MAX-PLANCK-GESELLSCHAFT

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Radiation reaction in classical and quantum electrodynamics

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OUTLINE

- Introduction to classical and quantum electrodynamics in intense laser fields
- The problem of radiation reaction in classical electrodynamics
- The quantum origin of radiation reaction
  - Stochasticity effects in quantum radiation reaction
- Conclusions

For more information see the recent reviews:

1. F. Ehlotzky, K. Krajewska, and J. Z. Kaminski, Rep. Prog. Phys. **72**, 046401 (2009)
2. A. Di Piazza, C. Müller, K. Z. Hatsagortsyan, and C. H. Keitel, Rev. Mod. Phys. **84**, 1177 (2012)

Electromagnetic interaction

- The electromagnetic interaction is one of the four "fundamental" interactions in Nature. It is the interaction among electric charged particles (e.g., electrons and positrons) and it is mediated by the electromagnetic field (photons, in the "quantum" language)
- Both classically and quantum mechanically it is described theoretically by a Lagrangian/Hamiltonian which depends on two parameters:
  - Electron mass $m=9.1\times10^{-28}$ g
  - Electron charge e, with $|e|=4.8\times10^{-10}$ esu
- The typical scales of classical electrodynamics (CED) are determined by the parameters m and e and by the speed of light c. Whereas, those of quantum electrodynamics (QED) are also determined by the (reduced) Planck constant $\hbar$.

Typical scales of CED and QED

	CED	QED
Energy scale	Electron's rest energy: $\varepsilon_0 = mc^2 = 0.5$ MeV	
Momentum scale	$p_0 = \varepsilon_0/c = 0.5$ MeV$/c$	
Length scale	Classical electron's radius: $r_0 = e^2/mc^2 = 2.8\times10^{-13}$ cm (from the Thomson cross section)	Compton wavelength: $\lambda_C = \hbar/p_0 = 3.9\times10^{-11}$ cm (from the Heisenberg uncertainty principle)
Time scale	$r_0/c = 1.0\times10^{-23}$ s	$\lambda_C/c = 1.3\times10^{-21}$ s
Field scale	Critical field of CED: $\mathcal{E}_0 = \varepsilon_0/\lvert e\rvert r_0 = 1.8\times10^{18}$ V/cm, $\mathcal{B}_0 = \varepsilon_0/\lvert e\rvert r_0 = 6.0\times10^{15}$ G	Critical fields of QED: $E_{cr} = \varepsilon_0/\lvert e\rvert \lambda_C = 1.3\times10^{16}$ V/cm, $B_{cr} = \varepsilon_0/\lvert e\rvert \lambda_C = 4.4\times10^{13}$ G
Intensity scale	$\mathcal{I}_0 = c\mathcal{E}_0^2/4\pi = 8.6\times10^{33}$ W/cm^2	$I_{cr} = cE_{cr}^2/4\pi = 4.6\times10^{29}$ W/cm^2

Note that if $\alpha = e^2/\hbar c \approx 1/137$ is the fine-structure constant then $r_0 = \alpha\lambda_C \approx \lambda_C/137$ and $(\mathcal{E}_0, \mathcal{B}_0) = (E_{cr}, B_{cr})/\alpha \approx 137(E_{cr}, B_{cr})$

