Numerical Simulations of Singularities

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(1) harmonic coordinate numerical method

(2) approach to the singularity

(3) asymptotically flat spacetimes

(4) conclusions and future projects

numerical simulations

replace functions with their values on a lattice of points
replace differential equations with finite difference equations

difficulties with simulating general relativity

(1) numerical instabilities

(2) form of the equations

(3) curvature singularities

(4) coordinate singularities

(5) outer boundary conditions

(6) constraints

(7) black holes

Harmonic coordinate numerical method

Make Einstein's equation look like the wave equation by using (generalized) harmonic coordinates

$$\nabla_a \nabla^a x^\mu = H^\mu$$

$$R_{\mu\nu} = -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{\mu\nu} + L_{\mu\nu}(g, \partial g)$$

$$-\partial_{(\mu} H_{\nu)} + \Gamma^\alpha_{\mu\nu} H_\alpha$$

time coordinate can go null. Source terms may postpone or eliminate this behavior

variables $g_{\mu\nu}$ and $P_{\mu\nu} \equiv \partial_t g_{\mu\nu}$

constraints $g^{\alpha\beta}\Gamma^\mu_{\alpha\beta} + H^\mu = 0$

evaluate spatial derivatives using centered differences

$$\frac{\partial F}{\partial x} \to \frac{F_{i+1} - F_{i-1}}{2\Delta x}$$

$$\frac{\partial^2 F}{\partial x^2} \to \frac{F_{i+1} + F_{i-1} - 2F_i}{(\Delta x)^2}$$

Evolve in time using 3 step ICN $\partial_t S = W$ implemented as

$$S^{n+1} = S^n + \frac{\Delta t}{2}[W(S^n) + W(S^{n+1})]$$

Approach to the singularity

Singularity theorems give very little information on the nature of the singularity

Approach to the singularity might be simple

A combination of numerical and mathematical results indicates that the singularity is local, spacelike and

(i) oscillatory in the vacuum case

(ii) non-oscillatory in the Einstein-scalar field case

Gowdy spacetimes

$$ds^2 = e^{(t-\lambda)/2}[-e^{-2t}dt^2 + dx^2]$$

$$+e^{-t}\left[e^P(dy + Qdz)^2 + e^{-P}dz^2\right]$$

Vacuum Einstein equations

$$P_{tt} - e^{-2t}P_{xx} - e^{2P}(Q_t^2 - e^{-2t}Q_x^2) = 0$$

$$Q_{tt} - e^{-2t}Q_{xx} + 2(P_tQ_t - e^{-2t}P_xQ_x) = 0$$

results

Numerical simulations
(Berger, Moncrief, . . .)
$P \to v(x)t$ and $Q \to Q(x)$ as $t \to \infty$
(but spikes at isolated points)

Global results
(Isenberg, Moncrief, Chrusciel)

Local, near singularity results
(Rendall, Kichenassamy)

more general spacetimes

U(1) spacetimes

Numerical simulations
(Berger, Moncrief)
Local Mixmaster behavior

No symmetry

Local, near singularity result
for Einstein-scalar equations
(Rendall, Andersson)
Local Kasner

Numerical simulations
(Garfinkle, Miller, Berger, Duncan)
work in progress

Code for approach to the singularity has been tested using comparison to a Gowdy (1+1) code and using a convergence test.

results for Einstein-scalar code (initial data found algebraically)

work in progress on vacuum case (initial data found by solving an elliptic equation)

Mark Miller has written a parallel version of the code using Cactus

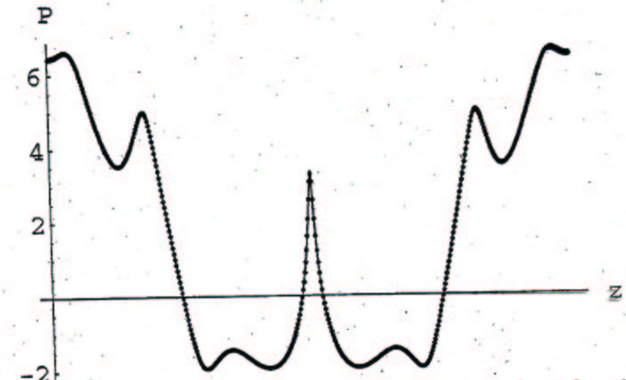


FIG. 1. comparison of the quantity P as found using a Gowdy code and the 3+1 harmonic code

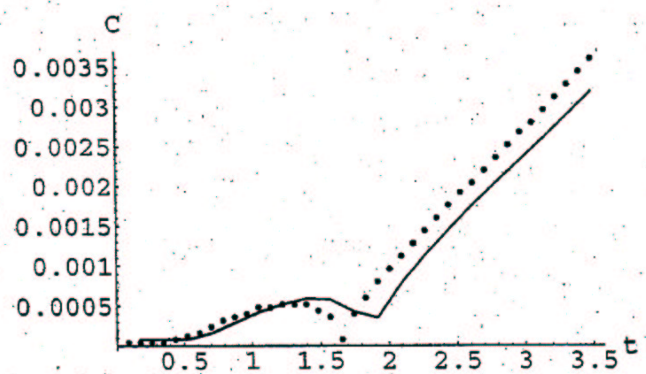
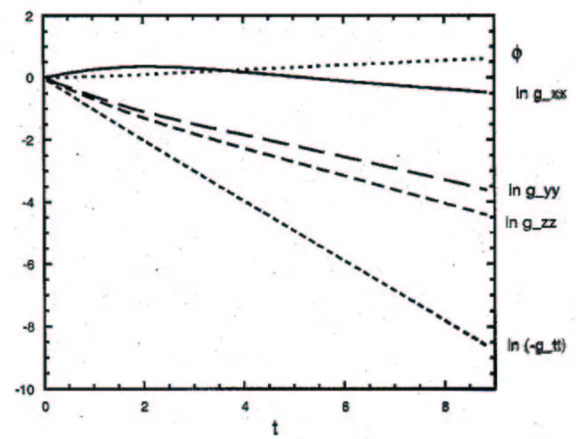


FIG. 2. convergence test involving the constraint

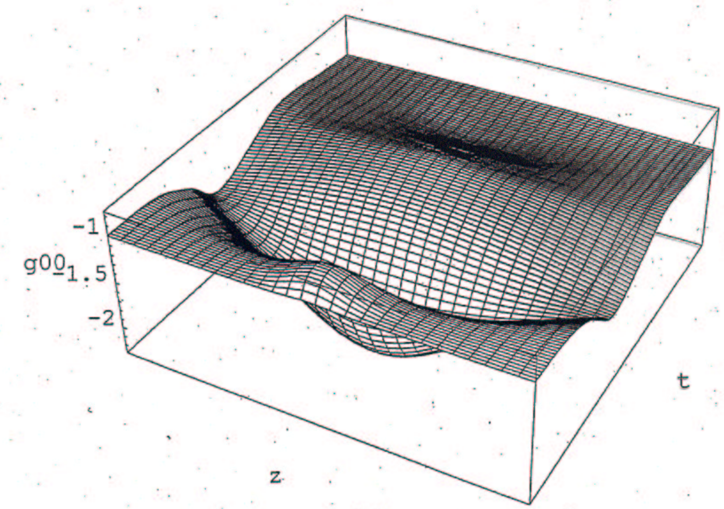


Asymptotically flat case

Initial data found using
a conjugate gradient method

coordinate source terms needed for
very strong fields

simple outer boundary condition
works well



Conclusion

harmonic coordinates seem to yield a useful numerical method

future projects

thorough examinations of the approach to the singularity.

examination of the collapse of gravitational waves

excision methods