New SUSY String Compactifications from Twisted Tori

Michael Schulz
November 2002

Work performed in collaboration with:

S. Kachru, S. Trivedi hep-th/0201028
S. Kachru, X. Liu, S. Trivedi hep-th/0205108
S. Kachru, P. Tripathy, S. Trivedi (in progress)

Plan:

Part I

IIB on $T^6/\mathbb{Z}_2$ with 3-form flux
Moduli stabilization
SUSY vacua
(hep-th/0201028)

Part II

T-dual description
Twisted Tori
The IIA Mirror
SUSY
(Work in progress)

Why study IIB backgrounds with (3-form) flux?

New $\mathcal{N} = 1$ SUSY vacua in 4d with striking advantages over pert het constructions:

Perturbative Heterotic Theory

- Dilaton-axion, cpx str and Kähler moduli un-fixed perturbatively. Need to invent mechanism for masses, or face mathematically intractable effects from WS instantons and wrapped NS5 Branes.
- Only description of non-pert phenomena (like χSB, $\langle\lambda\lambda\rangle$) is effective field theory.

IIB Flux Backgrounds

- Lift dilaton-axion, all cpx str and some Kähler moduli. (Mirror flux $\rightarrow$ remaining Kähler?)
- Holography—geometrical dual descriptions of nonperturbative field theory effects.

KS: $R(S^3) \leftrightarrow \langle\lambda\lambda\rangle$

GKP: $W_{\text{pert}} \rightarrow e^{-K/K'}$ hierarchy.

(Related to Randall-Sundrum and its stringy embedding by H. Verlinde).

Why study IIB on the T^6/Z_2 orientifold?

T^6/Z_2 is:

- the simplest 4-dimensional IIB background admiting flux
- highly computable.

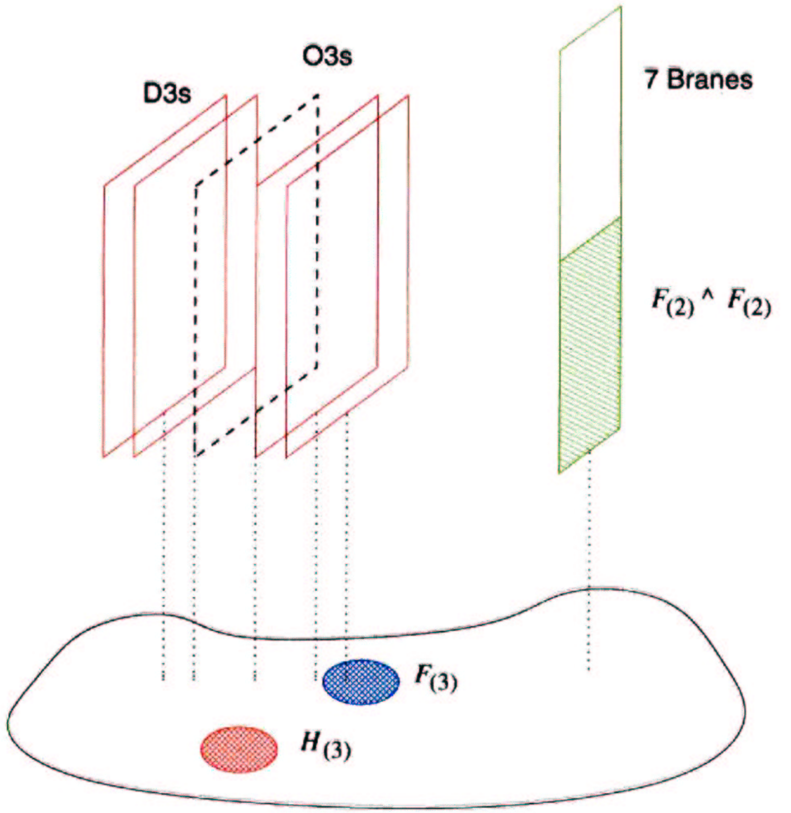
Previous results were:

- for CY orientifolds or F-theory
- soluble only in an approximate analysis near conifold points.

Here:

- can compute superpotential and D-flatness conditions explicitly and globally on the moduli space.

