

# **Recent progress in topological fluid dynamics: from helicity to Jones polynomials**

**Renzo L. Ricca**

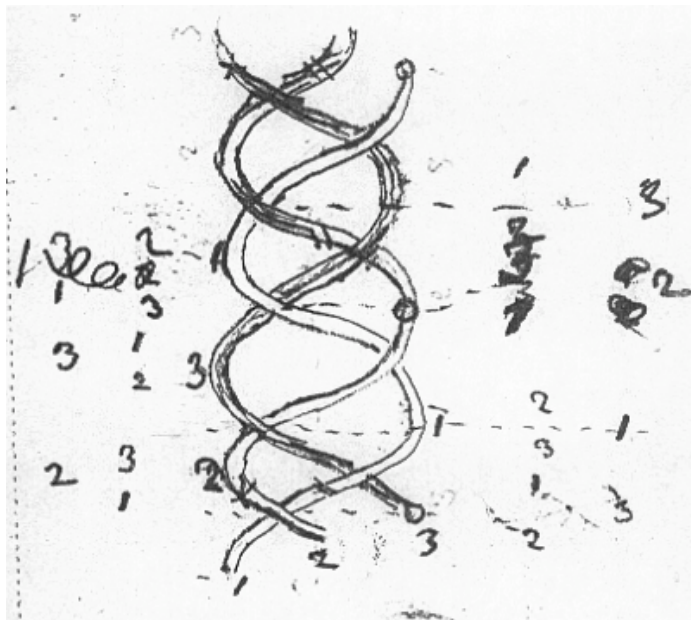
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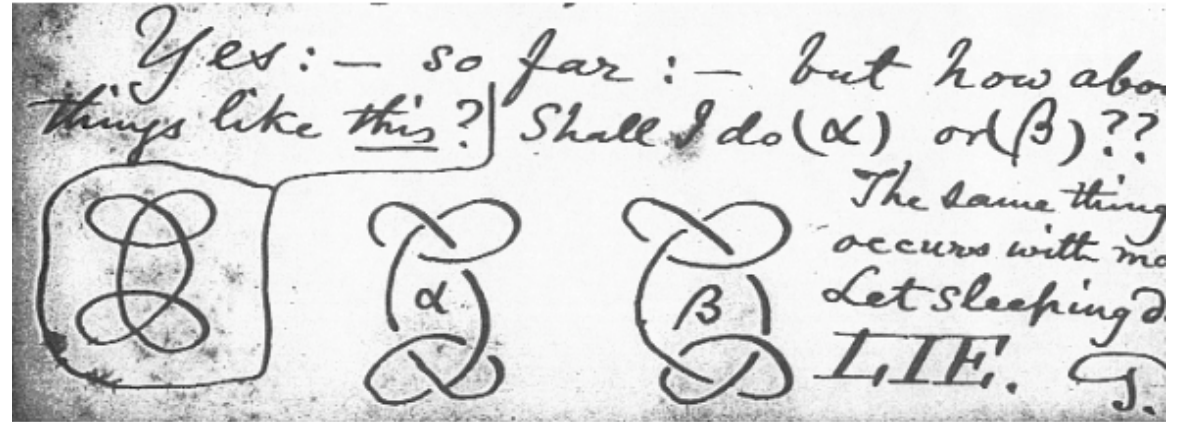
## **Outline**

- **150 years of topological dynamics:**
  - **diffeomorphisms of frozen fields and helicity.**
- **Magnetic fields under energy relaxation:**
  - **from inflexional knots to inflexion-free braids;**
  - **groundstate energy spectrum of first 250 prime knots;**
  - **new lower bounds on energy.**
- **Vortex dynamics:**
  - **torus knot solutions under LIA and Biot-Savart law;**
  - **tangle analysis on energy-complexity relations;**
  - **Jones polynomial for fluid knots from helicity.**

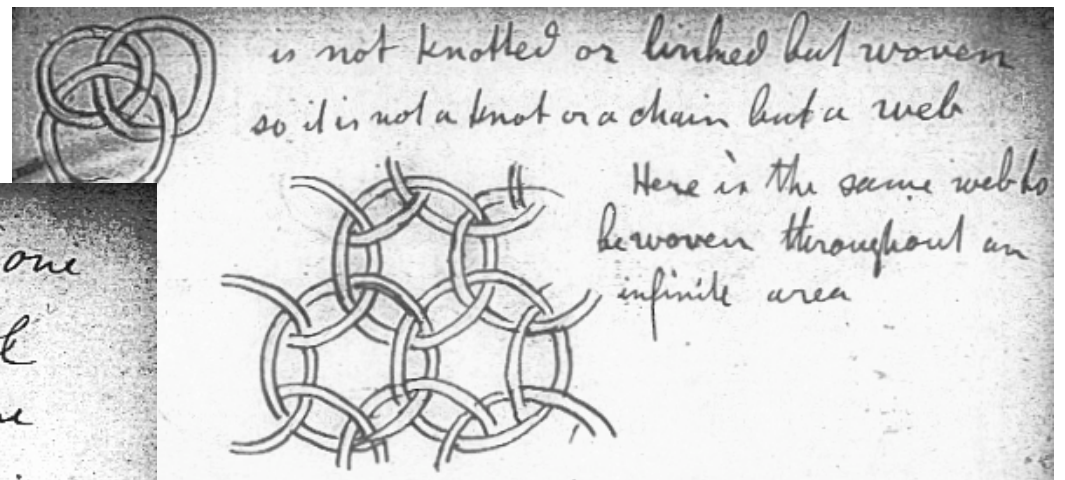
# Maxwell behind the scene



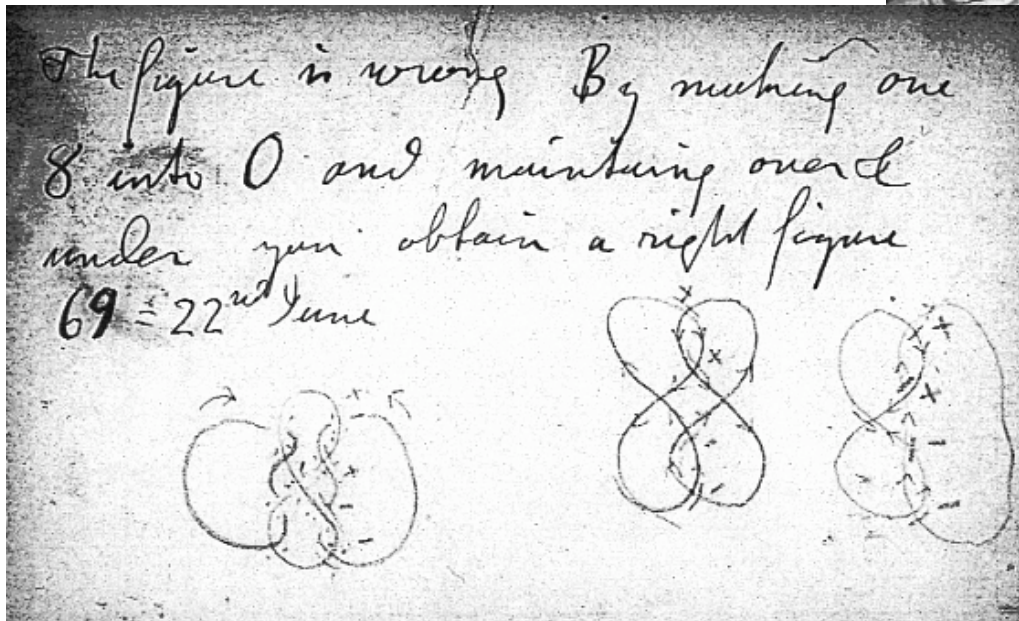
(Lord Kelvin, 1867)



(Tait, 1884)



(Maxwell, 1877)



(Ricca & Weber, in preparation)

# 150 years of topological dynamics

**Linking number formula**

**(Gauss, 1833)**

**Knot tabulation**

**(Tait, 1877)**

**Applications to magnetic fields**

**(Maxwell, 1867)**

**Applications to vortices**

**(Kelvin, 1867)**

**“topological fluid mechanics”**

● **Knotted solutions to Euler’s equations**

● **Energy relaxation methods**

● **Dynamical systems and  $\mu$ -preserving flows**

● **Change of topology**

- 3-D fluid topology
- vortex solutions
- fluid invariants
- topological stability

- magnetic knots
- “charged” knots
- groundstate energy

- $\exists$  Theorems for vector fields
- closed and chaotic orbits
- Hamiltonian structures

- reconnection mechanisms
- singularity formation

## Diffeomorphisms of frozen fields

- }

**ideal, incompressible**  
**perfectly conducting**

**fluid in  $\mathbb{R}^3$ :**

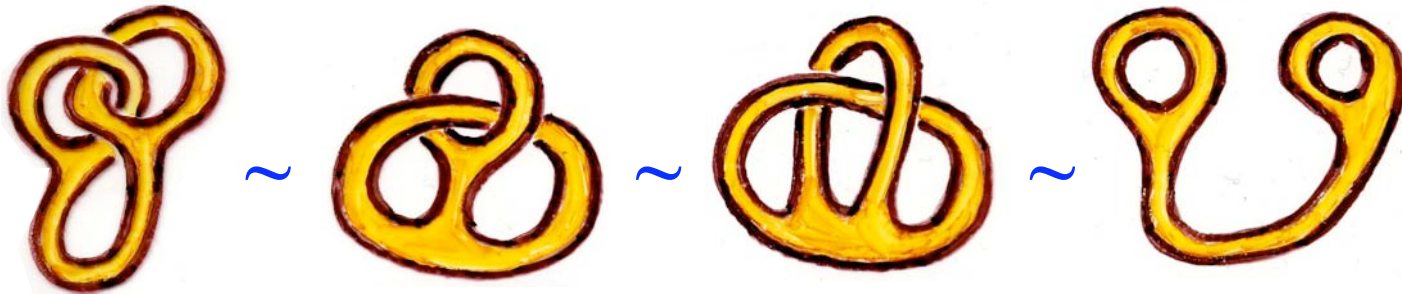
$$\mathbf{u} = \mathbf{u}(\mathbf{X}, t) \begin{cases} \nabla \cdot \mathbf{u} = 0 & \text{in } \mathbb{R}^3 \\ \mathbf{u} = 0 & \text{as } \mathbf{X} \rightarrow \infty \end{cases}$$

- frozen field evolution:**

$$\mathbf{B}(\mathbf{X}, t) \in \left\{ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) \wedge \nabla \cdot \mathbf{B} = 0; L_2 - \text{norm} \right\}$$

- topological equivalence class:**

$$B_i(\mathbf{X}, t) = B_j(\mathbf{X}_0, 0) \frac{\partial X_i}{\partial X_{0j}} \quad : \quad \mathbf{B}(\mathbf{X}_0, 0) \sim \mathbf{B}(\mathbf{X}, t)$$



## Physical knots and links as tubular embeddings

Let  $\mathcal{T}_i = \mathcal{S}_i \otimes \mathcal{C}_i$  and  $V_i = V(\mathcal{T}_i)$ :

$$\mathcal{T}_i \rightarrow \mathcal{K}_i \quad \text{in } \mathbb{R}^3$$

- **magnetic embedding:**

$$\mathcal{K}_i := \text{supp}(\mathbf{B})$$

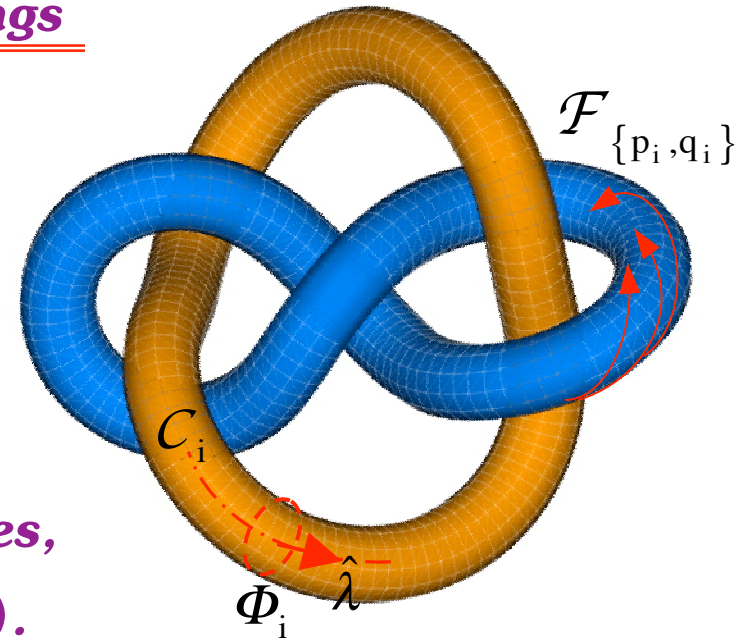
by a standard foliation  $\mathcal{F}_{\{p_i, q_i\}}$  of the  $\mathbf{B}$ -lines, such that  $\mathbf{B} \cdot \hat{\nu} = 0$  on  $\partial\mathcal{T}_i$  (material surface).

