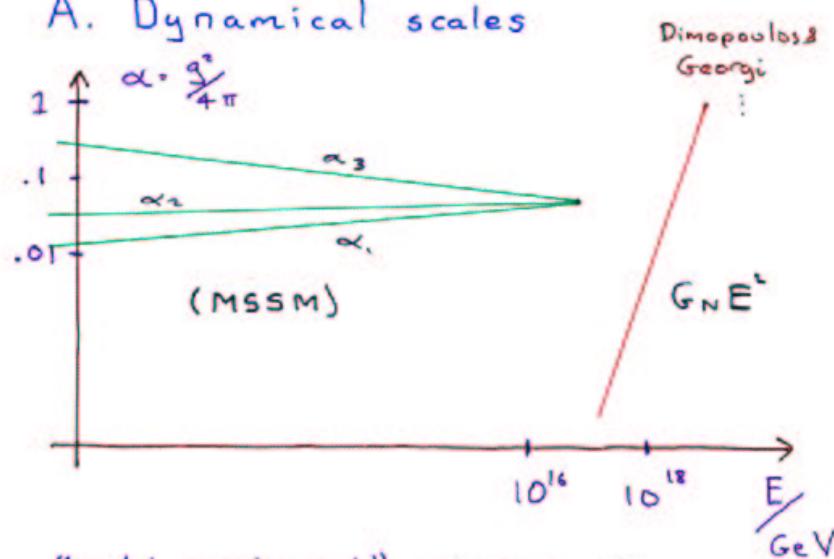


## I. String theory and experiment

### A. Dynamical scales



"old school" string theory

Quantum theory of gravity

$$m_{\text{pl}} \cdot (8\pi G_N)^{-1/2} \sim 2.4 \times 10^{18} \text{ GeV}$$

Unified theory

$$m_{\text{out}} \sim 2 \times 10^{16} \text{ GeV}$$

both  $\gg$  TeV!

### Two frameworks:

#### i) "Large extra dimensions"

fundamental dynamical scale:

$$m_* \sim m_{\text{EW}} \sim \text{TeV}$$

d-dimensional spacetime:

$$X_d \cdot \mathbb{R}^+ \times \tilde{X}_{d-4}$$

$$S_{\text{grav}}^{(d)} = m_{\text{pl},d}^{d-2} \int d^d x \sqrt{g} R$$

$$m_{\text{pl},d} \sim m_*$$

$$S^{(4)} = \underbrace{m_{\text{pl},d}^{d-2} V_X}_{m_{\text{pl},4}^2} \int d^4 x \sqrt{g} R$$

$$d=2: V_X \sim (mm)^2$$

gravity at LHC!

Arkani-Hamed,  
Dimopoulos  
Dvali

## 2) "Conventional" unified models

## a) d=10 string theories

$$\text{String loop} \quad \text{Tension} \sim \frac{1}{l_s} \equiv m_s^2$$

$$X_{10} = \mathbb{R}^4 \times \tilde{X}_6 - \text{"Calabi-Yau"} \\ \text{s/ N=1 SUSY} \\ \text{in } d=4$$

dynamical scales:

$$m_{KK} \sim \sqrt{\epsilon}^{-\frac{1}{6}} \quad \left. \right\} \sim m_{GUT} \\ m_s, m_{p_{10}}$$

 $m_{p_{10}}$  still derived

## b) d=11 M-theory

$$X_{11} = \mathbb{R}^4 \times \tilde{X}_7$$

N=1 SUSY:

$$\text{i) } \tilde{X}_7 = X_6^{CY} \times \xrightarrow[E_8]{L} \xleftarrow[E_8]{} \quad \begin{matrix} \text{Hořava} \\ \text{Witten} \end{matrix}$$

$$\text{ii) } \tilde{X}_7 \text{ "G}_2 \text{ manifold"}$$

$$\text{gauge dynamics on } d=7 \\ \text{"defect"} \quad \mathbb{R}^4 \times \sum_{d=3} \quad \begin{matrix} \text{Witten} \\ \text{Acharya} \\ \vdots \end{matrix}$$

dynamical scales:

$$m_{p_{11}} \gtrsim m_{GUT}$$

$$m_{KK} \lesssim m_{GUT} \quad \text{s/ CY, G}_2$$

$$L \sim 100/m_{GUT}$$

## B. Experiments and $M_{\text{GUT}}$

### i) Indirect tests

#### a) Extrapolate -/RG

Assume SUSY at  $M_2 \sim 90 \text{ GeV}$

Assume "desert"  $M_{\text{EW}} < \Lambda < M_{\text{GUT}}$   
couplings unify

#### b) Wait a long time

e.g. proton decay - via  
exchange of particles  
w/ mass  $M$

$$\tau = \# \frac{M^{20-8}}{m_p^{20-7}} \gtrsim 10^{32} \text{ yrs}$$

$$\rightarrow M \sim M_{\text{GUT}} \quad (\Delta = 6)$$

rules out some GUTs!

