Minimal Flux Vacua

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February, 2005

Klebanov, Maldacena and N.S.	hep-th/0309168
N.S. and Shih	hep-th/0312170
Maldacena, Moore, N.S. and Shih	hep-th/0408039
N.S. and Shih	hep-th/0412315

Motivation

Minimal string theory is a simple and tractable theory. It has a precise non-perturbative definition in terms of a dual matrix model.

It exhibits interesting phenomena like existence of D-branes, open/closed duality and holography.

Type 0 minimal strings can have in addition RR flux (flux vacua) and charged branes.

Here, we will focus on the simplest type 0 string theory.

Perspectives

- Target space – physical interpretation is clear, but computations are impossible

- Matrix model/Integrable hierarchies – easy to calculate, but the target space physics is obscure

- Worldsheet – good for semiclassical limit (can include α' corrections), but hard to study the quantum corrections

Outline

- Target space description of the closed string theory (with RR flux)
- Exact (matrix model) description of the closed strings
- The charged branes
- Exact description of the charged branes
- Target space interpretation

Target space description of closed string

Target space – one dimension, ϕ

Linear dilaton – string coupling $g_s = e^{\phi}$

$\phi \to -\infty$ $\qquad\qquad$ $\phi \to +\infty$

Weak coupling ⟵⟶ Strong coupling

The observables of the theory correspond to changing the boundary conditions at the weak coupling end: $\phi \to -\infty$.

There can be localized D-branes (ZZ branes) at the strong coupling end: $\phi \to +\infty$.

The target space fields

NS-NS: Closed string "tachyon" – $\langle T(\phi) \rangle = \mu e^{\phi}$

RR scalar – $C(\phi)$ with Lagrangian

$$\mathcal{L}_C = \frac{1}{2} e^{-2T} (\partial_\phi C)^2$$

Symmetries:

RR shift symmetry: $C(\phi) \to C(\phi) + \text{const.}$

Charge conjugation: $C(\phi) \to -C(\phi)$

Conserved current – flux

$$q = e^{-2T}\partial_\phi C$$

Solution of equation of motion with flux q

$$\langle C(\phi)\rangle = q\int^{\phi} e^{2T} = q\int^{\phi} e^{2\mu e^{\phi}}$$

$\mu > 0$ – diverges as $\phi \to +\infty$ – this solution arises from a charged brane localized there

$\mu < 0$ – converges as $\phi \to +\infty$ – no charged brane there. The flux q is specified by boundary conditions as $\phi \to -\infty$ (vertex operator in the worldsheet).

Exact description – matrix model

This system is described by the Gross-Witten model

$$V(M) = -M^2 + gM^4$$

with M an $N \times N$ hermitian matrix.

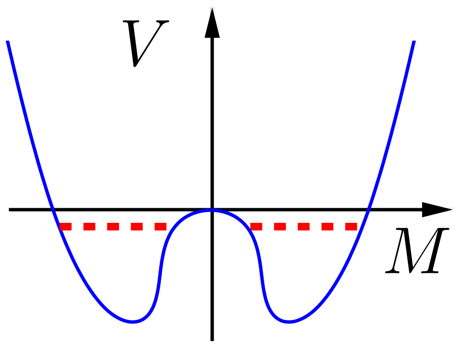
Study the partition function

$$\mathcal{Z} = e^{-N^2 F} = \int dM e^{-N\,\mathrm{Tr}\,V(M)}$$

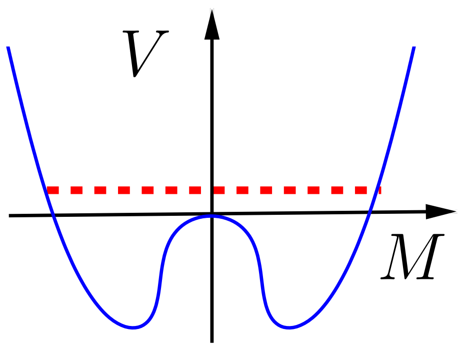
in the $N \to \infty$ limit

Large N phase transition

The eigenvalues of M are distributed along “cut(s).” As the parameters vary ($g \approx g_c$): two cuts/one cut transition



$\mu < 0$



$\mu > 0$

The continuum/scaling limit focuses on the transition.