- **Definition:** A physical knot/link is a smooth immersion into  $\mathbb{R}^3$  of finitely many disjoint standard solid tori  $\mathcal{T}_i$ , such that

$$\text{supp}(\mathbf{B}) := \bigcup_i \mathcal{K}_i \rightarrow \mathcal{L}_n \quad (i=1, \dots, n).$$

- **volume and flux-preserving diffeomorphism:**

$$V = V(\mathcal{L}_n), \quad \Phi_i = \int_{A(S_i)} \mathbf{B} \cdot \hat{\lambda} \, d^2\mathbf{x} : \quad \text{signature } \{V, \Phi_i\} \text{ constant.}$$



## Helicity and linking numbers

- **Magnetic helicity**  $H(t)$  :  
$$H(t) := \int_{V(\mathcal{L}_n)} \mathbf{A} \cdot \mathbf{B} \, d^3\mathbf{X}$$

where  $\mathbf{B} = \nabla \times \mathbf{A}$ , with  $\nabla \cdot \mathbf{A} = 0$  in  $\mathbb{R}^3$ .

- **Theorem (Woltjer 1958)**. *In ideal fluid magnetic helicity is frozen in the flow, that is*

$$\frac{d}{dt} H(\mathcal{L}_{n,\varphi}) = 0 \quad \Rightarrow \quad H(t) = H.$$

- **Theorem (Moffatt 1969; Moffatt & Ricca 1992)**. *Let  $\mathcal{L}_n$  be an essential magnetic link in an ideal fluid. Then, we have*

$$\begin{aligned} H &= \int_{V(\mathcal{L}_n)} \mathbf{A} \cdot \mathbf{B} \, d^3\mathbf{X} = \sum_i Lk_i \Phi_i^2 + 2 \sum_{i \neq j} Lk_{ij} \Phi_i \Phi_j \\ &= \sum_i (Wr + Tw) \Phi_i^2 + 2 \sum_{i \neq j} Lk_{ij} \Phi_i \Phi_j . \end{aligned}$$

## Lorentz force on magnetic flux tubes in ideal MHD

**Ideal magnetohydrodynamics (MHD): tubular knot**  $K : V(K) = \pi a^2 \cdot L$  ;

- **magnetic field in cylindrical coordinates**  $(r, \vartheta, s)$  :

$$\mathbf{B} = \mathbf{B}_m + \mathbf{B}_a = (0, B_\vartheta(r), B_s(r)) ;$$

**in terms of fluxes**  $\Phi_P, \Phi_T$  :

$$\mathbf{B} = \left( 0, \frac{1}{L} \frac{d\Phi_P}{dr}, \frac{1}{2\pi r} \frac{d\Phi_T}{dr} \right) + \left( 0, \frac{\partial \tilde{\psi}}{\partial s}, -\frac{\partial \tilde{\psi}}{\partial \vartheta_R} \right) ,$$

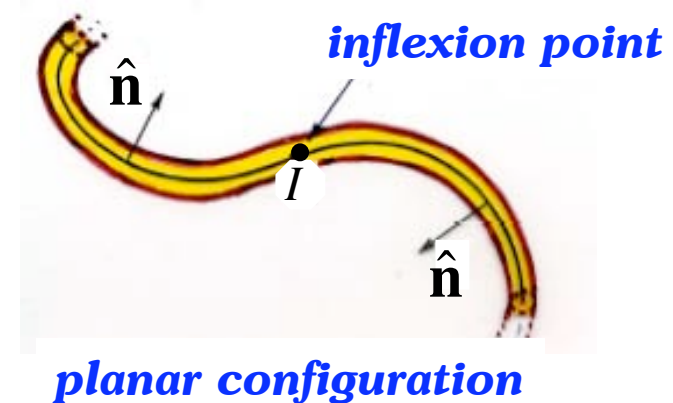
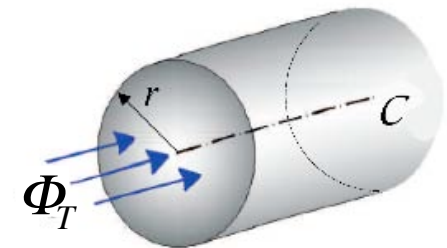
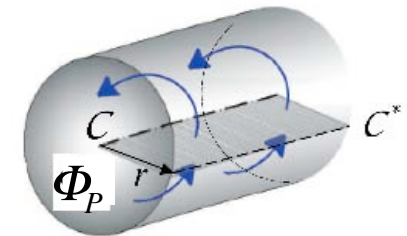
**with twist parameter**  $h = \Phi_P / \Phi_T$  .

- **Lorentz force:**  $\mathbf{F} = \mathbf{J} \times \mathbf{B} = (\nabla \times \mathbf{B}) \times \mathbf{B}$  .

**If**  $\mathbf{B}_m \ll \mathbf{B}_a$  , **then:**

$$\underline{\underline{\mathbf{F} \propto c \hat{\mathbf{n}}}}$$

**“curvature flow”**



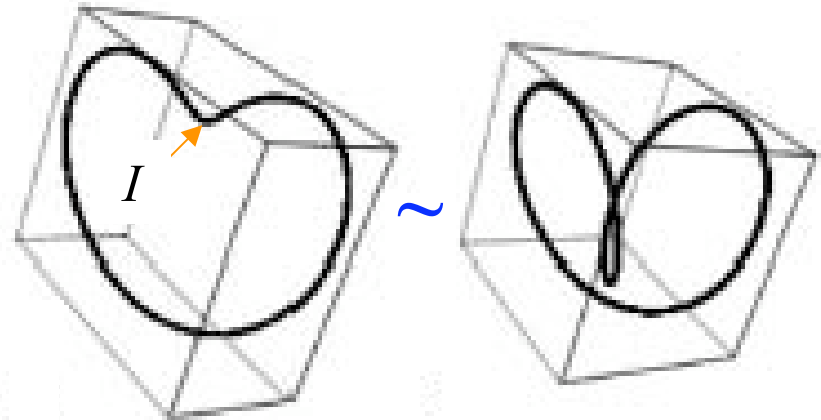
## Inflexional configuration and Reidemeister type I move

- **Inflexion at  $I$  (in isolation):**  $c = 0$

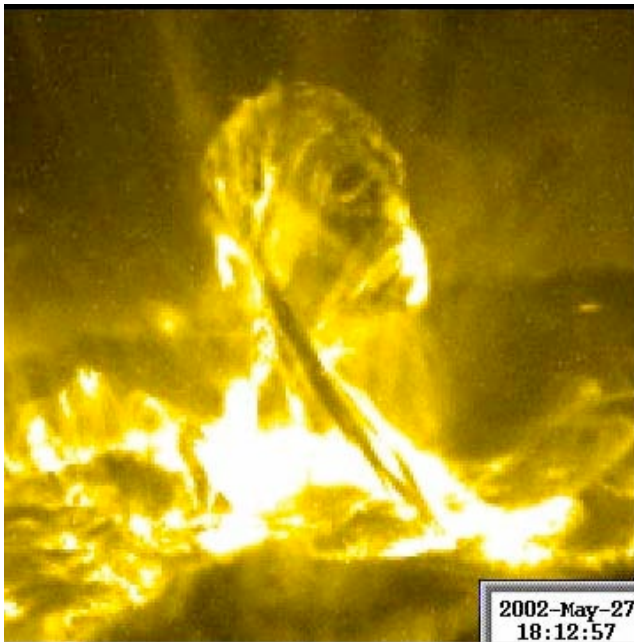
**generic behaviour in  $\mathbb{R}^3$ :**

