S-matrix test of longitudinal string spreading,

or

How your past catches up with you (the harder you run away from it).

With M. Dodelson (to appear).

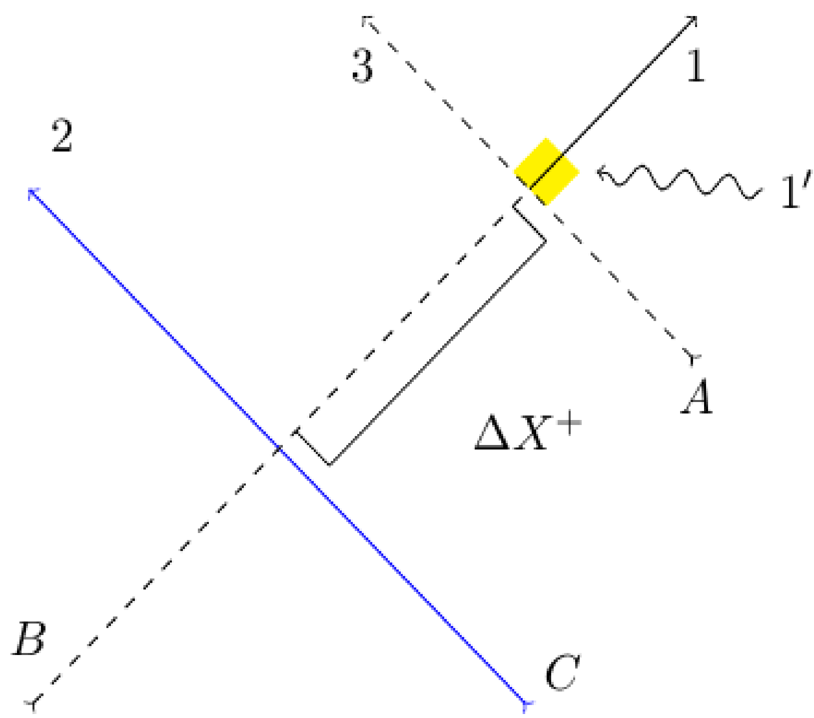


Table of Contents

String Spreading

Susskind '94 + Lowe Polchinski Thorlacius Uglum '95...

Light Cone gauge $X^- \sim p^- \tau$,

Constraint determines X^+ in terms of $X^\perp$

$$\langle\psi|(X_\perp - x_\perp)^2|\psi\rangle = \sum_n^{n_{max}} \frac{1}{n} = \log\frac{n_{max}}{n_0} + O\left(\frac{1}{n_{max}}\right)$$

$$\langle\psi|(X^+ - x^+)^2|\psi\rangle \simeq \frac{1}{(p^-)^2}\sum^{n_{max}} n \simeq \frac{n_{max}^2}{(p^-)^2}$$

$n_{max} \leftrightarrow$ light cone time resolution

Predicts $\Delta X^+ \sim \frac{k^+_{det}}{k_\perp^2} \sim E$ $\leftarrow$ if $> \frac{1}{\alpha'}$

A fun clarification: two quadratically divergent sums appear in light cone string theory calculations. In spreading, the transverse Virasoro generators imply

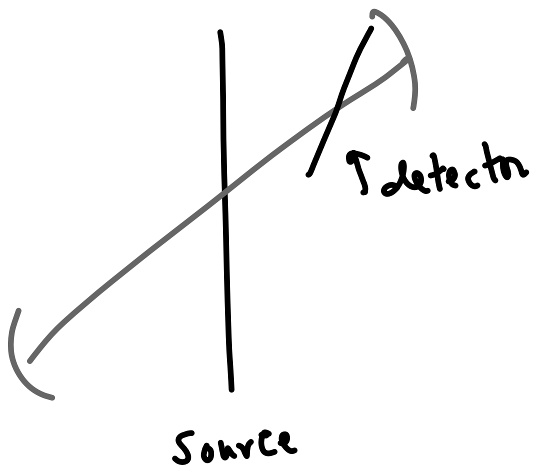
$$\langle (X^+(\sigma,0) - x^+)^2 \rangle \propto \frac{c_\perp}{(p^-)^2} \sum_n n \sim \frac{n_{\max}^2}{(p^-)^2}$$

On the other hand, in the mass shell formula we have a quadratically divergent sum which is subtracted (local counterterm tunes away the worldsheet c.c.):

$$\sum\nolimits_{n>0} n$$

In the superstring, the two are cleanly separated: the quadratically divergent worldsheet cosmological constant cancels, but the central term in the X^+ modes (transverse Virasoro generators) does not cancel.

Light cone time resolution (refined prediction)



Note that this effect is causal, just non-local (the string is spread out, so can potentially interact before its center reaches the detector).

Measurement degrades as $\uparrow p_\perp$: conservatively

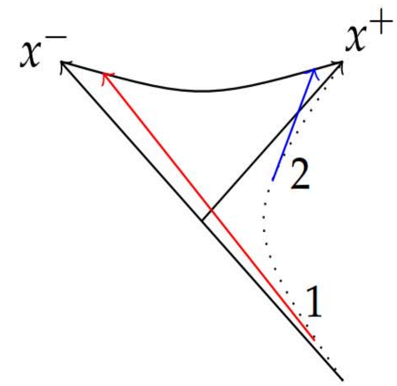
$$\Delta X^- \sim \frac{p_\perp^2 + m^2}{p^+_{det}} \, , \quad n_{max} \sim \frac{p_s^- p_d^+}{p_\perp^2 + m^2}$$

$$\Rightarrow \boxed{\Delta X^+_{spr,\,RMS} \sim \cancel{\frac{E_{det}}{p_\perp^2}}}$$

Agrees precisely with BPST '06 2->2 calculation, and with Gross-Mende

*One motivation: weakly curved geometries with a horizon: evolution of trajectories of (say) two probes sent in with modest energy leads to a large *nonlocal* invariant energy in the near horizon region. cf info loss problem...AMPS...

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2$$
$$= -\frac{2r_s}{r} e^{1-r/r_s} dx^+ dx^- .$$



How nonlocal is string theory in the longitudinal direction of relative motion between a string and a second object (detector)? Interesting beyond GR physics.

cf string production Bachas, McAllister/Mitra, Senatore et al, Polchinski, ES '14, Puhm, Rojas, Ugajin '16.

This is an old problem, but previously was applied to support complementarity. We note that it is a potential source of `drama'. There is extensive previous work on string scattering, but not conclusive on this question.

D. Lowe, J. Polchinski, L. Susskind, L. Thorlacius, J. Uglum, "Black hole complementarity versus locality," Phys. Rev. D **52** 6997-7010 (1995) [hep-th/9506138].

J. Polchinski, "String theory and black hole complementarity," In *Los Angeles 1995, Future perspectives in string theory* 417-426 [hep-th/9507094].

S. B. Giddings, D. J. Gross and A. Maharana, "Gravitational effects in ultrahigh-energy string scattering," Phys. Rev. D **77**, 046001 (2008) [arXiv:0705.1816 [hep-th]].

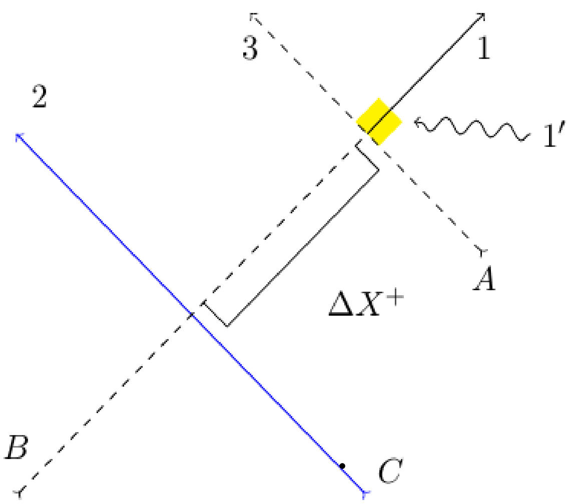
D. Amati, M. Ciafaloni and G. Veneziano, "Superstring Collisions at Planckian Energies," Phys. Lett. B **197**, 81 (1987).

