

A Generalised, Manifestly Gauge Invariant Exact Renormalisation Group for $SU(N)$ Yang-Mills

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Outline

- 1 Introduction
- 2 Generalised ERGs
 - Scalar Field Theory
 - Gauge Theory
- 3 Regularisation for $SU(N)$ Gauge Theory
- 4 $SU(N)$ Gauge Theory
 - Regularised Flow Equation
 - Diagrammatics
 - Perturbative Diagrammatics
 - Future Directions

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Motivation

Why the ERG?

- Renormalisation is built in
- Huge freedom in the construction
 - manifest gauge invariance
 - the gauge field need not renormalise
 - simple, strong constraints on vertices
 - Gribov copies entirely avoided
 - universal, diagrammatic computation
 - straightforward renormalisation to all loops

Status of the Manifestly Gauge Invariant ERG

β_2 Computed (OJR, 2004)

- consistency confirmed
- ready for (non)-perturbative computation

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The Basic Idea

Set-up

- scalar field, φ
- Euclidean dimension, D

Integrating Out

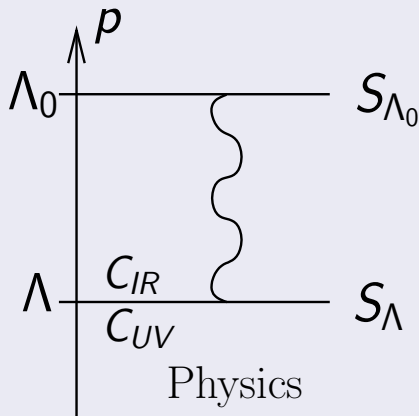
p
 Λ_0 S_{Λ_0}
 Λ C_{IR} S_{Λ}
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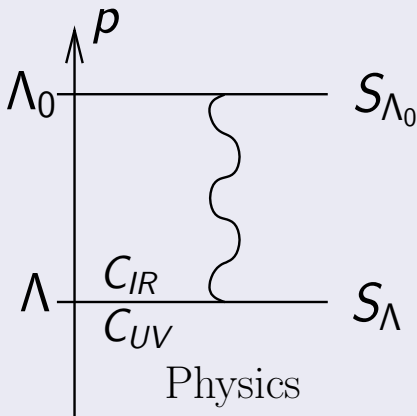
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Definitions

- Bare
- Effective
- IR cutoff function
- UV cutoff function
 - $C_{UV}(z) \rightarrow 0$
as $z \rightarrow \infty$
 - $C_{UV}(0) = 1.$

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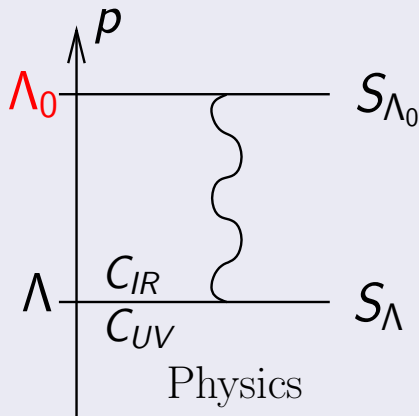
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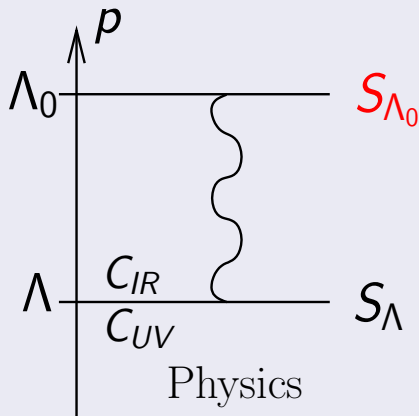
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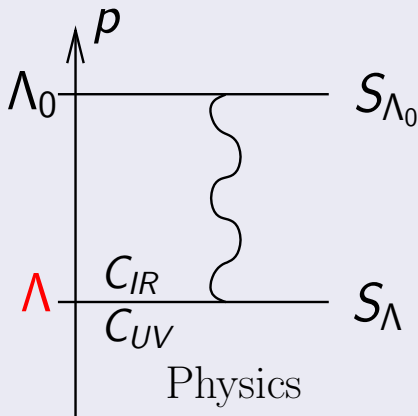
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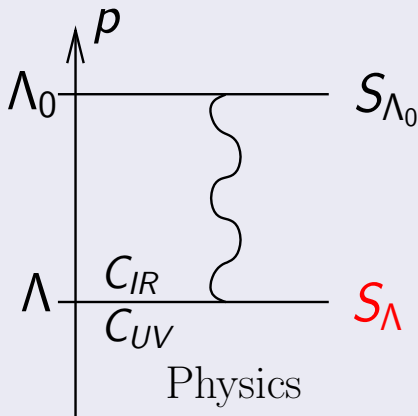
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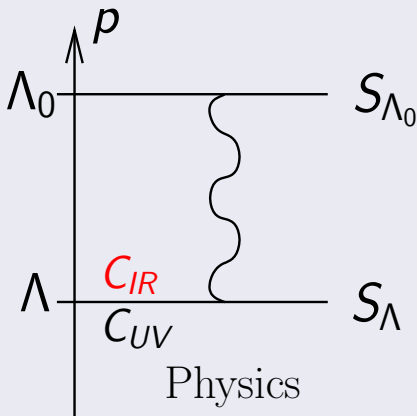
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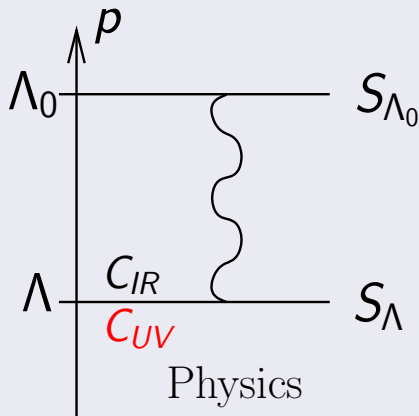
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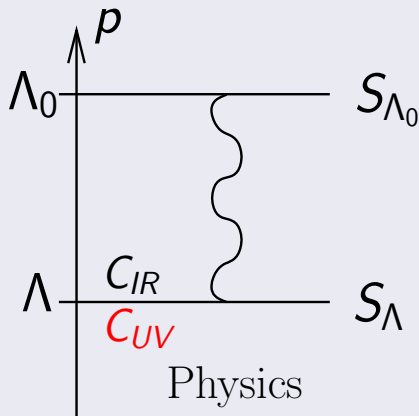
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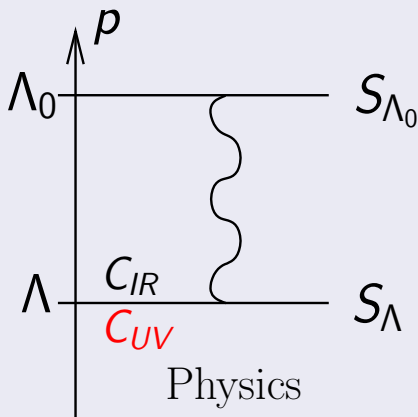
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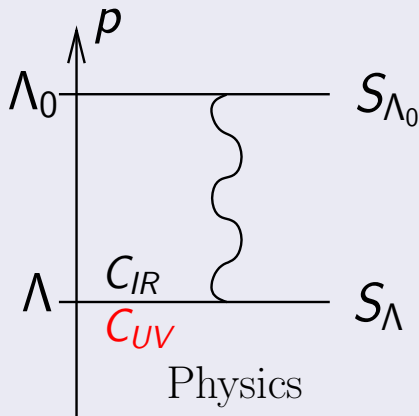
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Effective UV Cutoff