It is described by 2×2 matrices of differential operators P and Q satisfying

$$[P, Q] = 1$$

P and Q involve ∂_μ and two functions of μ: r and β.

The free energy F is

$$\partial_\mu^2 F = \frac{1}{2} r^2$$

[Periwal, Shevitz, Crnkovic, Douglas and Moore...]

$[P, Q] = 1$ leads to the closed string equations

$$\partial_\mu^2 r - \mu r - r^3 + (\partial_\mu \beta)^2 r = 0$$
$$r^2 \partial_\mu \beta = q$$

Here q appears as an integration constant. This is a modified version of the Painlevé II equation.

Symmetries:
RR shift symmetry: $\beta \to \beta + \text{const.}$
Charge conjugation: $\beta \to -\beta$

q is RR flux. More below.

Because of the β dependence, the combinations

$$Z_{\pm}(\mu, q) = r(\mu, q)e^{\mp\beta(\mu,q)}$$

have RR charges ± 1, while the free energy F is neutral

$$\partial_{\mu}^{2} F = \frac{1}{2} r^2 = \frac{1}{2} Z_+ Z_-$$

A surprising identity

$$Z_{\pm}(\mu, q) = e^{F(\mu,q) - F(\mu,q\pm 1)}$$

relates solutions with different q.

Hence, $Z_{\pm}$ is interpreted as the expectation value of an operator which changes $q \to q \pm 1$.

Classical limit: $|\mu| \to \infty$

$$F_{cl} = -\frac{\mu^3}{12} \qquad \mu < 0$$
$$F_{cl} = 0 \qquad \mu > 0$$

Third order transition at $\mu = 0$

But the exact answer, given by the differential equation, is smooth!

Semiclassical expansion

$$F = \sum_{h,r\geq 0} |\mu|^{3(1-h)} \left(\frac{q^2}{|\mu|^3}\right)^r \qquad \mu < 0$$

$$F = \sum_{h\geq 0,\ b\geq 1} \mu^{3(1-h)} \left(\frac{|q|}{\mu^{\frac{3}{2}}}\right)^b \qquad \mu > 0$$

(suppressed coefficients, when the power of μ vanishes replace by $\log\mu$)

Worldsheet interpretation:

h – number of handles

b – number of boundaries

$2r$ – number of insertions of RR-flux vertex operator

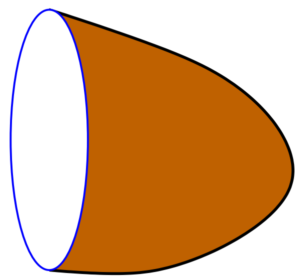
$\mu < 0$ phase: q is pure RR flux
$\mu > 0$ phase: the flux q arises from $|q|$ charged D-branes

This confirms the target space picture.

Since no phase transition, **smooth interpolation between branes and flux!**

Similar phenomenon in critical/topological string.

Extended branes (a.k.a macroscopic loops or FZZT branes)



Extended branes are described by worldsheet boundaries with boundary interaction depending on the open string "tachyon"

