Aspects of Lifshitz scaling in string theory

K. Narayan
Chennai Mathematical Institute

- Lifshitz scaling, gravity duals, AdS null deformations
- Lifshitz scaling, hyperscaling violation
- Lifshitz singularities in string constructions

[arXiv:1005.3291, Koushik Balasubramanian, KN, 1103.1279, KN, 1202.5935, KN, 1204.3506, KN, work in progress, . . .]

[see also AdS/CFT cosmo singularities: hep-th/0602107, 0610053, Das, Michelson, KN, Trivedi; arXiv:0711.2994, Awad, Das, KN, Trivedi; arXiv:0807.1517, Awad, Das, Nampuri, KN, Trivedi.]

Holography and Lifshitz scaling

Interesting to explore holography with reduced symmetries.

Generalizations of AdS/CFT to nonrelativistic systems

$\rightarrow$ holographic condensed matter, ...

[Son; Balasubramanian,McGreevy; Adams et al; Herzog et al; Maldacena et al; ...]

$\rightarrow$ Phases of string theory with non-relativistic symmetries.

Lifshitz symmetries: t, x_i-translations, x_i-rotations,

scaling $t \rightarrow \lambda^z t,\ x_i \rightarrow \lambda x_i$ [z: dynamical exponent].

[smaller than Galilean symmetries: $e.g.$ Galilean boosts broken]

Landau-Ginzburg action (free $z = 2$ Lifshitz): $S = \int d^3x((\partial_t\varphi)^2 - \kappa(\nabla^2\varphi)^2)$.

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Lifshitz spacetime: $ds_4^2 = -\frac{dt^2}{r^{2z}} + \frac{dx_i^2 + dr^2}{r^2}$. Kachru,Liu,Mulligan

Scaling: $t \rightarrow \lambda^z t,\ x_i \rightarrow \lambda x_i,\ r \rightarrow \lambda r$ [$z = 1$: AdS].

Solution to 4-dim gravity with $\Lambda < 0$ and massive gauge field $A \sim \frac{dt}{r^z}$
(or alternatively gauge field + 2-form: dualize to get A-mass) Taylor

[Focus on zero temperature solutions]

Lifshitz scaling in string theory

Many natural guesses appear difficult to realize explicitly.

String construction for $z = 2$ Lifshitz [Balasubramanian,KN]:
x^+-DLCQ of relativistic $\mathcal{N}$=4 SYM $\longrightarrow$

$z = 2$ nonrelativistic (Galilean) 2+1-dim system.

Gauge coupling $g_{YM}^2(x^+) = e^{\Phi(x^+)}$ varying in lightlike x^+-direction
$\longrightarrow$ breaks x^+-shift reducing to 2+1-dim Lifshitz symmetries.

Concrete bulk realization: null deformations of $AdS_5 \times S^5$ (more generally, $AdS \times X$) sourced by lightlike scalar ($e.g.$ dilaton in IIB).

$$ds^2_{Einst} = \frac{1}{r^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}r^2(\partial_+\Phi)^2(dx^+)^2] + \frac{dr^2}{r^2} + d\Omega_5^2.$$

Lightlike dilaton: $\Phi = \Phi(x^+)$ (also 5-form). [Das,KN,Trivedi,et al]

[The scalar Φ could also arise from the compactification: other interpretations, $e.g.$ fermion condensate etc? (recall Hartnoll,Polchinski,Silverstein,Tong, ...)]

AdS null deformations and Lifshitz

$$ds^2_{Einst} = \frac{1}{r^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}r^2(\partial_+\Phi)^2(dx^+)^2] + \frac{dr^2}{r^2} + d\Omega_5^2, \qquad \Phi = \Phi(x^+)\,.$$

$\Phi = const \;\; i.e. \;\; g_{++} = 0$: DLCQ $_{x^+}$ of AdS in lightcone coordinates — nonrelativistic, Schrodinger (Galilean) symmetries

[Goldberger, Barbon et al, Maldacena et al].

Regard $x^- \equiv t$ (time), $x^+ \equiv$ compact coordinate ($g_{++} \sim (\Phi')^2 > 0$).

Strictly: $x^+ = const \equiv$ null surfaces and $x^- = const$ surfaces spacelike ($g^{--} < 0$).

