Quantum gravity and quantum chaos

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Quantum chaos and quantum gravity

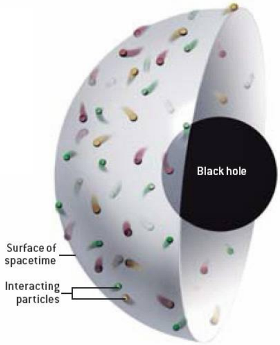
Quantum chaos $\leftrightarrow$ Quantum gravity

Black holes are thermal

- Strong chaos underlies thermal behavior in ordinary systems
- Quantum black holes are thermal
- They have entropy [Bekenstein]
- They have temperature [Hawking]
- Suggests a connection between quantum black holes and chaos

AdS/CFT

- Gauge/gravity duality: AdS/CFT
- Thermal state of field theory on boundary
- Black hole in bulk
- Chaos in thermal field theory ↔ what phenomenon in black holes?



[Maldacena, Sci. Am.]

$2+1$ dim boundary
$3+1$ dim bulk

Butterfly effect

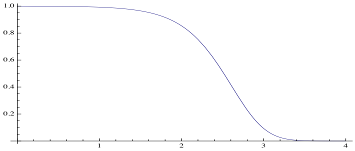
- Strong chaos – sensitive dependence on initial conditions, the "butterfly effect"
- Classical mechanics
- $v(0), v(0) + \delta v(0)$ two nearby points in phase space
- $|\delta v(t)| \sim e^{\lambda_L t} |\delta v(0)|$
- λ_L a Lyapunov exponent
- What is the "quantum butterfly effect"?

Quantum diagnostics

- Basic idea [Larkin-Ovchinnikov]
- General picture [Almheiri-Marolf-Polchinsk-Stanford-Sully]
- $W(t) = e^{iHt} W(0) e^{-iHt}$
- Forward time evolution, perturbation, then backward time evolution
- Chaos causes a lack of cancellation, $W(t)$ a complicated operator ("precursor")
  [Polchinski-Susskind-Toumbas, Roberts-Stanford-Susskind]
- $C(t) = -\langle [W(t), V(0)]^2 \rangle$, increases with time

Quantum diagnostics

- $C(t) = -\langle [W(t), V(0)]^2 \rangle$,   four terms
- Significant time dependence from the **out-of-time order correlator (OTOC)**
- $D(t) = \langle W(t) V(0) W(t) V(0) \rangle$
- $D(t)$ decreases with time due to chaos



Measuring D

- $D(t) = \langle W(t)V(0)W(t)V(0)\rangle$
- To measure D one must evolve forward, then backward, in time.
- Or change the sign of H
- Many body version of spin echo–Loschmidt echo.
  [Pastawski et al. ...]
- Proposed experiments in cavity QED
  [Swingle-Bentsen-Schleier-Smith-Hayden]
- Initial experiments (in ion and NMR systems) have been done...
  Li-Fan-Wang-Ye-Zeng-Zhai-Peng-Du;
  Garttner-Bohnet-Safavi-naini-Wall-Bollinger-Rey]

Holographic calculation of D

- Holographic calculation of OTOCs, and related quantities. [SS-Stanford; Kitaev]
- **The onset of chaos is dual to a high energy gravitational collision near the black hole horizon.**
- Why high energies?
- t becomes a lightlike direction at the horizon. Implies an enormous redshift between the horizon and a far away observer.
- For a wavepacket to start very near the horizon at time 0 and end up far away with thermal energy T at time t, it must start with enormous energy $E \sim T \exp\left(\frac{2\pi}{\beta} t\right)$.

Holographic calculation of D, contd.

- Gravitational scattering $\sim G_N E_{\mathrm{com}}^2$
- $G_N \sim$ inverse number of degrees of freedom $\sim 1/N^2$
- $D \sim c_0 - \frac{c_1}{N^2} \exp\left(\frac{2\pi}{\beta} t\right) + \ldots$
- $\exp\left(\frac{2\pi}{\beta} t\right) \to \exp\left(\lambda_L t\right)$ [Kitaev]
- $\lambda_L = \frac{2\pi}{\beta} = 2\pi T$

A bound on chaos

- A number of results suggest that this gravitational result should be a universal bound (for systems with large numbers of degrees of freedom and "reasonable" interactions) [Maldacena-SS-Stanford]
- $\lambda_L \leq \frac{2\pi}{\beta} + \mathcal{O}(\frac{1}{N^2})$
- In the spirit of the KSS conjecture, $\eta/\bar{s} \geq \frac{1}{4\pi}$
- A numerically precise refinement of the Fast Scrambling Conjecture [Sekino-Susskind]
- One of a growing number of bounds motivated by gravity, firmly established on general grounds

The chaos bound–flavor of argument

[Maldacena-SS-Stanford]

