\rangle$$

$$\approx 0.5 \left\langle\left\langle \frac{B^2}{8\pi} \right\rangle\right\rangle$$



- Gas Pressure
  - Joule Heating
  - Fluctuation-Dissipation Relation

$$\left\langle\left\langle \frac{\partial \epsilon}{\partial t} \right\rangle\right\rangle \propto \left\langle\left\langle -\frac{B_R B_\phi}{4\pi} \right\rangle\right\rangle$$

Sano & Inutsuka 2001

Effect of Ohmic Dissipation

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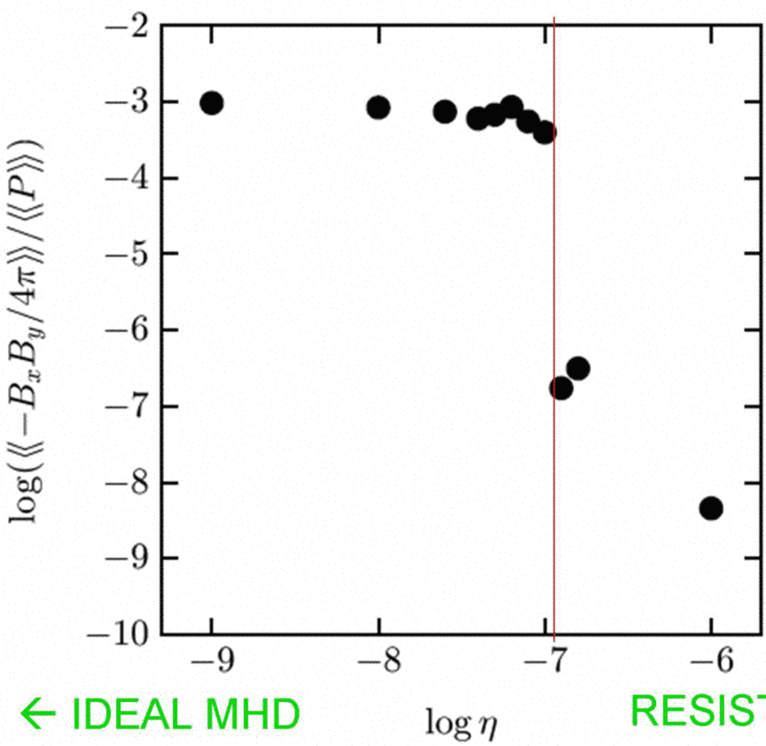
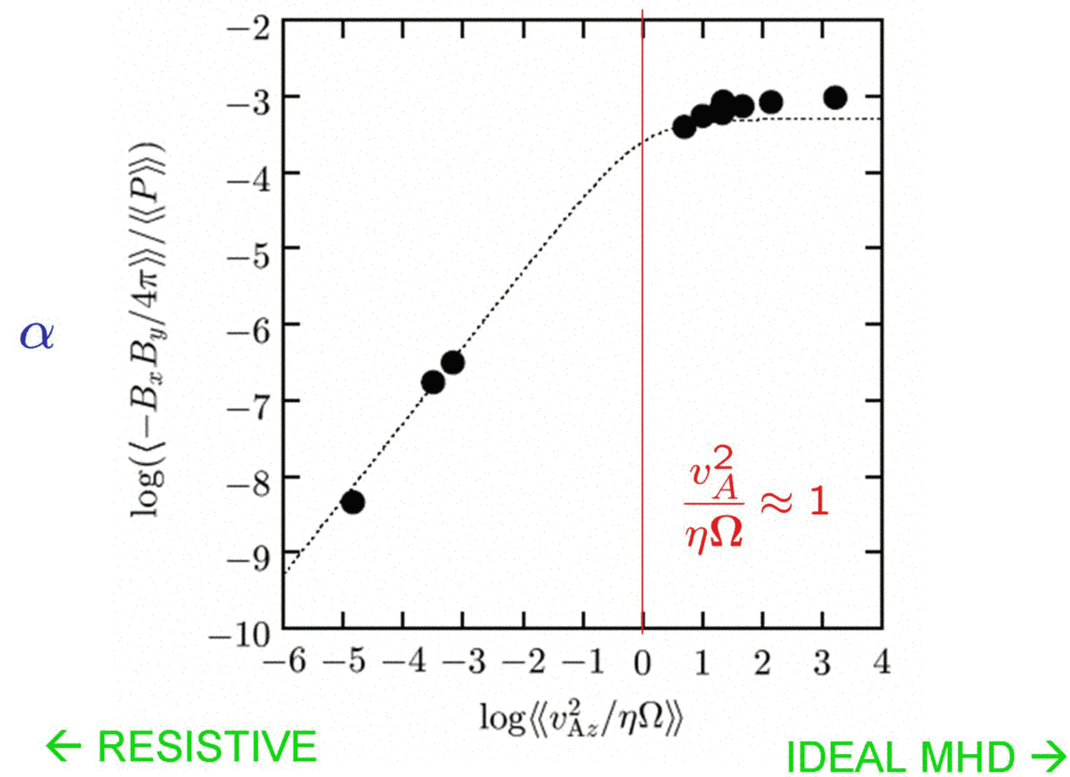