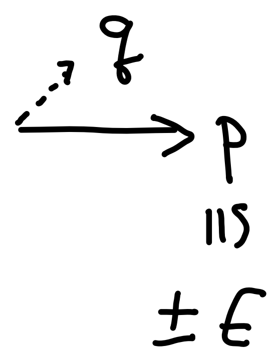
D. Amati, M. Ciafaloni and G. Veneziano, "Can Space-Time Be Probed Below the String Size?," Phys. Lett. B **216**, 41 (1989).

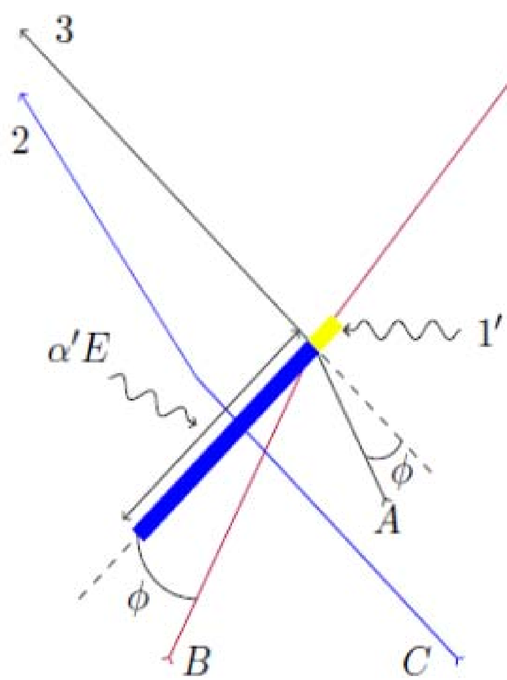
Our goal is to conclusively answer the subtle yes/no question, using gauge-invariant observables (S-matrix).

A similar setup to the black hole appears at six points in tree level flat space string scattering

Direct test of longitudinal interaction.







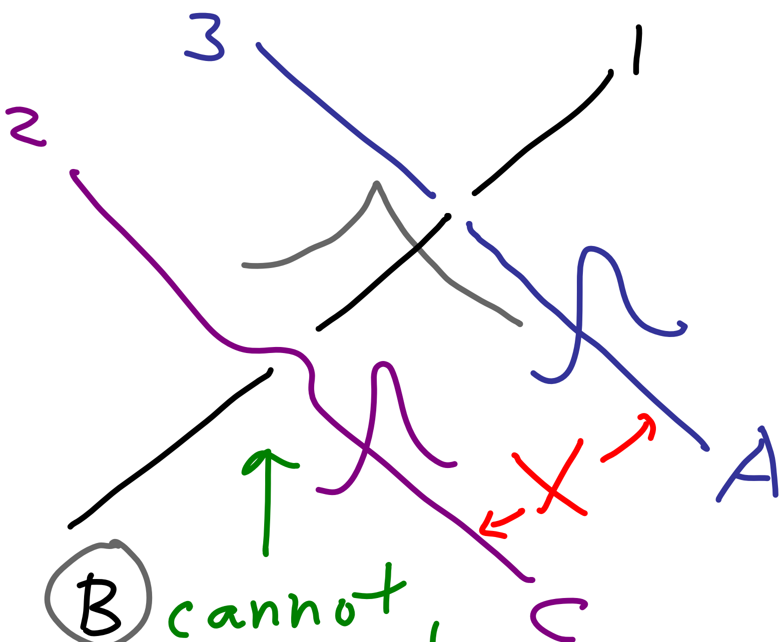
= 1 + c + 2

$$\Delta X^+_{\text{spreading}} \overset{?}{\sim} \frac{k^+_{1'}}{k^2_{1'}} \sim \frac{1}{k^-_{1'}} \sim \frac{E_1}{k^2_{1'}}.$$

Systematics and Noise

$$A(x) \equiv \int \underbrace{\psi_{x,p_0}}_{\text{wavepacket peaked at}}(\tilde{p})\, A(\tilde{p})$$

$x = x_c - x_A \, , \quad p = p_0$



cannot interact directly by gauge invariance

Open string orderings A3C21B, A3B1C2

We include wavepackets that keep the momenta in the longitudinal direction, while providing good spatial resolution to test $\sim E\alpha'$ spread.

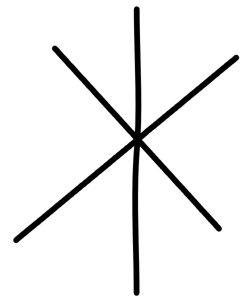
We can simply keep outgoers in momentum eigenstates; energy-momentum δ function is part of the momentum-space amplitude. This, along with the wavepacket, determines support in p_C conjugate to X.

In fact will first focus on a small momentum range in which the amplitude exhibits a certain structure, finding the effect there, then generalize.

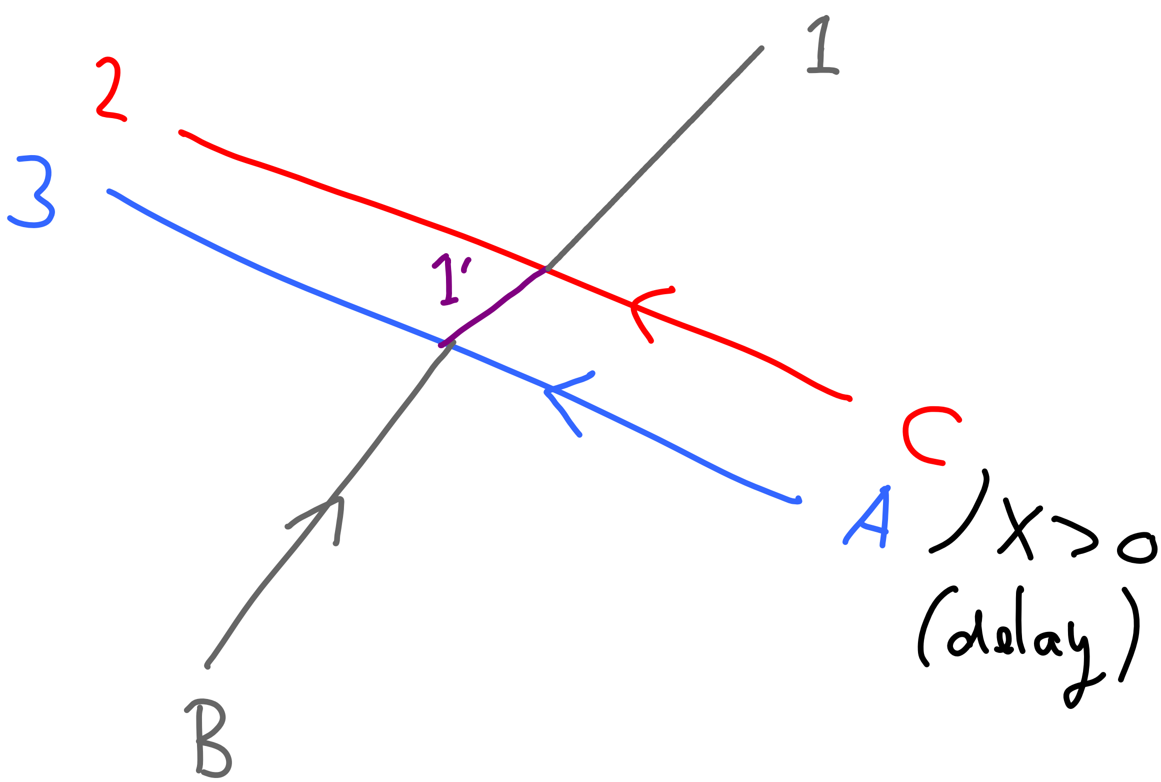
Tree-level QFT provides a control group, analyze with same wavepackets.