### 2) Direct measurements?

Look for  $E \sim M_{\text{GUT}}$  processes  
in the sky

### Inflation:

- Simplest models:  $V_{\text{inf}} \sim M_{\text{GUT}}^4$

- Quantum fluctuations

$$\lambda \sim (10^{14} \text{ GeV})^{-1}$$

↓  
CMBR anisotropies  $3^\circ K + \underline{\underline{ST}}$

Large scale structure

New high-precision CMBR measurements:

- MAP satellite  
(Microwave Anisotropy Probe)  
taking data now
- Planck Surveyor satellite  
est. launch 2007
- Polarization measurements

Accuracy in  $\delta T / T$  @ fundamental limits (only one universe)

May be sensitive to dynamics at  $E \sim 10^{14} \text{ GeV}$  ?  
 $M_{\text{GUT}}$

## II. Inflation and the CMBR

### A. CMBR and the "horizon problem"

• When universe  $\sim 10^5$  yrs old

Temp  $\sim 1 \text{ eV} \sim 10^4 \text{ K}$

$\gamma + e^- + \text{nuclei} - \text{black body}$



$\gamma + (\text{neutral}) \text{ atoms}$   
"recombination"

•  $\gamma$  propagates freely, redshifts as universe expands

2.7° K black body radiation

"Cosmic Microwave Background Radiation"  
COBE etc

Horizon problem:

At angles  $\delta\theta \gtrsim 0.8^\circ$  on sky:

2 objects at  $t_{\text{recombination}}$   
are causally disconnected

But  $\frac{sT}{T} \sim 10^{-4}$  at  
all  $\delta\theta$  on sky

Structure formation:

Density fluctuations

$\delta\rho/\rho$  at  $t_{\text{recombination}}$

gravitational collapse

Large scale structure

galaxies, clusters, ...

Temperature fluctuations

$\frac{sT}{T}$  in CMBR  
 $T_{\text{observed}}$

So we need to generate  
small  $\delta\rho/\rho$

## B. Inflation

- Assume "cosmological constant" when universe  $\sim 10^{-38}$  sec. dominates

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

$$H^2 = \frac{\Lambda}{3M_{Pl}^2}$$

$$\cdot \|\delta x(t)\| = e^{Ht} \|\delta x(0)\|$$



$\|\delta x\| > \frac{1}{H}$  - causally disconnected  
"horizon"

Correlated objects rapidly separate

- If universe grows

$$\sim e^{60}$$

"60 e-foldings"  $\sim 10^{-35}$  sec

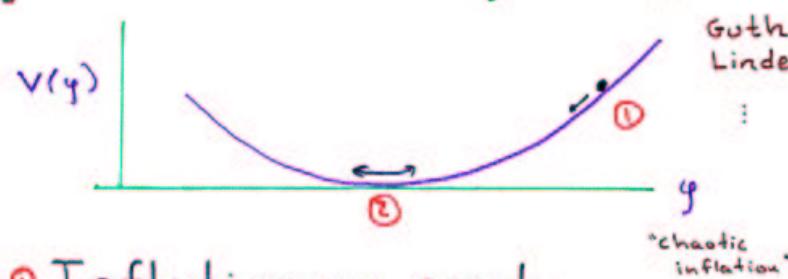
Solves the horizon problem

Leads to flat universe



Eventually "ordinary" matter/radiation must dominate

$$\dot{\phi} \text{ : "inflaton"} \quad \mathcal{L} - (\partial\phi)^2 - V(\phi)$$



