# Numerical Simulations Of Relativistic Jets In Gamma-Ray Bursts

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# Outline

- Special Relativistic Numerical Hydrodynamics
- Gamma-Ray Bursts
- Collapsar Model
- Jet Formation & Accretion Disk
- Jet Breakout
- Transition of Relativistic to Newtonian

# Equations of SRHD

$$\frac{\partial \mathbf{U}}{\partial t} + \sum_{j=1}^{3} \frac{\partial \mathbf{F}^j}{\partial x^j} = 0,$$

where the conserved variable $\mathbf{U}$ is given by

$$\mathbf{U} = (D, S^1, S^2, S^3, \tau)^T,$$

and the fluxes are given by

$$\mathbf{F}^j = (Dv^j, S^1 v^j + p\delta^{1j}, S^2 v^j + p\delta^{2j}, S^3 v^j + p\delta^{3j}, S^j - Dv^j)^T$$

$$\begin{aligned} D &= \rho W \\ S^j &= \rho h W^2 v^j \\ \tau &= \rho h W^2 - p - \rho W, \end{aligned}$$

# Semidiscretization

$$\frac{d\mathbf{U}_{i,j,k}}{dt} = -\frac{\mathbf{F}^x_{i+1/2,j,k} - \mathbf{F}^x_{i-1/2,j,k}}{\Delta x} - \ldots$$

PDE => ODE

ODE Solver : Runge-Kutta

# Flux

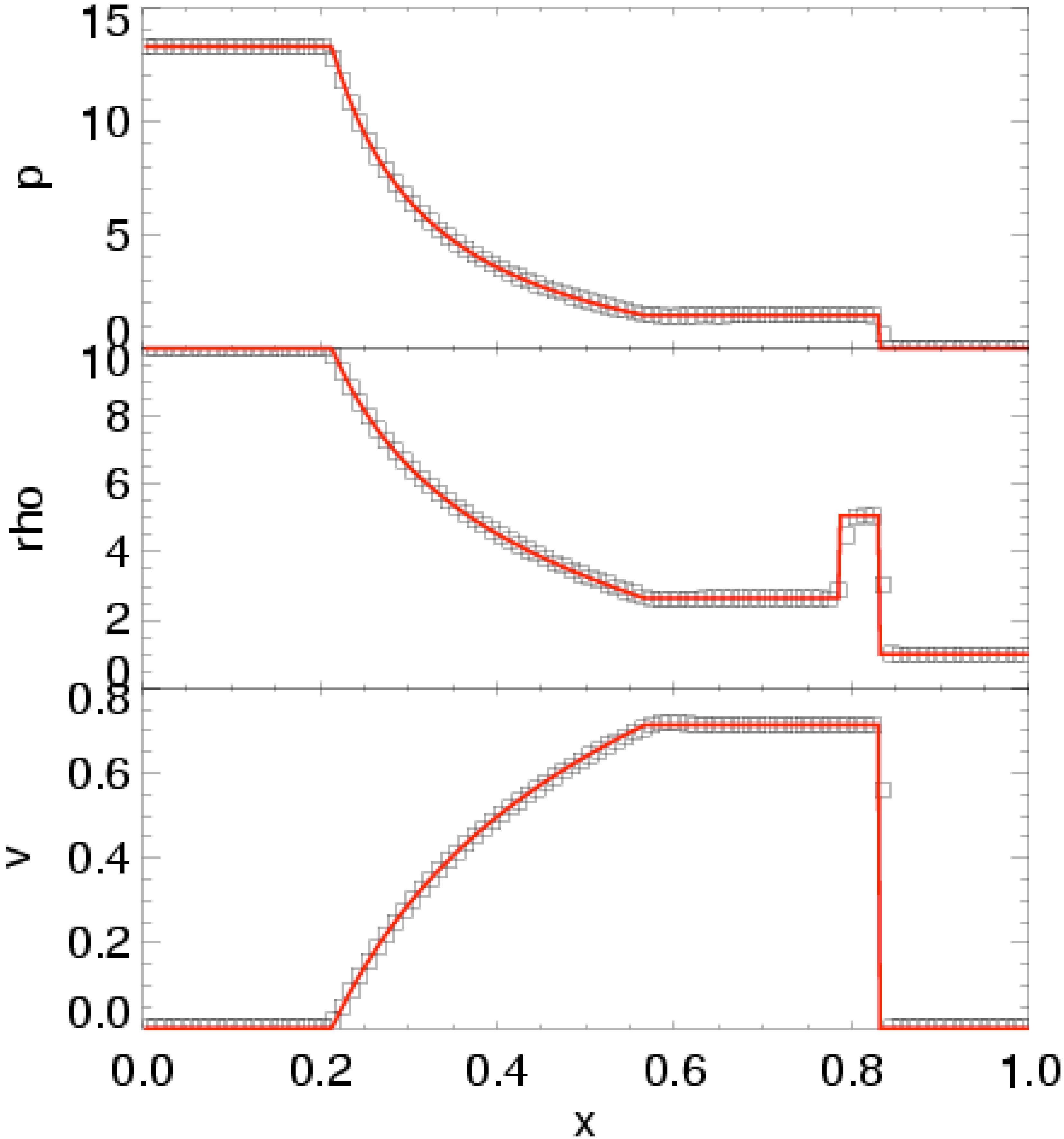
## Scheme 1: Reconstruction on U and then Riemann Solver

Given two neighbor cells, compute the flux at the interface. Riemann was the first to consider this. But he did it wrong! His solution had entropy problems)

## Scheme 2: Reconstruction on F

Spectral Decomposition Based on eigenvectors of Jacobian Matrix

$$\frac{\partial \mathbf{F}}{\partial \mathbf{U}}$$

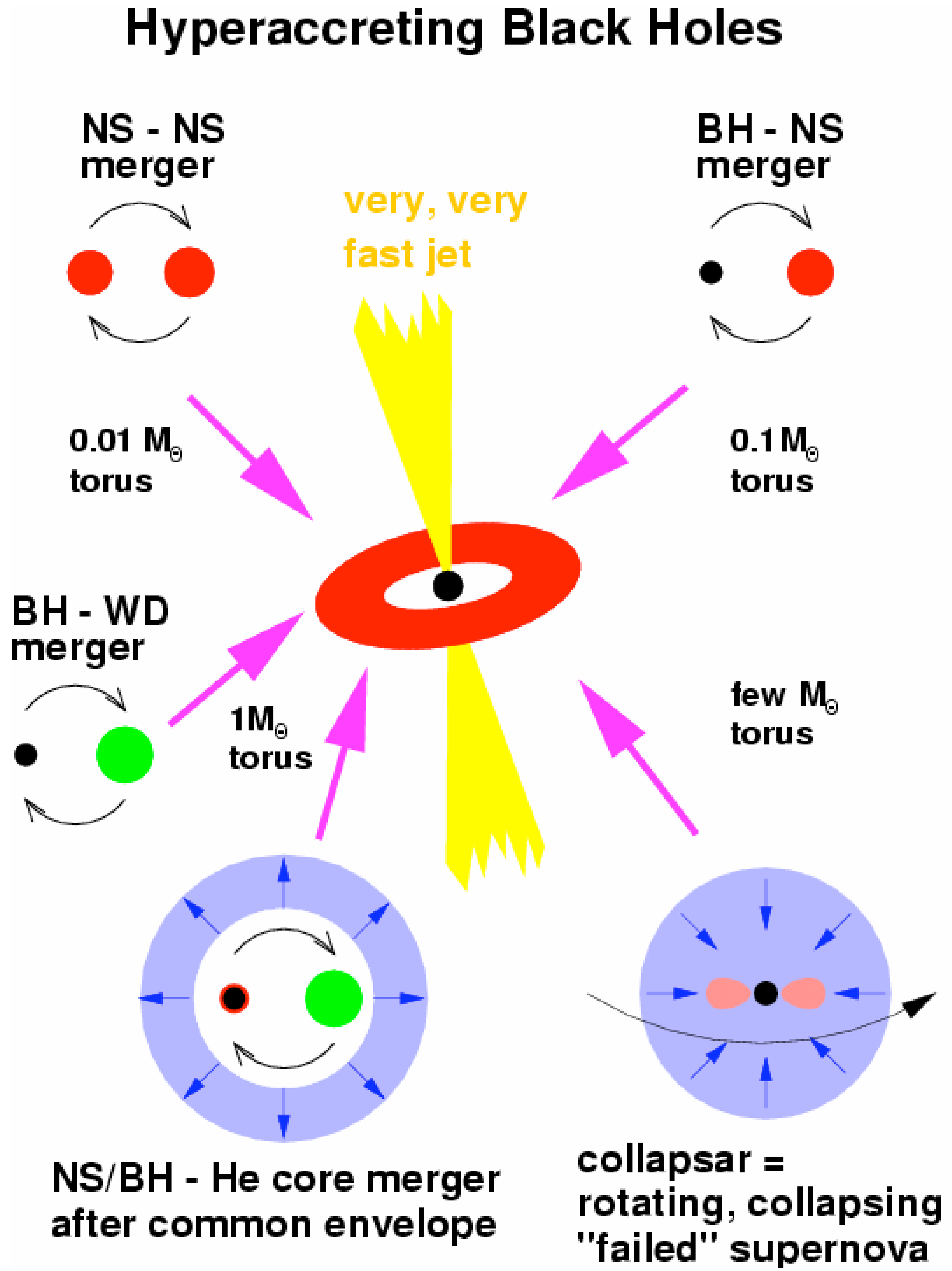


15
10
5
0
p
10
8
6
4
2
0
rho
0.8
0.6
0.4
0.2
0.0
v
0.0
0.2
0.4
0.6
0.8
1.0
x

# Fireball Shock Model

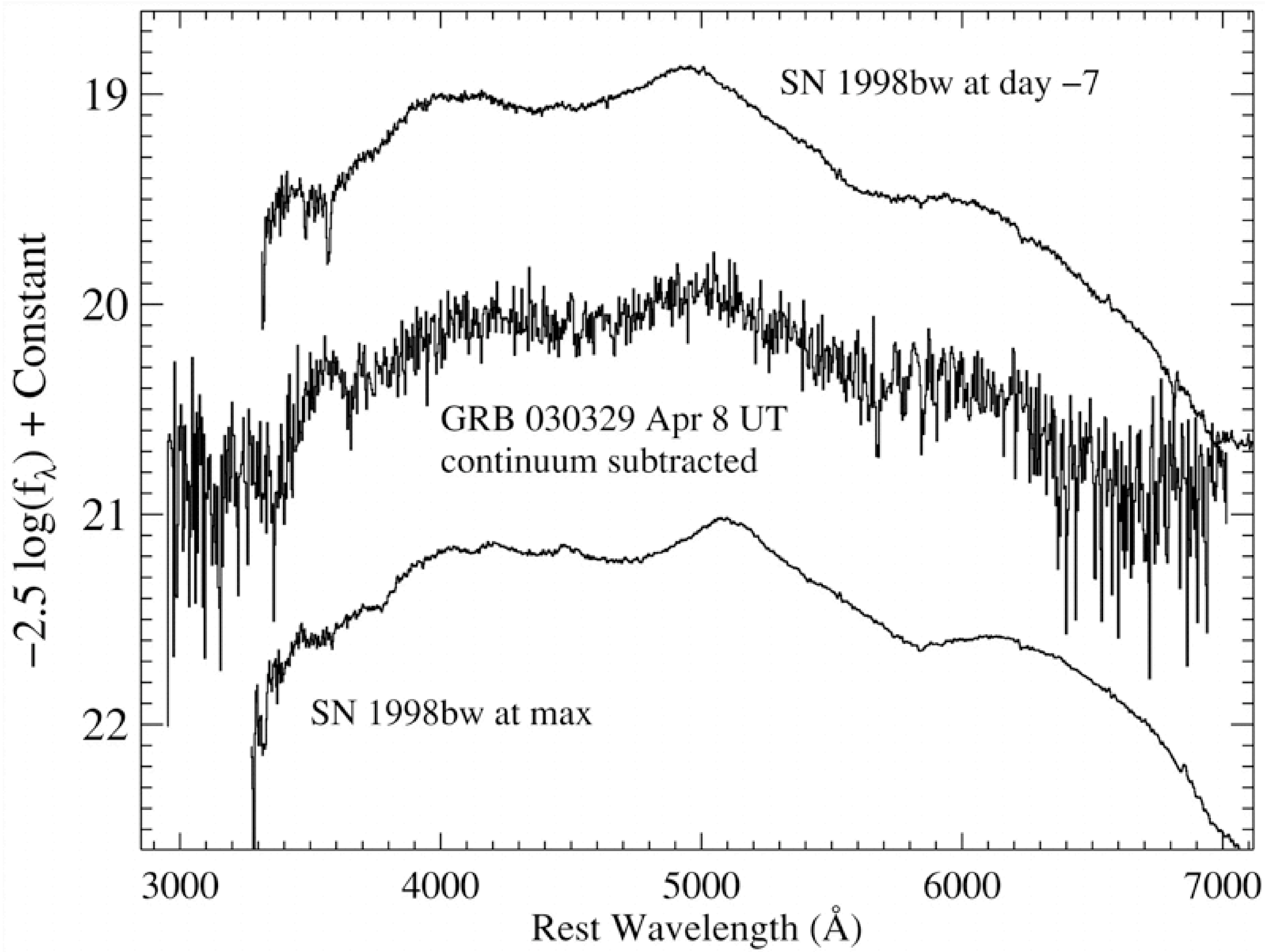
- Ultra-relativistic outflows (>100)
- Internal Shocks : gamma-rays
- External Shocks : afterglows
- Sideways Expansion : Break in light curves

# Central Engines



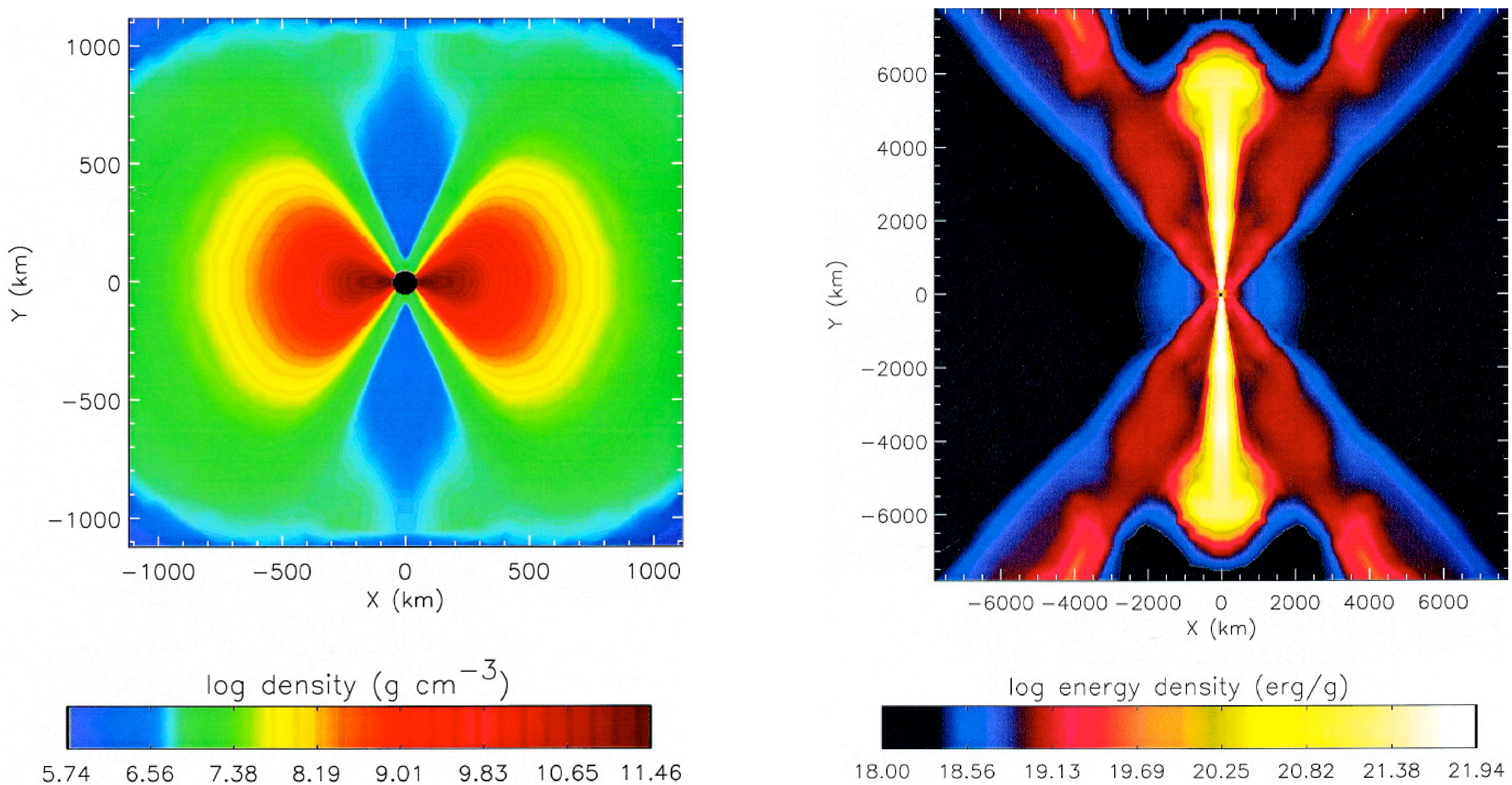
# Death of Massive Stars

(at least some long soft GRBs)




Stanek et al. 2003

# Collapsar Model

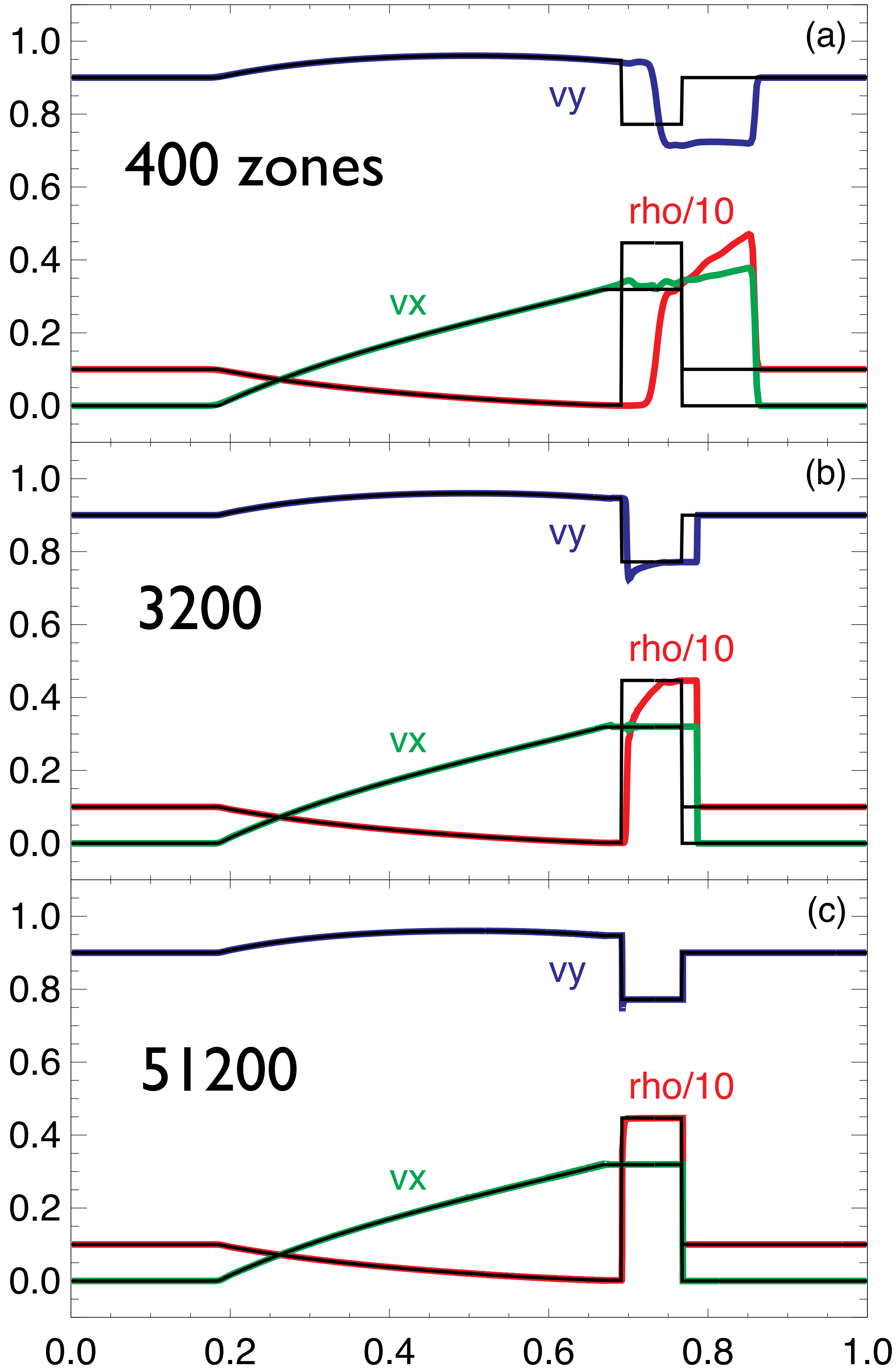



MacFadyen & Woosley 1999

Massive (>40) + Rotation + no Hydrogen
Core Collapse => BH + AD => Jets

# RAM : a Relativistic code with Adaptive Mesh

- special relativistic hydro
- rotation, viscosity
- nuclear physics: photodisintergration
- neutrino emission
- EOS (Ideal nucleons, radiation, relativistic degenerate electrons & positrons)
- post-Newtonian Gravity