Symmetries, 2+1-dim Lifshitz: x^-, x_i-translations, x_i-rotations,

$z = 2$ scaling $\quad x^- \equiv t \rightarrow \lambda^2 t, \;\; x_i \rightarrow \lambda x_i, \;\; r \rightarrow \lambda r \quad$ (x^+, no scaling).

x^+ compact $\Rightarrow$ lightlike boosts broken.

Galilean boosts $x_i \rightarrow x_i - v_i x^-, \;\; x^+ \rightarrow x^+ - \frac{1}{2}(2v_i x_i - v_i^2 x^-)$:

broken by g_{++}. Also broken $z = 2$ special conformal symmetry.

Nontrivial x^+-dependence $\Rightarrow z = 2$ Galilean broken to Lifshitz.

AdS null deformations, Lifshitz

$$ds^2_{Einst} = \frac{1}{r^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}r^2(\partial_+\Phi)^2(dx^+)^2] + \frac{dr^2}{r^2} + d\Omega_5^2, \quad \Phi = \Phi(x^+).$$

- (Das,KN,Trivedi,et al) Coord. transfmn. $r = we^{-f/2}, \quad x^- = y^- - \frac{r^2 f'}{4} \ \rightarrow$

$$ds^2 = \frac{1}{w^2}[e^{f(x^+)}(-2dx^+dy^- + dx_i^2) + dw^2] + d\Omega_5^2\,, \quad \Phi(x^+).$$

EOM: $R_{++} = \frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\Phi')^2$. Function-worth of solutions (any $\Phi(x^+)$).

- Dual: 4-d $\mathcal{N}$=4 super Yang-Mills theory with gauge coupling lightlike-deformed $g^2_{YM}(x^+) = e^{\Phi(x^+)} \longrightarrow$ DLCQ $_{x^+}$.

Lightlike (chiral) deformation $\Rightarrow$ various physical observables ($e.g.$ trace anomaly, anomalous dims) unaffected. 2-point correlator (conf. coords): operators $\mathcal{O}$ dual to massive scalars $\varphi \ \rightarrow$

$$\langle\mathcal{O}(x_i)\mathcal{O}(x_i')\rangle \sim \frac{1}{[\sum_i(\Delta x_i)^2]^{\Delta}}\,, \quad \text{and} \ \langle\mathcal{O}(t)\mathcal{O}(t')\rangle \sim \frac{1}{(\Delta x^-)^{\Delta}}\,.$$

Agrees with equal-time 2-pt fn of $2+1$-dim Lifshitz theory (Kachru,Liu,Mulligan).

Calculation difficult in form with $g_{++} \neq 0$: scalar wave eqn not straightforward to solve (later).

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- Can replace $S^5 \rightarrow X^5$ (Sasaki-Einstein). Preserve half susy.

These solutions can be generalized to a large family of solutions with axion-dilaton, 3-form field strength turned on (Donos,Gauntlett, various groups, ...).

- There also exist $AdS_4 \times X^7$ null deformations in M-theory, with scalar arising from G-flux on X^7: presumably dual to lightlike deformations of Chern-Simons (ABJM-like) theories arising on M2-brane stacks. Also AdS_7 null deformations from M5-branes.

More on AdS null defmns, Lifshitz

$$ds^2 = \frac{1}{r^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}r^2(\Phi')^2(dx^+)^2] + \frac{dr^2}{r^2} + d\Omega_5^2, \quad \Phi = \Phi(x^+)\,.$$

- Long wavelength geometry seen by massless bulk scalar: Lifshitz.

[action $S = \frac{1}{G_5}\int d^5x\,\sqrt{-g}\,g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ for modes with no x^+-dependence ($\partial_+\varphi = 0$),

$$S \to \frac{1}{G_5}\int \frac{d^4x}{r^5}\left[-r^4\left(\frac{\int dx^+(\Phi')^2}{4}\right)(\partial_-\varphi)^2 + r^2L(\partial_i\varphi)^2 + r^2L(\partial_w\varphi)^2\right]$$

L: compactification size, rescale $x^- \to \sqrt{\frac{L}{\int dx^+(\Phi')^2}}\,x^-$.]