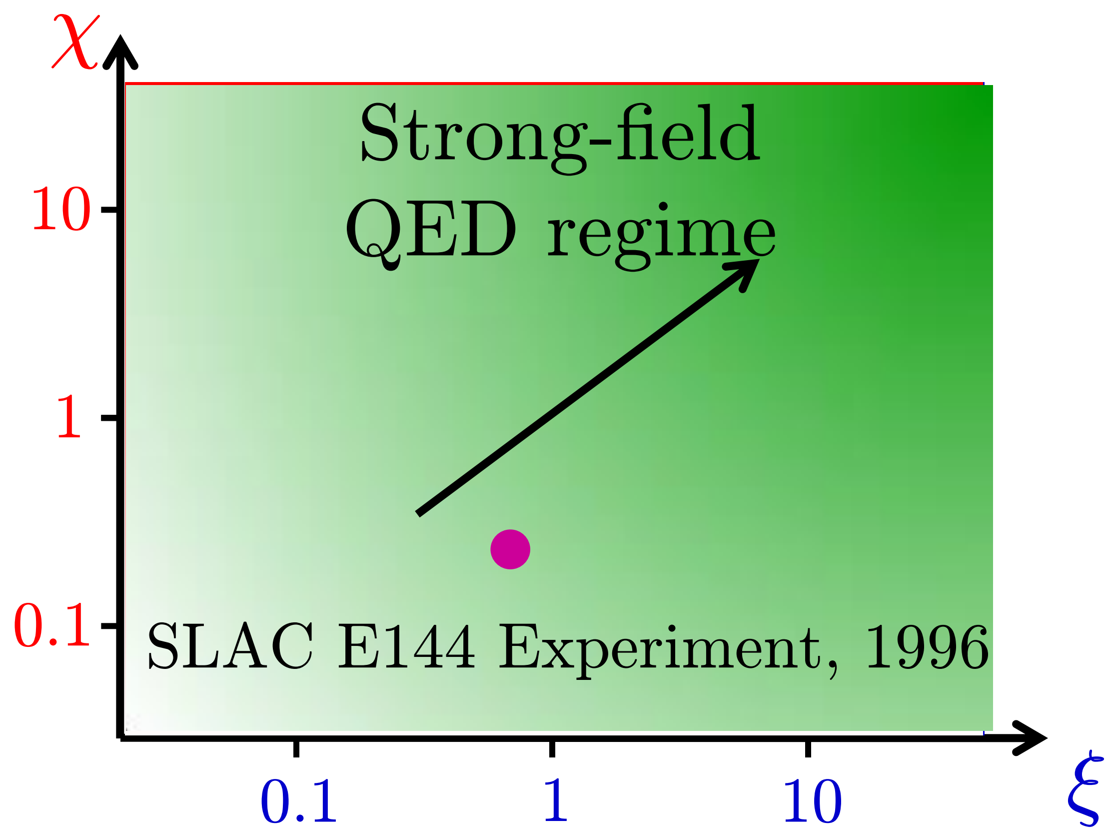
Regimes of QED in a strong laser field

An electron with energy ε head-on collides with a plane wave with amplitude E_L and angular frequency ω_L (wavelength λ_L)

Relevant Lorentz- and gauge-invariant parameters (Ritus 1985, Di Piazza et al., 2012):

$$\xi = \frac{1}{2\pi}\frac{|e|E_L\lambda_L}{mc^2} = \frac{|e|E_L\lambda_C}{\hbar\omega_L}$$

$$\chi = \left.\frac{E_L}{E_{cr}}\right|_{\text{rest frame}}$$



Optical laser and electron accelerator technology

Optical laser technology ($\hbar\omega_L$=1 eV)	Energy (J)	Pulse duration (fs)	Spot radius (μm)	Intensity (W/cm^2)
State-of-art (Yanovsky et al., Opt. Express 2008)	10	30	1	2×10^{22}
Soon (2015) (APOLLON, Vulcan, Astra-Gemini, BELLA etc...)	10÷100	10÷100	1	$10^{22}\div10^{23}$
Near future (2020) (ELI, XCELS)	10^4	10	1	$10^{25}\div10^{26}$

Electron accelerator technology	Energy (GeV)	Beam duration (fs)	Spot radius (μm)	Number of electrons
Conventional accelerators (PDG)	10÷50	$10^3\div10^4$	10÷100	$10^{10}\div10^{11}$
Laser-plasma accelerators (Wang et al., Nature. Commun. 2013)	0.1÷1	50	5	$10^9\div10^{10}$

$$\xi = 7.5\frac{\sqrt{I_L[10^{20}\ \mathrm{W/cm^2}]}}{\hbar\omega_L[\mathrm{eV}]}$$

$$\chi = 5.9\times10^{-2}\,\varepsilon[\mathrm{GeV}]\sqrt{I_L[10^{20}\ \mathrm{W/cm^2}]}$$

Present technology allows in principle the experimental investigation of strong-field QED

Radiation reaction in CED

What is the equation of motion of an electron in an external, given electromagnetic field $F^{\mu\nu}(x)$?

