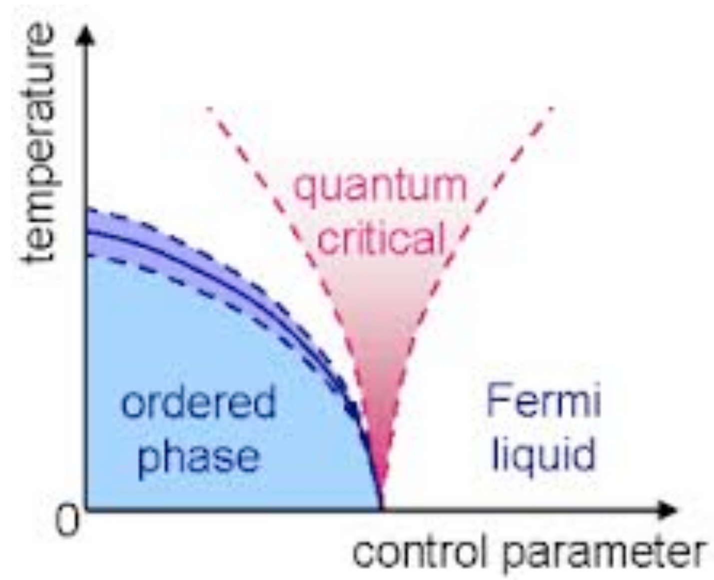
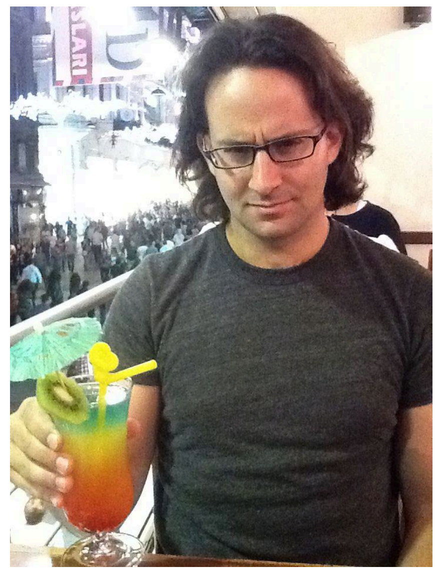
Field theories of quantum critical metals



Talk for "JoeFest," February 2014

Shamit Kachru (based on several collaborative works!)

My thinking about this subject has been shaped by collaborators on three recent papers:

arXiv 1307.0004
arXiv 1312.3321
arXiv 1402.xxxx

with Liam Fitzpatrick
Jared Kaplan
Sri Raghu
and (most recent one)
Steve Kivelson

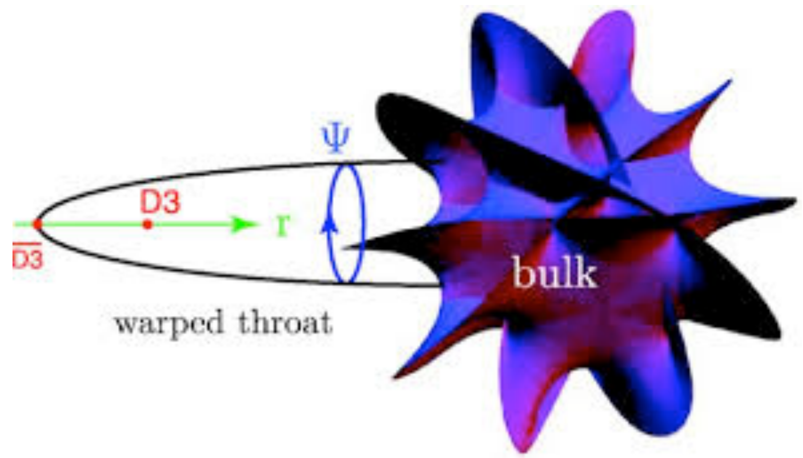
This subject is a fitting one for the occasion.

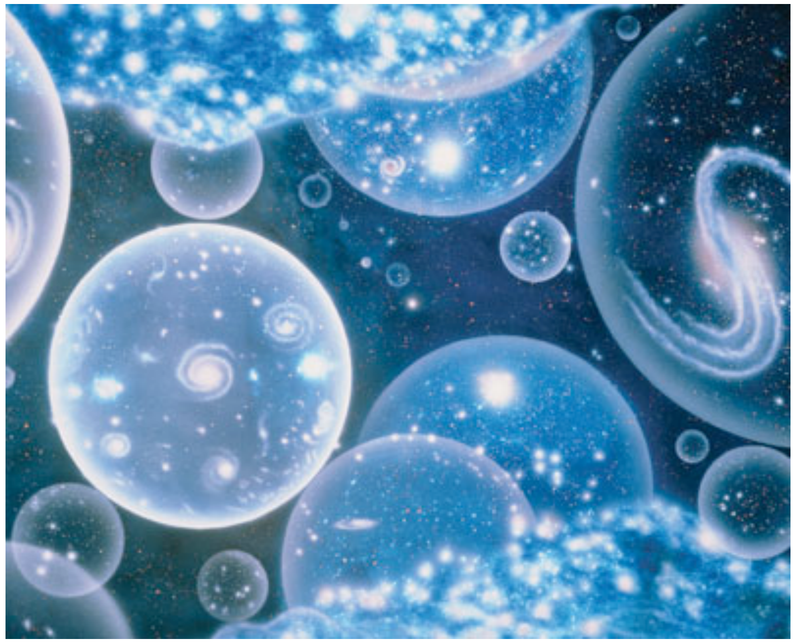
I first met Joe at TASI 1992, which he was co-directing.