$$\mathbf{X}(s,t) = \left( s - \frac{2}{3}t^2s^3, -ts^2, s^3 \right)$$

**(Ricca & Moffatt, 1992).**



- **Reidemeister type I move in action:**

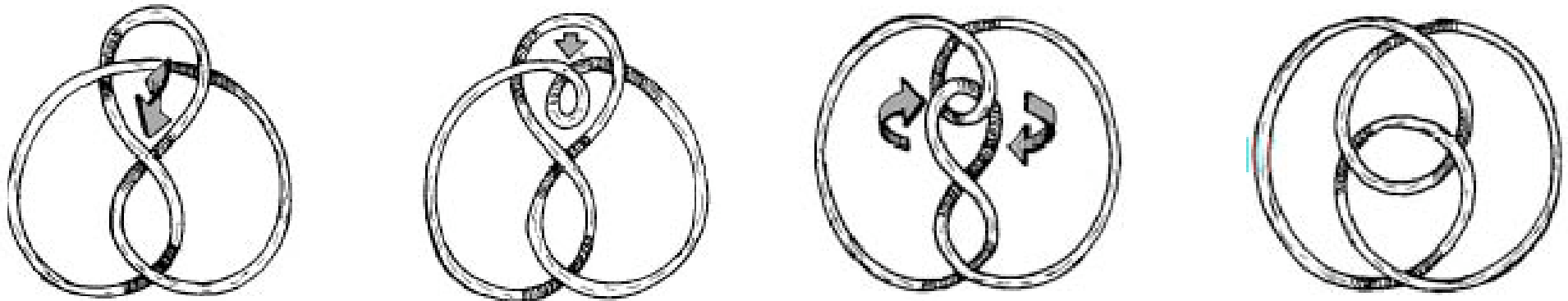


**(TRACE, 2002)**

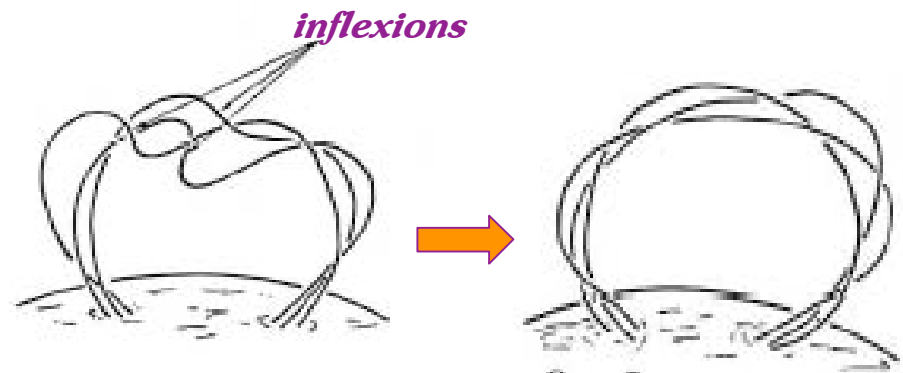


## From inflexion-free knots to braids

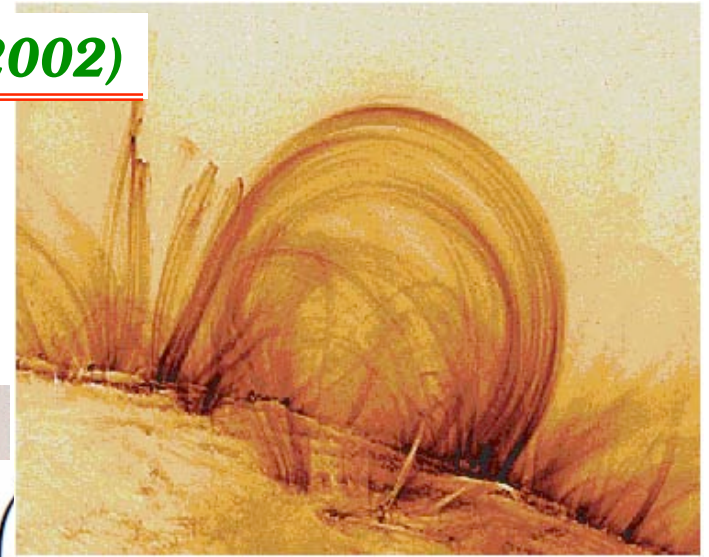
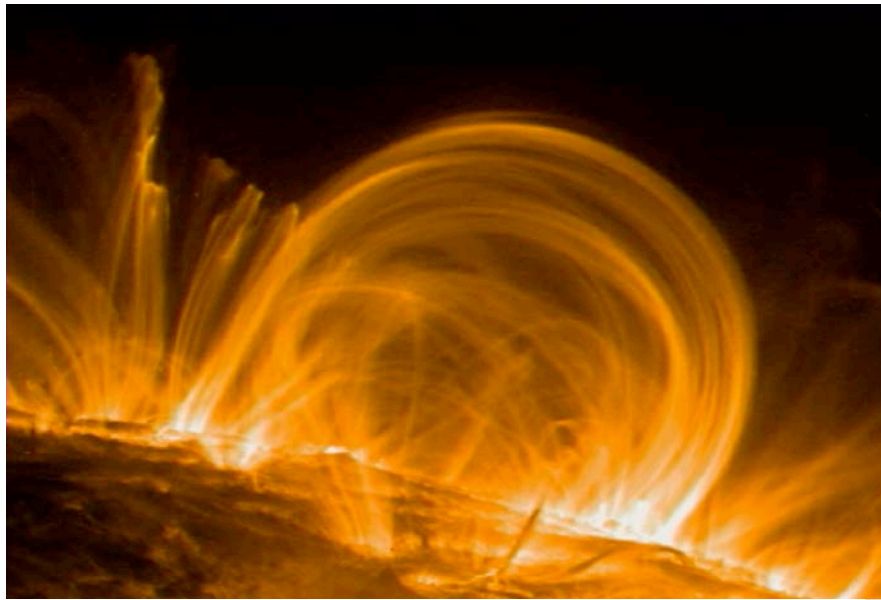
- **Definition.** A **spiral knot** is a knot free from inflexion points in isolation.
- **Figure 8 knot in braid presentation:**



- **Theorem (Ricca, 2005).** Let  $\tilde{\mathcal{K}}_{t_0}$  denote a loose magnetic knot in inflexional state. Then  $\tilde{\mathcal{K}}_{t_0}$  is in inflexional disequilibrium.
- **Corollary.**  $\tilde{\mathcal{K}}_{t_0}$  is naturally isotoped to a spiral knot  $\mathcal{K}_t$  for any  $t > t_0$ .



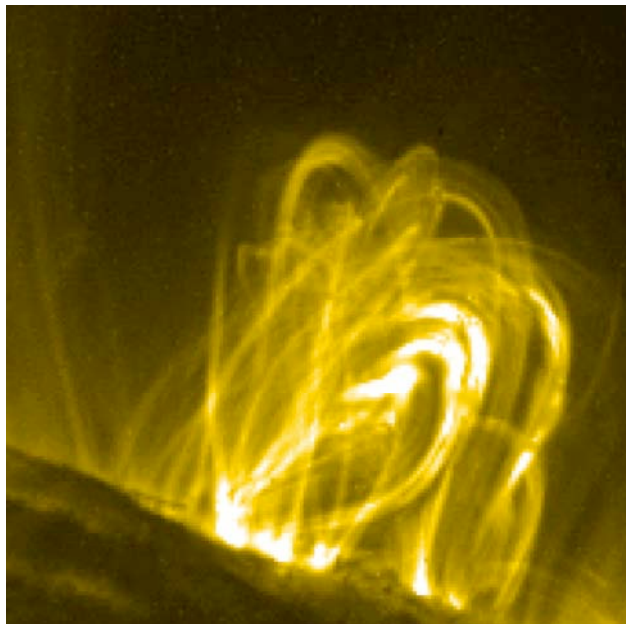
# Magnetic braids in the solar corona (TRACE, 2002)



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

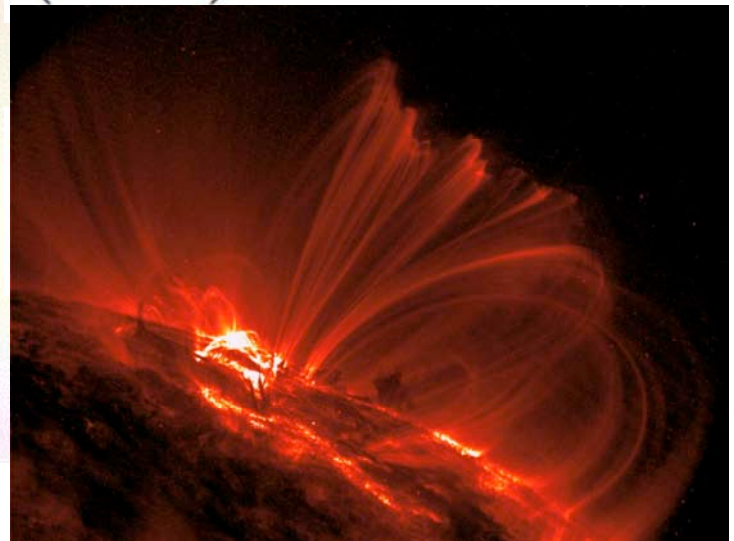
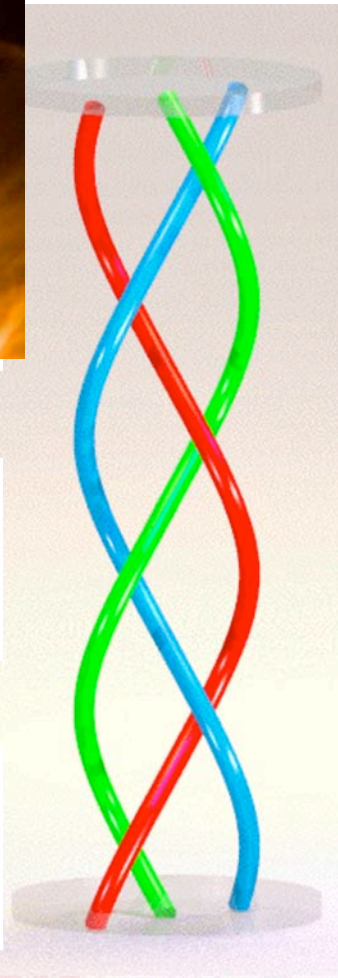
**Mitch Berger**  
**(U. Exeter)**

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$



## Magnetic relaxation

Let  $\mathcal{L}_n$  be a zero-framed magnetic link:  $\begin{cases} n \text{ components} & Lk_i = 0 \quad \forall i \\ \text{equal flux} & \Phi_i = \Phi \end{cases}$

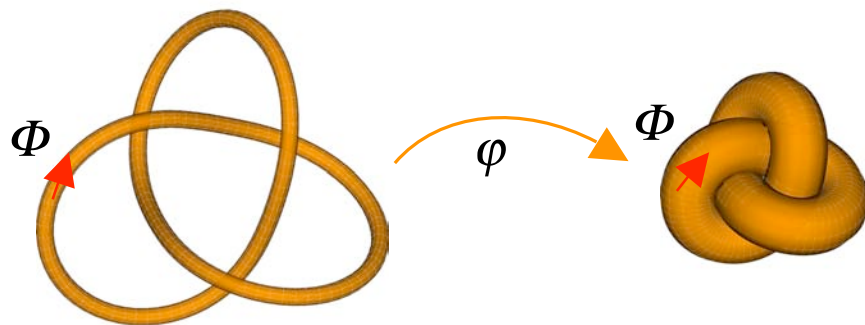
- **Magnetic energy:**

$$M(t) = \frac{1}{2} \int_{V(\mathcal{L}_n)} \|\mathbf{B}\|^2 d^3\mathbf{x} \quad ,$$

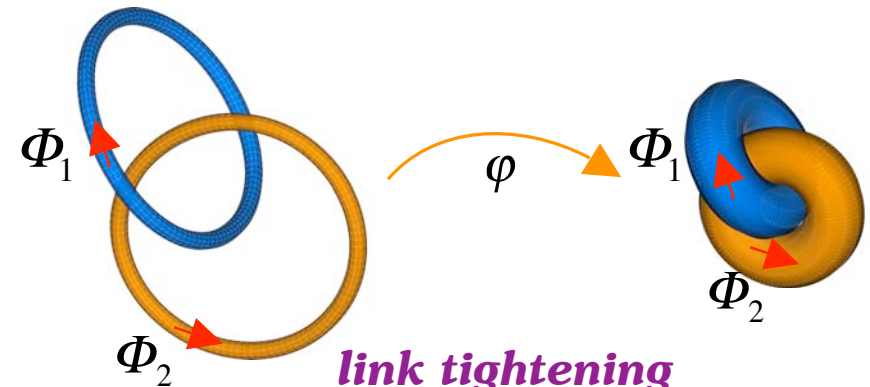
- **Magnetic helicity:**

$$H = \int_{V(\mathcal{L}_n)} \mathbf{A} \cdot \mathbf{B} d^3\mathbf{x} = 2\Phi^2 \sum_{i \neq j} Lk_{ij} \quad .$$

- **Magnetic relaxation (Moffatt, 1985) under  $\{V, \Phi_i\}$ -preserving flow:**



knot tightening



link tightening

- **Theorem 1 (Arnold, 1974; Freedman, 1988; Moffatt, 1990; Freedman & He, 1991; Ricca, 2008).** Let  $\mathcal{L}_n$  be a zero-framed link. Then,