Ingredients of IIB Backgrounds with Flux



The T^6/Z_2 Orientifold

$$T^6: \quad x^i \cong x^i + 1,\ y^i \cong y^i + 1, \quad i = 1, 2, 3,$$

modulo

$$Z_2: \quad \Omega R(-1)^{F_L},$$

where

$$\Omega: \quad X(\sigma, t) \to X(-\sigma, t),$$
$$R: \quad (x^i, y^i) \to (-x^i, -y^i),$$
$$(-1)^{F_L}: \quad -1 \text{ on L Ramond states.}$$

Points to note:

- $T^6/Z_2 \leftrightarrow$ Type I, via T-duality on T^6,
- 16 D9s in Type I $\leftrightarrow$ 16 D3s in T^6/Z_2,
- $\exists$ O3 planes at 2^6 fixed points of the Z_2,
- low-energy theory is $\mathcal{N} = 4$ $SO(32)$ SYM coupled to $\mathcal{N} = 4$ supergravity.

Tadpole Cancellation

The EOM/Bianchi Identity for $F_{(5)}$ implies a tadpole cancellation condition:

$$N_{\mathrm{D3}} + \frac{1}{2(2\pi)^4\alpha'^2}\int_{T^6} H_{(3)} \wedge F_{(3)} = \frac{1}{4}N_{\mathrm{O3}}.$$

($+ \frac{1}{24}\chi$, for F-theory)

The standard vacuum has:

$$N_{\mathrm{D3}} = 16,$$

$$N_{\mathrm{O3}} = 64,$$

$$H = F = 0.$$

But, there are other possibilities:

we can trade off D3s for quantized H and F to make new vacua.

Massless Fields of IIB on T^6/Z_2

The massless bosonic fields of 10d IIB sugra are:

$$g_{MN}, B_{MN}, \phi, \qquad C_{(0)}, C_{(2)}, C_{(4)}.$$

These fields tranform as:

	Ω	$(-1)^{F_L}$		Ω	$(-1)^{F_L}$
g_{MN}	$+$	$+$	$C_{(0)}$	$-$	$-$
B_{MN}	$-$	$+$	$C_{(2)}$	$+$	$-$
ϕ	$+$	$+$	$C_{(4)}$	$-$	$-$

Bosonic fields that survive the orientifold projection are:

graviton:	$g_{\mu\nu}$	(1)
scalars:	$g_{ab}, \phi, C_{(0)}, C_{abcd}$	(21+1+1+15=38)
vectors:	$B_{a\mu}, C_{a\mu}$	(6+6=12)

Also, H_{abc} and F_{abc} are:

- discrete: H, F in $(2\pi)^2\alpha' H^3(T^6, Z)$
- non-dynamical: B_{ab}, C_{ab} projected out.

4D Multiplets

These fields are organized into $\mathcal{N} = 4$ sugra multiplets as:

1 gravity mult	$g_{\mu\nu}$, 6 A_μ, $C_{(0)} + \frac{i}{g_s}$, ferms,
6 vector mults	A_μ, 6 scalars, ferms.

Each D3 gives an additional vector mult.

The Fluxes:

- generate a potential for scalars
- correspond to charges under $U(1)^{12}$.

$\Rightarrow$
- SUSY higgs mechanism,
- $\mathcal{N} = 4 \to \mathcal{N} < 4$.

Ferrara et al.

Scalar Potential

$$G_{(3)} = F - \varphi H, \quad \varphi = C_{(0)} + i/g_s.$$

$$\mathcal{L}_{\text{10d IIB}} \supset \mathcal{L}_1 = -\frac{1}{4\kappa_{10}{}^2} \int d^6y \frac{G \wedge \star_6 \bar{G}}{\text{Im}\tau}.$$

Define:

$$G = G^{\text{ISD}} + G^{\text{IASD}}, \quad \begin{array}{l} \star_6 G^{\text{ISD}} = +iG^{\text{ISD}} \\ \star_6 G^{\text{IASD}} = -iG^{\text{IASD}} \end{array}.$$

Then

$$-\mathcal{L}_1 \propto \underbrace{i \int G \wedge \bar{G}}_{\text{topological}} + \underbrace{2 \int G^{\text{IASD}} \wedge \star_6 \overline{G^{\text{IASD}}}}_{\mathcal{V}_{\text{Scalar}}}.$$

So,

$$\mathcal{V}_{\text{scalar}} = 0 \Leftrightarrow G \text{ ISD}.$$

Here:

- G depends on φ,
- G^{ISD} vs. G^{IASD} depends on metric,
- $\mathcal{V} \geq 0$ from no-scale structure:

$$\mathcal{V} \propto \sum_{\text{fields } f} |D_f W|^2 - 3|W|^2 = \sum_{f \neq \text{Kähler}} |D_f W|^2,$$

- scale of $\mathcal{V}$ is $\alpha'/R^3 \ll m_{KK}, m_s$ (R large).