- Regularised propagator

$$\frac{1}{p^2} = \frac{C_{UV}}{p^2} = \Delta_{UV}$$
- Modes above Λ effectively cutoff

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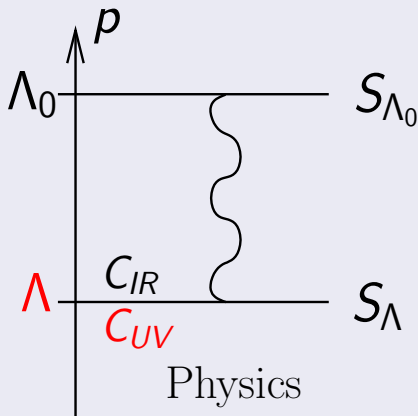
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- Choice of $\Psi \Rightarrow$ ERG

A Scalar Flow Equation

Example

$$\Psi_x = \frac{1}{2} \int_y \dot{\Delta}_{xy} \frac{\delta \Sigma_1}{\delta \varphi(y)}$$

- $\dot{\Delta}$ is an ERG Kernel
- $\Sigma_1 = S - 2\hat{S}$

$$\begin{aligned} -\Lambda \partial_\Lambda S &= a_0[S, \Sigma_1] - a_1[\Sigma_1] \\ &= \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma_1}{\delta \varphi} \end{aligned}$$

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What is it?

- A non-universal input which controls the flow
- Same structure and symmetries as S

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- \hat{S} has just regularised kinetic term, $\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi$
- flow equation is **Polchinski's equation** (up to discarded vacuum energy)

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Universal quantities cannot depend on non-universal details

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Isn't a General \hat{S} just Scaffolding?

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Gauge Theory

$U(1)$ Gauge Theory

- Simply replace φ with A_μ
- $$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta A_\mu} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma_1}{\delta A_\mu} - \frac{1}{2} \frac{\delta}{\delta A_\mu} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma_1}{\delta A_\mu}$$
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Covariantisation + Pauli-Villars = Regularisation

- Embed physical $SU(N)$ in spontaneously broken $SU(N|N)$
- Heavy fields act as Pauli-Villars!
- Defining rep.: $\mathcal{A}_\mu = \begin{pmatrix} A_\mu^1 & B_\mu \\ \bar{B}_\mu & A_\mu^2 \end{pmatrix} + \mathcal{A}_\mu^0 \mathbb{1}$
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local invariance

$$\delta\mathcal{A}_\mu = [\nabla_\mu, \Omega(x)] + \lambda_\mu(x)\mathbb{1}$$

- local $SU(N|N)$ invariance
- 'no- \mathcal{A}^0 symmetry'

Symmetry Breaking: $SU(N|N) \rightarrow SU(N) \times SU(N) \times U(1)$

- Superscalar $\mathcal{C} = \begin{pmatrix} C^1 & D \\ \bar{D} & C^2 \end{pmatrix}$
- Mass of B tracks Λ if
 - \mathcal{C} **dimensionless**
 - Effective Potential of \hat{S} has min. at

$$\langle \mathcal{C} \rangle = \sigma := \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$$
- S has min. at σ

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Symmetry Breaking: $SU(N|N) \rightarrow SU(N) \times SU(N) \times U(1)$

- Superscalar $\mathcal{C} = \begin{pmatrix} C^1 & D \\ \bar{D} & C^2 \end{pmatrix}$
- Mass of B tracks Λ if
 - \mathcal{C} dimensionless
 - Effective Potential of \hat{S} has min. at

$$\langle \mathcal{C} \rangle = \sigma := \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$$
- S has min. at σ

Symmetries

local invariance

$$\delta\mathcal{A}_\mu = [\nabla_\mu, \Omega(x)] + \lambda_\mu(x)\mathbb{1}$$

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The Broken Phase

Particle Content

- Massive Higgs fields C^1 and C^2
- Composite field $F_R = (B_\mu, D)$
 - five index
 - B and D gauge transform into each other
 - B eats D in unitarity gauge
- Massless fields A^1 and A^2
- Ignore $\mathcal{A}^0 \Rightarrow$ convenient diagrammatic prescription