# Riemann Problem With Transverse Velocity

P_L = 1000

P_R = 0.01

rho_L = 1.0

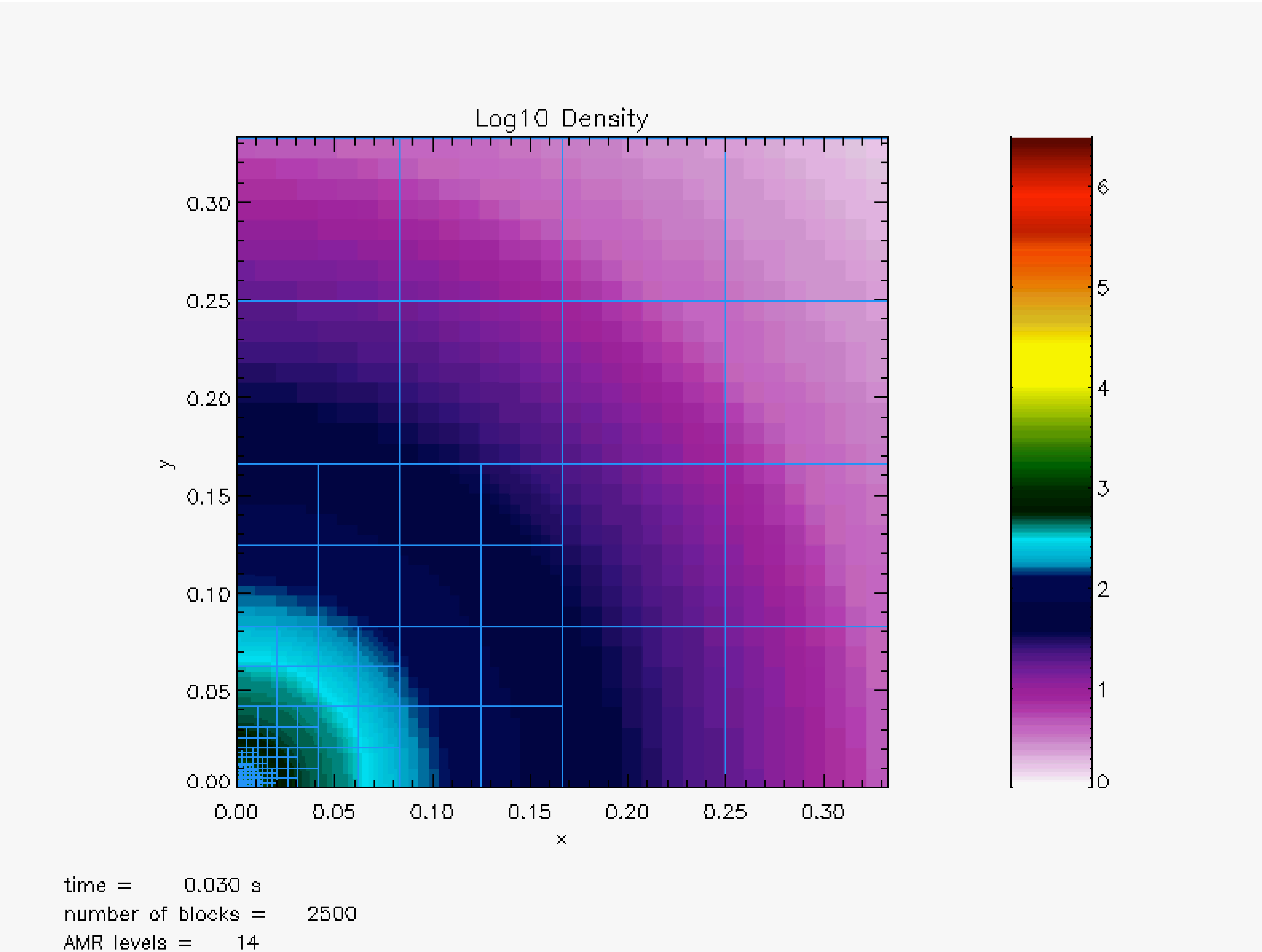
rho_R = 1.0

v_x = 0.0

v_y = 0.9

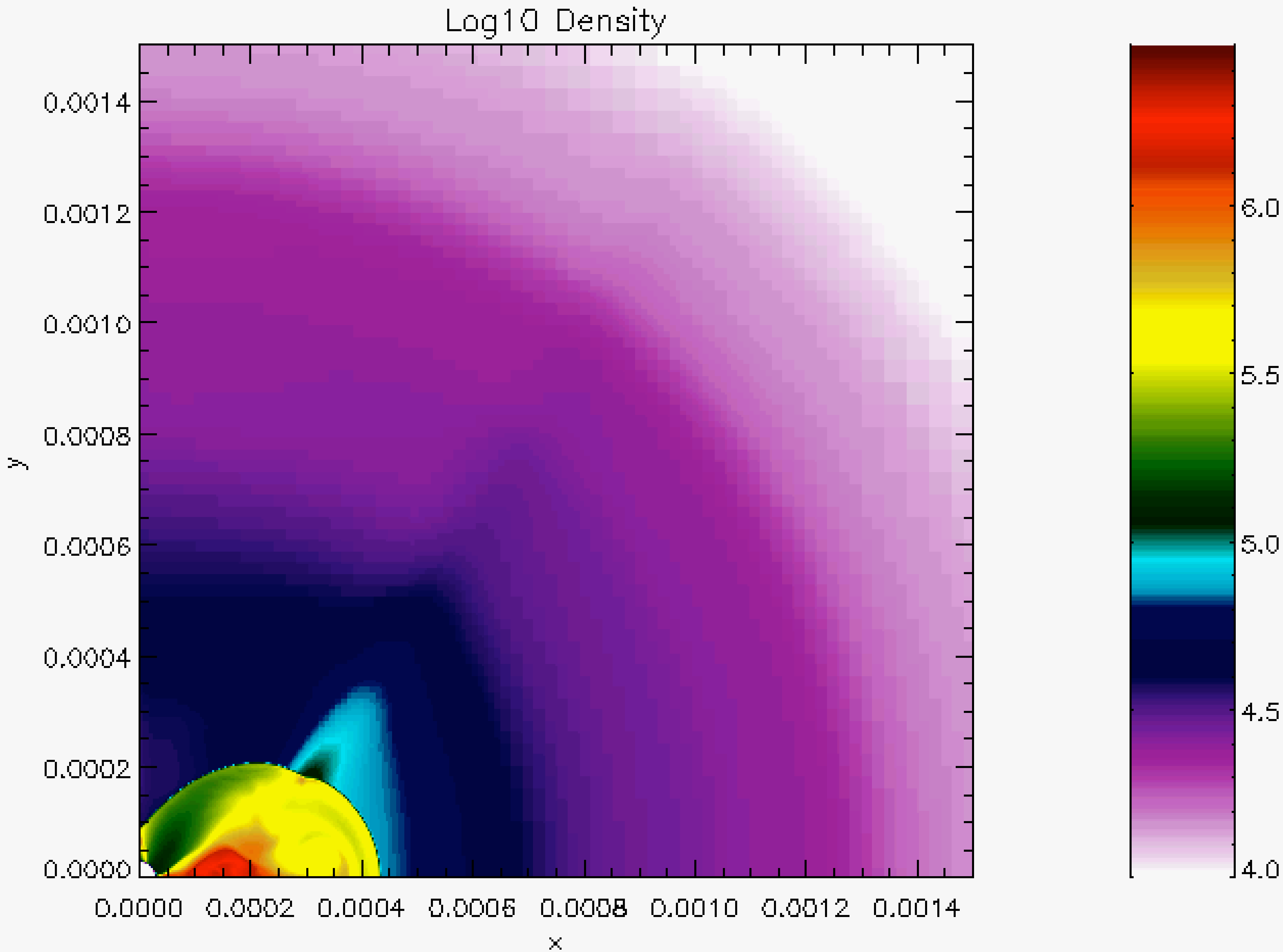
Really Need AMR!

