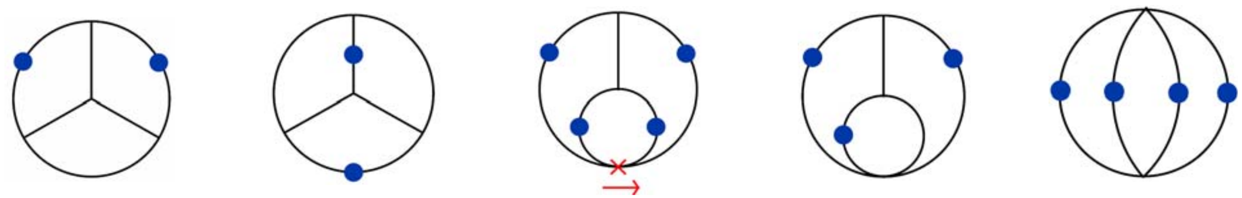
Is N=8 Supergravity Finite?



Z. Bern, L.D., R. Roiban, hep-th/0611086

Z. Bern, J.J. Carrasco, L.D., H. Johansson,
D. Kosower, R. Roiban, hep-th/0702112

U.C.S.B. High Energy and Gravity Seminar
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Introduction

• Quantum gravity is nonrenormalizable by power counting, because the coupling, Newton's constant, $G_N = 1/M_{\mathrm{Pl}}^2$ is dimensionful

• String theory cures the divergences of quantum gravity by introducing a new length scale, the string tension, at which particles are no longer pointlike.

• Is this necessary? Or could enough supersymmetry allow a point particle theory of quantum gravity to be perturbatively ultraviolet finite?

Maximal $\mathcal{N}=8$ Supergravity

DeWit, Freedman (1977); Cremmer, Julia, Scherk (1978); Cremmer, Julia (1978,1979)

- The most supersymmetry allowed, for maximum particle spin of 2, is $\mathcal{N}=8$

- This theory has $2^8 = 256$ massless states.
- Multiplicity of states, vs. helicity, from coefficients in binomial expansion of $(x+y)^8$ – 8th row of Pascal's triangle

$$\mathcal{N}=8: \quad 1 \leftrightarrow 8 \leftrightarrow 28 \leftrightarrow 56 \leftrightarrow 70 \leftrightarrow 56 \leftrightarrow 28 \leftrightarrow 8 \leftrightarrow 1$$

$$\text{helicity}: \quad -2 \quad -\tfrac{3}{2} \quad -1 \quad -\tfrac{1}{2} \quad 0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 2$$

$$h^- \quad \psi_i^- \quad v_{ij}^- \quad \chi_{ijk}^- \quad s_{ijkl} \quad \chi_{ijk}^+ \quad v_{ij}^+ \quad \psi_i^+ \quad h^+$$

SUSY charges Q_a, a=1,2,…,8 shift helicity by 1/2 $\leftrightarrow$

Is (N=8) Supergravity Finite?

A question that has been asked many times,
over many years.

Reports of the death of supergravity are exaggerations. One year everyone believed that supergravity was finite. The next year the fashion changed and everyone said that supergravity was bound to have divergences even though none had actually been found.

S. Hawking (1994)

Early Opinions

If certain patterns that emerge should persist in the higher orders of perturbation theory, then … N=8 supergravity in four dimensions would have ultraviolet divergences starting at three loops. Green, Schwarz, Brink, (1982)

Unfortunately, in the absence of further mechanisms for cancellation, the analogous N=8 D=4 supergravity theory would seem set to diverge at the three-loop order.

Howe, Stelle (1984)

It is therefore very likely that all supergravity theories will diverge at three loops in four dimensions. Marcus, Sagnotti (1985)

Thus, the onset of divergences in N=8 supergravity occurs at the three-loop order. Howe, Stelle (1989)

More Opinions

Our cut calculations indicate, but do not yet prove, that there is no three-loop counterterm for N=8 supergravity, contrary to the expectations from superspace power-counting bounds. On the other hand … we infer a counterterm at five loops with nonvanishing coefficient. Bern, LD, Dunbar, Perelstein, Rozowsky (1998)

The new estimates are in agreement with recent results derived from unitarity calculations ….
For N=8 supergravity in four dimensions, we speculate that the onset of divergences may … occur at the six loop level.

Howe, Stelle (2002)

Can also find arguments why first divergence might be at 7, 8, and 9 loops 8 loops: Kallosh (1981); Howe, Lindstrom (1981)

More Recent Opinions

… it is striking that these arguments suggest that maximally extended supergravity has no ultraviolet divergences when reduced to four dimensions Green, Russo, Vanhove (2006)

… we discussed evidence that four-dimensional N=8 supergravity may be ultraviolet finite. Bern, LD, Roiban (2006)

… recently discovered nonrenormalization properties of … the four-graviton amplitude in type II superstring theory [Berkovits], subject to an important smoothness assumption, [imply that] the four graviton amplitude of N=8 supergravity has no ultraviolet divergences up to at least eight loops.

Green, Russo, Vanhove (2006/7)

Basis for These Opinions?

1. Power-counting arguments, relying on superspace formalisms maintaining various amounts of supersymmetry
   Howe, Stelle, Townsend

2. Duality arguments, using the fact that N=8 supergravity is a compactified low energy limit of 11 dimensional M theory
   Green, Vanhove, Russo

3. Explicit calculation of four-graviton scattering, first in string theory [Green, Schwarz, Brink], and with Feynman diagrams in related theories [Marcus, Sagnotti]. More recently using unitarity method.
   [Bern, LD, Dunbar, Perelstein, Rozowsky (1998)]
   Also via zero-mode counting in pure spinor formalism for string theory [Berkovits, hep-th/0609006]

What about Ordinary Gravity?

On-shell counterterms in gravity should be generally covariant, composed from contractions of Riemann tensor $R_{\mu\nu\sigma\rho}$.
Terms containing Ricci tensor $R_{\mu\nu}$ and scalar R removable by nonlinear field redefinition in Einstein action

Since $R^{\mu}_{\nu\sigma\rho} \sim \partial_\rho \Gamma^{\mu}_{\nu\sigma} \sim g^{\mu\kappa}\partial_\rho\partial_\nu g_{\kappa\sigma}$
has mass dimension 2, and the loop-counting parameter $G_N = 1/M_{Pl}^2$ has mass dimension -2, every additional $R_{\mu\nu\sigma\rho}$ requires another loop, by dimensional analysis

One-loop → $R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$
However, $R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$
is Gauss-Bonnet term, total derivative in four dimensions.
So pure gravity is UV finite at one loop (but not with matter)

't Hooft, Veltman (1974)

Ordinary Gravity at Two Loops

Relevant counterterm,

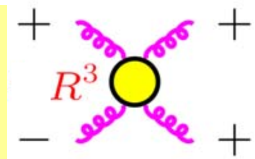
$$R^3 \equiv R^{\lambda\rho}{}_{\mu\nu} R^{\mu\nu}{}_{\sigma\tau} R^{\sigma\tau}{}_{\lambda\rho}$$

is nontrivial. By explicit Feynman diagram calculation
it appears with a nonzero coefficient at two loops

Goroff, Sagnotti (1986); van de Ven (1992)

4D Pure Supergravity:
Divergences Begin at Three Loops

R^3 cannot be supersymmetrized – it produces a helicity amplitude (-+++) forbidden by supersymmetry



Grisaru (1977); Tomboulis (1977)

However, at three loops, there is a perfectly acceptable counterterm, even for N=8 supergravity: The square of the Bel-Robinson tensor, abbreviated R^4 , plus (many) other terms containing other fields in the N=8 multiplet.