Decoupling in the limit $\Lambda \rightarrow \infty$

- A^1 and A^2 communicate via massive fields
- Theory renormalisable in $D \leq 4$
- Lowest dimension effective interaction is irrelevant
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A Manifestly Gauge Invariant, Regularised Flow Equation

Requirements

- $SU(N|N)$ Invariance at high energies
- No- \mathcal{A}^0 Invariance
- Treats A^1 and A^2 asymmetrically

Construction

$$-\Lambda \partial_\Lambda S = a_0[S, \Sigma_g] - a_1[\Sigma_g],$$

- $a_0[S, \Sigma_g] = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} + \frac{1}{2} \frac{\delta S}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}$
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Diagrammatics for the Action

$$\begin{aligned}
 S = & \frac{1}{2} S^{AA} \text{str} AA + \frac{1}{3} S^{AAA} \text{str} AAA + \frac{1}{4} S^{AAAA} \text{str} AAAA \\
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- vertex coefficient function
- supertrace over fields

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 S = & \frac{1}{2} S^{AA} \text{str} AA + \frac{1}{3} S^{AAA} \text{str} AAA + \frac{1}{4} S^{AAAA} \text{str} AAAA \\
 & + \frac{1}{8} S^{AA,AA} \text{str} AA \text{str} AA + \frac{1}{2} S^{CC} \text{str} CC + \frac{1}{2} S^{C,C} \text{str} C \text{str} C \\
 & + \frac{1}{3} S^{ACC} \text{str} ACC
 \end{aligned}$$



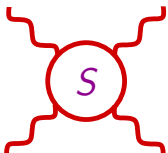
Diagrammatics for the Action

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 & + \frac{1}{3} S^{ACC} \text{str} ACC
 \end{aligned}$$

- $\text{str} \mathcal{A} = 0$
- $S^{\mathcal{A}C} = 0$ (charge conjugation)

Diagrammatics for the Action

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 S = & \frac{1}{2} S^{AA} \text{str} AA + \frac{1}{3} S^{AAA} \text{str} AAA + \frac{1}{4} S^{AAAA} \text{str} AAAA \\
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 & + \frac{1}{3} S^{ACC} \text{str} ACC
 \end{aligned}$$



- $\text{str} A = 0$
- $S^{AC} = 0$ (charge conjugation)

Diagrammatics for the Action

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 \end{aligned}$$



- $\text{str} A = 0$
- $S^{AC} = 0$ (charge conjugation)

Symmetric Phase Diagrammatics Flow Equation

The Classical Term

$$-\Lambda\partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{AA} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} + \frac{1}{2} \frac{\delta S}{\delta \mathcal{C}} \{ \dot{\Delta}^{CC} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}} - a_1[\Sigma_g]$$

Symmetric Phase Diagrammatics Flow Equation

The Classical Term

$$-\Lambda\partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} + \frac{1}{2} \frac{\delta S}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}} - a_1[\Sigma_g]$$

- S and Σ_g are composed of supertraces

Symmetric Phase Diagrammatics Flow Equation

The Classical Term

$$-\Lambda\partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} + \frac{1}{2} \frac{\delta S}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}} - a_1[\Sigma_g]$$

- S and Σ_g are composed of supertraces
- $\frac{\delta}{\delta \mathcal{A}_\mu}$, $\frac{\delta}{\delta \mathcal{C}}$ break open a supertrace

Symmetric Phase Diagrammatics Flow Equation

The Classical Term

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} + \frac{1}{2} \frac{\delta S}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}} - a_1[\Sigma_g]$$

- S and Σ_g are composed of supertraces
- $\frac{\delta}{\delta \mathcal{A}_\mu}$, $\frac{\delta}{\delta \mathcal{C}}$ break open a supertrace
- **The covariantisation glues everything back together**

Symmetric Phase Diagrammatics Flow Equation


The Classical Term

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu}$$

Symmetric Phase Diagrammatics Flow Equation

The Classical Term

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu}$$


$$-\Lambda \partial_\Lambda \left[\frac{1}{2} \text{str} \mathcal{A} \mathcal{A} \right] =$$


The diagram shows a red circle with a purple 'S' inside. Two red wavy lines extend vertically from the top and bottom of the circle, representing a trace operation over the gauge field.

Symmetric Phase Diagrammatics Flow Equation

The Classical Term


$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu}$$


$$-\Lambda \partial_\Lambda \left[\frac{1}{2} \text{str} \mathcal{A} \mathcal{A} \right] = \frac{1}{2}$$


Symmetric Phase Diagrammatics Flow Equation

The Classical Term

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu}$$


$$-\Lambda \partial_\Lambda \left[\frac{1}{2} \text{str} \mathcal{A}\mathcal{A} \right] = \frac{1}{2}$$


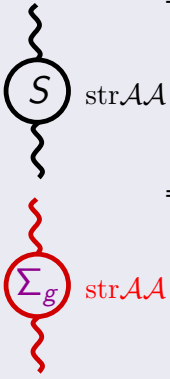
$$\left[\frac{1}{2} \text{str} \mathcal{A}\mathcal{A} \right]$$


Symmetric Phase Diagrammatics Flow Equation

The Classical Term

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu}$$

$$-\Lambda \partial_\Lambda \left[\frac{1}{2} \text{str} \mathcal{A} \mathcal{A} \right] = \frac{1}{2}$$


$$\left[\begin{array}{c} \frac{1}{2} \text{str} \mathcal{A} \mathcal{A} \\ \frac{1}{2} \text{str} \Sigma_g \end{array} \right]$$