Units with $\hbar$=c=1

- The Lorentz equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu$$

does not take into account that while being accelerated the electron generates an electromagnetic radiation field and it loses energy and momentum

- One has to solve self consistently the coupled Lorentz and Maxwell equations (Dirac 1938, Barut 1980)

$$m_0\frac{du^\mu}{ds} = eF_T^{\mu\nu}u_\nu \qquad\qquad m_0\frac{du^\mu}{ds} = e(\partial^\mu A_T^\nu - \partial^\nu A_T^\mu)u_\nu$$

$$\partial_\mu F_T^{\mu\nu} = e\int ds\delta(x - x(s))u^\nu \quad \xrightarrow{\text{Lorenz gauge}} \quad \Box A_T^\nu = e\int ds\delta(x - x(s))u^\nu$$

where now m_0 is the electron's bare mass and $F_{T,\mu\nu}$=$\partial_\mu A_{T,\nu}$–$\partial_\nu A_{T,\mu}$ is the total electromagnetic field (external field plus the one generated by the electron)

- One first solves the inhomogeneous wave equation exactly with the Green's-function method

$$\Box A_T^\nu = e\int ds\delta(x-x(s))u^\nu = j^\nu(x) \longrightarrow A_T^\nu(x) = A^\nu(x) + \int dx'\mathcal{D}(x-x')j^\nu(x')$$

and then re-substitute the solution into the Lorentz equation:

$$(m_0+\delta m)\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

where δm is a quantity which diverges for a pointlike charge

- After "classical mass renormalization" one obtains the Lorentz-Abraham-Dirac (LAD) equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

- The LAD equation is plagued by serious inconsistencies: runaway solutions. Consider its three-dimensional non-relativistic limit

$$m\frac{d\boldsymbol{v}}{dt} = e(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}) + \frac{2}{3}e^2\frac{d^2\boldsymbol{v}}{dt^2}$$

In the free case $\boldsymbol{E}$=$\boldsymbol{B}$=$\mathbf{0}$, it admits the solution $\boldsymbol{a}(t)$=$\boldsymbol{a}_0e^{t/\tau_0}$, where τ_0=$(2/3)e^2/m$~10^{-24} s. Note: the solution is non-perturbative in e

- Avoiding the runaways: integro-differential LAD equation (Rohrlich 1961)

$$m\frac{du^\mu}{ds}=\frac{e^{s/\tau_0}}{\tau_0}\int_s^\infty ds' e^{-s'/\tau_0}\left[eF^{\mu\nu}u_\nu+\frac{2}{3}e^2\frac{du^\nu}{ds'}\frac{du_\nu}{ds'}u^\mu\right]$$

- Problem: preacceleration at time scales of the order of τ_0
- If $\max|F^{\mu\nu}(x)|_{\text{inst. rest-frame}} \ll \mathcal{F}_0{}^{\mu\nu}=(\mathcal{E}_0,\mathcal{B}_0)$, one can replace the four-acceleration du^μ/ds in the radiation-reaction force in the LAD equation

$$m\frac{du^\mu}{ds}=eF^{\mu\nu}u_\nu+\boxed{\frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2}+\frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)}$$

with the zero-order four-acceleration $eF^{\mu\nu}u_\nu/m$ (Landau and Lifshitz 1947)

- Since $(\mathcal{E}_0,\mathcal{B}_0)=(E_{cr},B_{cr})/\alpha\approx 137(E_{cr},B_{cr})$, the above condition is always fulfilled by definition in the realm of CED
- The resulting equation

$$m\frac{du^\mu}{ds}=eF^{\mu\nu}u_\nu+\frac{2}{3}e^2\left[\frac{e}{m}(\partial_\alpha F^{\mu\nu})u^\alpha u_\nu-\frac{e^2}{m^2}F^{\mu\nu}F_{\alpha\nu}u^\alpha+\frac{e^2}{m^2}(F^{\alpha\nu}u_\nu)(F_{\alpha\lambda}u^\lambda)u^\mu\right]$$

is known as Landau-Lifshitz (LL) equation

- Three important remarks:
  1. The LL equation is "exact" in the realm of CED
  2. The LL equation is free from inconsistencies and it includes all physical solutions of the LAD equation (Spohn 2000)
  3. The LL equation can be directly derived from QED (Krivitsky and Tsytovich 1991)
- The LAD equation is "too exact" (but in an inconsistent way):

$$m\frac{du^\mu}{ds} = a_1^\mu e + a_2^\mu e^2 + a_3^\mu e^3 + a_4^\mu e^4 + \cdots$$

