Entanglement and Geometry

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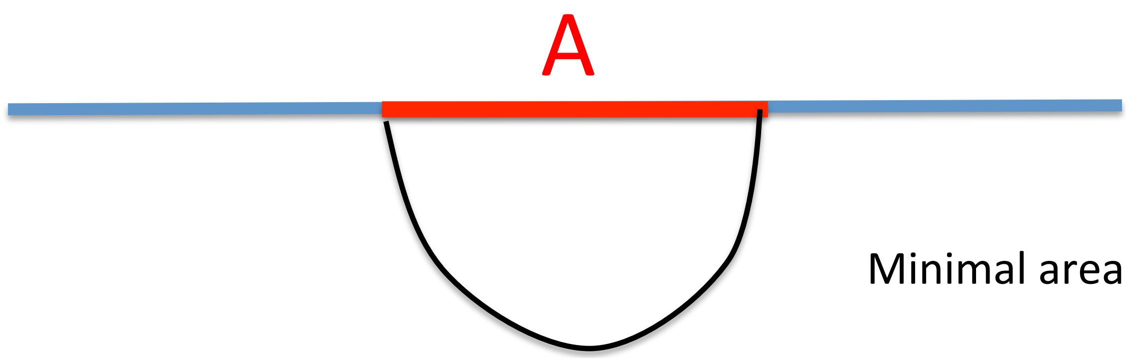
Entanglement in QFT

- Governed by the Renormalization Group
- Has been useful in Condensed Matter
- Has been useful as a tool to prove the c, f theorems. Casini, Huerta
- Has been useful to prove the Bekenstein bound and 2nd Law for black holes. Casini, Wall

$$\Delta S_{\text{ent}} \leq 2\pi \Delta K_{\text{Rindler}}$$

Entanglement from gravity



Minimal area surface in the bulk

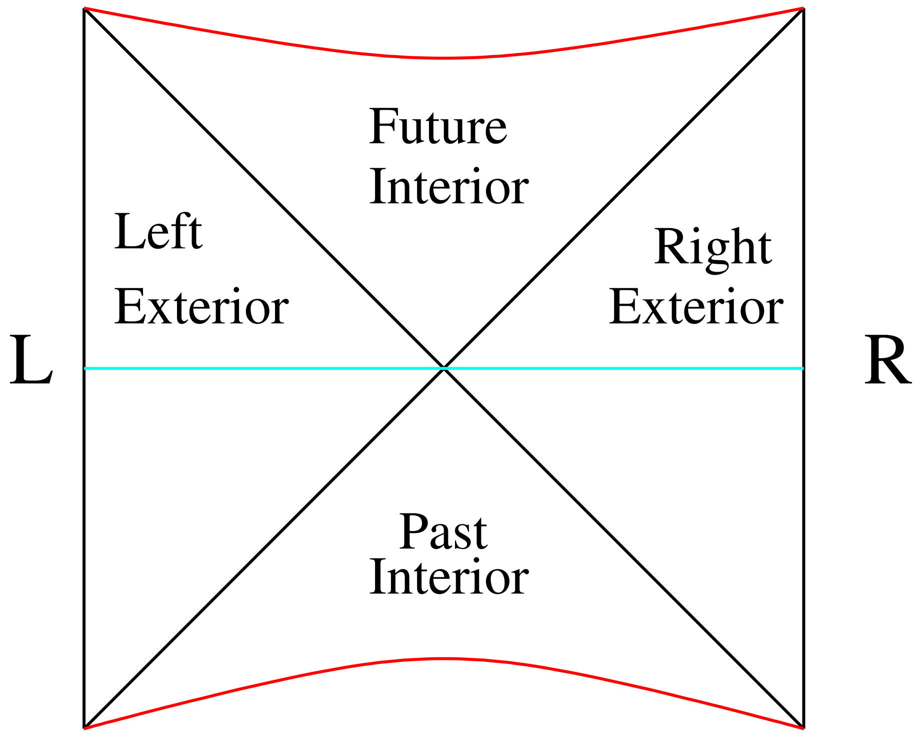
$$S_A = \frac{A_{\rm minimal}}{4G_N}$$

Ryu-Takayanagi
(Hubeny, Rangamani ..)

