Anton Kapustin

$$\mathcal{M}_{Hit}({}^LG, C)_J \xleftrightarrow{\text{mirror}} \mathcal{M}_{Hit}(G, C)_K$$

DO $\longleftrightarrow$ Lagrangian A-brane

moduli space
of deformations of
special Lagrangian

To determine SYZ fibration
need extra info

$$G = {}^LG = GL(N)$$

$$\mathrm{tr}\, \phi_z^k \longleftrightarrow \mathrm{tr}\, {}^L\phi_z^k$$

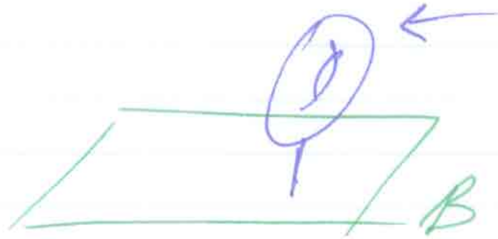
For G_2 and F_4 map is non-trivial

$$\mathrm{tr}\, \phi_z^{\ell} \in H^0(K_C^{\ell})$$

$$D_{\bar{z}}\, \phi = 0$$

$$(A, \phi) \longrightarrow \mathrm{tr}\, \phi^{\ell}$$

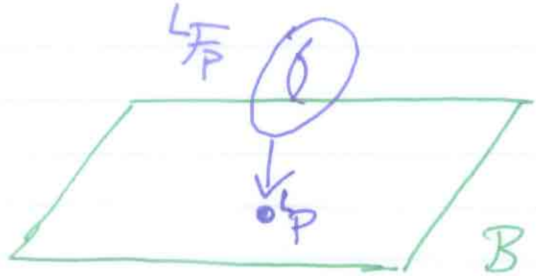
$$\mathcal{M}_{Hit}(G, C)$$
$$\downarrow$$
$$\bigoplus_{\ell=1}^{N} H^0(K_C^{\ell}) = \mathcal{B}$$

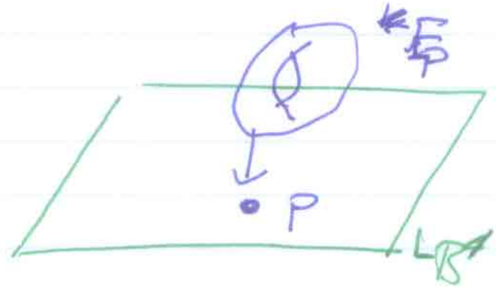


← Lagrangian wrt $\Omega_I = \omega_J + i\omega_K$

onto

$\sim \int \mathrm{Tr}\, \delta A \wedge \delta \phi$





moduli space must map
moduli space of
point

moduli space of flat
connections

$$\begin{array}{c} \mathrm{Jac}(F_p) \simeq {}^L F_p \\ \| \\ \mathcal{M}_{\substack{flat \\ unitary}}(F_p) \end{array}$$

Hitchin system on $\mathbb{P}^1$ w/ ramification
Arinkin

$\mathcal{N}=2$ gauge theory generalization

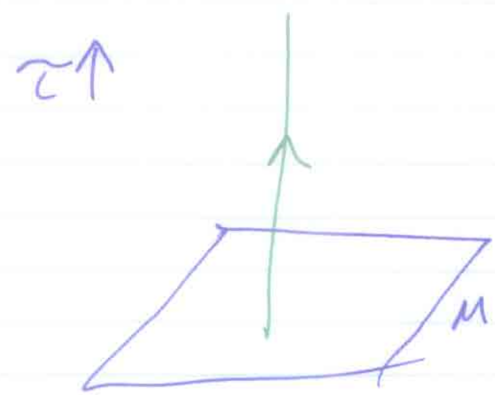
$t = i$
${}^L G$

Wilson loop:
$$W_{\gamma^L, R} = \mathrm{Tr}_R \, \mathrm{Hol}_\gamma(A + i\phi)$$

Wilson line
$R(\mathrm{Hol}_\gamma(A+i\phi))$

How do we understand when not gauge invariant?

