Higher Derivative Corrections in M-theory via Local Supersymmetry

Yoshifumi Hyakutake (KITP,UCSB)

Based on the works with S. Ogushi

- hep-th/0508204
- hep-th/0601092; JHEP0602(2006)068

1. Introduction

11D Supergravity : Low energy limit of M-theory

Various supersymmetric solutions

Flat, AdS $\times$ S, PP-wave (32)

p-brane, plane-wave, KK-monopole, … (16)

Intersecting branes, supertube, … (8)

Are these exact solutions in M-theory?

Yes for Flat, AdS $\times$ S, PP-wave and plane-wave.

But for other cases, it is not so clear.

We need to know the higher derivative corrections to equations of motion in supergravity.

Especially we consider the higher derivative corrections in M-theory

Because

- There are 32 supercharges
- Field contents are simple

Since the perturbative definition of the M-theory is not known yet, the local supersymmetry is the key to construct the higher derivative corrections in M-theory.

Is local supersymmetry enough to determine the structure of the effective action of the M-theory?

1. Introduction
2. R^4 Corrections in Type IIA Supergravity
3. Review of 11D Supergravity
4. Higher Derivative Corrections: an Overview
5. Higher Derivative Corrections: Details
6. Conclusions and Discussions

2. R^4 Corrections in Type IIA Supergravity

There are several methods to study R^4 corrections to type II supergravity

- String scattering amplitude
- Non linear sigma model — RR b.g. is difficult
- Superfield method
- Duality
- Noether's method — Hard calculation
- …

So far the complete form of the eight derivative action is not known. Known bosonic part (R^4 part) is as follows.

$$\mathcal{L}_{(\alpha')^3} \sim e^{-2\phi} I_{\text{tree}} + c\, I_{\text{1-loop}},$$
$$I_{\text{tree}} = t_8 t_8 e R^4 + \frac{1}{4\cdot 2!}\epsilon_{10}\epsilon_{10} e R^4,$$
$$I_{\text{1-loop}} = t_8 t_8 e R^4 - \frac{1}{4\cdot 2!}\epsilon_{10}\epsilon_{10} e R^4 - \frac{1}{6}\epsilon_{10} t_8 B R^4$$

$t_8 t_8 R^4$

This expression is obtained by calculating the on-shell 4 graviton amplitude.

$$A^{\text{closed}}_{4pt} \sim -g^2 \frac{\Gamma(-s/8)\Gamma(-t/8)\Gamma(-u/8)}{\Gamma(1+s/8)\Gamma(1+t/8)\Gamma(1+u/8)} \times t_8^{ijklmnop} t_8^{abcdefgh} f^1_{ij} f^2_{kl} f^3_{mn} f^4_{op} \otimes \tilde{f}^1_{ab} \tilde{f}^2_{cd} \tilde{f}^3_{ef} \tilde{g}^4_{gh}$$

Green, Schwarz

$f_{ij} \equiv \frac{1}{2}(k_i \zeta_j - k_j \zeta_i)$

Gamma functions are expanded as

$$-\frac{2^9}{stu} - 2\zeta(3) + \cdots$$

The first term is also obtained by the tree level scattering amplitude for 4 gravitons in type II supergravity.

By making the identification of $f_{ij} \otimes \tilde{f}_{ab} = R_{ijab}$
we obtain the corrections to the type II supergravity

$$S = \int \mathcal{L}_{\text{sugra}} + (const.)\zeta(3)\alpha'^3 \int e e^{-2\Phi} t_8 t_8 R^4 + \cdots$$

Gross, Witten
Gross, Sloan

$\epsilon_{10}\epsilon_{10}R^4$

This expression is obtained by calculating the beta functions of non-linear sigma model in 4 loop order.

$$\delta^{[ijklmnop]}_{[abcdefgh]} R^{ab}{}_{ij} R^{cd}{}_{kl} R^{ef}{}_{mn} R^{gh}{}_{op}$$

Grisaru, Zanon

$B \wedge \mathrm{tr}(R \wedge R) \wedge \mathrm{tr}(R \wedge R)$

In heterotic supergravity, the 3-form field strength is defined as

$$H = dB - \alpha' \mathrm{tr}\left(\omega d\omega + \tfrac{2}{3}\omega^3\right)$$
$$dH = -\alpha' \mathrm{tr}(R \wedge R)$$

Bianchi id.

