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Supersymmetry of the chiral-antichiral de Rham complex in the Calabi-Yau case

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$bc - \beta\gamma$

$$b(z)\,c(w) \sim \frac{1}{z-w}$$

$$\beta(z)\,\gamma(w) \sim \frac{1}{z-w}$$

$$b \longleftrightarrow c$$

$$\gamma(z)\gamma(w) \sim 0$$

Let $g(\gamma)$.

$$b \longrightarrow g\,c$$
$$c \longrightarrow g^{-1} b$$
$$\gamma \longrightarrow \gamma$$
$$\beta \longrightarrow \beta$$

still automorphism of $bc\beta\gamma$ syst?

$$g(\gamma) = \text{const}$$

Can I complete $b \to gc$, $c \to g^{-1}b$, $\gamma \to \gamma$, $\beta \to ?$.

$$b^i(z)\, c^j(w) = \frac{\delta_{ij}}{z-w}$$

$$\beta^i(z)\, \gamma^j(w) = \frac{\delta_{ij}}{z-w}$$

$$g_{ij}(\gamma), \quad g^{ij}(\gamma)$$

$$b^i \rightarrow g^{ij} c_j$$

$$c_i \rightarrow g_{ij} b^j$$

$$\gamma \rightarrow \gamma$$

$$\beta \rightarrow ?$$

Step I glue on general manifold

Step II find $N = 1, 2$ SUSY

hol. curve/$\mathbb{C}$ $\quad \Sigma \longrightarrow M \quad$ complex manifold

$$\begin{array}{ccc} \Sigma & \longrightarrow & M \\ \downarrow & & \downarrow \\ z, \bar{z} & & X_a, X_{\bar{a}} \end{array}$$

All fields on CDR depend only on z

but we have fields assoc. to both $X_a, X_{\bar{a}}$.

$$\overline{X_a(z)} = \text{"}X_{\bar{a}}(\bar{z})\text{"}$$

$$X_{\bar{a}}(z)$$

$$\Sigma = \operatorname{Spec} \mathbb{C}((z))$$

locally on M, x_i coords $\longrightarrow \gamma^i(z)$

$$\partial_{x_i} \longrightarrow c_i(z)$$

$$dx^i \longrightarrow b^i(z)$$

$\beta_i(z)$ transform in a strange way.

SUSY on bc-$\beta\gamma$ system

$$c_i \longrightarrow \beta_i$$

$$\gamma^i \longrightarrow b^i$$

$\theta^2 = 0$

$$\Psi_i(z,\theta) = c_i(z) + \theta\beta_i(z)$$

$$B^i(z,\theta) = \cancel{b(z)+\theta} \; \gamma^i(z) + \theta b^i(z)$$

B^i — transform as coordinates

Ψ_i — transform as vector fields

$\Sigma = \operatorname{Spec} \mathbb{C}((z,\theta))$, $\theta^2 = 0$

Do symmetries of Σ act on our theory?

Constructed CDR_M get $H^*(M, CDR_M) \supseteq \operatorname{Vect} \Sigma$

$\Sigma = \operatorname{Spec} \mathbb{C}((z,\theta))$

$$0 \to \mathbb{C} \to \underset{Lie}{N=2} \to \operatorname{Vect}_\Sigma \to 0$$

$N=2$ chiral algebra

$J(z,\theta)$ $\qquad H(z,\theta) = G^-(z) + \theta L(z)$

$\|$

$U(1)$ current

G^+

Tensors on $M \longrightarrow$ sections in CDR

vector fields $v^i \partial_{x_i} \to v^i(B^i)\psi_i$

$\Lambda^2 T_M \longrightarrow$ CDR

$$V^i \partial_{x_i} \longrightarrow V^i(B^i)\chi_i$$

$$V^{ij}\partial_{x^i}\wedge\partial_{x^j} \in \Lambda^2 T \longrightarrow V^{ij}(B)(\chi_i\,\chi_j)$$

$$B^i \to \tilde{B}^j \to \frac{\partial\tilde{B}^i}{\partial B^{i'}}\frac{\partial\tilde{B}^j}{\partial B^{j'}}\tilde{V}^{ij}(B)\cdot$$

$$\cdot\left(\partial B\,\tilde{\chi}_i\right)\left(\frac{\partial B}{\partial\tilde{B}}\,\tilde{\chi}\right)$$