Supersymmetry

To preserve $\mathcal{N} \geq 1$ SUSY, need

$$\delta\lambda^{\alpha} = 0, \quad \delta\psi^{\alpha}{}_{\mu} = 0 \text{ (for at least one } \psi).$$

This implies (Graña-Polchinski)

- $G_{(3)}$ of type (2,1)
- $G_{(3)}$ primitive ($J \wedge G = 0$).

(These conditions $\Rightarrow$ ISD$\Rightarrow$ $\mathcal{V}_{\text{Scalar}} = 0$).

CY case: primitivity trivial, (2,1) imposed by

$$W = \int G \wedge \Omega. \qquad \text{(Beckers, GVW, GKP)}$$

For torus, $\exists$ non-primitive 3-forms:

$$J \wedge dz^i, \quad J \wedge d\overline{z}^i, \qquad i = 1, 2, 3,$$

so, have D-terms as well.

Can study the structure of these D-terms, but we'll just impose primitivity by hand.

An Example

Fluxes:

$$\frac{1}{(2\pi)^2\alpha'}F = 4dx^1 \wedge dx^2 \wedge dy^3 + 4dy^1 \wedge dy^2 \wedge dy^3,$$
$$\frac{1}{(2\pi)^2\alpha'}H = 4dx^1 \wedge dx^2 \wedge dx^3 + 4dy^1 \wedge dy^2 \wedge dx^3.$$

Complex Structure:

$$dz^i = dx^i + \tau^i{}_j dy^j, \qquad \tau^i{}_j = \text{"period matrix,"}$$
$$\Omega = dz^1 \wedge dz^2 \wedge dz^3.$$

Kähler form:

$$J = i\rho_{i\bar{j}} dz^i \wedge dz^{\bar{j}}.$$

Superpotential:

$G = F - \varphi H$

$$\frac{1}{(2\pi)^2\alpha'}W = \frac{1}{(2\pi)^2\alpha'}\int G \wedge \Omega$$
$$= -4\Big(\big[(\text{cof}\,\tau)_3{}^3 + 1\big] + \varphi\big[\det\tau + \tau^3{}_3\big]\Big).$$

EOM:

$$\begin{array}{lcr} D_{\rho_{i\bar{j}}} W = 0 & & W = 0 \\ D_{\varphi} W = 0 & \xrightarrow{\text{No-Scale}} & \partial_{\varphi} W = 0 \\ D_{\tau^i{}_j} W = 0 & & \partial_{\tau^i{}_j} W = 0 \end{array}$$

Find $\tau^i{}_j$ diagonal, and

$$\varphi \tau^3{}_3 + 1 = 0$$
$$\tau^1{}_1 \tau^2{}_2 + 1 = 0.$$

So, $\mathcal{M}_{\text{cpx}}$ param'd by φ, $\tau^1{}_1$. Plugging back,

$$G = iA(\varphi, \tau^1{}_1)(dz^1 \wedge d\bar{z}^2 \wedge dz^3 + d\bar{z}^1 \wedge dz^2 \wedge dz^3).$$

This is:

- $(2,1)$ as required,
- primitive, for

  $J \wedge G = 0$,

  $J = i\rho_{i\bar{j}}\, dz^i \wedge dz^{\bar{j}}$

  $\rho_{1\bar{3}} = \rho_{2\bar{3}} = \rho_{1\bar{2}} + \rho_{2\bar{1}} = 0$ (6d space),
- but non-generic:

  $\exists$ an inequivalent complex str. s.t.

  G still $(2,1)$ and primitive: $z^{1,2} \to \bar{z}^{1,2}$

  $\Rightarrow \mathcal{N} = 2$ susy.

(3 ineq. cpx str's $\Rightarrow \mathcal{N} = 3$ (Frey-Polchinski)).