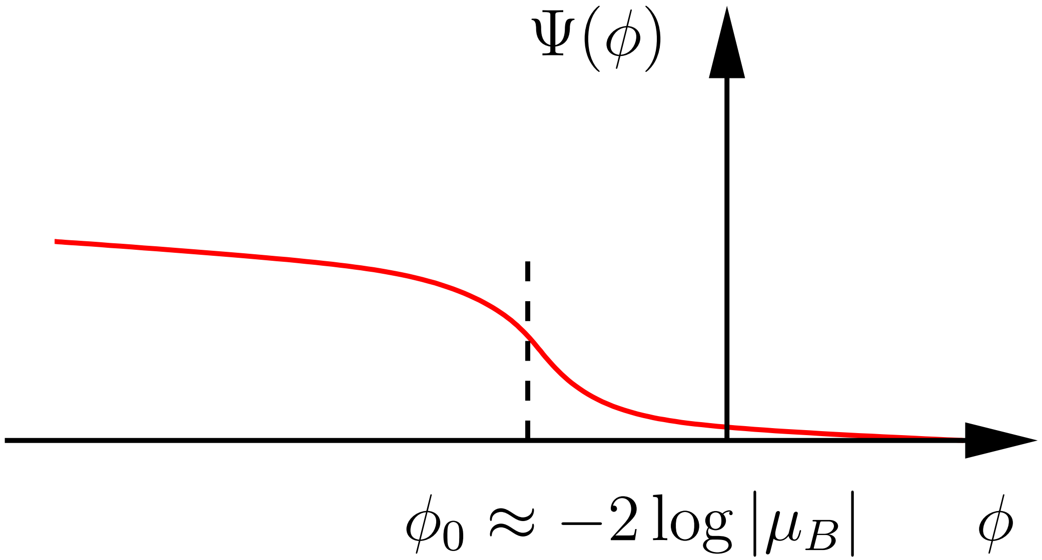
$$T_{open}(\phi) = \mu_B e^{\phi/2}$$

The branes are labelled by μ_B, which can be taken to be either real or complex.

Minisuperspace wavefunction

$$\Psi(\phi) = e^{-(T_{open})^2} = e^{-\mu_B^2 e^{\phi}}$$

The brane comes from infinity and dissolves at $\phi_0 \approx -2\log|\mu_B|$.



Motivated by the unstable branes of the critical string, the effective Lagrangian on the brane includes the term

$$C(\phi)G\left(T_{open}(\phi)\right) \approx \frac{C}{2}\mathrm{sign}\left(T_{open}\right)\delta(\phi-\phi_0)$$

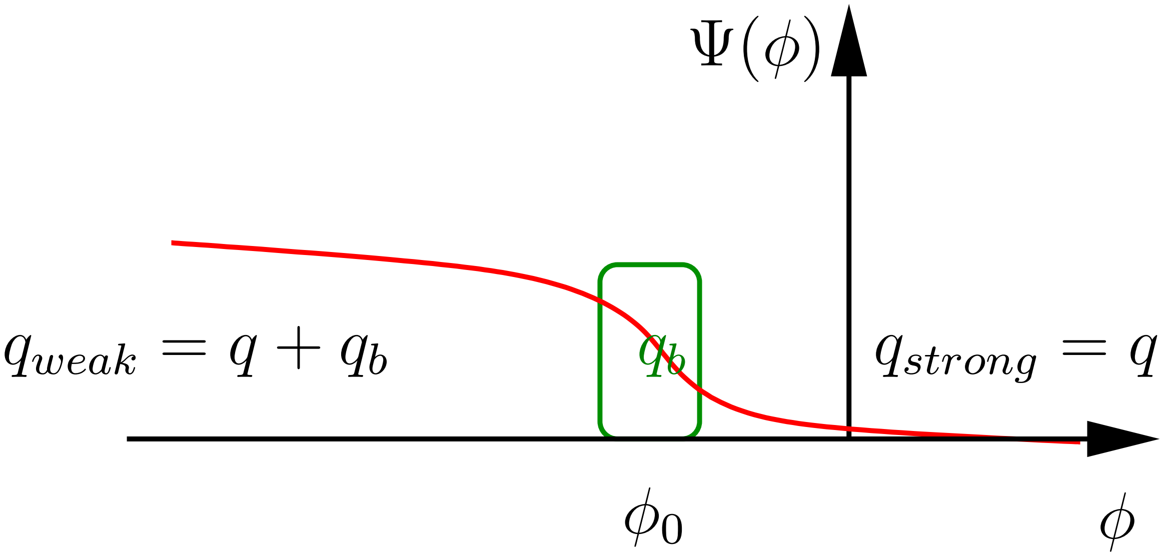
(C and T_{open} are charge conjugation odd).

Therefore, the brane has charge

$$q_b = \frac{1}{2}\mathrm{sign}(\mu_{\mathrm{B}})$$

It is localized around ϕ_0.

The charge changes the RR-flux in the weak coupling region



Note: the closed string background parameter q is the flux in the strong coupling region – not in the weak coupling asymptotic region (more below).

Similar to unstable branes in critical string, where a kink of open tachyons is charged.

Except:

- Our branes are stable because T_{open} is massive
- Since T_{open} varies from 0 to $\pm\infty$, the brane is like "half a kink", and hence its charge is $\pm\frac{1}{2}$.