← IDEAL MHD RESISTIVE →

Critical Value

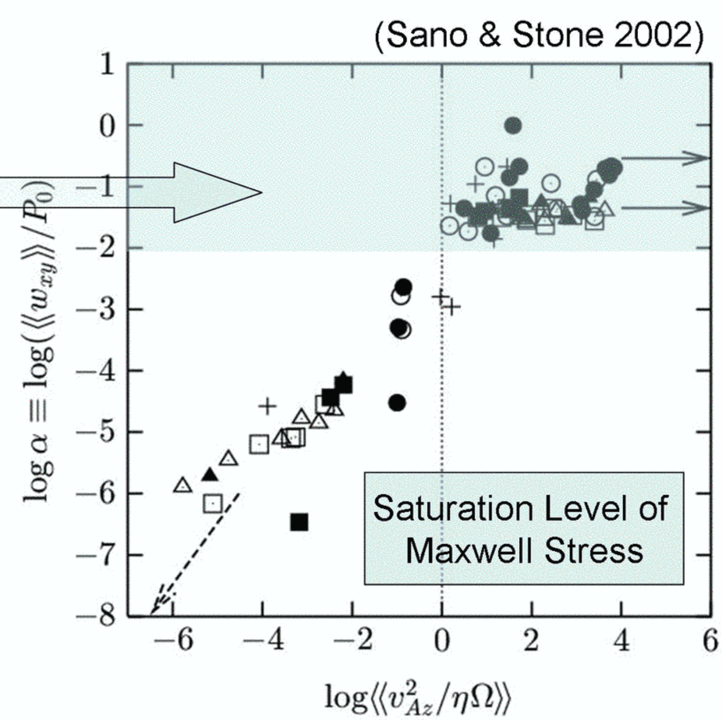


Dependence on Diffusivity

- Accretion Disk Theory

$$\frac{\langle\langle w_M \rangle\rangle}{P} > 0.01$$

- Field Geometry
  - Zero-Net
  - Uniform Bz
  - Uniform By
- Adiabatic or Isothermal
- Hall Term
- Different Scheme

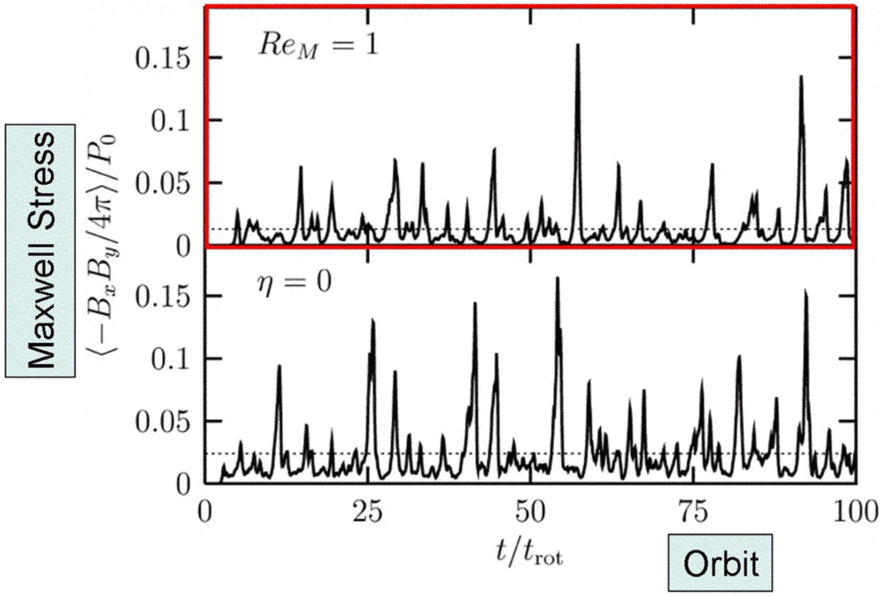


Time Variability

(Sano & Inutsuka 2001)