Generalities

Complex Structure Moduli:

$$W = \partial_{\varphi} W = \partial_{\tau^i{}_j} W = 0 \quad \text{for SUSY}$$

(11 cpx eqs in 10 unknowns φ, $\tau^i{}_j$).

So, SUSY soln exists only when eqs are redundant (non-generic flux).

- 1 redundancy $\Rightarrow$ unique soln, $\mathcal{N} = 1$,
- more $\Rightarrow$ unfixed cpx moduli, $\mathcal{N} > 1$.

Kähler Moduli:

$$\begin{array}{ll} J \in H^{1,1} & h^{1,1} = 9, \\ J \wedge G \in H^{3,2} & 2\, h^{3,2} = 6, \quad 9 - 6 = 3. \end{array}$$

So, generically $J \wedge G = 0$ for a 3d space of Kähler moduli.

Counting holds up in $\mathcal{N} = 1$ case, but fails for $\mathcal{N} > 1$. ($J \wedge G$ doesn't fill out all of $H^{3,2}$).

Summary: Generic SUSY case is $\mathcal{N} = 1$, all cpx moduli lifted, 6 of 9 Kähler moduli lifted.

The Mirror and T-dual Theories

Why study the mirror ?

Premotivation :

Generically, fluxes lift all cpx str, but not all Kähler moduli.

Mirror symmetry : Kähler $\leftrightarrow$ cpx str.

So, try to lift Kähler moduli with dual flux.

SYZ : Mirror symmetry = T-duality on T^3 fibers.
T-duality : NS-flux $\leftrightarrow$ geometry.
$\Rightarrow$ New geometrical terms in W_{IIB} ?

No. Geometrical "twists" in $T^6/\mathbb{Z}_2$ are incompatible with the IIB orientifold projection. No missing terms in superpotential.

Postmotivation :

New SUSY compactifications on non-Kähler, non-Calabi-Yau geometries.

(See also Dasgupta-Rajesh-Sethi
K. Becker - Dasgupta).

The Twisted T^3

IIB Flat T^3, $H^{NS} \neq 0$ $\xrightarrow{\text{T-duality}}$ IIA Twisted T^3, $H^{NS} = 0$

More explicitly,

IIB: Consider a T^3 in the xyz directions with $H^{NS} = N\, dx \wedge dy \wedge dz$,

$$ds^2 = dx^2 + dy^2 + dz^2, \quad B^{NS} = N x\, dy \wedge dz.$$

Now T-dualize in the z-direction.

Buscher rules $\Rightarrow$

IIA: Twisted T^3

- $ds^2 = dx^2 + dy^2 + (dz + N x\, dy)^2, \quad B^{NS} = 0, \quad H^{NS} = dB^{NS} = 0.$
- $\eta^{(1)} = dx,\ \eta^{(2)} = dy,\ \eta^{(3)} = dz + N x\, dy$ are globally well-defined one-forms.
- So, $y \cong y+1$, $z \cong z+1$ at fixed x, but
$\eta^{(3)} = dz + Nx\,dy = d(z - Ny) + N(x+1)\,dy$

$$\Rightarrow (x, y, z) \cong (x+1, y, z - Ny).$$

The Dual of NS Flux is Spin-Connection (or KK Magnetic Flux)

$ds^2 = dx^2 + dy^2 + (dz + N x\, dy)^2$ is

① a T^2 bundle over S^1
("dw" $= dz + \tau dy$, $\tau(x) = Nx + i$).

② an S^1 ($U(1)$ principal) bundle over T^2:

$ds^2 = \delta_{mn} dx^m dx^n + (dz + A^{(z)}_m dx^m)^2$, $x^m = x, y$,

$A^{(z)} = U(1)$ KK gauge field for z-translations,
$F^{(z)} = dA^{(z)} = N dx \wedge dy$.

From spin-connection $\omega_{(1)}{}^a{}_b$, can form

$\omega_{(3)} = \omega_{(1)ab} \wedge \eta^{(a)} \wedge \eta^{(b)}$.

