Two-Time Physics

The Unified View From
Higher Dimensional Space and Time

Itzhak Bars, USC

- One of the most important questions today: what is space-time?
- String theory: consistent Quantum Mechanics + Gravity <u>requires</u> extra space dimensions (Kaluza-Klein idea: small, curled up,…).
- How about extra time dimensions? (easy to ask, not so easy to realize!!)

Not much discussion of more time-like dimensions – why?

Because the road to 2T is dangerous and scary:

1) Ghosts!
   negative probability

2) Murderers!
   Causality violation (cause and effect disconnect)
   → time machines:
   You can kill your grandmother before
   your mother is born, thus preventing your birth!

It took ½ century to learn how to overcome such inconsistencies for the first time dimension.
More time dimensions make it much worse !! How will such issues be solved with more T's?

But in search of the mysterious theory there has been some attraction to more T's
Extended SUSY of M-theory is (10+2) SUSY (Bars -1995),
F-theory (10+2 or 11+1), S-theory (11+2), U-theory (other signatures), etc.

A solution to the inconsistencies is a new gauge symmetry (1998) → 2T-physics .

The new fundamental principle (1998)

X^M=(Position & Time)
P_M= (Momentum & Energy)

indistinguishable
at any instant.

A new gauge symmetry of the Laws of Mechanics
Applied to all motion

Surprising and far reaching preliminary conclusion:

The ordinary formulation of Physics in 3+1 LARGE dimensions is incomplete.
One extra timelike plus one extra spacelike LARGE dimensions are needed to provide a more complete view of Nature at ALL SCALES of physics.

Predictions

New relationships in 3+1 dimensions (dualities, hidden symmetries);
New unification directions; Testable predictions at all scales of physics.

Clues for the fundamental principle
Position ←→ Momentum GLOBAL symmetry

- **position/momentum are at same level of importance** before a specific Hamiltonian is chosen in classical or quantum mechanics
  - Boundary conditions, or any measurement.
  - Poisson brackets or quantum commutators
  - Any Lagrangian: $L = \dot{X}\cdot P - H \cdot$ or $\frac{1}{2}(\dot{X}\cdot P - \dot{P}\cdot X) - \cdots$

Symmetry
X→P, P→-X

- **More general: continuous GLOBAL symmetry: Sp(2,R)**

$$\begin{pmatrix} X^M \\ P^M \end{pmatrix}' = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} X^M \\ P^M \end{pmatrix}, \quad ad - bc = 1$$

$$X_i^M \equiv \begin{pmatrix} X^M \\ P^M \end{pmatrix}, \quad i = 1,2$$

Invariant $\frac{1}{2}\varepsilon^{ij}\dot{X}_i^M X_j^N \eta_{MN}$

- **Even more general is GLOBAL canonical transformations**

$$\delta_\varepsilon X^M = \{\varepsilon(X,P), X^M\} = \frac{\partial\varepsilon(X,P)}{\partial P_M}, \quad \delta_\varepsilon P_M = \{\varepsilon(X,P), P_M\} = -\frac{\partial\varepsilon(X,P)}{\partial X^M}$$

$$\delta_\varepsilon(\dot{X}\cdot P) = \frac{d}{d\tau}\left(P_M\frac{\partial\varepsilon(X,P)}{\partial P_M} - \varepsilon(X,P)\right)$$

New Gauge Principles

Require local Sp(2,R) symmetry

X←→P indistinguishable continuously

at EVERY INSTANT for ALL MOTION

Sp(2,R) doublet: $\begin{pmatrix} X^M(\tau) \\ P^M(\tau) \end{pmatrix} \equiv X_i^M \quad i = 1,2$

$D_\tau X_i^M = \partial_\tau X_i^M - A_i^{\ j} X_j^M$

Generalizes τ reparametrization

$$S = \frac{\eta_{MN}}{2}\int d\tau \left(\varepsilon^{ij}\partial_\tau X_i^M X_j^N - A^{ij}\, X_i^M X_j^N\right)$$

$$\partial_\tau x^\mu p_\mu - \frac{1}{2} e p_\mu p_\nu \eta^{\mu\nu}$$

Sp(2,R) generators : $X{\cdot}X,\ X{\cdot}P,\ P{\cdot}P,\ \rightarrow X_i{\cdot}X_j = 0$

Generalized Sp(2,R) generators $Q_{ij}(X,P)$ instead of the simpler $Q_{ij}(X,P)=X_i{\cdot}X_j=(X^2,P^2,X.P)$.

$$\mathcal{L}_{2T} = \partial_\tau X^M P_M - \frac{1}{2} A^{ij} Q_{ij}(X,P)$$

With these we can include ALL possible background fields.

More generalizations: Particles with SPIN or SUSY; strings/branes (partial) and finally **Field Theory.**

How does it work?

1) New symmetry allows only highly symmetric motions.
   → little room to maneuver.
2) With only 1 time the highly symmetric motions impossible.
   → Collapse to nothing.
3) Extra 1+1 dimensions necessary
   → 4+2 !! No less and no more than 2T.
4) **Straightjacket** in 4+2 makes allowed motions **effectively 3+1** motions (like shadows on walls).

(Position & Time)
(Momentum & Energy)

Indistinguishable **at any instant**

Sp(2,R) doublet: $\begin{pmatrix} X^M(\tau) \\ P^M(\tau) \end{pmatrix} \equiv X_i^M \quad i = 1,2$

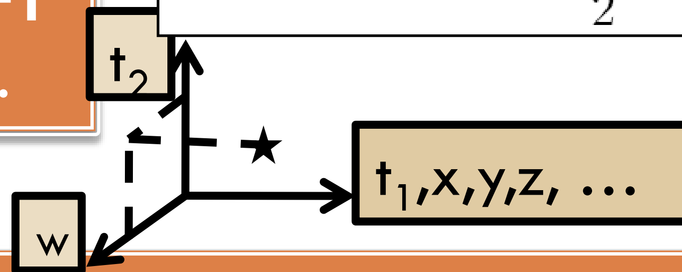
$D_\tau X_i^M = \partial_\tau X_i^M - A_i^{\,j} X_j^M$

$$S = \frac{\eta_{MN}}{2}\int d\tau \left(\varepsilon^{ij}\partial_\tau X_i^M X_j^N - A^{ij}\, X_i^M X_j^N\right)$$

Sp(2,R) generators : $X{\cdot}X,\ X{\cdot}P,\ P{\cdot}P,\ \rightarrow X_i{\cdot}X_j=0$

Constraints: generators vanish !!!

$$\mathcal{L}_{2T} = \partial_\tau X^M P_M - \frac{1}{2}A^{ij}Q_{ij}(X,P)$$



Non-trivial: Gauge fix 4+2 to 3+1 (have 3 gauge parameters)

<u>many</u> 3+1 shadows emerge for **<u>same</u> 4+2** spacetime history.