### ① Inflationary epoch

$$\dot{\phi}^2, \dot{\phi}\ddot{\phi} \ll V; \quad \dot{H} \ll H^2 \quad \text{"slow roll"}$$

$$H^2 = \frac{V}{3m_p^2}; \quad \dot{\phi} = -\frac{1}{3H} \partial_\phi V$$

### ② "Reheating" $\dot{\phi}^2, \dot{\phi}\ddot{\phi}$ dominate

$$\frac{1}{\omega_{\text{inf}}} < \frac{1}{H}$$

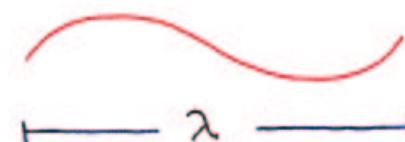
$\phi$  decays  $\rightarrow$  "ordinary" particles

C. Inflation  $\rightarrow \delta\rho(x), \delta T(x)$

$\frac{1}{\text{density}}$        $\frac{1}{\text{CMBR temp.}}$

- Definitions of wavelength

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$



$\lambda_c$  measured in coordinates  $\delta\vec{x}$

"comoving" wavelength

$\lambda_p$  measured in proper distances

"physical" wavelength grows

Comoving  $k_c \cdot \frac{1}{\lambda_c}$  conserved

- Quantum fluctuations of  $\varphi$

$$\lambda_{\text{phys}} \sim \frac{1}{H} \quad \text{quantum}$$

fluctuations "freeze into"  
classical perturbations  $\delta\varphi$

Hawking radiation

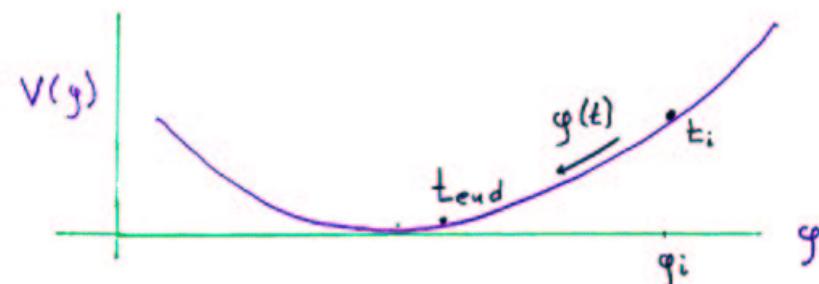
$$\langle \delta\varphi^2 \rangle \Big|_{\lambda_{\text{phys}} = \gamma_H} = \frac{H^2}{(2\pi)^2}$$

- These convert to density perturbations

$$\delta\rho = V'(\varphi) \delta\varphi$$

$$\delta\varphi^2 (\rho_{\text{phys}} = H) = \langle \varphi(\rho) \varphi(-\rho) \rangle$$

- Scale of fluctuations now



- $\delta\varphi_{\text{ee}}, \delta\rho$  generated at  $t_i$

$$\lambda_{\text{phys}}^{-1}(t_i) = p_{\text{phys}} + H(t_i)$$

$$\delta\varphi_{\text{ee}} = \langle \varphi(p_i) \varphi(-p) \rangle \Big|_{p_i} = H$$

- Fluctuation stretches

$$\lambda_{\text{end}}(t_i) = \frac{1}{H(t_i)} \exp \left\{ \int_{t_i}^{t_e} H(t) dt \right\}$$

late  $t_i \longrightarrow$  small  $\lambda_e$

- Angular scale on sky

$\delta\theta(\lambda_e)$  monotonic,  $\sim \lambda_e$  for small  $\lambda_e$

$\delta\varphi$  continually produced, stretched  
All angular scales

• Density inhomogeneities

$$\frac{\delta \rho}{\rho} \Big|_{\text{end}} (\lambda_e) = C \frac{H}{g} \delta g (t_i(\lambda_e)) \\ = \tilde{C} \frac{\sqrt{3/2}}{m_p^3 \partial_g V} (g(t_i(\lambda_e)))$$

$$\frac{\delta T}{T_{\text{CMBR}}} (\delta \theta) = \underbrace{S(\lambda_e(\theta))}_{\substack{\text{computable,} \\ \text{depends on} \\ \text{known physics}}} \frac{\delta \rho}{\rho} \Big|_{\text{end}} (\lambda_e)$$

↑  
measured

CMBFAST

$$\frac{\delta \rho}{\rho} \sim 5 \times 10^{-5}$$

COBE  
:

D. Primordial gravity waves

$$\delta h_{\mu\nu} = e_{\mu\nu}^1 \psi^1 + e_{\mu\nu}^2 \psi^2$$

physical polarizations  
(after gauge fixing)