modifies Hilbert space

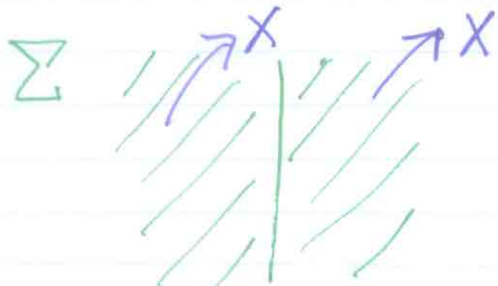


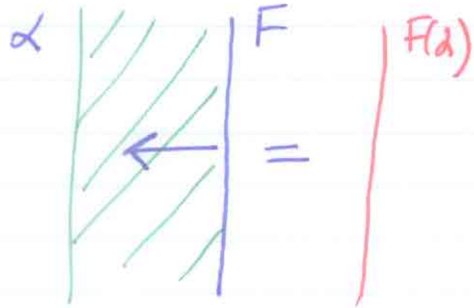
$\Psi(A, \dots) \in$ pre-weight function

$(\mathcal{H} \otimes V_R)^{\mathrm{inv}}$

Gauge transformations
G-valued functions on initial time slice

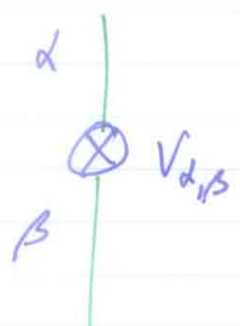
Line operators in 2d field theory



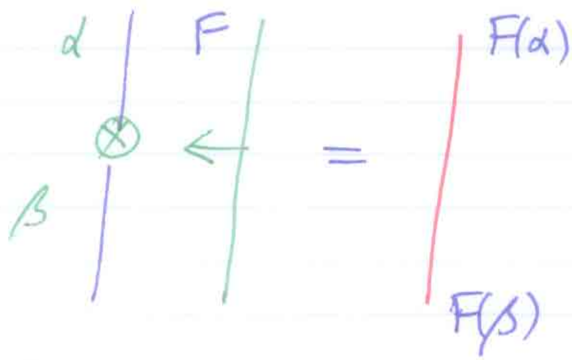


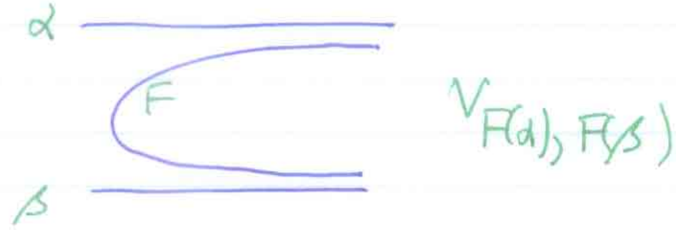
Composite boundary condition

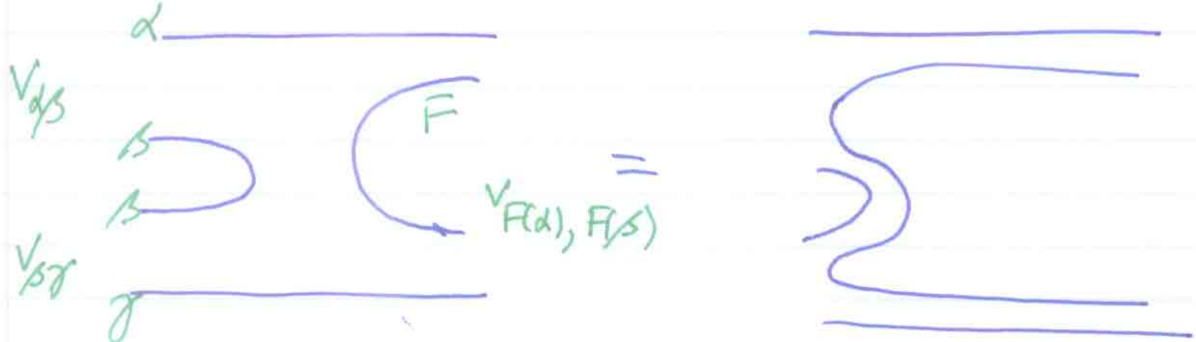
Boundary conditions form a category



$V_{\alpha,\beta}$ = space of FT on $I_{\alpha\beta} \times \mathbb{R}$







Boundary conditions form category

<u>Ex</u> B-model with target X

Category of line operators

$$D^b(\mathrm{Coh}(X, X))$$

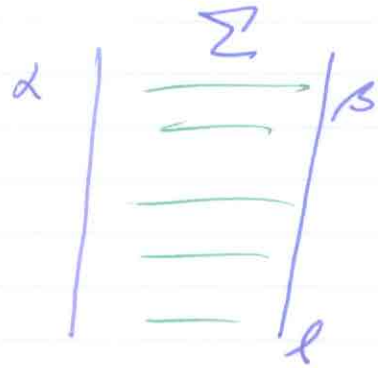
$$\begin{tikzcd} & X \times X \arrow[dl] \arrow[dr] & \\ X & & X \end{tikzcd}$$

