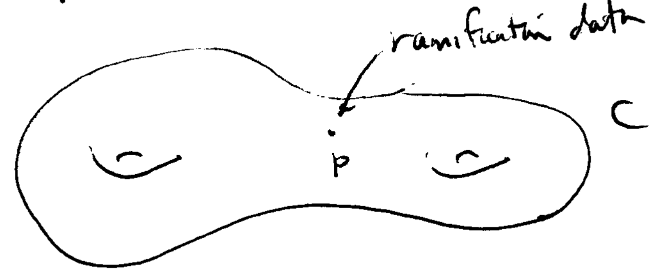
25 July 2008

S. Gukov

Surface Operators and Ramification, III

surface operators (ramification)



solutions to BPS eqs (Hitchin eqs) based on

$SU(2)$ triple (J_1, J_2, J_3)

classified by $SU(2)$ embeddings

$$SU(2) \longrightarrow G/\text{conj}$$

+ deformations $(\alpha, \beta, \gamma, \eta)$

Thm (Jacobson - Morosov)

$SU(2)$ embeddings $SU(2) \to G$ $\Longleftrightarrow$ unipotent conjugacy classes $\mathcal{C} \subset G_{\mathbb{C}}$

Ex: $G = SU(2)$

$$A = \begin{pmatrix} z & x \\ y & -z \end{pmatrix} \in sl(2,\mathbb{C}) = \mathfrak{g}_{\mathbb{C}}$$

$\mathcal{O} = G_{\mathbb{C}} \cdot A$ $\qquad xy - z^2 = \text{const}$

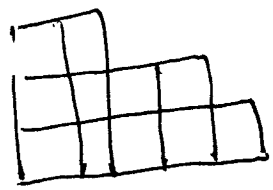
$xy - z^2 = 0$ in $\mathbb{C}^3$ is $\mathbb{C}^2/\mathbb{Z}_2$

Fact: For classical groups

$SU(N)$, $SO(N)$, $Sp(N)$

$SU(2)$ embeddings are labeled by partition of N (w/ certain conditions)

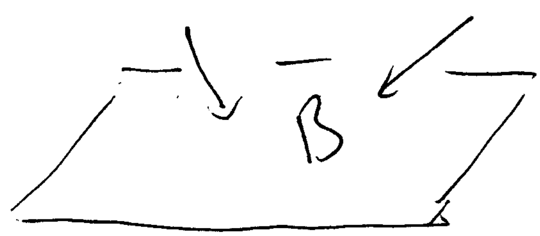
For $G = SU(N)$, all partition of N



$N = 3+3+2+2+1$

For $G = SO(7)$		${}^LG = Sp(6)$	
[7]		[6]	
~~[6,1]~~		~~[5,1]~~	
[5,1,1]	$\xleftrightarrow{\text{S-duality}}$	[4,2]	
[3,3,1]		[4,1,1]	[3,3]
[3,2,2]		[2,2,2]	
[3,1,1,1,1]	$\xleftrightarrow{\text{S-duality}}$	[2,2,1,1]	
[2,2,1,1,1]		[2,1,1,1,1]	$\mathbb{C}^2/\mathbb{Z}_2$

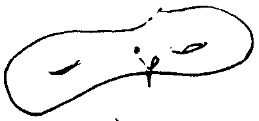
$\mathcal{M}_H(G, C, \text{ramification})$ $\qquad$ $\mathcal{M}_H({}^LG, C, {}^L\text{ramification})$