- OTOCs are analytic in time.
- Expect that chaotic decrease should persist for small amounts of imaginary time
- For $F(t)$, a special OTOC, the chaotic decrease in time persists when $t \to t + i\tau$, at least up to $|\tau| = \beta/4$. (Hard part)
- For simplicity assume $F(t) \sim 1 - \frac{1}{N^2} e^{\lambda_L t}$
- If λ_L is very large, the exponential will oscillate fast in τ, changing $-$ to $+$.
- $|\tau| \leq \beta/4 \implies \lambda_L \leq \frac{2\pi}{\beta} \; (+\mathcal{O}(\frac{1}{N^2}))$

Sachdev-Ye-Kitaev Model

- What systems saturate the chaos bound?
- A variant of the Sachdev-Ye model ! (The Sachdev-Ye-Kitaev model) [Kitaev]
- Quantum mechanics of N species of Majorana fermions $\{\chi_a, \chi_b\} = \delta_{ab}$
- $H = \sum_{a,b,c,d} J_{abcd} \chi_a \chi_b \chi_c \chi_d$ J random
- J_{abcd} gaussian distributed around 0, $\langle J_{abcd}^2 \rangle = \frac{1}{N^3} J^2$
- $\dim \mathcal{H} = 2^{N/2} = L$

Sachdev-Ye-Kitaev model

- $H = \sum_{a,b,c,d} J_{abcd}\chi_a\chi_b\chi_c\chi_d$
- Vectorlike model. Solvable in a large N expansion, but not almost integrable !
- $\lambda_L \to \frac{2\pi}{\beta} \quad \beta\overline{J}, N \to \infty$
- Gravitational sector in the bulk, but many other states as well ($G_N \sim 1/N$)
  [Maldacena-Stanford-Yang]
- What can we do with this new tool?

Finite black hole entropy from the bulk

- Black holes have finite entropy equal to the area of the horizon in Planck units
- What accounts for this finite entropy–from the bulk point of view?

A diagnostic

- A simple diagnostic [Maldacena]. Let O be a bulk (smeared boundary) operator.

$$\langle O(t)O(0)\rangle = \mathrm{tr}\left(e^{-\beta H}O(t)O(0)\right)/\mathrm{tr}e^{-\beta H}$$
$$= \sum_{m,n} e^{-\beta E_m}|\langle m|O|n\rangle|^2 e^{i(E_m-E_n)t}/\sum_n e^{-\beta E_n}$$

- At short times can treat the spectrum as continuous. $\langle O(t)O(0)\rangle$ generically decays exponentially.

- Perturbative quantum gravity–quasinormal modes. [Horowitz-Hubeny]

A diagnostic, contd.

$$\langle O(t)O(0)\rangle = \sum_{m,n} e^{-\beta E_m} |\langle m|O|n\rangle|^2 e^{i(E_m - E_n)t} / \sum_n e^{-\beta E_n}$$

- But we expect the black hole energy levels to be discrete (finite entropy) and generically nondegenerate (chaos).
- Then at long times $\langle O(t)O(0)\rangle$ oscillates in an erratic way. It is exponentially small and no longer decreasing.
- What accounts for the end of smooth relaxation from the bulk point of view?
- A nonperturbative effect in quantum gravity.
  (See also [Dyson-Kleban-Lindesay-Susskind; Barbon-Rabinovici])

Another diagnostic, $Z(t)Z^*(t)$

- To focus on the oscillating phases remove the matrix elements. Use a related diagnostic: [Papadodimas-Raju]

$$\sum_{m,n} e^{-\beta(E_m+E_n)} e^{i(E_m-E_n)t} = Z(\beta+it)Z(\beta-it) = Z(t)Z^*(t)$$

- The "spectral form factor"

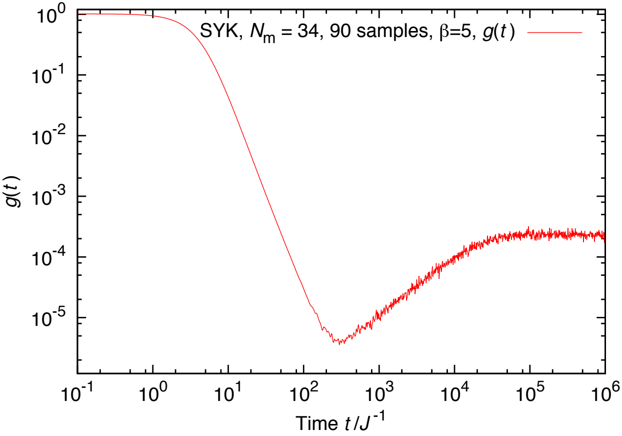
- (Essentially) the fourier transform of the energy eigenvalue pair distribution function $\rho^{(2)}(E,E')$

- How does $Z(t)Z^*(t)$ decrease exponentially in magnitude to its asymptotic (averaged) value?