- Standard Kaluza-Klein x^+-reduction does not work here in general: nontrivial x^+-dependence thro' $(\Phi')^2$ means no clear separation of scales. However an argument for dimensional reduction of "off-shell" metric family containing these solutions suggests long wavelength geometry is $z = 2$ Lifshitz.

- Φ'=const: linear dilaton. Not x^+-periodic. Compactify in a "non-geometric" way, upto S-duality of IIB theory.

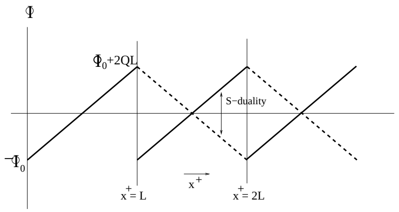
This can also be done in a linear axion solution.

(Interesting to study the noncompact solution independently: later.)

Non-geometric DLCQ

Linear dilaton not x^+-periodic. Compactify upto S-duality:

$\Phi(x^+)$ continuous, not periodic: note S-duality symmetry, $\tau \rightarrow -\frac{1}{\tau}$, $i.e.$ $\Phi \rightarrow -\Phi$.



Piecewise linear dilaton: $\Phi = \Phi_0 + 2Qx^+, \quad x^+ \in [0, L]\,,$

$\Phi = \Phi(L) - 2Q(x^+ - L), \quad x^+ \in [L, 2L], \quad \ldots$

Φ periodic upto S-duality if $\Phi(x^+ + L) = -\Phi(x^+)$, $i.e.$

$$\Phi(L) = -\Phi(0) \quad \Rightarrow \quad \Phi_0 + 2QL = -\Phi_0\,, \quad i.e. \quad g_s = e^{\Phi_0} = e^{-QL}.$$

[Einstein metric $ds^2 = \frac{1}{r^2}(-2dx^+dx^- + dx_i^2 + r^2Q^2(dx^+)^2 + dr^2)$ smooth.]

Asymptotic string coupling fixed: large string corrections?

Lightlike deformations of $AdS_5 \times S^5$: no nonzero contractions (only $\partial_+\Phi$ nonzero, $g^{++} = 0$), likely no higher derivative corrections. Preserves half susy. Dilaton bounded, no singularities.

Lower dim Lifshitz symmetries $\rightarrow$ lightlike structure in 5-dim, possibly controlled corrections.

$\Rightarrow$ non-geometric construction for dual $\mathcal{N}{=}4$ SYM too.

Linear axion: $\quad c_0 = c_0^0 + 2Qx^+, \qquad ds^2 = ds^2_{AdS\times S} \; + \; (\partial_+ c_0)^2 (dx^+)^2.$

($x^+ \rightarrow x^+ + L$): $c_0 \rightarrow c_0 + 2QL$ maps to $\tau \rightarrow \tau + 1$ shift if $QL = \frac{1}{2}$.

Axion $\rightarrow$ θ-angle in dual gauge theory.

Balasubramanian,McGreevy: Lifshitz-Chern-Simons gauge theory duals for some of these.

x^+-noncompact, anisotropic Lifshitz

$ds^2 = \frac{1}{r^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}r^2(\Phi')^2(dx^+)^2] + \frac{dr^2}{r^2} + d\Omega_5^2,$

$\Phi = \Phi(Qx^+),$ $\quad Q$ parameter of mass dim one.

Symmetries: time x^--translations, spatial x_i-translations/rotations (x^+-translations broken by nontrivial x^+-dependence), and anisotropic Lifshitz scaling $r \to \lambda r, \quad x_i \to \lambda x_i, \quad x^- \to \lambda^2 x^-, \quad x^+ \to \lambda^0 x^+$,

$i.e. \quad z = 2 \ \ \{x_i, x^-\}$: $\quad x^- \to \lambda^2 x^-, \quad x_i \to \lambda x_i$,

$\qquad z = \infty \ \ \{x^+, x^-\}$: $\quad x^- \to \lambda^2 x^-, \quad x^+ \to \lambda^0 x^+$.

In addition, dilaton $\Phi(Qx^+)$ acts as spatial x^+-potential.

[Recall $z = 0$ Schrodinger systems: $ds^2 = -dt^2 + \frac{dx_i^2 + dtd\xi + dr^2}{r^2}$.
Here $x^- \equiv$ time (const-x^- surface is spacelike).]

Also recall D3-D7 anisotropic Lifshitz systems + radial scalars (Azeyanagi,Li,Takayanagi).