His lectures on "Effective field theory and the Fermi surface" were aimed at educating high energy theorists about the fun mysteries of high Tc.

(They are now a standard reference for the QFT approach to Fermi liquid theory.)

Much later, in 2001, I was lucky to collaborate with Joe (and Steve Giddings) on a paper about flux compactification of string theory and warped geometries.





This collaboration shaped the direction of my research for 5 years and was a tremendous learning experience for me.

1. The basic problem and setup

Since this is a mixed audience, let me review the basic physics we're trying to understand. Many metals are well described by adiabatic modifications of a free electron model:

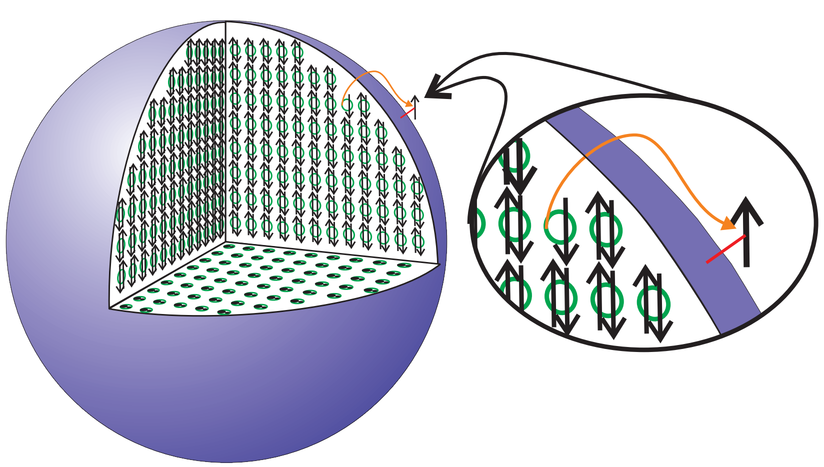


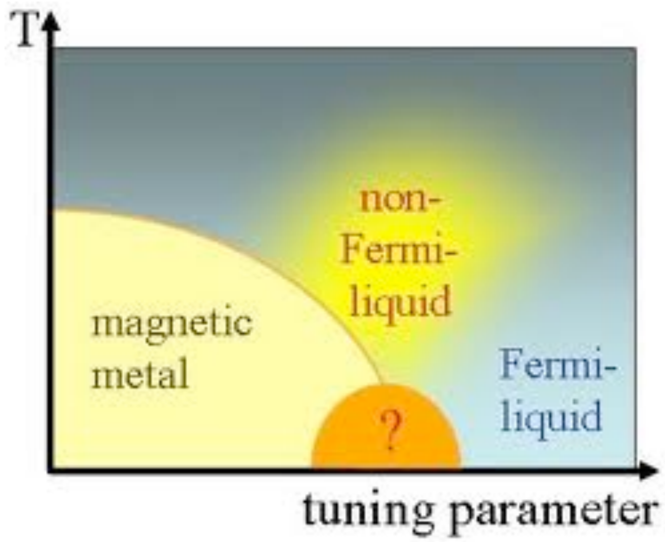
Figure 1: The ground state of the free Fermi gas in momentum space. All the states below the Fermi surface are filled with both a spin-up and a spin-down electron. A particle-hole excitation is made by promoting an electron from a state below the Fermi surface to an empty one above it.

Fairly robustly in the simplest setting, fermions contribute:

$$C_V \sim T$$

$$\rho \sim T^2$$

There are setting in modern materials, which many people in Santa Barbara understand better than me, where this breaks down:



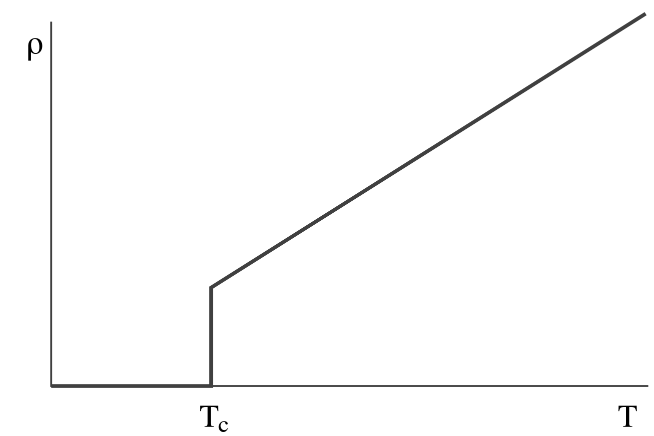


Figure 6: Resistivity versus temperature in a typical high-T_c material: zero below T_c, and linear above.

This is hard to incorporate in the minimal effective theory that e.g. Joe studied. Lets start with his theory (i.e. Landau's Fermi liquid, ala Polchinski and Shankar).