Deser, Kay, Stelle (1977); Kallosh (1981); Howe, Stelle, Townsend (1981)

R^4 produces first subleading term in low-energy limit of 4-graviton scattering in type II string theory:

$$\alpha'^3 R^4 \Rightarrow \alpha'^3 stu \underbrace{M_4^{\rm tree}(1,2,3,4)}_{\text{4-graviton amplitude in (super)gravity}}$$

Gross, Witten (1986)

Supergravity Scattering Amplitudes: Patterns Begin at One Loop

We can also study higher-dimensional versions of N=8 supergravity to see what critical dimension D_c they begin to diverge in, as a function of loop number L

Key technical ideas: BDDPR (1998)

- Unitarity to reduce multi-loop amplitudes to products of trees
  Bern, LD, Dunbar, Kosower (1994)
- Kawai-Lewellen-Tye (KLT) (1986) relations to express N=8 supergravity tree amplitudes in terms of simpler N=4 super-Yang-Mills tree amplitudes

Recall $\mathcal{N} = 8$ Spectrum

$2^8 = 256$ massless states, ~ expansion of $(x+y)^8$

$$\mathcal{N} = 8: \quad 1 \leftrightarrow 8 \leftrightarrow 28 \leftrightarrow 56 \leftrightarrow 70 \leftrightarrow 56 \leftrightarrow 28 \leftrightarrow 8 \leftrightarrow 1$$

$$\text{helicity}: \quad -2 \quad -\tfrac{3}{2} \quad -1 \quad -\tfrac{1}{2} \quad 0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 2$$

SUSY $\longleftrightarrow$

$$h^- \quad \psi_i^- \quad v_{ij}^- \quad \chi_{ijk}^- \quad s_{ijkl} \quad \chi_{ijk}^+ \quad v_{ij}^+ \quad \psi_i^+ \quad h^+$$

$$\mathcal{N} = 4: \quad 1 \leftrightarrow 4 \leftrightarrow 6 \leftrightarrow 4 \leftrightarrow 1$$

$2^4 = 16$ states ~ expansion of $(x+y)^4$

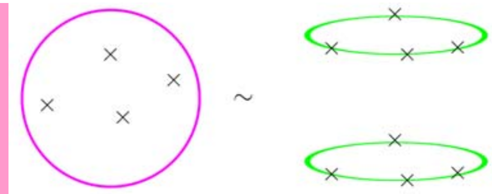
$$g^- \quad \lambda_A^- \quad \phi_{AB} \quad \lambda_A^+ \quad g^+$$

all in adjoint representation

$$\Rightarrow \quad [\mathcal{N} = 8] = [\mathcal{N} = 4] \otimes [\mathcal{N} = 4]$$

Kawai-Lewellen-Tye Relations

KLT, 1986



Derive from relation between open & closed string amplitudes.

Low-energy limit gives N=8 supergravity amplitudes $M_n^{\rm tree}$ as quadratic combinations of N=4 SYM amplitudes $A_n^{\rm tree}$, consistent with product structure of Fock space,

$$[\mathcal{N}=8] \;=\; [\mathcal{N}=4] \otimes [\mathcal{N}=4]$$

$$M_4^{\rm tree}(1,2,3,4) \;=\; -i\,\frac{st}{u}[A_4^{\rm tree}(1,2,3,4)]^2$$

$$M_5^{\rm tree}(1,2,3,4,5) \;=\; i\,s_{12}s_{23}\,A_5^{\rm tree}(1,2,3,4,5)\,A_5^{\rm tree}(2,1,4,3,5) + (2 \leftrightarrow 3)$$

$$M_6^{\rm tree}(1,2,3,4,5,6) \;=\; \cdots$$

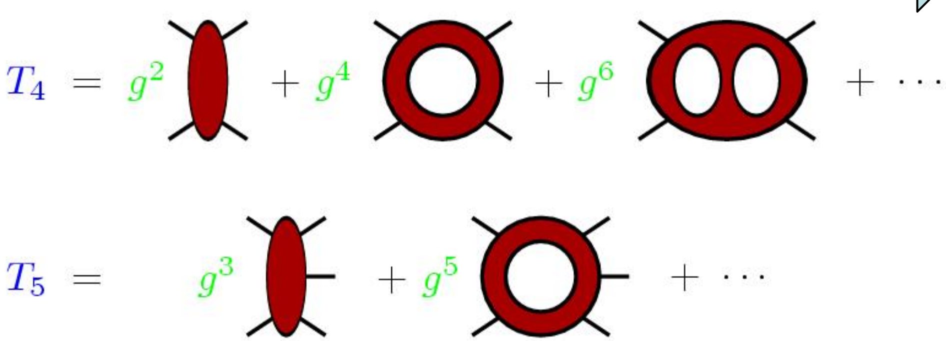
Amplitudes via Perturbative Unitarity

- S-matrix is a unitary operator between in and out states

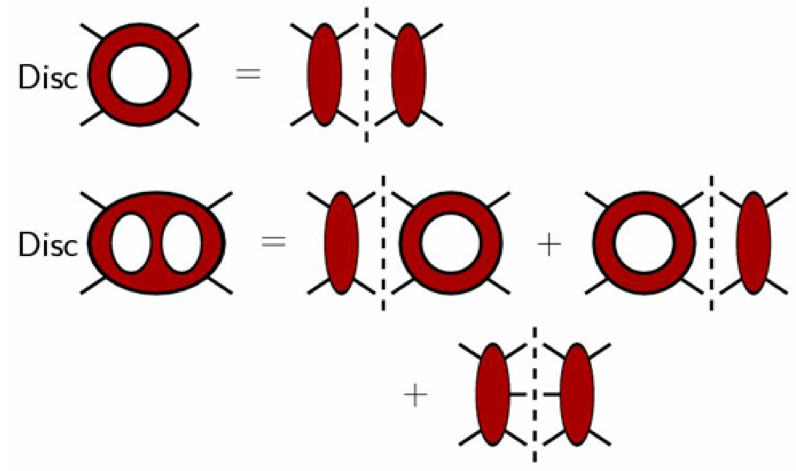
$$1 = S^{\dagger}S = (1 - iT^{\dagger})(1 + iT)$$