kernel of integral transform

B-model with target $\mathcal{M}_{Hit}({}^LG, C)_\gamma$

$M_4 = C \times \Sigma$

Σ = half plane



$\gamma \hookrightarrow M_4$

$\gamma_p = \ell \times p$

$p \in C$

$\rightsquigarrow W_{{}^LR,p}$ (a line operator for the B-model) $= \otimes\, {}^LR(\mathcal{E}_p)$

$\mathcal{E} \to \mathcal{M}_{Hit}({}^LG, C) \times C$

If ${}^LR$ transforms trivially under center of ${}^LG$

same X but w/ different (flat) B field
discrete

obstruction ~~to~~ $\mathcal{E} \to$ discrete B field

$$W_{{}^LR_1,p} \circ W_{{}^LR_2} = W_{{}^LR_1,p \otimes {}^LR_2,p}$$

Q. Do the p's have to be the same?
A. No

Why

Quantizing theory on $M_3 \times \mathbb{R}$
get vector space of states

Can take M_3 to be compact

Take M_3 = half-space

How do I do quantization?

Compact: Moduli space of vacua
$\rightsquigarrow$ cohomology

<u>Ex</u> $G = PSU(2)$ $\quad\quad\quad$ ${}^L G = SU(2)$

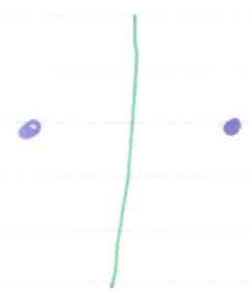
${}^L R$ = 2-dim rep

BPS equations $\rightsquigarrow$ Bogomolny equations

$$F = * d\phi_0$$

Both Higgs and gauge fields have singularities

Doubling trick



two singularities

Weinberg $\quad T^* \mathbb{P}^1 \quad$ two dirac monopoles

w/o singularities $\mathbb{R}^3 \times S^1$

S^1 fiber over $\mathbb{R}^3$ w/ two places where S^1

$\rightarrow$ Symmetry $\quad$ restricts to zero-section

Moduli space $\mathbb{P}^1$

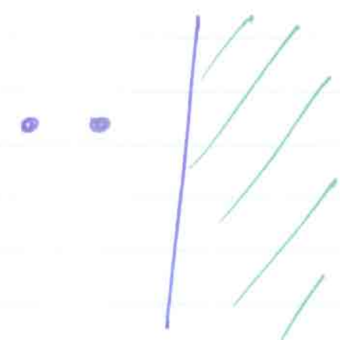
Quantize $\rightarrow$ take cohomology

In general

moduli space non-compact
take L^2 cohomology
can show L^2 cohomology has right dim

Ex
$^L R$ = 3-dim of $SU(2)$
Moduli space TP^1

L^2 cohomology 3-dim



Moduli space decomposes into sum

$^L R_1 = {}^L R_2 = 2$-dim

$2 \times 2 = 1 + 3$

Need to solve Bogomolny equations w/ 2 singularities on a half-space

$\mathbb{P}\left(T\mathbb{P}^1 \oplus \mathcal{O}_{\mathbb{P}^1}\right)$ $\mathbb{P}^1$ bundle over $\mathbb{P}^1$

Charge fixed by boundary conditions

$$\mathbb{P}(T\mathbb{P}^1 \oplus \mathcal{O}_{\mathbb{P}^1}) = \widetilde{W\mathbb{P}^2_{1,1,2}}$$

We can understand what happens when these singularities approach each other

The weight functions (3 of them) become spread out correspond to triplet

More powerful L^2 cohomology $\rightsquigarrow$ Intersection Cohomology

Schubert Cell in affine Grassmanian
Satake correspondence

Can use Nahm transform