# Jet Birth



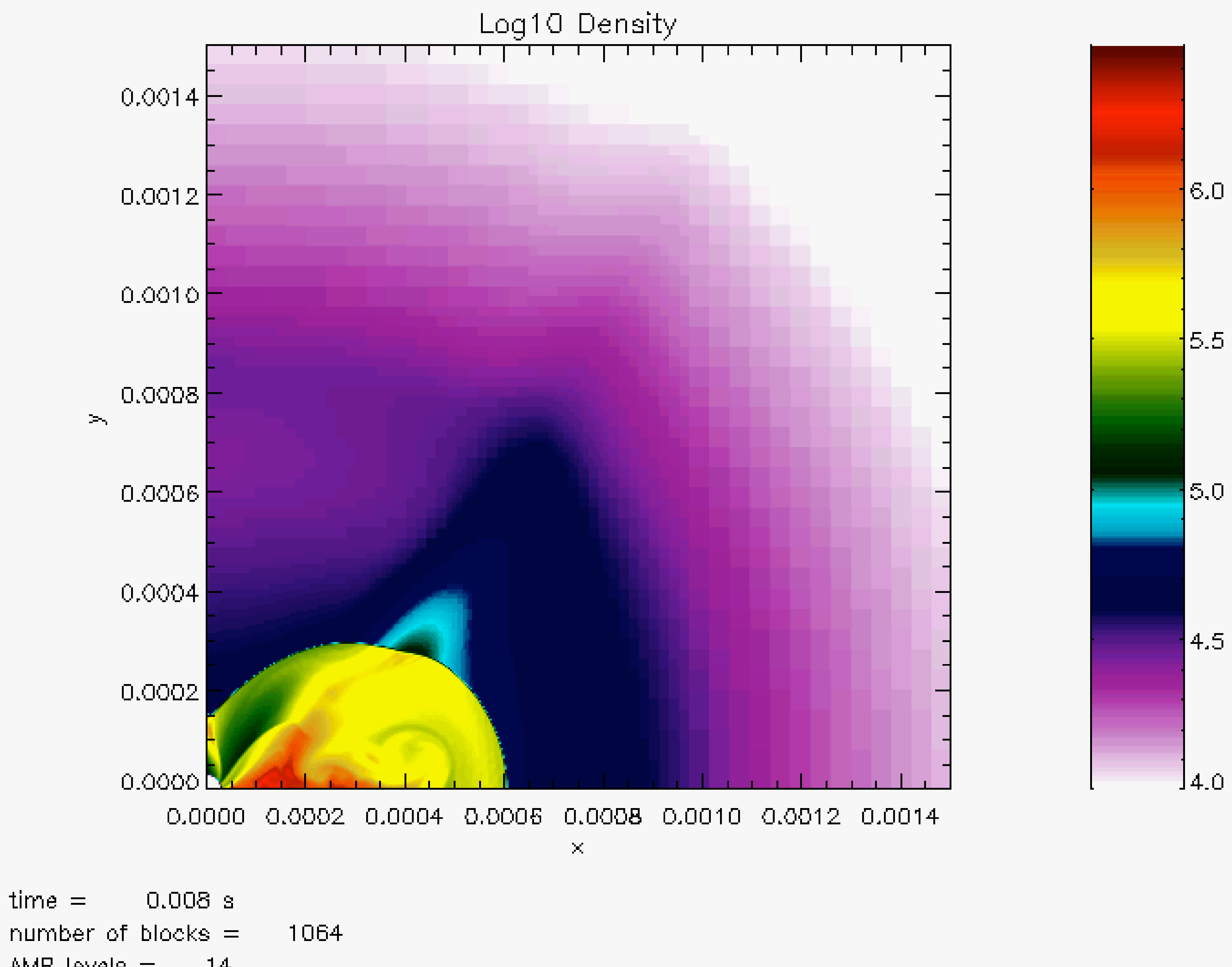

Star
~10^11cm

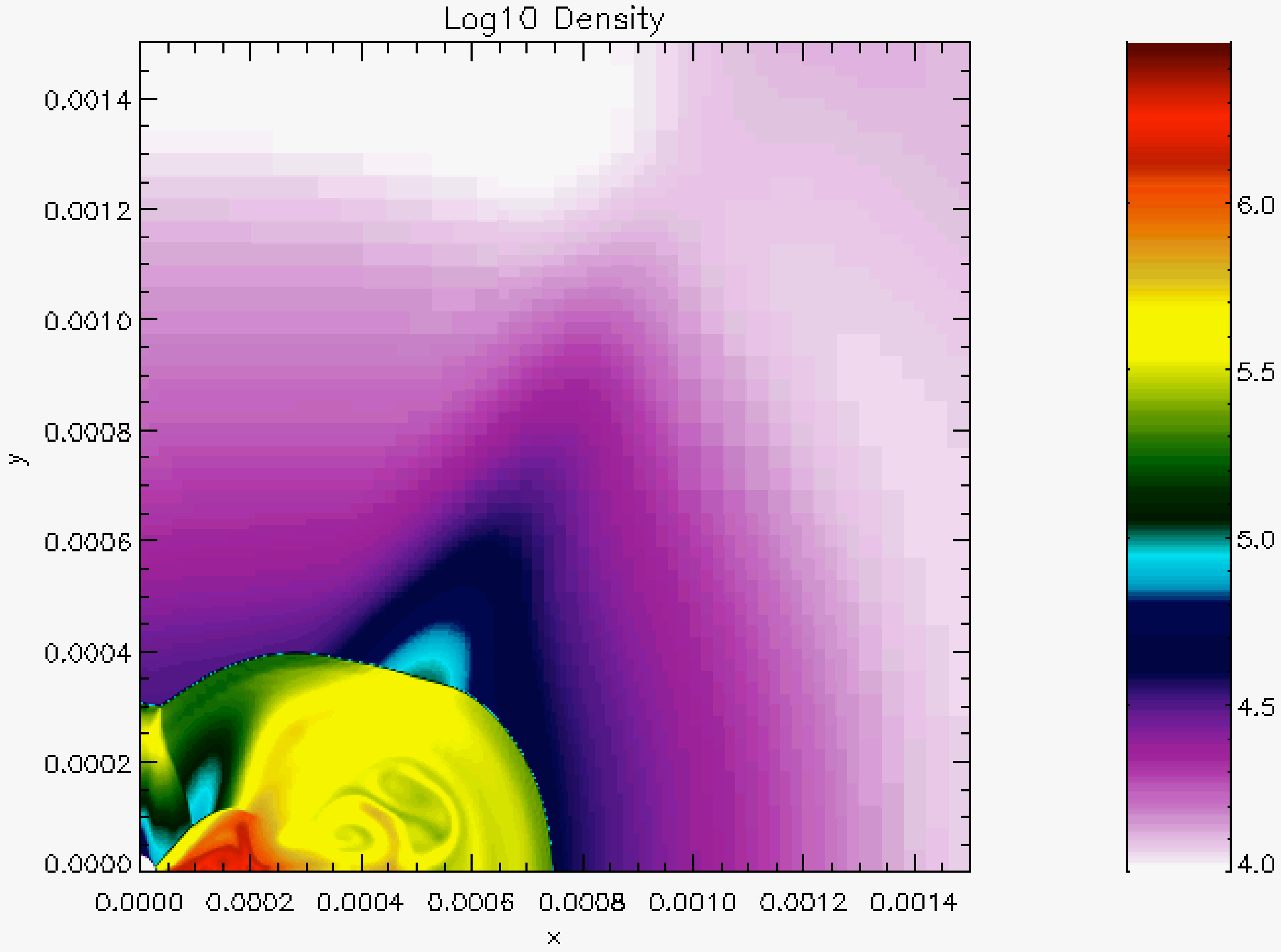
Hole
~ 10^6 cm



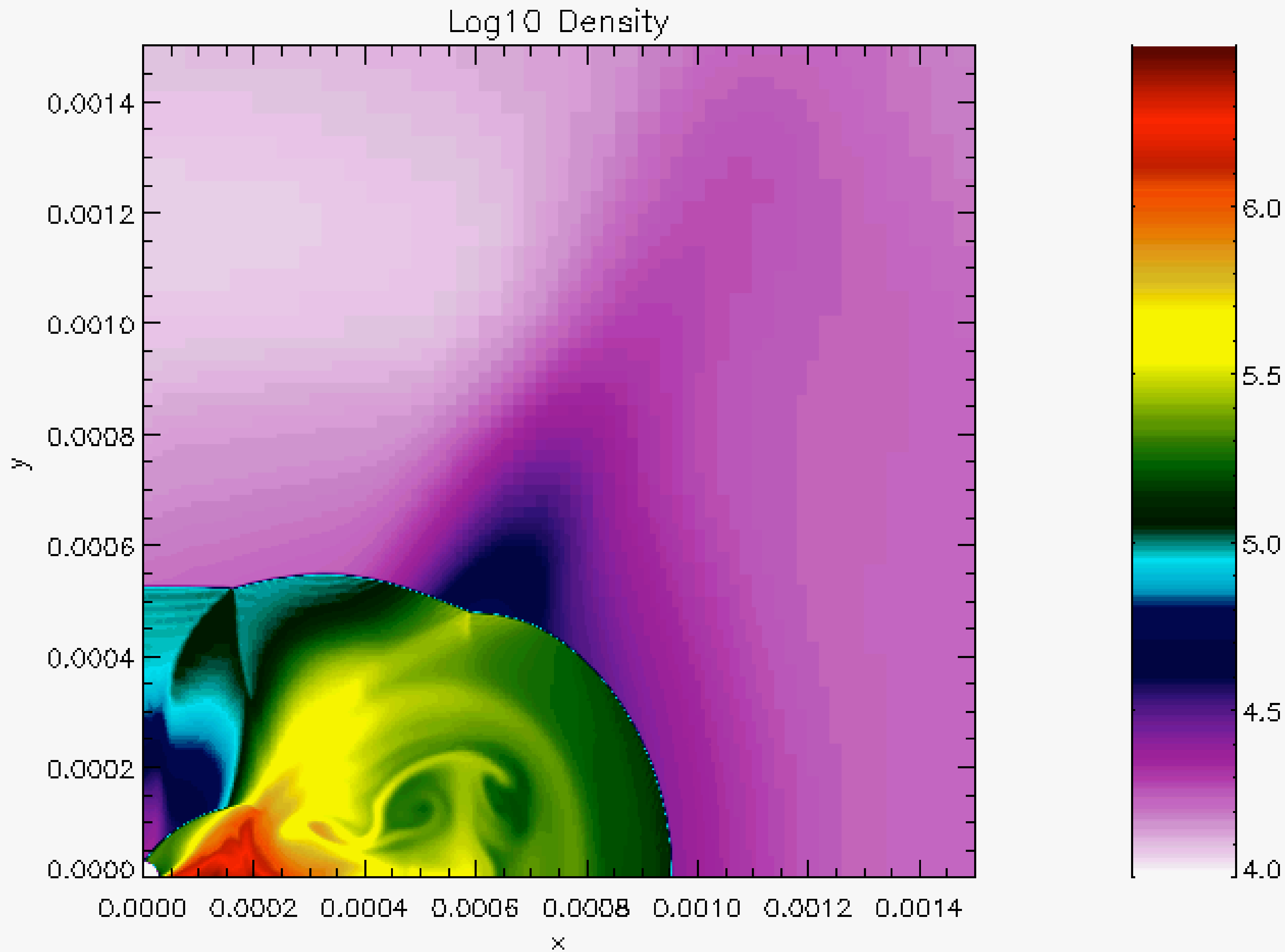

time = 0.004 s

number of blocks = 880

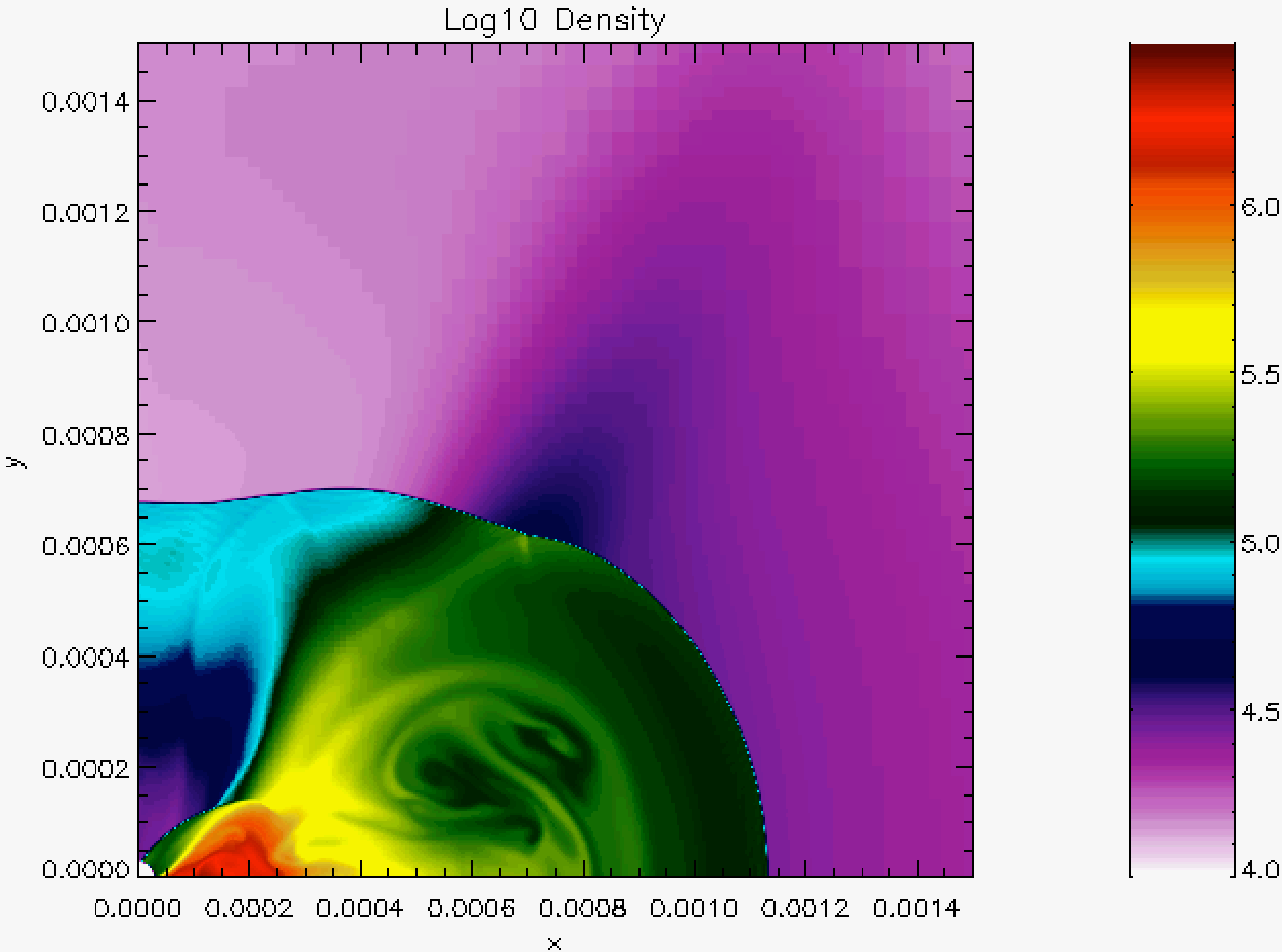

Log10 Density
0.0014
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
y
0.0000
0.0002
0.0004
0.0006
0.0008
0.0010
0.0012
0.0014
x
6.0
5.5
5.0
4.5
4.0
time = 0.008 s
number of blocks = 1064