$$2\,\mathrm{Im}\,T = T^{\dagger}T$$

- Expand T-matrix in g



Unitarity relations (cutting rules) for amplitudes

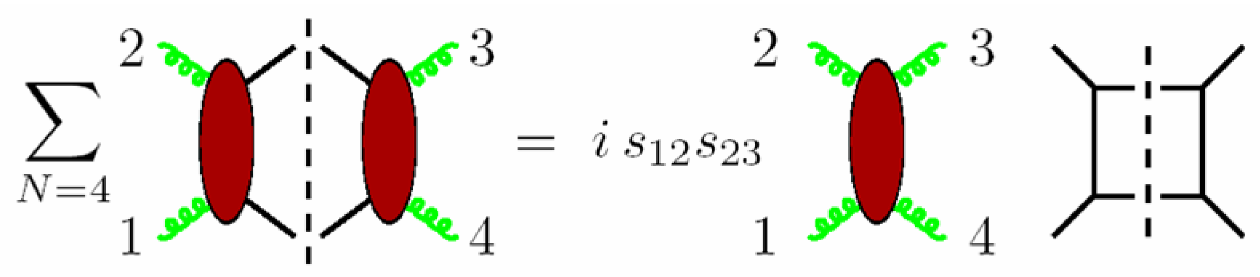


- Very efficient due to simple structure of tree helicity amplitudes

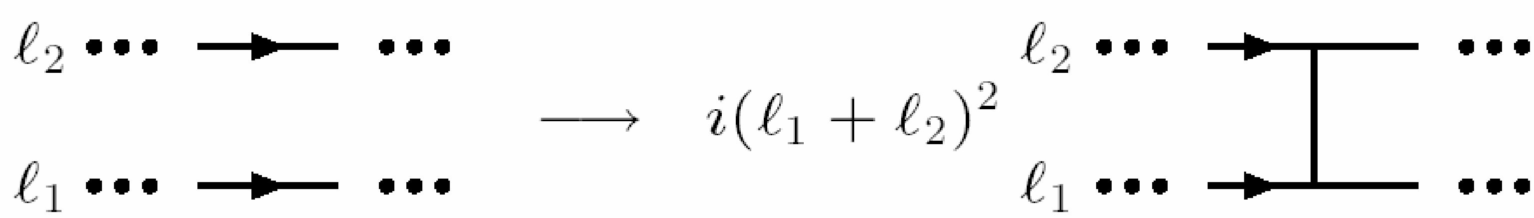
Bern, LD, Dunbar, Kosower (1994)

Unitarity and N=4 SYM

Many higher-loop contributions to *gg* → *gg* scattering deduced from a simple property of the 2-particle cuts at one loop Bern, Rozowsky, Yan (1997)



Leads to "rung rule" for easily computing all contributions which can be built by iterating 2-particle cuts



Unitarity and N=8 Supergravity

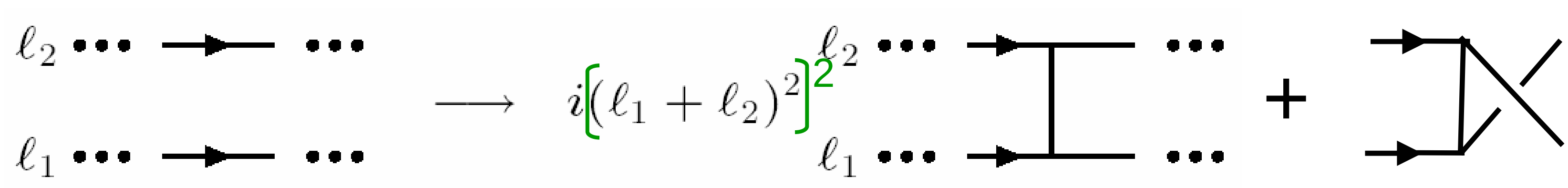
Using

$$M_4^{\text{tree}}(1,2,3,4) \;=\; -i\,\frac{st}{u}[A_4^{\text{tree}}(1,2,3,4)]^2$$

and the N=4 SYM 2-particle cutting equation,
yields an N=8 SUGRA 2-particle cutting equation,
which can also be written as a rung rule, but with a squared numerator factor.

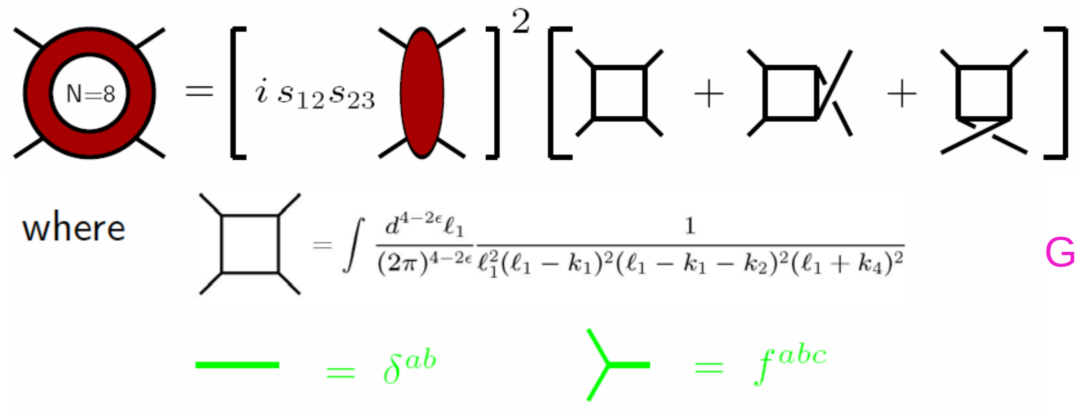
BDDPR

N=8 SUGRA rung rule



Resulting Simplicity at 1 and 2 Loops

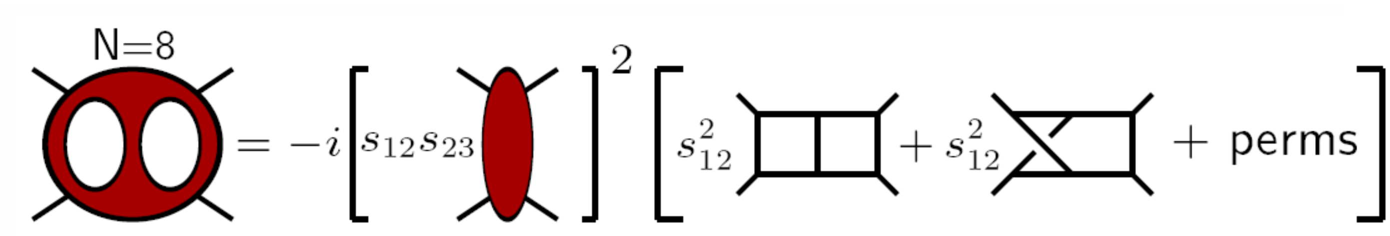
- 1 loop:



Green, Schwarz, Brink (1982)

"color dresses kinematics"

- 2 loops:



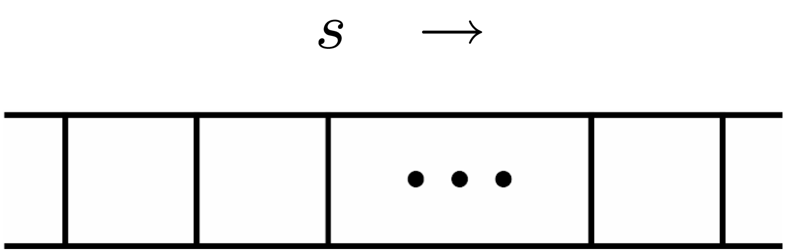
Bern, Rozowsky, Yan (1997); Bern, LD, Dunbar, Perelstein, Rozowsky (1998)

N=8 supergravity: just remove color, square prefactors!