$\psi^i$  - essentially scalar fields

Classical perturbations  
generated just as for  $\delta g$

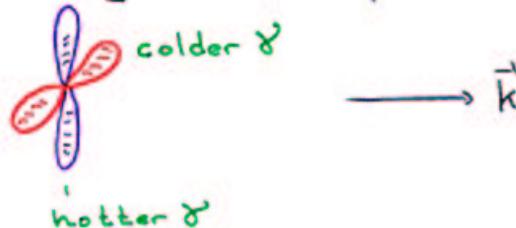
$$(\delta \psi_{ca}^i)^2 = \langle \psi^i(p_{\text{phys}}) \psi^i(-p) \rangle \Big|_{p_{\text{phys}}} = H \\ = \frac{H^2}{(2\pi m_{p,\perp})^2}$$

normalization of  $\psi$

- $\delta h_{\mu\nu}$  has distinct signature

generally,  $\delta_{\text{CMB}}$  polarized by  
Thomson scattering  
off  $e^-$  before  
recombination

Gravity waves in plasma:



Spin-2 character of  $\delta h$ :  
unique polarization  
signature

Measurable by polarization-sensitive detectors

$$A_T(\delta \theta(\lambda_e)) = \tilde{\delta} \frac{\nabla}{m_p^2} (g(t(\lambda_e)))$$



Direct measurement  
of  $V_{\text{inf}}$  if we can  
measure  $A_T$

## E. Scales of inflationary dynamics

Simplest models

$$V(g) = M_1^4 f\left(\frac{g}{M_2}\right)$$

Banks  
et al  
:

1) Demand "slow roll" for  $\gtrsim 60$  efolds

$$M_2 \sim \text{few } M_{\text{Pl}}$$

$$\langle g \rangle \sim M_2$$

2)  $\delta_T \sim 5 \times 10^{-5}$  from COBE

$$M_1 \lesssim M_{\text{GUT}}$$

3) Upper bound from ~~polarization~~

$$M_1 \lesssim 1.7 \times 10^{16} \text{ GeV}$$

$$H \lesssim 7 \times 10^{13} \text{ GeV}$$

- Inflaton potential  $V(g)$   
"fine-tuned"

Dimensionless coupling

$$\lambda \sim \partial_g^4 V(g) \sim \left(\frac{M_1}{M_2}\right)^4$$

$$\ll 10^{-8} \quad (\sim 10^{-12})$$

Hard to explain small numbers!  
(quantum corrections)

SUSY?

### III. Effects of short-distance physics

$$\left(\frac{\delta\rho}{\rho}\right)^2(\lambda_e) = D \frac{H^2(t_e(\lambda_e))}{\dot{\phi}^2} \langle \phi(p_{\text{phys}}) \phi(-p) \rangle_{p=H}$$

$H$  is relevant dynamical scale

Assume new physics at

$$\Lambda_{uv} \gg H$$

↓

"Integrate out" momenta  $p > H$

$$S_{\text{eff}}^H = \int^H d^4p \{ \phi(p) K(p; H, \Lambda_{uv}, \dots) \phi(-p) + \dots \}$$

use  $S_{\text{eff}}$  to compute

$$\langle \phi(p=H) \phi(-p) \rangle$$

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Example:

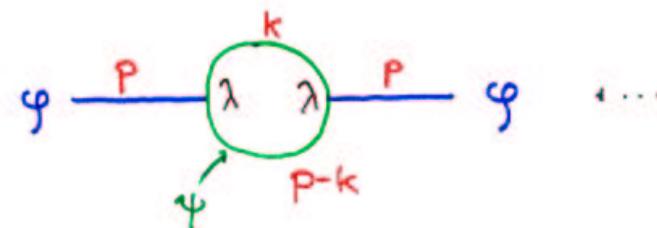
$$\mathcal{L}_{\text{int}}^{\Lambda_{uv}} = \lambda \phi \bar{\psi} \psi + m \bar{\psi} \psi + \dots$$