Fursaev, Headrick, Lewkowycz, JM

This a generalization of the Bekenstein-Hawking formula for black hole entropy

Eternal AdS black hole

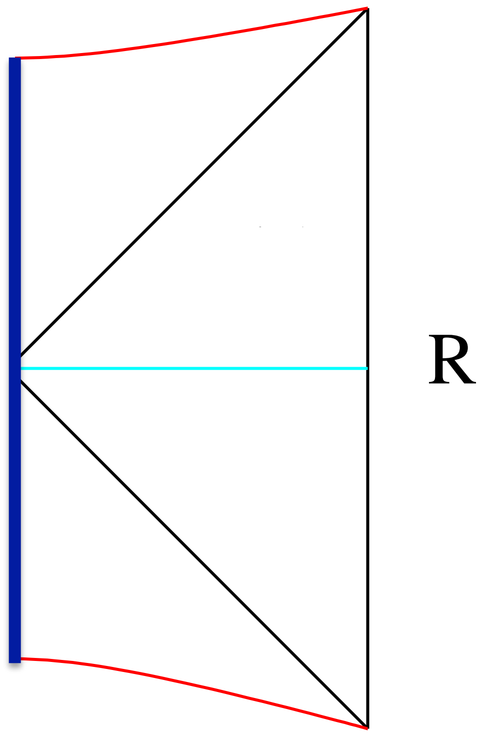


Entangled state in two <u>non-interacting</u> CFT's.

Israel
JM

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |E_n\rangle_L^{CPT} \times |E_n\rangle_R$$

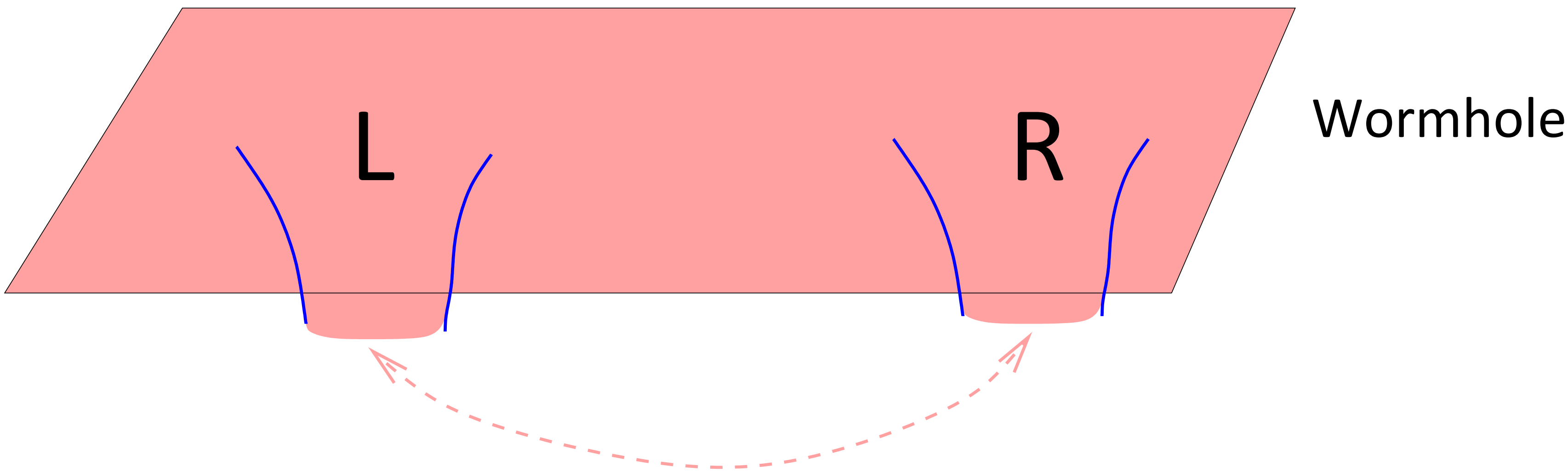
Pure State

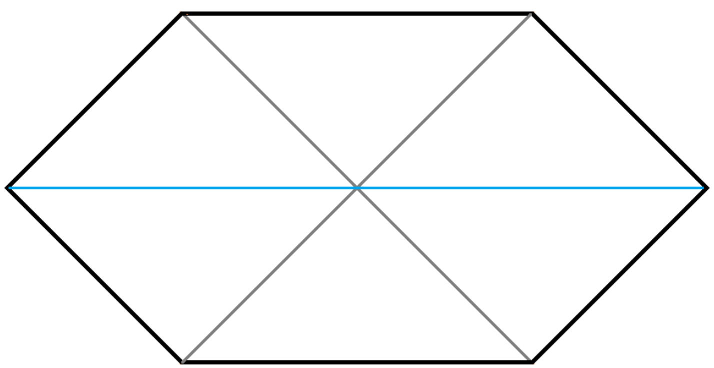


Spatial slice ends.