By the string-string duality between hetero/T^4 and IIA/K3,

$$d * e^{-2\Phi} H_{\mathrm{IIA}} = -\alpha' \mathrm{tr}(R \wedge R)$$

10 dimensional origin of this equation is given by

Vafa, Witten

$$B \wedge \mathrm{tr}(R \wedge R) \wedge \mathrm{tr}(R \wedge R)$$

Again, known bosonic part (R^4 part) in type IIA is written as

$$\mathcal{L}_{(\alpha')^3} \sim e^{-2\phi} I_{\text{tree}} + c\, I_{\text{1-loop}},$$
$$I_{\text{tree}} = t_8 t_8 e R^4 + \tfrac{1}{4\cdot 2!}\epsilon_{10}\epsilon_{10} e R^4,$$
$$I_{\text{1-loop}} = t_8 t_8 e R^4 - \tfrac{1}{4\cdot 2!}\epsilon_{10}\epsilon_{10} e R^4 - \tfrac{1}{6}\epsilon_{10} t_8 B R^4$$

We obtain a part of the higher derivative corrections to the N=1, D=11 supergravity by lifting the type IIA result. Then there are two candidates which will be invariant under the local supersymmetry

$$t_8 t_8 e R^4 + \tfrac{1}{4!}\epsilon_{11}\epsilon_{11} e R^4, \quad t_8 t_8 e R^4 - \tfrac{1}{12}\epsilon_{11} t_8 A R^4$$

Our goal is to check these forms by local supersymmetry

3. Review of 11D Supergravity

Cremmer, Julia, Scherk

11D supergravity is the low energy effective action of the M-theory. The action is uniquely determined by the requirement of the local supersymmetry.

Field contents are quite simple

${e^\mu}_i$	: Vielbein	44
ψ_μ	: Majorana gravitino	128
$A_{\mu\nu\rho}$	: 3-form potential	84

The ansatz of the action should be constrained so as to preserve the local symmetries and the parity invariance.

1. General coordinate inv. and local Lorentz inv.
2. Gauge symmetry $A \to A + d\Lambda$
3. N=1 local supersymmetry
4. Parity inv. $x^{10} \to -x^{10}, A \to -A, \psi \to \gamma^{10}\psi$

The building blocks of the action should be tensors such as

$R_{ab\mu\nu}$,	$F_{\mu\nu\rho\sigma}$,	D_μ,	$\bar{\psi}_\mu \gamma^{(n)} \psi_\nu$
2	1	1	1

Chern-Simons term $A \wedge F \wedge F$ is an exception, which include the 3-form potential explicitly but is still gauge invariant.

Possible terms which have mass dimension 2 are expressed as

$$\mathcal{L} = \mathcal{L}[eR] + \mathcal{L}[e\bar{\psi}\psi_{(2)}] + \mathcal{L}[eF^2] + \sum_{i=1}^{4} c_i \mathcal{L}[eF\bar{\psi}\psi]_i + c_5 \mathcal{L}[AF^2],$$

$$\mathcal{L}[eR] = +eR,$$

$$\mathcal{L}[e\bar{\psi}\psi_{(2)}] = -\tfrac{1}{2} e\bar{\psi}_\rho \gamma^{\rho\mu\nu} \psi_{\mu\nu},$$

$$\mathcal{L}[eF^2] = -\tfrac{1}{2\cdot 4!} eF_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma},$$

Kinetic terms

The above ansatz is parity invariant automatically.

Local supersymmetry is checked by employing the Noether method. The susy transformations are written as

$$\delta e^{\mu}{}_i = [\bar{\epsilon}\psi], \quad \delta\psi_\mu = d_1[D\epsilon] + \sum_{i=2}^{3} d_i[F\epsilon]_i, \quad \delta A_{\mu\nu\rho} = \sum_{i=4}^{5} d_i[\bar{\epsilon}\psi]_i$$

Note that the right hand sides of the eqs. behave correctly under the parity invariance. Therefore supersymmetry does not mix parity even terms and odd terms.