Symmetric Phase Diagrammatics Flow Equation

The Classical Term

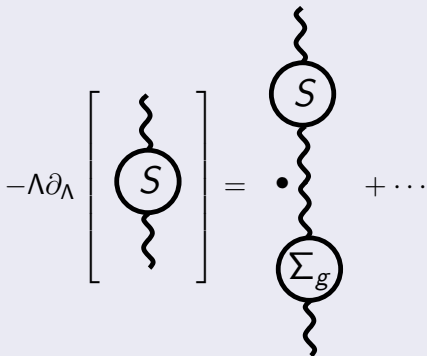
$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu}$$

$$-\Lambda \partial_\Lambda \left[\frac{1}{2} \text{str} \mathcal{A} \mathcal{A} \right] = \frac{1}{2} \left[\frac{\delta}{\delta \mathcal{A}} \right] \left[\frac{1}{2} \text{str} \mathcal{A} \mathcal{A} \right] + \frac{1}{2} \left[\frac{\delta}{\delta \mathcal{A}} \right] \left[\frac{1}{2} \text{str} \Sigma_g \mathcal{A} \mathcal{A} \right]$$

Symmetric Phase Diagrammatics Flow Equation

The Classical Term

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu}$$



Symmetric Phase Diagrammatics Flow Equation

The Quantum Term

$$-\Lambda\partial_\Lambda S = a_0[S, \Sigma_g] - \frac{1}{2} \frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} - \frac{1}{2} \frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}$$

Symmetric Phase Diagrammatics Flow Equation

The Quantum Term

$$-\Lambda\partial_\Lambda S = a_0[S, \Sigma_g] - \frac{1}{2} \frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} - \frac{1}{2} \frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}$$

- For Consistency take

- $\frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} = 0$
- $\frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} = 0$

Symmetric Phase Diagrammatics Flow Equation

The Quantum Term

$$-\Lambda\partial_\Lambda S = a_0[S, \Sigma_g] - \frac{1}{2} \frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{AA} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} - \frac{1}{2} \frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}$$

- For Consistency take
 - $\frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{AA} \} = 0$
 - $\frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} = 0$
- The **pairs of derivatives**

Symmetric Phase Diagrammatics Flow Equation

The Quantum Term

$$-\Lambda\partial_\Lambda S = a_0[S, \Sigma_g] - \frac{1}{2} \frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{AA} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} - \frac{1}{2} \frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{CC} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}$$

- For Consistency take

- $\frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{AA} \} = 0$

- $\frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{CC} \} = 0$

- The pairs of derivatives knock two fields out of Σ_g

Symmetric Phase Diagrammatics Flow Equation

The Quantum Term

$$-\Lambda\partial_\Lambda S = a_0[S, \Sigma_g] - \frac{1}{2} \frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{AA} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} - \frac{1}{2} \frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}$$

$$-\Lambda\partial_\Lambda \text{S} = \dots - \frac{1}{2} \left[\text{Diagram 1} + 2 \text{Diagram 2} - \text{Diagram 3} \right]$$

The diagrammatic equation shows the quantum term $-\Lambda\partial_\Lambda S$ represented by a circle with 'S' and two wavy external lines. This is equal to a series of terms in brackets, multiplied by $-\frac{1}{2}$. The terms are:

- Diagram 1: A circle with Σ_g and a dot, with two wavy external lines.
- Diagram 2: A circle with Σ_g and a dot, with one wavy external line and one wavy external line.
- Diagram 3: A circle with Σ_g and a dot, with two wavy external lines.

Symmetric Phase Diagrammatics Flow Equation

The Quantum Term

$$-\Lambda\partial_\Lambda S = a_0[S, \Sigma_g] - \frac{1}{2} \frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} - \frac{1}{2} \frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}$$

$$-\Lambda\partial_\Lambda \text{ (circle with } S \text{)} = \dots - \frac{1}{2} \left[\begin{array}{c} \text{(circle with } \Sigma_g \text{, red loop)} \\ \text{(circle with } \Sigma_g \text{, dot)} \\ \text{(circle with } \Sigma_g \text{, dot)} \end{array} \right]$$

The diagrammatic equation shows the quantum term as a sum of three diagrams. The first diagram is a circle with a wavy line on top and a wavy line on bottom, containing the letter 'S'. This is equal to a series of terms. The first term is a circle with a wavy line on top and a wavy line on bottom, containing the symbol Σ_g , with a red loop on top and a purple dot in the center. The second term is a circle with a wavy line on top and a wavy line on bottom, containing the symbol Σ_g , with a black dot in the center, multiplied by 2. The third term is a circle with a wavy line on top and a wavy line on bottom, containing the symbol Σ_g , with a black dot in the center, subtracted.

- Sum over kernels

Symmetric Phase Diagrammatics Flow Equation

The Quantum Term

$$-\Lambda\partial_\Lambda S = a_0[S, \Sigma_g] - \frac{1}{2} \frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{AA} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} - \frac{1}{2} \frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}$$

$$-\Lambda\partial_\Lambda \text{ (circle with } S \text{)} = \dots - \frac{1}{2} \left[\text{ (circle with } \Sigma_g \text{ and dot)} + 2 \text{ (circle with } \Sigma_g \text{ and red wavy line)} - \text{ (circle with } \Sigma_g \text{ and red wavy line)} \right]$$

- Vertices of the covariantised kernels

Broken Phase Diagrammatic Flow Equation

$$-\Lambda \partial_\Lambda \left[\text{S} \right]_{\{f\}} = \frac{1}{2} \left[\begin{array}{c} \text{\Sigma}_g \\ | \\ \text{S} \end{array} - \text{\Sigma}_g \right]_{\{f\}}$$

The diagrammatic equation shows the flow of the self-energy S under a broken phase. The left-hand side is the derivative of the self-energy S with respect to the scale Λ , multiplied by $-\Lambda$. The right-hand side is a sum of two diagrams: a self-energy S with a \Sigma_g loop attached to its top, and a self-energy \Sigma_g with a tadpole loop attached to its top. The diagrams are enclosed in large square brackets with $\{f\}$ on the right.