ER = EPR

JM
Susskind





Non travesable

No signals

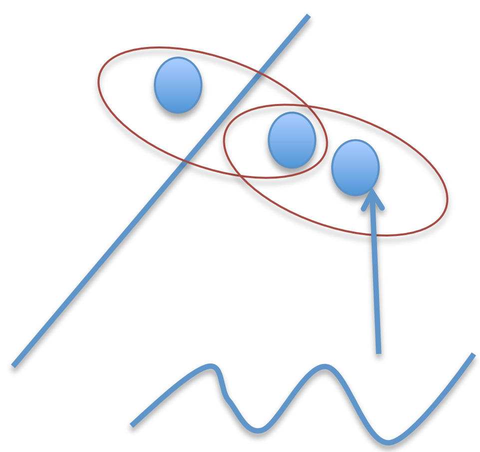
No causality violation

Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar

AMPS paradox

Almheiri, Marolf
Polchinski, Sully
Stanford
Mathur

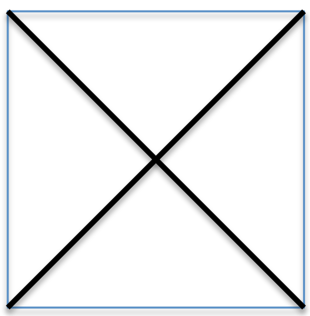
- Black hole entangled with radiation.
- In the radiation, distill the qubit that is entangled with an outside mode.
- Is the outside mode still entangled with the interior ?



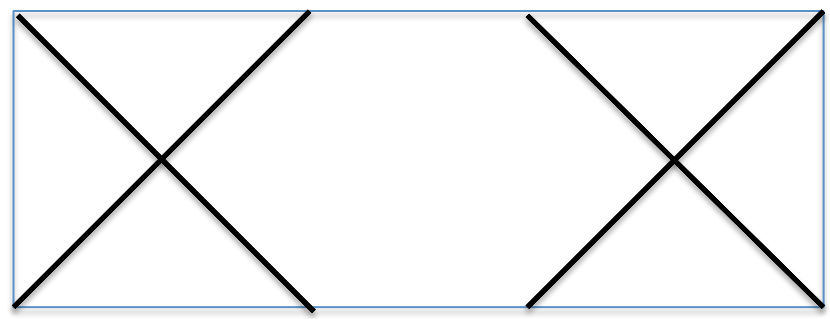
- Idea: Distillation process sends a signal through the wormhole to the interior and disrupts the entanglement.

- How are these wormholes and what is their geometry ?

Taxonomy



Thermofield double



More generic state

Shenker Stanford

More generic state ? → we expect it to be very long

Marolf Polchinski

What we expect

- Distilling is shortening the wormhole.
- Generic wormhole is very long and does not allow one to send signals doing simple operations.
- The region just behind the horizon is smooth for the generic wormhole.
- Wormhole is the construction of the entangled state using "simple" building blocks.

Tensor networks

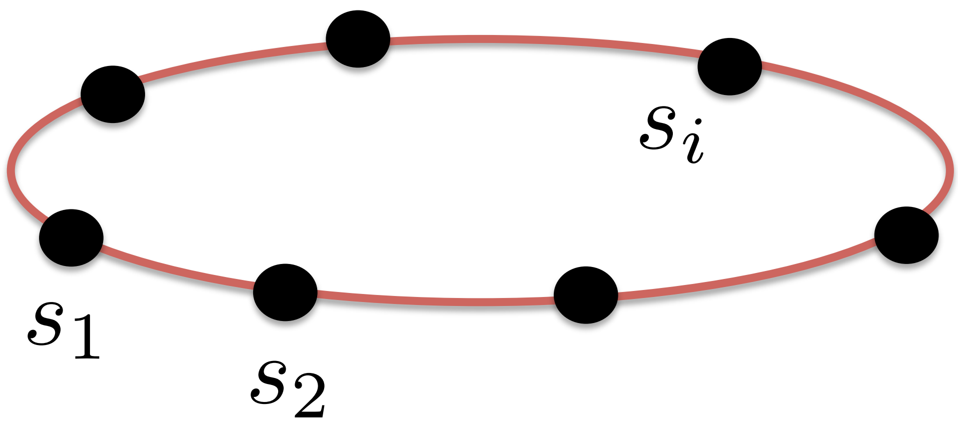
Condensed matter theorists found a convenient quantum mechanical, real time description of the wavefunction→ Tensor networks.

