Gravitational Condensate Stars

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Outline

- Classical Black Holes
- Quantum Black Holes
- Properties of Bose-Einstein Condensates
- A New Solution to Einstein's Eqs.
- Main Features
  - No Singularities
  - No Event Horizon
  - No Information Paradox
  - Maximum of Entropy $\Leftrightarrow$ Stable
- Implications and Conclusions

Astronomy Picture of the Day

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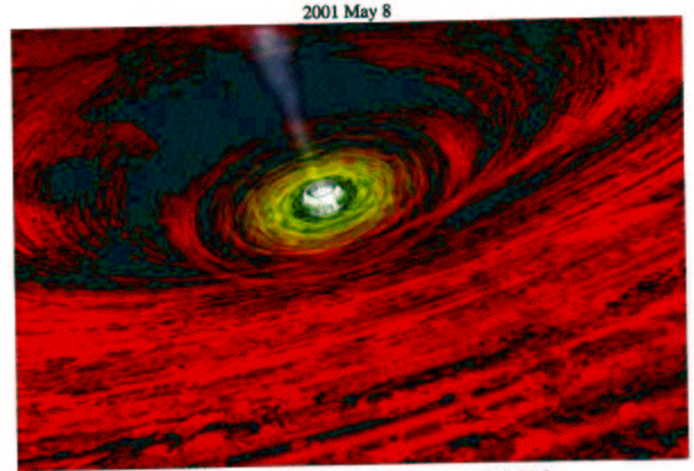
2001 May 8



GRO J1655-40: Evidence for a Spinning Black Hole
Drawing Credit: A. Hobart, CXC

Explanation: In the center of a swirling whirlpool of hot gas is likely a beast that has never been seen directly: a black hole. Studies of the bright light emitted by the swirling gas frequently indicate not only that a black hole is present, but also likely attributes. The gas surrounding GRO J1655-40, for example, has recently been found to display an unusual flickering at a rate of 450 times a second. Given a previous mass estimate for the central object of seven times the mass of our Sun, the rate of the fast flickering can be explained by a black hole that is rotating very rapidly. What physical mechanisms actually cause the flickering -- and a slower quasi-periodic oscillation (QPO) -- in accretion disks surrounding black holes and neutron stars remains a topic of much research.

Tomorrow's picture: Space Station Armed

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ON A STATIONARY SYSTEM WITH SPHERICAL SYMMETRY CONSISTING OF MANY GRAVITATING MASSES

By Albert Einstein

(Received May 10, 1939)

If one considers Schwarzschild's solution of the static gravitational field of spherical symmetry

$$ds^2 = -\left(1 + \frac{\mu}{2r}\right)^4 (dx_1^2 + dx_2^2 + dx_3^2) + \left(\frac{1 - \frac{\mu}{2r}}{1 + \frac{\mu}{2r}}\right)^2 dt^2 \qquad (1)$$

it is noted that

$$g_{44} = \left(\frac{1 - \frac{\mu}{2r}}{1 + \frac{\mu}{2r}}\right)^2$$

vanishes for $r = \mu/2$. This means that a clock kept at this place would go at the rate zero. Further it is easy to show that both light rays and material particles take an infinitely long time (measured in "coördinate time") in order to reach the point $r = \mu/2$ when originating from a point $r > \mu/2$. In this sense the sphere $r = \mu/2$ constitutes a place where the field is singular. (μ represents the gravitating mass.)

There arises the question whether it is possible to build up a field containing such singularities with the help of actual gravitating masses, or whether such regions with vanishing g_{44} do not exist in cases which have physical reality. Schwarzschild himself investigated the gravitational field which is produced by an incompressible liquid. He found that in this case, too, there appears a region with vanishing g_{44} if only, with given density of the liquid, the radius of the field-producing sphere is chosen large enough.

This argument, however, is not convincing; the concept of an incompressible liquid is not compatible with relativity theory as elastic waves would have to travel with infinite velocity. It would be necessary, therefore, to introduce a compressible liquid whose equation of state excludes the possibility of sound signals with a speed in excess of the velocity of light. But the treatment of any such problem would be quite involved; besides, the choice of such an equation of state would be arbitrary within wide limits, and one could not be sure that thereby no assumptions have been made which contain physical impossibilities.