- In the LAD equation the series in e is "summed" exactly in the realm of CED (essential non-perturbative effects in e are included)
- Lower-order quantum terms proportional to $\hbar$ are much larger than higher-order classical terms like $a_6^\mu e^6$

Radiation reaction in quantum electrodynamics

- Radiation reaction in CED: the Lorentz equation has to be modified as it does not account for the energy-momentum loss of the accelerating electron
- Thus one could be tempted to say that radiation reaction is automatically taken into account in QED already in the "basic" emission process

  because photon recoil, i.e., the energy-momentum subtracted by the photon to the electron is automatically included
- However, this cannot be the case because

  1. in the classical limit $\chi \to 0$, the spectrum of single-photon emission goes into the classical spectrum calculated via the Lorentz equation, i.e., without radiation reaction
  2. high-order corrections in χ would include $\hbar$
  3. in CED radiation reaction would always imply small corrections to the Lorentz dynamics, which is not the case

- Another conceptual reason: single-photon emission does not have a classical analog
  - Why the quantum origin of radiation reaction has been related to single-photon emission?
  - Why the single-photon emission spectrum $\hbar\omega dW/d^3k$ has a classical analog at all?
- Origin of confusion: in the classical limit, where photon recoil is negligible, subsequent photon emissions are independent processes and the emission probability of n photons follows the Poisson distribution (Glauber 1951)

$$w_n = \frac{1}{n!} W^n e^{-W}$$

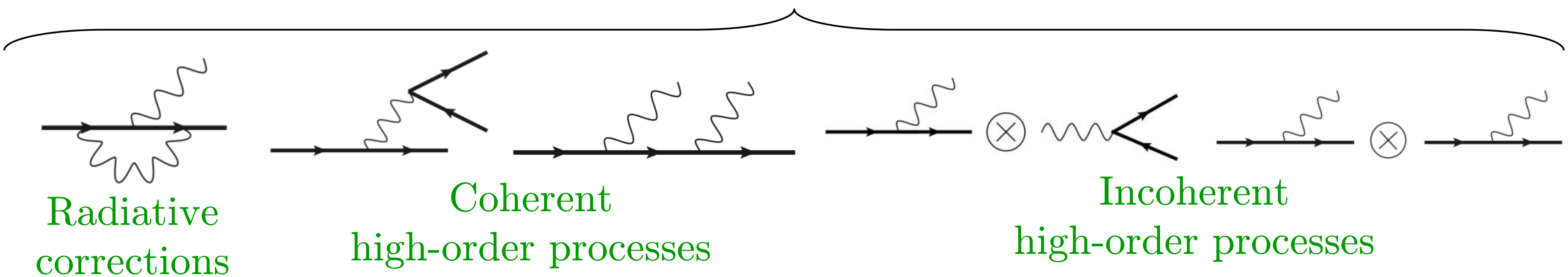
  where $W=\int d^3k/(2\pi)^3 dW/d^3k$ is the total probability of single-photon emission calculated at first order in α
- Thus, in the classical limit:
  1. W=average number of photons emitted
  2. $\mathcal{E}=\int d^3k/(2\pi)^3 \hbar\omega dW/d^3k$=average energy emitted by the electron
  3. $\hbar\omega dW/d^3k$ = average spectral energy emitted by the electron

- To determine the dynamics of the electron via the LAD equation amounts to solve self-consistently Maxwell and Lorentz equations

$$\begin{aligned} m_0\frac{du^\mu}{ds} &= eF_T^{\mu\nu}u_\nu \\ \partial_\mu F_T^{\mu\nu} &= e\int ds\,\delta(x-x(s))u^\nu \end{aligned} \quad \longleftrightarrow \quad m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}\alpha\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