Each shadow contains <u>only one timeline</u> (a mixture of all 4+2)

Emergent spacetimes and dynamics, hidden symmetries & dualities

from gauge fixing the 2T theory

Free or interacting systems, with or without mass, in flat or curved 3+1 spacetimes.
Analogy: multiple shadows on walls.

Massless
relativistic particle
$(p_\mu)^2=0$
conformal sy
Dirac

Twistors
SU(2,2)=SO(4,2)
twistor transform for all these

Particle in Robertson-Walker expanding Universe

Harmonic oscillator
2 space dims
mass = 3rd dim
SO(2,2)xSO(2)

2T-physics
Sp(2,R)
simplest example
$X^2=P^2=X\cdot P=0$.
Background flat 6=4+2 dims
SO(4,2) symmetry

Massive
relativistic
$(p_\mu)^2+m^2=0$
Non-relativistic
$H=\mathbf{p}^2/2m$

Particle in any Conformally flat Space singular OK, some Black Holes

H-atom
3 space dims
$H=\mathbf{p}^2/2m - a/r$
SO(4)xSO(2)
SO(3)xSO(1,2)

Particle in any Maximally Symmetric Space, e.g. $AdS_{4-n}\times S_n$

Hidden Symm.
SO(d,2), d=4
$C_2=1-d^2/4= -3$
singleton

Emergent parameters
mass, couplings, curvature, etc.

2T-physics predicts hidden symmetries and dualities (with parameters) among the "shadows".
1T-physics misses these phenomena.

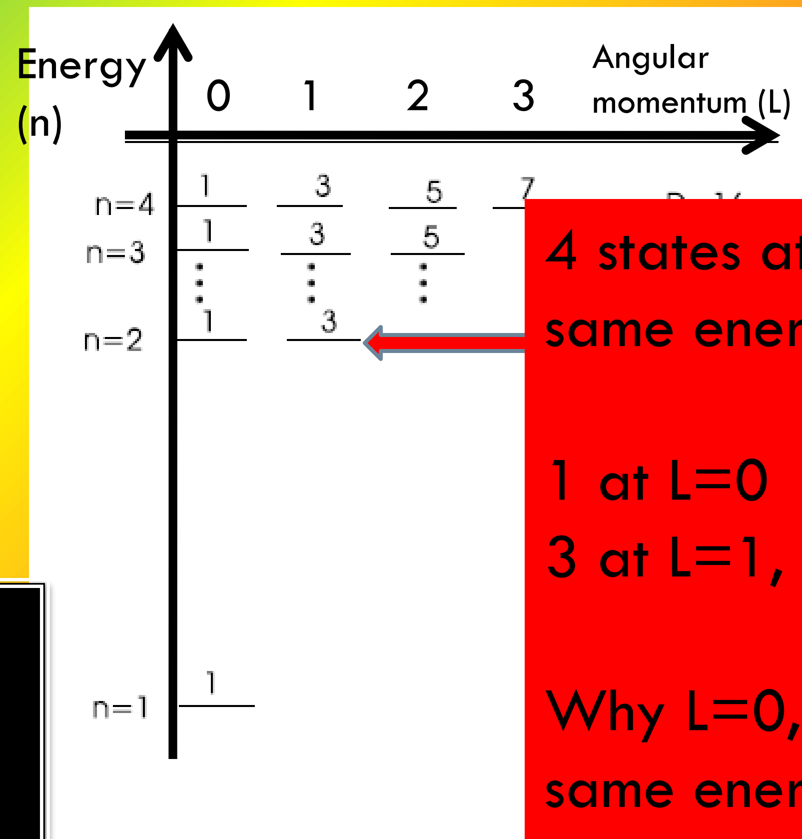
•Holography: These emergent holographic images are only some examples of **much broader phenomena.**

These emerge in 2T-field theory as well

Subtle effects in 3+1 dims: H-atom action is INVARIANT under SO(4,2).

"Seeing" 4+2 dimensions through the H-atom

H-atom orbitals



4 states at the same energy.

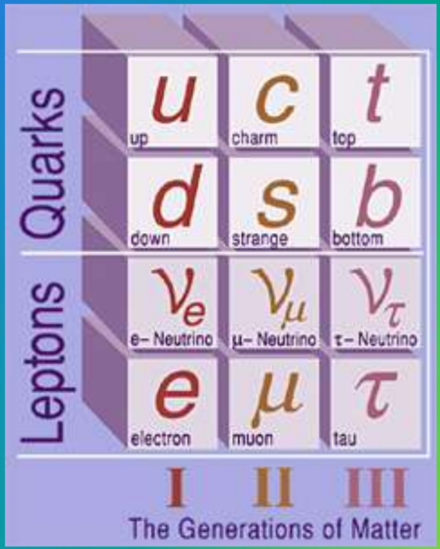
1 at L=0
3 at L=1, etc.

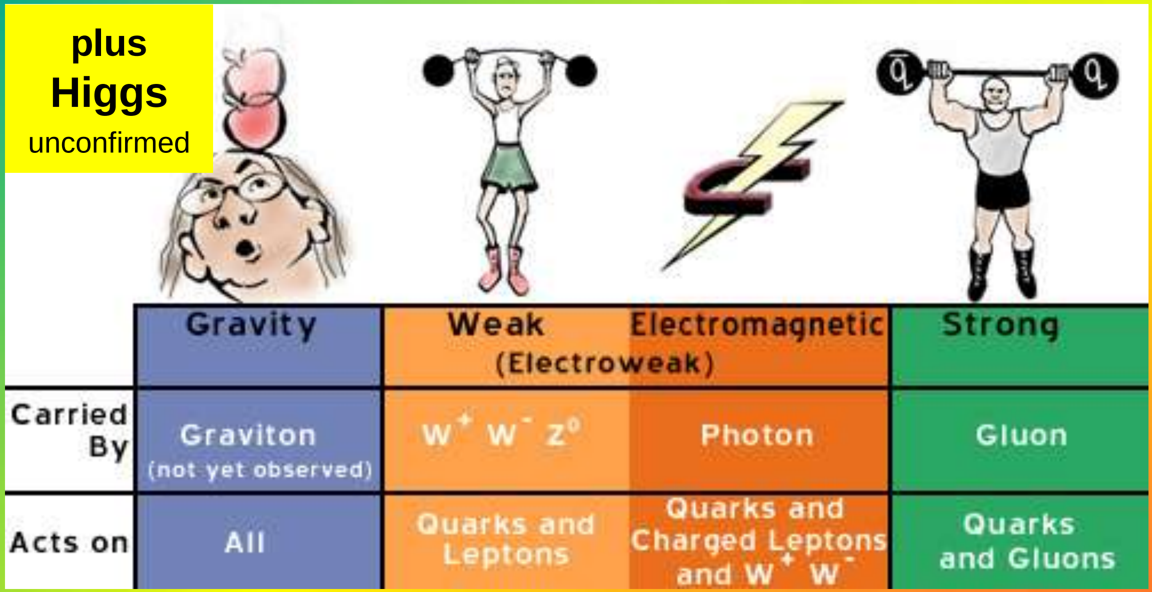
Why L=0,1 same energy?