Broken Phase Diagrammatic Flow Equation

$$-\Lambda \partial_\Lambda \left[\text{S} \right] \{f\} = \frac{1}{2} \left[\begin{array}{c} \text{\Sigma}_g \\ | \\ \text{S} \end{array} - \text{\Sigma}_g \right] \{f\}$$

- The fields $\{f\}$
 - Any of $A^1, A^2, C^1, C^2, F, \bar{F}$

Broken Phase Diagrammatic Flow Equation

$$-\Lambda \partial_\Lambda \left[\text{S} \right] \{f\} = \frac{1}{2} \left[\begin{array}{c} \text{\Sigma}_g \\ | \\ \text{S} \end{array} \cdot - \text{\Sigma}_g \right] \{f\}$$

- The fields $\{f\}$
 - Any of $A^1, A^2, C^1, C^2, F, \bar{F}$
 - Distributed in all independent ways

Broken Phase Diagrammatic Flow Equation

$$-\Lambda \partial_\Lambda \left[\text{S} \right] \{f\} = \frac{1}{2} \left[\begin{array}{c} \text{\Sigma}_g \\ | \\ \text{S} \end{array} - \text{\Sigma}_g \right] \{f\}$$

- The fields $\{f\}$
 - Any of $A^1, A^2, C^1, C^2, F, \bar{F}$
 - Distributed in all independent ways
- Prescription for evaluating group theory

Broken Phase Diagrammatic Flow Equation

$$-\Lambda \partial_\Lambda \left[\text{S} \right] \{f\} = \frac{1}{2} \left[\begin{array}{c} \text{\Sigma}_g \\ | \\ \text{S} \end{array} \cdot - \text{\Sigma}_g \right] \{f\}$$

- The fields $\{f\}$
 - Any of $A^1, A^2, C^1, C^2, F, \bar{F}$
 - Distributed in all independent ways
- Prescription for evaluating group theory
- **Internal fields label the kernels**

Broken Phase Diagrammatic Flow Equation

$$-\Lambda \partial_\Lambda \left[\text{Diagram } S \right] \{f\} = \frac{1}{2} \left[\text{Diagram } \Sigma_g \text{ --- } S - \text{Diagram } \Sigma_g \text{ with dot} \right] \{f\}$$

- The fields $\{f\}$
 - Any of $A^1, A^2, C^1, C^2, F, \bar{F}$
 - Distributed in all independent ways
- Prescription for evaluating group theory
- Internal fields label the kernels
- ERG sufficiently general to allow $\dot{\Delta}^{A^1 A^1} \neq \dot{\Delta}^{A^2 A^2}$

Weak Coupling Limit

Expansions

Weak Coupling Limit

Expansions

- $$S = \sum_{i=0}^{\infty} (g^2)^{i-1} S_i = \frac{1}{g^2} S_0 + S_1 + \dots$$

Weak Coupling Limit

Expansions

- $S = \sum_{i=0}^{\infty} (g^2)^{i-1} S_i = \frac{1}{g^2} S_0 + S_1 + \dots$
 - S_0 : classical effective action

Weak Coupling Limit

Expansions

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 - $S_{i>0}$: *ith-loop corrections*

Weak Coupling Limit

Expansions

- $S = \sum_{i=0}^{\infty} (g^2)^{i-1} S_i = \frac{1}{g^2} S_0 + S_1 + \dots$
 - S_0 : classical effective action
 - $S_{i>0}$: i th-loop corrections
- $\hat{S} = \sum_{i=0}^{\infty} g^{2i} \hat{S}_i$

Weak Coupling Limit

Expansions

- $S = \sum_{i=0}^{\infty} (g^2)^{i-1} S_i = \frac{1}{g^2} S_0 + S_1 + \dots$
 - S_0 : classical effective action
 - $S_{i>0}$: i th-loop corrections
- $\hat{S} = \sum_{i=0}^{\infty} g^{2i} \hat{S}_i$
- Define $\alpha = g_2^2/g^2$

Weak Coupling Limit

Expansions

- $$S = \sum_{i=0}^{\infty} (g^2)^{i-1} S_i = \frac{1}{g^2} S_0 + S_1 + \dots$$

- S_0 : classical effective action
- $S_{i>0}$: i th-loop corrections

- $$\hat{S} = \sum_{i=0}^{\infty} g^{2i} \hat{S}_i$$

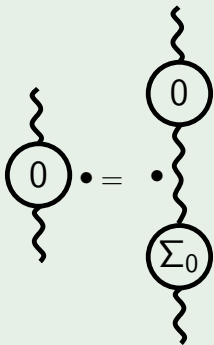
- Define $\alpha = g_2^2/g^2$

$$\beta \equiv \Lambda \partial_{\Lambda} g = \sum_{i=1}^{\infty} g^{2i+1} \beta_i(\alpha)$$

$$\gamma \equiv \Lambda \partial_{\Lambda} \alpha = \sum_{i=1}^{\infty} g^{2i} \gamma_i(\alpha).$$

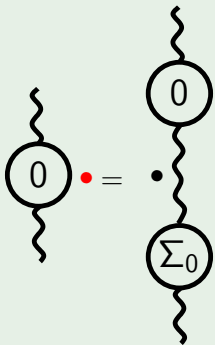
The Classical Flow Equation

Example (Classical $A^1 A^1$ Vertex)



The Classical Flow Equation

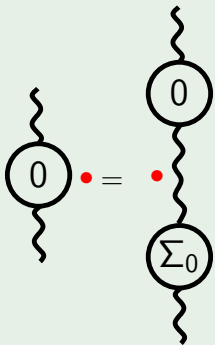
Example (Classical $A^1 A^1$ Vertex)



$$\bullet \bullet = -\Lambda \partial_\Lambda |_\alpha$$

The Classical Flow Equation

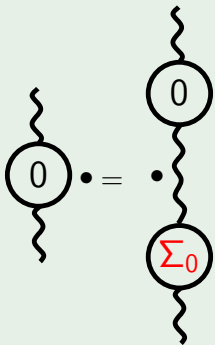
Example (Classical $A^1 A^1$ Vertex)