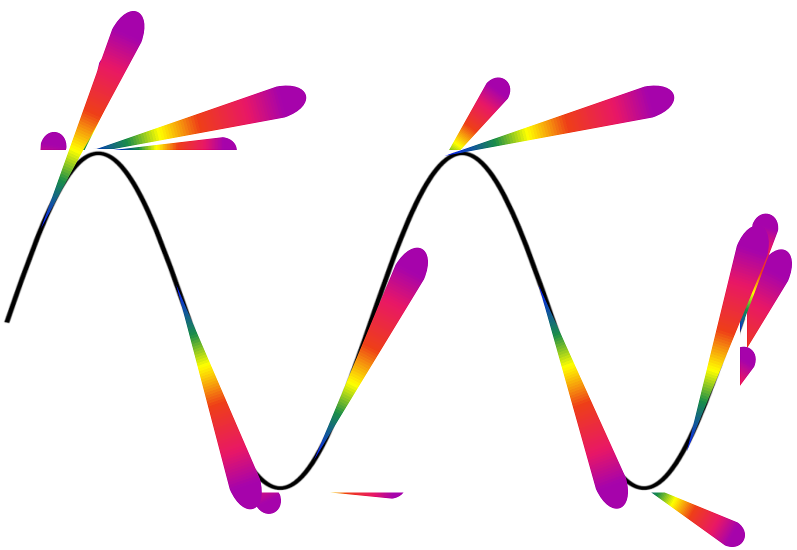
- This corresponds in QED to determine the evolution of a single-electron state in background field+radiation field generated by the electron

$$|t=-\infty\rangle = |e^-\rangle \longrightarrow \text{Complete evolution operator } (S\text{-matrix}) \longrightarrow |t=+\infty\rangle$$



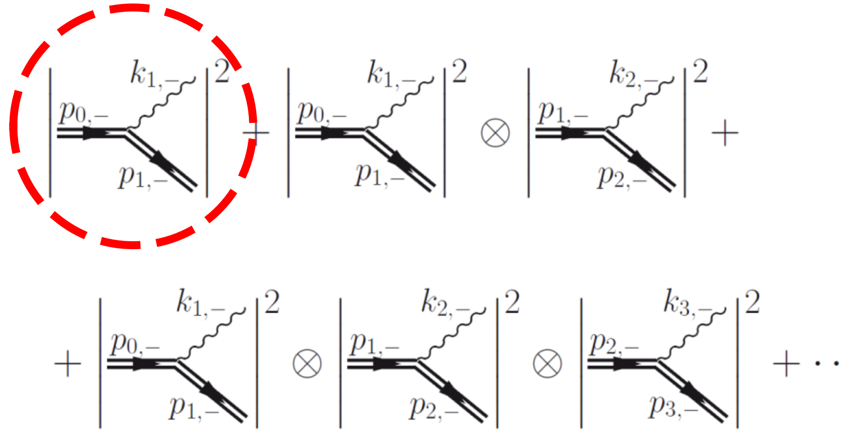
- In strong-field QED including “radiation reaction” amounts in accounting for all possible quantum processes arising with an electron in the initial state

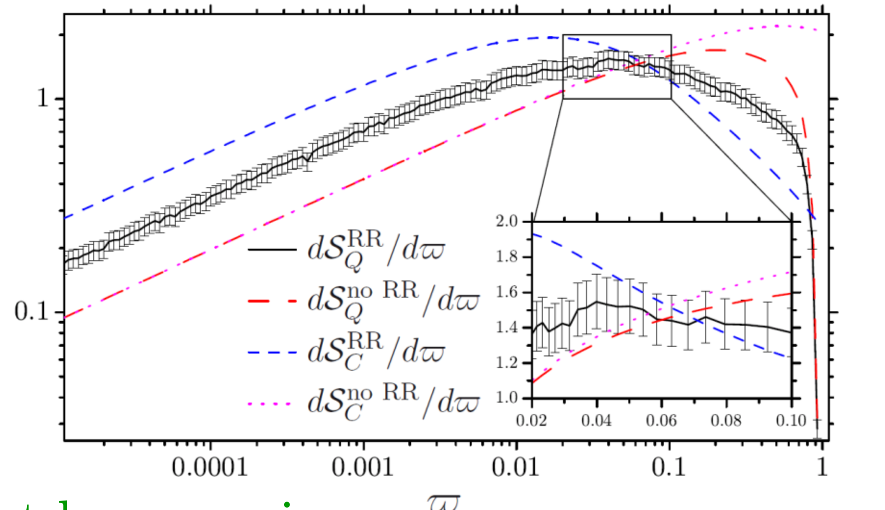
- The problem is in general multiparticle due to electron-positron pair production
- Moderately quantum regime (ADP, K. Z. Hatsagortsyan, and C. H. Keteil, PRL 2010): at $\xi \gg 1$ and $\chi \lesssim 1$, when still electron-positron pair production is negligible, the problem is still single-particle and the multiple incoherent emission of photons gives the main contribution to radiation reaction