Seeing 1 space (the 4th), and 2 times :
SO(4,2) > SO(3) x SO(1,2)
At fixed angular momentum (3-space) the energy towers are patterns of space-time symmetry group SO(1,2)

Demo : the 4th dimension - shadow of rotating system. SO(4,2) > SO(4)xSO(2)

The Standard Model of Particles and Forces





Four interactions or forces + Higgs (?) govern all known phenomena in the Universe.

Exquisite agreement with experiment. Describes Nature down to 10^{-18} m.

Computational framework : Quantum Field Theory.

This St.Mod. emerges as a shadow of a 4+2 field theory, with **improved features !!** AND it has more shadows ←→ duals

$X\cdot X = X\cdot P = P\cdot P = 0$ $\quad ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'}dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu} \quad X^{\pm'} = \frac{1}{\sqrt{2}}\left(X^{0'} \pm X^{1'}\right)$

		+'	-'	μ
Relativistic massless particle	$X^M =$	1	$\frac{1}{2}x^2$	x^μ
$p^2 = 0$	$P^M =$	0	$x\cdot p$	p^μ $\quad X\cdot X = X\cdot P = P\cdot P = 0$
Relativistic massive particle	$X^M =$	$\frac{1+a}{2a}$	$\frac{x^2 a}{1+a}$	$x^\mu \quad a \equiv \sqrt{1+\frac{m^2x^2}{(x\cdot p)^2}}$
$p^2 + m^2 = 0$	$P^M =$	$\frac{-m^2}{2ax\cdot p}$	$a\ x\cdot p$	p^μ
Non-relativistic massive particle	$X^M =$	t	$\frac{\mathbf{r}\cdot\mathbf{p}-tH}{m}$	$X^0 = \pm\left\lvert\mathbf{r}-\frac{t}{m}\mathbf{p}\right\rvert,\ X^i = \mathbf{r}^i$
$H - \frac{\mathbf{p}^2}{2m} = 0$	$P^M =$	m	H	$P^0 = 0,\ P^i = \mathbf{p}^i$
Maximally Symmetric Spaces	$X^M =$	$1+\sqrt{1-Kx^2}$	$\frac{x^2/2}{1+\sqrt{1-Kx^2}}$	$x^\mu \quad g_{\mu\nu} = \eta_{\mu\nu} + \frac{K}{1-Kx^2}x_\mu x_\nu$
$p^2 - \frac{K\ (x\cdot p)^2}{1-Kx^2} = 0$	$P^M =$	0	$\frac{\sqrt{1-Kx^2}}{1+\sqrt{1-Kx^2}}x\cdot p$	$p^\mu - \frac{K x\cdot p\ x^\mu}{1+\sqrt{1-Kx^2}}$
$\mathrm{AdS}_{d-n}\times \mathrm{S}^n$	$X^M =$	$\frac{R_0^2}{\lvert\vec{y}\rvert}$	$\frac{1}{2\lvert\vec{y}\rvert}(x^2 + \vec{y}^{\,2})$	$\frac{R_0}{\lvert\vec{y}\rvert}x^\mu,\ \frac{R_0}{\lvert\vec{y}\rvert}\vec{y}^i$
$\vec{y}^2(p^2 + \vec{k}^2) = 0$	$P^M =$	0	$\frac{\lvert\vec{y}\rvert}{R_0^2}(x\cdot p + \vec{y}\cdot\vec{k})$	$\frac{\lvert\vec{y}\rvert}{R_0}p^\mu,\ \frac{\lvert\vec{y}\rvert}{R_0}\vec{k}^i$
Free function $\alpha(x)$	$X^M =$	$x^2 + \alpha(x)$	$\frac{x^2/2}{x^2+\alpha(x)}$	$x^\mu \quad g_{\mu\nu} = \eta_{\mu\nu} - \frac{4\alpha(x)}{(x^2+\alpha(x))^2}x_\mu x_\nu$
$p^2 + \frac{4\alpha(x)(x\cdot p)^2}{(x^2-\alpha(x))^2} = 0$	$P^M =$	0	$\frac{x\cdot p}{\alpha(x)-x^2}$	$p^\mu - \frac{2x\cdot p}{x^2-\alpha(x)}x^\mu$