Ladder Diagrams (Regge-like)

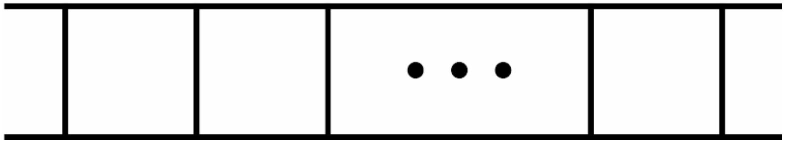
In N=4 SYM

$$st\, A_4^{\text{tree}} \times s^{L-1}$$



In N=8 supergravity

$$stu\, M_4^{\text{tree}} \times s^{2(L-1)}$$



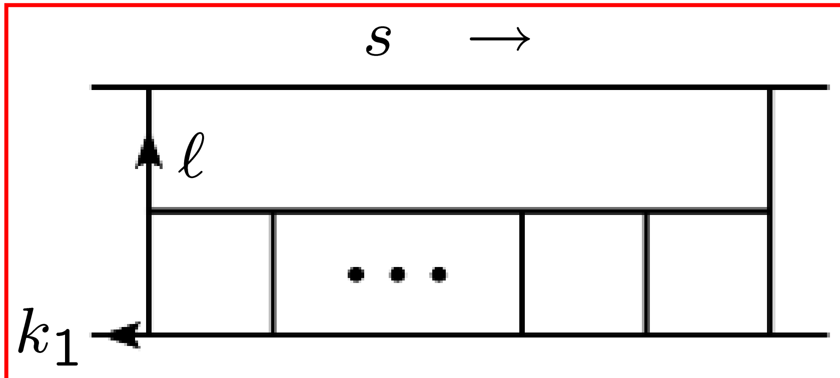
Extra s^L in gravity from "charge" = energy

Schnitzer, hep-th/0701217

More UV Divergent Diagrams

N=4 SYM

$$st\, A_4^{\text{tree}} \times t \times [(\ell + k_1)^2]^{L-2}$$



N=8 supergravity

$$stu\, M_4^{\text{tree}} \times t^2 \times [(\ell + k_1)^2]^{2(L-2)}$$

Integral in *D* dimensions scales as

$$\mathcal{I} \sim \int d^{DL}\ell \, \frac{(\ell^2)^{2(L-2)}}{(\ell^2)^{3L+1}}$$

→ Critical dimension D_c for log divergence obeys

$$\frac{D_c L}{2} + 2(L-2) = 3L + 1 \quad \Rightarrow$$

$$D_c = 2 + \frac{10}{L}$$ N=8

$$D_c = 4 + \frac{6}{L}$$ N=4 SYM

BDDPR (1998)

Is This Power Counting Correct?

$D_c = 2 + \frac{10}{L}$ N=8 $\Leftrightarrow$ $stu\, M_4^{\text{tree}} \times t^2 \times [(\ell + k_1)^2]^{2(L-2)}$

$\Leftrightarrow$ $\partial^4 R^4$ potential counterterm at every loop order $L \geq 2$

$\Leftrightarrow$ *D*=4 divergence at five loops

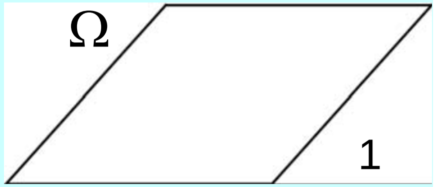
Reasons to reexamine whether it might be too conservative:

- Superspace-based speculation that *D=4* case diverges only at *L=6*, not *L=5* Howe, Stelle, hep-th/0211279
- Multi-loop string calculations seem not to allow $\partial^4 R^4$ past *L=2*. Berkovits, hep-th/0609006; Green, Russo, Vanhove, hep-th/0611273
- String/M duality arguments with similar conclusions, suggesting possibility of finiteness Green, Russo, Vanhove, hep-th/0610299
- No triangle hypothesis for 1-loop amplitudes Bjerrum-Bohr et al., hep-th/0610043

M Theory Duality

Green, Vanhove, hep-th/9910055, hep-th/0510027; Green, Russo, Vanhove, hep-th/0610299

- N=1 supergravity in *D*=11 is low-energy limit of M theory
- Compactify on two torus with complex parameter Ω
- Use invariance under $\Omega \rightarrow \frac{a\Omega + b}{c\Omega + d}$



  combined with threshold behavior of amplitude in limits leading to type IIA and IIB superstring theory
- Conclude that at *L* loops, effective action is $\sim \; D^{2L} R^4$
- **If** all dualities hold, and **if** this result survives the compactification to lower *D* – i.e. there are no cancellations between massless modes and Kaluza-Klein excitations – then it implies $D_c = 4 + \frac{6}{L}$ – same as in N=4 SYM – for all *L*

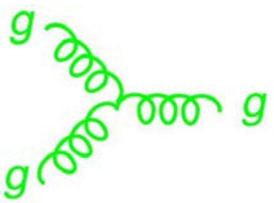
Zero-mode Counting in String Theory

Berkovits, hep-th/0609006; Green, Russo, Vanhove, hep-th/0611273

- New pure spinor formalism for type II superstring theory keeps spacetime supersymmetry manifest
- Allows generic properties of multi-loop 4-graviton amplitude in string theory to be extracted from a zero mode analysis.
- Conclude that at *L* loops, for *L* < 7, effective action is $\sim \; D^{2L} R^4$
- For *L* = 7 and higher, run out of zero modes, and can only argue for $\sim \; D^{12} R^4$
- **If** this result survives the both the low-energy limit, $\alpha' \to 0$, **and** compactification to *D*=4 – i.e. there are no cancellations between massless modes and either stringy or Kaluza-Klein excitations – then it implies finiteness through 8 loops

No-triangle power counting at one loop

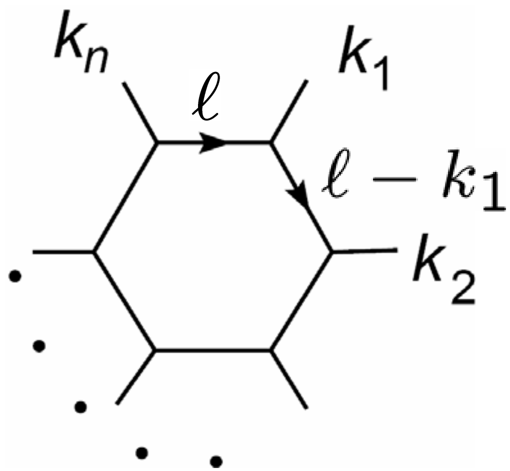
generic gauge theory (spin 1)



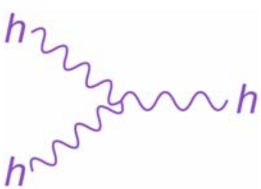
$$\supset \ \ell^{\mu}\,\eta^{\nu\rho} + \cdots$$

$$\Rightarrow \ (\ell^{\mu})^{n}$$

N=4 SYM $\Rightarrow \ (\ell^{\mu})^{n-4}$



generic gravity (spin 2)



$$\supset \ \ell^{\mu_1}\,\ell^{\mu_2}\,\eta^{\nu_1\rho_1}\,\eta^{\nu_2\rho_2} + \cdots$$

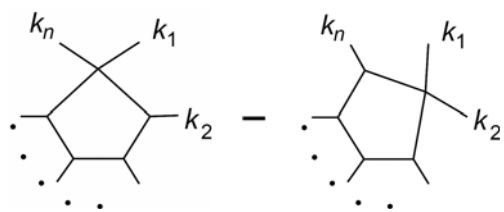
$$\Rightarrow \ (\ell^{\mu})^{2n}$$

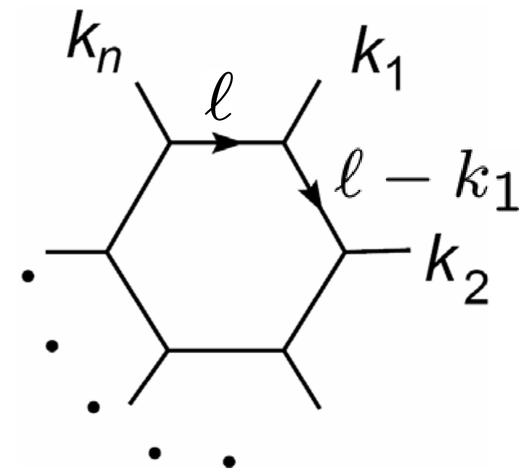
N=8 supergravity $\overset{??}{\Rightarrow} \ (\ell^{\mu})^{2(n-4)}$

evidence that it is better

No-triangle power counting (cont.)