QFT comparison model(s)

$$A_{\mathrm{QFT0}} \sim \lambda^{(6)} \delta^{(3)}(\sum \tilde{k}_I)$$



$$A_{\mathrm{QFT1}} \sim \lambda_1^{(4)} \lambda_2^{(4)} \frac{1}{k_{1'}^2 - i\epsilon} \delta^{(3)}(\sum \tilde{k}_I)$$



Basic result (1st example below):

A(X) in ST exhibits properties:

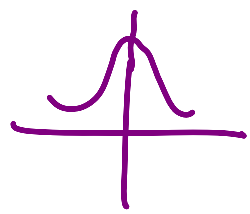
*Beats QFT at X ~-Eα'

*Support peaked there

*Not ascribable to wavepacket tails

*Amplitude not convergent sum of QFT propagators

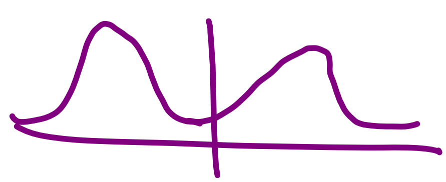
$$A^{(X)}_{QFT} \sim \frac{1}{k_{1,0}^2}\lambda_1^{(4)}\lambda_2^{(4)} e^{-\frac{X^2\sigma^2}{2}}$$



$$A_{ST}(X) \sim 2^n A_1^{(4)} A_2^{(4)} e^{-\frac{(X+X_*)^2\sigma^2}{2}} e^{i\pi n}$$

$$- 2^n A_1^{(4)} A_2^{(4)} e^{-\frac{(X-X_*)^2\sigma^2}{2}} e^{-i\pi n}$$

$$X_* = 4\pi\epsilon_1$$



The momentum-space amplitude is tractable near (not on) one pole. One form:

$$IJ = 1 + 2\alpha' k_I \cdot k_J = 1 + K_{IJ}$$

$$A_{\mathrm{ST}} = \frac{1}{A3} B(\alpha' k_{1'}^2 - 1, B1) B(C2, 12) {}_3F_2(1 - B2, \alpha' k_{1'}^2 - 1, C2; \alpha' k_{1'}^2 - 1 + B1, 12 + C2; 1)$$

(A3C21B)

$$A = \frac{1}{A3}(B(C1, B1) B(A23, -\eta) {}_3F_2(B1, A23, k_{1'}^2 - 1 + \eta, 1 + \eta, C1 + B1)$$
$$+ B(C1, C2) B(k_{1'}^2 - 1, \eta) {}_3F_2(C2, k_{1'}^2 - 1, A23 - \eta, 1 - \eta, C1 + C2))$$

(A3B1C2)

In both orderings, it can be reduced in our kinematic regime to

$$A_4^{(1)} A_4^{(2)} B(k_{1'}^2 - 1, \eta)$$

$\eta \equiv B1 - C2$ non-integer (poles cancel at integer values)

plus a piece that does not vary significantly with p_C (the conjugate variable to X).

Kinematics comment:

$$q_I \ll E_I$$

$$K_{A3} = (q_B + q_C + q_1 + q_2)^2 = (q_C + q_1 + q_2)^2 \simeq \delta q^2$$

$$K_{C2} = (q_C + q_2)^2 \simeq q_1^2 - 2 q_1 \delta q$$

$$K_{B1} = (q_B + q_1)^2 = q_1^2$$

$$k_{1'}^2 = K_{C2} + 4E_1(E_C - E_2) + 2q_1(q_C + q_2) + \frac{E_1}{E_2}(q_C^2 - q_2^2)$$

$$\eta = K_{B1} - K_{C2} = 2 q_1 \delta q$$

$$\Downarrow$$

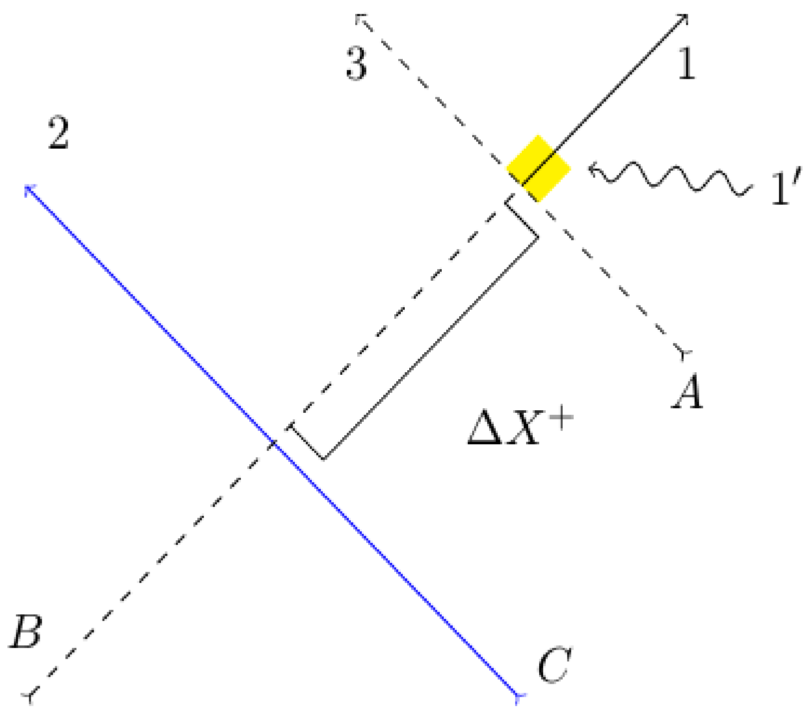
$$A3 = \delta q^2 > 0$$

$$A_4^{(1)} A_4^{(2)} B(k_{1'}^2 - 1, \eta)$$

$\frac{1}{A3}$

$B(c2, \underset{c1}{\overset{\text{or}}{12}}) \sim (E_1 E_c)^{-k_{c2}}$

Regge



Either of the $A^{(4)}$'s can be auxiliary (cf time-reverse)

The active ingredient will be the factor (in the superstring)

$$B(k_{1'}^2, \eta)$$

$$k_{1'}^2 = \frac{K_{C2}}{\alpha'} + 4E_1(E_C - E_2) + \frac{(E_C q_1 + E_1 q_C)^2}{E_1 E_C} - \frac{(E_2 q_1 - E_1 q_2)^2}{E_1 E_2}$$

$$= 4E_1 \delta p_C + k_{1'0}^2$$

Conjugate to $X = x_c - x_A$

inverse detector resolution

$$\Delta X^+_{lc} \sim \frac{E_1}{k^2_{1'0}}$$

$$\boxed{k^2_{1'}}$$

$\eta = B1 - C2 > 0$:

$$B(k_{1'}^2, \eta) = \sum_{n=0}^{\infty} \frac{(1-\eta)_n}{n!} \frac{1}{k_{1'}^2 + n - i\varepsilon}$$

convergent

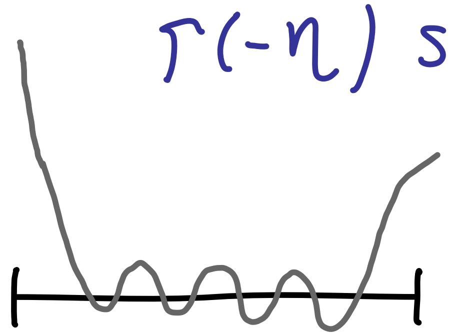
$A(\underline{\underline{X}})$ spread but can't exclude tail