Gauge choice	M	$0'$	0	$I=(1',i)$
Robertson-Walker $r<R_0$ (closed universe)	$X^M=$	$a(t)\cos(\int^t \frac{dt'}{a(t')})$	$a(t)\sin(\int^t \frac{dt'}{a(t')})$	$X^i=\mathbf{r}^i a(t)/R_0$ $X^{1'}=\pm a(t)\sqrt{1-\frac{r^2}{R_0^2}}$
$-H^2+\frac{R_0^2}{a^2(t)}(\mathbf{p}^2-\frac{(\mathbf{r}\cdot\mathbf{p})^2}{R_0^2})=0$	$P^M=$	$-H\sin(\int^t \frac{dt'}{a(t')})$	$H\cos(\int^t \frac{dt'}{a(t')})$	$P^i=\frac{R_0}{a(t)}(\mathbf{p}^i-\frac{\mathbf{r}\cdot\mathbf{p}}{R_0^2}\mathbf{r}^i)$ $P^{1'}=\mp\frac{\mathbf{r}\cdot\mathbf{p}}{a(t)}\sqrt{1-\frac{r^2}{R_0^2}}$
Robertson-Walker $r>0$ (open universe)	$X^M=$	$a(t)\sinh(\int^t \frac{dt'}{a(t')})$	$(\pm)'a(t)\sqrt{1+\frac{r^2}{R_0^2}}$	$X^i=\mathbf{r}^i a(t)/R_0$ $X^{1'}=\pm a(t)\cosh(\int^t \frac{dt'}{a(t')})$
$-H^2+\frac{R_0^2}{a^2(t)}(\mathbf{p}^2+\frac{(\mathbf{r}\cdot\mathbf{p})^2}{R_0^2})=0$	$P^M=$	$\pm H\cosh(\int^t \frac{dt'}{a(t')})$	$(\pm)'\frac{\mathbf{r}\cdot\mathbf{p}}{a(t)}\sqrt{1+\frac{r^2}{R_0^2}}$	$P^i=\frac{R_0}{a(t)}(\mathbf{p}^i+\frac{\mathbf{r}\cdot\mathbf{p}}{R_0^2}\mathbf{r}^i)$ $P^{1'}=H\sinh(\int^t \frac{dt'}{a(t')})$
Cosmological constant $\Lambda\equiv\frac{3}{R_0^2}>0$	$X^M=$	$\sqrt{R_0^2-r^2}\sinh\frac{t}{R_0}$	R_0	$X^i=\mathbf{r}^i$ $X^{1'}=\pm\sqrt{R_0^2-r^2}\cosh\frac{t}{R_0}$
$-H^2(1-\frac{r^2}{R_0^2})+(\mathbf{p}^2+\frac{(\mathbf{r}\cdot\mathbf{p})^2}{R_0^2-r^2})=0$	$P^M=$	$\pm\frac{H}{R_0}\sqrt{R_0^2-r^2}\cosh\frac{t}{R_0}$	$\frac{R_0\mathbf{r}\cdot\mathbf{p}}{R_0^2-r^2}$	$P^i=\mathbf{p}^i+\frac{\mathbf{r}\cdot\mathbf{p}}{R_0^2-r^2}\mathbf{r}^i$, $P^{1'}=\frac{H}{R_0}\sqrt{R_0^2-r^2}\sinh\frac{t}{R_0}$
Cosmological constant $\Lambda\equiv-\frac{3}{R_0^2}<0$	$X^M=$	$\sqrt{R_0^2+r^2}\sin\frac{t}{R_0}$	$\mp\sqrt{R_0^2+r^2}\cos\frac{t}{R_0}$	$X^i=\vec{r}^{\,i}$ $X^{1'}=R_0$
$-H^2(1+\frac{r^2}{R_0^2})+(\mathbf{p}^2-\frac{(\mathbf{r}\cdot\mathbf{p})^2}{R_0^2+r^2})=0$	$P^M=$	$\pm\frac{H}{R_0}\sqrt{R_0^2+r^2}\cos\frac{t}{R_0}$	$\frac{H}{R_0}\sqrt{R_0^2+r^2}\sin\frac{t}{R_0}$	$P^i=\mathbf{p}^i-\frac{\mathbf{r}\cdot\mathbf{p}}{R_0^2+r^2}\mathbf{r}^i$, $P^{1'}=-\frac{R_0\mathbf{r}\cdot\mathbf{p}}{R_0^2+r^2}$
(d–1)-sphere×time	$X^M=$	$R_0\cos\frac{t}{R_0}$	$R_0\sin\frac{t}{R_0}$	$R_0\hat{n}^I=$ $X^i=\mathbf{r}^i$ $X^{1'}=\pm\sqrt{R_0^2-r^2}$
$-H^2+(\mathbf{p}^2+\frac{(\mathbf{r}\cdot\mathbf{p})^2}{R_0^2-r^2})=0$	$P^M=$	$-H\sin\frac{t}{R_0}$	$H\cos\frac{t}{R_0}$	$P^i=\mathbf{p}^i$ $P^{1'}=\mp\frac{\mathbf{r}\cdot\mathbf{p}}{\sqrt{R_0^2-r^2}}$
H-atom, $H<0$	$X^M=$	$r\cos u$ $u(t)\equiv\frac{\sqrt{-2mH}}{m\alpha}(\mathbf{r}\cdot\mathbf{p}-2mHt)$	$r\sin u$	$X^i=\mathbf{r}^i-\frac{r}{m\alpha}\mathbf{r}\cdot\mathbf{p}\ \mathbf{p}^i$ $X^{1'}=-\frac{r}{m\alpha}\sqrt{-2mH}\,\mathbf{r}\cdot\mathbf{p}$
$H=\frac{\mathbf{p}^2}{2m}-\frac{\alpha}{r}$	$P^M=$	$-\frac{m\alpha}{r\sqrt{-2mH}}\sin u$	$\frac{m\alpha}{r\sqrt{-2mH}}\cos u$	$P^i=\mathbf{p}^i$ $P^{1'}=\frac{1}{\sqrt{-2mH}}\left(\frac{m\alpha}{r}-\mathbf{p}^2\right)$
H-atom, $H>0$	$X^M=$	$r\cosh u$ $u(t)\equiv\frac{\sqrt{2mH}}{m\alpha}(\mathbf{r}\cdot\mathbf{p}-2mHt)$	$\frac{r}{m\alpha}\sqrt{2mH}\,\mathbf{r}\cdot\mathbf{p}$	$X^i=\mathbf{r}^i-\frac{r}{m\alpha}\mathbf{r}\cdot\mathbf{p}\ \mathbf{p}^i$ $X^{1'}=r\sinh u$
	$P^M=$	$\frac{m\alpha}{r\sqrt{2mH}}\sinh u$	$\frac{1}{\sqrt{2mH}}\left(\frac{m\alpha}{r}-\mathbf{p}^2\right)$	$P^i=\mathbf{p}^i$ $P^{1'}=\frac{m\alpha}{r\sqrt{2mH}}\cosh u$

Table2 : Parametrization of X^M, P^M for $M=(0',0,I)$

An example: Massive relativistic particle gauge

$$X^{\pm'} = \frac{1}{\sqrt{2}}\left(X^{0'} \pm X^{1'}\right) \qquad ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'}dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu}$$

$$X^M = \left(\overset{+'}{\frac{1+a}{2a}},\ \overset{-'}{\frac{x^2 a}{1+a}},\ \overset{\mu}{x^\mu}\right),\ a \equiv \sqrt{1+\frac{m^2 x^2}{(x\cdot p)^2}}$$

$$P^M = \left(\frac{-m^2}{2(x\cdot p)a},\ (x\cdot p)\,a\ ,\ p^\mu\right),\ P^2 = p^2 + m^2 = 0.$$

embed phase space in 3+1 into phase space in 4+2

Make 2 gauge choices solve 2 constraints $X^2 = X.P = 0$

τ reparametrization and one constraint remains.