$$\mathcal{I}_n[2\ell\cdot k_1] \equiv \int \frac{d^D\ell \;\; 2\ell\cdot k_1}{\ell^2(\ell-k_1)^2\cdots}$$
$$= \int \frac{d^D\ell \;\; [\ell^2-(\ell-k_1)^2]}{\ell^2(\ell-k_1)^2\cdots}$$
$$= \mathcal{I}^{(1)}_{n-1}[1] - \mathcal{I}^{(2)}_{n-1}[1]$$





N=4 SYM, $(\ell^\mu)^{n-4}$

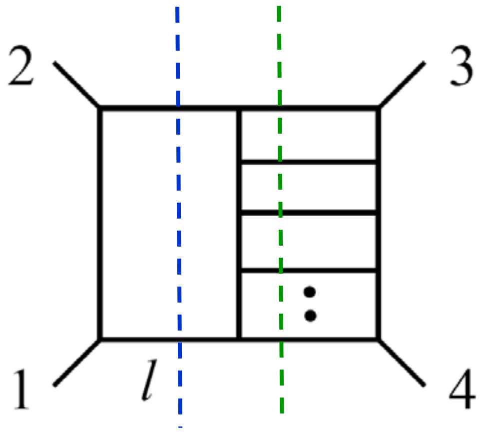
a pentagon linear in ℓ^μ → scalar box with no triangle

a generic pentagon quadratic in ℓ^μ → linear box → scalar triangle

But all N=8 amplitudes inspected so far, with 5,6,~7,… legs,
contain no triangles → more like $(\ell^\mu)^{n-4}$ than $(\ell^\mu)^{2(n-4)}$

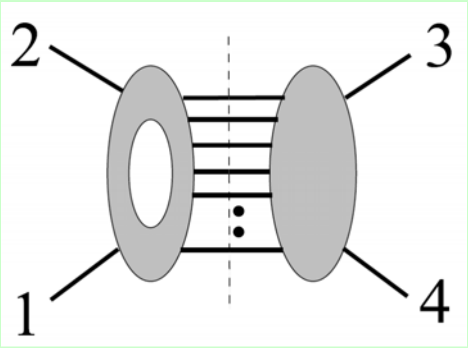
Bjerrum-Bohr et al., hep-th/0610043

A key L-loop topology



2-particle cut exposes Regge-like ladder topology, containing numerator factor of $[(l+k_4)^2]^{2(L-2)}$

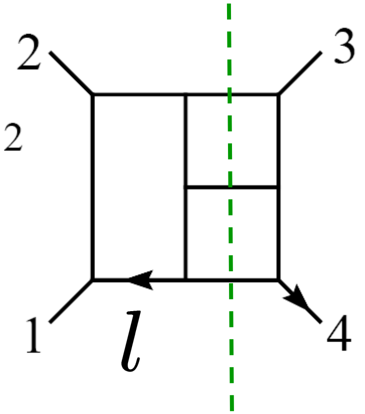
L-particle cut exposes one-loop (L+2)-point amplitude – but $[(l+k_4)^2]^{2(L-2)}$ would (heavily) violate the no-triangle hypothesis



Three-loop case

3 loops very interesting because it is first order for which N=4 SYM and N=8 supergravity might have a different D_c

$$s^2\left[(l+k_4)^2\right]^2$$



3-particle cut exposes one-loop 5-point amplitude with numerator of

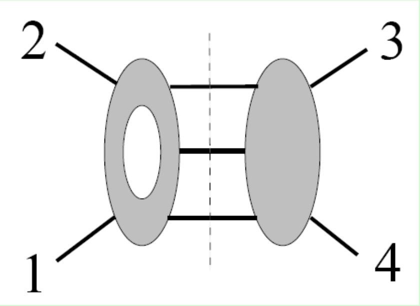
$$[(l+k_4)^2]^2 \approx [(l\cdot k_4)^2]^2$$

at least quadratic in l^μ

– violates no-triangle hypothesis

– which for 5-point case is a fact

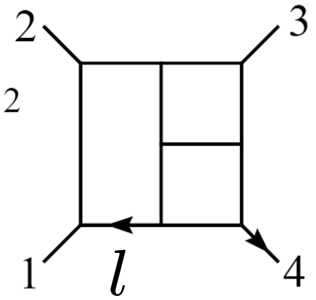
Bern, LD, Perelstein, Rozowsky, hep-th/9811140



Three-loop case (cont.)

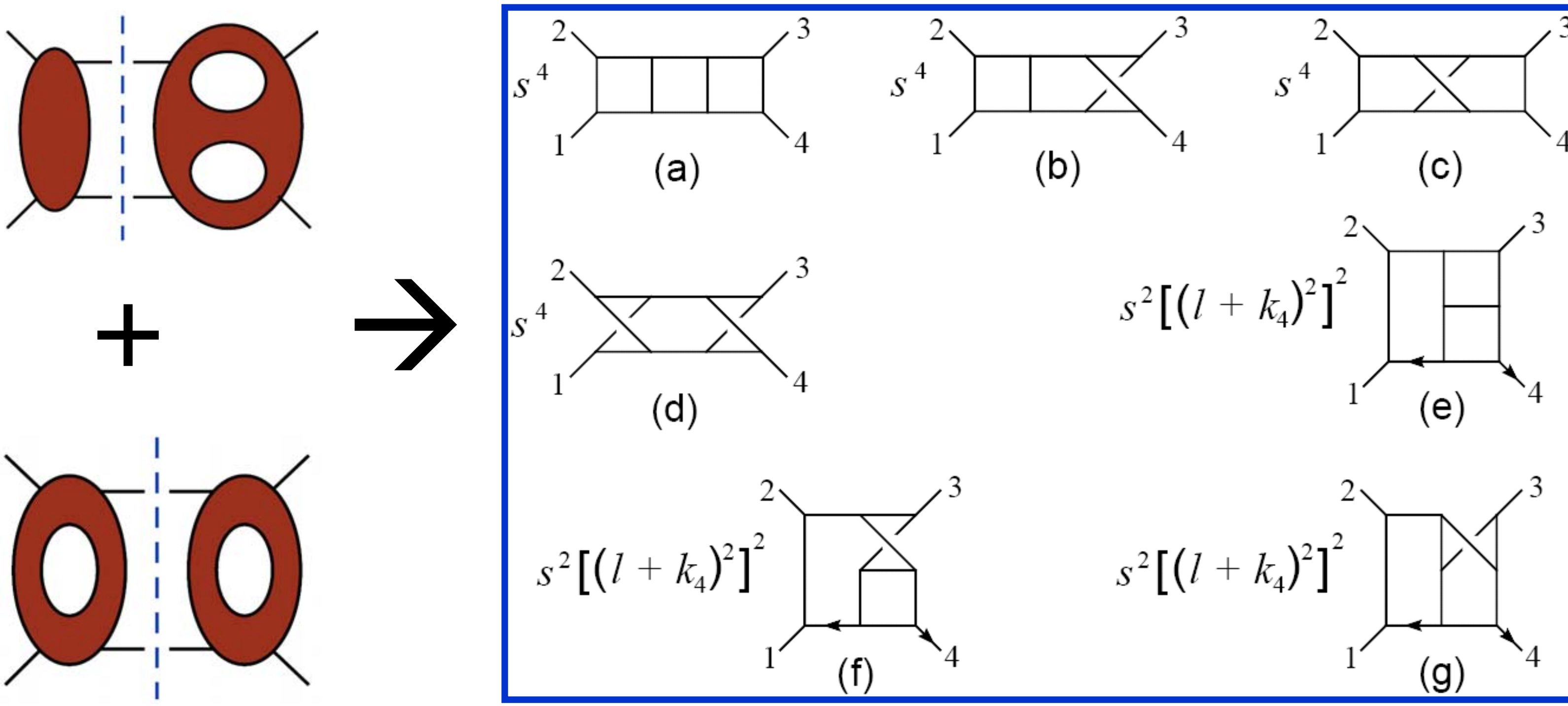
Something else must cancel the bad "left-loop" behavior of this contribution. But what?