One is thus led to ask whether matter cannot be introduced in such a way that questionable assumptions are excluded from the very beginning. In fact this can be done by choosing, as the field-producing mass, a great number of

Particles of finite size in the gravitational field

1962

By P. A. M. Dirac, F.R.S.
St John's College, Cambridge

I should like to speak briefly about some work that I am engaged on, although it is not yet complete. The object of this work is to set up a theory of the gravitational field interacting with particles. I want this theory to be in agreement with Einstein's theory of gravitation, and also I insist that it shall follow from an action principle. I do insist on this because I believe that Nature works according to an action principle, and if we have an action principle, we certainly have a first step towards quantization.

Now in considering a particle interacting with the gravitational field, the first thing one would think of would be a point particle, but here one runs into a difficulty, because if one keeps to physically acceptable ideas, one cannot have a particle smaller than the Schwarzschild radius, which provides a sort of natural boundary to space. The mathematicians can go beyond this Schwarzschild radius, and get inside, but I would maintain that this inside region is not physical space, because to send a signal inside and get it out again would take an infinite time, so I feel that the space inside the Schwarzschild radius must belong to a different universe and should not be taken into account in any physical theory. So from the physical point of view, the possibility of having a point singularity in the Einstein field is ruled out. Each particle must have a finite size no smaller than the Schwarzschild radius.

I tried for some time to work with a particle with radius equal to the Schwarzschild radius, but I found great difficulties, because the field at the Schwarzschild radius is so strongly singular, and it seems that a more profitable line of investigation is to take a particle bigger than the Schwarzschild radius and to try to construct a theory for such a particle interacting with the gravitational field. There we have quite a definite problem, and we can get some help by considering the analogous problem in electrodynamics. Previous speakers have called attention to the close analogies between the electromagnetic field and the gravitational field, and I am going to follow in their footsteps and start off by considering the corresponding problem in the electromagnetic field.

This is the problem of setting up a theory of an extended electron in an electromagnetic field. We must make some basic assumptions about this extended electron, and I make the simplest ones which give a reasonable physical theory.

I assume that the electron has a definite surface—a definite boundary—outside which the field is described by Maxwell's equations.

I assume the surface itself to be a perfect conductor, so that there is no field inside the surface.

I assume that the potentials are continuous at the surface, while their first derivatives may have discontinuities, so that the field conditions just outside the

Classical Black Holes

Schwarzschild Metric (1916)

$$ds^2 = -dt^2\, f(r) + \frac{dr^2}{h(r)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$f(r) = 1 - \frac{2GM}{r} = h(r)$$

Classical Singularities:

- $r = 0$: Infinite Tidal Forces, Breakdown of Gen. Rel.
- $r \equiv R_s = 2GM$ $(c = 1)$: Event Horizon, Infinite Blueshift, Change of sign of f, h

Trapping of light inside the horizon is what makes a black hole

BLACK

The $r = R_s$ singularity is purely kinematic, removable by a coordinate transformation iff $\hbar = 0$

Quantum Black Holes

Consequences of $\hbar \neq 0$

- Infinite Blueshift of ω Means Infinite Energy $\hbar\omega$
- Gravitational Coupling $\frac{GE^2}{\hbar c^5}$ Grows Near Horizon
- Semi-Classical Hawking Radiation is Thermal
  $T_H = \frac{\hbar c^3}{8\pi G k_B M}$
- Negative Specific Heat Capacity ⇔ Unstable
- First Law $dE = T\, dS$ implies a HUGE Entropy
  $S_{BH} = \frac{4\pi k_B M^2}{\hbar c} \simeq 10^{77} k_B \left(\frac{M}{M_\odot}\right)^2$
- Information Paradox $S = k_B \ln\Omega$??
- Bekenstein-Hawking Entropy is Non-Extensive
- Information Loss, Non-conservation of Probability → Change Quantum Mechanics??

BUT is the semi-classical calculation correct? What really happens near $r = R_s$?

