What is Quantum Mechanics? A Minimal Formulation

Pierre Hohenberg in homage to Walter Kohn

- Introduction:
  - Why, after ninety years, are the foundations of quantum mechanics still a matter of controversy?
- Formulating classical mechanics: microscopic theory (MICM)
  - Phase space: states and properties
- Microscopic formulation of quantum mechanics (MIQM):
  - Hilbert space: states & properties/quantum incompatibility
- The measurement problem
- Macroscopic quantum mechanics (MAQM):
  - Testing the theory. Not new principles, but consistency checks
- Other treatments: not wrong, but laden with excess baggage
- The ten commandments of quantum mechanics
- The future of Foundations of QM
- Walter's beneficial influence on my life and career

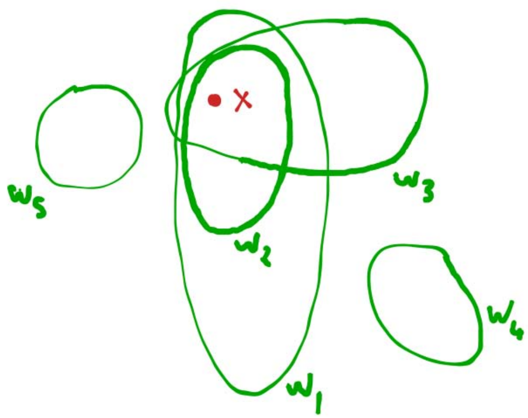
Foundations of Quantum Mechanics

- I. What is quantum mechanics (QM)? How should one formulate the theory?
- II. Testing the theory. Is QM the whole truth with respect to experimental consequences?
- III. How should one interpret QM?
    - Justify the assumptions of the formulation in I
    - Consider possible assumptions and formulations of QM other than I
    - Implications of QM for philosophy, cognitive science, information theory,…
- IV. What are the physical implications of the formulation in I?

- In this talk I shall only be interested in I, which deals with foundations
- II and IV are what physicists do. They are not foundations
- III is interesting but too often not central to physics
- Why is there no field of Foundations of Classical Mechanics?
    - We wish to model the formulation of QM on that of CM

Formulating nonrelativistic classical mechanics (CM)

- Consider a system S of N particles, each of which has three coordinates (x_i, y_i, z_i) and three momenta (p_{xi}, p_{yi}, p_{zi}) .
- Classical mechanics represents this closed system by objects in a Euclidean phase space of $6N$ dimensions.
- A state of the system is a point x in phase space
- A property is a subset w of points in phase space, e.g. the set of all points for which the energy E of the system has the value E_1.
- If the state is x_0 at time $t = t_0$ then the Hamiltonian determines the trajectory $x(t)$ of the system for all $t < t_0$ and $t > t_0$.
- Predictions: the property w is true at time t if $x(t) \epsilon w$ and false otherwise.



If the state is x then the properties w_1 , w_2, w_3 are simultaneously true, and the properties w_4, and w_5 are simultaneously false.

- Determining which physically interesting properties are true given the state at any time is the full logical content of classical mechanics.

Comments regarding classical mechanics:

- The preceding formulation is what we call microscopic classical mechanics (MICM), since it applies to any closed system of arbitrary size *N*, and uses only concepts pertaining to the system itself.
- The state is assumed to exist. It is not observed. Only properties are observed, by noting whether they are true or false.
- MICM makes no direct reference to how states are prepared, nor how the predictions might be tested.
- Those questions can be answered by macroscopic classical mechanics (MACM), which is not a separate theory, but a special case ($N \rightarrow \infty$) of MICM, in which one assumes the existence of macroscopic measurement or preparation devices, which interact with, but are external to, the system under study.
- Note that MICM is logically complete by itself.
- We wish to formulate quantum mechanics (QM) in as close analogy as possible to classical mechanics.

Microscopic (nonrelativistic) quantum mechanics (MIQM)

- MIQM also defines states and properties, but these are now objects in Hilbert space, rather than phase space.