Linear dilaton, anisotropic Lifshitz

$ds^2 = \frac{1}{r^2}[-2dx^+dx^- + dx_i^2 + r^2Q^2(dx^+)^2] + \frac{dr^2}{r^2}, \quad \Phi = \Phi_0 + 2Qx^+.$

Now metric also has x^+-translations: linear dilaton acts as spatial x^+-potential.

Dual gauge theory: d=4 $\mathcal{N}=4$ SYM theory living on flat spacetime (flat boundary), gauge coupling $g_{YM}^2(x^+) = g_s e^{2Qx^+}$ $(g_s = e^{\Phi_0})$.

$x^+ \to -\infty$: weakly coupled gauge theory.

Consistent: string frame metric $ds^2_{str} = e^{\Phi/2} ds^2$ highly curved.

Momentum space 2-pt correlator (operators dual to bulk scalars):

[Lorentzian calculation, closed form bulk-bndry propagator involving conf.hypergeom.fns.]

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim -\nu 2^\nu \alpha^\nu \frac{\Gamma(-\nu)}{\Gamma(\nu)} \frac{\Gamma(\frac{1+\nu}{2} + \frac{k^2}{4iQk_-})}{\Gamma(\frac{1-\nu}{2} + \frac{k^2}{4iQk_-})} \qquad (\nu \notin \mathbb{Z}),$$

$$\sim \quad ((k_i^2 - 2k_+k_-)^2 + 4Q^2k_-^2)\,[\log(iQk_-) + \psi(\tfrac{3}{2} + \frac{k_i^2 - 2k_+k_-}{4iQk_-})]$$

$$(\nu = 2\ \ (\Delta = 4)). \quad [k^2 = -2k_+k_- + k_i^2,\ \ \Delta = 2 + \sqrt{4 + m^2} = 2 + \nu].$$

Structurally similar to Kachru,Liu,Mulligan: for $k_+ \sim 0$, (after Euclidean continuation) this agrees with KLM, as expected from dim.redn. of 5-d background to Lifshitz.

Linear dilaton, Lifshitz, correlators

$$\langle\mathcal{O}(k)\mathcal{O}(-k)\rangle \sim -\nu 2^\nu \alpha^\nu \frac{\Gamma(-\nu)}{\Gamma(\nu)} \frac{\Gamma(\frac{1+\nu}{2}+\frac{k^2}{4iQk_-})}{\Gamma(\frac{1-\nu}{2}+\frac{k^2}{4iQk_-})} \qquad (\nu \notin \mathbb{Z}),$$

$$\sim \left((k_i^2-2k_+k_-)^2+4Q^2k_-^2\right)\left(\log(iQk_-)+\psi(\tfrac{3}{2}+\frac{k_i^2-2k_+k_-}{4iQk_-})\right). \quad [\Delta=4]$$

[Consistent with Lifshitz scaling: $(x^-, x^+, x_i) \rightarrow (\lambda^2 x^-, \lambda^0 x^+, \lambda x_i)$.]

[For $Q \to 0$, $\psi() \to \log()$: recover AdS 2-point correlation fn $k^4 \log k^2$.]

Note: $(k_i^2-2k_+k_-)^2+4Q^2k_-^2 = (k_i^2-2(k_+-iQ)k_-)(k_i^2-2(k_++iQ)k_-)$

Effective x^+-momentum shift: $k_+ \to k_+ \pm iQ$ $\quad (e^{ik_+x^+} \to e^{ik_+x^+}e^{\pm Qx^+})$

— reminiscent of Liouville-like wall in $c=1$ string theory.

Also, for $k_i=0$: $(k_i^2-2k_+k_-)^2+4Q^2k_-^2 \to (k_+^2+Q^2)k_-^2$,

$i.e.$ effective mass-gap in x^+-direction.

Free SYM: $S=\int \frac{d^4x}{g_{YM}^2(x^+)} TrF^2 \;\rightarrow\; \int d^4x\, e^{-\Phi(x^+)}[(\partial_j A_i)^2 - 2(\partial_+A_i)(\partial_-A_i)]$.