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The Classical Flow Equation

Example (Classical $A^1 A^1$ Vertex)

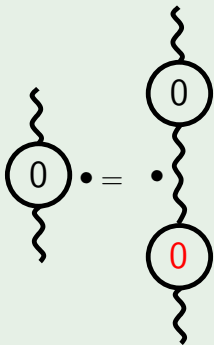


- $\bullet = -\Lambda \partial_\Lambda |_\alpha$

- $\Sigma_0 = S_0 - 2\hat{S}_0$

The Classical Flow Equation

Example (Classical $A^1 A^1$ Vertex)

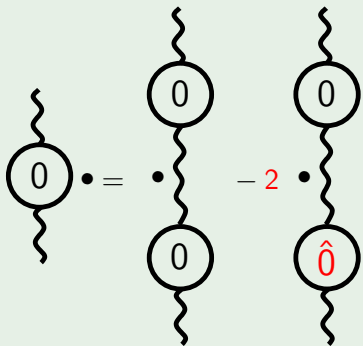


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The Classical Flow Equation

Example (Classical $A^1 A^1$ Vertex)

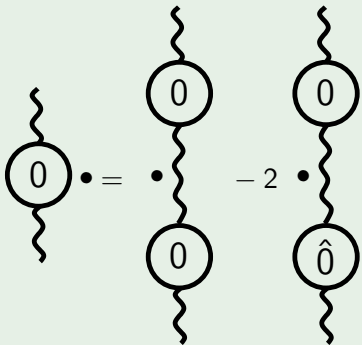


- $\bullet = -\Lambda \partial_\Lambda |_\alpha$

- $\Sigma_0 = S_0 - 2\hat{S}_0$

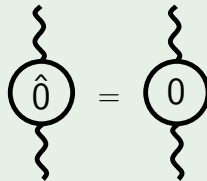
The Classical Flow Equation

Example (Classical $A^1 A^1$ Vertex)



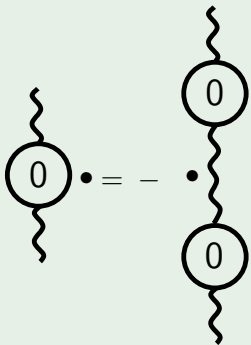
$$\bullet \bullet = -\Lambda \partial_\Lambda |_\alpha$$

Choose



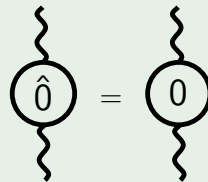
The Classical Flow Equation

Example (Classical $A^1 A^1$ Vertex)



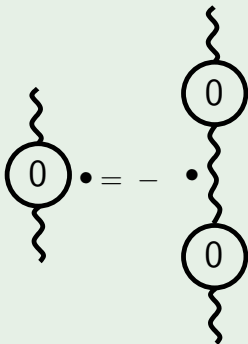
$$\bullet \bullet = -\Lambda \partial_\Lambda |_\alpha$$

Choose



The Classical Flow Equation

Example (Classical $A^1 A^1$ Vertex)

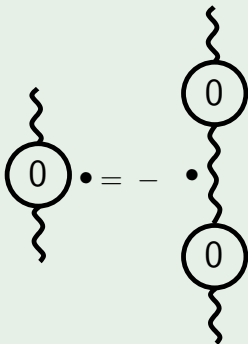


- Gauge Invariance:

$$\bullet = -\Lambda \partial_\Lambda |_\alpha$$

The Classical Flow Equation

Example (Classical $A^1 A^1$ Vertex)



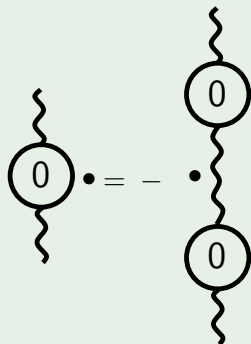
- Gauge Invariance: $p_\mu S_{0\mu\nu}^{11}(p) = 0$

- $\bullet = -\Lambda \partial_\Lambda |_\alpha$

- 1 stands for A^1

The Classical Flow Equation

Example (Classical $A^1 A^1$ Vertex)



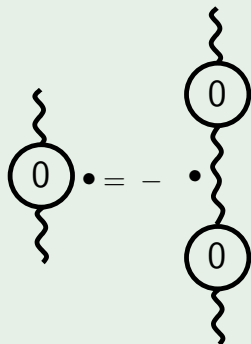
- Gauge Invariance: $p_\mu S_{0\mu\nu}^{11}(p) = 0$
- Lorentz Invariance and Dimensions:

- $\bullet = -\Lambda \partial_\Lambda |_\alpha$

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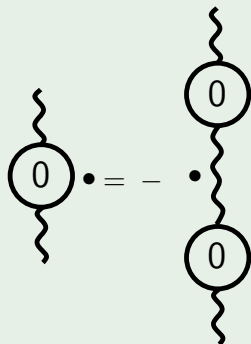
- Gauge Invariance: $p_\mu S_{0\mu\nu}^{11}(p) = 0$
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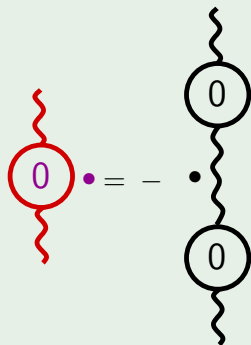
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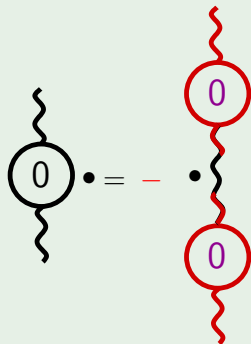