Recall that the charge of the brane is

$$q_b = \frac{1}{2}\mathrm{sign}(\mu_{\mathrm{B}})$$

Semiclassically, there are two branes:

- Start with $\mu_B > 0$, and hence $q_b = +\frac{1}{2}$, and analytically continue to $\mu_B < 0$

- Start with $\mu_B < 0$, and hence $q_b = -\frac{1}{2}$, and analytically continue to $\mu_B > 0$

Explicit worldsheet calculations (not done here) confirm this semiclassical expectation.

Preview

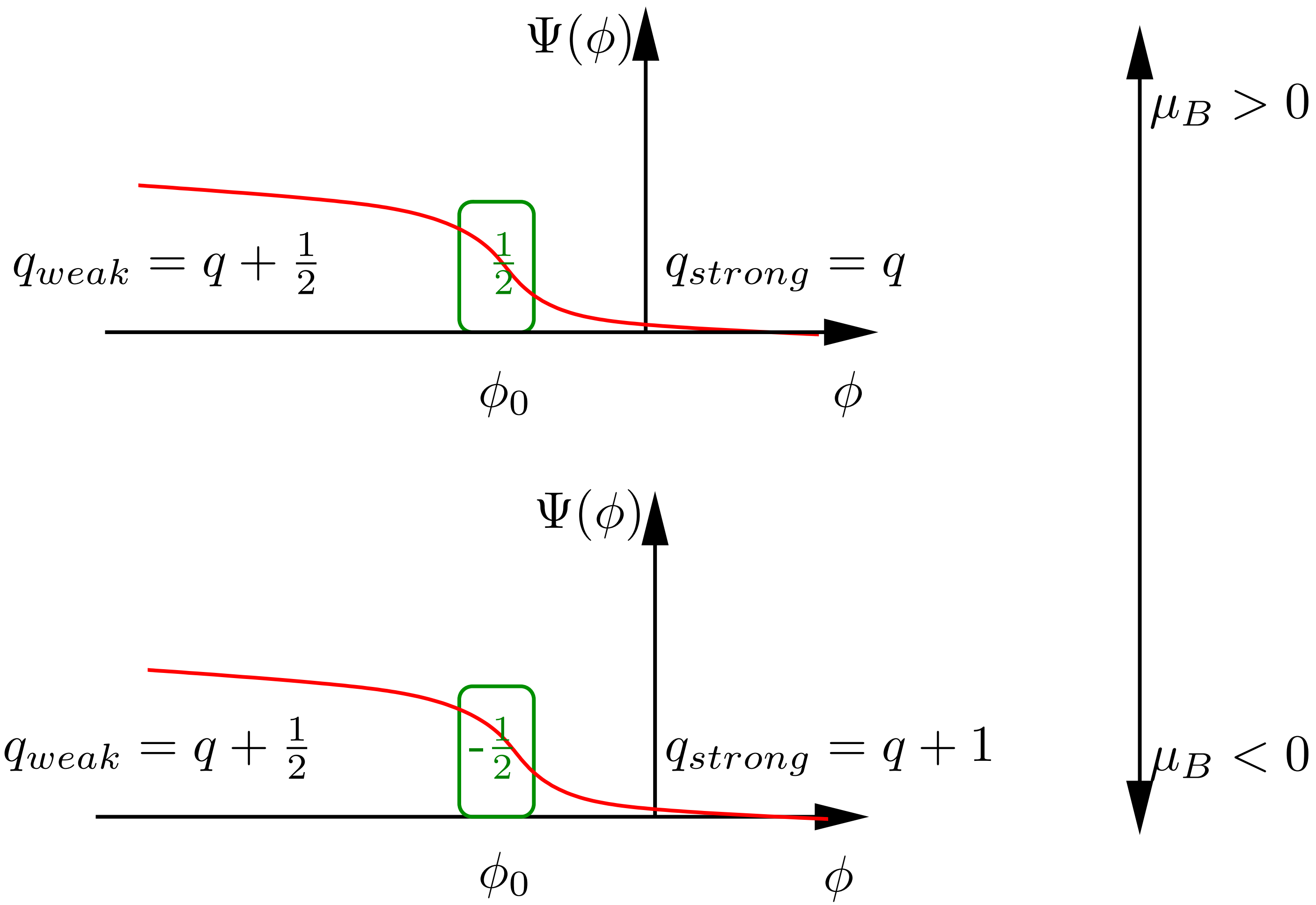
We will show that this is not true in the exact theory!