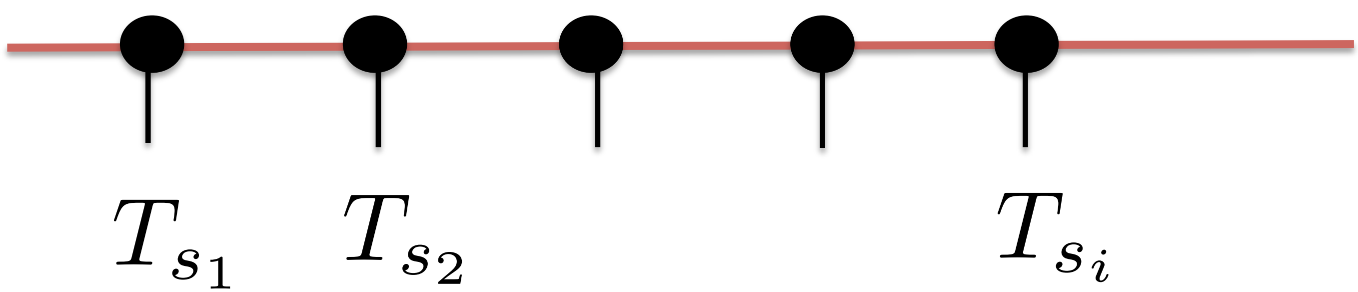
It is based on the entanglement pattern of the state.

There is a resemblance between the structure of the tensor network and the bulk

Swingle

$$\Psi(s_1, \cdots, s_n) = Tr[T_{1,s_1} T_{2,s_2} \cdots T_{n,s_n}]$$

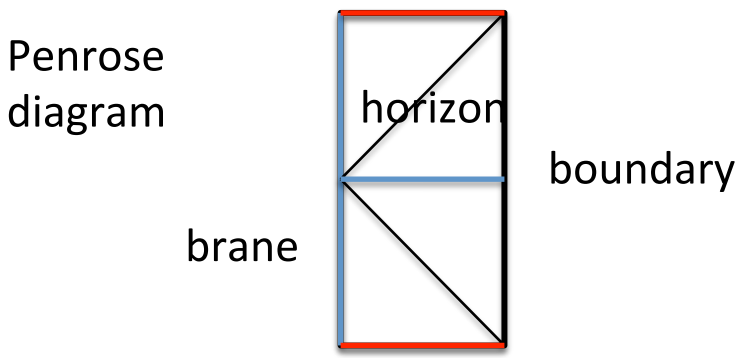


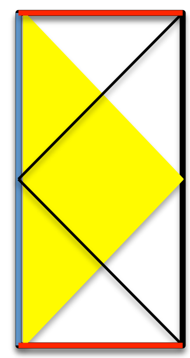


Time dependence

Start with a state with a gap and evolve it.