The variation of the ansatz under the local susy is sketched as

$$\begin{array}{lll}
\delta\mathcal{L}[eR] & \sim & [eR\bar{\epsilon}\psi], \\
\delta\mathcal{L}[e\bar{\psi}\psi_{(2)}] & \sim & [eR\bar{\epsilon}\psi] \oplus [eF\bar{\epsilon}D\psi], \\
\delta\mathcal{L}[eF^2] & \sim & [eDF\bar{\epsilon}\psi] \oplus [eF^2\bar{\epsilon}\psi], \\
\delta\mathcal{L}[eF\bar{\psi}\psi] & \sim & [eF\bar{\epsilon}D\psi] \oplus [eDF\bar{\epsilon}\psi] \oplus [eF^2\bar{\epsilon}\psi], \\
\delta\mathcal{L}[AF^2] & \sim & [eF^2\bar{\epsilon}\psi]
\end{array}$$

The cancellation of the variation determines c_i and d_i completely.

4. Higher Derivative Corrections : An Overview

Roo,Suelmann,Wiedemann
Peeters,Vanhove,Westerberg

Let us consider the ansatz for the higher derivative corrections in 11 dimensions. In general there are many possible ways to contract indices, so we should make the ansatz as simple as possible.

First restrict our corrections to the order of ℓ_p^6

$$\mathcal{L}(\Phi) = \mathcal{L}_0(\Phi) + \ell_p^6 \mathcal{L}_1(\Phi) \qquad \Phi = (e, \psi, A)$$

Now let us consider the field redefinitions

$$\Phi \to \Phi' = \Phi + \ell_p^6 \Delta\Phi$$

The Lagrangian changes like $\quad E(\Phi) = e^{-1}\frac{\delta\mathcal{L}_0}{\delta\Phi}$

$$\mathcal{L}(\Phi) \to \mathcal{L}'(\Phi) = \mathcal{L}_0(\Phi) + \ell_p^6(\mathcal{L}_1(\Phi) + eE(\Phi)\Delta\Phi)$$

Thus the corrections which depend on the field equations of the supergravity can be removed by the appropriate field redefinitions.

As like the action, the supersymmetric transformation rules for the fields should be modified from the order of ℓ_p^6

$$\delta\Phi = \delta_0\Phi + \ell_p^6\delta_1\Phi$$

Then the variation of the Lagrangian under the supersymmetric transformations is expressed as

$$\begin{aligned}\delta\mathcal{L} &= \delta_0\mathcal{L}_0 + \ell_p^6\Big\{\delta_0\mathcal{L}_1 + e\delta_1\Phi E(\Phi)\Big\} \\ &= \delta_0\mathcal{L}_0 + \ell_p^6\Big\{V + e(X + \delta_1\Phi)E(\Phi)\Big\}\end{aligned}$$

where V is independent of the field equation

$$V = \delta_0\mathcal{L}_1 - eXE(\Phi)$$

The cancellation of the leading term, i.e., the supergravity part is just checked in the previous section. So the local supersymmetry is achieved when

$$V = 0, \qquad \delta_1\Phi = -X$$

5. Higher Derivative Corrections : Details

Higher derivative effective action should have mass dimension 8. The possible terms are written as

$$\mathcal{L}[eR^4], \quad \mathcal{L}[e\epsilon_{11}AR^4], \quad \mathcal{L}[eD^2R^3], \quad \mathcal{L}[eD^4R^2], \quad \mathcal{L}[eD^6R],$$
$$\mathcal{L}[eDR^3\bar{\psi}\psi], \quad \mathcal{L}[eD^3R^2\bar{\psi}\psi], \quad \mathcal{L}[eD^5R\bar{\psi}\psi], \quad \mathcal{L}[eD^7\bar{\psi}\psi]$$

To apply the Noether method it is necessary to reduce the number of terms in practice. One way is to classify the variations of the ansatz under the number of the covariant derivatives.

$$\mathcal{L}[eR^4], \quad \mathcal{L}[e\epsilon_{11}AR^4],$$
$$\mathcal{L}[eR^3\bar{\psi}\psi_{(2)}], \quad \mathcal{L}[eR^2\bar{\psi}_{(2)}D\psi_{(2)}]$$

The variation of this ansatz contain one covariant derivative explicitly at most.