Quantum Effects Near $r = R_S$

- Large Vacuum Stresses of Matter Fields

$$\langle T^t_{\ t}\rangle \sim \langle T^r_{\ r}\rangle \sim \left(1 - \frac{2GM}{r}\right)^{-1}$$

- Extreme Behavior of r, t Components Imply an Effective One-Dimensional Relativistic Fluid

$$p = \rho$$

- Critical Region where Sound Speed = Light Speed

$$c_S^2 = \frac{dp}{d\rho} = c^2$$

  Any Additional Increase in Pressure Would Violate Causality: Onset of Superluminal Modes is the Signature of a Relativistic Phase Transition
- Back Reaction of Hawking Radiation not Negligible

$$S = 4\,\frac{\kappa+1}{7\kappa+1}\,S_{BH}$$

  becomes $S = S_{BH}$ for $\kappa = 1$ ('t Hooft)
  Semi-Classical Limit Requires $p = \rho$
- Conformal Phase of Gravity with $p = \rho$ obtained from the Stress Tensor of Effective Action of the Quantum Trace Anomaly
- A Critical Region is Essential for Joining Interior (de Sitter) to the Exterior (Schwarzschild)

Bose-Einstein Condensation

- Bose-Einstein statistics imply any number of particles can occupy the same single particle state.
- At high enough densities and/or low enough temperatures the free energy of a boson gas is minimized by a finite fraction of all the particles in the lowest energy (ground) state.
- This tendency of bosons to condense takes place in the absence of interactions or even with (not too strong) repulsive interactions. Attractive interactions make it all the more favorable.
- Bose-Einstein Condensation is a generic macroscopic quantum phenomenon, observed in Superfluids, ^4He (even ^3He by fermion pairing), Superconductors, and Atomic Gases, ^{85}Rb.
- Relativistic Quantum Field Theory exhibits a similar phenomenon in Spontaneous Symmetry Breaking, in both the strong and electroweak interactions at their characteristic energy or temperature scale. $\langle \Phi \rangle \neq 0$

Gravitational Vacuum Condensates

- Gravity is a theory of spin-2 bosons
- Interactions are attractive
- Interactions are strong near $r = R_s$
- Energy of any scalar order parameter must couple to gravity with vacuum eq. of state:

$$p_V = -\rho_V = -V(\phi)$$

- Relativistic Entropy Density s is (for $\mu = 0$):

$$Ts = p + \rho = 0 \quad \text{if} \quad p = -\rho$$

- Zero entropy density for a single macroscopic quantum state: $k_B \ln \Omega = 0$ for $\Omega = 1$
- Such eq. of state does not satisfy the energy condition $\rho + 3p \geq 0$ if $\rho_V > 0$ needed to prove the classical singularity theorems
- Acts as a repulsive core

A BEC phase transition can stabilize a high density, compact cold stellar remnant to further gravitational collapse

A New Soln. to Einstein Eqs.

$$R_a^{\ b} - \frac{1}{2}R\,\delta_a^{\ b} = 8\pi G\,T_a^{\ b}$$

- $1 - \frac{d(r\,h)}{dr} = 8\pi G\,\rho\,r^2$
- $\frac{r\,h}{f}\frac{df}{dr} + h - 1 = 8\pi G\,p\,r^2$
- $\frac{dp}{dr} + \frac{p+\rho}{2f}\frac{df}{dr} = 0 \qquad (\nabla_b T_r^{\ b} = 0)$

Other components follow by differentiating these

Define $h \equiv 1 - \frac{2Gm(r)}{r} \equiv 1 - \frac{\mu}{r}$

Then $\frac{dm}{dr} = 4\pi\,\rho\,r^2$ and

$$\frac{dp}{dr} = -\frac{G(\rho+p)(m+4\pi p r^3)}{r\,(r-2Gm)} \qquad \text{(TOV eq.)}$$