Find that

$$\underset{(\mathrm{IIB})}{H^{NS}_{(3)} = N dx \wedge dy \wedge dz} \xrightarrow{\text{T-duality}} \underset{(\mathrm{IIA})}{\omega_{(3)} = N dx \wedge dy \wedge dz}.$$

General rule:

$$\omega_{(3)} = \sum_{\text{isometries } k} F^{(k)}_{(2)} \wedge k + \omega_{(3)\,\text{background geometry}}.$$

I.e., NS (winding) flux $\rightarrow \omega_{(3)}$ (KK flux).

From T^3 to SUSY Vacua

Start with IIB on $T^6/\mathbb{Z}_2$
+ $\mathcal{N} \geq 1$ SUSY flux.

Choose gauge with B^{NS} indep of say x^9.

Buscher rules $\Rightarrow$

IIA: $ds^2 = g_{ab}\, \eta^{(a)} \eta^{(b)}$,
$\eta^{(a)} = dx^a + \gamma^a_{bc} x^b dx^c$, $x^a \in \{x^i, y^i\}$.
($\gamma^a_{bc} \neq 0$ only for $a = 9$, $b, c \neq 9$).

Superpotential:

H^{NS}_{ij9} (IIB) $\rightarrow \omega_{(3)\,ij9}$ (IIA)

So, $\int H \wedge \Omega \subset W_{\mathrm{IIB}} \rightarrow \int (H - \omega_{(3)}) \wedge \Omega \subset W_{\mathrm{IIA}}$.

RR-fluxes? ~~[illegible]~~ We'll get there.

N.B. Can't introduce twist in IIB since dx^a and $x^b dx^c$ have opposite parity.

IIB		IIA
$\mathbb{Z}_2 : \Omega (-1)^{F_L} R_6$	$\rightarrow$	$\mathbb{Z}_2 : \Omega R_5$
O3	$\rightarrow$	O4 wrapped on x^9.

Twisted Tori as Scherk-Schwarz Compactifications

Scherk-Schwarz refresher:

In standard compactifications,

$$\phi^\alpha(x^\mu, x^i) = \tilde{\phi}^\alpha(x^\mu).$$

Scherk and Schwarz generalized this to

$$\phi^\alpha(x^\mu, x^i) = U^\alpha{}_\beta(x^i)\tilde{\phi}^\beta(x^\mu).$$

Here, $U^\alpha{}_\beta(x^i)$ = map from the internal manifold to a group G of appropriate dim.

$U^\alpha{}_\beta(x^i)$ single-valued only up to an element of the "U-duality" group.

Simplest case: Compactify on S^1. $G = U(1)$.
Monodromy = axion shift.
(fluxes...).

If only metric moduli are involved,

$$ds^2 = \sum_i \eta^{(i)} \otimes \eta^{(i)}, \quad \text{with}$$

$$d\eta^{(i)} = -\tfrac{1}{2} \underline{\underline{f^i_{jk}}}\, \eta^{(j)} \wedge \eta^{(k)}.$$

The f^i_{jk} are <u>str. consts.</u> of the SS group G.

Scherk-Schwarz G for Twisted T^3

Here, $ds^2 = \sum_i \eta^{(i)} \otimes \eta^{(i)}$;

$$\eta^{(1)} = dx$$
$$\eta^{(2)} = dy$$
$$\eta^{(3)} = dz + Nx\,dy.$$

So,

$$\begin{aligned} d\eta^{(3)} &= N\eta^{(1)}\wedge\eta^{(2)} \\ d\eta^{(1)} &= 0 \\ d\eta^{(2)} &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} [T_1, T_2] &= -NT_3 \\ [T_1, T_3] &= 0 \\ [T_2, T_3] &= 0 \end{aligned}.$$

Corresponding Scherk-Schwarz group is the 3D Heisenberg group:

$$g(x,y,z) = \begin{pmatrix} 1 & y & -\frac{1}{N}z \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{w. matrix mult.}$$

The twisted T^3 is just $G(\mathbb{R})/G(\mathbb{Z})$!

This gives the same identifications deduced earlier.

The $\eta^{(i)}$ in ds^2 are the rt-inv one-forms of G.

More math facts:

Isometry group is commutant of $G(\mathbb{Z})$ in $G(\mathbb{R})$.
$\Rightarrow$ $U(1)$ of z-translations.

$\pi_1(\text{twisted } T^3) = G(\mathbb{Z})$, $\quad H^1(\mathbb{Z}) = \mathbb{Z}^2 \times \mathbb{Z}_N$

$\Rightarrow$ two directions w. conserved winding (+ one conserved mod N).