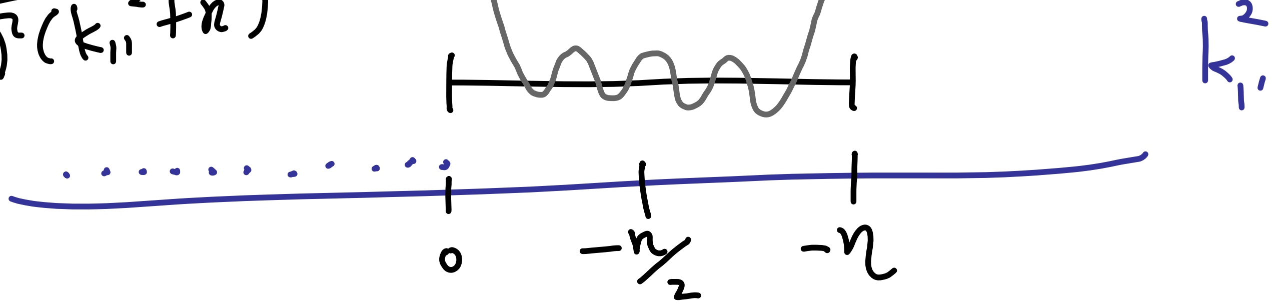
$\eta = B1 - C2 < 0$: no such expansion

$$B(k_{1'}^2, \eta) \cong \frac{\sin\pi(k_{1'}^2+\eta)\,\Gamma(k_{1'}^2)\,\Gamma(-k_{1'}^2-\eta)}{\Gamma(-\eta)\,\sin\pi\eta}$$

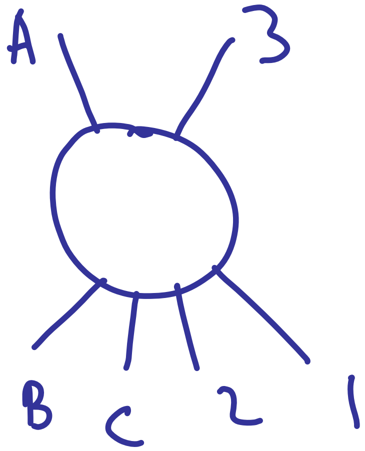
$$= \frac{\Gamma(k_{1'}^2)\Gamma(\eta)}{\Gamma(k_{1'}^2+\eta)}$$

> 0

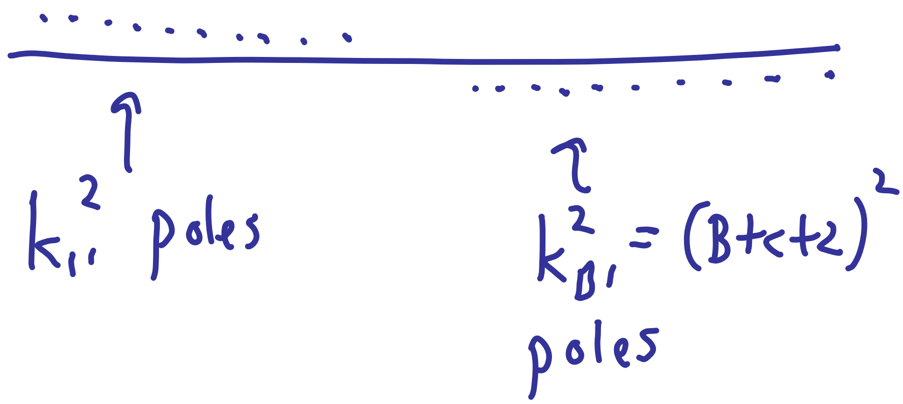


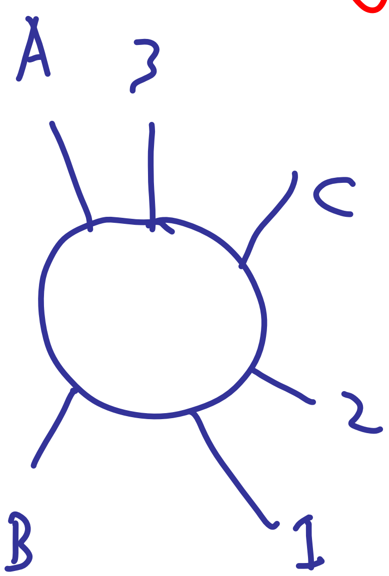


BC systematic and pole structure



$$A \propto B(k_{1'}^2, k_{B'}^2)$$





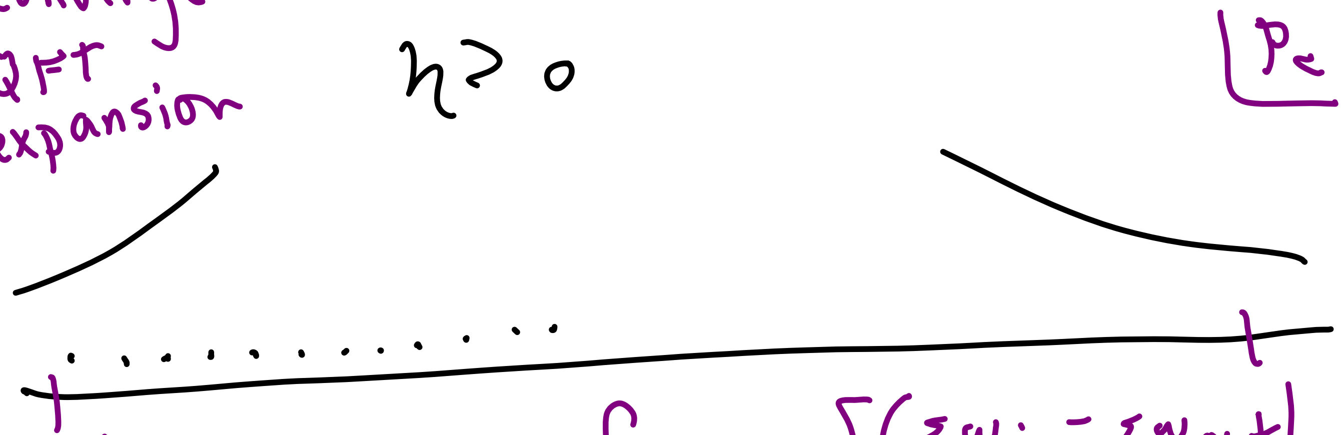
$$A \propto B(k_{1'}^2, \eta)$$

Only delayed interactions? Not a priori clear: positive energies lead to endpoint contributions, must explicitly compute and assess wavepacket tail contributions (next slide). Also, related case without poles but similar results (integer $\eta<0$).