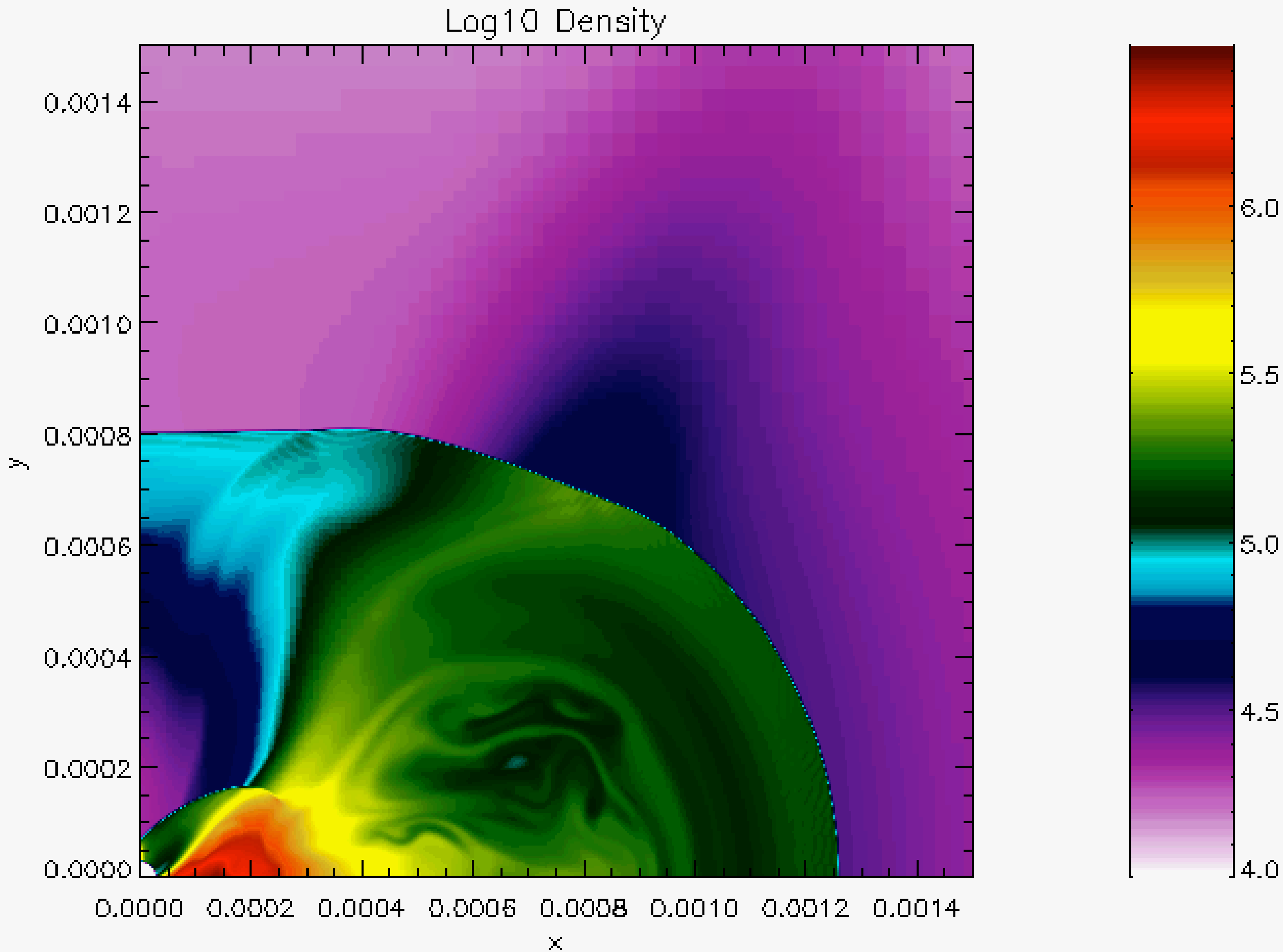

time = 0.012 s
number of blocks = 1340
AMR levels = 14



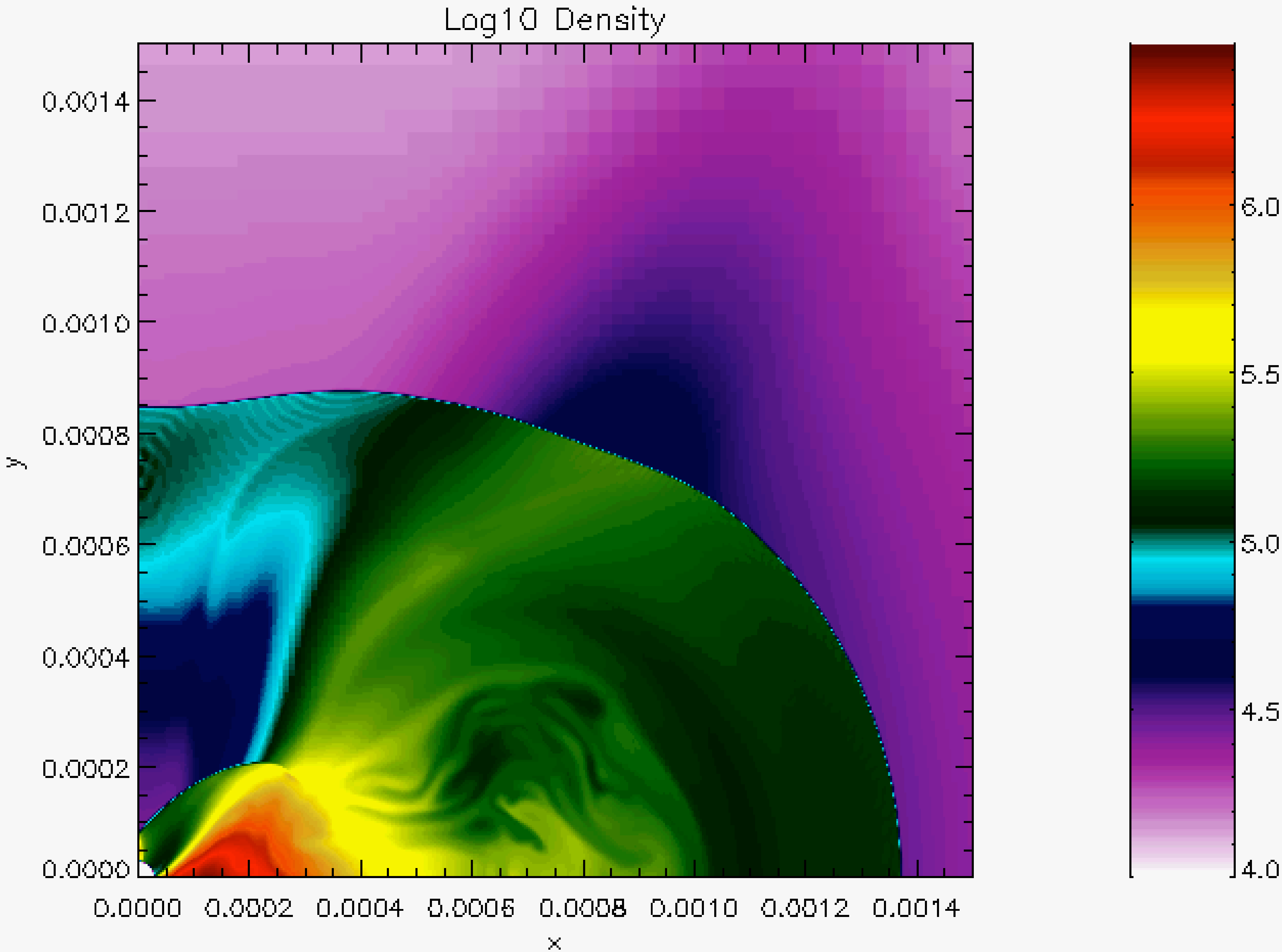

time = 0.016 s
number of blocks = 1616
AMR levels = 14



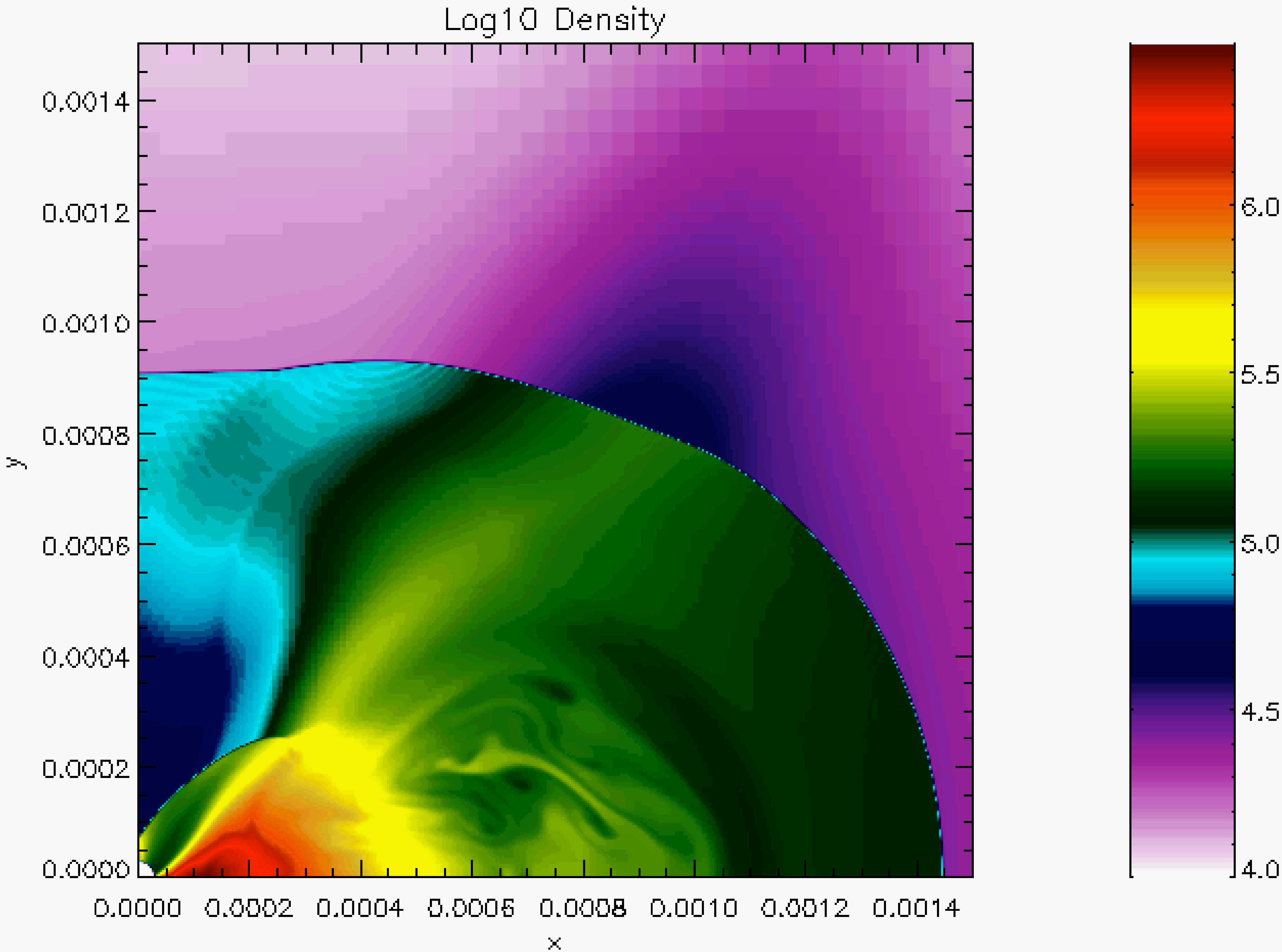

time = 0.020 s
number of blocks = 2012
AMR levels = 14



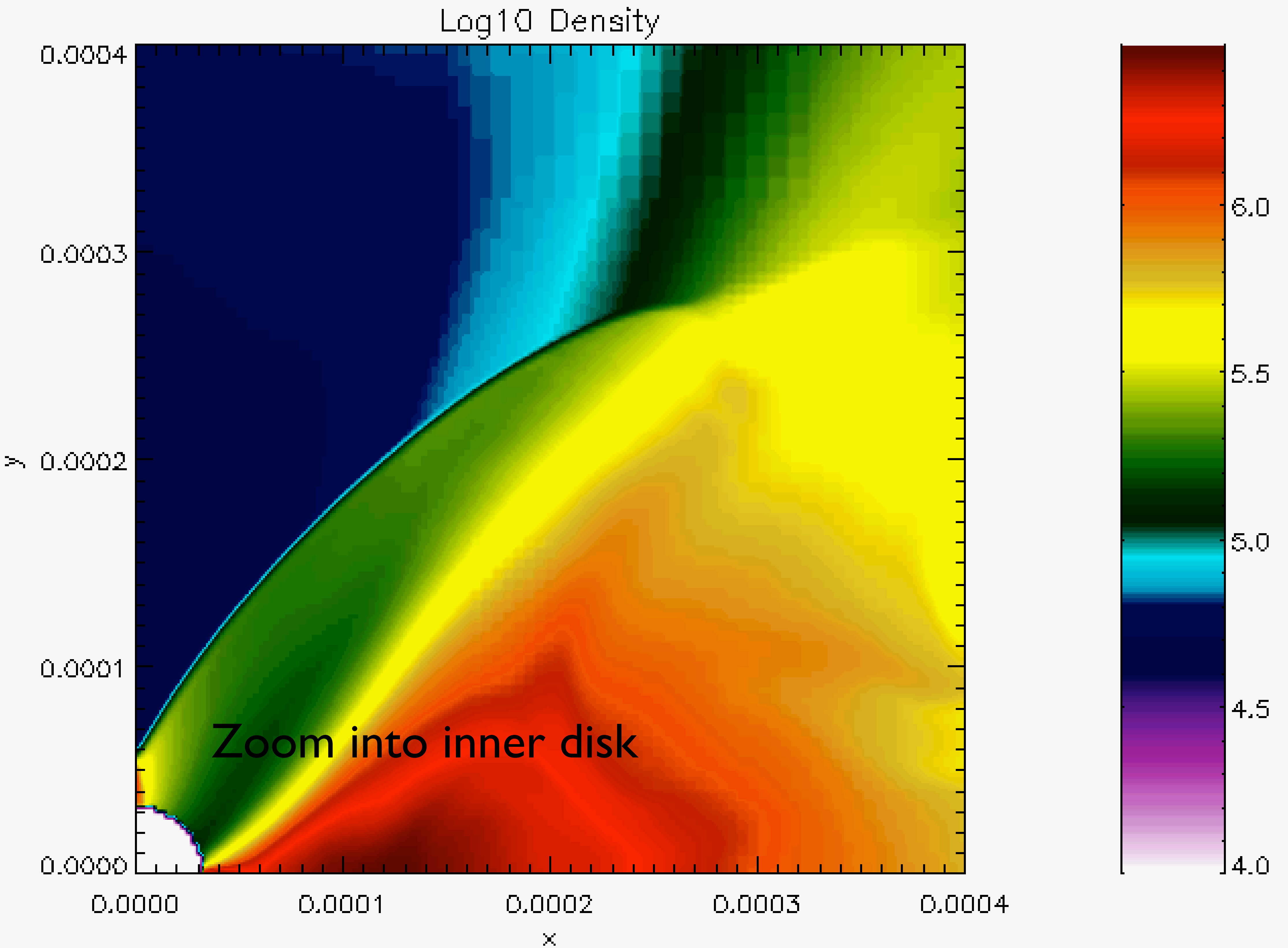

time = 0.024 s
number of blocks = 2348




time = 0.028 s
number of blocks = 2528
AMR levels = 14

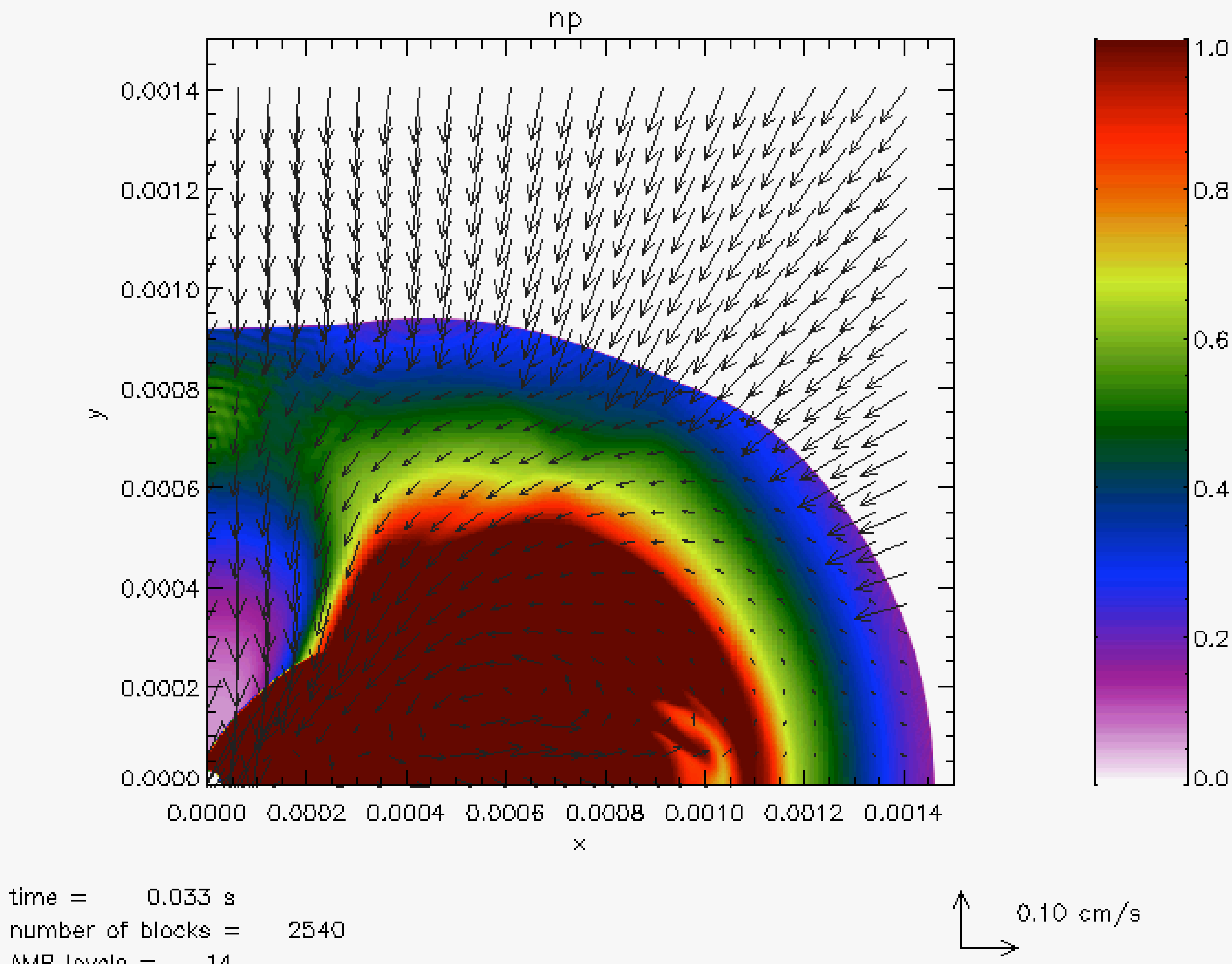



time = 0.032 s
number of blocks = 2528
AMR levels = 14

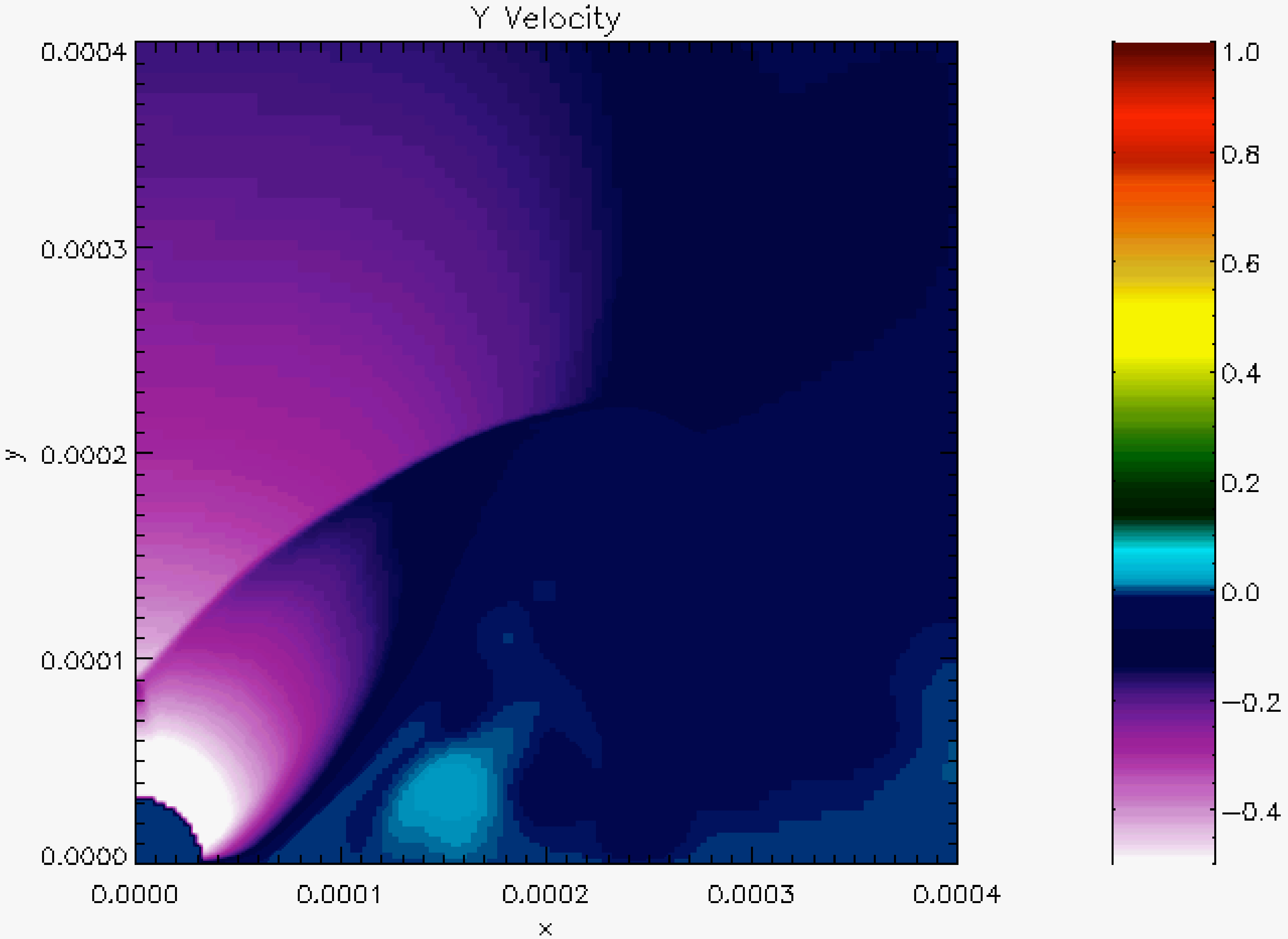



time = 0.033 s
number of blocks = 2540

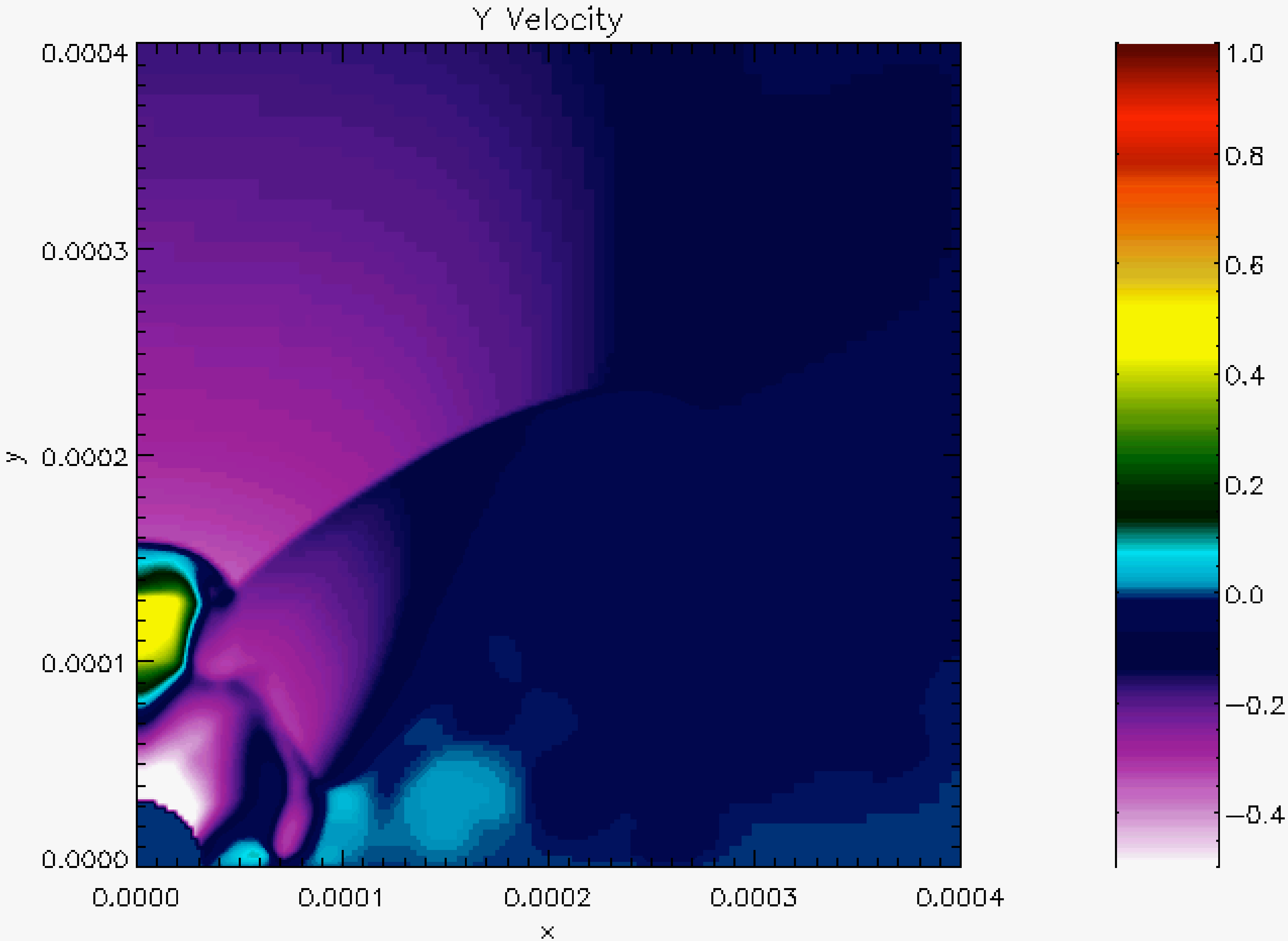
# Free Nucleon (=> URCA neutrinos)