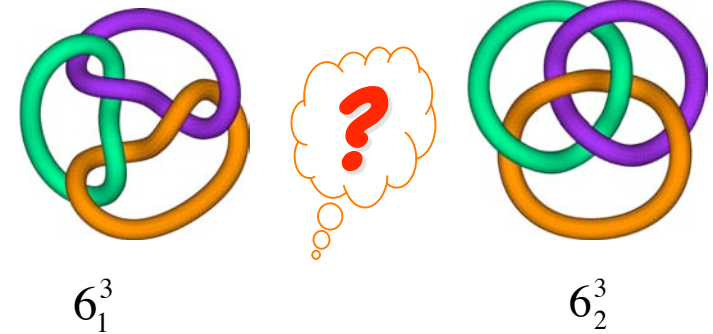
i)  $M(t) \geq \left( \frac{2}{\pi V} \right)^{1/3} |H| \quad ;$

ii)  $M_{\min} = \left( \frac{2\Phi^6}{\pi V} \right)^{1/3} c_{\min} \quad .$

## Constrained minimization of magnetic energy of knots

Under signature-preserving flows,  
we have:

$$M_{\min} \propto C_{\min}$$



- **Assumptions:**

- **tubular knot**  $\mathcal{K}$  :  $V(\mathcal{K}) = \pi a^2 L$  ; **Mercier (orthogonal) system:**  $(r, \vartheta, s)$

- **magnetic field:**  $\mathbf{B} = (0, B_\vartheta(r), B_s(r))$  ( $\nabla \cdot \mathbf{B} = 0$ )

- **fluxes**  $\Phi_P, \Phi_T$  ; **twist parameter:**  $h = \Phi_P / \Phi_T$

- **Theorem (Maggioni & Ricca, 2009).** *Let us assume that*

- (i)  $\{V, \Phi\}$  *invariant* ( $V=1, \Phi=1$ );

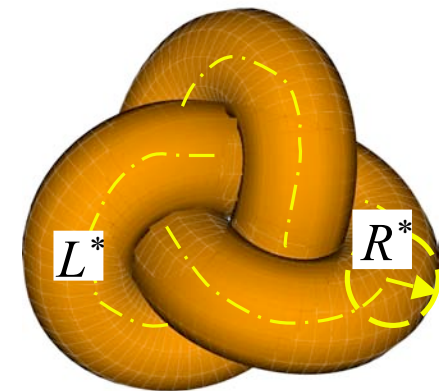
- (ii) *circular cross-section independent of arc-length;*

- (iii)  $\tilde{\psi}$  *independent of arc-length;*

- (iv)  $L$  *independent of internal twist.*

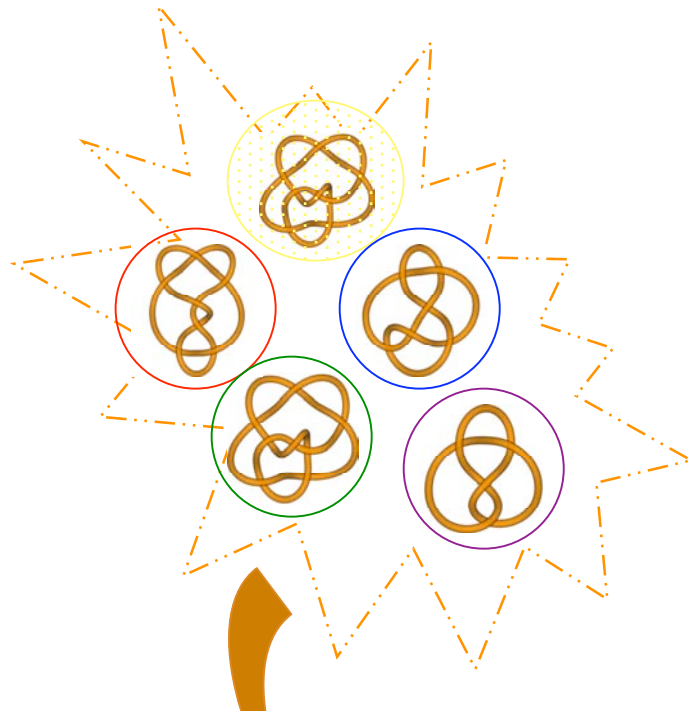
*Then,*

$$M_\lambda^*(h) = \frac{(\lambda)^{4/3}}{2\pi^{2/3}} + \frac{\pi^{4/3} h^2}{(\lambda)^{2/3}} \cdot$$



**ropelength:**  $\lambda = L^* / R^*$

# Groundstate energy spectrum: averaging over complexity

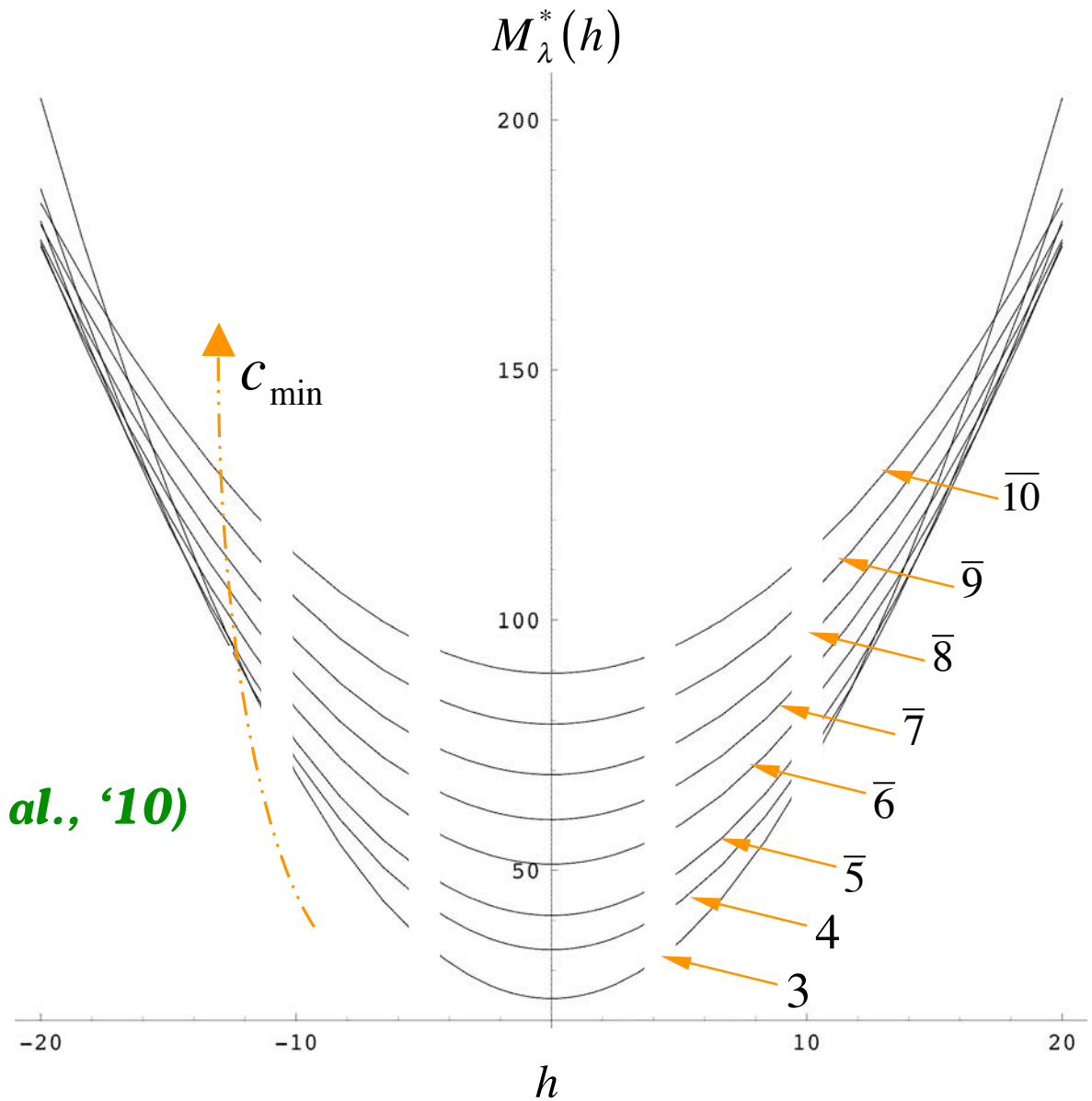


**tightening** ( $h = 0$ )

**SONO** (Pieranski, '98)

**RIDGERUNNER** (Ashton et al., '10)

$V = 1, \Phi = 1$

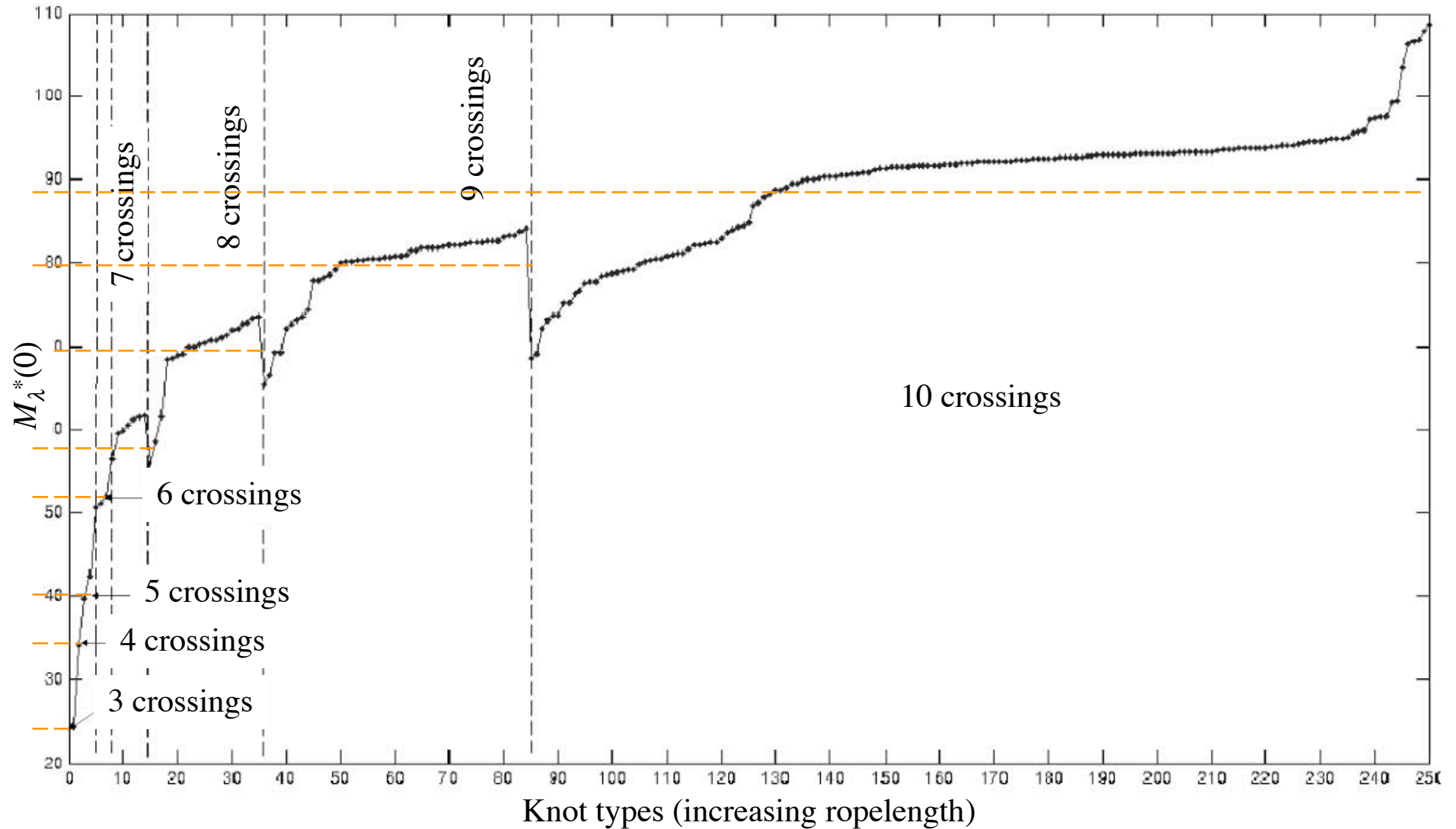


(Maggioni & Ricca, Proc. Roy. Soc. A 465, 2009)