The mistake in the semiclassical reasoning is in the analytic continuation to the other sign of μ_B.

Instead, as expected from the target space picture and from the analogy with the unstable brane, there is only one brane for every μ_B.

The charge of the brane changes as μ_B is varied!

Continued preview: a transition in the charge of the brane



Exact analysis – matrix model

In the matrix model the branes are constructed using the observable $\det(M - i\mu_B)$ (exponentiated macroscopic loop).

Recall that in the scaling limit the closed string sector is controlled by 2×2 matrices of differential operators in μ satisfying

$$[P, Q] = 1$$

Given $[P, Q] = 1$, it is natural to consider the (Baker-Akhiezer) functions $\psi_\pm(\mu, q, \mu_B)$ satisfying

$$Q \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \mu_B \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$P \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = -\partial_{\mu_B} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$(\psi_+ \pm \psi_- \sim \lim_{N\to\infty}\langle \det(M - i\mu_B)\rangle$ with N even or odd.)

Physical interpretation:

There are two branes $B_\pm(\mu_B)$ with partition functions

$$\langle B_\pm(\mu_B)\rangle_{\mu,q} = \psi_\pm(\mu, q, \mu_B)e^{-F(\mu,q)}$$

The functions $\psi_\pm$ are the brane partition functions normalized with the closed string partition function $\mathcal{Z} = e^{-F(\mu,q)}$

The equations for the branes can be written as

$$\partial_\mu \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} \mu_B & \frac{Z_+}{\sqrt{2}} \\ \frac{Z_-}{\sqrt{2}} & -\mu_B \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$\partial_{\mu_B} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} -4\mu_B^2 + r^2 + \mu & -\sqrt{2}(2\mu_B Z_+ + Z'_+) \\ -\sqrt{2}(\mu_B Z_- - Z'_-) & 4\mu_B^2 - r^2 - \mu \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

Recall that $Z_\pm(\mu, q) = r(\mu, q) e^{\mp\beta(\mu,q)}$ are determined by the closed string equations.

(The details are not important!)

From the structure of the equations:

- $B_\pm$ and $\psi_\pm = \frac{\langle B_\pm \rangle}{e^{-F}}$ have charges $\pm\frac{1}{2}$ (recall that $Z_\pm(\mu, q)$ have charges ± 1)

- The exact solutions of these differential equations are smooth functions of μ and μ_B!

Naively, this agrees with the semiclassical picture: two branes for each μ_B.

But, explicit worldsheet calculations (not done here) agree with the semiclassical limit of $\psi_+(\mu_B)$ ($\psi_-(\mu_B)$) only for $\mu_B > 0$ ($\mu_B < 0$). This will be explained soon.