- The classical limit is a double limit: the electron emits a large number of photons but all with a small recoil, in such a way that the average energy emitted is finite
- Unless the number of emitted photons and the photon recoil, the average emitted energy has a classical analog
- Quantum radiation dominated regime: multiple photon emission already in one laser period with a large recoil

- We calculated the average energy emitted per unit of electron energy (emission spectrum), by taking into account the incoherent emission of $N>1$ photons (quantum radiation reaction)





- Numerical parameters: electron energy 1 GeV, laser wavelength 0.8 μm, laser intensity 5×10^{22} W/cm^2 (ξ=150) corresponding to R_Q=1.1 and χ=1.8, laser pulse duration 5 fs

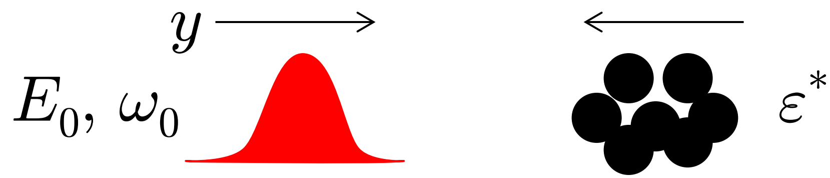
- Effects of radiation reaction:
  1. increase of the spectrum yield at low energies
  2. shift to lower energies of the maximum of the spectrum yield
  3. decrease of the spectrum yield at high energies

- Classical radiation reaction artificially amplifies all the above effects
- Classical spectra both without and with radiation reaction give unphysical results at high photon energies

A. Di Piazza, K. Z. Hatsagortsyan, and C. H. Keitel, Phys. Rev. Lett. **105**, 220403 (2010)

Kinetic approach to quantum radiation reaction

- If the electron emits sequentially a large number of photons, a kinetic approach is suitable to treat the problem
- Setup: an electron bunch head-on collides with a plane wave



- Parameters regime:
  1. the electron bunch is ultra-relativistic and it is barely deviated by the laser field from its initial direction of propagation: $\varepsilon^* \gg m\xi \gg m$ (Landau and Lifshitz 1975)
  2. Quantum effects are “moderately” important: $\chi^* = (2\varepsilon^*/m)(E_0/E_{cr}) \lesssim 1$ (Ritus 1985)
- Corresponding simplifying assumptions:
  1. the one-dimensional kinetic approach can be employed (transverse motion: see Jiang-Xin Li et al. PRL 2014)
  2. electron-positron pair production can be neglected
- It is convenient to employ the coordinates: $\phi = t - y$, $T = (t + y)/2$ and $\boldsymbol{r}_\perp = (x, z)$, and the corresponding momenta components $P = (\varepsilon + p_y)/2$, $p_- = \varepsilon - p_y$ and $\boldsymbol{p}_\perp = (p_x, p_z)$, as the field depends only on ϕ and the quantities p_- and $\boldsymbol{p}_\perp$ are constant of motions