$$|B(k_{1,1}^2,\eta)| \underset{|k_{1,1}^2| \gg |\eta|}{\sim} |k_{1,1}^2|^{\eta}$$

convergent QFT expansion

$\eta > 0$



endpoints from $\delta(\Sigma\omega_{in} - \Sigma\omega_{out})$

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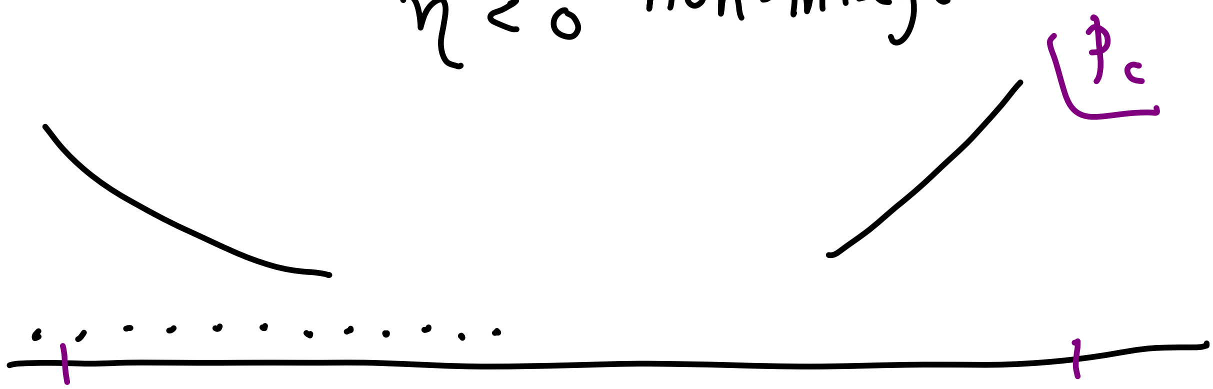
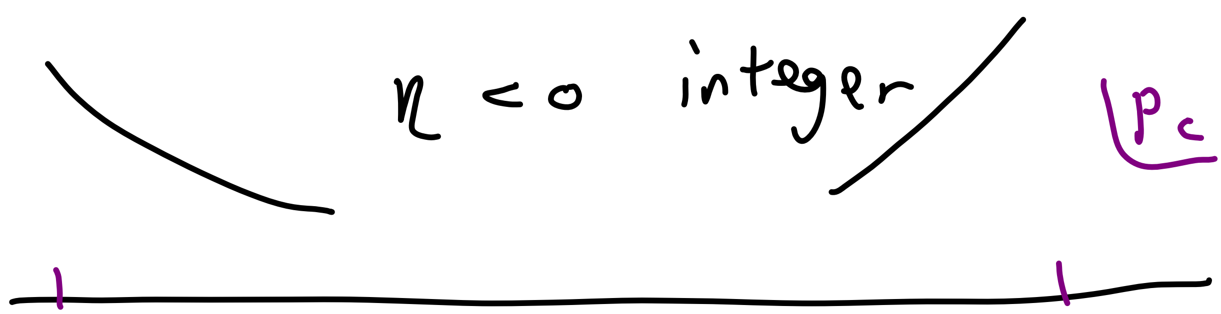
no QFT expansion

$\eta < 0$ non-integer



$\eta < 0$ integer



Two cases:

$q_A \neq q_C \quad (q_A = -q_C)$

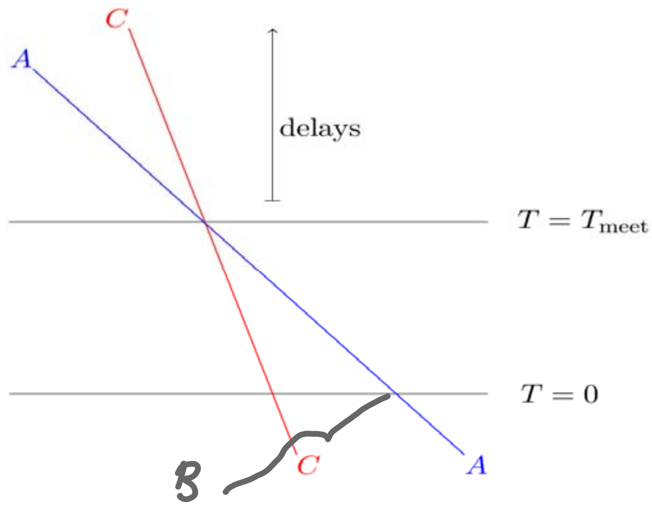
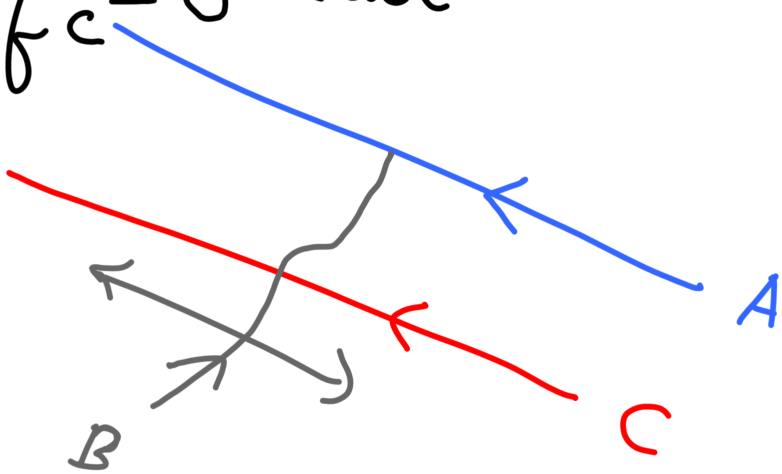


Figure 8: At the very late time $T_{\text{meet}} \sim E^3 X/(q^2(p_C - p_A))$ (5.5), the central trajectories of strings A and C meet. After this time, the interaction is purely delayed (not requiring any nonlocality). This region is accessible to the strings out on a highly suppressed power law tail of the wavefunction Ψ_B, as discussed in the text. String B propagates to the right in this picture, and the B wavefunction is peaked such that B meets A at $T = 0$.

As depicted in Figure 8, for $p_C \neq p_A$ the trajectories of A and C meet at a very late, but finite, timescale:

$$T_{\text{meet}} = \frac{2}{q^2} X \frac{p_A^2 p_C^2}{p_C^2 - p_A^2} \tag{5.5}$$

$q_A = q_C = 0$ case:



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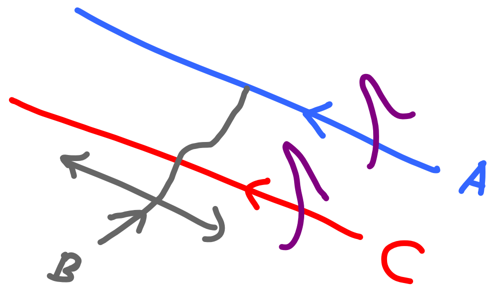
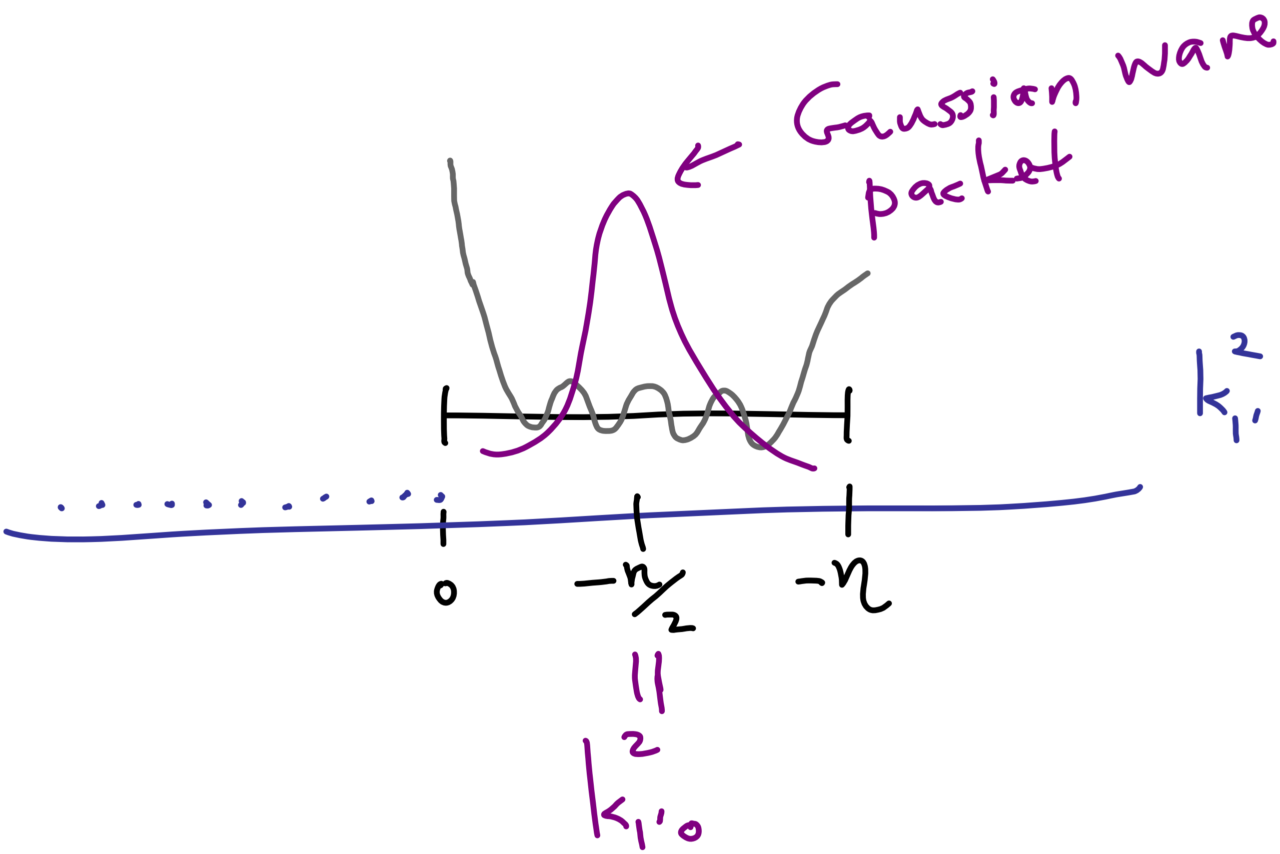