$B^i(z,\theta)$ transform as coords

$$(\partial_\theta + \theta\,\partial_z)\,B^i(z,\theta) =: SB^i$$

SB^i transforms as diff forms

$$\omega_{ij}\,dx^i\,dx^j \longrightarrow \omega_{ij}\,SB^i\,SB^j \in H^0(CDR)$$

End $T_M \longrightarrow CDR$

$$\omega_i^j\,dx^i\otimes\partial_{x^j} \longrightarrow \left(\omega_i^j\,SB^i\chi_j + \Gamma^i_{jk}\,\omega_i^j\,S^2B^k\right)$$

Christoffel symbols of Levi-Civita

$$SO(T_M \oplus T_M^*, \langle\ \rangle_+) \ni K = \begin{pmatrix} J & \alpha \\ \omega & -J^* \end{pmatrix}$$

$$J \in \mathrm{End}\, T_M, \quad \alpha \in \Lambda^2 T_M, \quad \omega \in \Lambda^2 T_M^*$$

$$K \to K(z,\theta) \in H^*(M, CDR)$$

Let (M, ω, g, J) — M: diff. manifold; ω: 2-form; g: Riem. metric; J: complex structure

$$J_1, J_2 \qquad J_1 = \begin{pmatrix} J & 0 \\ 0 & -J^* \end{pmatrix} \qquad J_2 = \begin{pmatrix} 0 & \omega^{ij} \\ \omega_{ij} & 0 \end{pmatrix}$$

<u>Thm</u> Using the embedding $SO(T \oplus T^*) \hookrightarrow H^*(CDR)$ get two superfields $J_1(z,\theta)$, $J_2(z,\theta)$

1) J_1 generates $N=2$ VA of central charge $c = 3\dim M$ iff M is CY

2) J_2 generates another $N=2$ $c = 3\dim M$ iff M is Kähler.

3) J_1 and J_2 generate $N_2 \otimes N_2$ $c = \frac{3}{2}\dim M$

$$J^{\pm} = \frac{1}{2}(J^1 \pm J^2)$$

F. Coeff of J^+ $\left\{ \begin{array}{l} c_i \xrightarrow{S} \beta_i \\ \gamma^i \xrightarrow{S} b^i \end{array} \right.$

$$\varphi_i = c_i + \theta \beta_i$$

Let's define S' be the coeff. of z^{-1} of J^-.

$$\gamma^i \longrightarrow g^{ij} c_j$$

$$g_{ij} b^j \longrightarrow ? =: S'(g_{ij} b^j)$$

~~$L^{\pm}(z)$ - Virasoro~~

~~$J^{\pm}(z)$~~

$\left. \begin{array}{l} L_{lr}(z) - \text{Virasoro} \\ J_{lr}(z) \quad - U(1) \\ G^+_{lr} \\ G^-_{lr} \end{array} \right\}$

$$G^+ \longleftrightarrow G^-$$
$$J \longrightarrow -J$$
$$L \longrightarrow L$$

<u>Thm</u> The automorphism S' acts as the identity on $\mathcal{F}_{N=2}$ and as the mirror involution on the $-N=2$.

$\cancel{SB^i \; S\psi_i}$

$$(\omega_i{}^j SB^i)\psi_j + \Gamma^i_{jk}\,\omega_i{}^j S^2 B^k = J_1$$

$$\frac{1}{2}\left(\omega_{ij} SB^i SB^j + \omega^{ij}\psi_i\psi_j\right) = \sigma_2$$

$$CDR_M \simeq CDR_{M^\vee} \;?$$

$$H_{BRST}(CDR_M, G_\ell) \simeq H_{BRST}(CDR_{M^\vee}, G_r^-)$$

Q

Pf B-field transform

$$\varphi_i \longrightarrow \varphi_i + H_{ij}\, S B^{\bar{j}}$$

$$H_{ij} \in \Lambda^2 T^*_M$$

Automorphism $H_{ij} = \sqrt{-1}\, \omega_{ij}$

$$G_\alpha^- = S\bar{B}^a \varphi_{\bar{a}}$$

$$\cdot \ \mathrm{Tr}\ g^{L_0} \tau^{J_0} H_{BRST}(CDR_M, G_\alpha^-)$$