MAX-PLANCK-GESELLSCHAFT

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

Recollision processes in strong-field QED

Antonino Di Piazza

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Outline

- Introduction to recollision processes in atomic physics
- Introduction to strong-field QED
- The polarization operator in an intense laser field
- Recollision processes in strong-field QED
- Conclusions

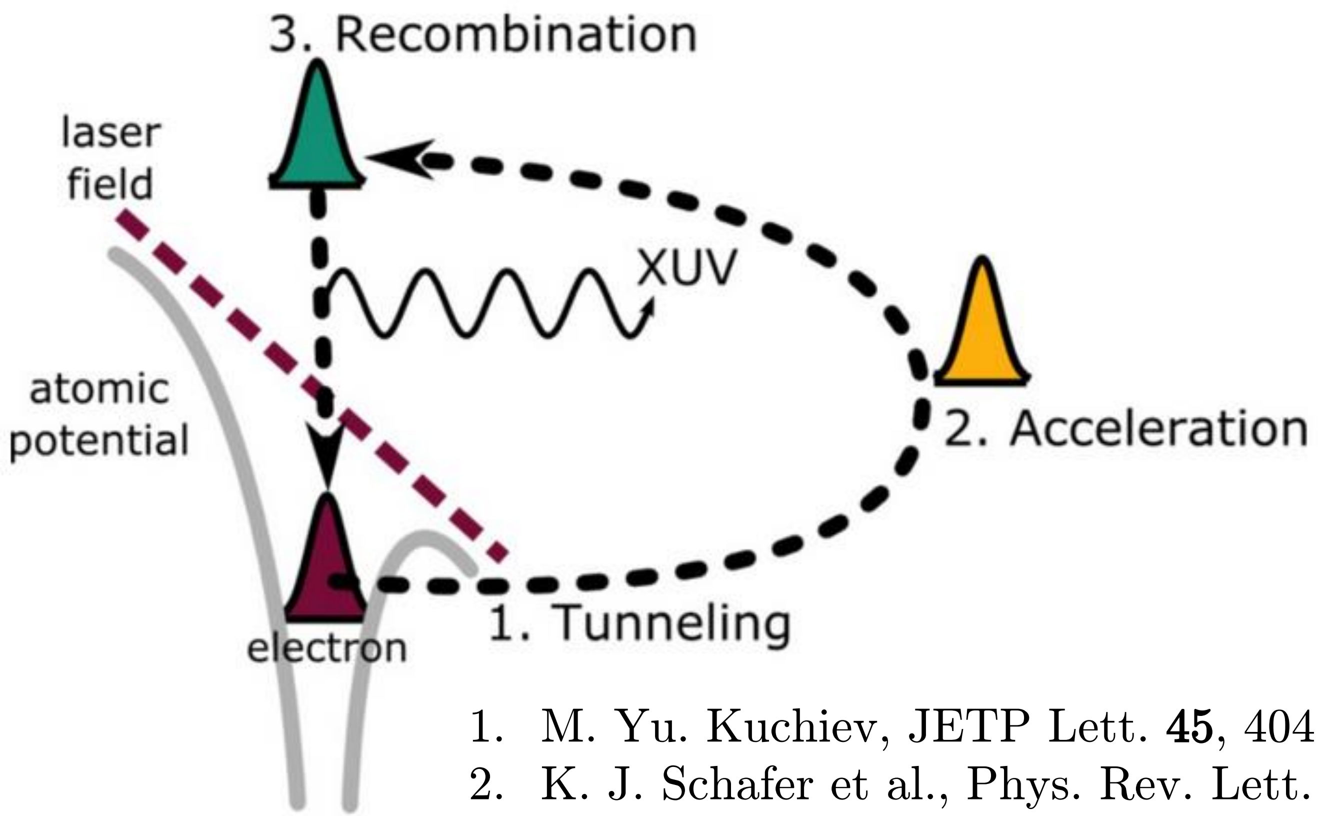
For more information see:

1. S. Meuren, C. H. Keitel, and A. Di Piazza, Phys. Rev. D **88**, 013007 (2013)
2. S. Meuren, K. Z. Hatsagortsyan, C. H. Keitel, and A. Di Piazza, arXiv:1407.0188

- Units with $\hbar = c = 1$ will be employed

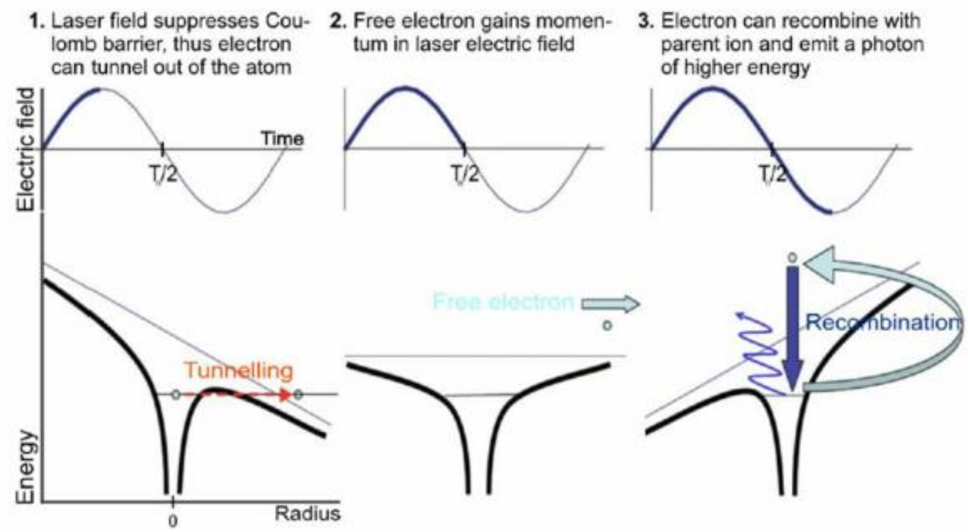
Recollision processes in atomic physics

- "In a recollision, the oscillating field of a laser pulse causes an electron to accelerate away from an atom or molecule and then, upon reversal of the field, careen back into its parent ion." P. B. Corkum, Physics Today, 2011



1. M. Yu. Kuchiev, JETP Lett. **45**, 404 (1987)
2. K. J. Schafer et al., Phys. Rev. Lett. **70**, 1599 (1993)
3. P. B. Corkum, Phys. Rev. Lett. **71**, 1994 (1993)

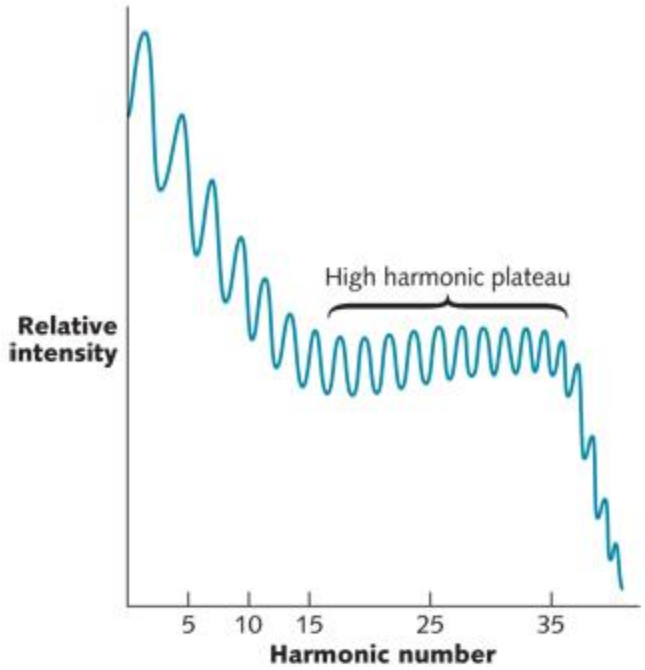
1. The coherent infrared field splits the electron's wave function into two coherently connected parts: a bound wave function and a ionized, tunneled wave packet