- Hilbert space contains vectors $|\psi>$ and operators O_1 , O_2. If you multiply two operators you get another operator…

- BUT in general: $O_1 O_2 \neq O_2 O_1$ Quantum incompatibility! 

- Any vector $|\psi>$ (more precisely any ray $\alpha|\psi>$) or the corresponding projector $[\psi]$ can be selected as the unique (pure) state. More generally, a state ρ (pure or mixed) is a positive operator of trace one, called the density matrix.

- Any vector $|\psi>$ or its projector $[\psi]$ can also represent a property. More generally a property is a subspace A of Hilbert space, or the projector $[A]$ satisfying $[A]^2 = [A]$. The subspace may have any dimension $d_H \geq d_A \geq 1$

- Incompatible properties A and B are ones whose projectors don't commute: $[A]\,[B] \neq [B]\,[A]$

States and properties (cont'd)

- In classical mechanics states confer truth on properties
- Bell/Kochen-Specker Theorem: in Hilbert space no truth function can be consistently defined to apply to arbitrary incompatible properties. It follows that contrary to the case in CM, the state cannot confer truth on all properties. It can at most confer the probability of being true.
- Gleason's Theorem: the only consistent way to define the probability that the state ρ confers truth on property A is via the Born rule:

$$P_\rho(A) = \mathrm{Tr}\,(\rho\,[A]), \quad \text{(Tr is the 'trace')}$$

- BUT: the Born rule does not define a probability function over the whole space of (possibly incompatible) properties since it violates the Kolmogorov condition:

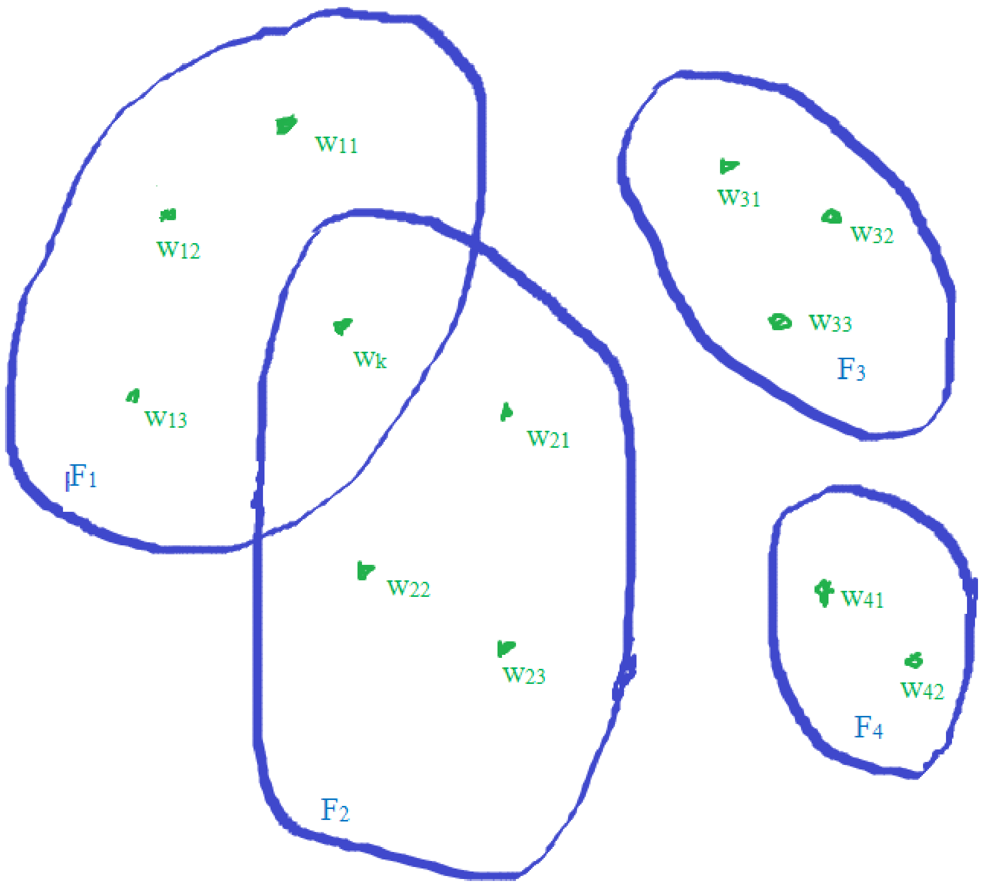
$$P_\rho(A \vee \mathrm{B}) + P_\rho(A \wedge \mathrm{B}) \neq P_\rho(A) + P_\rho(B) \quad \text{if} \quad [A][B] \neq [B][A].$$