## Groundstate energy spectrum of first 250 prime knots

●  $V = 1, \Phi = 1, h = 0 :$

$$M_{\lambda}^*(0) = \frac{(\lambda)^{4/3}}{2\pi^{2/3}}$$

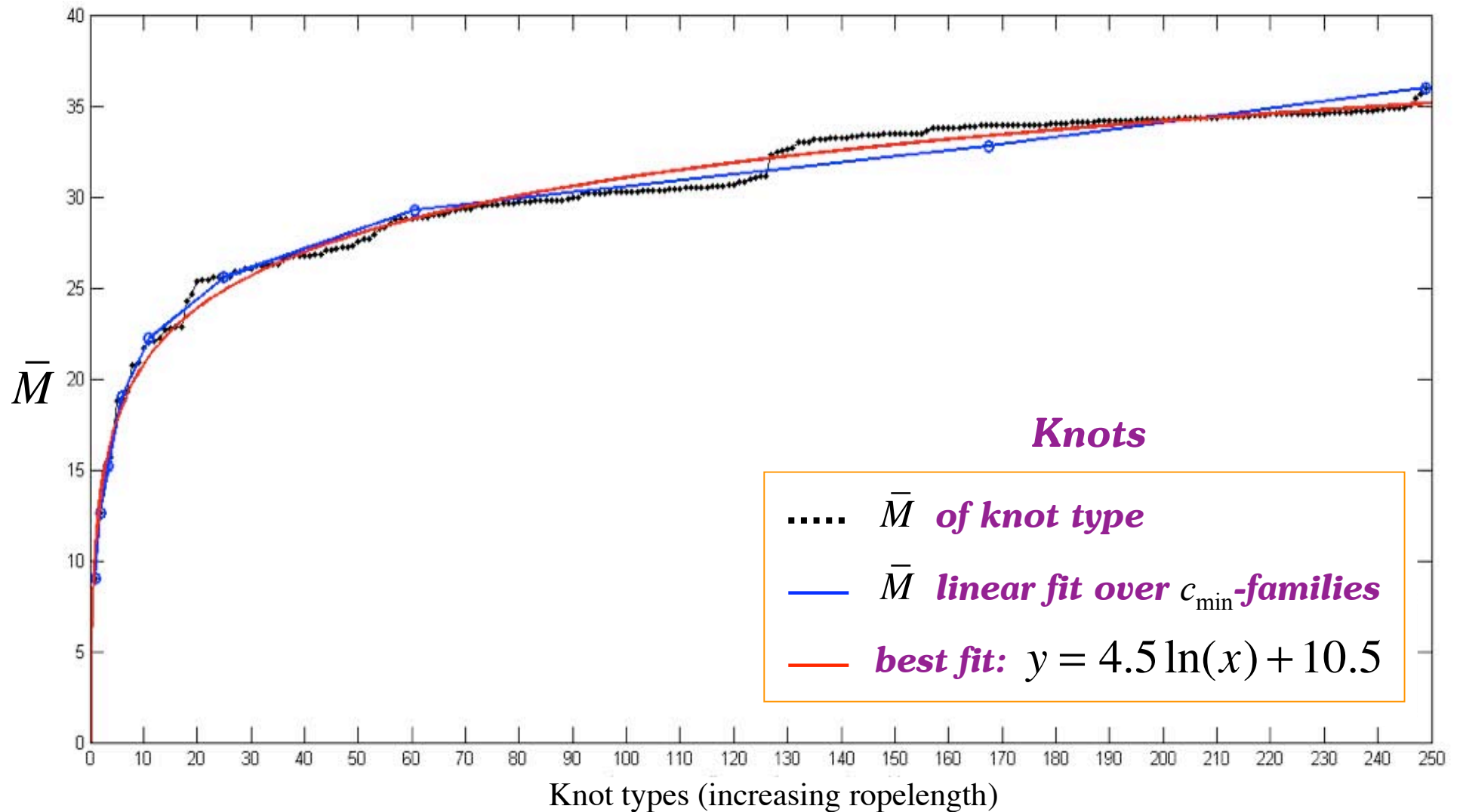


## Knot energy spectrum: normalized energy vs. ropelength

•  $V = 1, \Phi = 1, h = 0$

• **tight unknot:**  $M_o^* = (2\pi^2)^{1/3}$

$$\bar{M} = \frac{M_\lambda^*(0)}{M_o^*} = \left( \frac{\lambda}{2\pi} \right)^{4/3}$$

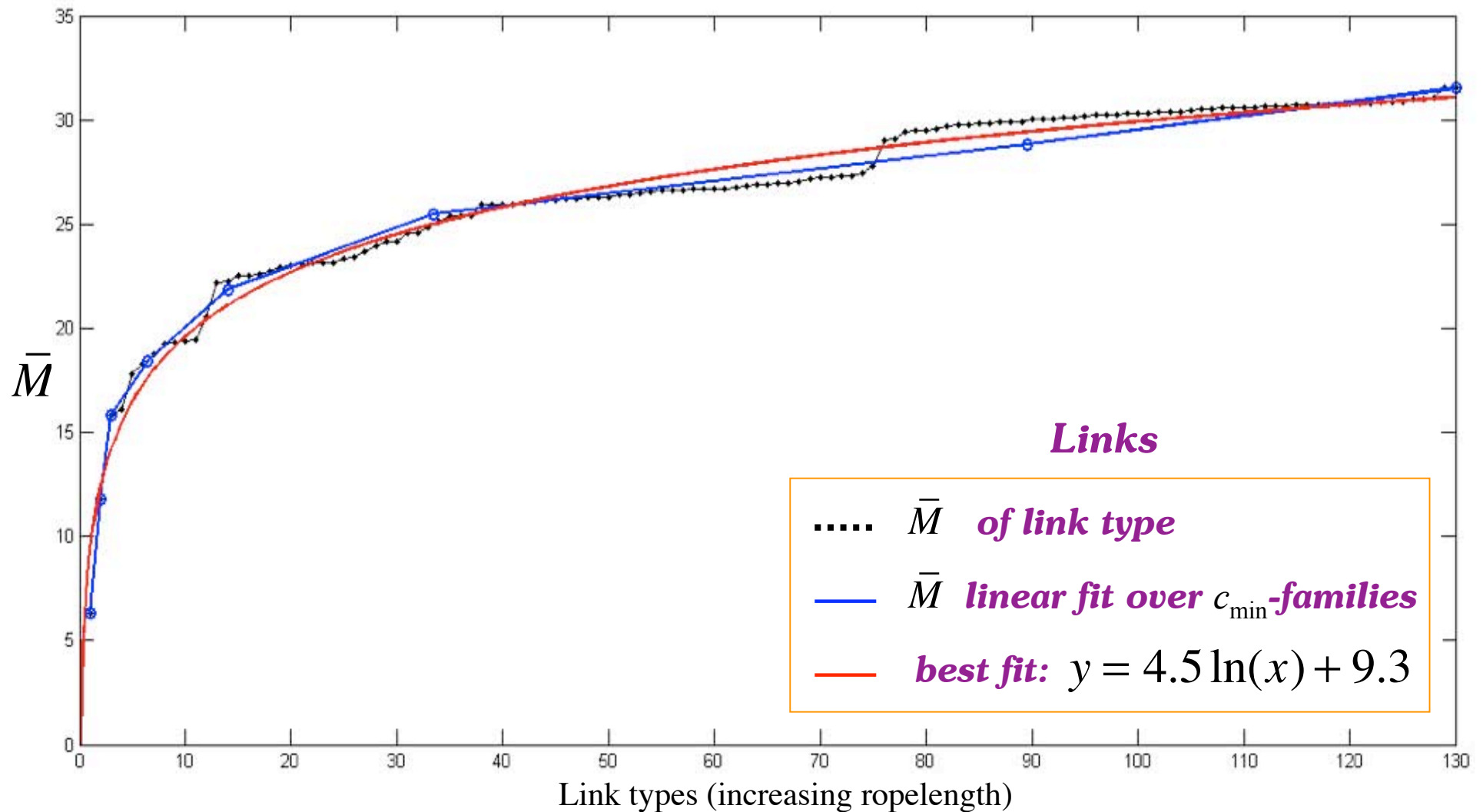


## Link energy spectrum: normalized energy vs. ropelength

- $V = 1, \Phi = 1, h = 0$

- **tight unknot:**  $M_o^* = (2\pi^2)^{1/3}$

$$\bar{M} = \frac{M_\lambda^*(0)}{M_o^*} = \left( \frac{\lambda}{2\pi} \right)^{4/3}$$





## New lower bounds for tight knots, links and braids

By comparing energy minima, we have:

$$M_{\lambda}^*(0) = \frac{\lambda^{4/3}}{2\pi^{2/3}} \geq M_{\min} = \left(\frac{2}{\pi}\right)^{1/3} c_{\min} ,$$

hence:

$$\lambda \geq (16\pi)^{1/4} c_{\min}^{3/4} \approx \underline{2.66} c_{\min}^{3/4} \quad \forall c_{\min} .$$

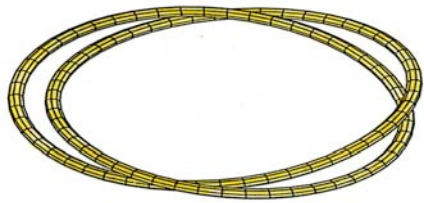
Best results for constant so far:

- for any  $c_{\min}$  : **Buck & Simon (1999)**:  $(4\pi/11)^{3/4} \approx \underline{1.10}$
- for small  $c_{\min}$  : **Denne, Diao & Sullivan (2000)**: ...  
**Cantarella, Kusner & Sullivan (2002)**: ...
- for  $c_{\min} = 3$  : **by SONO Baranska et al. (2004)**: 14.04
- **Minimal braids  $\mathcal{B}$** : by using **Ohyama (1993) inequality**, we have:

$$M_{\min} \geq \left(\frac{16}{\pi}\right)^{1/3} (b(\mathcal{B}) - 1), \quad b(\mathcal{B}): \text{braid index of } \mathcal{B} .$$