Wave modes $e^{ik_+x^+ + ik_-x^- + ik_ix^i} \;\rightarrow k_i^2+2(k_++iQ)k_-=0,\; i.e.$

$k_+ = -\frac{k_i^2}{2k_-} - iQ$. For generic k_i, k_-, x^+-momentum k_+ nonzero, $i.e.$ generic waves move along x^+-direction due to dilaton x^+-potential.

Recap: $z = 2$ Lifshitz, null defmns

$d = 4$ $\mathcal{N}$=4 super Yang-Mills theory with gauge coupling
lightlike-deformed as $g^2_{YM}(x^+) = e^{\Phi(x^+)}$ $\longrightarrow$ DLCQ $_{x^+}$.

Lightlike deformation $\rightarrow$ break Galilean symmetries $\rightarrow$ Lifshitz.

$AdS_5 \times S^5 \rightarrow$ null deformations with lightlike dilaton $\rightarrow$ DLCQ $_{x^+}$:

$$ds^2 = \frac{1}{r^2}[-2dx^+dx^- + dx_i^2] + \frac{1}{4}(\Phi')^2(dx^+)^2 + \frac{dr^2}{r^2} + d\Omega_5^2, \quad \Phi = \Phi(x^+) \, .$$

$z = 2$ scaling $\quad x^- \equiv t \rightarrow \lambda^2 t, \;\; x_i \rightarrow \lambda x_i, \;\; r \rightarrow \lambda r$ $\quad$ (x^+ compact, no scaling).

Note: so far $g_{++} \sim \; r^0$ — non-normalizable null deformation.
Normalizable null deformations of AdS? $\;\; e.g.$ $g_{++} \sim \; r^4$ $\;\; [AdS_5]$.
Dim'nal reduction $\longrightarrow$ Hyperscaling violation effects.

Hyperscaling violation

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2 + R^2 d\Omega_5^2$$

AdS shock wave: $R^4 \sim g_{YM}^2 N \alpha'^2$, Q $\sim$ energy-momentum density.

Has appeared in literature in many places. [recently: Singh]

(5-d) Scaling: $x_i \to \lambda x_i, \quad r \to \lambda r, \quad x^- \to \lambda^3 x^-, \quad x^+ \to \lambda^{-1} x^+$

$$ds^2 = R^2\big(-\frac{dt^2}{Qr^6} + \frac{dx_i^2+dr^2}{r^2} + Qr^2\big(dx^+ - \frac{dt}{Qr^4}\big)^2\big) \longrightarrow \text{dim.redn.}_{x^+} \to$$

4-dim Einstein metric ($x^- \equiv t$): $ds_E^2 = \frac{R^3\sqrt{Q}}{r}\left(-\frac{dt^2}{Qr^4} + dx_i^2 + dr^2\right)$,

Electric gauge field $A = -\frac{dt}{Qr^4}$, scalar $e^\phi \sim r$. Nontrivial IR scales R, Q.

$$[ds^2 = g_{\mu\nu}^D dx^\mu dx^\nu + h(x^\mu) d\sigma_{D_I}^2 \quad \longrightarrow \quad ds_E^2 = h^{D_I/(D-2)} g_{\mu\nu}^D dx^\mu dx^\nu]\,.$$

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$$[ds^2 = g_{\mu\nu}^D dx^\mu dx^\nu + h(x^\mu)d\sigma_{D_I}^2 \quad\longrightarrow\quad ds_E^2 = h^{D_I/(D-2)}g_{\mu\nu}^D dx^\mu dx^\nu]\,.$$

$$ds^2 = r^{2\theta/d}\Big(-\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2}\Big).$$

d = "boundary" spatial dim (x_i).

θ = hyperscaling violation exponent.

Conformal to Lifshitz. Arise in effective gravity+vector+scalar theories.
Thermodynamics reflects effective space dim $(d-\theta)$ [$e.g.\ S \sim T^{(d-\theta)/z}$].

Above: $d = 2, \quad z = 3, \quad \theta = 1 \quad (d_{eff} = d - \theta = 1)$.
(4-d) scaling: $t \to \lambda^z t, \quad x_i \to \lambda x_i, \quad r \to \lambda r, \quad ds \to \lambda^{\theta/d} ds\,.$

(Lifshitz symmetry broken.)

Kachru et al: dim'nal reduction of black Dp-branes $\to\; z = 1, \theta \neq 0$ solutions.