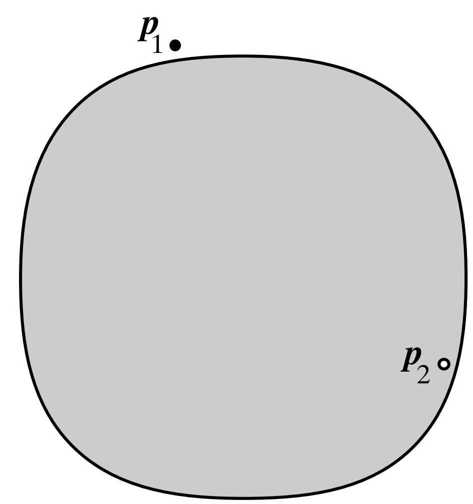


Figure 1: Fermi sea (shaded) with two low-lying excitations, an electron at $\mathbf{p}_1$ and a hole at $\mathbf{p}_2$.

The free action governing these basic quasiparticles is:

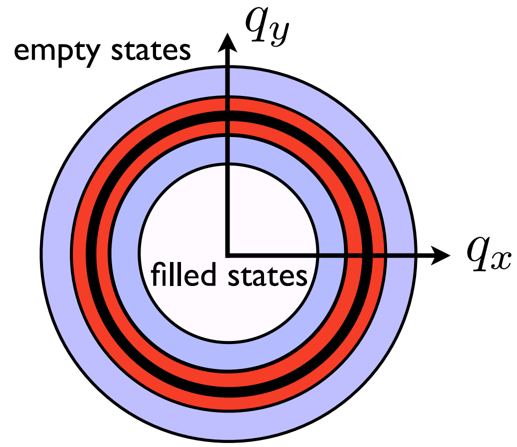
$$\int dt\, d^3\mathbf{p} \left\{ i\psi_\sigma^\dagger(\mathbf{p})\partial_t\psi_\sigma(\mathbf{p}) - (\varepsilon(\mathbf{p}) - \varepsilon_{\mathrm{F}})\psi_\sigma^\dagger(\mathbf{p})\psi_\sigma(\mathbf{p}) \right\}.$$

What about interactions? Lets study an RG and see if this free theory is stable or not.

The RG is a bit unusual for high energy theorists. One scales towards the Fermi surface:

$$\mathbf{p} = \mathbf{k} + \mathbf{l},$$

$$\varepsilon(\mathbf{p}) - \varepsilon_{\mathrm{F}} = l v_{\mathrm{F}}(\mathbf{k}) + O(l^2),$$



$$dt \to s^{-1}dt, \quad d\mathbf{k} \to d\mathbf{k}, \quad d\mathbf{l} \to s\,d\mathbf{l}, \quad \partial_t \to s\partial_t, \quad l \to sl,$$

yielding a Fermi field that scales as

$$\psi \to s^{-1/2}\psi.$$

So, what dangerous terms could arise to destabilize the Fermi liquid?

* μ could be shifted. No problem.

* The leading possible interactions are four-Fermi:

$$\int dt\, d^2\mathbf{k}_1\, d\mathbf{l}_1\, d^2\mathbf{k}_2\, d\mathbf{l}_2\, d^2\mathbf{k}_3\, d\mathbf{l}_3\, d^2\mathbf{k}_4\, d\mathbf{l}_4\, V(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$
$$\psi^\dagger_\sigma(\mathbf{p}_1)\psi_\sigma(\mathbf{p}_3)\psi^\dagger_{\sigma'}(\mathbf{p}_2)\psi_{\sigma'}(\mathbf{p}_4)\delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4).$$

Naively, these scale to zero $\sim s$.

This is too quick:

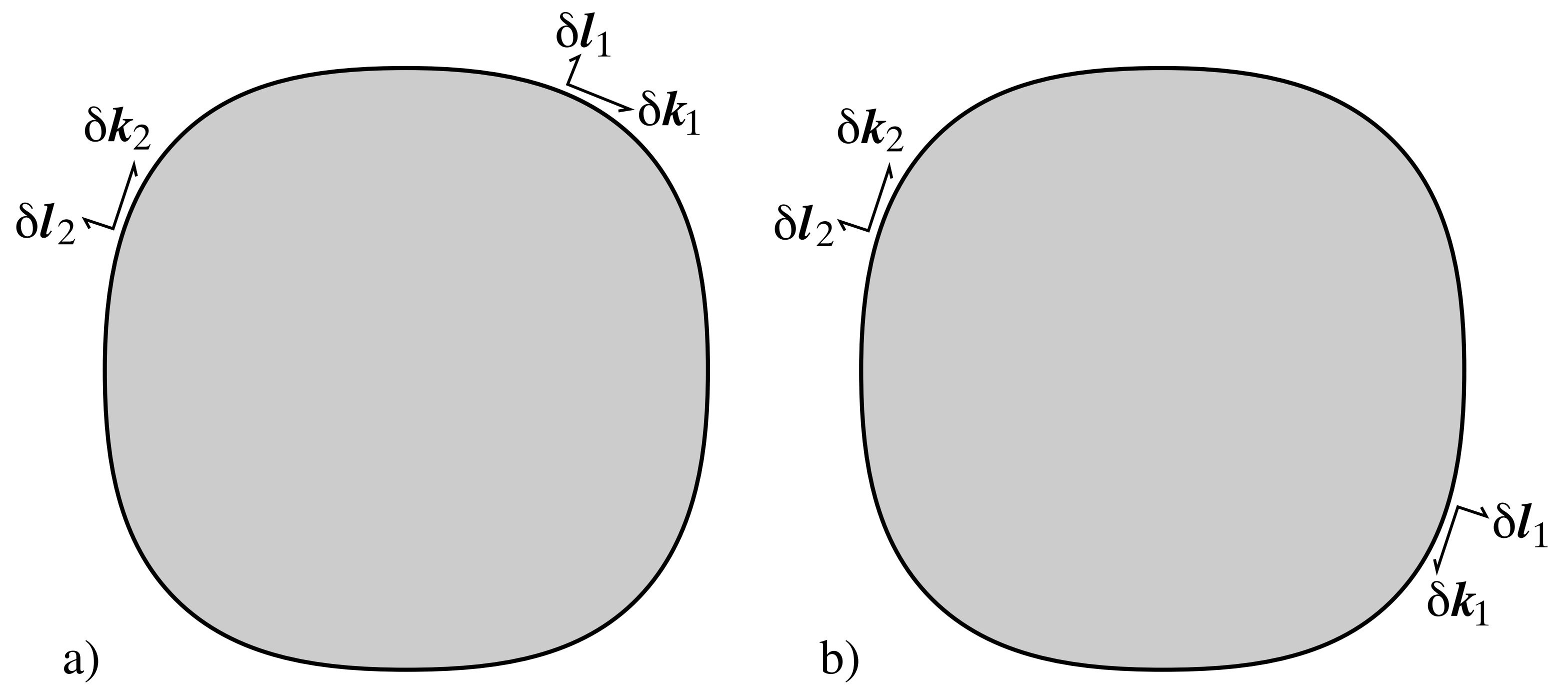


Figure 3: a) For two generic points near a two-dimensional Fermi surface, the tangents $\delta\mathbf{k}_i$ are linearly independent. b) For diametrically opposite points on a parity-symmetric Fermi surface, the tangents are parallel.

For antipodal momenta, the delta function scales as s^{-1} yielding a marginal "BCS" operator.

This gives the only instability of the fixed point.

Since the standard theory cannot explain the non-Fermi liquid scalings, what can we add?

These systems exhibit phase transitions (nematic order, magnetic order, ...). Natural to add a bosonic order parameter field. So, we (and others) have tried to find interesting behaviors in this theory:

$$
\begin{aligned}
S &= \int d\tau \int d^d x \, \{\mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_{\psi,\phi}\} \\
\mathcal{L}_\psi &= \bar{\psi}_\sigma \left[\partial_\tau + \mu - \varepsilon(i\nabla)\right] \psi_\sigma + \lambda_\psi \bar{\psi}_\sigma \psi_{\sigma'} \bar{\psi}_{\sigma'} \psi_\sigma \\
\mathcal{L}_\phi &= m_\phi^2 \phi^2 + (\partial_\tau \phi)^2 + c^2 \left(\vec{\nabla}\phi\right)^2 + \lambda_\phi (\phi\phi)^2
\end{aligned}
$$

The bosons and fermions are coupled by a Yukawa function:

$$S_{\psi,\phi} = \int \frac{d^{d+1}k d^{d+1}q}{(2\pi)^{2(d+1)}} g(\boldsymbol{k},\boldsymbol{q}) \bar{\psi}_\sigma(k)\psi_\sigma(k+q)\phi(q)$$

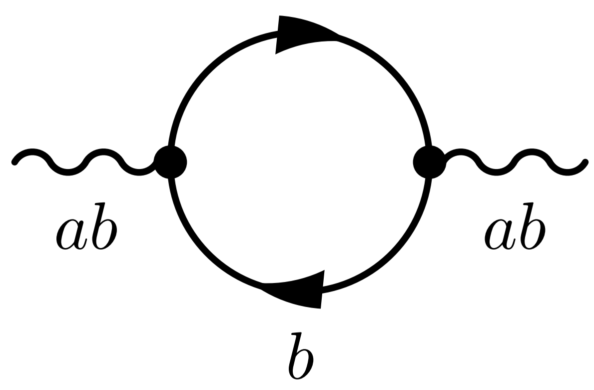
$$g(\mathbf{k}_F) \sim \cos(k_F^x) - \cos(k_F^y)$$

Can the addition of the bosonic order parameter to the story lead to interesting new (approximate) fixed points?

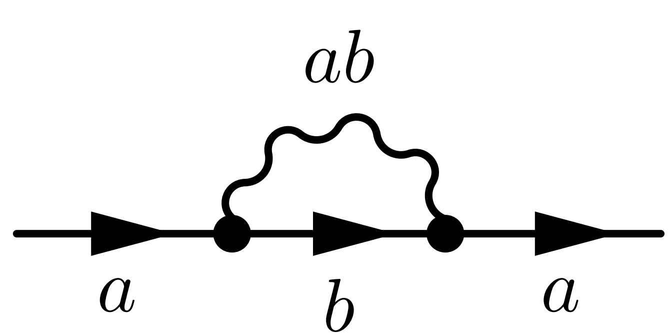
II. Basic conflict in the coupled system

The coupling leads to a titanic struggle between bosonic and fermionic degrees of freedom:

The fermions can damp the bosons (intuitively, a boson can decay into a fermion pair):



Meanwhile, the bosons can dress the fermions into a non-Fermi liquid:



A standard approach builds on the venerable "Hertz-Millis theory":

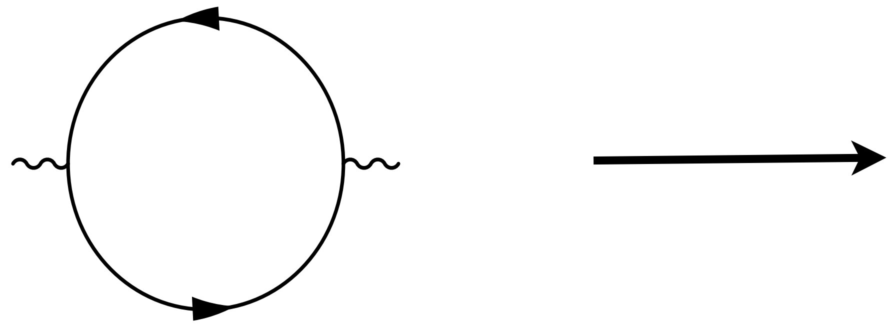
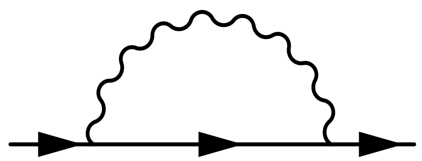


FIG. 2: The one-loop boson self energy.

$$D(q) = \frac{1}{N}\left(c_b\frac{|q_\tau|}{|q_y|} + \frac{q_y^2}{e^2} + r\right)^{-1}.$$

Extrapolating to very low energy, this gives a "z=3" boson. Feeding this back into the fermion self-energy:



can now yield a non-Fermi liquid, if one allows the correction to dominate at low-energy

A long line of work building in this direction exists:

Hertz, 1976

Millis, 1993

Polchinski, 1994

Nayak, Wilczek, 1994

Oganesyan, Kivelson, Fradkin 2001

Chubukov et al, 2006

S.S. Lee, 2009

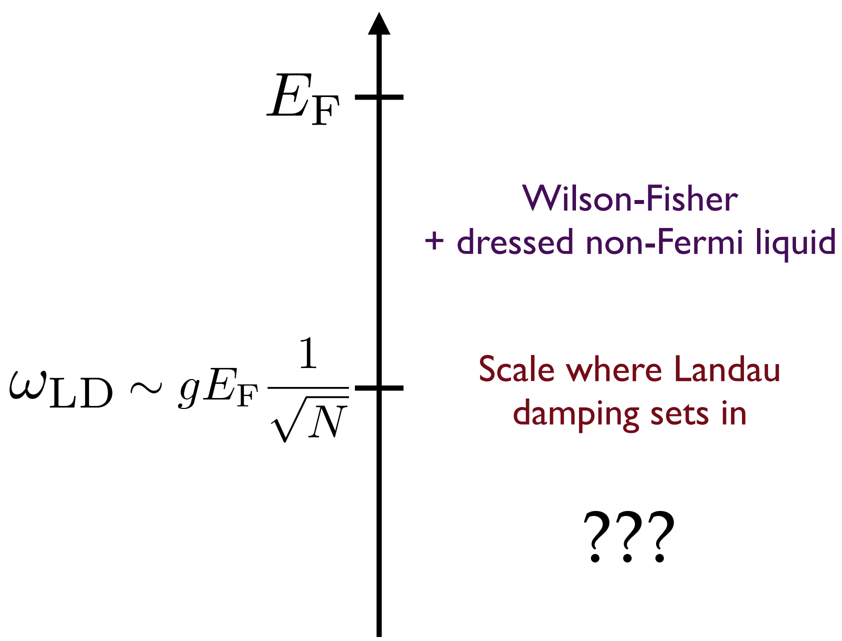
Metlitski, Sachdev 2010

Mross, McGreevy, Liu, Senthil 2010

....

We go in a different direction.

We will only do systematic perturbation theory about the UV, and look for non-Fermi liquid fixed points or intermediate scaling regimes that we can reliably access that way.



As a function of energy, we will estimate scales where corrections become so large that our analysis breaks down. Sometimes these can be pushed to zero.

But in a problem with IR "domes," finding an intermediate scaling regime governed by an approximate fixed point is just fine.

III. Our analysis: small v/c (and/or large N)

The theory we are perturbing by the Yukawa interaction (including the Yukawa itself)

$$
\begin{aligned}
S &= \int d\tau \int d^d x \, \{\mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_{\psi,\phi}\} \\
\mathcal{L}_\psi &= \bar{\psi}_\sigma \left[\partial_\tau + \mu - \varepsilon(i\nabla)\right] \psi_\sigma + \lambda_\psi \bar{\psi}_\sigma \psi_{\sigma'} \bar{\psi}_{\sigma'} \psi_\sigma \\
\mathcal{L}_\phi &= m_\phi^2 \phi^2 + (\partial_\tau \phi)^2 + c^2 \left(\vec{\nabla}\phi\right)^2 + \lambda_\phi (\phi\phi)^2
\end{aligned}
$$

has an upper critical dimension of d=3+1.

So, we study it in the epsilon expansion, and in perturbation theory in "g", with two additional handles to control it:

a) We can consider a matrix large N boson coupled to flavors of fermions (with boson self-interactions treated in such a way that they give an $O(N^2)$ Wilson-Fisher like model)

b) The system is non-relativistic, and characterized by a fermion velocity v and a boson velocity c. We will see that the ratio v/c can serve as a small parameter allowing us to see what happens at N=1.