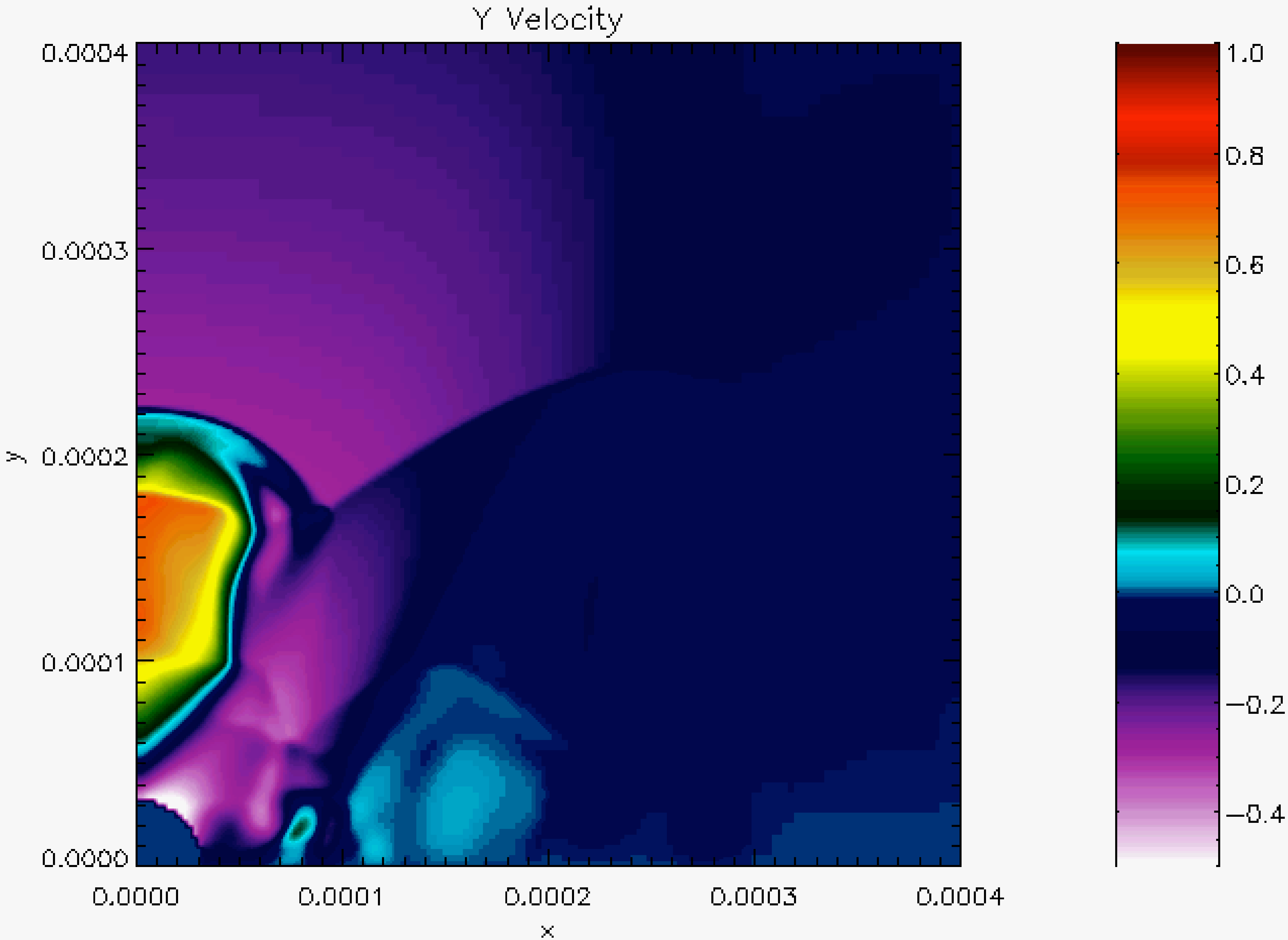
# Early Jet Propagation



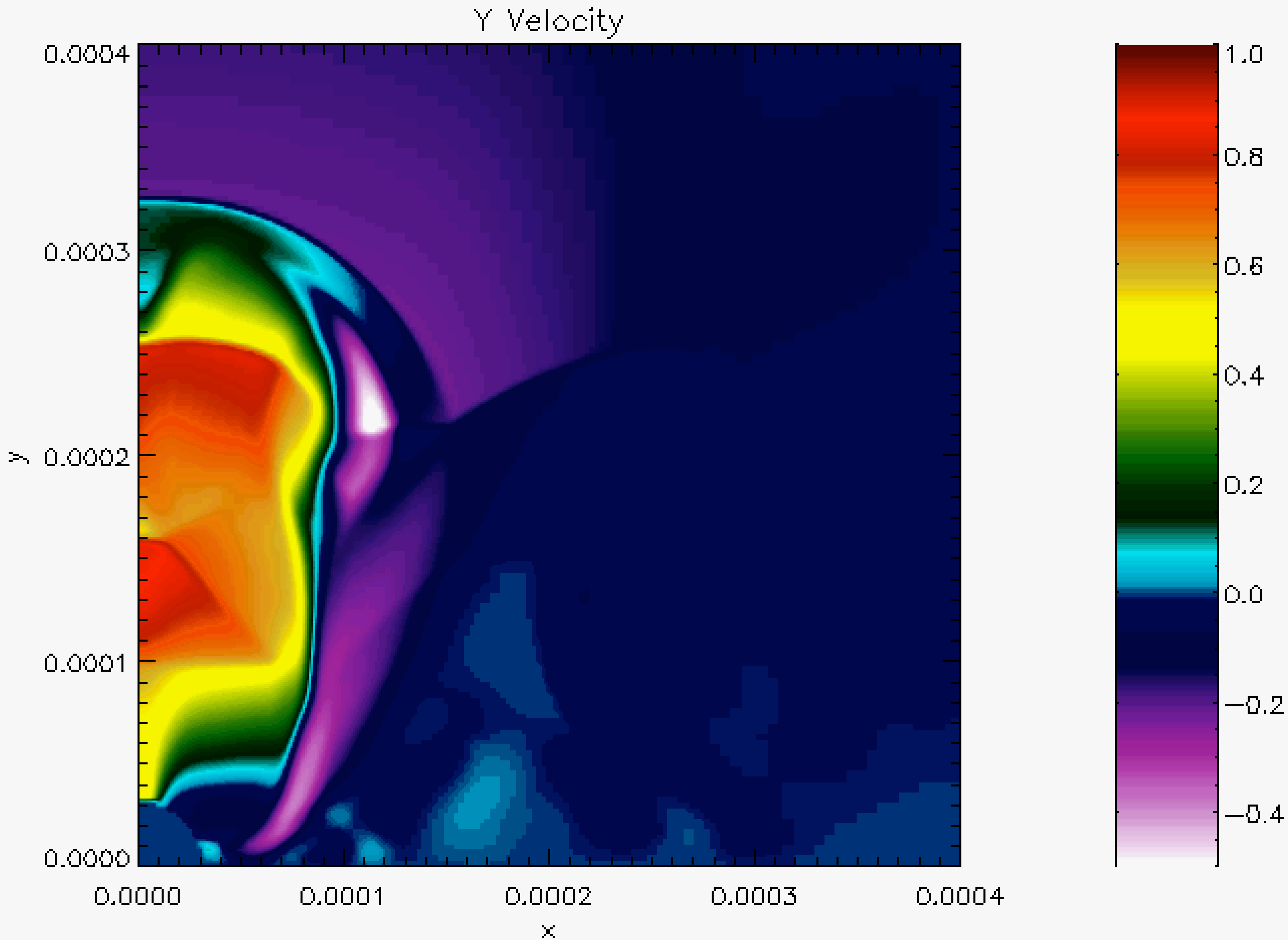

time = 0.030 s
number of blocks = 2500
AMR levels = 14



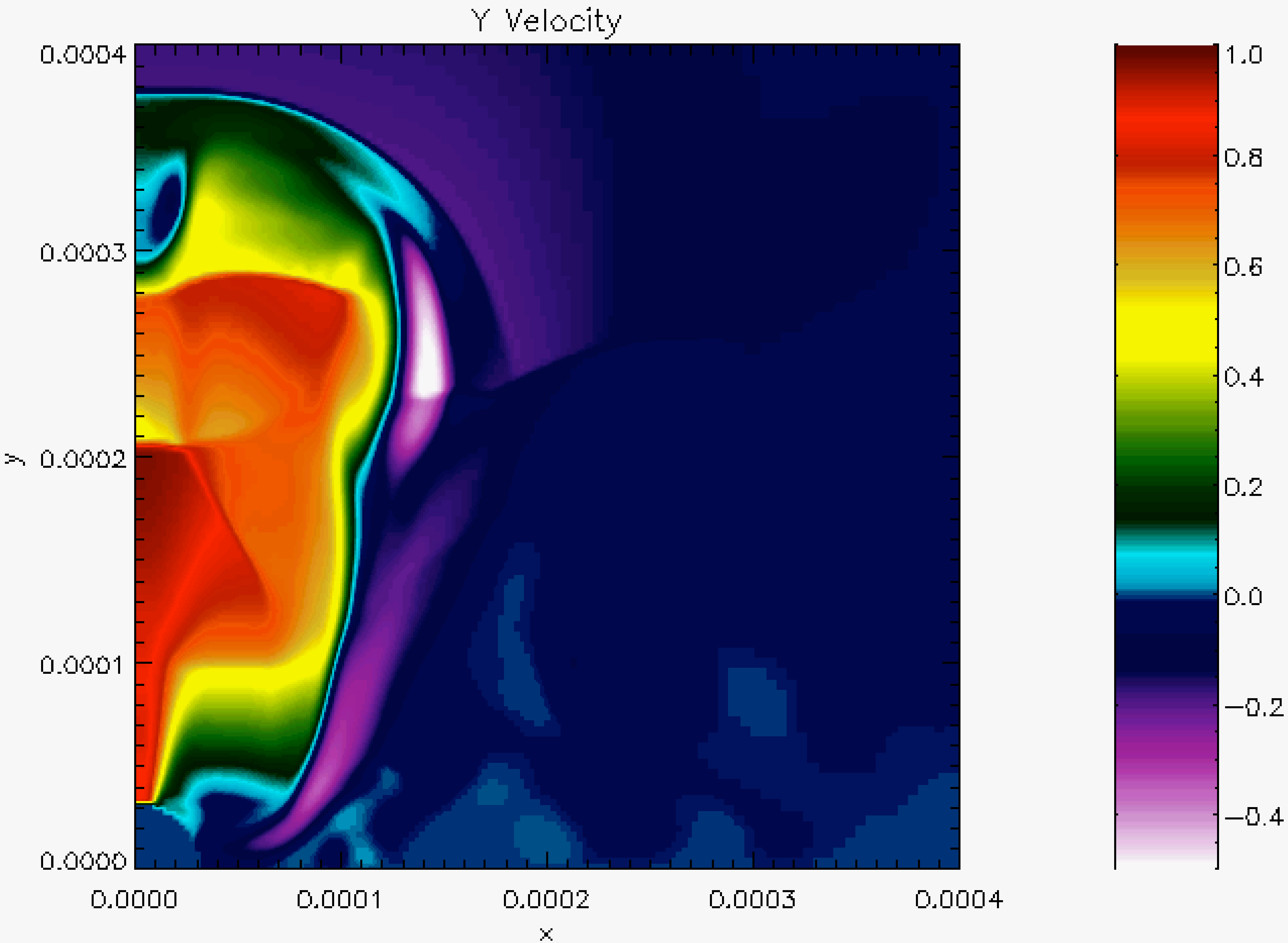

time = 0.031 s
number of blocks = 2476
AMR levels = 14




time = 0.031 s
number of blocks = 2476
AMR levels = 14



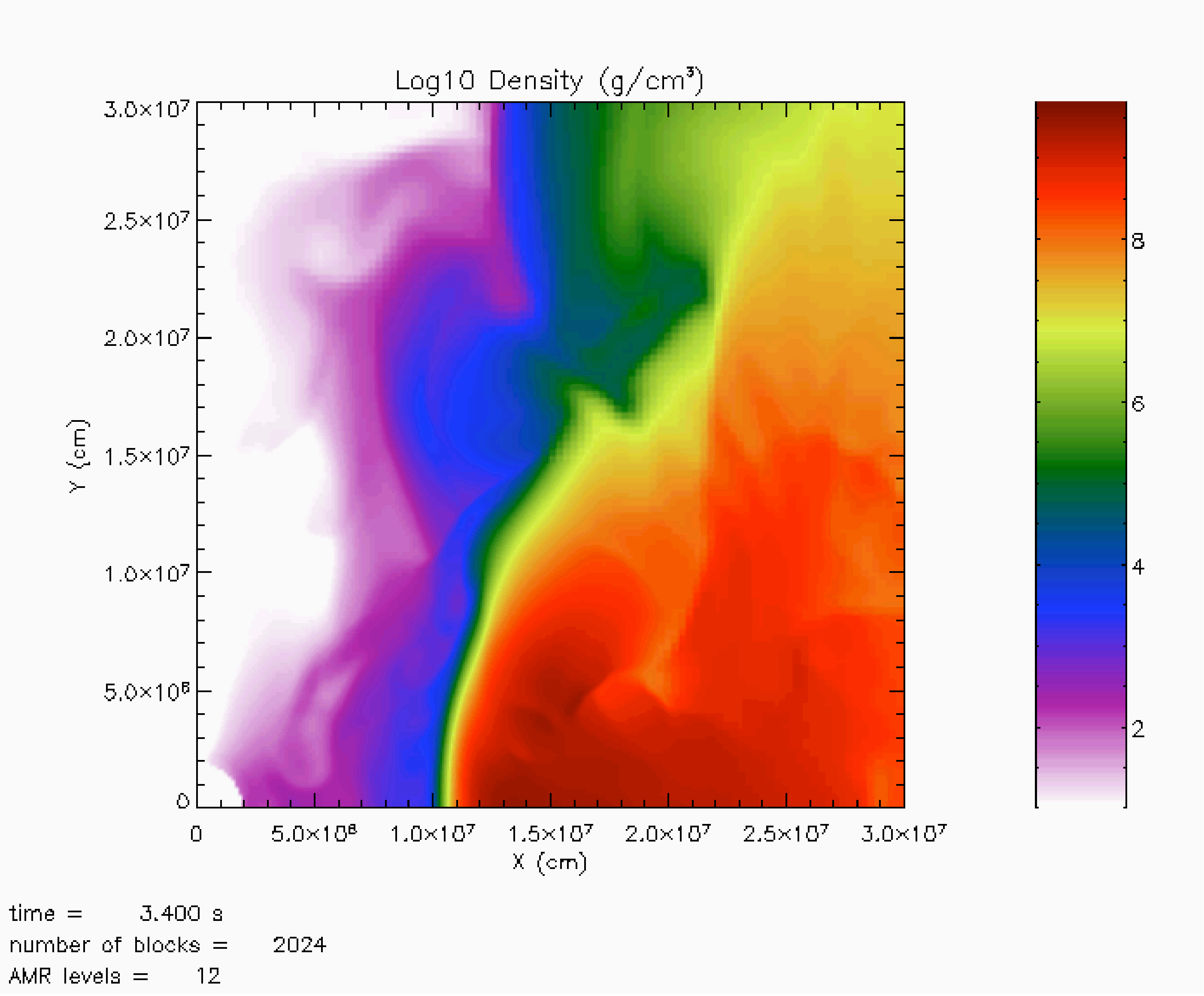

time = 0.032 s
number of blocks = 2564




time = 0.032 s
number of blocks = 2616

# Problem: Viscosity

alpha viscosity is not hyperbolic ==>
This causes problems especially for high-resolution cases
(1) information speed > c (2) numerically unstable




No viscosity
No accretion

MHD, MRI
or
hyperbolic viscosity

# Accretion Powered Supernova

- Supernova: Radioactive decay of Ni56
- No Ni56 ==> No Supernova
  SN 1998bw : 0.5 Msun Ni56
- Jet is too cold to make enough Ni56
- Accretion Disk Wind
  (MacFadyen & Woosley 2000, Kohri, Narayan, Piran 2005)
  Neutrino heating is more efficient in accretion disk than symmetric collapse!

# Relativistic Jet Propagation in Stars

- Pure special relativistic hydrodynamics
- No MHD  (may not be dynamically important)
- Put  jet by hand
- Can they penetrate massive stars?
- Still high Lorentz factor?
- Implications for observations

# Parameterized Jet

Numerically, it is a boundary condition.

- Power : 1e49 --- 1e51 erg/s
- Initial Lorentz factor : 5 --- 50
- Opening angle : 5 --- 20

These jets can break out of the stars if the central engine lasts long enough (+10s).

# Relativistic Jets From Collapsars

## S.E. Woosley's Group

Inital Model: he15

480 radial zones, 200 angular zones

Energy Deposition Rate: $10^{51}$ ergs/s

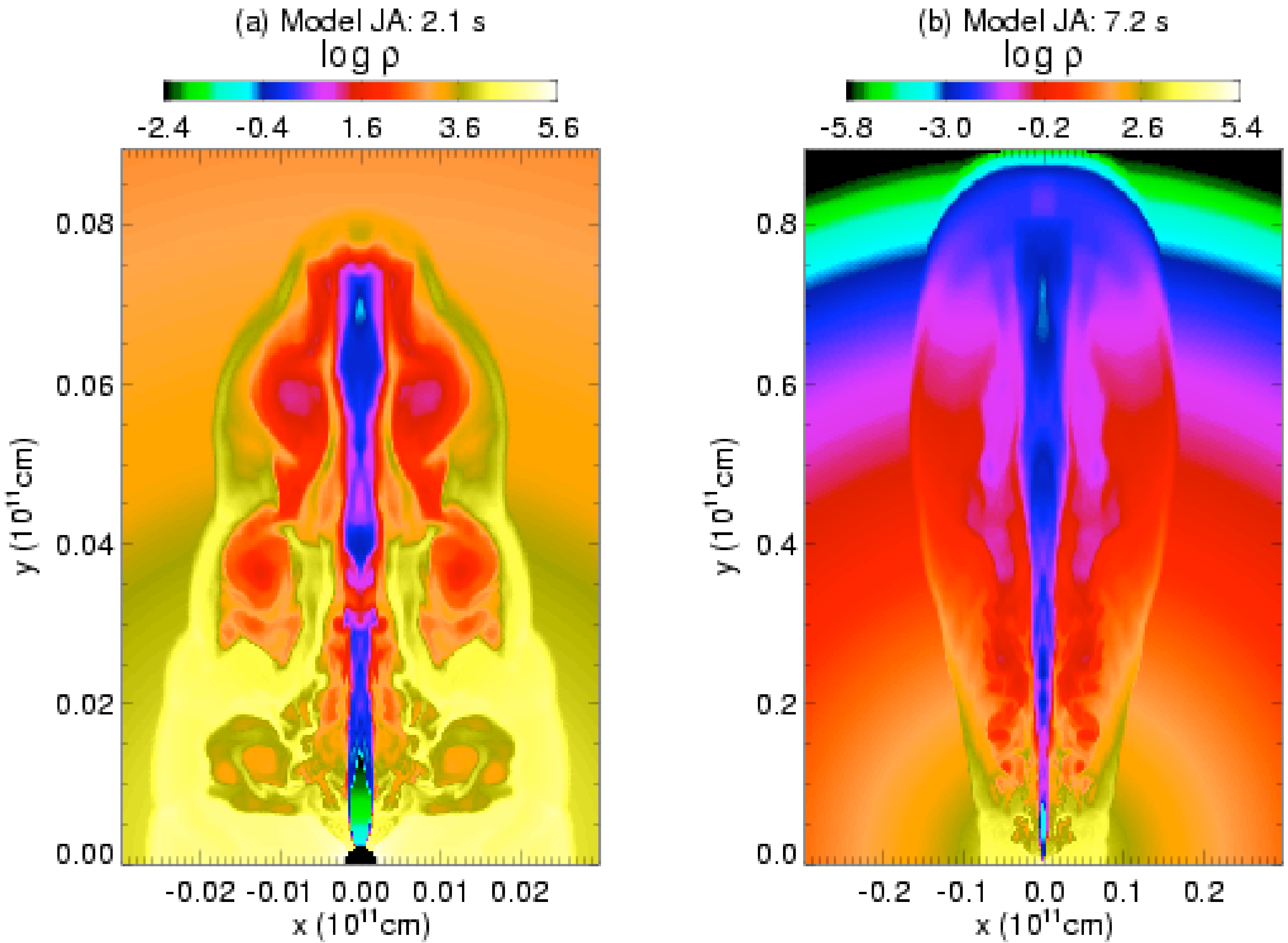
Half Opening Angle: 20

$f_e(E_{th}/E_{tot})$: 0.67

Lorentz Factor: 50

# Density Structure

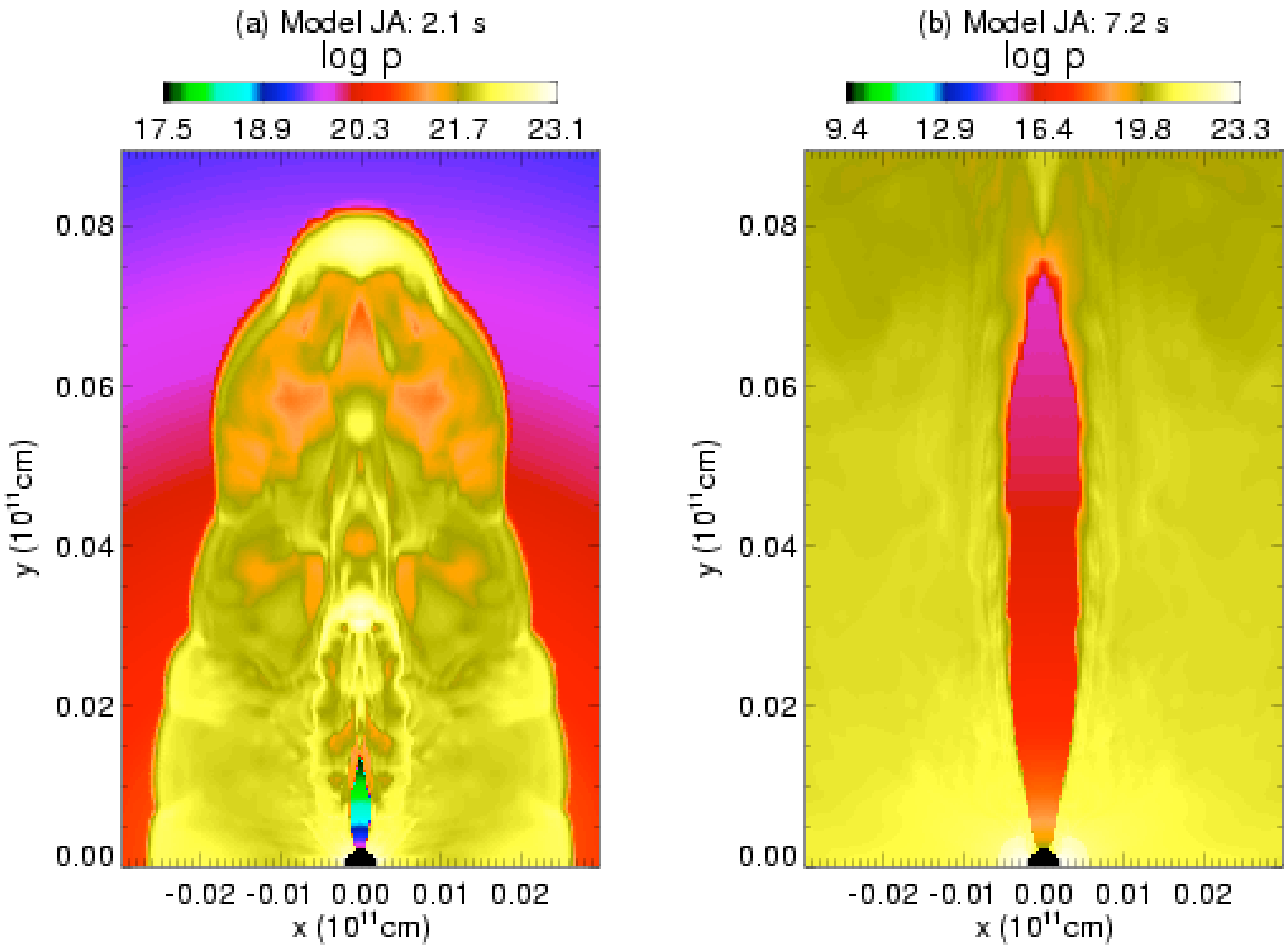
1/10
of star




Edge of
Star

Jet Beam, Cocoon, Bow Shock, Mach Disk, Backflows
Structure in jet beam and cocoon
break out at ~ 7s

# Pressure Structure




Pjet << Pstar < Pcocoon

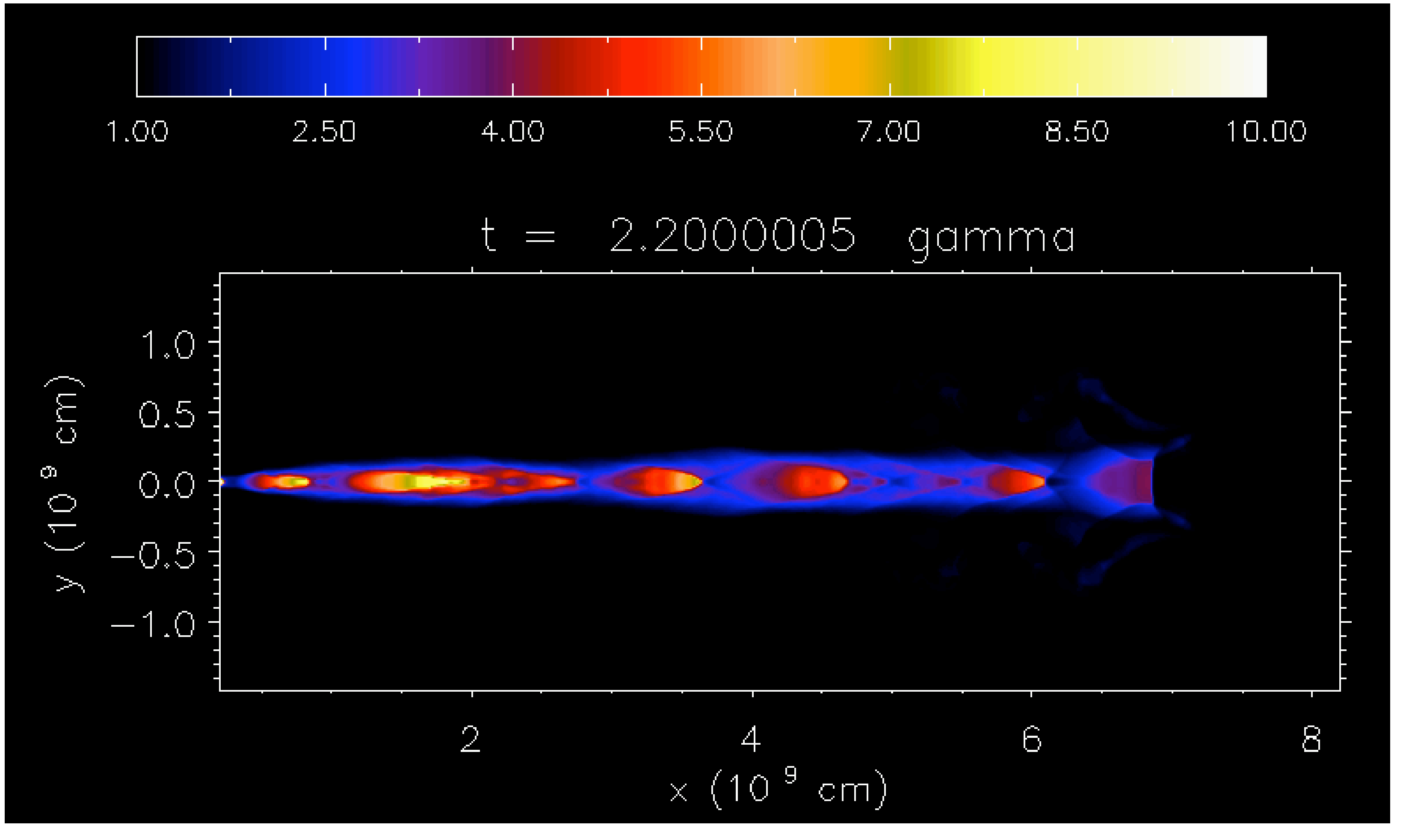
Jet beam shocked by pressure collimation shock

Angle : 20 ==> <5

# Origin of Variabilities in Lorentz Factor

Internal shocks ==> gamma-rays

- Born with variabilities (may be washed out)
- Instabilities during jet propagation inside star
- Outside the star : hard to develop