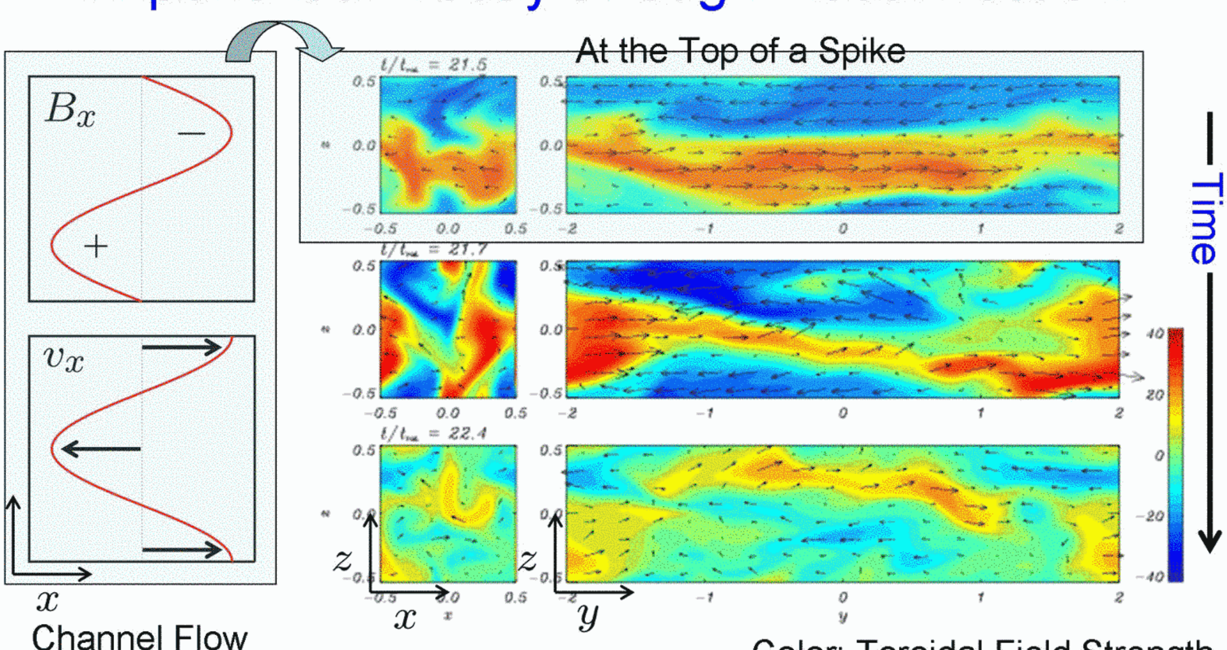
- Maxwell Stress in the Saturated State:
  - Highly Fluctuating
  - Spike-Shaped Excursions