Hyperscaling violation

$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2 + R^2 d\Omega_5^2$

AdS shock wave: $R^4 \sim g_{YM}^2 N\alpha'^2$, Q $\sim$ energy-momentum density. $\rightarrow$ dim.redn.$_{x^+}$

- Using Ryu-Takayanagi expression, $\theta = d - 1$ gives logarithmic violation of entanglement entropy.

Gravitational dual of hidden Fermi surfaces? Takayanagi et al, Sachdev et al

- Holographic stress tensor calculated upstairs using standard AdS/CFT prescription: $T_{++} = \frac{2Q}{8\pi G_5} \longrightarrow$ wave on boundary.

Lower dimensional point of view? Definition for holographic renormalization downstairs from upstairs definition?

- Often in condensed matter literature on critical phenomena, violation of hyperscaling relations is due to dangerously irrelevant operator that ruins simple scaling relations. Connections to present context?

Gauge theory interpretation: nontrivial shock wave state in CFT $\longrightarrow$ dim'nal reduction $\longrightarrow$ residual nontrivial IR scales. Explore ...

Hyperscaling violation

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2Qr^{D-3}(dx^+)^2$$

AdS_D shock wave, $D = d+3$, Q energy-momentum density, $(D-1)$-dim.

Scaling: $x_i \to \lambda x_i, \quad r \to \lambda r, \quad x^- \to \lambda^{2+d/2}x^-, \quad x^+ \to \lambda^{-d/2}x^+$

Dim'nal redn: $\quad ds_E^2 = \frac{R^2(R^2Q)^{1/(D-3)}}{r}\Big(-\frac{dt^2}{Qr^{D-1}} + dx_i^2 + dr^2\Big)$.

"boundary" spatial dimension $d = D-3$, $\quad z = \frac{d}{2}+2\,, \quad \theta = \frac{d}{2}$.

[General dim'nal reduction: $\int d^Dx\sqrt{-g^{(D)}}\,(R^{(D)} - 2\Lambda) =$
$\int dx^+d^{D-1}x\sqrt{-g^{(D-1)}}\,(R^{(D-1)} - \#\Lambda e^{-2\phi/(D-3)} - \#(\partial\phi)^2 - \#e^{2(D-2)\phi/(D-3)}F_{\mu\nu}^2)$

Lower dim electric gauge field $A = -\frac{dt}{r^{D-1}}$, scalar $g_{DD} = e^{2\phi} = r^{D-3}$.]

In particular, for the conformal branes of M-theory:

$M2$-brane stacks $\to$ AdS_4 deformations, $\quad d=1, \quad z=\frac{5}{2}, \quad \theta=\frac{1}{2}$.

$M5$-brane stacks $\to$ AdS_7 deformations, $\quad d=4, \quad z=4, \quad \theta=2$.

Phase structure of these M-brane solutions includes IIA string/sugra phases of D2, D4-brane solutions with null deformations (new z, θ).

(recall Dp-brane phases, Itzhaki,Maldacena,Sonnenschein,Yankielowicz)

Hyperscaling violation

More generally (along IMSY lines): Conformal M-theory brane stack $ds^2 = ds^2_{AdS\times S} + g_{++}(dx^+)^2$, for g_{++} (non-)normalizable deformations, gives rise in appropriate IIA phase to hyperscaling violation and new z,θ with $ds^2_E = r^{2\theta/d}(-\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2})$.

$ds^2_{M-th} \rightarrow$ 10-d ds^2_{st} + dilaton $\Phi \rightarrow ds^2_E \rightarrow$ dim.redn.$^{\text{sphere}S} \rightarrow$ null-deformed solution, non-compact $x^+ \rightarrow$ dim.redn.$^{x^+} \longrightarrow z,\theta$.

[Note: 10-d solutions can be checked independently in IIA supergravity.]

Dp-branes $\rightarrow d = p,\ z = 1,\ \theta \neq 0$. (Dong,Harrison,Kachru,Torroba,Wang)

M2 $\rightarrow$ D2 null non-normalizable: $d = 1,\ z = \frac{7}{3},\ \theta = 0$.

M2 $\rightarrow$ D2 null normalizable: $d = 1,\ z = 3,\ \theta = \frac{2}{3}$.

M5 $\rightarrow$ D4 null non-normalizable: $d = 3,\ z = 2,\ \theta = -1$.