# Lorentz factor at Breakout

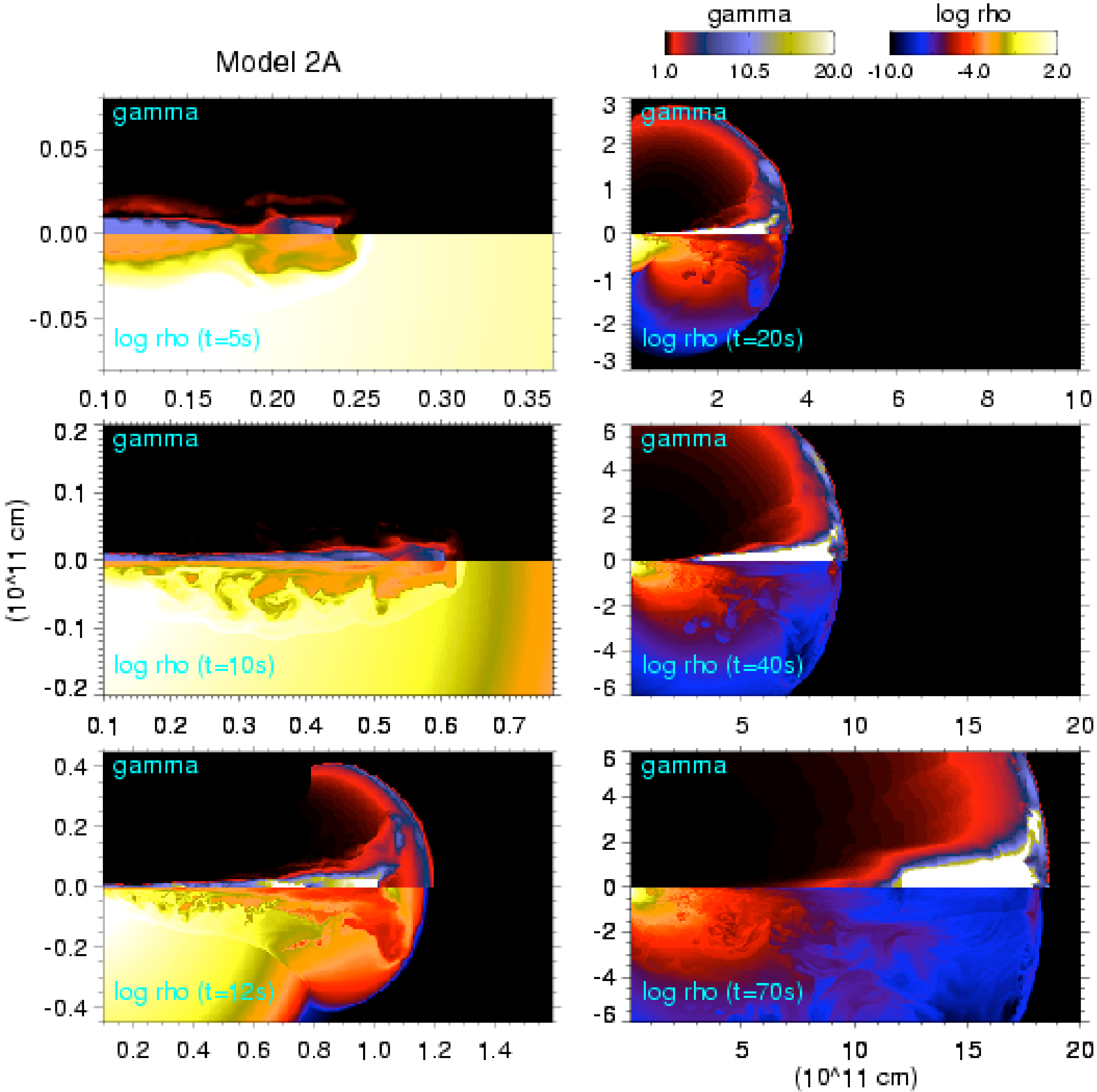
About 10 at breakout

Too low to make GRB

Internal energy : 10

10 + 10 = 200

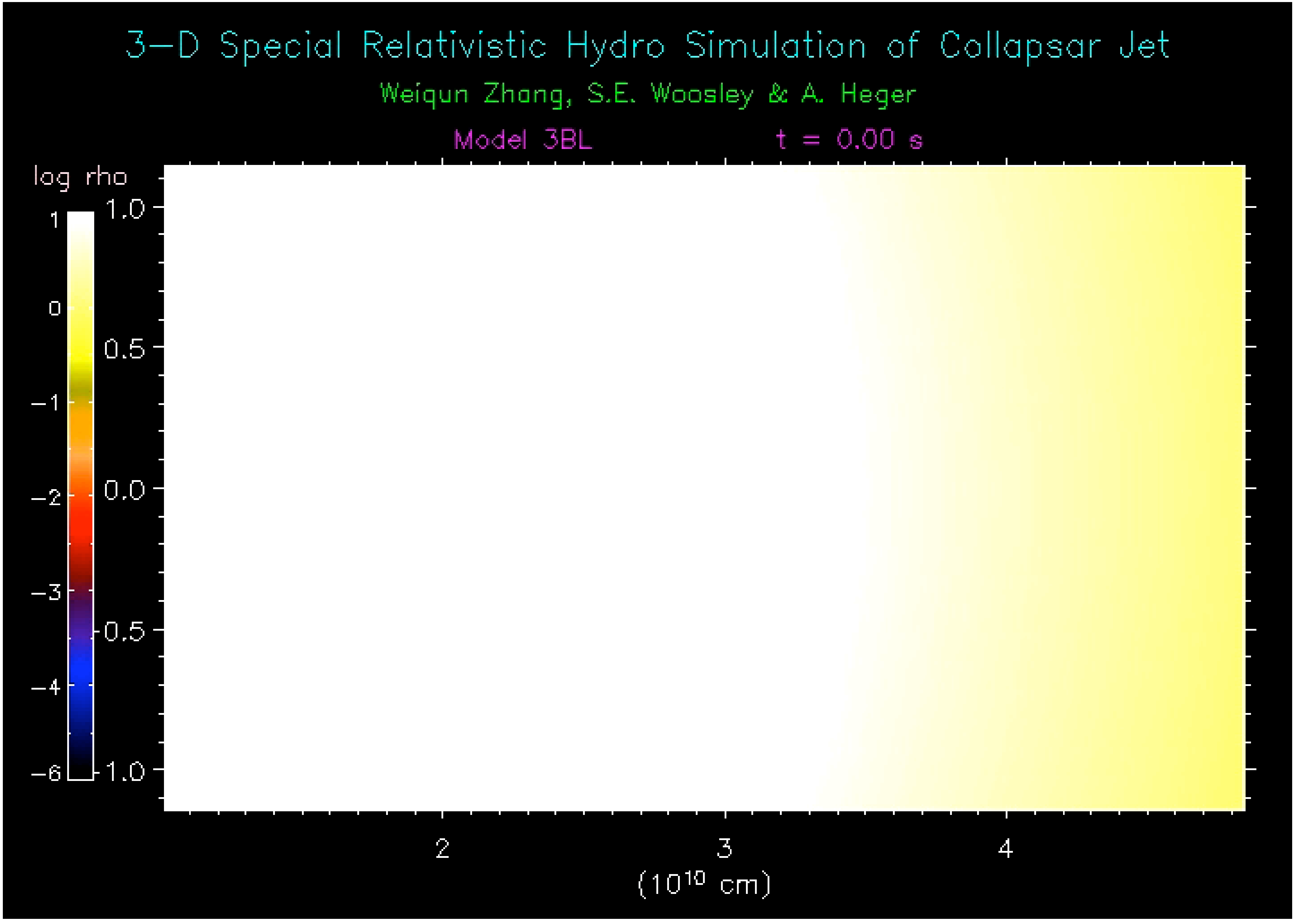
# After Jet Breakout



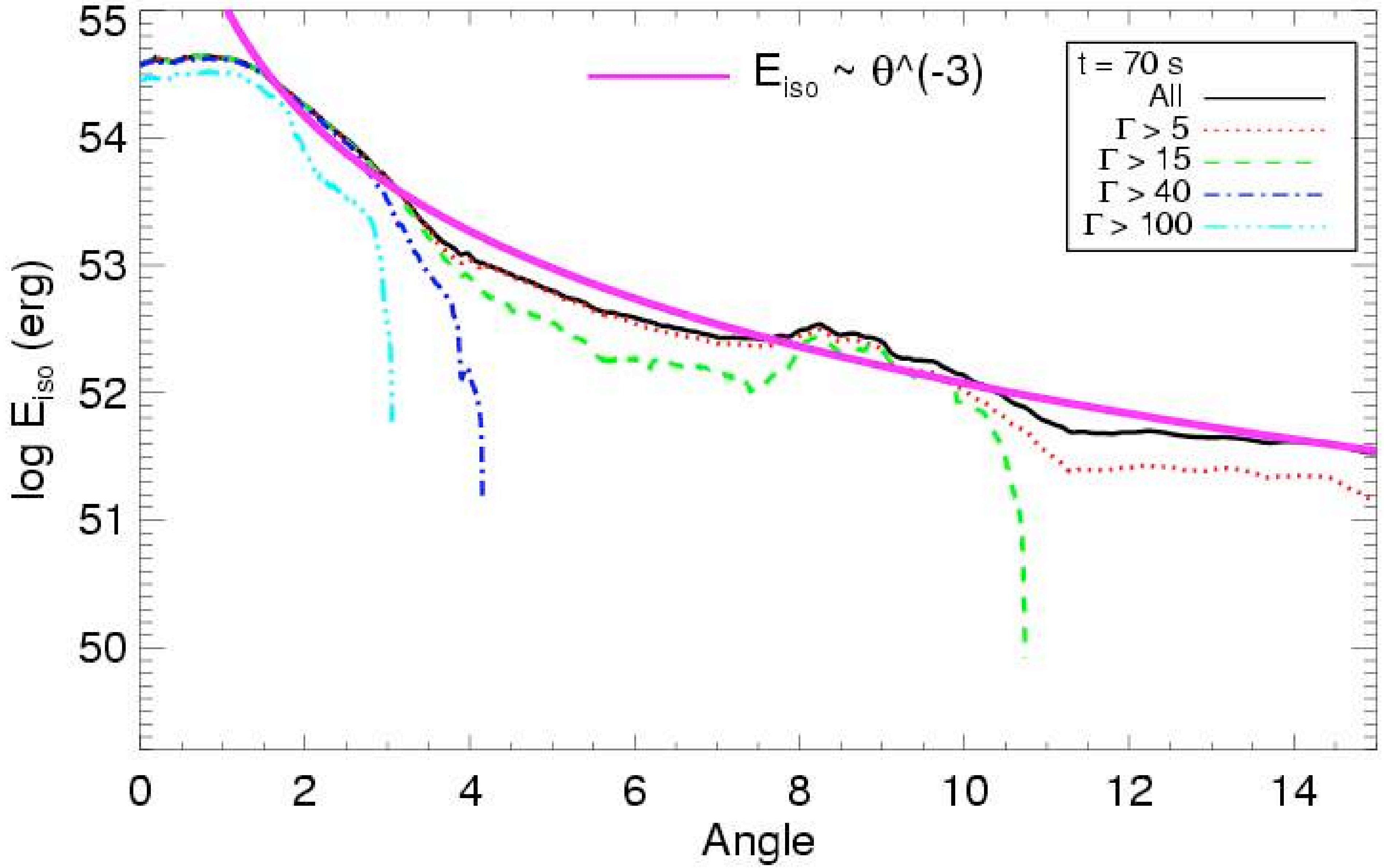

Lorentz factor:
10 => 100+

Explosion of Cocoon

Gamma ~ 10-20
off-axis 10 deg


3-D Special Relativistic Hydro Simulation of Collapsar Jet
Weiqun Zhang, S.E. Woosley & A. Heger
Model 3BL
t = 0.00 s
log rho
1
0
-1
-2
-3
-4
-6
1.0
0.5
0.0
0.5
1.0
2
3
4
(10^10 cm)

# Equivalent Isotropic Energy



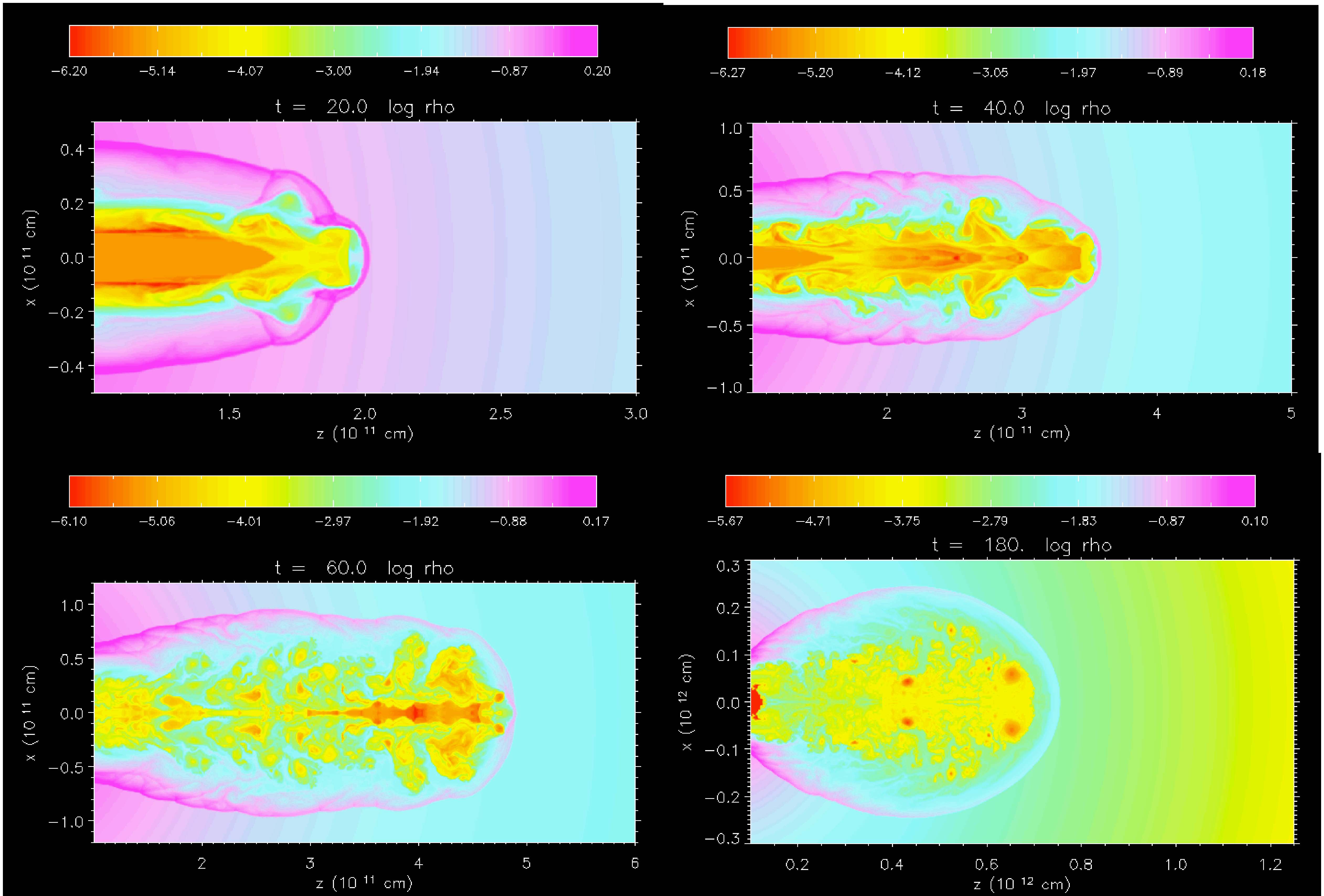

> 2 deg : ~ theta^(-3)      Large off-axis E_iso

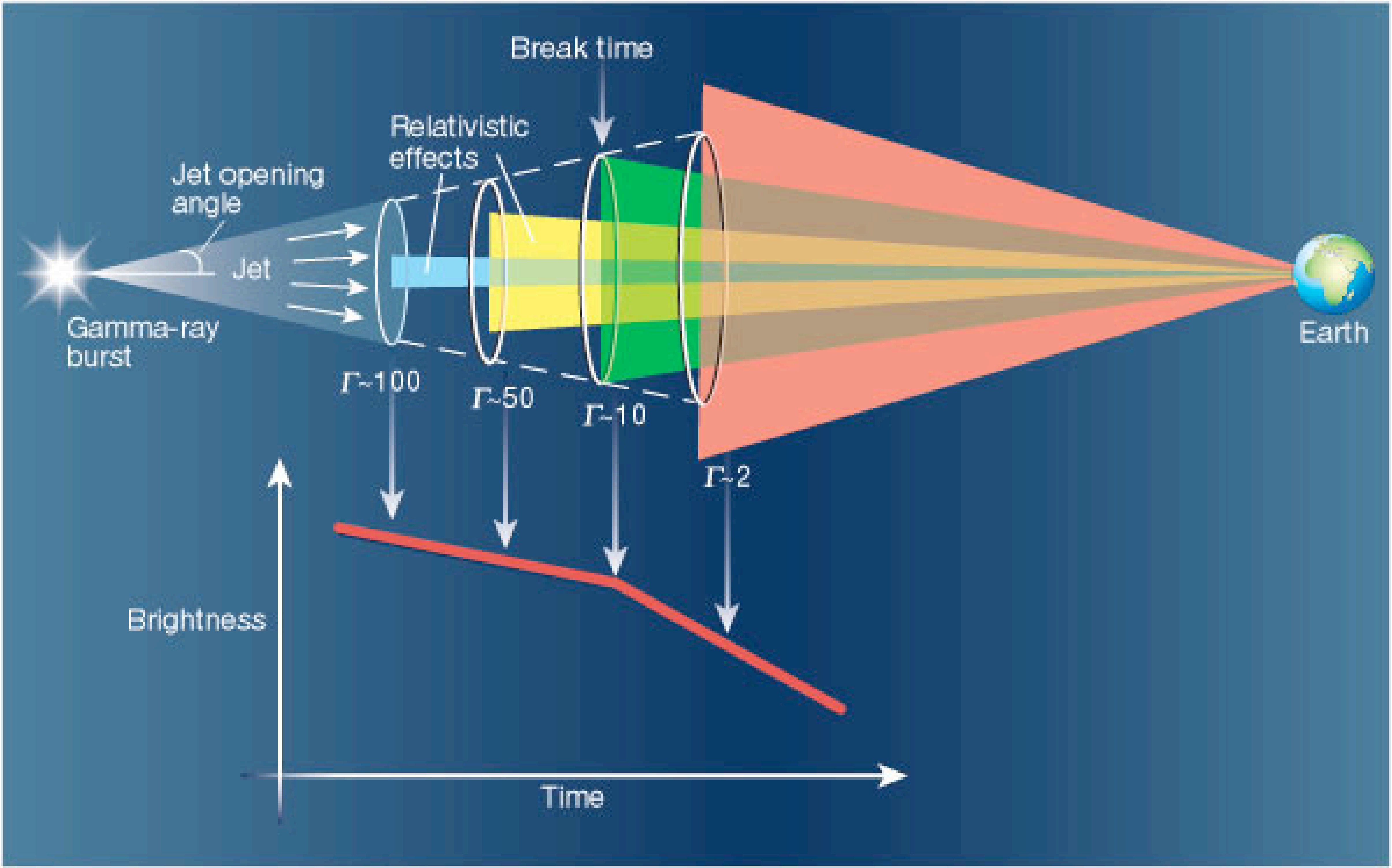
Lorentz factor: highly relativistic core + mildly relativistic wing

# Viewing Angle

- On Axis : Normal GRBs
- Off-Axis : X-Ray Flashes, UV Flashes(?)
- Equator : Supernovae