$\mathcal{L}[eR^4]$

- Quartic terms of the Riemann tensor. No Ricci and scalar curvature.

- Includes the bilinear terms of the Majorana gravitino through the supercovariant spin connection

$$\hat{R}^{ab}{}_{\mu\nu}(\hat{\omega}) = \partial_\mu \hat{\omega}_\nu{}^{ab} - \partial_\nu \hat{\omega}_\mu{}^{ab} + \hat{\omega}_\mu{}^a{}_c \hat{\omega}_\nu{}^{cb} - \hat{\omega}_\nu{}^a{}_c \hat{\omega}_\mu{}^{cb}$$

$$\delta\hat{\omega} = [\bar{\epsilon}\psi_{(2)}]$$

- 13 terms in total with coefficients $b^1_{1\sim13}$. Purely bosonic parts are expanded by 7 terms

- Variations under the local supersymmetry are sketched like

$$\delta[eR^4] = V[eR^4\bar{\epsilon}\psi] \oplus V[eR^2 DR\bar{\epsilon}\psi_{(2)}]$$

$\mathcal{L}[e\epsilon_{11}AR^4]$

- Anomaly cancellation terms. 2 possible terms with coefficients $b^2_{1\sim2}$.

$$A \wedge \mathrm{tr}(R^2) \wedge \mathrm{tr}(R^2), \quad A \wedge \mathrm{tr}(R^4)$$

- Gauge invariant due to the Bianchi identity

$$dR + \hat{\omega} \wedge R - R \wedge \hat{\omega} = 0$$

- Variations under the local supersymmetry are sketched like

$$\delta[e\epsilon_{11}AR^4] = V[eR^4\bar{\epsilon}\psi]$$

$\mathcal{L}[eR^3\bar{\psi}\psi_{(2)}]$

- Do not include field equations of

$$\gamma^\mu\psi_{\mu\nu}, \quad \gamma^z D_z\psi_{ab}, \quad D^z\psi_{za}$$

- There are 92 terms in total with coefficients $f^1_{1\sim 92}$. These are classified into 3 types.

$$eR^3\bar{\psi}_z\gamma^{z\cdots}\psi_{(2)}, \quad eR^3\bar{\psi}_z\gamma\psi^{za}_{(2)}, \quad eR^3_z\bar{\psi}_z\gamma\psi_{(2)}$$

- Variations under the local supersymmetry are sketched like

$$\delta[eR^3\bar{\psi}\psi_{(2)}] = V[eR^4\bar{\epsilon}\psi] \oplus V[eR^2DR\bar{\epsilon}\psi_{(2)}] \oplus V[eR^3\bar{\epsilon}D\psi_{(2)}] \oplus XE(\psi)$$

- Supersymmetric transformation is modified as

$$X = \delta_1\psi = [D(R^3\epsilon)]$$

$\mathcal{L}[eR^2\bar{\psi}_{(2)}D\psi_{(2)}]$

- Do not include field equations of

$$\gamma^\mu\psi_{\mu\nu}, \quad \gamma^z D_z\psi_{ab}, \quad D^z\psi_{za}$$

- There are 25 terms in total with coefficients $f^2_{1\sim 25}$.