2. The ionized wave packet is then accelerated by the laser field
   – If the laser field is circularly polarized, the center of the wave packet drifts on the plane where the electric field of the laser rotates and never returns to the parent ion (exceptions…)
3. If the laser field is linearly polarized, part of the electron's wave packet, after being accelerated, can drift back and recollides with the parent ion within one laser period

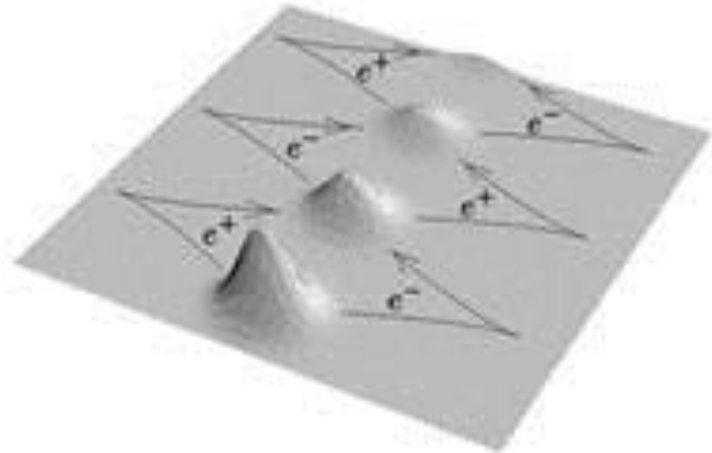
- In the process of recollision, the electron recombines with the parent ion by releasing the acquired energy as a high-energy photon (high-harmonic generation) or by striking out another electron (non-sequential double ionization)

- Extension of the harmonic spectrum: $\hbar\omega_M \approx I_p + 3.17U_p$, where $U_p = m\xi^2/4 = e^2E_L{}^2/4m\omega_L{}^2$
- Scaling of the ratio of the high-harmonic yield in the plateau region and the lowest-order harmonics $1/\xi^{5.5}$ (Murnane's talk)
- In order to produce higher and higher harmonics one would try to go to larger and larger values of ξ

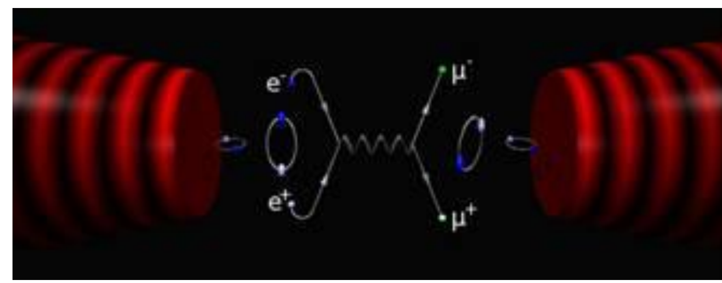


- Atomic recollision processes are suppressed in the relativistic regime: the $(\boldsymbol{E}\times\boldsymbol{B})$-drift prevents the electron to recombine with parent ion (difference in the charge/mass ration between the nucleus and the electron)
- Possible solutions: preaccelerated ions (Chirila et al. PRL 2004), standing wave with $\boldsymbol{E}\times\boldsymbol{B}=\boldsymbol{0}$ (Milosevic et al. PRL 2004), asymmetric initial electron state (Fischer et al. PRL 2006), tailored laser pulses (Klaiber et al. PRA 2006)

- Using positronium atoms: electron and positron have the same charge/mass ratio and undergo the same relativistic drift (Henrich et al. PRL 2004)
- Problem: wave-packet spreading suppresses the harmonic yield



- A microscopic collider based on laser-driven positronium atoms (Hatsagortsyan et al. EPL 2006, Müller et al. PLB 2008)



- Production of high-energy particles from the recollision of a laser-accelerated electron-positron pair directly created from vacuum in the presence of the laser field and a nucleus (Kuchiev PRL 2007)
- In PRL 2007 a semiquantitative description of the recollision process is provided, not based on strong-field QED calculations
- How can we describe such recollision processes within the formalism of strong-field QED?