$m \gg H$

$$\langle \phi(p) \phi(-p) \rangle =$$

$$\phi \xrightarrow{P} \phi$$

+



Compute in de Sitter space

$$\text{corrections for } k \gg H \sim \frac{H^2}{k^2}$$

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Locality at  $p \lesssim H$



$S_{\text{eff}}^{(H)}$  analytic in

$$p^2, R_{\mu\nu 2p} \sim \partial^2 g \sim H^2$$

$\mathcal{L}_{\text{eff}}^{(H)}$

↓

$$g(p)g(-p) \left\{ p^2 + 2H^2 + c_1 \frac{H^2}{\Lambda_{\text{UV}}} p^2 + c_2 \frac{1}{\Lambda_{\text{UV}}} p^4 \right\}$$

( $\Lambda \sim m_p$  in example?) + ...

$$\langle g(p=H)g(-p) \rangle = \frac{H^2}{(2\pi)^2} \left( 1 + \chi_s \frac{H^2}{\Lambda_{\text{UV}}^2} \right)$$

depends on  $c_1, c_2$

Effect on measurable quantities:

$$\frac{\delta\rho}{\rho} \equiv A_S = \tilde{C} \frac{H^2}{2\pi\dot{g}} \left( 1 + \frac{1}{2} \chi_s \frac{H^2}{\Lambda_{\text{UV}}^2} + \dots \right)$$

$$A_T = D \frac{H}{m_{p^2, 4}} \left( 1 + \frac{1}{2} \chi_T \frac{H^2}{\Lambda_{\text{UV}}^2} + \dots \right)$$

$$\text{Recall: } H^2 = \frac{V(g)}{3m_{p^2, 4}^2}$$

$$3H\dot{g} \sim -\partial_g V$$

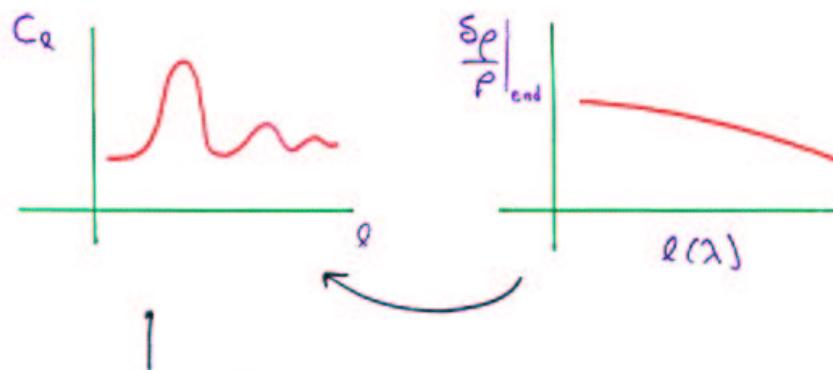
Can we extract  $H$ -dependence  
of  $A_S, A_T$  from measurements?

## IV. Experimental detection

### A. Scale dependence of $\frac{\delta T}{T}$

$$\frac{\delta T}{T} = \sum_{\ell m} Y_{\ell m}(\theta, \phi) C_\ell \quad \delta A_T$$

$$\ell = \ell(\lambda_{\text{end}}) \sim 1/\lambda$$



Final (observed) spectrum depends calculably on post-inflationary physics

$\Omega_b$ , dark matter,  $\Lambda$ , ...

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"Nearly scale-invariant spectrum"

Recall:  $\lambda_e - \lambda_e(q)$  invertible

$$l = l(\lambda_e) = l(q(\lambda_e))$$

Slow roll during inflation:

$$\dot{H} \ll H^2$$

$A_{S,T}(V, V')$  change slowly

$A_{S,T}$  depends weakly on  $l$

$$\ln A_S = \alpha_0 + (n_{S-1}) \ln l + v_S (\ln l)^2 + \dots$$

$$\ln A_T = \beta_0 + n_T \ln l + v_T (\ln l)^2 + \dots$$

$A_S, A_T, n_S, n_T, v_S$ :

"inflationary observables"

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• Functional form of  $V, A_S, A_T$

$l$ -dependence  $\Leftrightarrow g$ -dependence

Given:

a) Theoretical input  $A_S = A_S(V, \partial_g V)$   
 $A_T = A_T(V)$

b) Experimental input:

$$A_{S,T} = \sum_n a_{S,T}^{(n)} (ln l)^n$$

↓

• "Reconstruct"  $V(g)$

• "Consistency condition"

$$n_T = -\gamma_k \left( \frac{A_T}{A_S} \right)^2 \left( 1 + \chi_S \frac{H^2}{\lambda_W^2} + \dots \right)$$