- Variations under the local supersymmetry are sketched like

$$\delta[eR^2\bar{\psi}_{(2)}D\psi_{(2)}] = V[eR^2DR\bar{\epsilon}\psi_{(2)}] \oplus V[eR^3\bar{\epsilon}D\psi_{(2)}]$$

The cancellation mechanism for higher derivative terms is sketched like

			116	88	60
13	$\delta\mathcal{L}[eR^4]$	$\sim$	$[eR^4\bar{\epsilon}\psi]\oplus$	$[eR^2DR\bar{\epsilon}\psi_{(2)}]$,	
2	$\delta\mathcal{L}[e\epsilon_{11}AR^4]$	$\sim$	$[eR^4\bar{\epsilon}\psi]$,		
92	$\delta\mathcal{L}[eR^3\bar{\psi}\psi_{(2)}]$	$\sim$	$[eR^4\bar{\epsilon}\psi]\oplus$	$[eR^2DR\bar{\epsilon}\psi_{(2)}]\oplus$	$[eR^3\bar{\epsilon}D\psi_{(2)}]$,
25	$\delta\mathcal{L}[eR^2\bar{\psi}_{(2)}D\psi_{(2)}]$	$\sim$		$[eR^2DR\bar{\epsilon}\psi_{(2)}]\oplus$	$[eR^3\bar{\epsilon}D\psi_{(2)}]$,
20	0	$\sim$	$[eR^4\bar{\epsilon}\psi]\oplus$		$[eR^3\bar{\epsilon}D\psi_{(2)}]$

Note that the first terms $[eR^4\bar{\epsilon}\psi]$ and the third terms $[eR^3\bar{\epsilon}D\psi_{(2)}]$ are not independent because of the identity:

$$D_{[e}\psi_{cd]} = \frac{1}{4}\gamma^{ab}\psi_{[e}R_{cd]ab}$$

In fact we find 20 identities

$$0 = \sum_{i=n}^{20} i_n([R^4\bar{\epsilon}\psi]_n + [R^3\bar{\epsilon}D\psi_{(2)}]_n)$$

Calculations are done by mathematica.

Used GAMMA.m by U. Gran

Now under the local supersymmetry transformation, we obtain the 264 independent terms.

And the coefficients of the 264 terms are finally given by linear combinations of $b_n^1,\ b_n^2,\ f_n^1,\ f_n^2$ and i_n

Therefore by requiring the supersymmetry, we obtain the 264 linear equations among the 152 variables of $b_n^1,\ b_n^2,\ f_n^1,\ f_n^2$ and i_n

We find two solutions which relate bosonic and fermionic terms

First solution

$$(b_1^1, b_2^1, b_4^1, b_6^1, b_7^1, b_9^1, b_1^2, b_2^2) = a(1, 32, -32, -16, -16, -32, -\tfrac{1}{4}, 1)$$

Bosonic part is written as

$$\begin{aligned}\mathcal{L}_{boson} = a\Big(&+ eR_{abcd}R_{abcd}R_{efgh}R_{efgh} - 16eR_{abcd}R_{abce}R_{dfgh}R_{efgh} \\ &+ 2eR_{abcd}R_{abef}R_{cdgh}R_{efgh} + 16eR_{abcd}R_{aecg}R_{bfdh}R_{efgh} \\ &- 32eR_{abce}R_{abdg}R_{cfdh}R_{efgh} + 32eR_{abce}R_{adcg}R_{bfdh}R_{efgh} \\ &+ \tfrac{1}{24}\epsilon_{11}^{\mu_1\cdots\mu_{11}} A_{\mu_1\mu_2\mu_3} R_{ab\mu_4\mu_5} R_{ab\mu_6\mu_7} R_{cd\mu_8\mu_9} R_{cd\mu_{10}\mu_{11}} \\ &- \tfrac{1}{6}\epsilon_{11}^{\mu_1\cdots\mu_{11}} A_{\mu_1\mu_2\mu_3} R_{ab\mu_4\mu_5} R_{bc\mu_6\mu_7} R_{cd\mu_8\mu_9} R_{da\mu_{10}\mu_{11}} \Big) \\ = \tfrac{1}{12}a\Big(&t_8 t_8 eR^4 - \tfrac{1}{12}\epsilon_{11} t_8 AR^4\Big)\end{aligned}$$

Second solution

$$(b_7^1, b_9^1, b_{10}^1, b_{11}^1, b_{12}^1) = b(\tfrac{1}{2}, -1, \tfrac{1}{5}, \tfrac{1}{5}, \tfrac{1}{5})$$