I will focus on b), simply mentioning what N does when relevant .

The boson self-energy can be evaluated in a standard way:

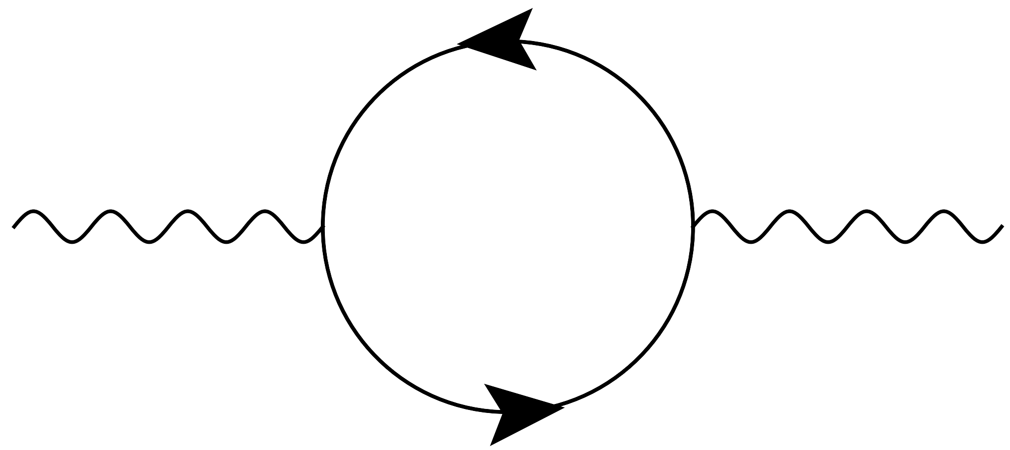


FIG. 1: The one-loop diagram contributing to Landau damping.

$$\Pi_{d=3}(q_0, q) = \frac{g^2 k_F^2}{2\pi^2 v}\left[1 + \frac{q_0}{2vq}\log\frac{q_0 - vq}{q_0 + vq}\right]$$

When the bosons are fast -- $v < c$ -- it is purely real. This reflects the kinematics that bosons cannot decay to fermions in this limit.

In fact, in the small v/c limit, the leading behavior is

$$\Pi(q_0, q) \sim v \frac{g^2 k_F^2}{2\pi^2} \frac{q^2}{q_0^2} + \mathcal{O}(v^2),$$

indicating that small v should broaden an energy range where the bosonic "dressing" of the fermions dominates over Landau damping.

Matrix large N will only help any such statement - the Landau damping is a 1/N effect relative to the fermion self-energy - so we will simply analyze the theory at N=1, with this in mind.

Now, we do a basic RG about the two decoupled UV fixed points. This fixes our scaling to be as in the picture:

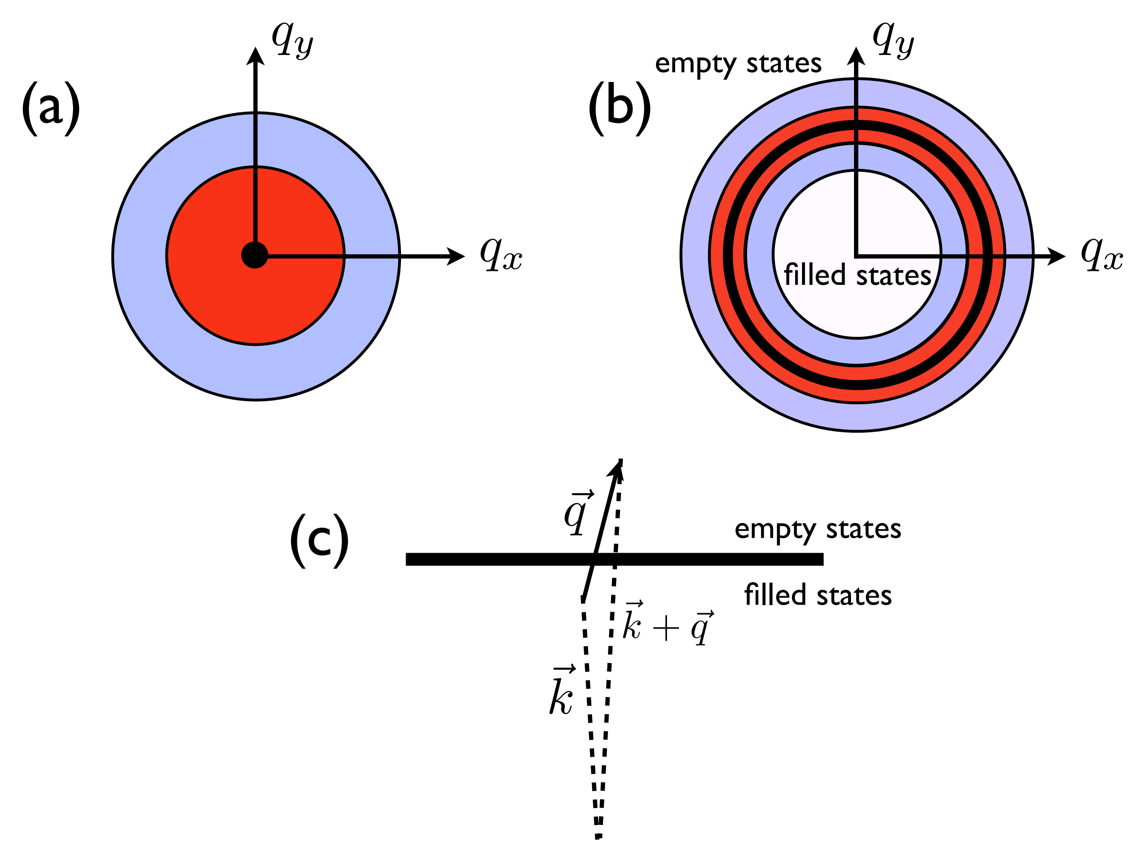


FIG. 2. Summary of tree-level scaling. High energy modes (blue) are integrated out at tree level and remaining low energy modes (red) are rescaled so as to preserve the boson and fermion kinetic terms. The boson modes (a) have the low energy locus at a point whereas the fermion modes (b) have their low energy locus on the Fermi surface. The most relevant Yukawa coupling (c) connects particle-hole states separated by small momenta near the Fermi surface; all other couplings are irrelevant under the scaling.

RG equations can be derived by decimating in energies and momenta in a Wilsonian manner. The resulting system of equations is as follows. First off:

$$\frac{dc}{dt} = 0, \qquad \frac{d\lambda_\phi}{dt} = -\beta_{\lambda_\phi} = \epsilon\lambda_\phi - \frac{3\lambda_\phi^2}{16\pi^2} + \mathcal{O}(\lambda_\phi^3)$$

The bosons flow to a Wilson-Fisher fixed point, with g not perturbing the flow at this order.

The other parameters flow in an interesting way:

$$\begin{aligned}
\frac{dv}{dt} &= -\beta_v = -\frac{g^2}{(2\pi c)^2}\mathcal{S}(v, w, ...), +\mathcal{O}(\lambda_\phi^2, g^2), \\
\frac{dg}{dt} &= -\beta_g = g\left(\frac{\epsilon}{2} - 2\frac{g^2}{4\pi^2 c^2(c + |v|)}\right) + \mathcal{O}(\lambda_\phi^2, g^2), \\
\frac{dw}{dt} &= -w[1 + \mathcal{O}(g^2)] \qquad (7)
\end{aligned}$$

Here, w is a new parameter we introduced:

$$\varepsilon(\mathbf{k}) - \mu = v\ell + w\ell^2 + \ldots$$

for reasons to become apparent shortly.

First, let us set w=0 and analyze the RG flow.

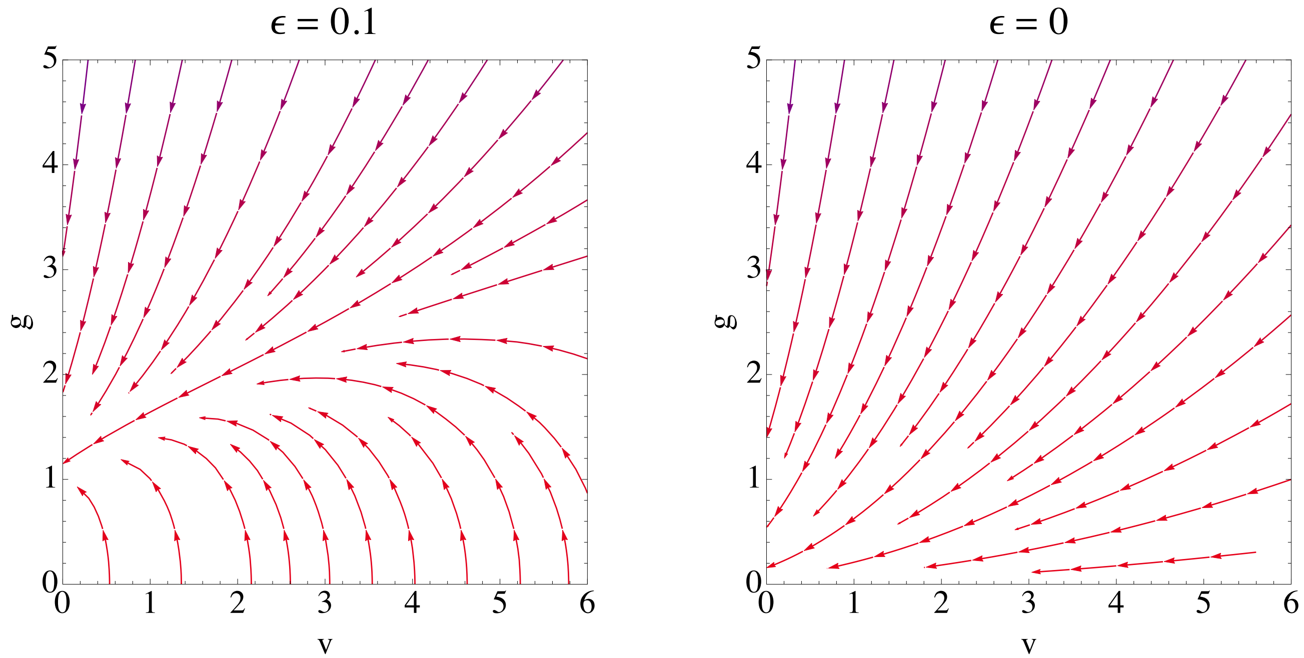


FIG. 2: Graphic depiction of RG flow for the parameters g and v when λ_ψ is (artificially) set to zero. The plot takes $d = 2.9$ ($d = 3$) on the left (right) plot, and units of $c = 1$. All flows point to $v = 0$, and g runs toward its fixed point value. Red (blue) indicates slower (faster) flow *(color online)*.