An interesting identity

Using the charge ± 1 objects

$$Z_\pm(\mu, q) = r(\mu, q) e^{\mp\beta(\mu,q)} = e^{F(\mu,q) - F(\mu,q\pm 1)}$$

there is a surprising identity

$$\psi_+(\mu, q, \mu_B) = Z_+(\mu, q)\psi_-(\mu, q+1, \mu_B)$$

which means

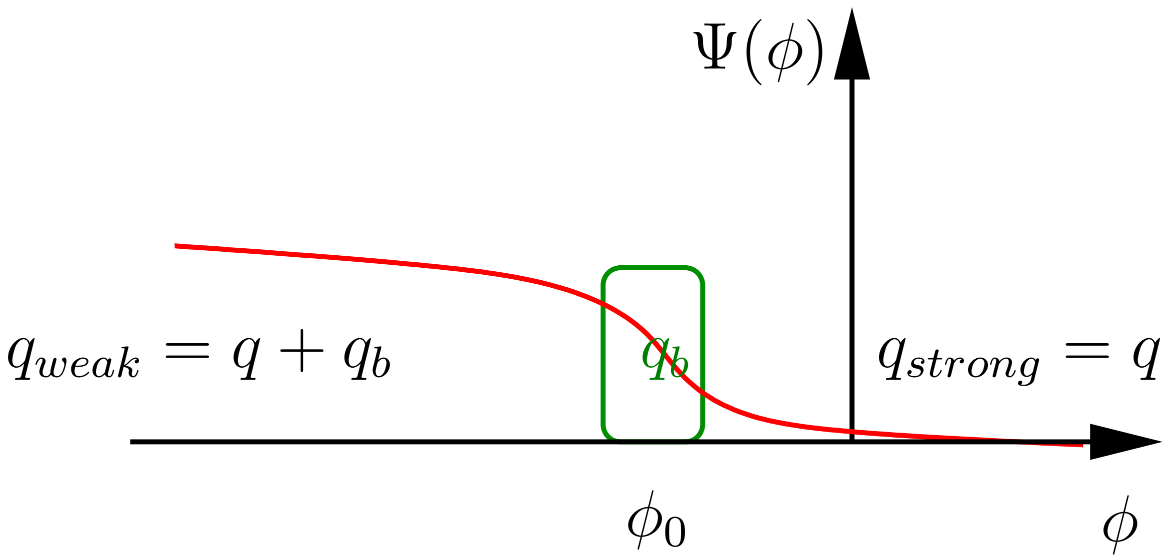
$$\langle B_+(\mu_B)\rangle_{\mu,q} = \langle B_-(\mu_B)\rangle_{\mu,q+1}$$

i.e. there are only half as many independent flux/brane configurations – the counting agrees with the target space picture.

Interpreting the exact answers for $\langle B_\pm \rangle$

The equations with the parameter q describe branes $B_\pm$ with $q_{weak} = q \pm \frac{1}{2}$.
The subscript in $B_\pm$ determines the flux at infinity and not "the charge of the brane."

B_+ has its natural charge ($q_b = +\frac{1}{2}$) for $\mu_B > 0$.
B_- has its natural charge ($q_b = -\frac{1}{2}$) for $\mu_B < 0$.
In these two situations the flux in the strong coupling region is $q_{strong} = q$.



Note, the parameter q in the equations is not the flux in the asymptotic weak coupling region.

Analytically continuing $B_\pm$ to the other sign of μ_B changes their charges $q_b \to -q_b$ and $q_{strong} \to q_{strong} \pm 1$, while preserving q_{weak}.

The surprising identity

$$\langle B_+(\mu_B)\rangle_{\mu,q} = \langle B_-(\mu_B)\rangle_{\mu,q+1}$$

equates two branes with the same $q_{weak} = q + \frac{1}{2}$.

Consider the identity for $\mu_B > 0$.
The brane B_+ in the LHS has its natural charge $q_b = +\frac{1}{2}$ and flux $q_{strong} = q$.
The brane B_- in the RHS is analytically continued from negative μ_B, where it has its natural charge $q_b = -\frac{1}{2}$ and flux $q_{strong} = q + 1$.