## B. Measurability

### i) Limits on detectability

a)  $A_T, n_S, n_T$  very small

competes with

- Detector noise

- Astrophysical backgrounds

### b) "Cosmic variance"

Only one observable universe!

$$\delta C_\ell \sim \left( \frac{2}{2\ell+1} \right)^{1/2}$$

(sample variance)

## 2) Polarization measurements

- Post-inflationary physics sharply reduces signal for  $\ell < 50, \ell > 150$

- Best (optimistic) case  
Cosmic variance-limited measurement of  $A_T$  for  $50 \leq \ell \leq 150$

Can detect  $r_T > 0.01$

$$r_T = -\chi \left( \frac{A_T}{A_s} \right)^2 \left( 1 + \chi_s \frac{H^2}{\Lambda^2} + \dots \right)$$

$$\left( \frac{A_T}{A_s} \right)^2 < \frac{1}{15}$$

"New physics" observable if  $\chi_s \frac{H^2}{\Lambda^2} \gtrsim 0.1 - 1$

## V. Specific models

### A. d=4 field theories (GUTs)

- Renormalizable interactions at  $\Lambda_{UV} \sim m_{GUT}$

$$\text{ex: } \mathcal{L}_{int} \sim \lambda g \bar{\psi} \psi + m \bar{\psi} \psi$$

$m \sim m_{GUT}$

now recall:  $g \sim \text{few} \times m_{Pl,+}$   
↙ during inflation

$$m_\phi \sim \lambda g + m \sim \lambda m_{Pl,+} + m$$

$$\lambda \frac{H^2}{m_\phi^2} \lesssim \frac{H^2}{m_{Pl,+}^2} \sim \underline{\underline{10^{-10}}}$$

This is unobservable!

- $\lambda m_{GUT}$  effects in 4D models generically unobservable

Notable exception:

$$\lambda g + m \sim \lambda'(g - g_0)$$

$g = g_0$  at some time during  
in inflationary epoch

Chung  
et al

Feature in  $A_T, A_s$   
at  $l(g_0)$

## B. Can we increase effect?

$$\chi_s \frac{H^2}{m_{\text{phys}}^2} \text{ big if } \chi_s \gg 1$$

eg: many particles couple to  $g$   
(not so natural in 4D GUTs)

## • Higher-dimensional theories:



loop integral includes  
sum over Kaluza-Klein  
momenta

This is highly divergent

Dominated by  $\text{km cut-off} \sim m_{\text{phys}}$

$m_{\text{phys}} \ll m_{\text{phys}}$	$\chi \frac{H^2}{m_{\text{phys}}^2} \sim \frac{H^2}{m_{\text{phys}}^2}$
---------------------------------------	---

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## C. String/M-theory GUTs

### 1) Weakly coupled string theory

$$\Lambda_{\text{UV}} \sim M_3 \sim M_{\text{GUT}}$$

$$\chi \frac{H^2}{m_s^2} \sim g_s^2 \frac{H^2}{m_s^2} \ll 10^{-6}$$

↑  
"string coupling"



### 2) M-theory models

$$\chi_7 = (CY_7 \times \mathbb{R}) \text{ or } G_2$$

$$\chi \frac{H^2}{M_{\text{GUT}}^2} \sim 10^{-6}$$

Better, but still unobservable

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## D. Observable models

Try to lower  $m_{pe,d}$

### Constraints

1) Correct  $m_{pe,4} = 2 \times 10^{13} \text{ GeV}$

$$m_{pe,+}^2 = m_{pe,d}^{d-2} V_{d-4}$$

2)  $\alpha_{\text{gauge}} = g^2 / 4\pi \sim 1/25$

to get correct gauge couplings  
at  $p \sim \text{TeV}$

3)  $m_{KK} > T_{ds} = \frac{H}{2\pi}$

quantum dynamics of  $g$  is  
four-dimensional

4) Energy densities  $E_{40}^+ = E_d^d V_{d-4}$

$$\frac{H^2}{m_{pe,d}^2} = \frac{1}{3} \left( \frac{E_d}{m_{pe,d}} \right)^d \lesssim \mathcal{O}(1)$$

### Problems

1) Lose precision grand  
unification

$$m_{\text{fund}} \sim m_{pe,11} \sim 0.05 \text{ M}_{\text{GUT}}$$

2) Proton decay

- dimension 6 operators:

$$\mathcal{Z}_p \sim \frac{m_{\text{fund}}^4}{m_p^6} \sim 10^{-12} \text{ too small}$$