Uniform Bz Cases



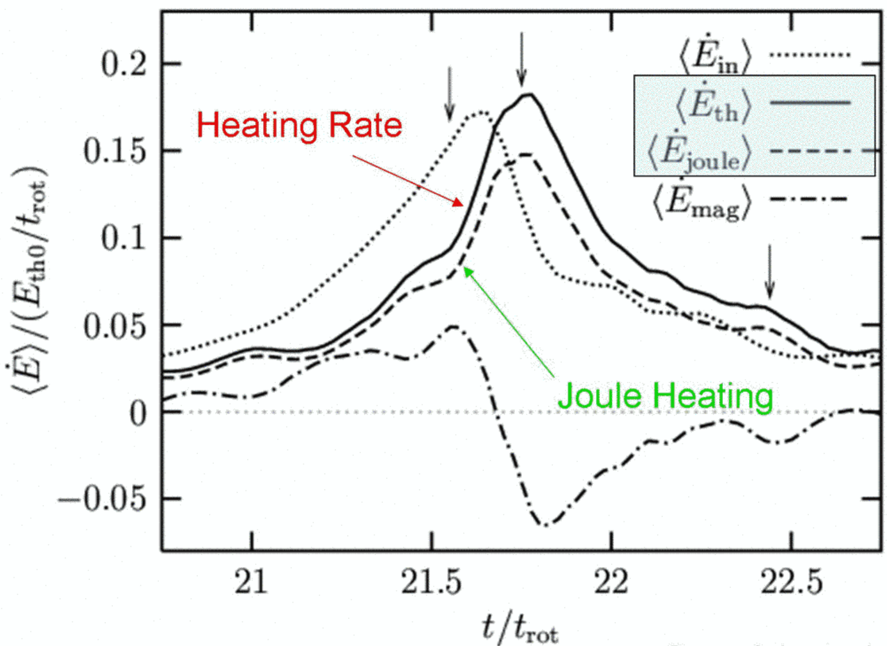
Spike-Shaped Variation

- Exponential Growth of MRI (Channel Flow)
- Exponential Decay through Reconnection



Heating

- Heating Rate = Joule Heating



Sano & Inutsuka (2001)

Characteristic Ratios

- (Stress)=0.5(Magnetic Pressure)

TABLE 4

Characteristic Ratios in MRI Turbulence

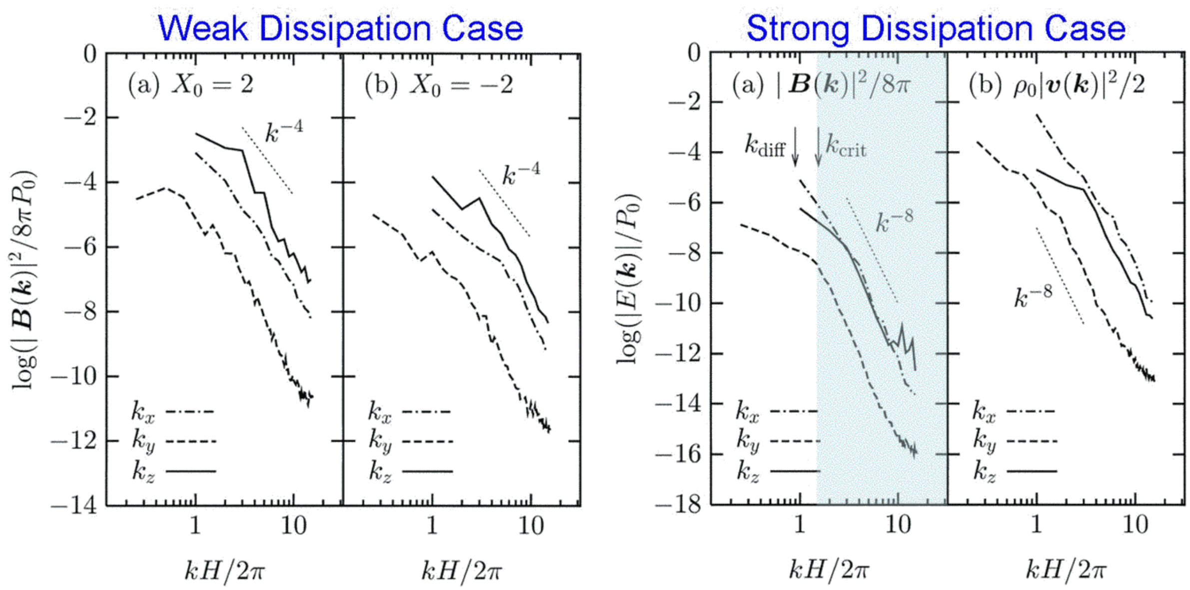
Quantity	Average
$\langle\langle -B_xB_y/4\pi\rangle\rangle/\langle\langle B^2/8\pi\rangle\rangle$	0.467 ± 0.040
$\langle\langle -B_xB_y/4\pi\rangle\rangle/\langle\langle \rho v_x \delta v_y\rangle\rangle$	5.19 ± 0.67
$\langle\langle B_x^2\rangle\rangle/\langle\langle B_z^2\rangle\rangle$	3.35 ± 0.28
$\langle\langle B_y^2\rangle\rangle/\langle\langle B_z^2\rangle\rangle$	23.7 ± 4.0
$\langle\langle v_x^2\rangle\rangle/\langle\langle v_z^2\rangle\rangle$	2.62 ± 0.48
$\langle\langle \delta v_y^2\rangle\rangle/\langle\langle v_z^2\rangle\rangle$	2.15 ± 0.34
$\langle\langle \delta E_{kin}\rangle\rangle/\langle\langle E_{mag}\rangle\rangle$	0.326 ± 0.036