- Every probability function requires a sample space of compatible properties $\{A_1, A_2, \ldots\}$ called a framework:

Framework: an Exhaustive Set of Exclusive Alternatives (ESEA)

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$A_1 \wedge A_2 \wedge \ldots = \mathrm{I} \qquad\qquad <A_1|\,A_2> = 0$$



- Frameworks $F_1 = \{w_{11}, w_{12}, \ldots\}$, $F_2 = \{w_{21}, w_{22}, \ldots\}$, … are mutually incompatible
- The properties $w_{11}, w_{12}, \ldots$ within the framework F_1 are mutually compatible
- Given a state ψ and a framework F_1 the probability function $P_{\psi, F1}(w_{ij})$ is defined in the sample space of mutually compatible properties $\{w_{11}, w_{12}, \ldots\}$ belonging to F_1

Conditioning and selection

- From the standard Bayesian definition of conditional probabilities one can prove that the state $\psi = |z^+\rangle$, conditioned on the truth of the property $[x^+]$ is a new state $\psi_x = |x^+\rangle$.
- The transition from ψ to ψ_x illustrates a fundamental principle:

Information is physical

- Conditioning on the property $[x^+]$ adds information to the state ψ and thereby changes the state physically. This is referred to as the 'collapse of the wavefunction' in the orthodox interpretation. It occurs in 'logical time', not in dynamical (physical) time and does not require the intervention of any external apparatus or agent.
- Note that the state ψ does not determine the truth, falsehood or even probability of any property, until a framework has been chosen. This breaking of framework symmetry is necessary before classical information about properties can be extracted from a quantum state.

The von Neumann-Lüders Rule

- More generally, conditioning a state ρ on a framework F_A defines a probability function $P_\rho(\{A_i\})$, with a unique outcome A_k, say. By unique we mean A_k or A_j, not A_k and A_j.

- Further conditioning on the outcome A_k (or equivalently selecting that outcome), produces the state

$$\rho_A = [A_k]\, \rho\, [A_k] / \mathrm{Tr}(\rho[A]) \qquad \text{(von-Neumann Lüders rule)}$$

- These conditioning operations are what we call 'microscopic measurement'. Similarly, conditioning and selection can be used to define 'microscopic state preparation'.

- All of these operations occur in logical time.

The Measurement Problem

- Since we have not talked about macroscopic measurements in MIQM, one might think we have avoided the measurement problem, which is usually phrased in terms of macroscopic measurements.
- We can, however, identify a 'microscopic measurement problem', by noting that the collapse mechanism ($\rho \rightarrow \rho_A$) of the von Neumann-Lüders rule violates the unitary dynamics of the Schrödinger equation.
- Our 'resolution' is to note that this rule is a theorem about conditioning and selection in Hilbert space. It is the only way for the state to confer truth on a property. The collapse is a direct consequence of the quantum incompatibility of ρ and A and the physical nature of information (i.e. of conditioning and selection).
- The transformation from ρ to ρ_A occurs in logical time, which can be simultaneous in dynamical time. The two are distinct.
- Thus the 'measurement problem' is neither about macroscopic measurements, nor is it a problem.
- This is QM made ESEA

Macroscopic quantum mechanics (MAQM)

- The preceding was a quick sketch of MIQM, with all its supposed weirdness and paradox, manifested in any closed system. It depends on a single fundamental assumption: Hilbert space.
- Just as in classical mechanics, in order to test the theory or to prepare the quantum state, the system must be put into contact with a macroscopic measurement or preparation apparatus.
- This is the domain of MAQM, a special case of MIQM, applicable to large systems, which can display classical behavior (the classical behavior of large systems can be considered a phenomenological assumption, but it can also be justified from the microscopic theory).
- Most standard 'interpretations' of QM concern only the above questions, involving the interaction of quantum systems with external classical devices, as well as the physical consistency of MIQM. We consider these to be applications of MIQM, not part of the foundations, since MIQM is itself logically complete.