$$s^2\left[(l+k_4)^2\right]^2$$



Only way to know for sure is to evaluate **all the cuts – 3-particle and 4-particle cuts** as well as 2-particle

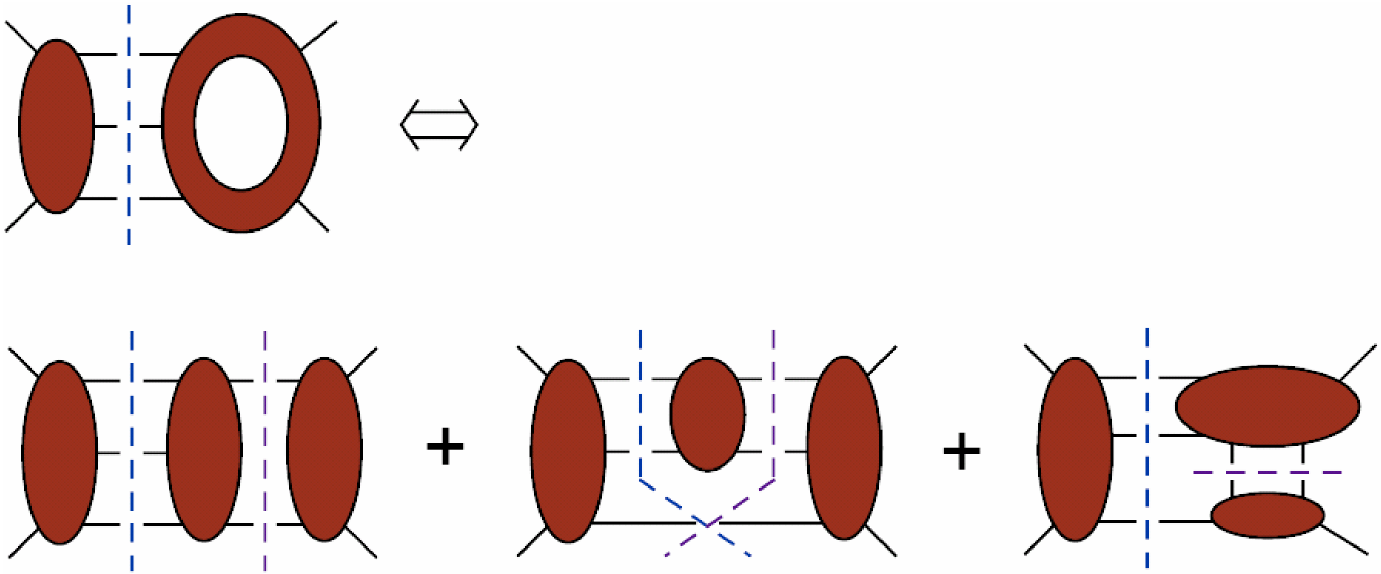
2-particle cuts → rung-rule integrals



all numerators here are precise squares of corresponding N=4 SYM numerators

3-particle cuts

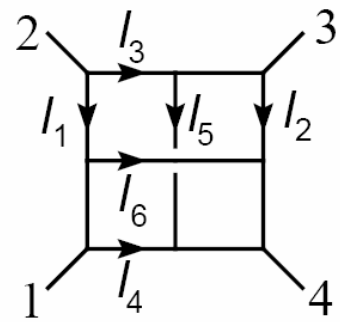
Chop 5-point loop amplitude further,
into (4-point tree) x (5-point tree), in all inequivalent ways:



Using KLT, each product of 3 supergravity trees decomposes into pairs of products of 3 (twisted, nonplanar) SYM trees.
To evaluate them, we needed the full, non-rung-rule, non-planar 3-loop N=4 SYM amplitude → 2 more integral topologies

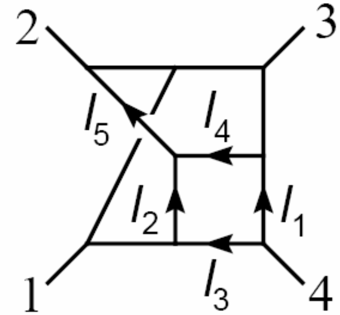
Non-rung-rule N=4 SYM at 3 loops

$$s_{12}\,(l_1+l_2)^2 + s_{23}\,(l_3+l_4)^2 - s_{12}\,l_5^2 - s_{23}\,l_6^2 - s_{12}\,s_{23}$$



(h)

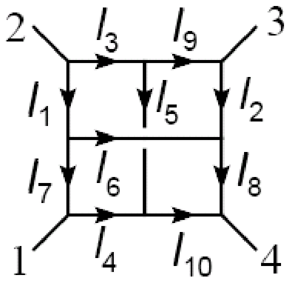
$$s_{12}\,(l_1+l_2)^2 - s_{23}\,(l_3+l_4)^2 - \frac{1}{3}\left(s_{12} - s_{23}\right) l_5^2$$



(i)

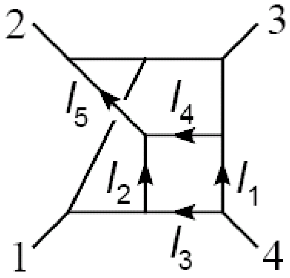
Non-rung-rule N=8 SUGRA at 3 loops

$$\left[s_{12}(l_1+l_2)^2 + s_{23}(l_3+l_4)^2 - s_{12}s_{23} \right]^2$$
$$- s_{12}^2 \left[2\left(((l_1+l_2)^2 - s_{23}\right) + l_5^2 \right] l_5^2$$
$$- s_{23}^2 \left[2\left(((l_3+l_4)^2 - s_{12}\right) + l_6^2 \right] l_6^2$$
$$-2 \left[s_{12}^2 \left(l_1^2 l_8^2 + l_2^2 l_7^2 \right) + s_{23}^2 \left(l_3^2 l_{10}^2 + l_4^2 l_9^2 \right) \right]$$
$$- s_{12}^2 \left(l_1^2 l_7^2 + l_2^2 l_8^2 \right) - s_{23}^2 \left(l_3^2 l_9^2 + l_4^2 l_{10}^2 \right)$$
$$+ 2 s_{12} s_{23} l_5^2 l_6^2$$



(h)

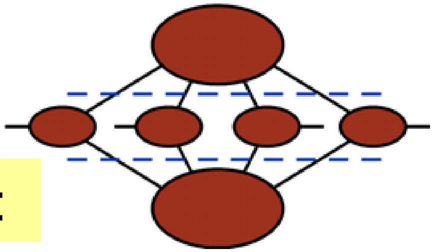
$$\left[s_{12}(l_1+l_2)^2 - s_{23}(l_3+l_4)^2 \right]^2$$
$$-\left[s_{12}^2 (l_1+l_2)^2 + s_{23}^2 (l_3+l_4)^2 + \frac{1}{3} s_{12} s_{23} s_{13} \right] l_5^2$$