Scherk - Schwarz for Twisted T^3 (continued)

Can also get math facts from physical arguments:

Before T-duality:

- T^3 with three winding directions, $H_{123} = N$.
- No isometry. Isometry in the v direction is broken by WS instantons with

$$\int_\Sigma v^i H_{ijk}\, dx^j \wedge dx^k \neq 0$$

(but, momentum still conserved mod N).

T-duality: isometry $\leftrightarrow$ winding.

After T-duality:

- Twisted T^3 with two winding directions, $H_{123} = 0$.
- One isometry.

(+ one direction w. conserved winding mod N).

Since $H_{123} = 0$, no additional breaking of isometry.

Multiple T-dualities

Can iterate and perform several T-dualities, but there is a caveat.

If $H_{ijk} \neq 0$ for some ijk, then T-dualizing in two of the ijk directions $\Rightarrow$ non-geometrical compactification. (SS monodromies mix metric moduli with other moduli).

Hellerman - McGreevy - Williams
Dabholkar - Hull

T^3: $ds^2 = dx^2 + dy^2 + dz^2$; $B = Nx\, dy \wedge dz$

$T^2_{\{yz\}}$ fibration over $S^1_{\{x\}}$ with (complexified) Kähler modulus $\rho = B + iJ$, cpx str τ on fiber. ($O(2,2;\mathbb{Z}) \sim SL(2)_\rho \times SL(2)_\tau$).

SS monodromy is in $U(1)_{\rho\text{-axion}} \subset SL(2)_\rho$.

T-duality in z-direction $\rightarrow$

$$ds^2 = dx^2 + dy^2 + (dz + Nx\,dy)^2;\quad B = 0, \quad (\tau = Nx + i).$$

SS monodromy is in $U(1)_{\tau\text{-axion}} \subset SL(2)_\tau$.

Additional T-duality in y-direction $\rightarrow$

$$ds^2 = dx^2 + \frac{1}{1+N^2x^2}(dy^2 + dz^2),\quad B = \frac{Nx}{1+N^2x^2}\, dy \wedge dz.$$

SS monodromy is in $SL(2)_\rho$ but not in $U(1)_{\rho\text{-axion}}$. ($\rho = -1/\rho_{\text{original } T^3}$).

We'll stick to geometrical compactifications.

An Honest $\mathcal{N}=2$ Example

Earlier, we discussed the $\mathcal{N}=2$ vacuum with

$$\left[\tfrac{1}{(2\pi)^2\alpha'}\right] F_{(3)} = 2\,dx^1\wedge dx^2\wedge dy^3 + 2\,dy^1\wedge dy^2\wedge dy^3$$

$$\left[\tfrac{1}{(2\pi)^2\alpha'}\right] H_{(3)} = 2\,dx^1\wedge dx^2\wedge dx^3 + 2\,dy^1\wedge dy^2\wedge dy^3$$

for IIB on $T^6/\mathbb{Z}_2$ with 64 O3-planes and no D3-branes.

We will now consider the dual theories related by one, two, and three T-dualities.

(Third case is of interest as a simple version of SYZ: CY Mirror Symmetry = T-duality of T^3 fibers).

Validity of sugra approximation:

Found $\phi\,\tau_3 = -1$, $\tau_1\tau_2 = -1$ (+ some Kähler moduli $\rho_{i\bar{j}}$).

For simplicity, restrict to ϕ, τ_i imaginary and $\rho_{i\bar{j}}$ diagonal. Then

$$ds^2 = \sum_{i=1}^{3}\left(R_{x^i}^2\,dx^{i\,2} + R_{y^i}^2\,dy^{i\,2}\right),$$

$$\tau_i = iR_{y^i}/R_{x^i},\quad \rho_{i\bar{i}} = iR_{x^i}R_{y^i},\quad \phi = i/g_{IIB}.$$

T-duality takes one $R \to (2\pi)^2\alpha'/R$, $g_s \to g_s/R$.

In each dual description, can satisfy constraints with large radii and small string coupling.