Bosonic part is written as

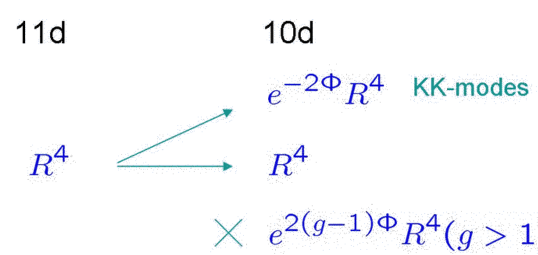
$$\begin{aligned}\mathcal{L}_{boson} &= b\Big(-\tfrac{1}{4}eR_{abce}R_{abdf}R_{cdgh}R_{efgh} + eR_{abce}R_{adcg}R_{bfdh}R_{efgh}\Big) \\ &= \tfrac{1}{24\times 32}b\Big(t_8 t_8 eR^4 + \tfrac{1}{4!}\epsilon_{11}\epsilon_{11} eR^4\Big)\end{aligned}$$

Vanishing theorem :

Only tree and one-loop amplitudes contribute to the R^4 terms

Proof :

In 11 dim. there are only two superinvariants which contain R^4 terms. These become tree level or one-loop terms in type IIA by Kaluza-Klein reduction. No terms more than one-loop.



6. Conclusions and Discussions

We obtained higher derivative corrections to 11D supergravity only by employing the N=1 supersymmetry

The bosonic terms exactly match with the result obtained by string theory.

We still have 13 parameters which relate only fermionic terms. These should be fixed by the cancellation of the variations which depend on 3-form potential.

Future direction

- Simplify the expression

  Probably the action will be simplified by using supercovariant quantities

  $$F \to \hat{F}, \qquad \delta \hat{F} = [\bar{\epsilon}\psi_{(2)}]$$

- Construction of terms like $e\hat{F}^2 R^3$

  A part of the variation contributes like

  $$\delta \mathcal{L}[eR^3 \bar{\psi}\psi_{(2)}] \sim V[eFR^3 \bar{\epsilon}\psi_{(2)}]$$

  This should be cancelled by introducing $e\hat{F}^2 R^3$

Some Application

F1-W in hetero

No horizon area in sugra → Finite horizon area in sugra + R^2

Entropy matches with the CFT calculation

D6-W in IIA

No horizon area in sugra → Finite horizon area in sugra + R^4 ?

Zero modes : $|i\rangle,\ |\dot{a}\rangle, \qquad i, \dot{a} = 1, \cdots, 8$

$$\langle i|j\rangle = \delta_{ij}, \quad \langle \dot{a}|\dot{b}\rangle = \delta_{\dot{a}\dot{b}} \qquad R_0^{ij} = \tfrac{1}{4} S_0 \gamma^{ij} S_0$$

$-iR_0^{ij}$ are generators of Lorentz algebra

$$[R_0^{ij}, R_0^{kl}] = -\delta^{ik} R_0^{jl} + \delta^{il} R_0^{jk} + \delta^{jk} R_0^{il} - \delta^{jl} R_0^{ik}$$

$$R_0^{ij}|k\rangle = \delta^{jk}|i\rangle - \delta^{ik}|j\rangle,$$

$$R_0^{ij}|\dot{a}\rangle = -\tfrac{1}{2}\gamma^{ij}_{\dot{a}\dot{b}}|\dot{b}\rangle$$

Now we can check that

$$\mathrm{Tr}_0(R_0^{ij}) = \langle i|R_0^{ij}|i\rangle - \langle \dot{a}|R_0^{ij}|\dot{a}\rangle = 0$$

$$\mathrm{Tr}_0(R_0^{ij} R_0^{kl}) = \langle m|R_0^{ij} R_0^{kl}|m\rangle - \langle \dot{a}|R_0^{ij} R_0^{kl}|\dot{a}\rangle = 2(-\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}) - 2(same) = 0$$

$$\mathrm{Tr}_0(R_0^{ij} R_0^{kl} R_0^{mn}) = 0$$

$$\mathrm{Tr}_0(R_0^{ij} R_0^{kl} R_0^{mn} R_0^{op}) = t_8^{ijklmnop}$$