$$S = \int d\tau \left(\dot{X}^M P^N - \frac{1}{2}A^{ij}X_i^M X_j^N\right)\eta_{MN} = \int d\tau\left(\dot{x}^\mu p_\mu - \frac{1}{2}A^{22}\left(p^2+m^2\right)\right)$$

Gauge invariants

$$L^{MN} = \varepsilon^{ij}X_i^M X_j^N = X^M P^N - X^N P^M$$

$$L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu, \quad L^{+'-'} = (x\cdot p)\,a,$$

$$L^{+'\mu} = \frac{1+a}{2a}p^\mu + \frac{m^2}{2\,(x\cdot p)\,a}x^\mu$$

$$L^{-'\mu} = \frac{x^2 a}{1+a}p^\mu - (x\cdot p)\,a x^\mu$$

Note a=1 when m=0

$\delta x^\mu = \omega_{MN}\{L^{MN}, x^\mu\}, \quad \delta p^\mu = \omega_{MN}\{L^{MN}, p^\mu\},$

conformal transformations deformed by mass is symmetry of the massive action.

Field equations in 2T-physics

Derived from Sp(2,R) in hep-th/0003100; also Dirac 1936 other approach

$$X^2|\Phi\rangle = 0,\ P^2|\Phi\rangle = 0,\ (X\cdot P + P\cdot X)\,|\Phi\rangle = 0.$$

Constraints = 0 on physical states
i.e. Sp(2,R) gauge invariant

$$\hat{\Phi}(X) = \langle X|\Phi\rangle$$

Probability amplitude is the field

$$X^2\hat{\Phi}(X) = 0,\ \partial_M\partial^M\hat{\Phi}(X) = 0,\ X^M\partial_M\hat{\Phi}(X) + \partial_M\left(X^M\hat{\Phi}(X)\right) = 0.$$

kinematic #1

$$\hat{\Phi}(X) = \delta\left(X^2\right)\Phi(X)$$

kinematic #2

$$\left(X\cdot\partial\Phi + \frac{d-2}{2}\Phi\right)_{X^2=0} = 0$$

Kinematic eom´s say how to embed d dims in d+2 dims.

dynamical eq. extended with interaction

$$\left[\partial^2\Phi - V'(\Phi)\right]_{X^2=0} = 0$$

3 equations in d+2 = KG in d

$$\delta_\Lambda\Phi = X^2\Lambda(X)$$

gauge symmetry

$$\Phi(X) = \Phi_0(X) + X^2\tilde{\Phi}(X)$$

Physical part of field

remainder

$$\Phi_0 \equiv [\Phi(X)]_{X^2=0}$$

Action for scalar field in 2T-physics

Obtain 3 equations not just one : 2 kinematic and 1 dynamic.

BRST approach for Sp(2,R). Like string field theory
I.B.+Kuo hep-th/0605267

After gauge fixing, eliminating redundant fields, and simplifications, boils down to a simplified partially gauge fixed form

Gauge fixed version is more familiar looking

$$S(\Phi) = 2\gamma \int d^{d+2}X \, \delta(X^2) \left[\frac{1}{2}\Phi \partial^2 \Phi - \lambda \frac{d-2}{2d} \Phi^{\frac{2d}{d-2}} \right]$$

Gauge symmetries

$$\delta_\Lambda \Phi = X^2 \Lambda(X)$$

Works only for unique V(Φ)

Minimizing the action gives two equations, so get all 3 Sp(2,R) constraints from the action

dynamical eq.

$$\delta S(\Phi) = 2\gamma \int d^{d+2}X \, \delta\Phi \left\{ \begin{array}{c} \delta(X^2)\left[\partial^2\Phi - V'(\Phi)\right] \\ +2\delta'(X^2)\left[X\cdot\partial\Phi + \frac{d-2}{2}\Phi\right] \end{array} \right\}$$

kinematic #1,2

Can gauge fix to Φ_0 which is independent of X^2

$$\Phi(X) = \Phi_0(X) + X^2 \tilde{\Phi}(X)$$

Homogeneous Φ_0 hence one less X, so alltogether two fewer dimensions

Action of the Standard Model in 4+2 dimensions

$$S\left(A,\Psi^{L,R},H,\Phi\right)=Z\int\left(d^6X\right)\ \delta\left(X^2\right)\ L\left(A,\Psi^{L,R},H,\Phi\right)$$

$$L\left(A,\Psi^{L,R},H,\Phi\right)=L\left(A\right)+L\left(A,\Psi^{L,R}\right)+L\left(\Psi^{L,R},H\right)+L\left(A,\Phi,H\right)$$

Gauge fields

SU(3) SU(2) U(1)

$$L\left(A\right)=-\frac{1}{4}Tr_3\left(G_{MN}G^{MN}\right)-\frac{1}{4}Tr_2\left(W_{MN}W^{MN}\right)-\frac{1}{4}B_{MN}B^{MN}.$$

quarks & leptons 3 families

$$\begin{aligned}L\left(A,\Psi^{L,R}\right)=&\frac{1}{2}\left(\bar{Q}^{L_i}\ \not{X}\overline{\not{D}}Q^{L_i}+\bar{Q}^{L_i}\overleftarrow{\not{D}}\overline{\not{X}}Q^{L_i}\right)+\frac{1}{2}\left(\bar{L}^{L_i}\ \not{X}\overline{\not{D}}L^{L_i}+\bar{L}^{L_i}\overleftarrow{\not{D}}\overline{\not{X}}L^{L_i}\right)\\&+\frac{1}{2}\left(\bar{d}^{R_j}\overline{\not{X}}\ \not{D}d^{R_j}+\bar{d}^{R_j}\overleftarrow{\overline{\not{D}}}\ \not{X}d^{R_j}\right)+\frac{1}{2}\left(\bar{e}^{R_j}\overline{\not{X}}\ \not{D}e^{R_j}+\bar{e}^{R_j}\overleftarrow{\overline{\not{D}}}\ \not{X}e^{R_j}\right)\\&+\frac{1}{2}\left(\bar{u}^{R_j}\overline{\not{X}}\ \not{D}u^{R_j}+\bar{u}^{R_j}\overleftarrow{\overline{\not{D}}}\ \not{X}u^{R_j}\right)+\frac{1}{2}\left(\bar{\nu}^{R_j}\overline{\not{X}}\ \not{D}\nu^{R_j}+\bar{\nu}^{R_j}\overleftarrow{\overline{\not{D}}}\ \not{X}\nu^{R_j}\right)\end{aligned}$$