Eqs. become closed when eq. of state is given:

$$p = \kappa\,\rho$$

with

$$\kappa = \begin{cases} -1, & r < r_1 \\ +1, & r_1 < r < r_2 \end{cases}$$

$$p = \rho = 0\,, \quad r_2 < r$$

Integration of final (conservation) eq. gives

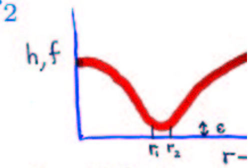
$$f(r) = \left(\frac{r}{r_1}\right)^2 \left(\frac{w_1}{w}\right) f(r_1) \simeq \left(\frac{w_1}{w}\right) f(r_1)$$

A consistent soln. matching at r_1 and r_2 is obtained if w is $\mathcal{O}(1)$ but $\Delta w \equiv w_2 - w_1 = \mathcal{O}(\epsilon)$. Then

$$r \simeq r_1 \simeq H_0^{-1} \simeq R_s \simeq r_2$$

barely changes in region II with

$$\Delta r \equiv r_2 - r_1 = \mathcal{O}(\epsilon^2)$$



and both f and h are of order ϵ in region II but nowhere vanishing. This means that the soln. has a globally defined time and **NO** event horizon. The physical meaning of $\epsilon \ll 1$ is that $\epsilon^{-\frac{1}{2}}$ is the order of the very large but finite redshift a photon emitted at the shell experiences in escaping to infinity.

The proper thickness of the shell is

$$\ell = \int_{r_1}^{r_2} dr\, h^{-\frac{1}{2}} \simeq R_s \epsilon^{\frac{1}{2}} \int_{w_1}^{w_2} dw\, w^{-\frac{3}{2}}$$

which is $\mathcal{O}(\epsilon^{\frac{3}{2}} R_s) \ll R_s$

$$\epsilon \sim (\ell/R_s)^{2/3} \ll 1$$

if ℓ independent of M

- I. Interior (Vacuum Condensate) de Sitter:
  $f(r) = Ch(r) = C\left(1 - H_0^2 r^2\right)$,
  $\rho_V = -p_V = \frac{3H_0^2}{8\pi G}$
- III. Exterior (Vacuum) Schwarzschild:
  $f(r) = h(r) = 1 - \frac{2GM}{r}$
  C, H_0 and M are (so far) arbitrary parameters
- II. Only Non-Vacuum Region:
  Thin shell with $p = \rho \to pf = const.$
  Let $w \equiv 8\pi G p r^2$ so the other two eqs. are
  - $\frac{dr}{r} = \frac{dh}{1-w-h} \simeq \frac{dh}{1-w}$
  - $\frac{dh}{h} = -\frac{1-w-h}{1+w-3h}\frac{dw}{w} \simeq -\frac{1-w}{1+w}\frac{dw}{w}$

If region II shell is **thin**, *i.e.* exists only near $r \simeq R_s \simeq H_0^{-1}$, then $h \ll 1$ in region II and h can be neglected on r.h.s. of •
Elementary Integration gives then

$$h \simeq \epsilon\, \frac{(1+w)^2}{w} \ll 1 \to \epsilon \ll 1, \text{ integ. const.}$$

$$r \simeq r_1 \left[1 - \epsilon \ln\left(\frac{w}{w_1}\right) + \epsilon\left(\frac{1}{w} - \frac{1}{w_1}\right)\right]$$

Likewise the energy in the thin shell of region II is

$$E_{II} = 4\pi \int_{r_1}^{r_2} \rho r^2 dr = \epsilon\, M \int_{w_1}^{w_2} dw$$

which is $\mathcal{O}(\epsilon^2 M) \ll M$

However, the entropy all resides in the shell since

$$p = \rho = \frac{a^2}{8\pi G}\left(\frac{k_B T}{\hbar}\right)^2$$ implies

$$s = \frac{p+\rho}{T} = k_B \frac{a}{\hbar}\left(\frac{p}{2\pi G}\right)^{\frac{1}{2}} = k_B \frac{a}{4\pi G\hbar}\frac{w^{\frac{1}{2}}}{r}$$ so

$$S_{II} = 4\pi \int_{r_1}^{r_2} s r^2 dr h^{-\frac{1}{2}} \simeq k_B \frac{aR_s^2}{\hbar G}\epsilon^{\frac{1}{2}} \int_{w_1}^{w_2} \frac{dw}{w}$$

which is of order

$$ak_B \frac{M}{\hbar}\epsilon^{\frac{3}{2}} R_s \sim ak_B \frac{M\ell}{\hbar} \ll S_{BH} = 4\pi k_B \frac{GM^2}{\hbar}$$

and **very** much smaller than S_{BH}.