Introduction to strong-field QED

- QED in vacuum is the most successful physical theory we have:

$$g(\text{experimental})/2 = 1.00115965218073(28)$$
$$g(\text{theoretical})/2 = 1.00115965218113(11)(37)(02)(77)$$

  where g is the gyromagnetic factor of the electron (Aoyama et al. 2012). Equivalent to measure the distance Earth-Moon with an accuracy of the order of the diameter of a single human hair!

- The experimental tests of QED in the presence of background electromagnetic fields are not comparably numerous and accurate

- The reasons are:
  1. on the theoretical side, exact analytical and numerical calculations are feasible only for a few background electromagnetic fields: constant and uniform electric/magnetic field, Coulomb field, plane-wave field
  2. on the experimental side, the "typical" electromagnetic fields of QED are very "large"

- Fields scale of QED:

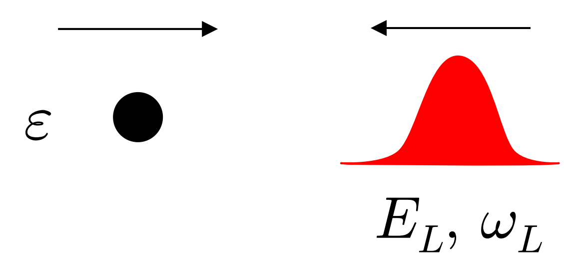
$$E_{cr}=\frac{m^2c^3}{\hbar|e|}=1.3\times10^{16}\ \mathrm{V/cm}$$

$$B_{cr}=\frac{m^2c^3}{\hbar|e|}=4.4\times10^{13}\ \mathrm{G}$$

$$I_{cr}=\frac{cE_{cr}^2}{4\pi}=4.6\times10^{29}\ \mathrm{W/cm^2}$$

Optical laser technology ($\hbar\omega_L$=1 eV)	Energy (J)	Pulse duration (fs)	Spot radius (μm)	Intensity (W/cm^2)
State-of-art (Yanovsky et al., Opt. Express 2008)	10	30	1	2×10^{22}

- An electron with energy ε head-on collides with a plane wave with amplitude E_L and angular frequency ω_L



- Relevant Lorentz- and gauge-invariant parameter:

$$\chi=E_L|_{\text{rest frame}}/E_{cr}=0.59\varepsilon[\mathrm{GeV}]\sqrt{I_L[10^{22}\mathrm{W/cm^2}]}$$

Electron accelerator technology	Energy (GeV)	Beam duration (fs)	Spot radius (μm)	Number of electrons
Laser-plasma accelerators (Wang et al., Nature. Commun. 2013)	0.1÷1	50	5	$10^9\div10^{10}$

Furry picture in strong-field QED

Lagrangian density of QED in the presence of a background field $A_{B,\mu}(x)$ (Furry 1951)

$$\mathscr{L}_{QED} = \mathscr{L}_e + \mathscr{L}_\gamma + \mathscr{L}_{\text{int}}$$

$$\mathscr{L}_e = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \qquad\longrightarrow\qquad \mathscr{L}_e = \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_{B,\mu}) - m]\psi$$

$$\mathscr{L}_\gamma = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} \qquad\longrightarrow\qquad \mathscr{L}_\gamma = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

$$\mathscr{L}_{\text{int}} = -e\bar{\psi}\gamma^\mu\psi(A_\mu + A_{B,\mu}) \qquad\longrightarrow\qquad \mathscr{L}_{\text{int}} = -e\bar{\psi}\gamma^\mu\psi A_\mu$$

- Only the interaction between the spinor and the radiation field is treated perturbatively

1. Solve the Dirac equation $[\gamma^\mu(i\partial_\mu - eA_{B,\mu}) - m]\psi=0$
   - find the "dressed" one-particle electron states and the "dressed" propagator (Green's function of the Dirac equation in the background field)
2. Write the Feynman diagrams of the process at hand
3. Calculate the amplitude and then the cross sections (or the rates) using "dressed" states and propagators