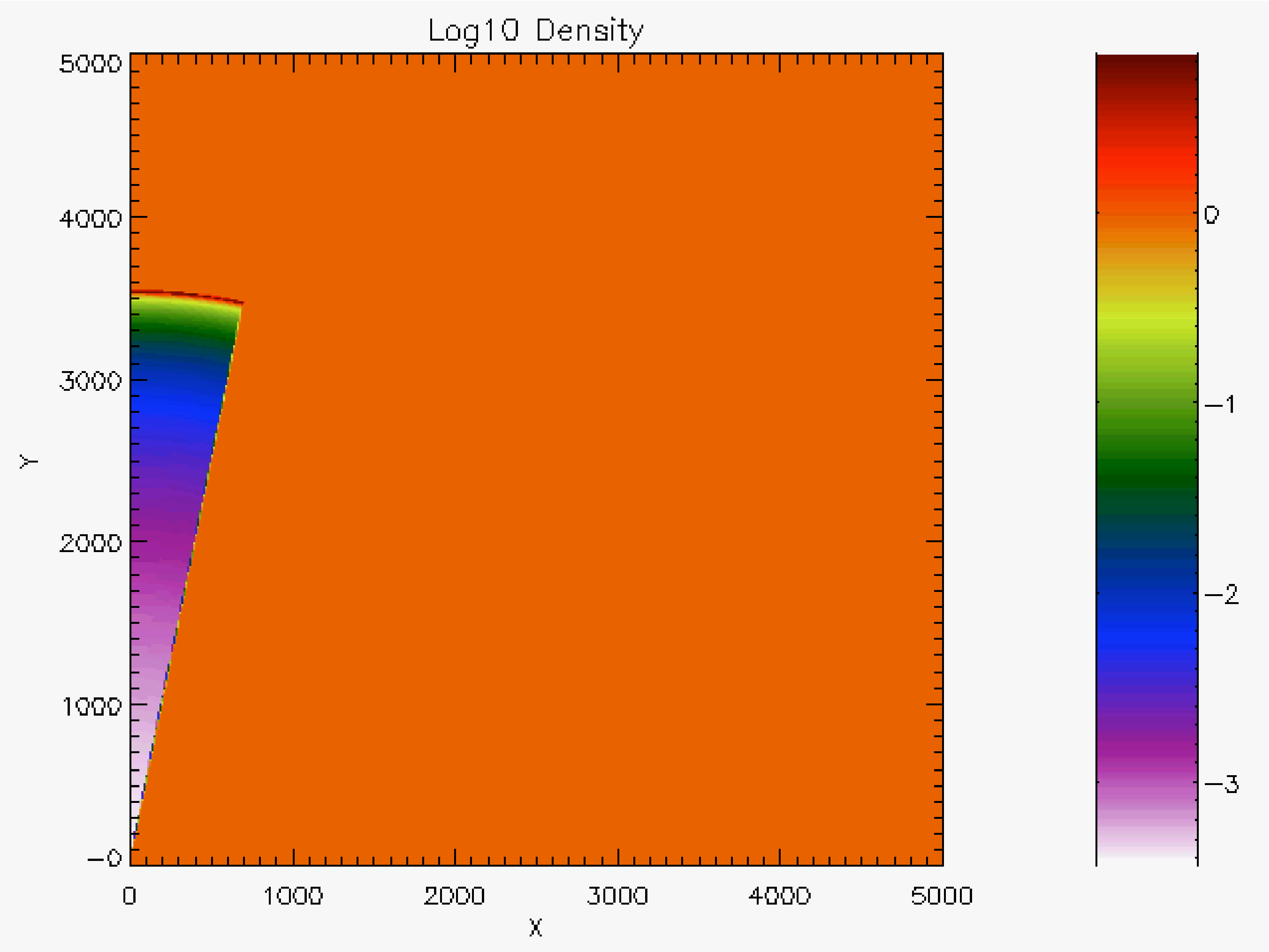
# Jets in Blue Super Giants






Woosley 2001

# From Relativistic To Newtonian



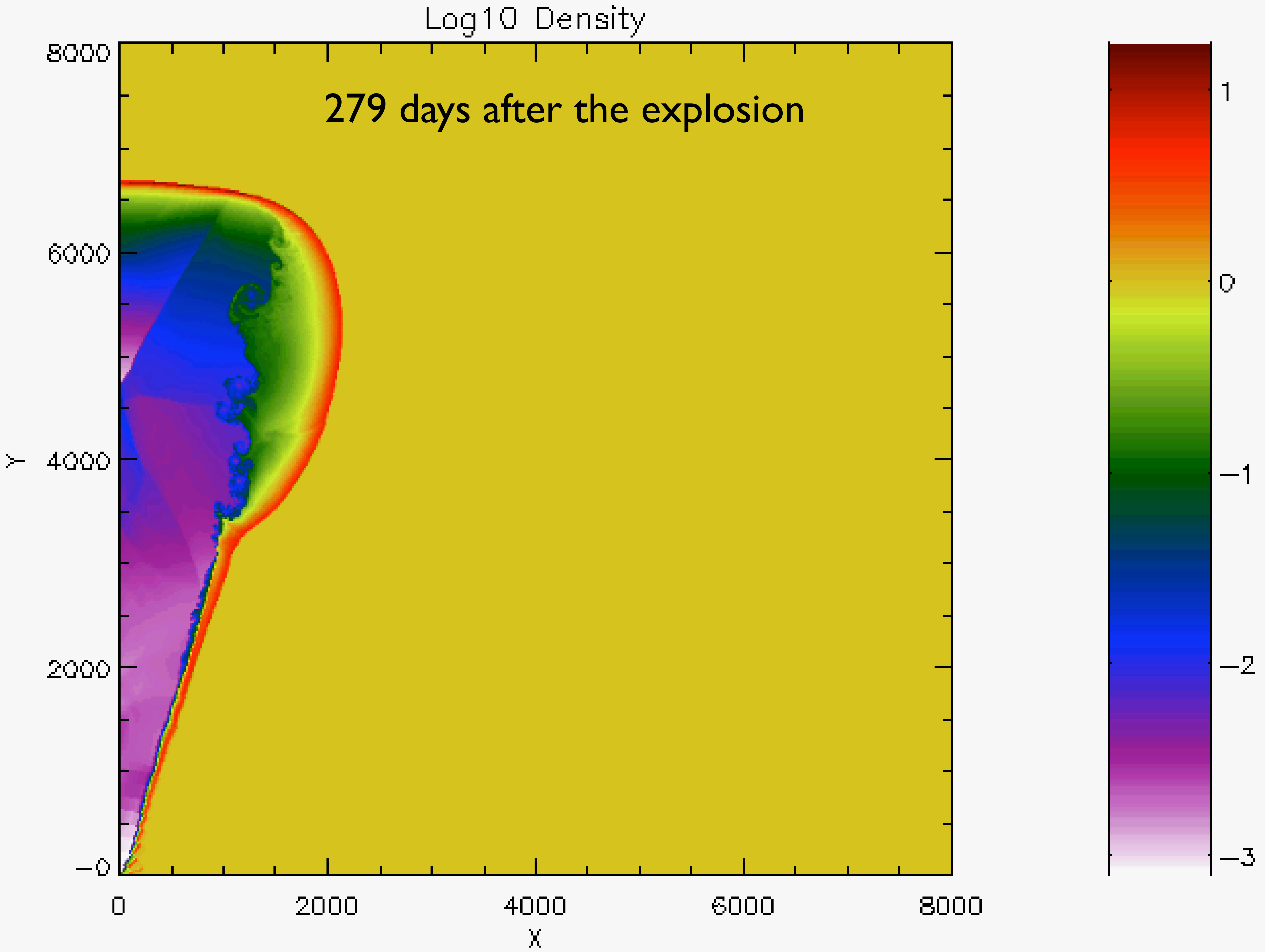

Gamma: 20

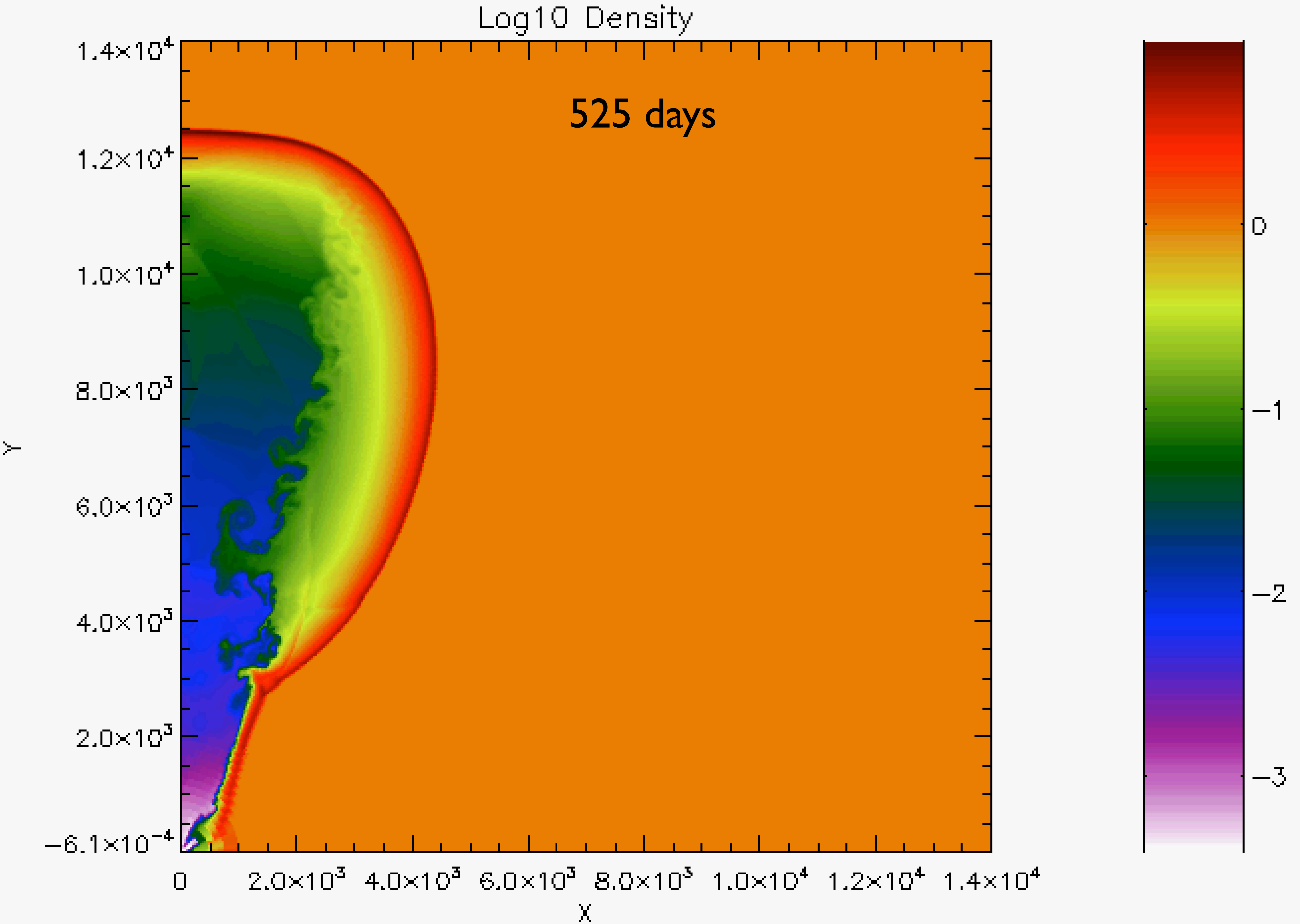
Opening angle : 0.2

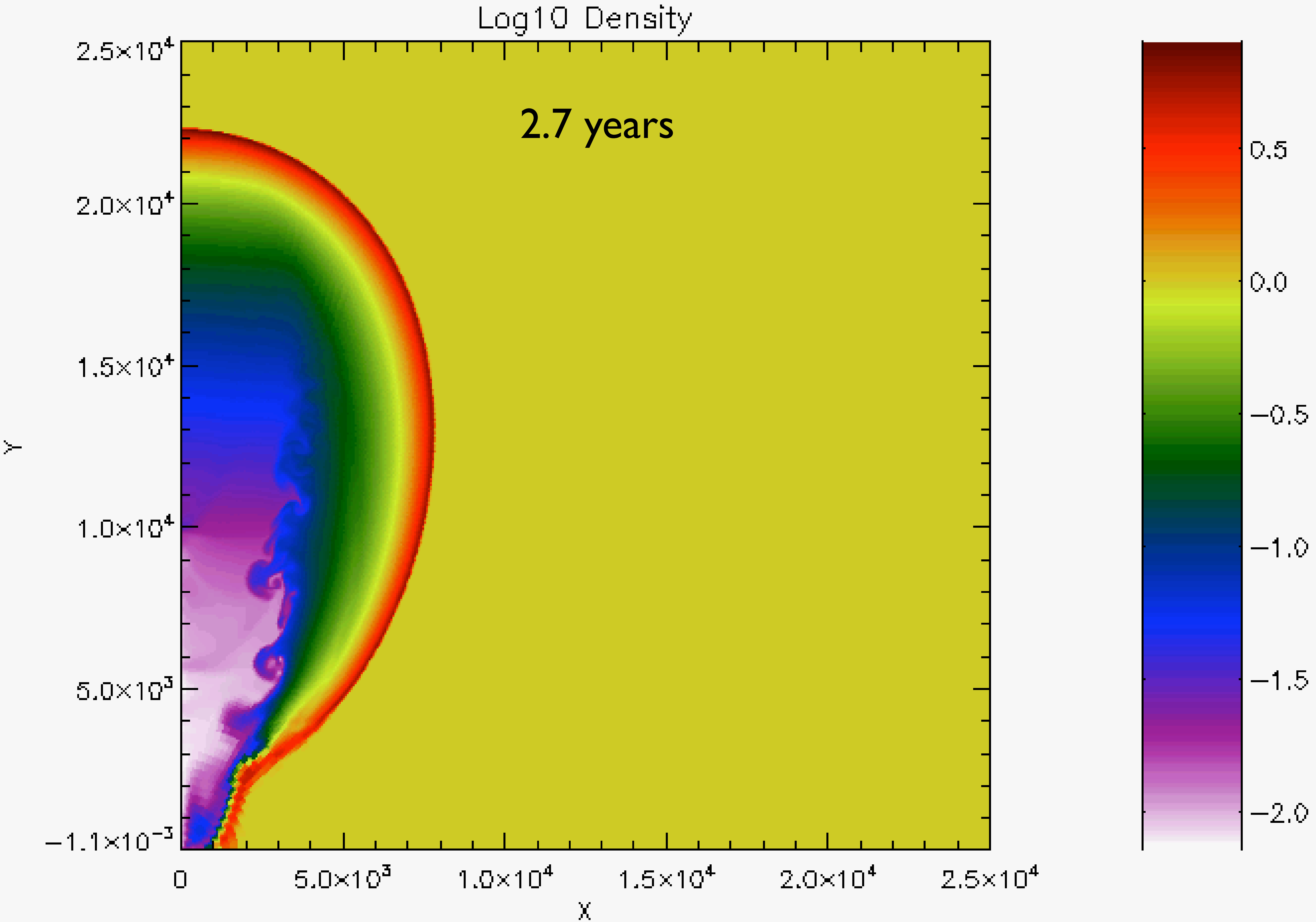
Energy: 2e51 erg

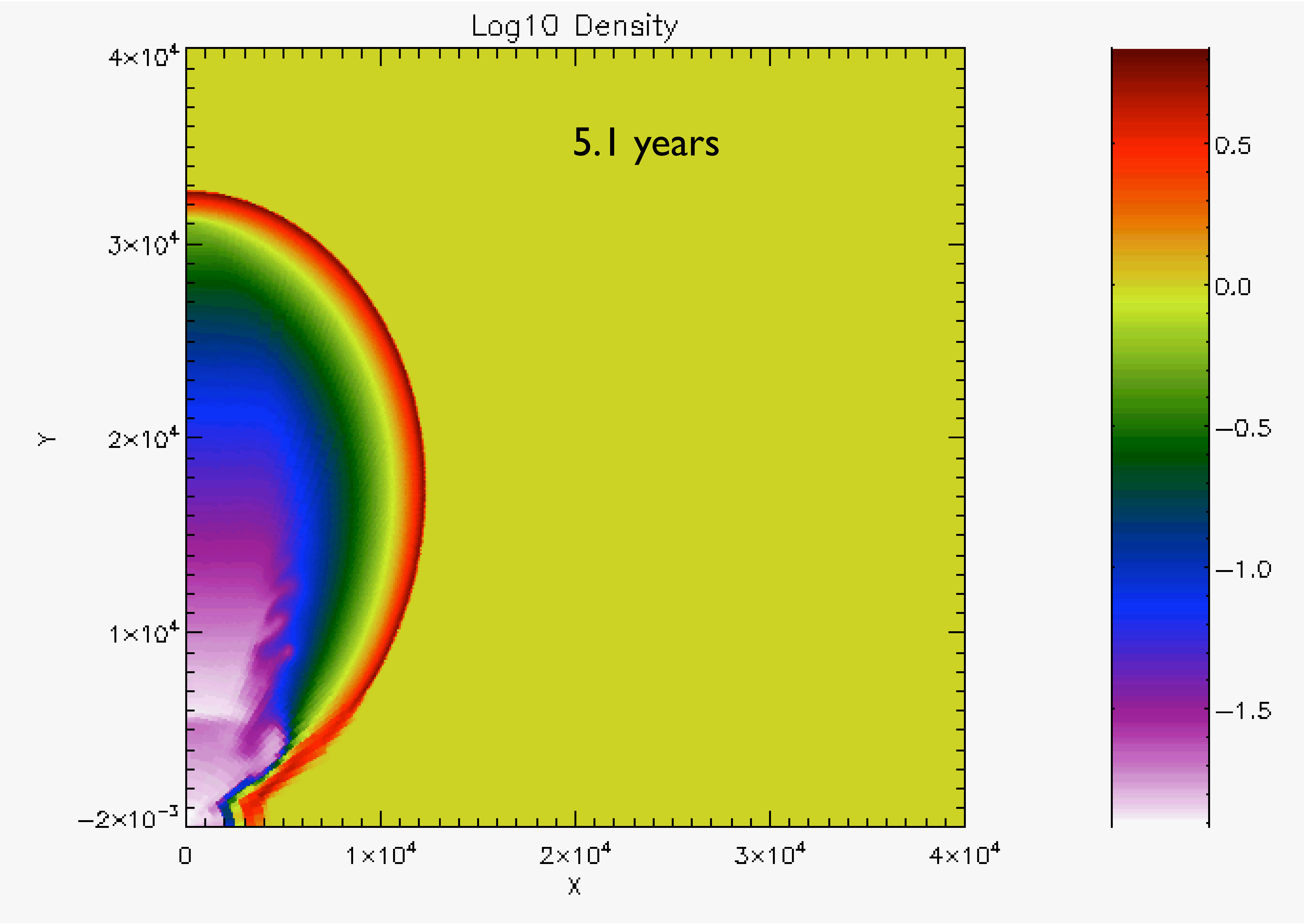
Medium:
1proton/cm^3

Initial: Blandford-McKee (Extremely Thin Shell!!!)