- dimension  $\geq 7$  OK

## VI. Conclusions

### A. Competing effects?

- Multiple scalar fields

$\varphi_{\text{inflaton}}, \sigma_i$

$$V = V(\varphi, \sigma_i)$$

$$\text{If } \frac{\partial^2}{\partial \sigma_i^2} V \ll H^2,$$

$\langle \delta \sigma_i^2 \rangle$  may contribute to  $\frac{8T}{T}$

$$n_T = -\chi \left( \frac{A_T}{A_s} \right)^2 \left( 1 + \chi \frac{H^2}{K^2} + f(\delta \sigma_{i-1}) \right)$$

Observable  $f$  requires additional fine tuning of  $\cdot V$

- initial conditions for  $\varphi, \sigma_i$

Linde  
Kofman  
  
Langlois  
Bartolo,  
Matarrese,  
Riotto

### B. Need better control of explicit models

M theory on  $G_2$  manifolds

Other "brane world" models

(F-theory, D-branes on  $X_6^{CY}$ )

Compute  $X_6 \xrightarrow{H^2} \mathbb{R}^2$  explicitly

Effects of other scalar fields

:

### C. Other astrophysical probes of new physics?

Example: dark matter

$$\Omega_{\text{baryon}} \sim 0.015$$

$$\Omega_{\text{dark matter}} \sim 0.3$$

CMBR

Galactic dynamics

Gravitational lensing data  
;

Nature of dark matter (cold, hot, ...)  
effects CMBR, structure formation

Detailed properties (interactions)  
effect (sub)galactic structure

Spergel & Steinhardt

SF Dimopoulos,  
Kapner,  
Kachru,  
Lawrence,  
Silverstein

## Short-distance physics and the cosmic microwave background

### I. Introduction

### II. Inflation and structure formation

### III. Impact of new physics

### IV. Experimental signatures

### V. Observability in 4d GUTs, string/M theory

Albion Lawrence  
SLAC Theory Group  
& Stanford Physics

In collaboration with:

Nemanja Kaloper

Matthew Kleban

Stephen Shenker

hep-th/0201158

0.1

"Brane world" scenarios

$$M_d = \mathbb{R}^4 \times M_{d-4}$$

Gauge d.o.f. propagate on  
codimension k  
brane/singularity

$$M_{d-k} = \mathbb{R}^4 \times M_{d-k-4}$$

$$\alpha \cdot \frac{g^2}{4\pi} \sim \frac{1}{m_{\text{Pl},d}^{d-k-4}} V_{d-k-4} \sim \lambda_{25}$$

$$m_{\text{Pl},+}^2 \sim m_d^{d-2} V_{d-4}$$

$$V_k \equiv \frac{V_{d-4}}{V_{d-k-4}}$$

$$\tilde{V}_k = m_{\text{Pl},d}^2 V_k$$

38.1

We can rewrite:

$$m_\phi^2 = m_d^2 \tilde{V}_k / \alpha \quad (*)$$

Recall:

$$m_{KK} \sim \frac{2\pi}{\sqrt{k}} \gtrsim T_{ds} = H / 2\pi$$

$\tilde{V}_{d-k+4}$  fixed

Places lower limit on  $m_d$  via (\*)

$k$  larger: can push  $m_{\text{Pl}, d}$  down farther

### M-theory examples

1)  $X_7 = X_6^{CY} \times \begin{array}{c} \xleftarrow{\hspace{1cm}} \\[-1ex] \longleftarrow L \rightarrow \\[-1ex] \xrightarrow{\hspace{1cm}} \end{array} \quad d=11, k=1$

$$\lambda \gtrsim 10^{12} \text{ GeV}$$

$$m_{\text{Pl}, 11} \gtrsim 6 \times 10^{15} \text{ GeV}$$

$$\frac{H^2}{m_{\text{Pl}, 11}^2} \lesssim 10^{-4}$$

2)  $X_7 = G_2 \text{ manifold} \quad d=11, k=4$

$$m_{11} \sim H \sim 7 \times 10^{13} \text{ GeV}$$

$$m_{KK} \gtrsim 2 \times 10^{12} \text{ GeV}$$

$$\frac{H^2}{m_{\text{Pl}, 11}^2} \sim \mathcal{O}(1)$$