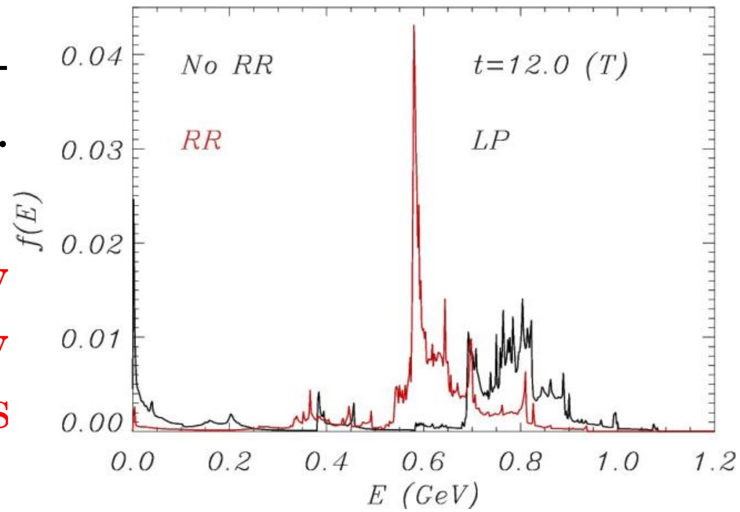
- The kinetic equations in our regime are (Baier et al. 1998)

$$\frac{\partial n_e(\varphi,p_-)}{\partial\varphi} = \int_{p_-}^{\infty} dp_{i,-}\, n_e(\varphi,p_{i,-})\frac{dP_{p_{i,-}}}{d\varphi dp_-} - n_e(\varphi,p_-)\int_0^{p_-} dk_-\frac{dP_{p_-}}{d\varphi dk_-}$$

$$\frac{\partial n_\gamma(\varphi,k_-)}{\partial\varphi} = \int_{k_-}^{\infty} dp_{i,-}\, n_e(\varphi,p_{i,-})\frac{dP_{p_{i,-}}}{d\varphi dk_-},$$

where $n_{e/\gamma}(\varphi,p_-)$=electron/photon distribution function, $\varphi=\omega_0\phi$=laser phase, $dP_{p-}/d\varphi dk_-$= photon emission probability

- The two equations are not coupled and we consider only the first
- Motivation: radiation reaction in classical electrodynamics acts as a beneficial cooling mechanism
- Example: energy spectrum of a laser-generated ion beam (Tamburini et al. NJP 2010)
- Cooling mechanism: high-energy particles emit more than low-energy ones and the phase space contracts (Tamburini et al. NIMA 2011)



- What happens when quantum effects come into play?

- The classical-quantum transition can be studied by expanding the kinetic equation at small values of the quantum parameter $\chi(\varphi,p_-)$

Order $\chi^2(\varphi,p_-)$: Liouville-like deterministic equation

$$\frac{\partial n_e}{\partial \varphi} = -\frac{\partial}{\partial p_-}\left(n_e \frac{dp_-}{d\varphi}\right), \quad \frac{dp_-}{d\varphi} = -\frac{I_{cl}(\varphi,p_-)}{\omega_0} = -\frac{2}{3}\alpha\frac{m^2}{\omega_0}\chi^2(\varphi,p_-)$$

corresponding to the LL dynamics (classical radiation reaction)

Order $\chi^3(\varphi,p_-)$: Fokker-Planck-like diffusion equation

$$\frac{\partial n_e}{\partial \varphi} = -\frac{\partial}{\partial p_-}[A(\varphi,p_-)n_e] + \frac{1}{2}\frac{\partial^2}{\partial p_-^2}[B(\varphi,p_-)n_e]$$

$$A(\varphi,p_-) = -\frac{I_{cl}(\varphi,p_-)}{\omega_0}\left[1 - \frac{55\sqrt{3}}{16}\chi(\varphi,p_-)\right]$$

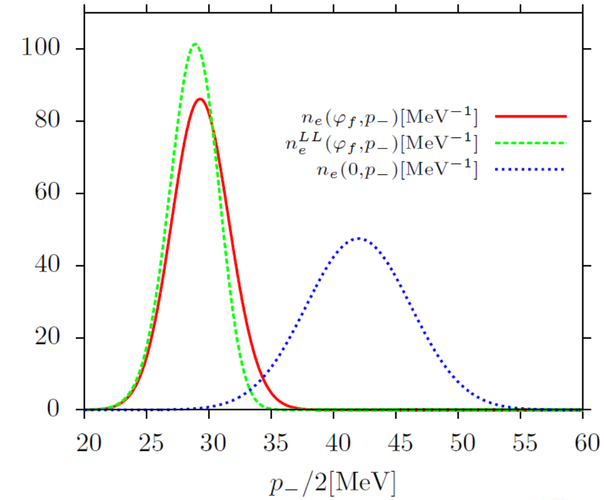
$$B(\varphi,p_-) = \frac{\alpha m^2}{3\omega_0}\frac{55}{8\sqrt{3}}p_-\chi^3(\varphi,p_-)$$