4*x4=adjoint

$$\bar{Q}^{L_i}\ \not{X}\overline{\not{D}}Q^{L_i}$$

Yukawa couplings to Higgs

$$L\left(\Psi^{L,R},H\right)=-i\left(\begin{array}{c}\left(g_u\right)_{ij}\bar{Q}^{L_i}\ \not{X}u^{R_j}H^c-\left(g_u^\dagger\right)_{ji}\bar{H}^c\bar{u}^{R_j}\overline{\not{X}}Q^{L_i}\\+\left(g_d\right)_{ij}\bar{Q}^{L_i}\ \not{X}d^{R_j}H-\left(g_u^\dagger\right)_{ji}\bar{H}\bar{d}^{R_j}\overline{\not{X}}Q^{L_i}\\+\left(g_\nu\right)_{ij}L^{L_i}\ \not{X}\nu^{R_j}H^c-\left(g_\nu^\dagger\right)_{ji}\bar{H}^c\bar{\nu}^{R_j}\overline{\not{X}}L^{L_i}\\+\left(g_e\right)_{ij}L^{L_i}\ \not{X}e^{R_j}H-\left(g_e^\dagger\right)_{ji}\bar{H}\bar{e}^{R_j}\overline{\not{X}}L^{L_i}\end{array}\right)$$

$$\bar{Q}^{L_i}\ \not{X}d^{R_j}H$$

4*x4*=6 vector

Higgs and dilaton

$$L\left(A,\Phi,H\right)=\frac{1}{2}\Phi\partial^2\Phi+\frac{1}{2}\left(H^\dagger D^2H+\left(D^2H\right)^\dagger H\right)-V\left(\Phi,H\right)$$

$$V\left(\Phi,H\right)=\frac{\lambda}{4}\left(H^\dagger H-\alpha^2\Phi^2\right)^2+V\left(\Phi\right)$$

quadratic mass terms not allowed

No F*F terms

Emergent scalars in 3+1 dimensions

lightcone type basis in $4+2$ dimensions

$$X^{\pm'} = \frac{1}{\sqrt{2}}\left(X^{0'} \pm X^{1'}\right)$$

$$ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'}dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu}$$

$$X^{+'} = \kappa,\ X^{-'} = \kappa\lambda,\ X^\mu = \kappa x^\mu \longleftarrow$$

Embedding of 3+1 in 4+2 defines emergent spacetime x^μ. This is analog of Sp(2,R) gauge fixing

$$\kappa = X^{+'},\quad \lambda = \frac{X^{-'}}{X^{+'}},\ x^\mu = \frac{X^\mu}{X^{+'}}$$

x^μ and λ are homogeneous coordinates

$X^2=0$

$$\left(d^6X\right)\ \delta\left(X^2\right) = \kappa^5 d\kappa\ d^4x\ d\lambda\ \delta\left(\kappa^2\left(2\lambda - x^2\right)\right)$$

Solve kinematic equations in extra dimensions

$$\left(X\cdot\partial + \tfrac{d-2}{2}\right)\Phi = \left(\kappa\tfrac{\partial}{\partial\kappa} + 1\right)\Phi = 0$$

$$\Phi_0 + X^2\tilde{\Phi}$$

$$\Phi\left(X\right) = \Phi\left(\kappa,\lambda,x^\mu\right) = \kappa^{-1}\underline{\Phi}\left(x,\lambda\right) = \kappa^{-1}\left[\phi\left(x\right) + \left(\lambda - \frac{x^2}{2}\right)\tilde{\phi}\left(x,\lambda\right)\right]$$

Remainder is gauge freedom, remove it by fixing the 2Tgauge-symmetry at any λ,κ,x

Result of gauge fixing and solving kinematic eoms is fields only in 3+1

$$\Phi\left(X\right) = \kappa^{-1}\phi\left(x\right)$$

Dynamics only in 3+1

$$\partial^M\partial_M\Phi\left(X\right) = \frac{1}{\kappa^3}\frac{\partial^2\phi\left(x\right)}{\partial x^\mu\partial x_\mu}$$

Emergent gauge bosons in 3+1 dimensions

start with YM axial gauge

$$X\cdot A=0$$

kinematic equation simplifies → homogeneous

$$X^N F_{NM}=\left(X\cdot\partial+1\right)A_M=\left(\kappa\partial_\kappa+1\right)A_M=0$$

There is leftover YM gauge symm.

$$X\cdot\delta_\Lambda A=0\ \rightarrow\ X\cdot\partial\Lambda=0$$

homogeneous Λ enough to gauge fix $A^{+'}=0$

$$A_{-'}=-\eta_{-'+'}A^{+'}=0$$

Solution of X.A=0

$$A^{-'}=-A_{+'}=\frac{1}{\kappa}x^\mu\underline{A}_\mu$$

Only independent

$$\rightarrow A^\mu\left(X\right)=\frac{1}{\kappa}\underline{A}^\mu\left(x^\mu,\lambda\right)$$

Use 2Tgauge symmetry to eliminate V_μ gauge freedom proportional to X^2

$$A_\mu\left(X\right)=\frac{1}{\kappa}\left[A_\mu\left(x\right)+\left(\lambda-\frac{x^2}{2}\right)V_\mu\left(x,\lambda\right)\right]=\frac{1}{\kappa}A^\mu\left(x\right)$$

(the term $\left(\lambda-\frac{x^2}{2}\right)V_\mu\left(x,\lambda\right)$ is crossed out)

$$F_{\mu\nu}\left(X\right)=\kappa^{-2}F_{\mu\nu}\left(x\right),\ \text{with}\ F_{\mu\nu}\left(x\right)=\partial_\mu A_\nu-\partial_\nu A_\mu-i\left[A_\mu,A_\nu\right]$$

$$F_{+'\mu}\left(X\right)=\kappa^{-2}x^\nu F_{\mu\nu}\left(x\right),\ \ F_{-'\mu}\left(X\right)=0,\ \ F_{+'-'}\left(X\right)=0.$$

← F_{MN} is YM gauge invariant but 2Tgauge dependent

result is standard 3+1 YM Lagrangian

$$L\left(A\left(X\right)\right)=-\frac{1}{4}Tr\left(F_{MN}F^{MN}\right)\left(X\right)=-\frac{1}{4\kappa^4}Tr\left(F_{\mu\nu}F^{\mu\nu}\right)\left(x\right)$$

Emergent fermions in 3+1 dimensions

$$\Psi^{L,R}(X) = \Psi_0^{L,R}(X) + X^2\tilde{\Psi}^{L,R}(X)$$

(the term $X^2\tilde{\Psi}^{L,R}(X)$ is crossed out)

choose $X^2\xi_1$
2Tgauge symm.