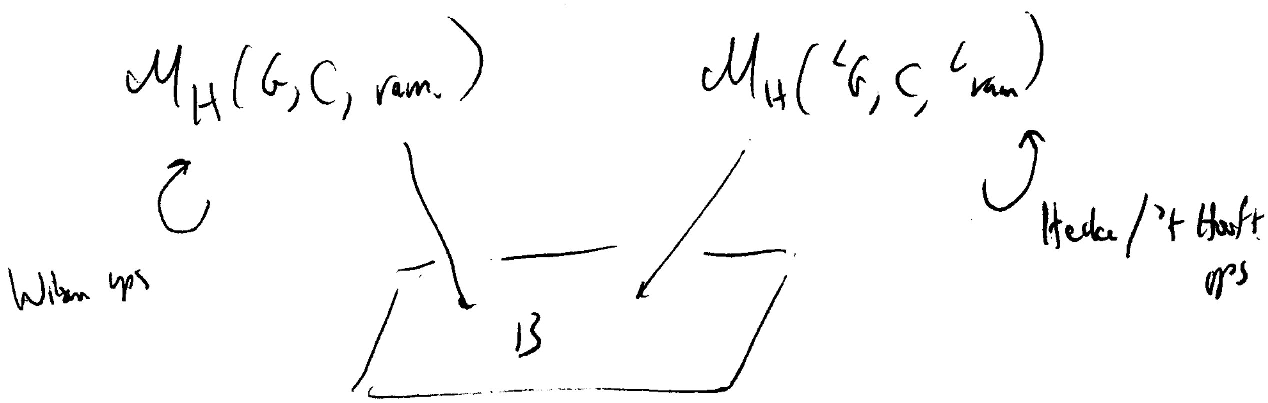
$SU(2)$ embedding

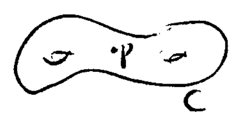
$(\alpha, \beta, \gamma, \eta)$

S-duality



	cplx parameters	Kähler parameters
I	$\beta + i\gamma$	$\alpha + i\eta$
J	$\gamma + i\alpha$	$\beta + i\eta$
K	$\alpha + i\beta$	$\gamma + i\eta$





$B_{\text{aff}} \circlearrowright D^b(\mathcal{M}_H(G, C, \alpha, \beta, \gamma))$

$\mathcal{H}_{\text{aff}} \circlearrowright K^{\mathbb{C}^*}(\mathcal{M}_H(\ldots))$

$W_{\text{aff}} \circlearrowright K(\mathcal{M}_H(\ldots))$

$W_{aff} = W \ltimes \Lambda_{cochar}$ for $G = SU(2)$, $W_{aff} = \underset{A}{\mathbb{Z}_2} \ltimes \underset{R}{\mathbb{Z}}$

$A^2 = 1$, $R: n \to n+1$

$A: n \to -n$

$B = AR$, $B^2 = 1$

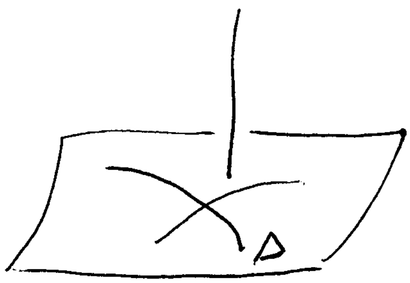
W_{aff} generated by simple reflection T_i, $T_i^2 = 1$

$T_i T_j T_i = T_j T_i T_j$ if $|i-j| = 1$

H_{aff}: replace $T_i^2 = 1$ by $(T_i - 1)(T_i + q) = 0$

B_{aff}: drop $T_i^2 = 1$, keep braid relation

$K(\mathcal{M}_H(G, C, \alpha, \beta, \gamma))$ or $H^*(\mathcal{M}_H(G, C, \alpha, \beta, \gamma))$



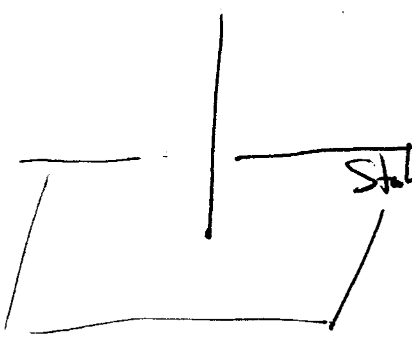
parameters $(\alpha, \beta, \gamma) \in (T \times \mathfrak{t} \times \mathfrak{t})/W = (\mathfrak{t} \times \mathfrak{t} \times \mathfrak{t})/W_{aff}$

$\pi_1(\text{parameters} \setminus \Delta) \circlearrowright H^*(\mathcal{M}_H(G, C, \alpha, \beta, \gamma))$

regular parameters $(\alpha, \beta, \gamma) \in (T \times \mathfrak{t} \times \mathfrak{t} - \Delta)/W = (\mathfrak{t} \times \mathfrak{t} \times \mathfrak{t} - \Delta)/W_{aff}$

$\pi_1(\text{parameters} - \Delta) = W_{aff}$

$D^b(\mathcal{M}_H(G, C, \alpha, \beta, \gamma))$



Stab $= \{(\beta, \eta)^{reg}\} = \mathfrak{t} \times {}^L T^{-\Delta}/W = {}^L T_{\mathbb{C}}^{-\Delta}/W$

$\pi_1({}^L T_{\mathbb{C}}^{reg}/W) = B_{aff}$

Ex. $\mathbb{C}^2/\mathbb{Z}_2$