- Quantum effects induce:
  1. a correction to the intensity of radiation in agreement with the expansion of the corresponding quantum intensity of radiation $I_q(\varphi,p_-)$ (Ritus 1985)
  2. the appearance of the diffusion term
- The diffusion term is related to the stochasticity of quantum radiation reaction. The Fokker-Planck equation can be related to the single-particle stochastic equation (Gardiner 2009)

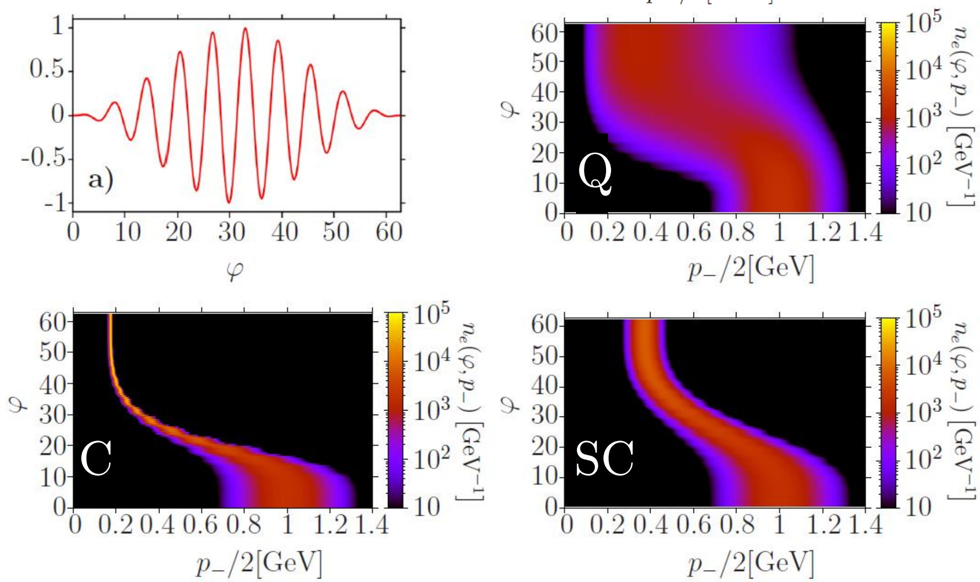
$$dp_- = -A(\varphi,p_-)d\varphi + \sqrt{B(\varphi,p_-)}dW$$

with dW being an infinitesimal stochastic function

- Numerical example: $\sin^2$-like optical (λ_0=0.8 μm) pulse with $I_0=4.3\times10^{20}$ W/cm^2 (ξ=10) and duration τ=4 ps and an electron bunch initially with ε^*=42 MeV ($\chi^*=5\times10^{-3}$) and $\Delta\varepsilon^*$=4.2 MeV
- Classical and quantum equations predict a reduction of $\Delta\varepsilon^*$ to 2.3 MeV



- Numerical parameters as above except $I_0=2.2\times10^{22}$ W/cm^2 (ξ=68), τ=30 fs, ε^*=1 GeV (χ^*=0.8) and $\Delta\varepsilon^*$=0.1 GeV
- SC: classical formulas with $I_{cl}(\varphi,p_-)\rightarrow I_q(\varphi,p_-)$



- Classical and quantum approaches give opposite results
- The semiclassical approach does not include stochasticity effects and cannot explain the broadening of the distribution function

N. Neitz and A. Di Piazza, Phys. Rev. Lett. **111**, 054802 (2013)

CONCLUSIONS

- Present and next-generation lasers offer a unique possibility of accessing new extreme regimes of interaction, where the radiation strongly affects the electron dynamics
- In such extreme regimes
  1. new effects can become measurable and long-standing problems, like the radiation-reaction problem, can be explored
  2. predictions of QED in the high-intensity domain, as complementary to the well-known high-energy one, can be addressed
  3. the quantum origin of radiation reaction can the investigated