Impose kinematical
eom in extra dimension

$$\left(X\cdot\partial+\frac{d}{2}\right)\Psi^{L,R} = \left(\kappa\frac{\partial}{\partial\kappa}+2\right)\Psi^{L,R} = 0$$

$$\Psi^{L,R}(X) = \kappa^{-2}\chi^{L,R}(x)$$

4 component
SU(2,2) chiral
fermions

choose ξ_2
2Tgauge symm.

$$\Gamma^{+'}\Psi^{L,R} = 0 \qquad \Psi^{L,R}(X) = \frac{1}{2^{1/4}\kappa^2}\begin{pmatrix}\psi^{L,R}(x)\\ 0\end{pmatrix}$$

2 component
SL(2,C) chiral
fermions

$$\overline{\not{D}}\Psi^L = \frac{1}{2^{1/4}\kappa}\begin{pmatrix}\bar\sigma^\mu D_\mu & -i\sqrt{2}\left(\kappa D_\kappa - \lambda\partial_\lambda - x^\mu D_\mu\right)\\ -i\sqrt{2}\partial_\lambda & -\sigma^\mu D_\mu\end{pmatrix}\begin{pmatrix}\frac{1}{\kappa^2}\psi^L(x)\\ 0\end{pmatrix} = \frac{1}{2^{1/4}\kappa^3}\begin{pmatrix}\bar\sigma^\mu D_\mu\psi^L(x)\\ 0\end{pmatrix}$$

4+2 Lagrangian
descends to 3+1
standard Lagrangian. No
explicit X.

$$\bar\Psi^L\,\not{X}\,\overline{\not{D}}\Psi^L = \frac{i}{\kappa^4}\bar\psi^L\bar\sigma^\mu D_\mu\psi^L \quad , \quad -i\,g\bar\Psi^L\,\not{X}\Psi^R H = \frac{g}{\kappa^4}\bar\psi^L\psi^R h$$

standard 3+1 kinetic term

standard 3+1 Yukawa term

Translation invariance in 3+1 comes
from rotation invariance in 4+2

Emergent Standard Model in 3+1 dimensions

Every term in the 4+2 action is
- proportional to κ^{-4} after solving kinematic eoms
- and is independent of λ after 2Tgauge fixing,

remainders proportional to X^2 eliminated by 2Tgauge

$$\Phi(X) = \Phi_0(X) + X^2\tilde{\Phi}(X)$$
$$A_M(X) = A_M^0(X) + X^2\tilde{A}_M(X)$$
$$\Psi^{L,R}(X) = \Psi_0^{L,R}(X) + X^2\Psi_1^{L,R}(X)$$

$$S = Z\int |\kappa|^5 d\kappa\, d^4x\, d\lambda\, \delta\left(\kappa^2\left(2\lambda - x^2\right)\right) \times \frac{1}{\kappa^4} L\left(A_\mu(x), \phi(x), h(x), \psi^{L,R}(x)\right)$$
$$= \left[Z\int d\kappa du\, \delta\left(2|\kappa|u\right)\right]\int d^4x L\left(A_\mu(x), \phi(x), h(x), \psi^{L,R}(x)\right)$$

Normalize to 1 (the bracketed factor)

Emergent Standard Model in 3+1 has dilaton in addition to usual matter

Emergent SM is Poincare invariant. More, it has hidden SO(4,2) symmetry

What is new in 3+1 ?

1. Resolution of the strong CP violation problem of QCD
2. Mass generation: a) new mechanisms, b) dilaton (perhaps observable phenomenology)

Resolution of the strong CP problem

strong CP problem in QCD

Take into account chiral rotations, instantons

$$\frac{\theta}{4!}\int dx^4 \varepsilon_{\mu\nu\lambda\sigma} Tr\left(G^{\mu\nu}G^{\lambda\sigma}\right) \text{ can be added to the QCD action in } 3+1$$

There is no observed CP violation in the strong interactions, so why is TOTAL θ so small, $\theta \leq 10^{-10}$?

θ can be made zero if there is an extra $U(1)_{PQ}$ suggested by Peccei & Quinn, but electroweak spontaneous breaking generates a Goldstone boson = the axion. It does not seem to exist !! So there is an outstanding fundamental problem.

The 4+2 Standard Model solves the strong CP violation problem of QCD

There is no term in 4+2 that can descend to the troublesome F*F terms in 3+1
No need for the Peccei-Quinn symmetry, and no elusive axion.

$J_{M_1M_2}$ homogeneous of degree 0, cannot give renormalizable θ term.

$$\int \left(d^6X\right)\,\delta\left(X^2\right)\boxed{X_{M_1}\partial_{M_2}} Tr\left(F_{M_3M_4}F_{M_5M_6}\right)\varepsilon^{M_1M_2M_3M_4M_5M_6} \longrightarrow 0$$

$$\int \left(d^6X\right)\, B_{M_1M_2} Tr\left(G_{M_3M_4}G_{M_5M_6}\right)\varepsilon^{M_1M_2M_3M_4M_5M_6} \longrightarrow 0$$

Non-renormalizable J_{MN} made from composite fields OK. Good for pion-decay, U(1) problem, etc.

topological term vanishes: $F_{+'-'}(X) = 0 \quad F_{-'\mu}(X) = 0,$

"Dilaton" driven Electroweak phase transition

The 4+2 Standard Model has 2Tgauge symmetry which forbids quadratic mass terms in the scalar potential. Only quartic interactions are permitted → Scale invariance in 3+1 ! Quantum effects break scale inv. But give insufficient mass to the Higgs (10 GeV).

$$V(\Phi, H) = \frac{\lambda}{4}\left(H^\dagger H - \alpha^2\Phi^2\right)^2 + V(\Phi)$$

$$\partial^2 H = \lambda H\left(H^\dagger H - \alpha^2\Phi^2\right)$$

$$\partial^2 \Phi = -2\alpha^2\Phi\left(H^\dagger H - \alpha^2\Phi^2\right) + V'(\Phi)$$

$$\langle H(\kappa, \lambda, x^\mu)\rangle = \frac{v}{\kappa}\begin{pmatrix}0\\1\end{pmatrix} \qquad \langle\Phi(X)\rangle = \pm\frac{v}{\kappa\alpha}.$$

Electroweak vev is probe of extra dimension

All space filled with vev. Makes sense to have dilaton & gravity & strings involved

small fluctuations. $V(\Phi, H) = \frac{1}{\kappa^4}V(h, \phi) = \frac{\lambda}{4\kappa^4}(h - \alpha\phi)^2(h + \alpha\phi + 2v)^2$

Goldstone boson due to spontaneous breaking of scale invariance

$$h = \frac{\tilde h + \alpha\tilde\phi}{\sqrt{1+\alpha^2}}, \quad \phi = \frac{-\alpha\tilde h + \tilde\phi}{\sqrt{1+\alpha^2}} \qquad V\left(\tilde h, \tilde\phi\right) = \frac{\lambda}{4}\tilde h^2\left(\left(1-\alpha^2\right)\tilde h + 2\alpha\tilde\phi + \sqrt{1+\alpha^2}\,2v\right)^2$$

Goldstone boson couples to everything the Higgs couples to, but with reduced $\alpha m_f/v$ strength factor α. It is not expected to remain massless because of quantum anomalies that break scale symmetry. Can we see it ? LHC? Dark Matter? Inflaton?