SYK model

- The Sachdev-Ye-Kitaev model is a promising system in which to investigate these questions
  [Jordan Cotler, Guy Gur-Ari, Masanori Hanada, Joe Polchinski, Phil Saad, Stephen Shenker, Douglas Stanford, Alex Streicher, Masaki Tezuka]
  See also
  [Garcia-Garcia–Verbaarschot]
- J average smooths out erratic behavior, giving a smooth function of time
- Finite dimensional Hilbert space. For $N = 34$, L = 128K

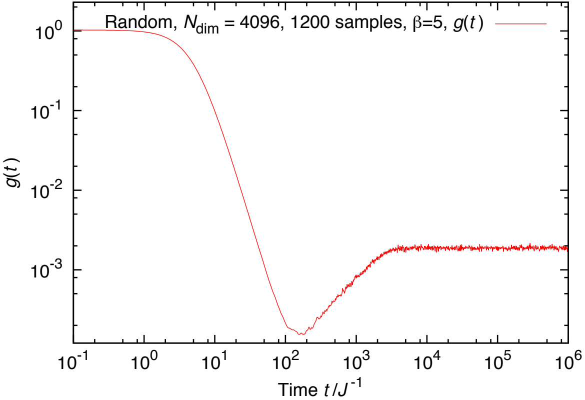
SYK $\langle Z(t)Z^*(t)\rangle$

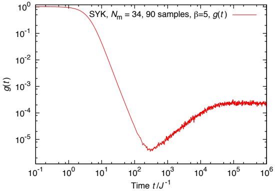


- The Slope
- The Dip
- The Ramp
- The Plateau
- What do they mean?

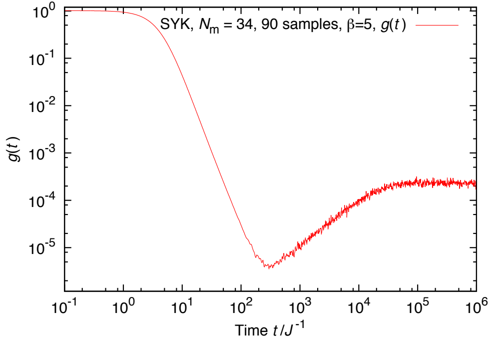
Random Matrix Theory

- Chaotic quantum systems typically have fine grained energy level statistics described by Random Matrix Theory (RMT) [Wigner; Dyson; Bohigas-Giannoni-Schmit, Berry...]
- Consider a simple model where $H \to M$, an $L \times L$ (hermitian) random matrix
  [You-Ludwig-Xu]
- The spectral form factor of RMT


Random, N_{dim} = 4096, 1200 samples, β=5, $g(t)$
$g(t)$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5 10^6
Time t/J^{-1}

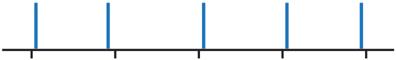

SYK, N_m = 34, 90 samples, β=5, $g(t)$
$g(t)$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5 10^6
Time t/J^{-1}

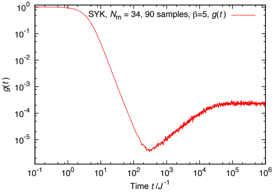
Meaning



- The Slope ↔ "gravity"
- The Plateau ↔ Eigenvalue repulsion
- The Ramp ↔ Spectral rigidity
- The Dip ↔ Exponential separation in scales

Spectral rigidity





- Energies roughly evenly spaced – eigenvalue repulsion, no crossing rule ("Dyson gas" – long range logarithmic interactions)
- Long range fluctuations suppressed $\langle \delta E_n \, \delta E_m \rangle \sim \log |n - m|$
- Nearest neighbor balls and springs $\langle \delta E_n \, \delta E_m \rangle \sim |n - m|$
- "Spectral rigidity"
- Conjecture that these phenomena generically describe the long time behavior of black hole horizon fluctuations for large AdS black holes
- Lessons for nonperturbative quantum gravity....

Lessons for quantum gravity

$$\rho^{(2)}(E, E') \sim 1 - \frac{\sin^2(L(E-E'))}{(L(E-E'))^2}$$

- The ramp is due to long range correlations: $\rho^{(2)} \sim 1/(L(E-E'))^2$
- This is a $1/L^2$ perturbative effect in RMT
- But in SYK and presumably more generally $L \sim e^{aN}$, so the ramp is of order e^{-2aN}, a nonperturbative effect of standard strength.

- The plateau is due to the oscillatory behavior $\sin^2(L(E-E'))$. Continuing to imaginary E this is an e^{-L} effect, the Andreev-Altshuler instanton of RMT.
- But in SYK, and presumably more generally, this is $\sim e^{-e^{aN}}$, a novel extremely small nonperturbative effect.

A research program

- A research program:

- In SYK there is an exact nonperturbative rewrite in terms of singlet bilocal fields: a proxy for a bulk theory.
- $G(t,t') = \frac{1}{N}\chi_a(t)\chi_a(t')$, $\Sigma(t,t')$ a Lagrange multiplier enforcing this identification

- $Z = \int dG(t,t')d\Sigma(t,t') \ \exp(-N \ I[G,\Sigma])$

- What part of the G, Σ functional integral accounts for the ramp and plateau?

- Does this give a hint for higher dimensional examples of AdS/CFT ?