Strong-field QED in an intense laser field

- The laser field is approximated by a plane-wave field: $A_B{}^\mu(x) = A_L{}^\mu(\phi)$, $\phi = (k_L x) = \omega_L(t - z)$ and $k_L{}^\mu = \omega_L(1,0,0,1)$
- One-particle states: Volkov states (Volkov 1936)

$$\psi_{p,\sigma} = \left[1 + \frac{e}{2(k_L p)}\hat{k}_L \hat{A}_L\right] \boxed{\frac{u_{p,\sigma}}{\sqrt{2p_0}} \exp\left\{-i(px)\right.} \left. - i\int_{-\infty}^{\phi} d\phi' \left[\frac{e(pA_L)}{(k_L p)} - \frac{e^2 A_L^2}{2(k_L p)}\right]\right\}$$

Four-momentum and spin at $t \to -\infty$

Spin term

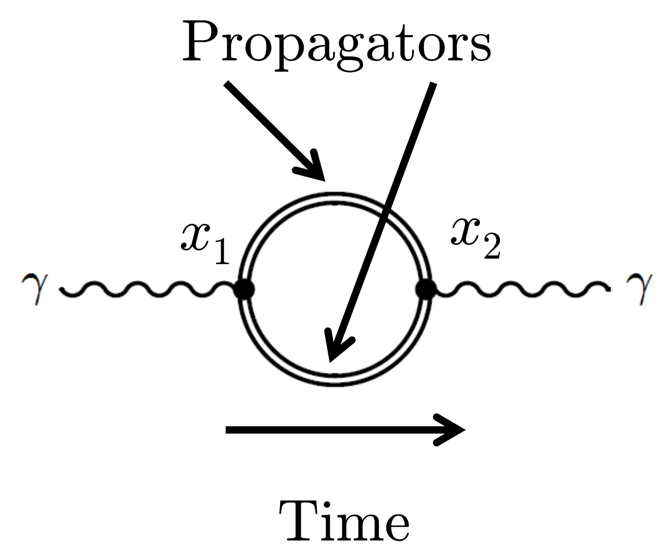
Free constant bi-spinor

i(Classical action)

- Technical notes:
  - Volkov states are quasiclassical
  - The average spin four-vector ζ^μ satisfies the "classical" Bargmann-Michel-Telegdi (BMT) equation $d\zeta^\mu/ds = (e/m)F^{\mu\nu}\zeta_\nu$
  - The vacuum in the presence of a plane wave is stable
- Analogously one can determine the Volkov propagator $G(x,x')$ (Green's function of the Dirac equation in the presence of a plane-wave field)

Polarization operator in strong-field QED

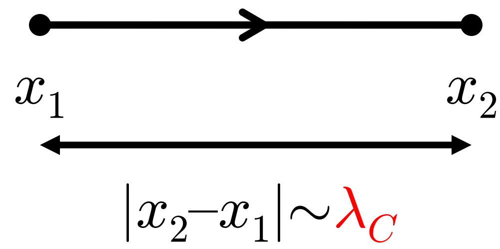
- The polarization operator corresponds to the Feynman diagram on the right: a photon transforms at x_1 into an electron-positron pair, which travels through the background field and then annihilates back into a photon at x_2



- The wavy lines indicate the photons
- The double lines indicate that the electron and positron propagate inside and interact with the external field
- The polarization operator seems to be the right object to investigate recollision processes in strong-field QED
- Why has this not been realized so far?
- The polarization operator describes an effective non-local and pure quantum interaction among photons and electromagnetic fields in vacuum (Di Piazza et al. 2012)