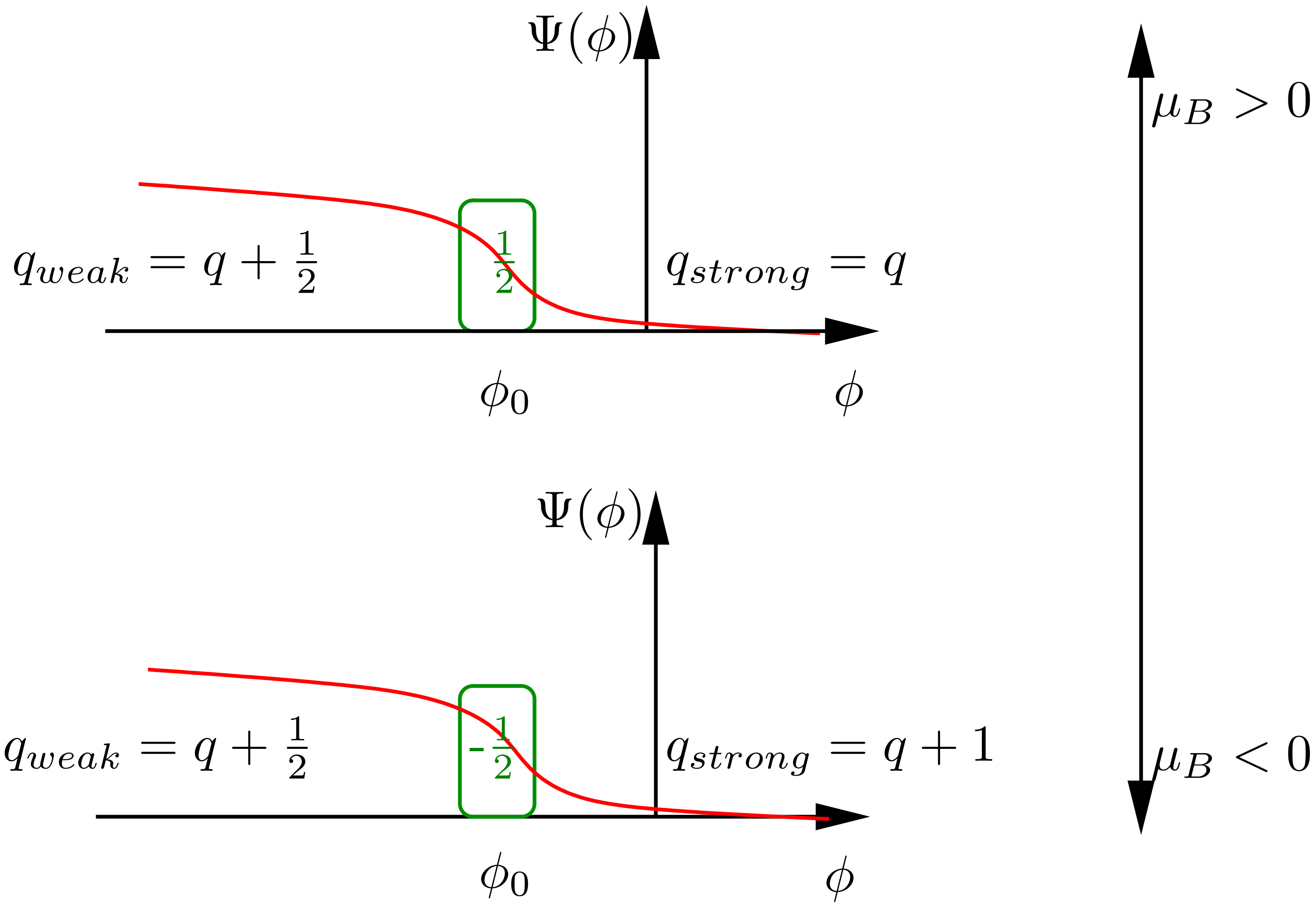
Conclude: the distinct flux/brane configurations are labelled by q_{weak} and μ_B (including its sign).

This agrees with the target space picture.

It also resolves the discrepancy with the semiclassical worldsheet computations – they are correct only before the transition; i.e. when $B_\pm$ have their natural charges $q_b = \pm\frac{1}{2}$ and $q_{strong} = q$ (Stokes' phenomenon).

The main surprise is that the transition which changes the charge is smooth.

Physical picture of the transition



Comments about the transition

- q_{weak} is well defined and does not fluctuate
  - The weak coupling region has infinite volume.

- q_b and q_{strong} are not meaningful but fluctuate. This allows smooth transitions changing q_b and q_{strong}.
  - The strong coupling region effectively has finite volume.
  - For $\mu_B \to \pm\infty$ the volume of the strong coupling region becomes large, and suppresses the fluctuations. q_b and q_{strong} become well defined.

Lessons

- We discussed a solvable model with RR flux q and charged branes.

- For $\mu > 0$ the flux is generated by charged D-branes in the strong coupling region $\phi = +\infty$.

- For $\mu < 0$ there are no such branes, but the flux still exists.

- There is a smooth transition between two semiclassical limits: $\mu \to \pm\infty$. It converts charged D-branes $\longleftrightarrow$ flux.

- Extended D-branes dissolve at $\phi_0 \approx -2\log(|\mu_B|)$, with localized charge near ϕ_0. Semiclassically, i.e. as $\mu_B \to \pm\infty$, it is $q_b = \frac{1}{2}\text{sign}(\mu_B)$.

- The classically meaningful charge q_b and flux in the strong coupling region q_{strong} fluctuate in the quantum theory. Only the flux in the weak coupling region q_{weak} is meaningful.

- There is a smooth transition as μ_B varies. It changes $q_b \longleftrightarrow -q_b$. Semiclassically, the brane picks up charge at $\phi = +\infty$ as it passes through $\mu_B = 0$.