$$B(k_{1,}^2, \eta) \cong \frac{\sin\pi(k_{1,}^2+\eta)\,\Gamma(k_{1,}^2)\,\Gamma(-k_{1,}^2-\eta)}{\Gamma(-\eta)\,\sin\pi\eta}$$

$$\cong \Big(e^{i\pi\eta}\,\underbrace{e^{i\pi k_{1,}^2}} - e^{-i\pi\eta}\,e^{-i\pi k_{1,}^2}\Big)\times(\ldots)$$

$$e^{i\pi(4\epsilon_1\delta p_c + k_{1,0}^2)}$$

$$\int d\delta p_c\, e^{-\frac{\delta p_c^2}{2\sigma^2}}\, e^{i\delta p_c(X+X_*)}\,(\ldots)$$

$$\cong B(k_{1,0}^2, \eta)\, e^{-\frac{(X+X_*)^2\sigma^2}{2}}$$

$$X_* = 4\pi\epsilon_1$$

ST: $$e^{-(X+X_*)^2\sigma^2/2}\frac{B(C2,12)}{A3}2^\eta$$

✓ at $X = -X_* = -4\pi E_1\alpha'$

QFT: $$e^{-X^2\sigma^2/2}\frac{1}{A3}\frac{1}{(-\eta/2)}$$

stripping off auxiliary 4pt.

$$\sigma \equiv \frac{\sqrt{c_\sigma(-\eta)}}{4E_1} \qquad \frac{2\log(2)}{\pi^2} < c_\sigma < 1$$

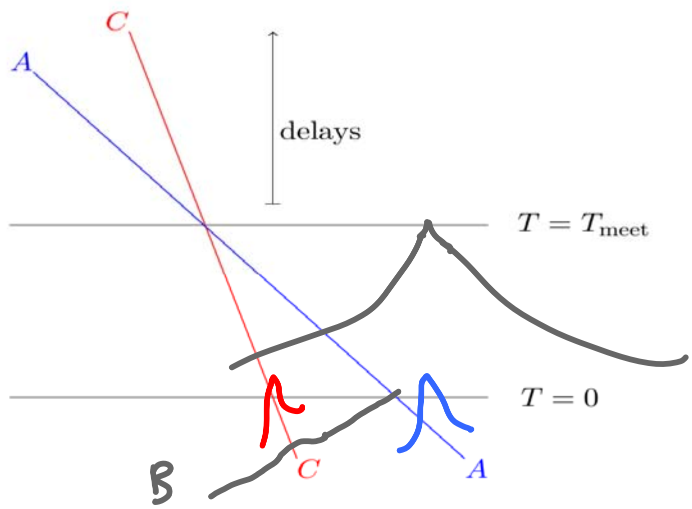
Could the ST answer be on the tail of the wavefunction? A priori, might take advantage of higher momentum-space amplitude elsewhere in p_C. But Gaussian wavepacket suppresses this.

How does this compare to light cone prediction? Consistent:

$$\Delta X^+_{\text{spreading}} \sim \frac{k^+_{1'}}{k^2_{1'}} \sim \frac{1}{k^-_{1'}} \sim \frac{E_1}{k^2_{1'}}.$$

$$\exp(-const\frac{\Delta X^+}{\Delta X^+_{spreading}}) \sim \exp(-const\frac{E_1\alpha'}{E_1\alpha'/k^2_{1'0}}) \sim 2^\eta$$

Similar result for more generic transverse motion of A and C



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A(X) computed in two bases. B wavefunction plus energy conservation restricts p_C to range with above peak structure (for $\eta<0$). In QFT explicitly tracks tail of wavefunction to good approximation. Not so in ST.

For $\eta<0$ no expansion purely into 1' propagators, but there is an expansion with extra structure in k1'^2:

$$B(k_{1'}^2,\eta) = B(k_{1'}^2,K)\frac{\Gamma(\eta)\Gamma(k_{1'}^2+K)}{\Gamma(k_{1'}^2+\eta)\Gamma(K)} = \sum_{n_{1'}} \frac{(1-K)_{n_{1'}}}{n_{1'}!}\frac{1}{k_{1'}^2+n_{1'}-i\epsilon}\frac{\sin\pi(k_{1'}^2+\eta)\Gamma(-k_{1'}^2-\eta)\Gamma(k_{1'}^2+K)}{\sin\pi\eta\Gamma(-\eta)\Gamma(K)}$$

Compared to QFT result on the tail, the string theory calculation has terms with shifted positions:

$$X \to X \pm 4\pi E_1\alpha' + 4iE_1\alpha'\log(\frac{K}{-\eta})$$

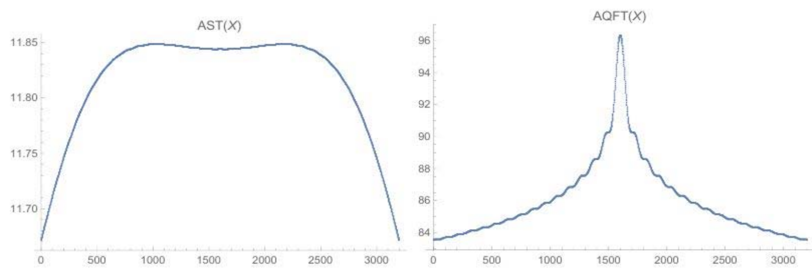
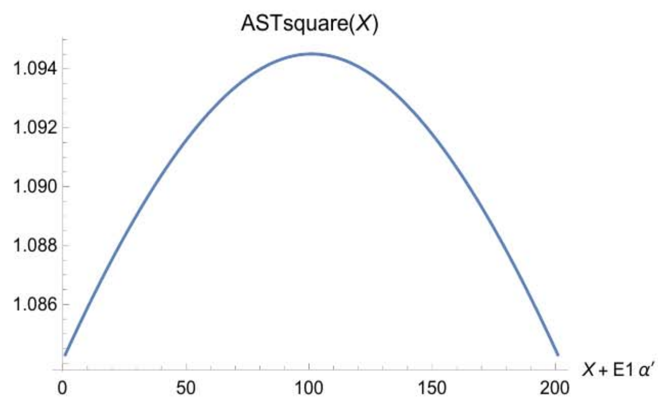
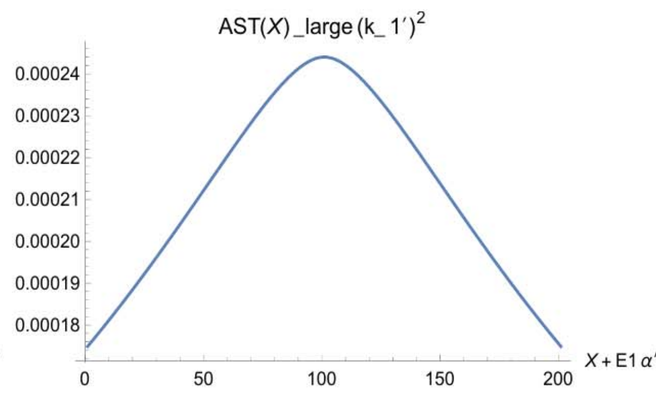