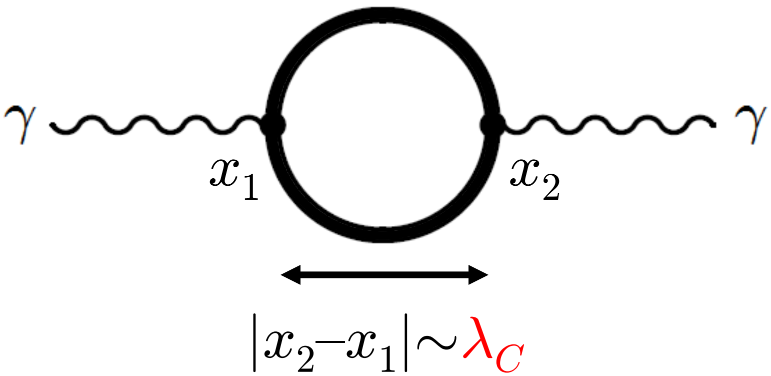
- The propagator in vacuum is

$$G_0(x_2 - x_1) = \int \frac{d^4p}{(2\pi)^4} \frac{\hat{p} + m}{p^2 - m^2 + i\epsilon} e^{-ip(x_2 - x_1)}$$

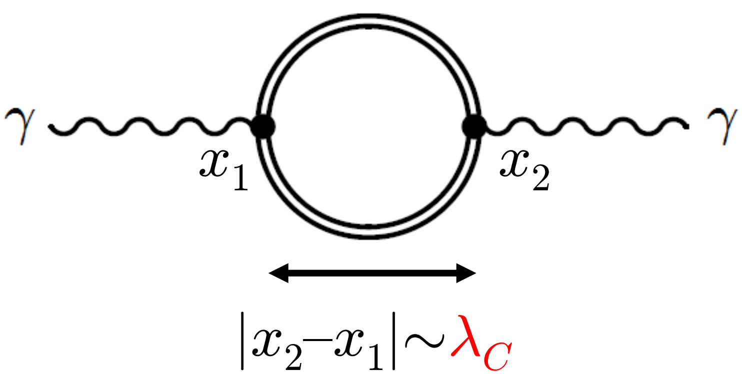


- The only length/time scale contained in $G_0(x_2 - x_1)$ is the Compton wavelength $\lambda_C = 1/m = 3.9 \times 10^{-11}$ cm $= 1.3 \times 10^{-21}$ s

- An analogous conclusion is drawn for the polarization operator: internal or "virtual" particles "live" for a microscopic time of the order of λ_C

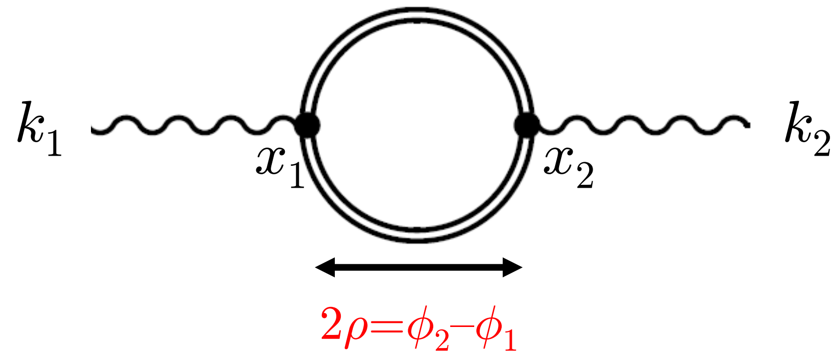


- Despite the external field can introduce new length and time scales, the same conviction has been extended to the polarization operator in an external field



Computation of the polarization operator

- Following the Feynman rules, the diagram of the polarization operator corresponds to the amplitude



$$\Pi(k_1,k_2) = -\int d^4x_1 d^4x_2\, \mathrm{Tr}[\hat{e}_{k_1,\lambda_1} e^{-i(k_1x_1)} G(x_1,x_2) \hat{e}^*_{k_2,\lambda_2} e^{i(k_2x_2)} G(x_2,x_1)]$$