$\mathcal{N}=2$ Example (p. 2 of 3)

Having shown that we can satisfy the (SUSY) constraints in the region of moduli space where sugra is valid, now set $R_{x^i} = R_{y^i} = R_i$ (i.e., $\tau_i = \phi = i$) for simplicity, and choose the gauge

$$B_{x^1x^3} = -2x^2, \qquad B_{y^1x^3} = -2y^2.$$

T-dualizing in the order x^1, y^1, y^3 avoids the caveat mentioned earlier.

One T-duality (x^1): IIA Dual

$$ds^2 = \frac{1}{R_1^2}(dx^1 - 2x^2dx^3)^2 + R_1^2\,dy^{1\,2} + \sum_{i=2,3} R_i^2\,(dx^{i\,2} + dy^{i\,2}),$$

$H = 2\,dy^1\wedge dy^2\wedge dx^3$, $B_{y^1x^3} = -2y^2$, (Non-Kähler: $h^1 = 5$)

$F_{(2)} = 2\,dx^2\wedge dy^3$, $F_{(4)} = 2(dx^1 - 2x^2dx^3)\wedge dy^1\wedge dy^2\wedge dy^3$,

O4 planes wrapped on x^1-direction.

Two T-dualities (x^1y^1): IIB Dual

$$ds^2 = \frac{1}{R_1^2}(dx^1 - 2x^2dx^3)^2 + \frac{1}{R_1^2}(dy^1 - 2y^2dx^3)^2 + \sum_{i=2,3} R_i^2\,(dx^{i\,2} + dy^{i\,2}),$$

$$B = 0, \qquad F_{(3)} = 2(dy^1 - 2y^2dx^3)\wedge dx^2\wedge dx^3 + 2(dx^1 - 2x^2dx^3)\wedge dy^2\wedge dy^3,$$

O5 planes wrapped on x^1y^1-directions.

$\mathcal{N}=2$ Example (p. 3 of 3)

Three T-dualities $(x^1 y^1 y^3)$: IIA "Mirror"

$$ds^2 = \frac{1}{R_1^2}(dx^1 - 2x^2 dx^3)^2 + \frac{1}{R_1^2}(dy^1 - 2y^2 dx^3)^2 + R_2^2 dx^{2\,2} + R_2^2 dy^{2\,2} + R_3^2 dx^{3\,2} + \frac{1}{R_3^2} dy^{3\,2},$$

$$B = 0, \qquad F_{(2)} = 2(dy^1 - 2y^2 dx^3)\wedge dx^2 + 2(dx^1 - 2x^2 dx^3)\wedge dy^2,$$

O6 planes wrapped on $x^1 y^1 y^3$-directions.

The "mirror" metric is not Calabi-Yau. Data is encoded in monodromies:

$T^2\,\{x^1 x^3\}$ undergoes an $SL(2,\mathbb{Z})$ monodromy around x^2.
$T^2\,\{y^1 x^3\}$ undergoes an $SL(2,\mathbb{Z})$ monodromy around y^2.

Also possible to describe the twisting via $SL(3,\mathbb{Z})$ monodromy of $T^3\,\{x^1 y^1 y^3\}$ as one moves around base $x^2 x^3 y^2$.

Would be interesting to find non-CY mirrors of more generic CY orientifolds with flux. Perhaps their "non-Calabi-Yau ness" will also be encoded in the $SL(3,\mathbb{Z})$ monodromies of the appropriate T^3 fiber.

M-Theory Lift:

Connection to $CY_3 \times S^1$ and G_2 Compactifications

The IIA mirror was of the form (w. help from P. Berglund and N. Warner)

$M_6 = T^3$ fibration over T^3,
$F_{(2)} = dA_{(1)}$, with $A_{(1)}$ in base directions,
O6 planes wrapped on fibre T^3,
8 moduli (2 cpx str + 6 Kähler in IIB).

The M-theory lift is purely geometrical since the only flux is $F_{(2)}$ (11D KK flux).

$F_{(2)}, A_{(1)} \longrightarrow$ twisting of M-theory S^1 over base T^3,
O6 planes $\xrightarrow[\text{locally}]{}$ Atiyah-Hitchin spaces in the directions of S^1 and base T^3.

This is a smooth 7D manifold that preserves 4D $\mathcal{N}=2$ SUSY $\Rightarrow CY_3 \times S^1$ (up to discrete identifications).