Figure 9: For $\eta<0$, the full string theory amplitude (summed over $n_{1'}$) pole is spread out by order $\sim 4\pi E\alpha'$ compared to the QFT1 model in the kinematic regime described in the text. The term by term spreading described in the text and in figure 8 survives the sum over the (deformed) propagators.

For $\eta>0$: A(X) spread out, but can't exclude tail interpretation. e.g. for nonzero q_C and q_A





In this case the dominant contribution in a position basis is a step function

$$\int_{p_{C,\min}}^{p_{C,\max}} d\tilde{p}_C e^{-i\tilde{p}_C X} \hat{A}(\tilde{p}_C) \sim \frac{1}{4E_1\alpha'} e^{-ip_{C,0}X} \sin\left(\frac{X}{8\alpha' E_1}\right)^{K_{B1}-K_{C2}} \theta(X) + \ldots$$

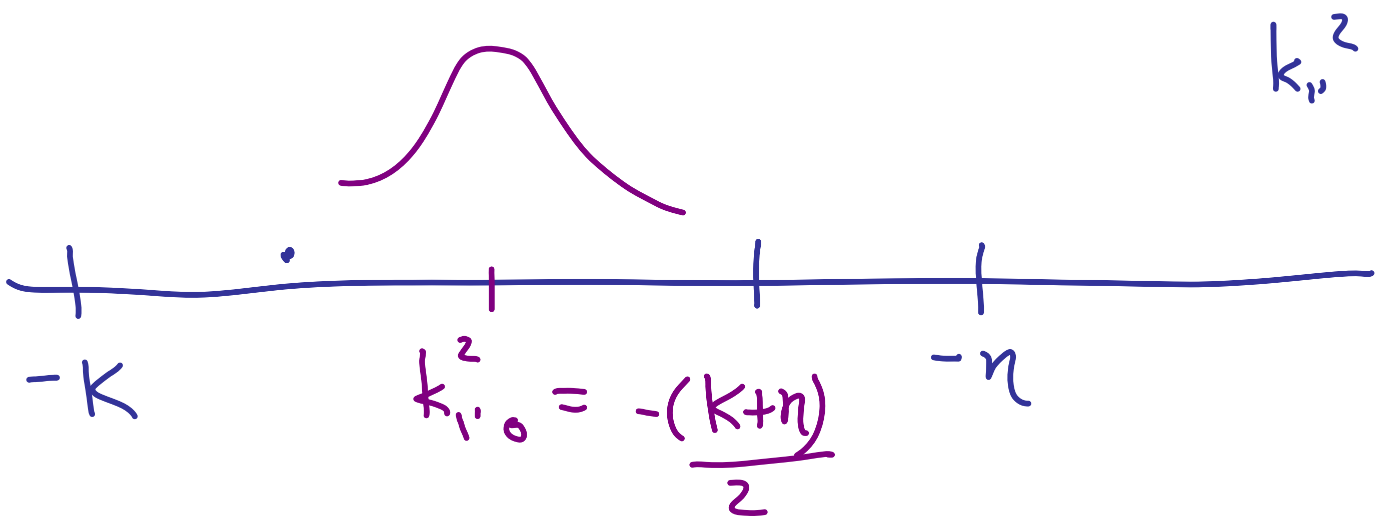
and as mentioned above, the amplitude can be written as a convergent sum over QFT propagators.

Generalization (wider wavepackets in momentum space): in progress

Again work with expansion

$$B(k_{1'}^2,\eta) = B(k_{1'}^2,K)\frac{\Gamma(\eta)\Gamma(k_{1'}^2+K)}{\Gamma(k_{1'}^2+\eta)\Gamma(K)} = \sum_{n_{1'}} \frac{(1-K)_{n_{1'}}}{n_{1'}!}\frac{1}{k_{1'}^2+n_{1'}-i\epsilon}\frac{\sin\pi(k_{1'}^2+\eta)\Gamma(-k_{1'}^2-\eta)\Gamma(k_{1'}^2+K)}{\sin\pi\eta\,\Gamma(-\eta)\Gamma(K)}$$

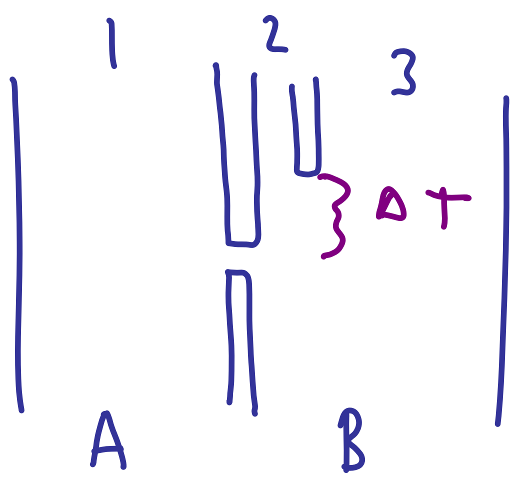
but use wider wavepacket, centered at the minimum of Γ factors. Each term integrated against the wavepacket is like in QFT problem, but shifted in X by X*. Effect survives the alternating sum over n1', with some suppression in size. But still this is giving a stronger effect than above, as well as finer position space amplitude



Next: assorted comments, applications, generalizations, current/future directions

Beyond S-matrix observables: e.g. interaction timescales in flat spacetime scattering

$(5 = 4+1)$ pt scattering



(Frame A & B moving transversely)

Mandelstam...

$$A + B \longrightarrow 1 + 3 \underbrace{+2}_{\text{Collinear radiation}}$$

$$\Delta T \sim E \alpha'$$

Connect to worldsheet relational observables? Captures worldsheet spacetimes scales up to appropriate uncertainties cf Gary/Giddings

*The size of the effect in the general case will dictate how strong the effect can be in BHs and in AdS.

*The simplest example confirming long-range interaction suggests beyond-EFT physics at BH horizons (cf M. Dodelson/ES '15, to be generalized to present examples).

*AdS: emergent (non-)locality (cf Engelhardt/Horowitz,...scattering and bulk points)

*ST is more than a sum of tree-level QFT parts.

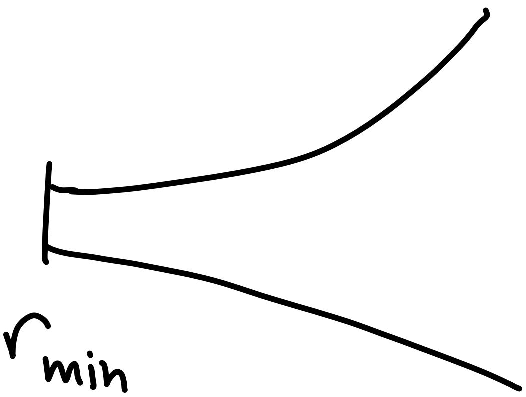
--No obvious way to parameterize beyond-GR physics as EFT expansion.