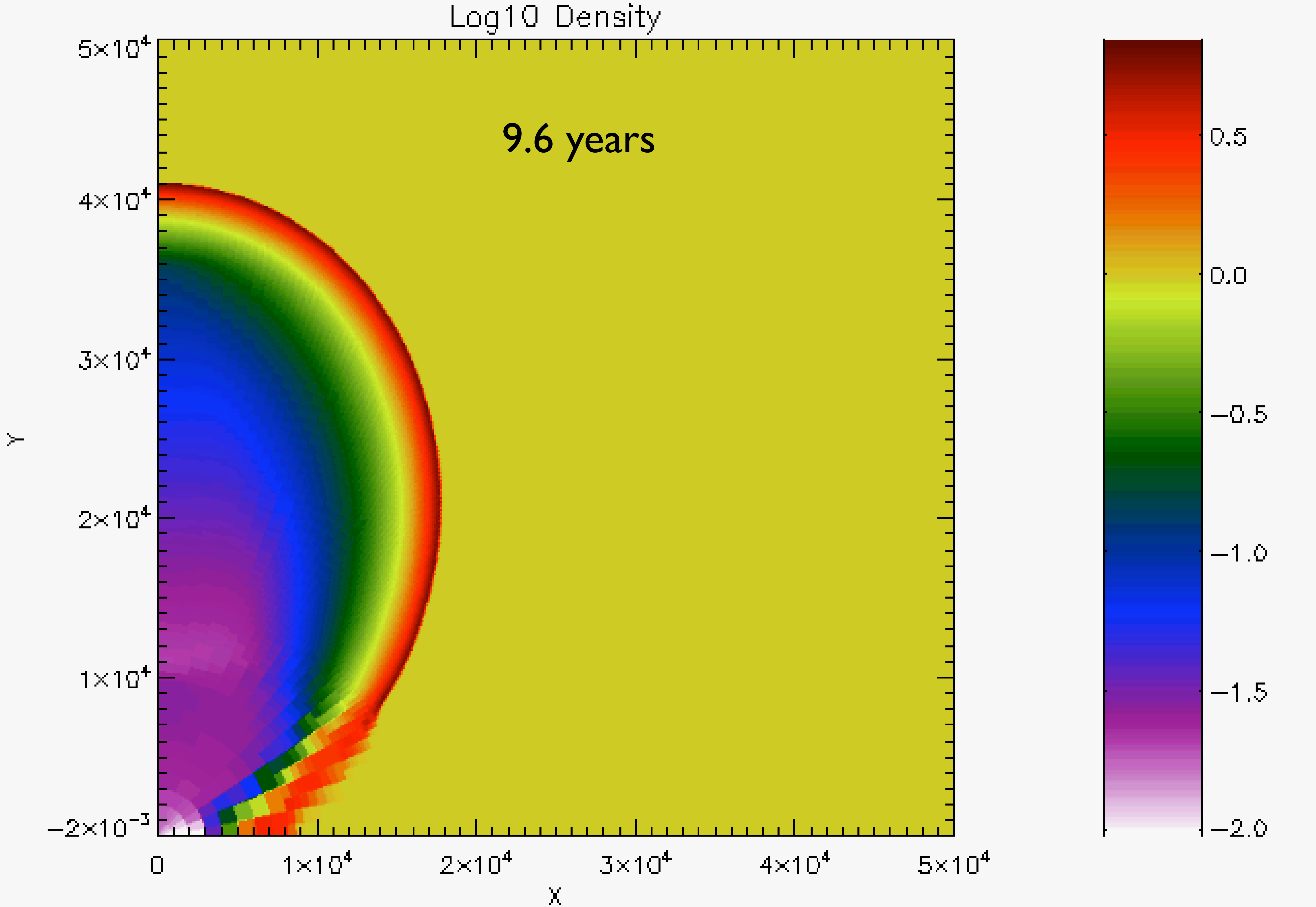

Log10 Density
279 days after the explosion
8000
6000
4000
2000
−0
Y
0
2000
4000
6000
8000
X
1
0
−1
−2
−3


Log10 Density
525 days
1.4×10⁴
1.2×10⁴
1.0×10⁴
8.0×10³
6.0×10³
4.0×10³
2.0×10³
−6.1×10⁻⁴
Y
0
2.0×10³
4.0×10³
6.0×10³
8.0×10³
1.0×10⁴
1.2×10⁴
1.4×10⁴
X
0
−1
−2
−3

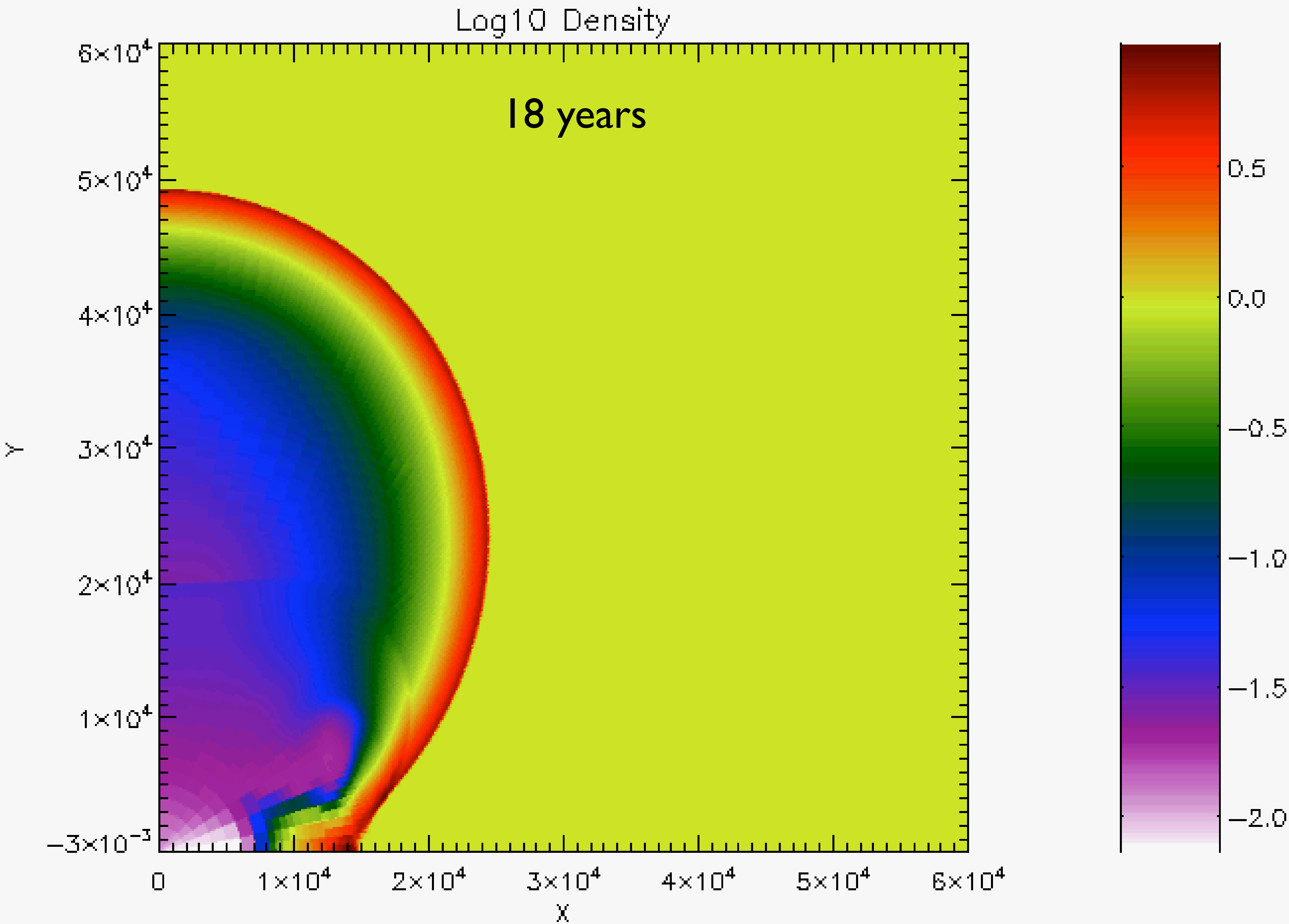

Log10 Density
2.7 years
2.5×10⁴
2.0×10⁴
1.5×10⁴
1.0×10⁴
5.0×10³
−1.1×10⁻³
Y
0
5.0×10³
1.0×10⁴
1.5×10⁴
2.0×10⁴
2.5×10⁴
X
0.5
0.0
−0.5
−1.0
−1.5
−2.0



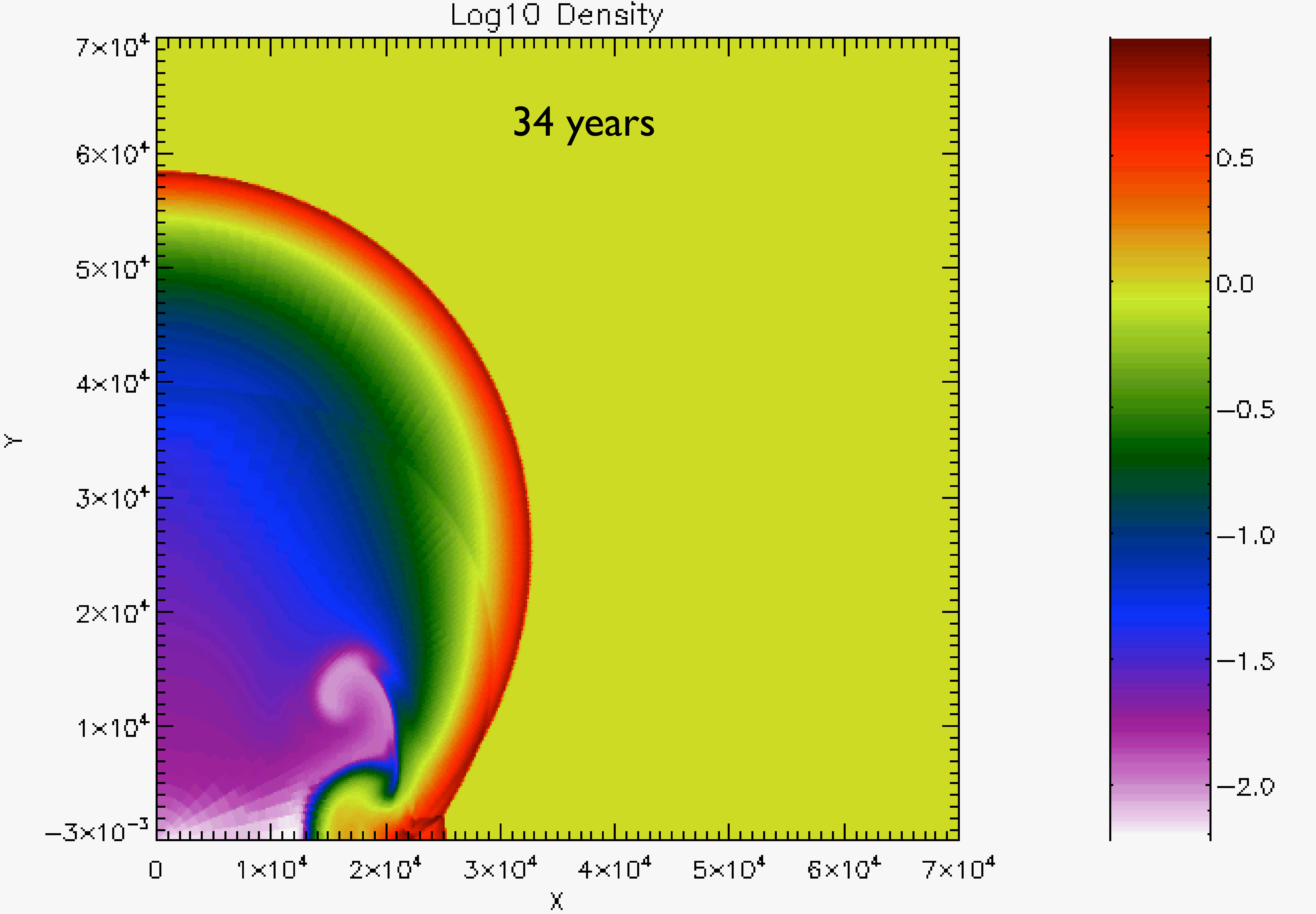
Log10 Density
5.1 years
4×10⁴
3×10⁴
2×10⁴
1×10⁴
−2×10⁻³
Y
0
1×10⁴
2×10⁴
3×10⁴
4×10⁴
X
0.5
0.0
−0.5
−1.0
−1.5



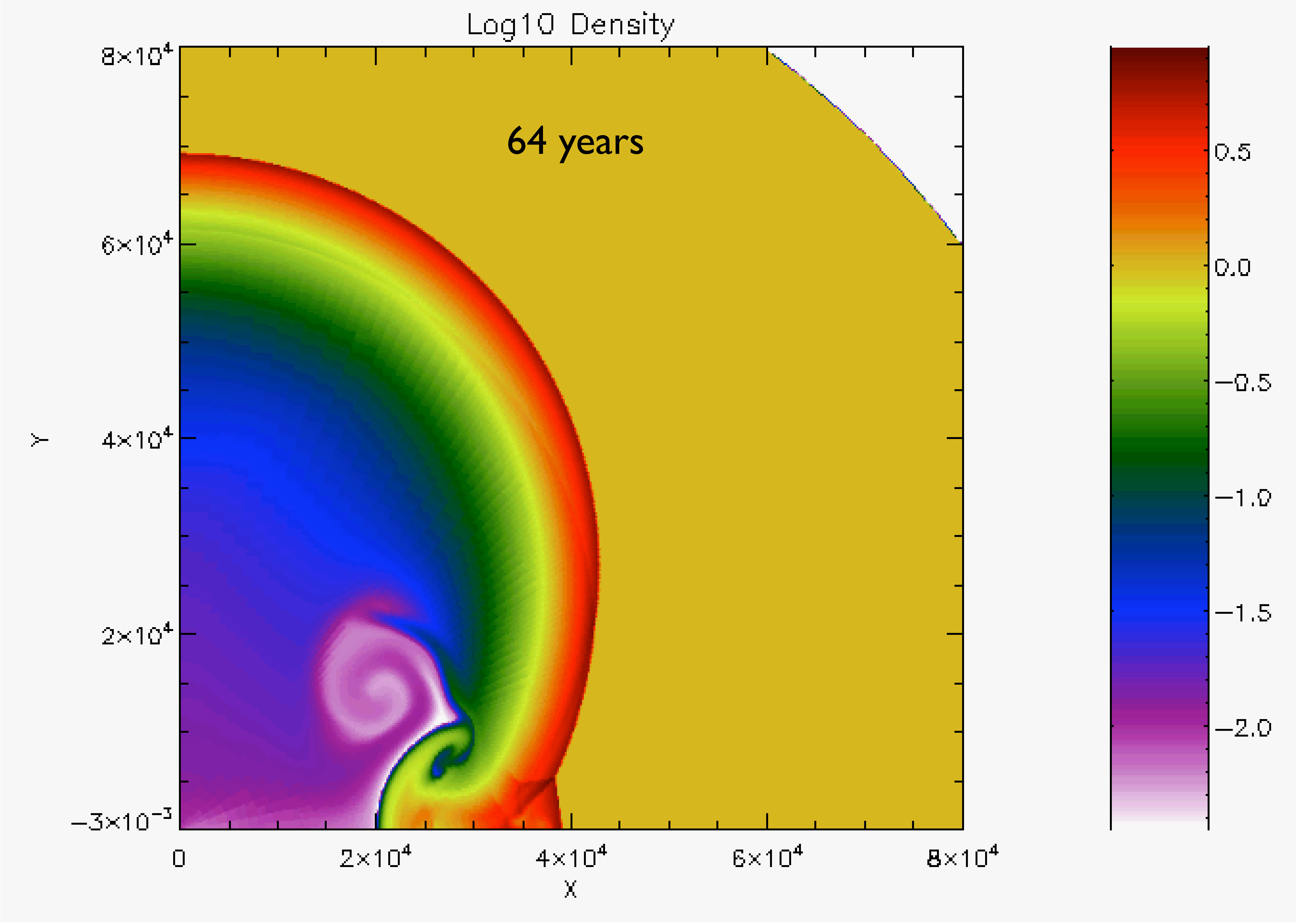
Log10 Density
9.6 years
5×10⁴
4×10⁴
3×10⁴
2×10⁴
1×10⁴
−2×10⁻³
Y
0
1×10⁴
2×10⁴
3×10⁴
4×10⁴
5×10⁴
X
0.5
0.0
−0.5
−1.0
−1.5
−2.0



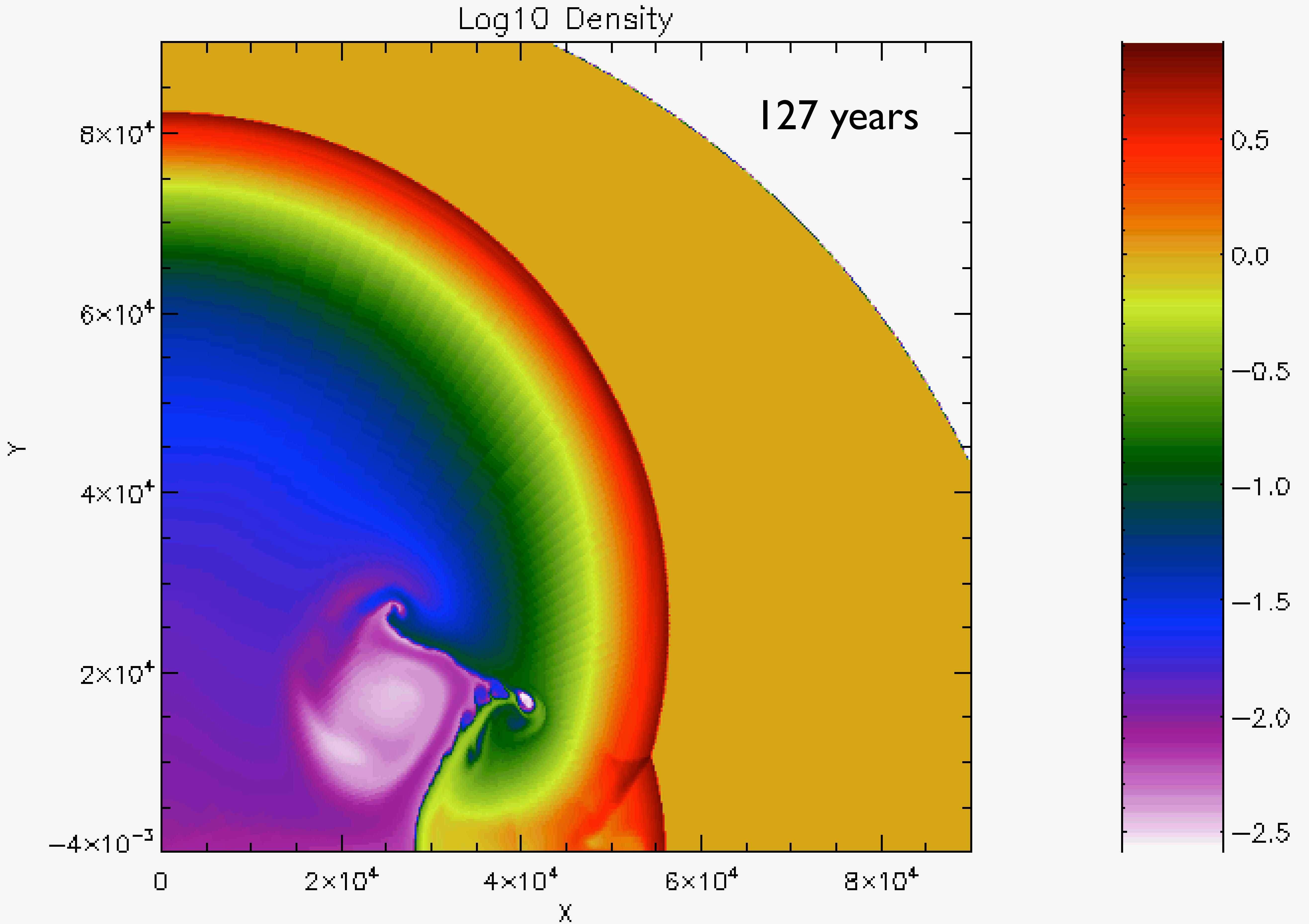
Log10 Density
18 years
6×10⁴
5×10⁴
4×10⁴
3×10⁴
2×10⁴
1×10⁴
−3×10⁻³
Y
0
1×10⁴
2×10⁴
3×10⁴
4×10⁴
5×10⁴
6×10⁴
X
0.5
0.0
−0.5
−1.0
−1.5
−2.0



Log10 Density
34 years
7×10⁴
6×10⁴
5×10⁴
4×10⁴
3×10⁴
2×10⁴
1×10⁴
−3×10⁻³
0
1×10⁴
2×10⁴
3×10⁴
4×10⁴
5×10⁴
6×10⁴
7×10⁴
X
Y
0.5
0.0
−0.5
−1.0
−1.5
−2.0



Log10 Density
64 years
8×10⁴
6×10⁴
4×10⁴
2×10⁴
−3×10⁻³
0
Y
X
0.5
0.0
−0.5
−1.0
−1.5
−2.0



Log10 Density
127 years
8×10⁴
6×10⁴
4×10⁴
2×10⁴
−4×10⁻³
Y
0
2×10⁴
4×10⁴
6×10⁴
8×10⁴
X
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5