Eg. Brane in Ads that falls into a black hole

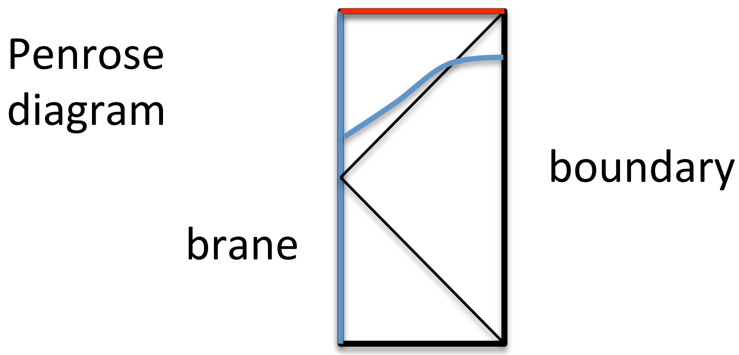


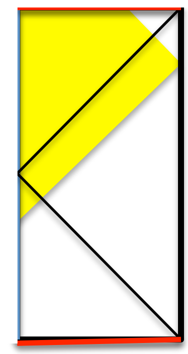


t=0

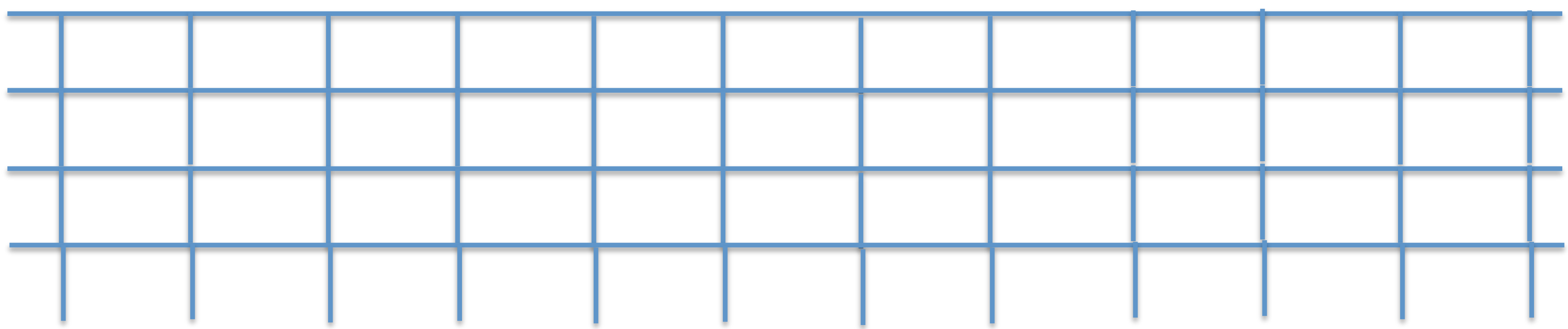
This wavefunction is <u>not</u> the ground state

Hartman & JM





t>0



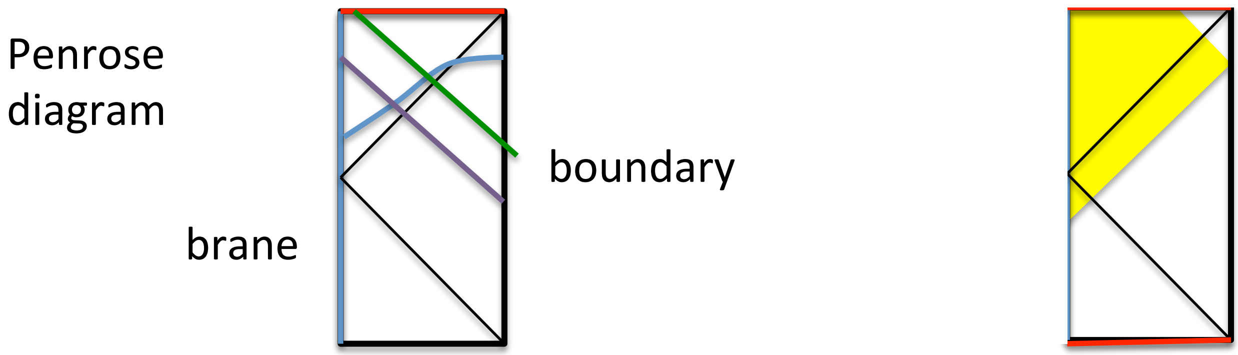
The wavefunction becomes more complicated since we did <u>not</u> start with the ground state.

This is the structure of the most complicated it can become, since the Hamiltonian is local.