Sano et al. (2004)

Power Spectrum

- Unisotropic; Smaller k has larger energy.
- Power Index: Mostly $\propto k^{-4}$

  Dissipation Region $\propto k^{-8}$



Summary

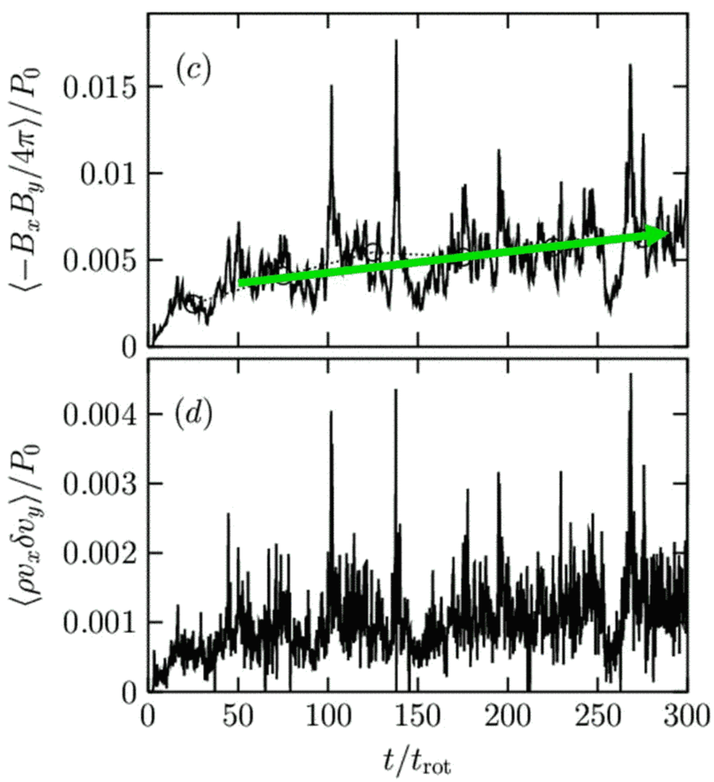
- Ohmic dissipation suppresses the MRI at the inner part of protoplanetary disks.
- Typical dead zone size is about a few tens of AU.
- The criterion for the instability is given by magnetic Reynolds number. $\frac{v_A^2}{\eta\Omega} > 1$
  - This condition includes the saturated field strength.
  - For accretion disk modeling, we should understand the relation between the sound speed and the saturated Alfven speed.

Saturation Level

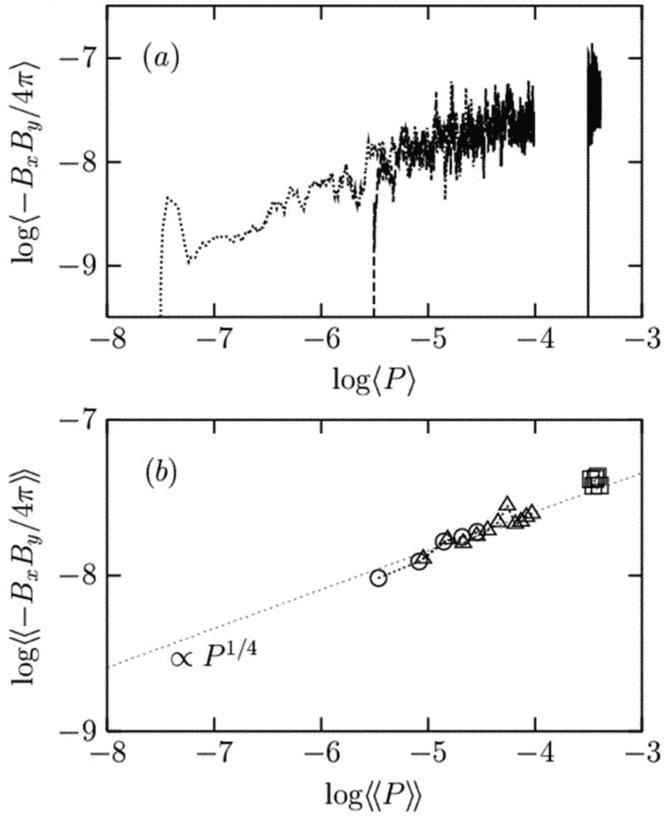
- Numerical Experiment to Understand the Saturation Mechanism
- Dependence on Physical Quantities:
  - Gas Pressure
  - Field Strength and Geometry
  - Box Size
  - Resolution