M5 $\rightarrow$ D4 null normalizable: $d = 3,\ z = 4,\ \theta = \frac{1}{3}$.

Note: some of these (with $d = 1$) have $d - 1 \leq \theta \leq d \rightarrow$
Violation of area law of entanglement entropy. Field theory duals nontrivial: explore?

Hyperscaling violation

(Following Dp-brane phase structure of Itzhaki,Maldacena,Sonnenschein,Yankielowicz):

These solutions are of the form of null-deformed Dp-brane systems, with rich phase structure. z, θ-values flow.

Null-deformed D2-brane phases: flow from DLCQ$_{x^+}$ of 2+1-dim perturbative SYM (UV) regime $\rightarrow$ IIA supergravity region with null-deformed D2-solution (valid in some intermediate regime where x^+-circle large relative to 11-th circle) $\rightarrow$ 11-dim AdS_4 null-deformed M2-brane IR phase (dual to DLCQ of null-defmn of Chern-Simons ABJM-like theory).

Null-deformed D4-brane phases: flow from AdS_7 null-deformed M5-brane UV phase (dual to null-deformation of $(2,0)$ M5-theory) $\rightarrow$ intermediate IIA supergravity region with null-deformed D4-solution $\rightarrow$ DLCQ$_{x^+}$ of 4+1-dim perturbative SYM IR phase.

Lifshitz singularities, string theory

Mild singularities present in Lifshitz spacetimes $ds_4^2 = -\frac{dt^2}{r^{2z}} + \frac{dx_i^2 + dr^2}{r^2}$: curvature invariants finite, diverging tidal forces as $r \to \infty$ (interior). Expect that these are zero temperature limits of Lifshitz black holes with regular horizons: however zero temperature limit singular.

[Kachru et al: stringy cloak, de-singularizing horizon by higher derivative corrections.]

Singularities also reflected in above string constructions exhibiting exact Lifshitz symmetries (Horowitz,Way): origins? resolution?

$$ds^2 = \frac{1}{r^2}[-2dx^+dx^- + dx_i^2] + \frac{1}{4}(\partial_+\Phi)^2(dx^+)^2 + \frac{dr^2}{r^2} + d\Omega_5^2, \quad \Phi(x^+).$$

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Recall coord. transfmn. $r = we^{-f/2}, \quad x^- = y^- - \frac{r^2 f'}{4} \ \rightarrow$

$$ds^2 = \frac{1}{w^2}[e^{f(x^+)}(-2dx^+dy^- + dx_i^2) + dw^2] + d\Omega_5^2\ , \quad \Phi(x^+).$$

EOM: $R_{++} = \frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\Phi')^2.$ (Das,KN,Trivedi et al)

This lies in general family of AdS-deformations (also 5-form)

$$ds^2_{Einst} = \frac{R^2}{w^2}\left(\tilde{g}_{\mu\nu}(x^\mu)dx^\mu dx^\nu + dw^2\right) + R^2 d\Omega_5^2, \quad \Phi = \Phi(x^\mu),$$

solutions if $\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi, \quad \tilde{\Box}\Phi \equiv \partial_\mu(\sqrt{-\tilde{g}}\ \tilde{g}^{\mu\nu}\partial_\nu\Phi) = 0.$

Above null solutions: $\tilde{g}_{\mu\nu} = e^{f(x^+)}\eta_{\mu\nu}.$

Lifshitz singularities, string theory

$$ds^2_{Einst} = \frac{R^2}{w^2}\left(\tilde{g}_{\mu\nu}(x^\mu)dx^\mu dx^\nu + dw^2\right) + R^2 d\Omega_5^2, \quad \Phi = \Phi(x^\mu)$$

Potential singularities on Poincare horizon $w \to \infty$:

$$R_{ABCD}R^{ABCD} = w^4 \tilde{R}_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta} + \ldots \qquad \text{diverges.}$$

For null solutions, invariants vanish: no nonzero contraction.

Diverging tidal forces as $w \to \infty$. Horowitz,Way

Holographic stress tensor (Awad,Das,KN,Trivedi)

$$T^{\mu\nu} \sim K^{\mu\nu} - Kh^{\mu\nu} - 3h^{\mu\nu} + \frac{1}{2}G^{\mu\nu} - \frac{1}{4}\partial^\mu\Phi\partial^\nu\Phi + \frac{1}{8}h^{\mu\nu}(\partial\Phi)^2$$

Vanishes identically for null solutions (either coords).