## Vortex knots and links

### ● *Knotted solutions to Euler equations*

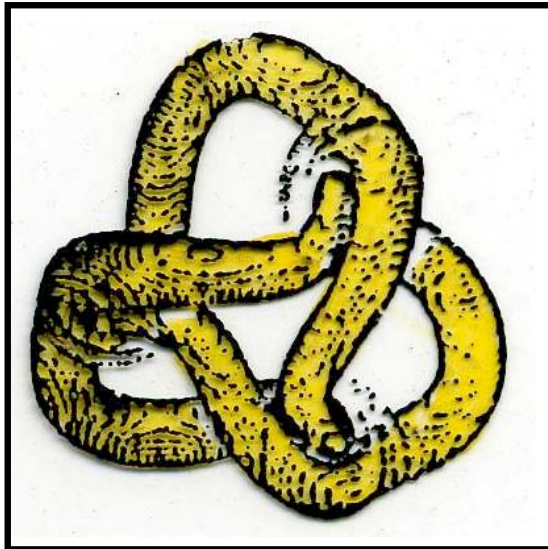


$$\begin{cases} r^2 = F_r(J) \\ \alpha = F_\alpha(E) \\ z = F_z(\Pi) \end{cases}$$

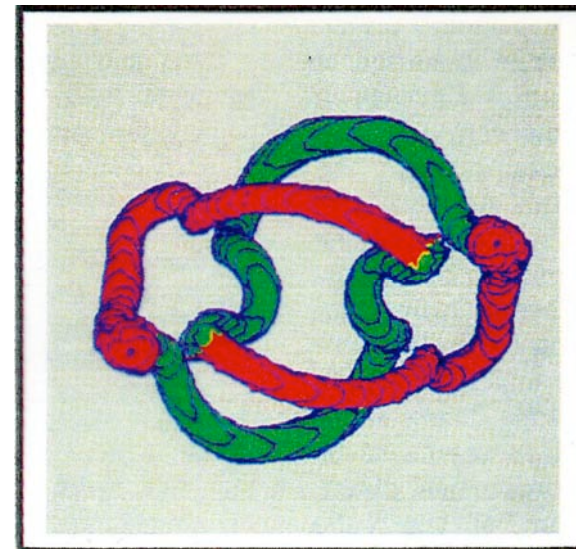
- *existence of torus knot solutions*
- *existence of knotted chaotic orbits*
- *stability criteria*
- *fluid invariants, soliton invariants, Hamiltonian structures & knot types*

*Kida, Keener, Ricca, MacKay, Ghrist, Sullivan, Fuentes, Enciso & Peralta-Salas, ...*

### ● *Visiometric approach to knotting and linking*

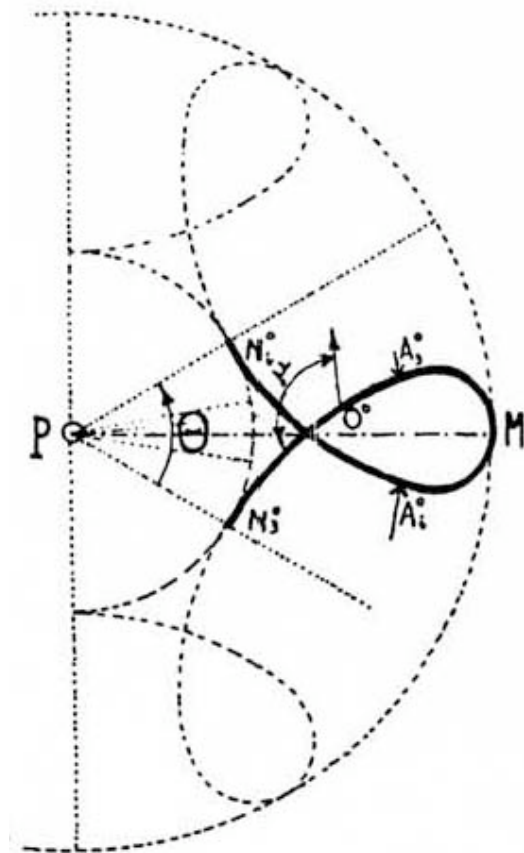


*(Kida & Takaoka, 1988)*



*(Aref & Zawadzki, 1991)*

## Steady torus knots under LIA ( $\mathbf{u}_{LIA} = c\hat{\mathbf{b}}$ )



(Levi-Civita, 1932  
Da Rios, 1933)

- **Kida's class: existence of torus knot solutions  $\mathcal{T}_{p,q}$  ( $p > 1, q > 1$  co-prime integers) in terms of incomplete elliptic integrals:**

(Kida, 1981)

$$\begin{cases} r^2 = F_r(J) \\ \alpha = F_\alpha(E) \\ z = F_z(\Pi) \end{cases}$$

- **Solutions  $\mathcal{T}_{p,q}$  in explicit analytic closed form:**

(Ricca, 1993)

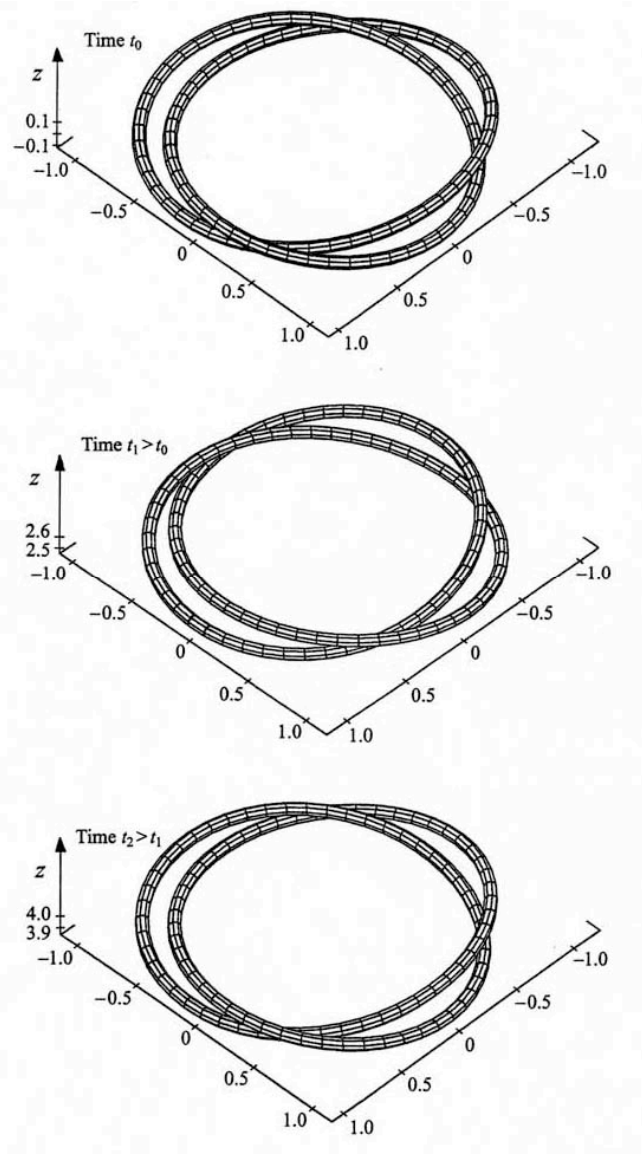
$$\begin{cases} r = r_0 + \varepsilon \sin \frac{w\xi}{r_0} \\ \alpha = \frac{s}{r_0} + \frac{\varepsilon}{wr_0} \cos \frac{w\xi}{r_0} \\ z = \frac{\bar{t}}{r_0} + \varepsilon \left(1 - \frac{1}{w^2}\right)^{1/2} \cos \frac{w\xi}{r_0} \end{cases}$$

- **Linear stability criterium: given  $w = q/p$ , then**

“ $\mathcal{T}_{p,q}$  steady & stable iff  $w > 1$ ”

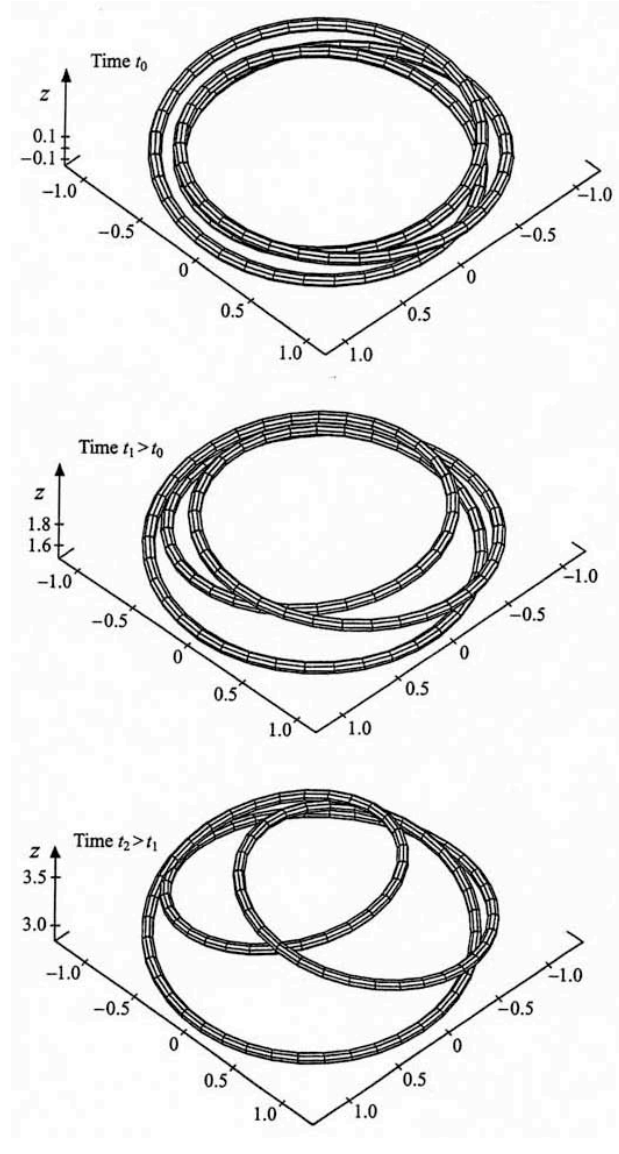
# Numerical evolution of $\mathcal{T}_{p,q}$ under LIA