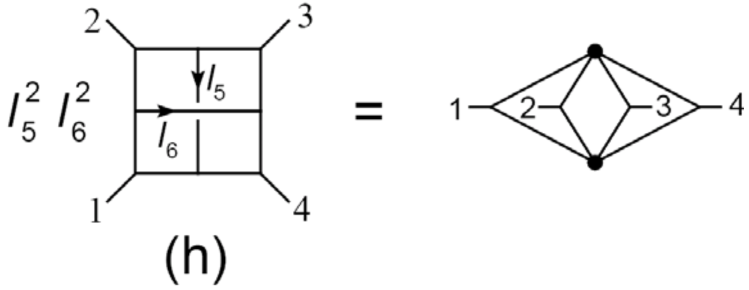
(i)

4-particle cuts

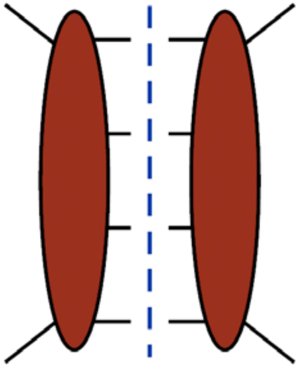
An important – but simple – subset of the full 4-particle cuts, of the form →



can be used to determine the $l_5^2\, l_6^2$ term in (h):

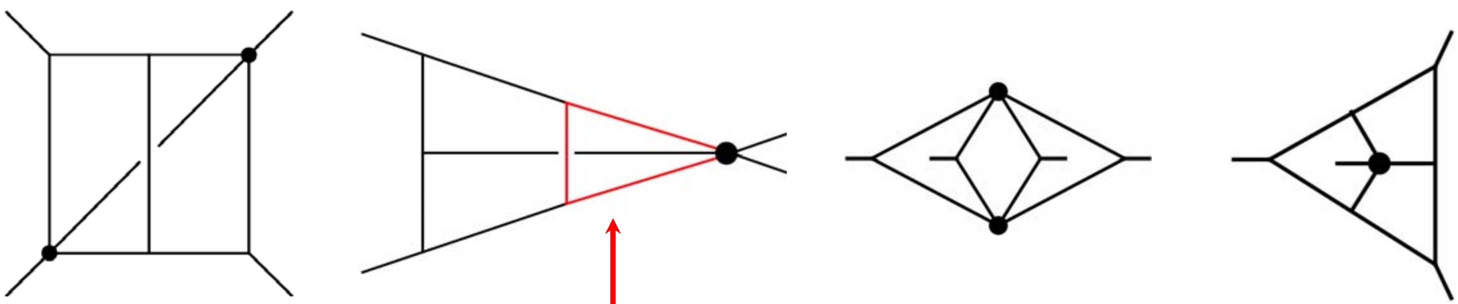


- Full 4-particle cuts are easy to draw → but more difficult to evaluate (72 different twisted SYM configurations).
- They work, with the above integrals (a)-(i), confirming our representation of the 3-loop N=8 supergravity amplitude.



Regarding the no-triangle hypothesis

- At 3 loops, it is manifested in an interesting way.
- Parts of the (h) and (i) contributions can be rewritten by cancelling propagator factors between numerator and denominator, as:



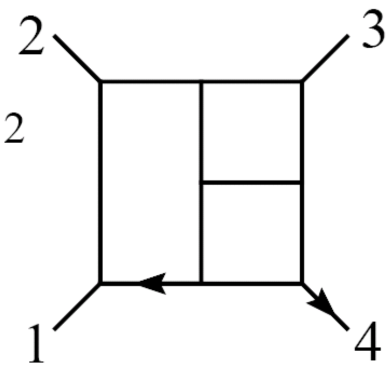
This one has a triangle, but its role is to cancel bad UV behavior of other topologies, e.g. pentagons with loop momentum in the numerator

Leading UV behavior

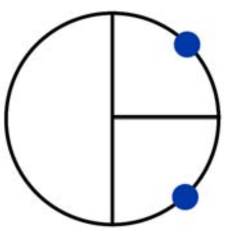
- Can be obtained by neglecting all dependence on external momenta, $k_i \longrightarrow 0$
- Each four-point integral becomes a vacuum integral

For example, graph (e) becomes

$$s^2\left[(l+k_4)^2\right]^2$$

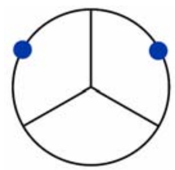
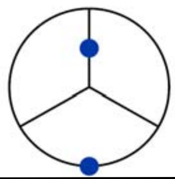
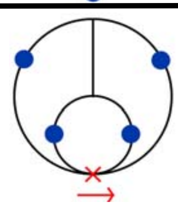
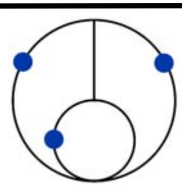
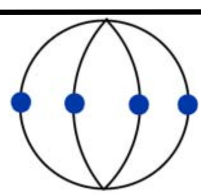


$$\Rightarrow \quad s^2$$



where ● denotes a doubled propagator

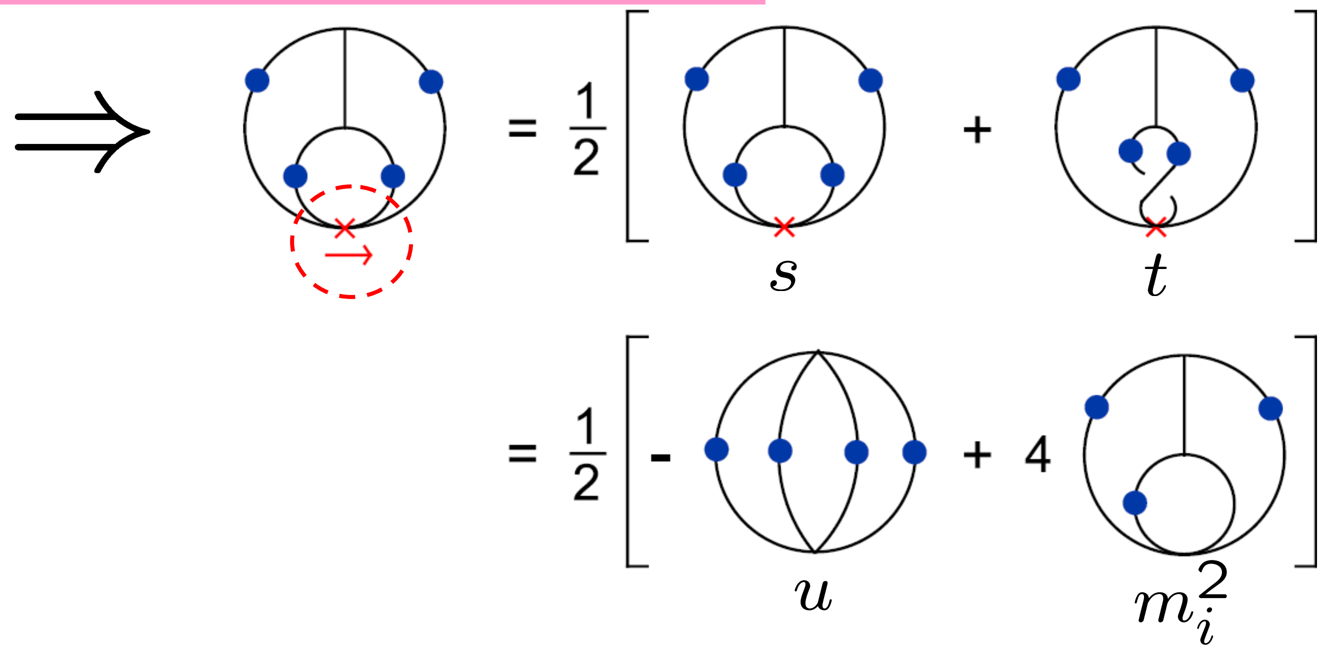
Sum up vacuum diagrams

	(e)	(f)	(g)	(h)	(i)	Total
	4	0	8	-4	-8	0
	0	4	0	-8	-4	0
	0	0	0	-4	0	-4
	0	0	0	0	8	8
	0	0	0	-2	0	-2