We just chose an unusual S^1 (fibre) for IIA = M on S^1.

$2+6 = 8$ moduli $\Rightarrow$ Which CY_3 ?

For $\mathcal{N}=1$, there is a similar story when only $F_{(2)} \neq 0$. In this case the 7D manifold is a G_2. Generically, we have $0+3 = 3$ moduli.

Supersymmetry

SUSY in the T-dual descriptions follows automatically from that of the original IIB $T^6/\mathbb{Z}_2$ orientifold with SUSY flux. If sugra valid → SUSY also in sugra.

But, would like to formulate SUSY conditions in dual theory, w/o reference to original IIB orientifold.

One T-duality (x^9-direction)

Saw $\int H \wedge \Omega \subset W_{IIB} \to \int (H - \omega_{(3)}) \wedge \Omega \subset W_{IIA}$.
($\omega_{(3)}$ term gives $V_{scalar} \sim \int d^6x \sqrt{g_6}\, R_6$).

Also, RR fluxes: $F_{(3)\,ijk} \longrightarrow F_{(4)\,ijk9}$,
$F_{(3)\,ij9} \longrightarrow F_{(2)\,ij}$.

Find $\int F_{(3)} \wedge \Omega \subset W_{IIB} \to \int (i_9 F_{(4)} + g_{99}\, dx^9 \wedge F_{(2)}) \wedge \Omega \subset W_{IIA}$.

(Note: Usual IIA superpot. $= \int F_{(4)} \wedge J + F_{(2)} \wedge J \wedge J$).

Putting it all together, $W_{IIA\,\text{total}} = \int G_{IIA} \wedge \Omega$, with G_{IIA} linear in $F_{(4)}, F_{(2)}, H^{NS}_{(3)}$, and $\omega_{(3)}$.

$\mathcal{N} \geq 1$ SUSY if G_{IIA} (2,1) and primitive.
(Can obtain same result from IIA spinor conditions).

But, relative to which complex structure? We've been vague about Ω.

Complex Structure

(Work in progress).

The complex structure relative to which the SUSY conditions above hold is a nonintegrable almost complex structure,*

$$\tilde{J}^a{}_b = \bar{\tilde{\varepsilon}}\, \Gamma^a \Gamma_b \tilde{\varepsilon},$$

where $\varepsilon^{IIA} = \varepsilon_L + i\varepsilon_R$ ($\varepsilon_{L,R}$ = 10D Majorana)
and $\tilde{\varepsilon}_R = \varepsilon_R$, $\tilde{\varepsilon}_L = (\frac{1}{\sqrt{g_{99}}}\Gamma_{11}\Gamma_9)\,\varepsilon_L$.

(* An almost complex structure J acts as $J\,dz = i\,dz$, $J\,d\bar{z} = -i\,d\bar{z}$ on local holomorphic coordinates).

This almost complex structure is natural from a IIB perspective, but is not the natural IIA almost complex structure,

$$J^a{}_b = \bar{\varepsilon}^{IIA}\, \Gamma^a \Gamma_b\, \varepsilon^{IIA}.$$

To do:

- Determine whether this J is integrable (i.e., compute Nijenhuis tensor $N^i{}_{jk}$).
- Formulate SUSY conditions relative to this (almost) complex structure. Is $W_{RR} = \int F_4 \wedge J + F_2 \wedge J \wedge J$, or do other calibrations than J^n still appear?

In Conclusion

- By turning on NS and RR flux through three-cycles in $T^6/\mathbb{Z}_2$, we can find a variety of novel IIB string vacua with $\mathcal{N} = 0, 1, 2, 3$ SUSY.
- For $\mathcal{N}=1$, we generically lift all of the complex structure moduli and 6 of the 9 Kähler moduli.
- The T-dual descriptions involve non-Calabi-Yau non-Kähler (non-complex?) twisted tori.
- In the mirror geometry, the "non-Calabi-Yau ness" is encoded in the $SL(3,\mathbb{Z})$ monodromies of the T^3 fibration.
- The mirror often has a purely geometrical lift to M-theory on $CY \times S^1$ ($\mathcal{N}=2$) or G_2 ($\mathcal{N}=1$). (Which CYs? Which G_2s?)

But, we still need to do some work to understand the IIA complex structure and SUSY conditions from an intrinsically IIA point of view.