- Zero-Net Flux Cases
- Uniform Vertical Field Cases

Long Time Evolution of Stress



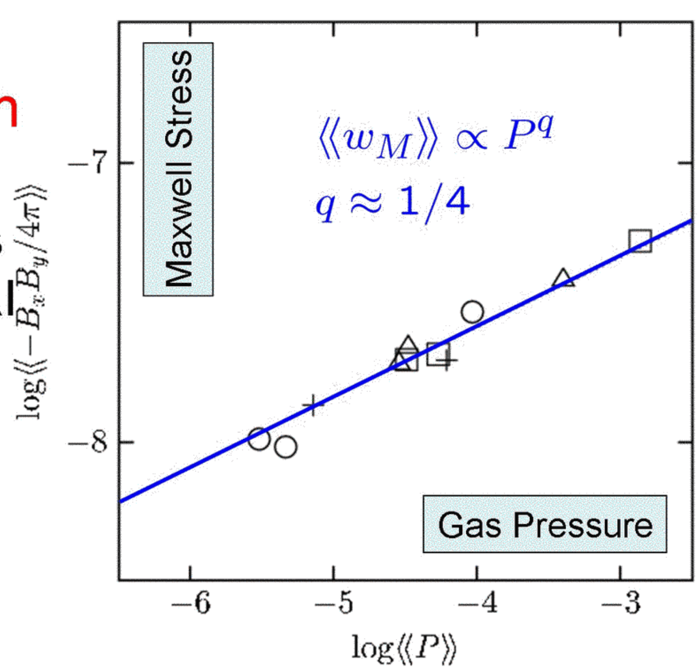
Stress-Pressure Relation



Gas Pressure Dependence

- Weak Power Law Relation
- Independent of Initial Field Strength
- Origin?
  - Gas pressure does not change the MRI growth rate.
  - Dissipation Process?

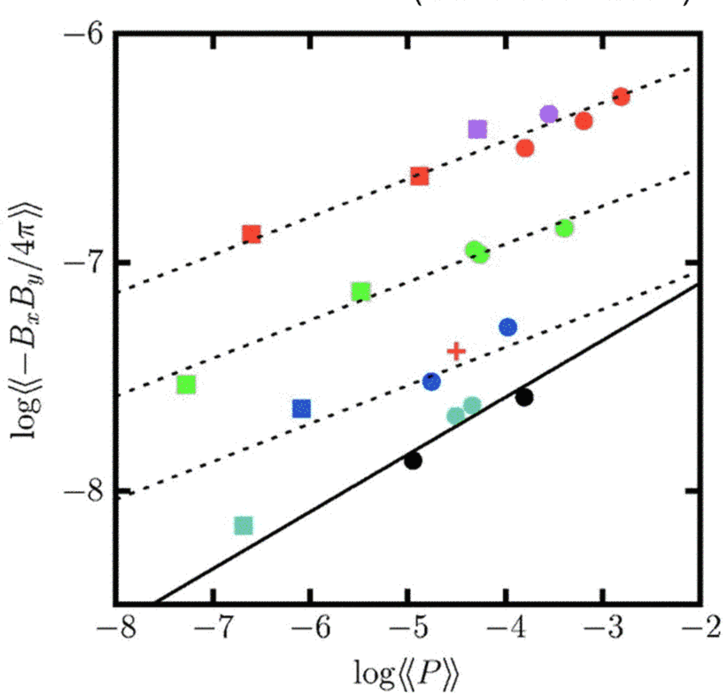
(Sano et al. 2004)



Uniform Bz Cases

- Saturation level is larger compared with zero-net flux models.
- Proportional to Bz
- Lower and Upper Limit
- No Correlation between Pgas and Pmag

(Sano et al. 2004)



Summary 2

- Saturation level of the Maxwell stress has a weak power-law relation with the gas pressure.
  - The origin may be in a dissipation process?
- Existence of net vertical flux enhances the saturation level.
  - Evolution of the net vertical flux will be essential. ← Global Model