Important point: v/c flows towards zero. As small v acts in some sense as a control parameter to prevent boson decay from being important, this is physically interesting.

But, is the analysis under control? A reasonably conservative control criterion would be to say that the analysis breaks down at a scale which is defined by:

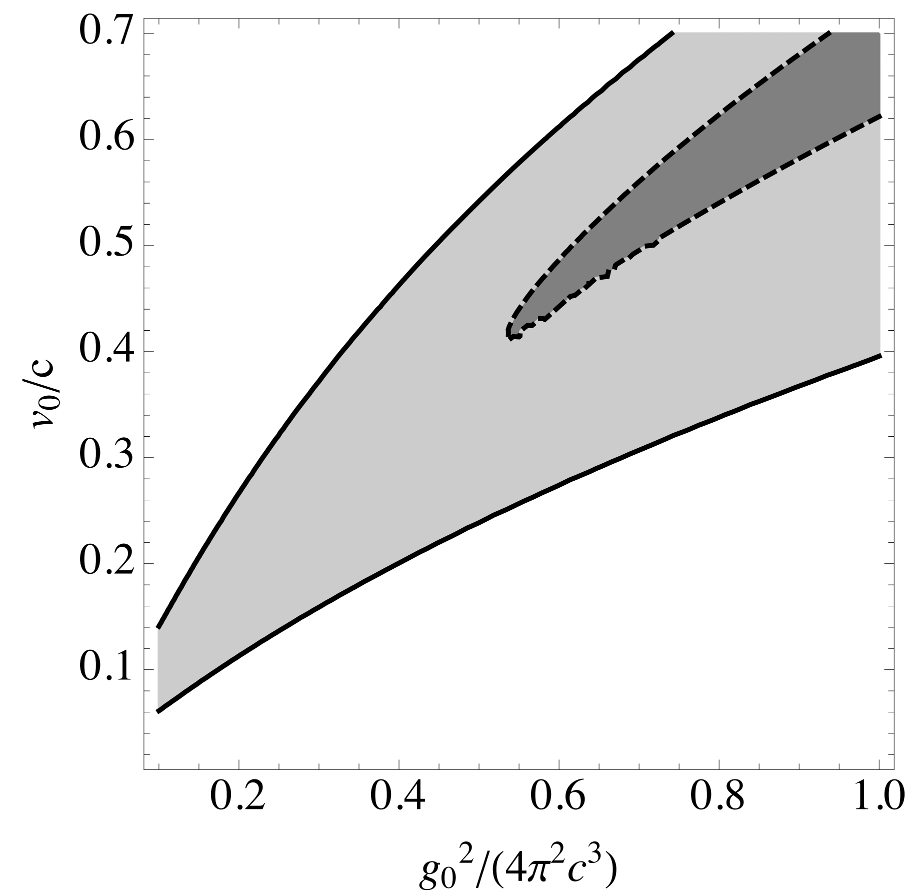
$$\frac{x^2|\Pi(\mu_{\rm LD}, \mu_{\rm LD}/cx)|}{\mu_{\rm LD}^2(1+x^2)} = 1$$

$$x = q_0/cq$$

i.e. one-loop correction competes in size with tree propagator

The natural thing to do would be to say x=1 (on-shell), but we are more conservative and choose x to maximize the ratio at any point along the flow.

Result:



Matrix "large" N rescales the correction by 1/N, the contours quickly become space-filling.

FIG. 3: The shaded region shows values of the UV fermion velocity v_0 for which $v(t)$ reaches zero before Landau damping would become important in $d = 3$. The light (dark) shaded gray region inside the solid (dashed) contour shows where the size of Landau damping is at most equal to (half of) the bare propagator along the RG trajectory into the IR.

However, in the shaded region where v hits zero before we ever encounter "large" damping corrections, there is eventually a very low energy scale where

$$v\mu \sim w\mu^2 \ .$$

re-introducing the "irrelevant" w...

A simple computation reveals:

$$\mu_w \sim \Lambda \exp[-v_0 c^2/b\bar{g}^2]$$

Above this scale, one is governed by the approximate fixed point of our RG equations. It is characterized by Wilson-Fisher bosons and a non-Fermi liquid with:

$$G_F \ = \ \frac{1}{\omega^{1-\frac{\epsilon}{2}}} \ f(\frac{\omega}{\ell})$$

scaling function f =1 at large N

By the end, this talk has devolved into some complicated figures. Therefore, I summarize the take-away messages:

1. Either small v/c or large (matrix) N allow one to access novel (intermediate?) fixed points in the natural field theory for quantum critical metals.

2. Because of the nature of the perturbative flows starting from the UV decoupled fixed point, small v becomes increasingly good in the IR.

3. At very low energy, the “irrelevant” w operator causes some new behavior. (Lifshitz transition?)