Vacuum diagram identity

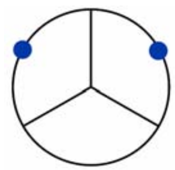
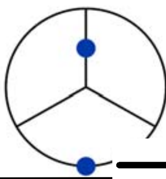
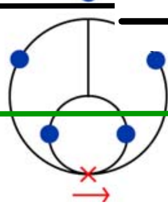
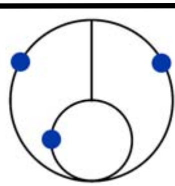
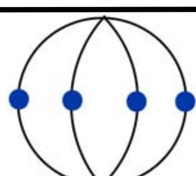
Apply $s + t + u \;=\; m_1^2 + m_2^2 + m_3^2 + m_4^2$

to the 4 legs surrounding "x"



Cancellations beyond no-triangle hypothesis!

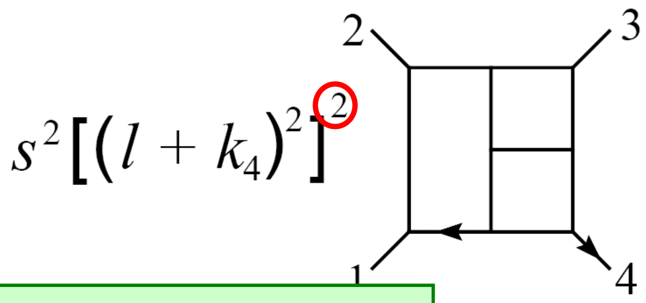
"Re-sum" vacuum diagrams

	(e)	(f)	(g)	(h)	(i)	Total
	4	0	8	-4	-8	0
	0	4	0	-8	-4	0
	0	0	0	-4	0	-4
	0	0	0	0	8	8 - 8 = 0
	0	0	0	-2	0	-2 + 2 = 0

⇒ leading UV divergence cancels perfectly!

N=8 SUGRA (again) no worse than N=4 SYM

Leading behavior of individual topologies in N=8 SUGRA is $\sim\ l^4$



But leading behavior cancels after summing over topologies → $\sim\ l^3\ \equiv\ (l^2)\, l^\mu$

Lorentz invariance → no cubic divergence
In fact there is a manifest representation $\sim\ l^2$
– same behavior as N=4 SYM

Evaluation of l^2 integrals → no cancellation at this level
→ At 3 loops, $D_c = 6$ for N=8 SUGRA as well as N=4 SYM

Conclusions & Outlook

• Old power-counting formula from iterated 2-particle cuts predicted

$$D_c = 2 + \frac{10}{L}$$ N=8

• New terms found from 3-particle cuts, and confirmed with the 4-particle cuts, exhibit no-triangle UV behavior of one-loop multi-leg N=8 amplitudes.

• There are two additional cancellations, which together reduce the overall degree of divergence, so that, not only is N=8 finite at 3 loops, but $D_c = 6$ at $L=3$, the same as for N=4 SYM!

• Will the same happen at higher loops, so that the formula

$$D_c = 4 + \frac{6}{L}$$ N=4 SYM

continues to be obeyed by N=8 supergravity as well?

• If so, it will represent a finite, pointlike theory of quantum gravity

• Not of direct phenomenological relevance, but could it point the way to other, more relevant finite theories?

Extra slides

Superspace Power Counting

Howe, Stelle, hep-th/0211279

• Write allowed counterterms as integrals over superspace that are allowed by superspace Feynman rules

- **Problem**: In maximally supersymmetric theories, cannot maintain full supersymmetry off-shell
- In N=4 SYM,
  - N=4 SUSY constraints → eqns. of motion
  - N=2 SUSY can be maintained off-shell
  - N=3 OK with extra bosonic variable (harmonic superspace)
- In N=8 SUGRA, N=4 SUSY can be maintained off-shell.
- This predicts the 3-loop counterterm $\int d^{16}\theta \, W^4 \sim R^4$

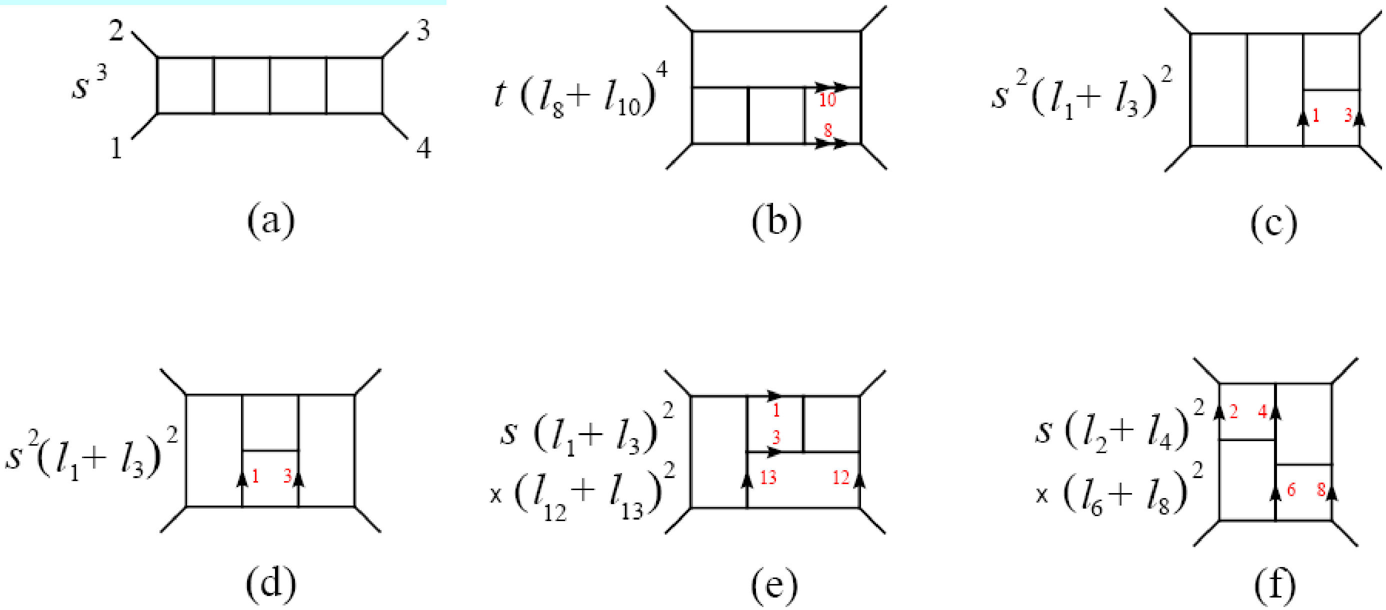
– not "good enough"

- **If** there is an N=7 harmonic superspace (not known) then predict finiteness up to 6 loops, counterterm $\int d^{28}\theta \, W^4 \sim D^6 R^4$

Integrals for 4-loop N=4 planar amplitude

BCDKS, hep-th/0610248

rung-rule diagrams



non-rung-rule diagrams