Other formulations or interpretations of QM

- Copenhagen and/or orthodox QM (textbooks): phenomenology
- Modern treatments: (Preskill, Bub, Kochen): very close to ours, except for language, primarily use of the term ‘measurement’ in MIQM.
- Many-worlds: geared to cosmology; unnecessarily complicated for a minimal theory.
- Consistent and decoherent histories: the ‘static’ theory is essentially our formulation. The ‘dynamical theory’ (multitime histories and consistency conditions) is also unnecessary (just as MW).
- These theories are not wrong: we claim our minimal formulation clarifies the language and eliminates excess baggage.
- Neoclassical theories: Bohmian or Spontaneous Collapse (GRW):

  These are neoclassical in the sense that they have a classical ontology, with nonlocal or stochastic dynamics to reproduce (some of) QM. These theories deny the primacy of Hilbert space. They are “not even wrong”.

The Ten Commandments of Quantum Mechanics

- Quantum mechanics (QM) does not require an interpretation. It requires a clear and unambiguous formulation. Such a minimal formulation exists for classical mechanics (CM), in which states confer truth on properties in Euclidean space.
- Both classical and quantum mechanics are first formulated microscopically, for closed systems of arbitrary size (MICM and MIQM).
- Quantum mechanics replaces Euclidean space with Hilbert space, in which ☠ quantum incompatibility ☠ (noncommutativity of operators) is fundamental.
- Quantum states do not in general confer truth on properties. They confer the probability of being true to subsets of compatible properties, called frameworks (Exhaustive Sets of Exclusive Alternatives, ESEA).
- A microscopic measurement consists of (i) selecting a state, (ii) selecting (choosing) a framework (breaking framework symmetry) and (iii) selecting the outcome that is true with some probability.

- Conditioning on the selected (true) outcome adds information and thereby changes the state: information is physical. This is the microscopic ‘collapse of the wavefunction’.
- Subsystems of quantum systems are in general entangled: the state of the composite system is in general incompatible with the states of the subsystems.
- The secret to ‘solving the measurement problem’ is the fact that states are not observable. Only the truth of properties is observable.
- In this way the microscopic theory (MIQM) is fully formulated for closed systems of any size, without reference to external apparatus or external agents, and with no paradoxes. It is logically complete.
- Experimental tests of the predictions of MIQM are described by the macroscopic theory (MAQM), which involves classical measurement apparatus or preparation devices. MAQM is an application of the theory, involving no new principles. It provides a test and a consistency check on the theory.

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The future of the Foundations of Quantum Mechanics