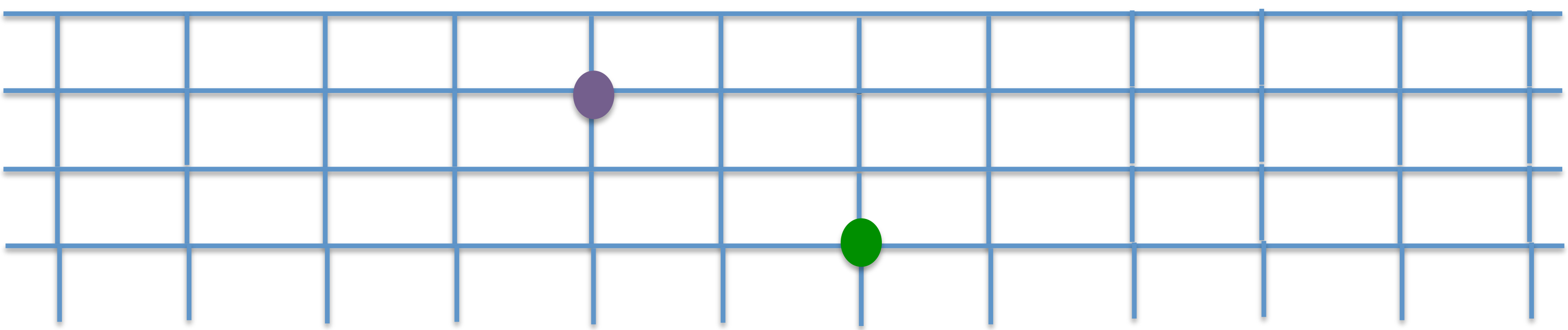
This looks similar to the nice slices that go through the black hole interior.

- That was the evolution of a particular state.
- More generic state?

Hartman & JM



t>0



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Formal Hilbert space → True one

- Changes in the tensors generate new states.
- We can superpose them.
- This looks like the bulk Hilbert space on the nice slices.
- We can have more states than the black hole entropy.
- This formal Hilbert space is projected onto the boundary one by evaluating the tensor network. This can lead to a reduction on the number of states.

- Similar story for the two sided black hole.

- This picture remains vague. But it has some qualitative features that we expect from the bulk geometry.

- Of course, the classical geometric picture would arise only for large N systems, so this needs to be incorporated in some way…

Comments on the emergence of a classical interior.

Euclidean black hole

- A classical solution is giving us a thermal partition function.

$$Z_{\text{singlet}} = \int DU \sum_R Tr_R[U] Z_R = \int D\rho e^{-N^2 f[\rho]}$$

Polyakov
Aharony, Marsano Minwalla,
Papadodimas, Van Raamsdonk

Lagrange multiplier U → becomes classical

The U-dependent part of the action for U arises only from the non-singlets.

For the deconfined phase, U is very localized → similar to forgetting about the integration over U and writing a classical value for it, $U \to U_0$.

- U dependent observables are not to be found among the observables of the singlet sector.
- They are Wilson loops which project onto the non-singlets → deformations of the original problem.

$$\langle Tr_R[e^{i\int A}]\rangle \sim Tr_R[U_0]$$

Thermofield double

- Similar story.

$$|TFD\rangle = \sum_{R,n} \int DV \, V_i^j |R^i, n\rangle_L |\bar{R}_j, n\rangle e^{-\beta E_n/2}$$

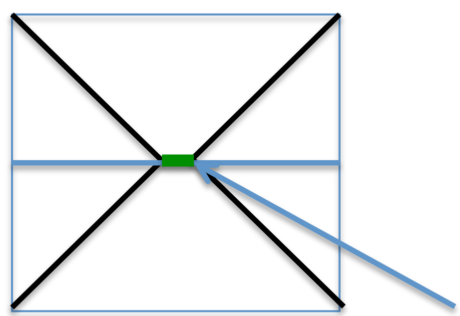
Projects onto singlets on each side.

For coarse observables, we sum over many states → we will generate saddle points for V.

If we consider the saddle point for V , now there seems to be only one gauge invariance and V connects the gauge indices on the two sides.

Gravity case

- In the case of gravity we might have a similar story.
- Gauge constraints of GR should act independently on both sides (microstates/"fuzzballs").
- The enforcer of this constraint is the interior, which, due to the sum over many microstates states, is now classical



Like the matrix V_i^j

Conclusions

- Entanglement patterns are an important feature of QFT.
- Entanglement patterns are encoded in the geometry of spacetime.
- A geometric connection can arise purely from entanglement, but horizons will prevent transfer of information.
- The tensor network formalism seems to have features similar to bulk geometry, including the ability to connect far away regions.
- Analogy between holonomy in large N gauge theories and the classical interior.

Happy birthday Joe