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Supersymmetry of the chiral-antichiral de Rham complex
in the Calabi-Yau case

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$b\epsilon - \beta\delta$

$$b(z)c(w) \sim \frac{1}{z-w}$$

$$\beta(z)\delta(w) \sim \frac{1}{z-w}$$

$$b \leftrightarrow c$$

$$\delta(z)\delta(w) \sim 0$$

Let $g(\delta)$.

$$b \rightarrow g c$$

$$c \rightarrow g^{-1} b$$

$$\delta \rightarrow \delta$$

$$\beta \rightarrow \beta$$

still automorphism of $b\epsilon\beta\delta$ syst?

$$g(\delta) = \text{const}$$

Can I complete

$$b \rightarrow g c$$

$$c \rightarrow g^{-1} b$$

$$\delta \rightarrow \delta$$

$$\beta \rightarrow ?$$

$$b^{\dot{\alpha}}(z) c^{\dot{\beta}}(w) = \frac{\delta_{\dot{\alpha}\dot{\beta}}}{z-w}$$

$$\beta^{\dot{\alpha}}(z) \gamma^{\dot{\beta}}(w) = \frac{\delta_{\dot{\alpha}\dot{\beta}}}{z-w}$$

$$g_{\alpha\dot{\beta}}(\gamma), g^{\dot{\alpha}\beta}(\gamma)$$

$$b^{\dot{\alpha}} \rightarrow g^{\dot{\alpha}\beta} c_{\beta}$$

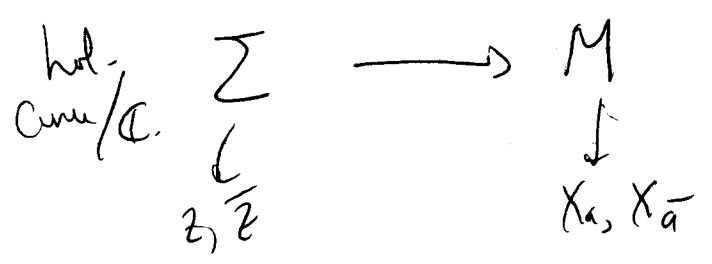
$$c_{\alpha} \rightarrow g_{\alpha\dot{\beta}} b^{\dot{\beta}}$$

$$\gamma \rightarrow \gamma$$

$$\beta \rightarrow ?$$

Step I give on general manifold

Step II find $N=2,2$ susy



Complex manifold

All fields on CDR depend only on z

But we have fields assoc. to both $X_{\alpha}, X_{\bar{\alpha}}$.

$$\overline{X_a(z)} = "X_{\bar{a}}(\bar{z})"$$

$$X_{\bar{a}}(z)$$

$$\Sigma = \text{Spec } \mathbb{C}((z))$$

locally on M , X_i coords $\longrightarrow \gamma^i(z)$

$$2X_i \longrightarrow C_i(z)$$

$$dX^i \longrightarrow b^i(z)$$

$\beta_i(z)$ transform in a strange way.

Susy in bc- $\beta\gamma$ system

$$C_i \longrightarrow \beta_i$$

$$\gamma^i \longrightarrow b^i$$

$\theta \ll z$

$$N_i(z, \theta) = C_i(z) + \theta \beta_i(z)$$

$$B^i(z, \theta) = ~~C_i(z)~~ \gamma^i(z) + \theta b^i(z)$$

B^i - transform as coordinates

N_i - transform as vector fields

$$\Sigma = \text{Spec } \mathbb{C}((z, \theta)), \theta^2 = 0$$

Do symmetries of Σ act on our theory?

Construct CDR_M get $H^*(M, \text{CDR}_M) \supset \text{Vect } \Sigma$

$$\Sigma = \text{Spec } \mathbb{C}((z, \theta))$$

$$0 \rightarrow \mathbb{C} \xrightarrow{\text{Lie}} N=2 \rightarrow \text{Vect } \Sigma \rightarrow 0$$

$N=2$ chiral algebra

$$J(z, \theta) \quad H(z, \theta) = G^-(z) + \theta U(z)$$

||

$U(1)$ current

G^+

Tensors on $M \rightarrow$ sections on CDR

vector fields $v^i \partial_{x^i} \rightarrow v^i(B^i) \psi_i$

$$\Lambda^2 T_M \hookrightarrow \text{CDR}$$

$$V^i \partial_{x^i} \longrightarrow V^i (B^i) \chi_i$$

$$V^{i,j} \partial_{x^i} \wedge \partial_{x^j} \in \Lambda^2 T \longrightarrow V^{i,j} (B) (\chi_i \chi_j)$$

$$B^i \longrightarrow \tilde{B}^j \longrightarrow \frac{\partial \tilde{B}^j}{\partial B^i} \frac{\partial \tilde{B}^k}{\partial B^l} V^{i,j} (B)$$

$$\cdot (\partial B \tilde{\chi}_i) \left(\frac{\partial B}{\partial \tilde{B}} \tilde{\chi}^j \right)$$