Duals of the Standard Model

0705.2834, IB + S.H. Chen + G. Quelin

- Instead of choosing the flat 3+1 spacetime gauge, choose any conformally flat spacetime gauge. These include
  - Robertson – Walker universe
  - Cosmological constant (dS_4 or AdS_4)
  - Any maximally flat spacetime
  - AdS(4), AdS(3)xS(1), AdS(2)xS(2)
  - A spacetime with singularities (free function $\alpha(x)$), etc.

More complicated gauge choices with $X^M(x^\mu, p_\mu)$ mixed parametrization e.g. massive particle, etc.
These have non-local field interactions, but approach local for small mass.

Conformally flat $g_{\mu\nu}=e_\mu^m(x)e_\nu^n(x)\eta_{mn}$	$X^M =$	$\pm e^{\sigma(x)}$	$\pm\frac{1}{2}e^{\sigma(x)}q^2(x)$	$\pm e^{\sigma(x)}q^m(x^\mu)$ $e_\mu^m(x)\equiv\pm e^{\sigma(x)}\frac{\partial q^m(x)}{\partial x^\mu}$
$g^{\mu\nu}(x)p_\mu p_\nu = 0$	$P^M =$	0	$q^m(x)e_m^\mu(x)p_\mu$	$e_m^\mu(x)p_\mu$

All these have hidden SO(4,2) (note this is more than usual Killing vectors).

All are dual transforms of each other as **field theories.**

Duality transformations: Weyl rescaling of background metric and general coordinate reparametrizations taking one field theory with backgrounds to another.

The dualities can possibly be used for practical computations.

The 2T field theory approach has been generalized to SUSY (I.B. + Y-C.Kuo)

N=1 : HEP-TH 0702089, HEP-TH 0703002

N=2 General, including coupling of hyper multiplets

N=4 Super Yang Mills

all in 4+2 dims, to appear soon.

Preparing to develop 2T field theory for:

SUGRA (9+1)+(1,1)=10+2 (see earlier connection hep-th/ 0208012)

Particle limit of M-theory (10+1) + (1+1) = 11+2

Expect to have a dynamical basis for earlier work on
algebraic S-theory in 11+2 (hep-th/9607112, 9608061)
M-theory type dualities, etc. (see hep-th/9904063)

Current Status

- Local Sp(2,R) → 2T-physics works!
  (X,P indistinguishable) is a fundamental principle that agrees with everything we know about Nature as embodied by the Standard Model.

- The Standard Model in 4+2 dimensions provides new guidance:
  1) Resolution of the strong CP violation problem of QCD.
  2) Dilaton driven electroweak spontaneous breakdown.
  Conceptually more appealing source for vev ; could relate to choice of vacuum in string theory
  Weakly coupled dilaton, possibly not very massive; LHC ? Dark Matter ? Inflaton?
  Can mass hierarchy problem be solved by conformal symmetry and/or 4+2 with remainders?

- Beyond the Standard Model

  GUTS, SUSY, (gravity); all can be elevated to 2T-physics in d+2 dimensions.
  Strings, branes; tensionless, and twistor superstring, 2T OK. Tensionful incomplete.
  M-theory; expect 11+2 dimensions → OSp(1|64) global SUSY, S-theory.

- New technical tools
  Emergent spacetimes and dynamics; unification; holography; duality; hidden symms.
  Non-perturbative analysis of field theory, including QCD? But wait until we develop quantum field theory directly in 6D.

- There is more to space-time than can be garnered with 1T-physics.
  New physical predictions and interpretations . It is more than a math trick.

In conclusion …

These old Greeks eventually discovered the concept of the atom.

Similarly, you can't ignore 2T-physics.

"Of course the elements are earth, water, fire and air. But what about chromium? Surely you can't ignore chromium."

Hidden information is predicted by 2T-physics. A new idea on <u>unification</u>.

1T-physics on its own cannot capture these hidden symmetries and dualities, which actually exist. 1T is OK only with additional guidance.

**A lot more remains to be done with 2T-physics.
New testable predictions at every scale of physics are expected from the hidden dualities and symmetries.**

2T-physics works
down to 10^{-18} meters
at least !

... and through work in progress
we hope to the extend
its domain of validity
to solve the remaining mysteries!!

The End

So far 2T-physics ⇔ known physics, but in new light with new predictions related to 1+1 dims.
Are there predictions of disagreements with 1T-Physics?? Yes, only in hitherto unknown realms.

Unsolved Mysteries:

•CP (time reversal) violation problem in the strong interactions (but axion not seen?)
→ 2T-physics solves this (a property of 4+2 dims.) – no need for axion!

•Why these degrees of freedom? And why in these patterns? (nothing new here)

•25 parameters : masses, couplings, mixings – is there a theory that determines them from first principles? Is Higgs the answer to the origin of mass ? LHC 2008 !!
→ 2T-physics modifies Higgs (adds dilaton). Also new possibilities for mass !

•Is there Supersymmetry (to resolve the mass hierarchy problem: stability) ?
→a) If SUSY exists, there are new constraints from 2T-physics ! Tests !
→b) 2T-physics may provide an alternative (conformal symmetry, 6 dims.!!) ?

•What is dark matter ? 25% of the matter in the Universe.

•What is dark energy? 70% of the energy in the Universe.

Possible candidates for these exist among the degrees of freedom above.

•How do we solve the Quantum Gravity problem (strings 9+1 dims, M-theory 10+1 dims, curled up dimensions) ? This is a framework ! Not a theory yet !
→ 2T-physics requires extra 1-space + 1-time. M-theory in 11 space + 2 times !!

NewScientist

October 13, 2007

TOP STORY

Time gains an extra dimension NS Exclusive

Will adding a second dimension to time lead to a 'theory of everything'? Could it lead to time travel? Only time will tell