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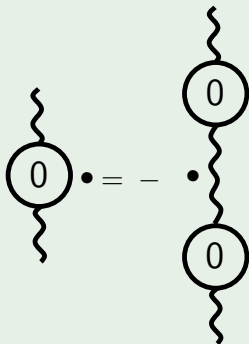
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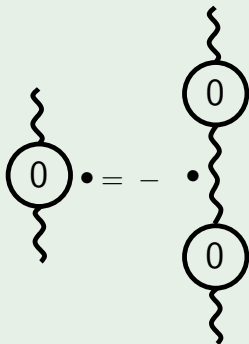
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The Classical Flow Equation

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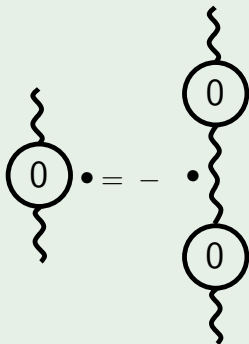
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The Classical Flow Equation

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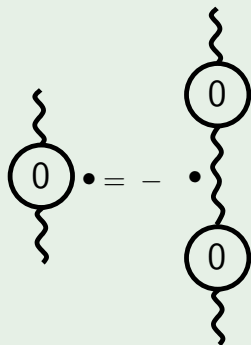
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$\Delta^{11}(p)$ is an Effective Propagator

$$S_{0\mu\nu}^{11}(p)\Delta^{11}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

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 - Relationship to $S_{0\mu\nu}^{11}(p)$ down to choice of \hat{S}

The Effective Propagator Relation

Diagrammatics

- The effective propagator relation works in all sectors

$$\text{---} \bigcirc 0 \text{---} \text{---} \left. \begin{array}{l} | \\ | \end{array} \right\} = \text{---} \left. \begin{array}{l} | \\ | \end{array} \right\} - \text{---} \blacktriangleright \left. \begin{array}{l} | \\ | \end{array} \right\}$$

The Effective Propagator Relation

Diagrammatics

- The effective propagator relation works in all sectors

$$\text{Red line} \text{---} \text{Red circle with } 0 \text{---} \text{Black line} = \text{Black line} \text{---} \text{Vertex} - \text{Black line} \text{---} \text{Vertex with triangle}$$

- Two-point, classical vertex

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- ERG sufficiently general for $\Delta^{11}(p) \neq \Delta^{22}(p)$**
 - $S_{0\mu\nu}^{11}(p) \neq S_{0\mu\nu}^{22}(p)$

Universal Computation

Renormalisation Condition for g :

- $$S[\mathcal{A} = A^1, \mathcal{C} = \sigma] = \frac{1}{2g^2} \text{str} \int d^D x (F_{\mu\nu}^1)^2 + \dots$$

Universal Computation

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 - The $\mathcal{O}(p^2)$ part of $S_{0\mu\nu}^{11}(p)$
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 - The A^1 gauge remainders
- **All non-universal contributions cancel, diagrammatically!**

β_{n+1} Diagrammatics

$$-8\beta_{n+1}\square_{\mu\nu}(p) + \dots = \left[\begin{array}{c} \text{Diagram with a dot on a top loop} \\ \Sigma_n \end{array} \right]^{11}$$

β_{n+1} Diagrammatics

$$-8\beta_{n+1}\square_{\mu\nu}(p) + \dots = \left[-2 \left(\text{Diagram 1} \right) + \left(\text{Diagram 2} \right) \right]^{11}$$

The equation shows the expansion of a term in the beta function. The first term is $-8\beta_{n+1}\square_{\mu\nu}(p)$, followed by an ellipsis. This is equal to a bracketed sum of two diagrams, raised to the power of 11. The first diagram is a circle with a dot at the top and the label \hat{n} inside. The second diagram is a circle with a dot at the top and the label n inside. Both diagrams have two external lines forming a loop at the top.

β_{n+1} Diagrammatics

$$-8\beta_{n+1}\square_{\mu\nu}(p) + \dots = \left[-2 \begin{array}{c} \text{Diagram 1} \\ \hat{n} \end{array} + \begin{array}{c} \text{Diagram 2} \\ n \end{array} + \begin{array}{c} \text{Diagram 3} \\ n \end{array} \right]^{11}$$

The equation shows the expansion of a term in the beta function. The left side is $-8\beta_{n+1}\square_{\mu\nu}(p) + \dots$. The right side is a sum of three diagrams enclosed in large square brackets with a superscript 11. The first diagram is a circle with a dot inside, labeled \hat{n} . The second diagram is a circle with an open circle inside, labeled n . The third diagram is a circle with a dot inside a smaller circle, labeled n .

β_{n+1} Diagrammatics

$$-8\beta_{n+1}\square_{\mu\nu}(p) + \dots = \left[-2 \begin{array}{c} \text{loop with } \hat{n} \text{ and } \bullet \\ \text{loop with } n \text{ and } \circ \\ \text{loop with } n \text{ and } \odot \end{array} \right]^{11}$$

- Must be decorated

β_{n+1} Diagrammatics

$$-8\beta_{n+1}\square_{\mu\nu}(p) + \dots = \left[-2 \begin{array}{c} \text{circle with top loop and dot} \\ \hat{n} \end{array} + \begin{array}{c} \text{circle with top loop and open circle} \\ n \end{array} + \begin{array}{c} \text{circle with top loop and red dot} \\ n \end{array} \right]^{11}$$

- Must be decorated
- Cannot be decorated $\Rightarrow -\Lambda\partial_\Lambda|_\alpha\Delta$

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$$\left[\begin{array}{c} \text{loop with } \odot \\ n \end{array} \right]^{11} =$$