--What about large-N gauge theory? (work in progress with B. Kang).

AdS glueball scattering

w/B. Kang et al, cf Polchinski/Strassler, C.-I. Tan et al, ...

Perform same scattering experiment for holographic `glueballs' along Poincare slices of AdS.



$$\hat{K}_{IJ} = K_{IJ}\frac{R^2}{r^2}$$

proper

$$ds^2 = \frac{r^2}{R^2}\,dX^2 + \frac{R^2}{r^2}\,dr^2 + R^2 d\Omega^2$$

$$A_{AdS}(p) = \int dr\, d^5\Omega \sqrt{-g}\,\Pi_i \psi_i(r) A(\hat{p})$$

$$\psi(t,x,r) = e^{-iwt+ikx} r^{-d/2} J_\nu(\sqrt{w^2-k^2}R^2/r) \equiv e^{-iwt+ikx}\psi(r)$$

$$\Delta = \nu + d/2$$

- $A(\hat{p}) \sim 2^{\hat{\eta}} = 2^{\eta \frac{R^2}{r^2}}$

  $(\eta < 0)$ shuts off $r \to 0$

$$\to \int \frac{dr}{r^{\sum d + \Delta \equiv \gamma}} \, 2^{\eta \frac{R^2}{r^2}}$$

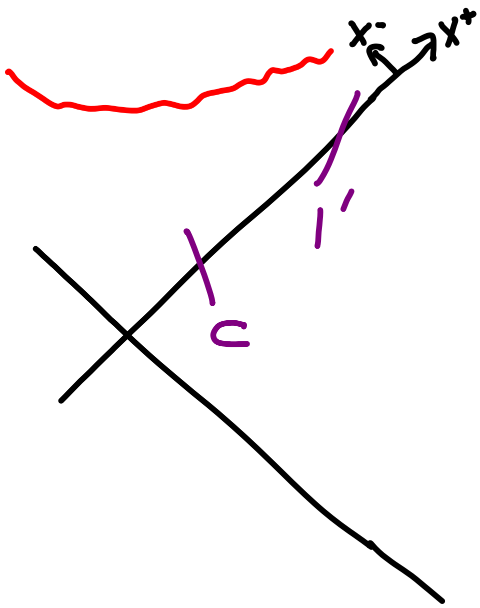
saddle $\frac{\gamma}{r_*} \sim -2\eta \frac{R^2}{r_*^3}$ $\quad \hat{\eta}\Big|_{r_*} \sim -\frac{\gamma}{2}$

Longitudinal spreading could come from $\begin{cases} r \ll r_* \\ \gamma \sim \Delta \gg 1 \end{cases}$

- $A(\hat{p}) \sim (\hat{k}^2_{1,0})^{-\hat{\eta}}$ would give strong effect at $r = r_{min}$

BH/rindler horizons and AdS/CFT constraints

cf M.Dodelson/ES '15, Shenker/Stanford, Fitzpatrick/Kaplan,...



Near horizon.

$$E^2_{n.h.} \sim 2 p^+_{1',h}\, p^-_{c,h} \sim e^{\frac{\Delta t}{2r_s}} m^2$$

$$x^+_{1'} - x^+_c \propto p^+_{1'} \propto e^{\frac{\Delta t}{2r_s}}$$

- $B(C2,12) \sim (E^2)^{-K_{c2}} \sim e^{-\frac{\Delta t}{2R}K_{c2}}$

But that could be auxiliary.

$A \sim 2^{\eta}$ Δt-independent

$(k_{11}^2)^{-\eta}$ grows with Δt

Black Hole Complementarity *vs.* Locality

David A. Lowe,[a] Joseph Polchinski,[b] Leonard Susskind,[c]
Lárus Thorlacius,[b] and John Uglum[c]

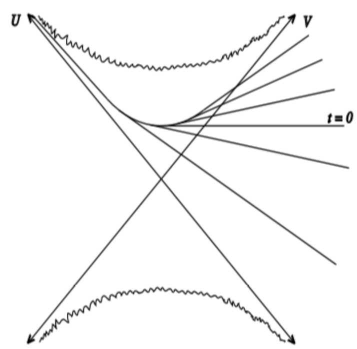


Fig. 2: A family of nice slices.

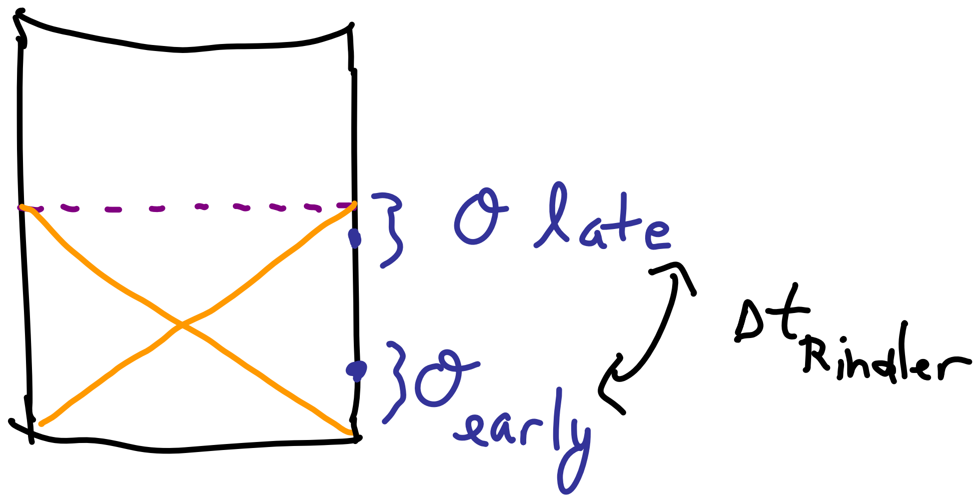
Nonvanishing SFT commutators.

Finally, one should compare the above findings to those obtained from string S-matrix calculations [14]. There one finds a degree of nonlocality far smaller than that present in the light-front string field commutator. Although certain S-matrix elements computed in [14] do display nonlocal effects over macroscopic separations, these amplitudes were found to be highly suppressed. This is to be expected, otherwise an observer crossing an event horizon would necessarily experience a large scale violation of the equivalence principle. It is an important open question whether an interpolating field could be constructed which is "more local" than the light-front string fields.

13

The longitudinal spreading effect can support drama (as opposed to complementarity)

AdS Rindler horizon and OPEs



OPE $\quad \mathcal{O}_e \, \mathcal{O}_\ell \sim e^{-(\Delta t)\Delta_{CFT}}$

- $B(C2,12) \sim (\epsilon^2)^{-K_{C2}} \sim e^{-\frac{\Delta t}{2R} K_{C2}}$

But that could be auxiliary.

- $A \sim 2^n$ $\quad \Delta t$ - independent

  $(k_{\parallel}^2)^{-n}$ grows with Δt

Local ops correspond to particular bulk wave packets, would need to check effect of that.

Bootstrap constraints:

Fitzpatrick/Kaplan: eternal BH version of info problem not solved by any finite sum of conformal blocks at leading order in 1/c.

*They find resolution at finite c via explicit calculations in CFT2/AdS3

*At large c but finite λ, there is room for stringy effects. Our results involve amplitudes without convergent series in QFT propagators, perhaps some connection/application.

Summary:

Longitudinally separated interactions pass strong S-matrix test, at scale ~ $E\alpha'$.

This raises interesting questions and suggests wide array of applications, including interactions between separated horizon infallers.