Recall: source for bulk field turned on $\Rightarrow$ generically expect response.

Here metric source $\tilde{g}_{\mu\nu}$, so expect $\langle T^{\mu\nu}\rangle \neq 0$. At variance with above.

Requiring vanishing $T^{\mu\nu}$ gives constraint $\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi$ above.

Lifshitz singularities, string theory

$$ds^2_{Einst} = \frac{R^2}{w^2}\left(\tilde{g}_{\mu\nu}(x^\mu)dx^\mu dx^\nu + dw^2\right) + R^2 d\Omega_5^2, \quad \Phi = \Phi(x^\mu)$$

Consider Fefferman-Graham expansion of asymptotically locally AdS space (Poincare coords) about boundary $w = 0$:

$$ds^2 = \frac{dw^2}{w^2} + \frac{1}{w^2}\left[g^0_{\mu\nu}(x^\mu) + w^2 g^2_{\mu\nu}(x^\mu) + \ldots\right] dx^\mu dx^\nu$$

In this perspective, above solutions seem constrained: $g^n_{\mu\nu} = 0,\ n > 0$.

In general, with leading source $g^0_{\mu\nu} = \tilde{g}_{\mu\nu}$, holographic RG methods give relations between $g^n_{\mu\nu}, \ldots$, stress tensor etc. Skenderis et al

Iteratively solve $R_{MN} = -4g_{MN} + \frac{1}{2}\partial_M\Phi\partial_N\Phi$, using Fefferman-Graham expn for 5-d metric and massless scalar $\Phi = \Phi^0 + r^2\Phi^2 + \ldots$:

$$g^2_{\mu\nu} \sim R^0_{\mu\nu} - \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2(d-1)}\left(R - \frac{1}{2}(\partial\Phi)^2\right)g^0_{\mu\nu}, \qquad \Phi^2 \sim \Box^0\Phi^0.$$

$$g^2_{\mu\nu} = 0 \ \Rightarrow\ R^0_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi, \qquad \Phi^2 = 0 \ \Rightarrow\ \Box^0\Phi^0 = 0.$$

Higher order coefficients also vanish.

[Analogous arguments for AdS-cosmologies (Das,KN,Trivedi et al) and also for similar deformations of de Sitter space (Poincare slices) with late-time singularity (1204.3506; Das,KN, in progress): initial conditions appear fine-tuned.]

Lifshitz singularities, string theory

$ds^2_{Einst} = \frac{R^2}{w^2}\left(\tilde{g}_{\mu\nu}(x^\mu)dx^\mu dx^\nu + dw^2\right), \quad \Phi = \Phi(x^\mu): \quad g^0_{\mu\nu}, \Phi^0$ nonzero.

$g^2_{\mu\nu} = 0 \;\Rightarrow\; R^0_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi, \qquad \Phi^2 = 0 \;\Rightarrow\; \Box^0\Phi^0 = 0.$ Higher coeffs vanish.

The above conditions on the Fefferman-Graham coefficients $g^n_{\mu\nu}, \phi^n, n > 0$ vanishing in these string configurations are non-generic. Appear to be nontrivial constraints fine-tuning dual gauge theory state. These arguments also apply in the earlier coordinates with $g_{++} \neq 0$.

This suggests that the Lifshitz state in these string constructions is a "constrained" state, leading to the bulk singularity in the interior. [Unclear if higher derivative corrections become large: dilaton regular.] Speculation: Lifshitz state unstable? recall Gregory-Laflamme.

Technical resolution: turn on the subleading coefficients $g^n_{\mu\nu}$. Physical interpretations? Lifshitz symmetries break. New (zero temperature) phases, perhaps with hyperscaling violation, clumped phases? work in progress …

Conclusions, open questions

- AdS null deformations $\rightarrow$ dim'nal redux

  $\rightarrow$ Lifshitz scaling, hyperscaling violation.

Lifshitz state appears constrained in string constructions.

- Finite temperature?
- Lifshitz singularities in string constructions? Resolutions?
- Explore hyperscaling violation from field theory point of view.

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