β_{n+1} Diagrammatics

$$\begin{aligned}
 -8\beta_{n+1}\square_{\mu\nu}(p) + \dots &= \left[-2 \begin{array}{c} \text{circle with top dot} \\ \text{circle with } \hat{n} \end{array} + \begin{array}{c} \text{circle with top hole} \\ \text{circle with } n \end{array} + \begin{array}{c} \text{circle with top hole and dot} \\ \text{circle with } n \end{array} \right]^{11} \\
 \left[\begin{array}{c} \text{circle with top hole and dot} \\ \text{circle with } n \end{array} \right]^{11} &= \left[\left[\begin{array}{c} \text{circle with top dot} \\ \text{circle with } n \end{array} \right] - \begin{array}{c} \text{circle with top dot} \\ \text{circle with } n \end{array} \right]^{11}
 \end{aligned}$$

β_{n+1} Diagrammatics

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 \left[\begin{array}{c} \text{circle with top dot} \\ \text{circle with } n \end{array} \right]^{11} &= \left[\begin{array}{c} \left[\begin{array}{c} \text{circle with top dot} \\ \text{circle with } n \end{array} \right] \cdot \\ + 2 \begin{array}{c} \text{circle with } 0 \\ \text{circle with } \hat{n} \end{array} \\ + \dots \end{array} \right]^{11}
 \end{aligned}$$

β_{n+1} Diagrammatics

$$\begin{aligned}
 -8\beta_{n+1}\square_{\mu\nu}(p) + \dots &= \left[-2 \begin{array}{c} \text{loop with } \hat{n} \text{ and } \bullet \\ \text{loop with } n \end{array} + \begin{array}{c} \text{loop with } n \text{ and } \circ \\ \text{loop with } n \end{array} + \begin{array}{c} \text{loop with } n \text{ and } \odot \\ \text{loop with } n \end{array} \right]^{11} \\
 \left[\begin{array}{c} \text{loop with } n \text{ and } \odot \\ \text{loop with } n \end{array} \right]^{11} &= \left[\begin{array}{c} \left[\begin{array}{c} \text{loop with } n \text{ and } \bullet \\ \text{loop with } n \end{array} \right]^{\bullet} \\ \dots \end{array} \right]^{11} + 2 \begin{array}{c} \text{loop with } \hat{n} \text{ and } \bullet \\ \text{loop with } \hat{n} \end{array} - 2 \begin{array}{c} \text{loop with } \hat{n} \text{ and } \bullet \\ \text{loop with } \hat{n} \end{array}
 \end{aligned}$$

β_{n+1} Diagrammatics

$$\begin{aligned}
 -8\beta_{n+1}\square_{\mu\nu}(p) + \dots &= \left[-2 \begin{array}{c} \text{diagram with } \hat{n} \text{ and a dot, crossed out} \\ \hat{n} \end{array} + \begin{array}{c} \text{diagram with } n \text{ and a dot} \\ n \end{array} + \begin{array}{c} \text{diagram with } n \text{ and a dot} \\ n \end{array} \right]^{11} \\
 \left[\begin{array}{c} \text{diagram with } n \text{ and a dot} \\ n \end{array} \right]^{11} &= \left[\begin{array}{c} \left[\begin{array}{c} \text{diagram with } n \text{ and a dot} \\ n \end{array} \right]^{\bullet} \\ + 2 \begin{array}{c} \text{diagram with } \hat{n} \text{ and a dot, crossed out} \\ \hat{n} \end{array} \\ - 2 \begin{array}{c} \text{diagram with } \hat{n} \text{ and a dot, with arrow} \\ \hat{n} \end{array} \\ + \dots \end{array} \right]^{11}
 \end{aligned}$$

Iterating the Diagrammatic Procedure

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Cancellations to all orders

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- All explicit instances of \hat{S}

Iterating the Diagrammatic Procedure

The Diagrammatic Procedure

- Isolate and process any diagrams which can be manipulated using the flow equation
- Employ the effective propagator relation
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Cancellations to all orders

- All explicit instances of \hat{S}
- **All explicit details of the covariantisation**

A Universal β Function?

What does β_n depend on?

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Cancellation of Remaining non-Universality

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- Trivial, at one loop
- Iterative, diagrammatic procedure at two loops

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- **Iterative, diagrammatic procedure at n loops??**

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- gauge remainders

Cancellation of Remaining non-Universality

- Trivial, at one loop
- Iterative, diagrammatic procedure at two loops
- Iterative, diagrammatic procedure at n loops??
- **Strong suggestion that all β_n can be arranged to depend only on the universal details of this ERG**

Future Directions

Perturbative

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- Complete analysis of universality of β_n

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- **Truncations**

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- **Strong coupling expansion**

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- **A more direct framework**

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- Quarks

Evaluating the Λ -Derivative Terms

Strategy

Return

$$\beta_1 \square_{\mu\nu}(p) = \int_k [\mathcal{D}_1(k, p)]_{p^2}^\bullet$$

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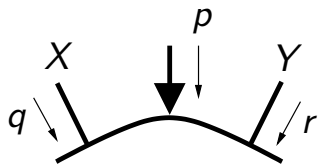
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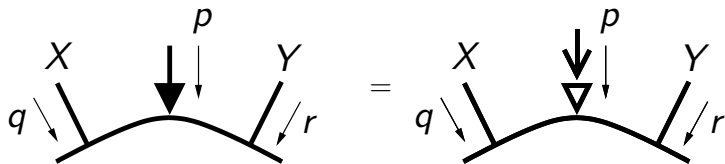
Diagrammatics for Gauge Remainders

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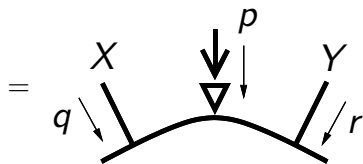
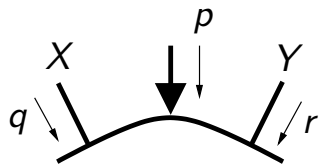
Diagrammatics for Gauge Remainders

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$$\triangleright = \begin{cases} p_\rho & A^i \\ p_R = (p_\rho, 2) & F \bar{F} \\ \text{null} & C^i \end{cases}$$

Diagrammatics for Gauge Remainders

[Return](#)