30-35 radii



$$\mathcal{T}_{2,3} \quad (w > 1)$$

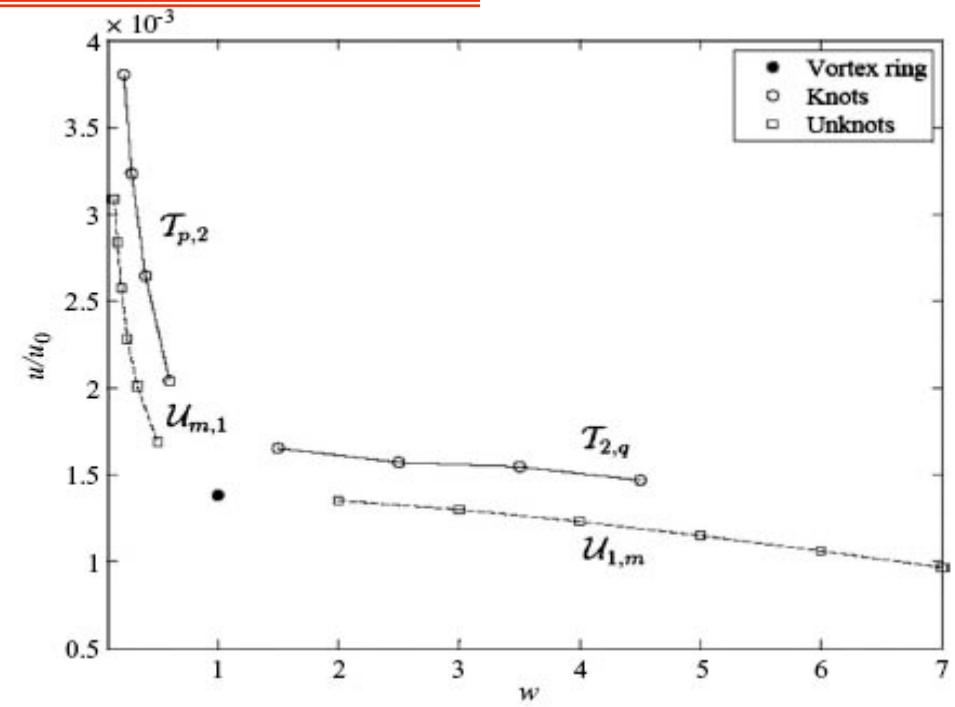
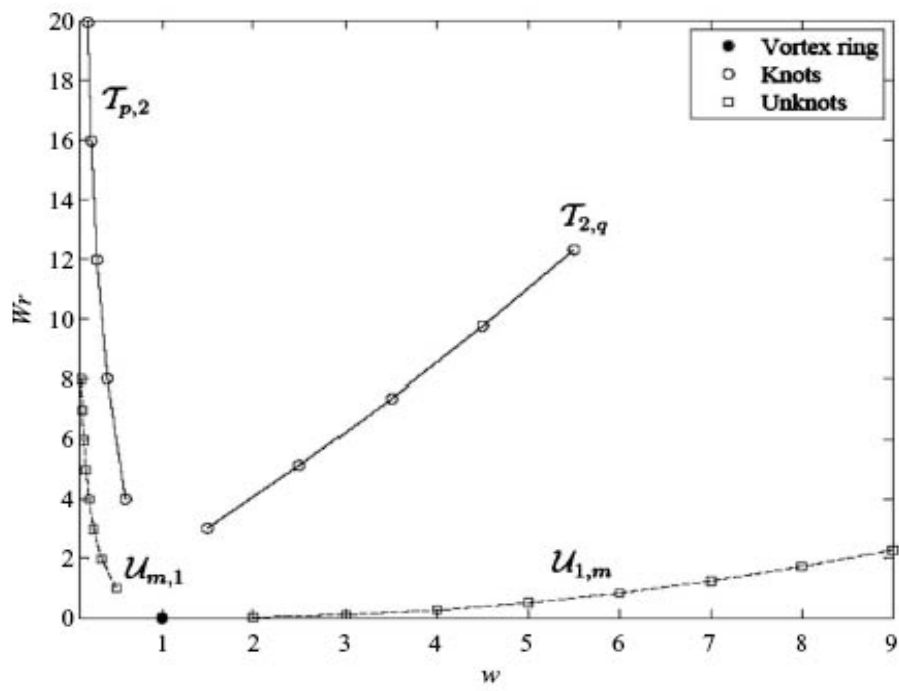
3-5 radii



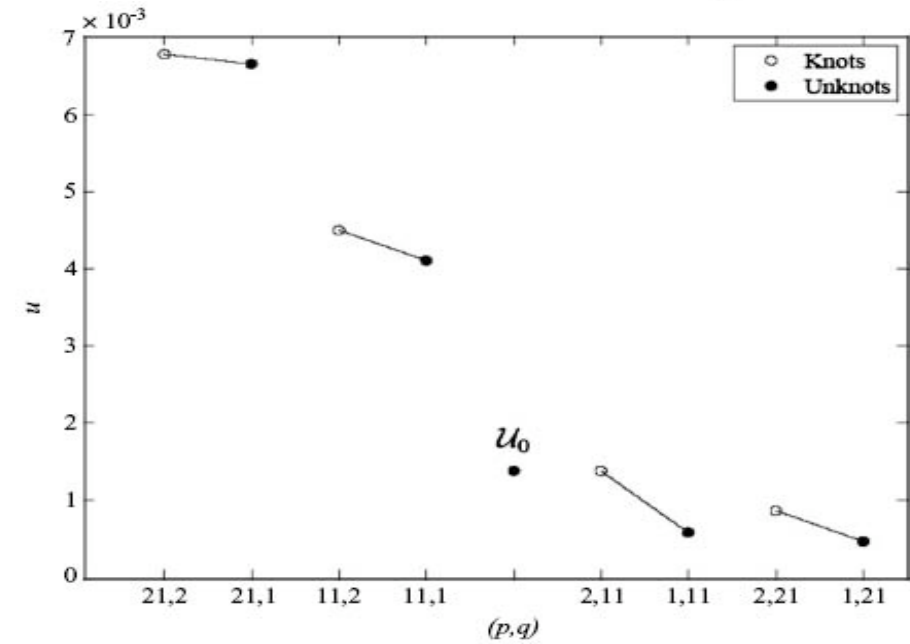
$$\mathcal{T}_{3,2} \quad (w < 1)$$

(Ricca et al., JFM 391, 1999)

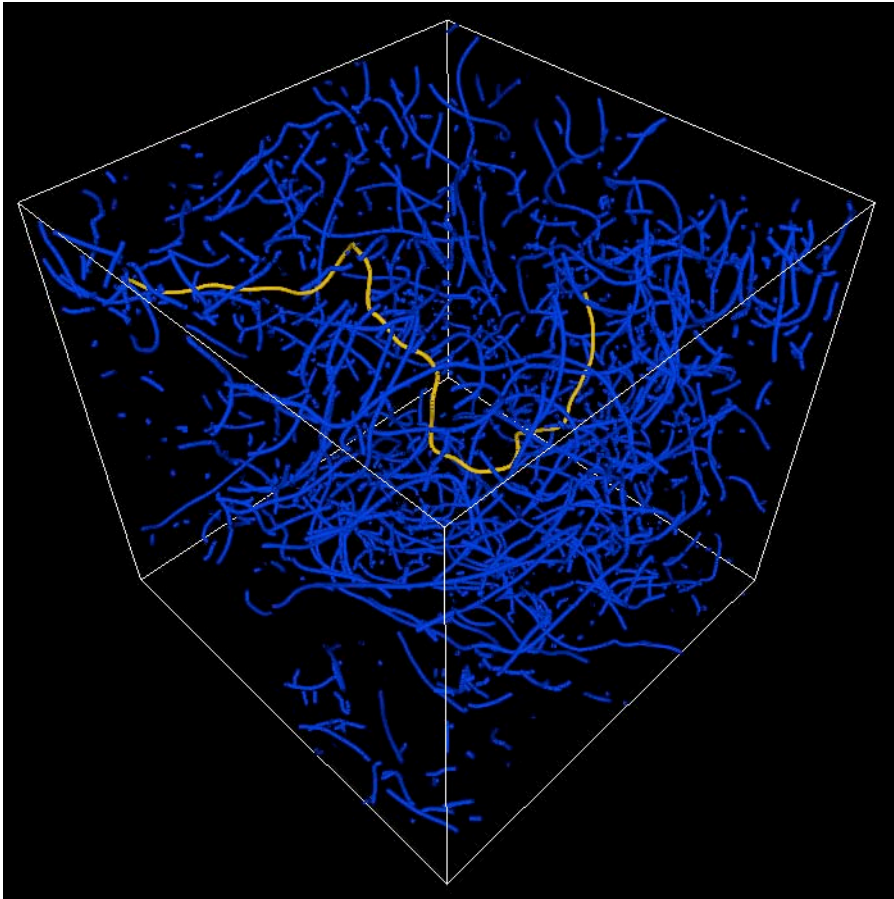
# LIA knots and unknots: writhing and velocity vs. $w = q/p$



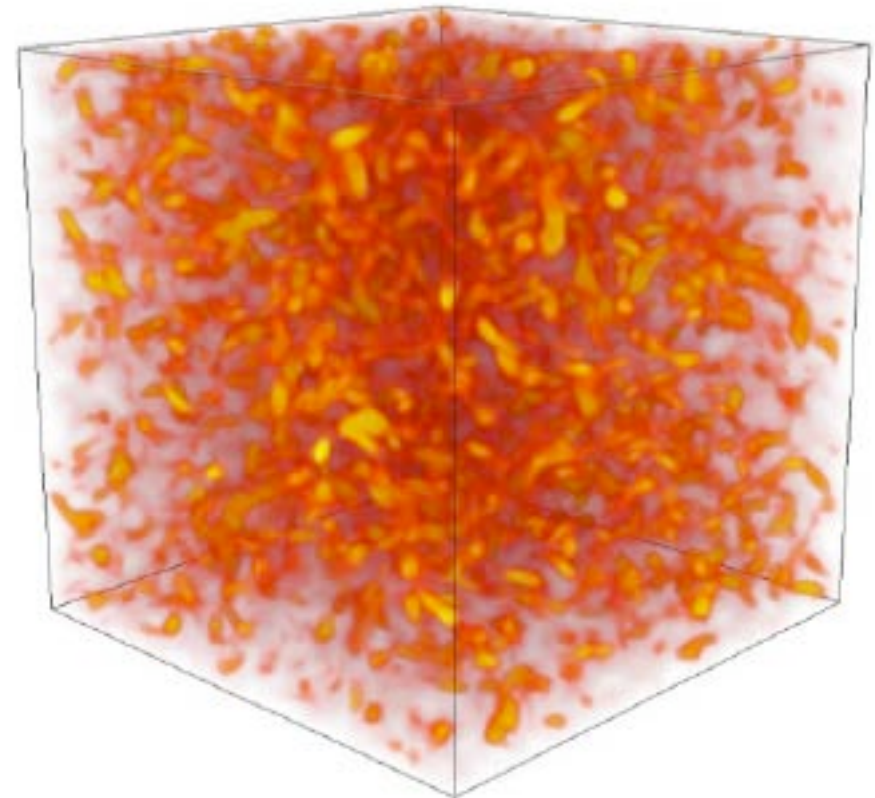
*Maggioni et al.,  
Phys Rev. E 82, 2010*