$B^i(z, \theta)$ transform as coords

$$(\partial_\theta + \theta \partial_z) B^i(z, \theta) =: S B^i$$

$S B^i$ transforms as diff forms

$$\omega_{ij} dx^i dx^j \longrightarrow \omega_{ij} S B^i S B^j \in H^0(CDR)$$

End $T_M \longrightarrow CDR$

$$\omega_{ij} dx^i \otimes \partial_{x^j} \longrightarrow (\omega_{ij} S B^i) \chi_j + \Gamma_{jk}^i \omega_{ij} S^2 B^k$$

↙
Christoffel symbols of Levi-Civita

$$SO(T_M \oplus T_M^*, \langle \rangle_+) \ni K = \begin{pmatrix} J & \cancel{A} \\ \omega & -J^* \end{pmatrix}$$

$$J \in \text{End } T_M, \cancel{A} \in \Lambda^2 T_M, \omega \in \Lambda^2 T_M^*$$

$$K \rightarrow K(\mathcal{Z}, \theta) \in H^*(M, \mathbb{C}DR)$$

Let (M, ω, g, J) Riem. metric
Complex structure
 |
 diff. manifold 2-fold

$$J_1, J_2 \quad J_1 = \begin{pmatrix} J & 0 \\ 0 & -J^* \end{pmatrix} \quad J_2 = \begin{pmatrix} 0 & \omega J^* \\ \omega J & 0 \end{pmatrix}$$

Then Using the embedding $SO(T \oplus T^*) \hookrightarrow H^*(\mathbb{C}DR)$

get two superfields $J_1(\mathcal{Z}, \theta), J_2(\mathcal{Z}, \theta)$

- 1) J_1 generates $N=2$ VA of central charge $c = 3 \dim M$
iff M is CY
- 2) J_2 generates another $N=2$ $c = 3 \dim M$ iff
 M is Kähler.
- 3) J_1 and J_2 generate $N_2 \oplus N_2$ $c = \frac{3}{2} \dim M$

$$J^\pm = \frac{1}{2} (J^1 \pm J^2)$$

F. Coeff of J^+ $\left\{ \begin{array}{l} c_i \xrightarrow{S} \beta_i \\ \gamma_i \xrightarrow{S} b_i \end{array} \right.$

$$\psi_i = c_i + \theta \beta_i$$

Let's define S' be the coeff of z^{-1} of J^- .

~~γ^i~~ $\rightarrow g^{ij} \delta_{ij} c_j$
 $g_{ij} b^j \rightarrow ? =: S'(g_{ij} b^j)$

~~$L(z)$~~ - Virasoro
 ~~$J^\pm(z)$~~

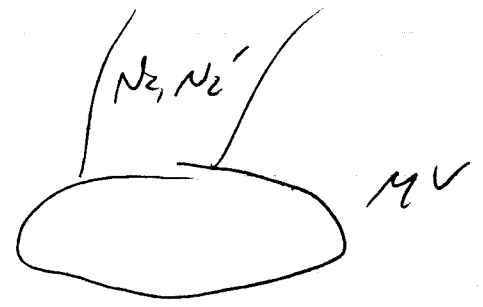
$L_{Lr}(z)$ - Virasoro	}	$G^+ \rightarrow G^-$
$J_{Lr}(z)$ - U(1)		$J \rightarrow -J$
G_{Lr}^+		$L \rightarrow L$
G_{Lr}^-		

Then The automorphism S' acts as the identity on $\mathcal{H}_{N=2}$ at the mirror location on the $-N=2$.

~~$S B^i S^{-1}$~~

$$(\omega_i \circ S B^i) \gamma_j + \gamma_{0k}^i \omega_i \circ S^2 B^k = J,$$

$$\frac{1}{2} (\omega_{ij} S B^i S B^j + \omega^{ij} \gamma_i \gamma_j) = \sigma_2$$



$$CDR_M \cong CDR_{M'} ?$$

$$H_{BRST}(CDR_M, F_2^-) \cong H_{BRST}(CDR_{M'}, F_2^-)$$

\uparrow
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PF B-field transform

$$\psi_i \rightarrow \psi_i + H_{ij} S B^{\hat{j}}$$

$$H_{ij} \in \Lambda^2 T^* M$$

Automorphism $H_{ij} = \sqrt{-1} \omega_{ij}$

$$G_{\mu}^{-} = S B^a \psi_{\bar{a}}$$

$$\text{Tr } g^{\text{Lo}}_{\tau} J_0 H_{\text{BNS}} (CDM, G_{\mu}^{-})$$