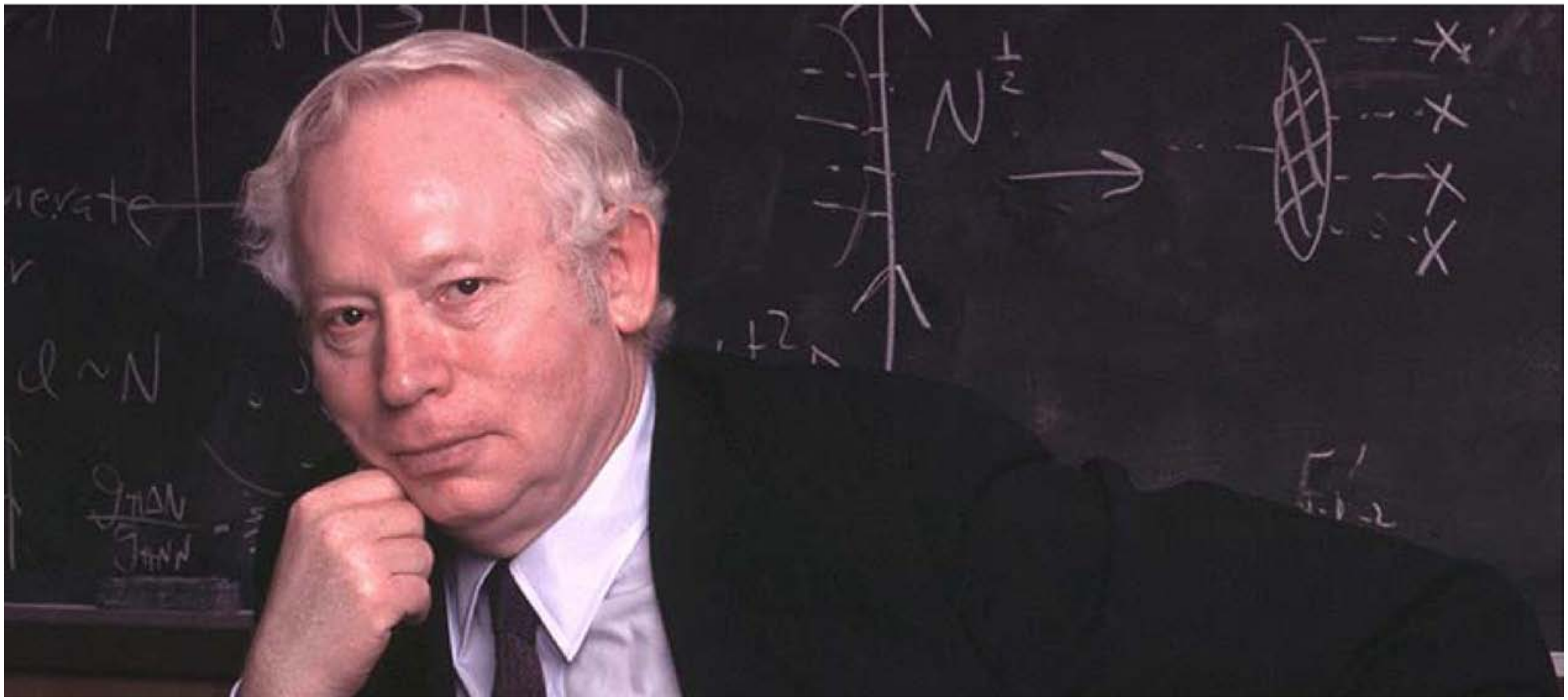
- Pierre’s quixotic dream is that the Foundations of QM should rest in peace alongside the Foundations of Classical Mechanics, allowing physicists to pursue ‘what physicists do’, namely testing the theory and examining its physical consequences.

R.I.P.

Why quantum mechanics might need an overhaul

Nobel laureate Steven Weinberg says current debates suggest need for new approach to comprehend reality

By Tom Siegfried 3:37pm, November 4, 2016



Nobel laureate Steven Weinberg, a physicist at UT Austin, was once happy with quantum mechanics. But now he thinks that some more general theory may be needed to resolve long-standing disputes about the meaning of quantum mechanical math.

Walter's beneficial influence on my life and career

- 1963: Sharing an office and a desk with Walter in the office of Phillipe Nozières led to an invitation to collaborate and to a paper entitled "The inhomogeneous electron gas", which was the birth of Density Functional Theory.
- 1964: Walter suggested I might be interested in a further postdoc at Bell Labs, an institution I had barely heard of. I followed his advice and spent the next 30 years at Bell Labs.
- 1979: Having just been appointed as Director of ITP, Walter invited Jim Langer and me to organize one of the first full ITP programs, which lasted 15 months, on "Dynamics and patterns outside of equilibrium". I believe it is fair to say this program had considerable impact on the field.
- On a more personal note, Walter reminded me of my father, who was born and raised in Vienna and emigrated as a young man to Paris. Like my father Walter was a great francophile.