## Vorticity localization in classical and quantum fluids



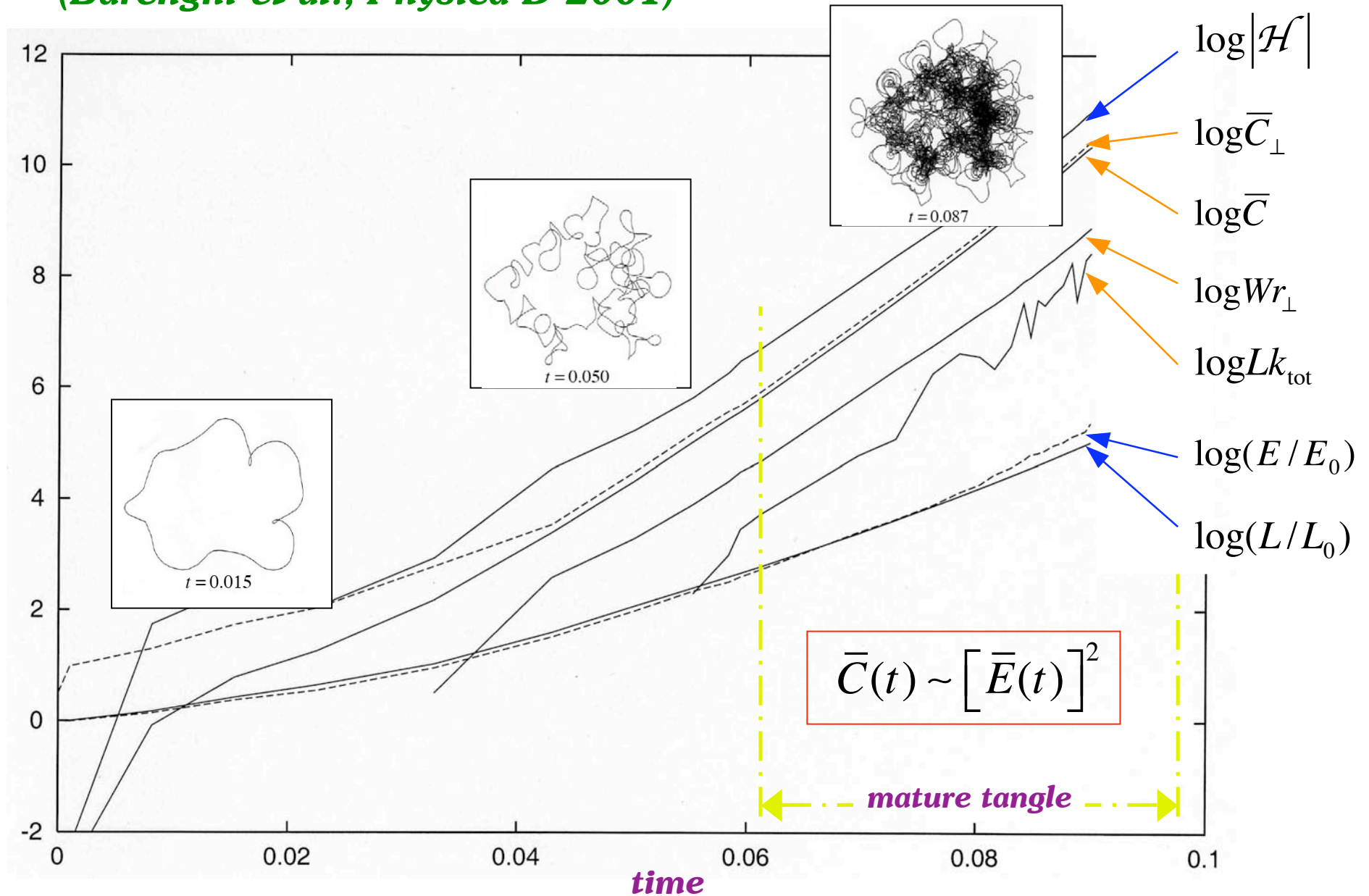
*Kida et al.*  
*(Toki-Kyoto 2002)*



*Baggaley et al.*  
*(EPL 2012)*

# Energy-complexity relation for vortex tangles

(Barenghi et al., Physica D 2001)



## Jones polynomial of fluid knots from helicity

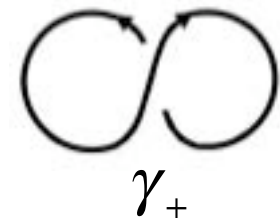
Let  $H \equiv \int_{V(\mathcal{K})} \mathbf{u} \cdot \boldsymbol{\omega} d^3 X = \sum_i \kappa_i \oint_{\mathcal{K}_i} \mathbf{u}_v \cdot d\mathbf{l}$  and set  $\kappa_i = 1 \quad \forall i$ . Then

- **Theorem (Liu & Ricca, 2012).** Let  $\mathcal{K}$  denote a fluid knot. If the helicity of  $\mathcal{K}$  is  $H = H(\mathcal{K})$ , then

$$e^{H(\mathcal{K})} = e^{\oint_{\mathcal{K}} \mathbf{u}_v \cdot d\mathbf{l}}$$

appropriately re-scaled, satisfies the skein relations of the Jones polynomial  $V = V(\tau)$ .

- In general:  $\tau = e^{-4\lambda H(\gamma_+)}$ ,  $(0 \leq \lambda \leq 1)$ .



- For a homogeneous tangle of knots and links of same circulation  $\mathcal{K}$ , we have

$$\bar{\lambda} = \langle \lambda \rangle = \frac{1}{2}, \quad \langle H(\gamma_+) \rangle = \frac{\mathcal{K}^2}{2},$$

hence

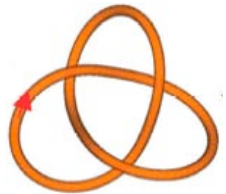
$$\underline{\tau = e^{-\mathcal{K}^2}}$$



## Tackling structural complexity by knot polynomials

$$V(\mathcal{K}(\tau)) \rightarrow V(\mathcal{K}(\kappa)) = f(\mathcal{K}; \kappa)$$

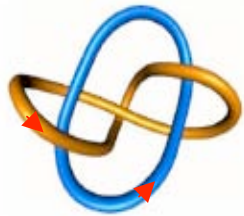
### ● **Examples: (i) trefoil knots**



(a)  $T^L$  (left-handed):  $V(T^L) = e^{\kappa^2} + e^{3\kappa^2} - e^{4\kappa^2}$

(b)  $T^R$  (right-handed):  $V(T^R) = e^{-\kappa^2} + e^{-3\kappa^2} - e^{-4\kappa^2}$

### (ii) Whitehead link:



(a)  $W_+$  = (b)  $W_- = W$  :

$$V(W) = e^{-\frac{3}{2}\kappa^2} \left( -1 + e^{\kappa^2} - 2e^{2\kappa^2} + e^{3\kappa^2} - 2e^{4\kappa^2} + e^{5\kappa^2} \right)$$

### ● **Future work:**

- extension to Vassiliev finite type invariants, Kouhanov homology and Heegaard Floer theory;
- implementation of topological analysis in advanced visiometrics for predictive diagnostics of vortical flows.

# Fluid knots in the press

IOP <http://iopscience.iop.org/1751-8121>

ISSN 1751-8113

## Journal of Physics A

### Mathematical and Theoretical

Volume 45 Number 20 25 May 2012

**BRAIN FACTOR**

**I polinomi della complessità**

Li hanno chiamati "i polinomi che governano" anziano Filippo Ricci (foto) dell'Università di Milano-Bicocca. Le nuove formule matematiche messe a punto dai ricercatori coordinati da Renzo Ricci del dipartimento di Matematica e Applicazioni dell'università di Milano-Bicocca. Le nuove formule (polinomi) consentono di misurare fenomeni come i vortici e di prevedere l'evoluzione di eventi naturali complessi come le turbolenze di una cascata d'acqua o le disordinate interazioni dei campi magnetici sul Sole o nelle altre stelle. Grazie a loro si apre la strada alla misurazione diretta di strutture complesse, come appunto i vortici, non più sperimentando ma quantificando il cambiamento in tempo reale, man mano che il fluido si muove davanti ai nostri occhi. Lo studio è pubblicato sul Journal of Physics e condotto in collaborazione con il gruppo di Xin Liu, dell'università di Sydney.

I ricercatori descrivono la scoperta di nuove tecniche matematiche per affrontare lo studio di complessi grovigli di fluidi annodati, sfruttando i più recenti progressi nella teoria matematica dei nodi (un setto-

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#### Tackling the complexity of fluid knots

Fluid flows may exhibit very complex behaviors that escape traditional modeling. Recent progress in topological techniques offers new tools to tackle this difficult outstanding problem.

How thick smoke rings into the air is not as hard as it seems, but trying to blow smoke rings to form knots and links is a mathematical challenge. Due to the formidable progress in geometric and topological knot theory, for the first time we have new tools to tackle the fundamental aspects of complex fluid motion. Knot theory, for instance, provides us with helicity, a conserved quantity of fluid motion. On the other hand, it provides us with helicity, a conserved quantity of fluid motion.

On the other hand, we now have the chance to take advantage of both worlds. We have derived one of the Jones polynomials, in terms of the helicity of fluid knots, hence providing new means to study knots in terms of measurable quantities. This is done by demonstrating that the skein relations that define the Jones polynomial can be worked out from the local contributions to the helicity of fluid strands. By applying a standard reduction scheme, we can reduce the Jones polynomial of fluid knots and links of any topological complexity (see the figure).

Figure. Reduction schemes used to compute the Jones polynomial. Top diagrams: (a) left-handed and (b) right-handed trefoil knots. The bottom diagrams are obtained by switching an over-crossing (pictured in the dashed circle in top diagrams) into an under-crossing and a non-crossing of parallel strands. By using this reduction scheme recursively, any complex knot or link can be reduced to simple, known configurations.

This result opens up new scenarios and directions of work, both from a theoretical viewpoint, where foundational aspects of classical field theory merge with topology, and from a more applied aspect, where knot theoretical information can be used to guide direct numerical simulations of fluid flows to perform a real-time analysis of energy-complexity

ly reported in *J. Phys. A: Math. Theor.* **45** 205501.

Journal of Mathematical Physics in physics and applications of the Caltech and Lagrange Senior Fellow of exchange. His main research is in fluid dynamics, particularly vortex dynamics, and structural dynamics. He received a PhD from Cambridge University for a research fellow and a member of the Institute of Mathematics and Statistics of Sydney. Liu's main research interests are in classical field theory. He received a PhD from the University of Queensland on topics of mathematical physics. He has published more than 20 papers.

Renzo L. Ricci and Xin Liu.

financial support from the LISVD Postdoctoral Fellowship program of the University of Sydney. He is kind hospitality of the Ennio De Giorgi Mathematical Research Center of the Scuola Normale Superiore. Part of this work was carried out.

## Ricerca italoaustraliana. Studiosi al lavoro sui polinomi

# Eventi naturali 'predetti' da formule matematiche

MILANO — Nuovi software per prevedere l'evoluzione di incontrollabili fenomeni naturali potranno essere sviluppati grazie alle nuove formule matematiche messe a punto dai ricercatori coordinati da Renzo Ricci del dipartimento di Matematica e Applicazioni dell'università di Milano-Bicocca. Le nuove formule (polinomi) consentono di misurare fenomeni come i vortici e di prevedere l'evoluzione di eventi naturali complessi come le turbolenze di una cascata d'acqua o le disordinate interazioni dei campi magnetici sul Sole o nelle altre stelle. Grazie a loro si apre la strada alla misurazione diretta di strutture complesse, come appunto i vortici, non più sperimentando ma quantificando il cambiamento in tempo reale, man mano che il fluido si muove davanti ai nostri occhi. Lo studio è pubblicato sul Journal of Physics e condotto in collaborazione con il gruppo di Xin Liu, dell'università di Sydney.

I ricercatori descrivono la scoperta di nuove tecniche matematiche per affrontare lo studio di complessi grovigli di fluidi annodati, sfruttando i più recenti progressi nella teoria matematica dei nodi (un setto-

re della topologia che si occupa dello studio qualitativo delle forme). Il disordinato turbino di flussi, fluidi e campi magnetici forma strutture fluide complesse, in cui le forze e l'energia si distribuiscono come in un groviglio di fili che si intrecciano e disfanno continuamente. Per descrivere questa dinamica non sono sufficienti i metodi classici per elaborare i classici modelli matematici, basati sullo studio di equazioni differenziali semplificate, modelli statistici o geometria elementare. Grazie a formidabili progressi fatti in questi ultimi anni, in particolare dalla teoria dei nodi, è ora possibile identificare e seguire nel tempo l'annodamento e lo sprodimento di filamenti fluidi nello spazio.

Il risultato della ricerca non solo apre un nuovo orizzonte in quel che si chiama "dinamica topologica", ma offre nuove possibilità per studiare fenomeni complessi, sia in aspetti fondamentali della ricerca fisica e biologica, sia nel futuro campo delle applicazioni. Fra queste, lo sviluppo di software per rendere sempre più precisa e raffinata la predicibilità di fenomeni naturali.

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