Eg. If $a \sim 1,\ \ell \sim L_{planck},\ M \sim M_\odot$,

$$S \simeq 10^{38} k_B \ll S_\odot \simeq 10^{58} k_B \ll S_{BH} \simeq 10^{77} k_B$$

Stability

The Entropy Principle

- Entropy $S = 4\pi \int s\, r^2\, dr\, h^{-\frac{1}{2}}$ where
- $s = (\rho + p)/T$ is the local entropy density
- Local form of the first law, $d\rho = T ds$ and
- Eq. of state, $p = \kappa\rho$ imply
- $s = \frac{dp}{dT} \propto \rho^{\frac{1}{1+\kappa}} \propto \left[\frac{1}{4\pi r^2}\frac{dm}{dr}\right]^{\frac{1}{1+\kappa}}$

  so S can be expressed completely in terms of one function $\mu(r) = 2Gm(r)$
- Extremization of the Entropy w.r.t. μ

$$\delta S = 0$$

is satisfied iff the TOV eq. for static equilibrium is satisfied. Moreover, the soln. is stable iff the second variation is negative semi-definite,

$$\delta^2 S \leq 0$$

i.e. iff the Entropy is Maximized

- Regions I and III are vacuum regions with $S_I = S_{III} = 0$ which require no new analysis.
- In the shell region II, $\kappa = 1$ and

$$S = \frac{ak_B}{\hbar G}\int_{r_1}^{r_2} r\,dr \left(\frac{d\mu}{dr}\right)^{\frac{1}{2}} \left(1-\frac{\mu}{r}\right)^{-\frac{1}{2}}$$

- Second Variation of the Entropy is

$$\delta^2 S = \frac{ak_B}{4\hbar G}\int_{r_1}^{r_2} r\,dr \left(\frac{d\mu}{dr}\right)^{-\frac{3}{2}} \left(1-\frac{\mu}{r}\right)^{-\frac{3}{2}} \times \left\{-\left[\frac{d(\delta\mu)}{dr}\right]^2 + \frac{d\mu}{dr}\frac{1}{r^2}\left(1-\frac{\mu}{r}\right)^{-2}\left(1+\frac{d\mu}{dr}\right)(\delta\mu)^2\right\}$$

- Associated Sturm-Liouville diff. op. has zero solns.

$$\mathcal{L}\chi_0 = 0$$

corresponding to varying the endpoints r_1, r_2

- Let $\delta\mu(r) \equiv \chi_0(r)\psi(r)$ with $\psi(r_1) = \psi(r_2) = 0$, integrate by parts &

$$\delta^2 S = -\frac{ak_B}{4\hbar G}\int_{r_1}^{r_2} r\,dr \left(\frac{d\mu}{dr}\right)^{-\frac{3}{2}} \left(1-\frac{\mu}{r}\right)^{-\frac{1}{2}} \chi_0^2 \left(\frac{d\psi}{dr}\right)^2 < 0$$

Entropy is **Maximal**, Solution is **Stable**

Gra(vitational) Va(cuum) Stars

Gravastars as Astrophysical Objects

- Cold, Dark, Compact, Arbitrary M
- Accrete Matter like a Black Hole
- But Matter does not Disappear down a Hole
- May be Re-emitted by Ultra-relativistic Shell
- Possible More Efficient Central Engine for Sporadic Gamma Ray Bursters, High-Energy Cosmic Rays, Other Sources?
- Formation could be Violent 'Bosenova'
- Should Support Angular Momentum, Magnetic Fields
- Gravitational Wave Signatures?
- Alternative to Black Holes for the Final State of Gravitational Collapse

- Cosmological Models of Dark Energy
- Much to be Done...