- The plane wave depends on $t-z$, the integrals in $\boldsymbol{x}_\perp$ and $t+z$ provide the conservation laws: $\boldsymbol{k}_{1,\perp}=\boldsymbol{k}_{2,\perp}$, and $\omega_1-k_{1,z}=\omega_2-k_{2,z}$
- The remaining expression can be written as

$$\Pi(k_1,k_2) = \int_{-\infty}^{\infty} d\phi_2\, e^{in\phi_2} \int_0^{\infty} d\rho\, F(\rho,\phi_2) e^{-i\Phi(\rho,\phi_2)}$$

where

- $2\rho=\phi_2-\phi_1$=phase difference between annihilation and creation points
- n="number" of laser photons absorbed by the electron-positron pair ($k_2{}^\mu=k_1{}^\mu+nk_L{}^\mu$)
- $F(\rho,\phi_2)$=complicated function varying on a time scale $\sim T_L=2\pi/\omega_L$

- Note that $n\sim\Phi(\rho,\phi_2)$

- The phase $\Phi(\rho,\phi_2)$ reads

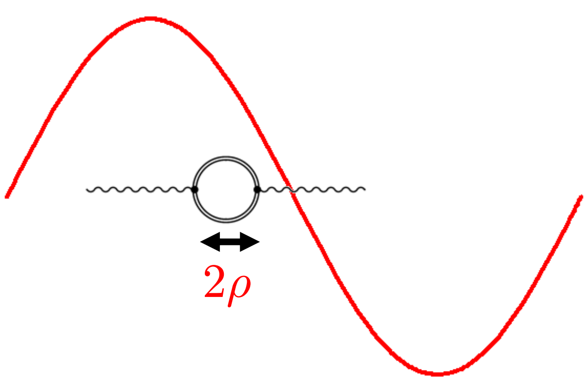
$$\Phi(\rho, \phi_2) = \frac{4\xi\rho}{\chi}[1 + \xi^2(J - I^2)]$$

where $\chi=(2\omega_1/m)E_L/E_{cr}$ and

$$I = \int_0^1 d\lambda\, v(\phi_2 - 2\rho\lambda) \quad J = \int_0^1 d\lambda\, v^2(\phi_2 - 2\rho\lambda)$$

with $v(\phi)$ being the laser temporal profile (note that $J\geq I^2$)

- At $\xi\gg 1$ and $\chi\lesssim 1$, the phase $\Phi(\rho,\phi_2)$ becomes very large
- The main contribution to the integral in $\rho\in[0,\infty[$ comes from the region of small $\rho=(\phi_2-\phi_1)/2\lesssim 1/\xi\ll 1$, where $\Phi(\rho,\phi_2)\sim 1\sim n$
- The dominant "quasistatic" contribution indicates that only a few laser photons can be efficiently absorbed in general in processes involving electron-positron loops and external photons even at ultrarelativistic laser intensities (Di Piazza 2013)



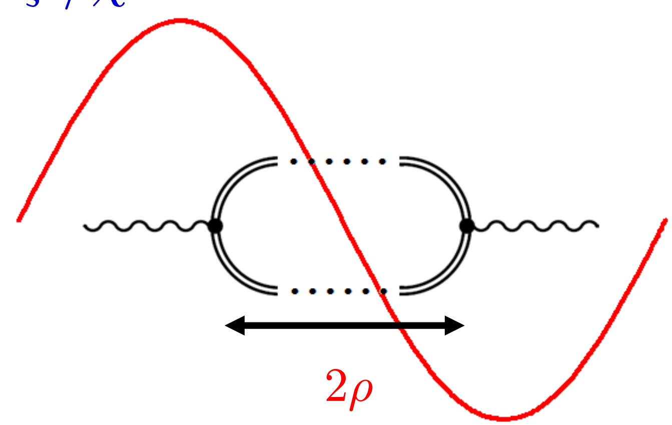
- Saddle points of the phase

$$\Phi(\rho,\phi_2)=\frac{4\xi\rho}{\chi}[\cancel{1}+\xi^2(J-I^2)]$$

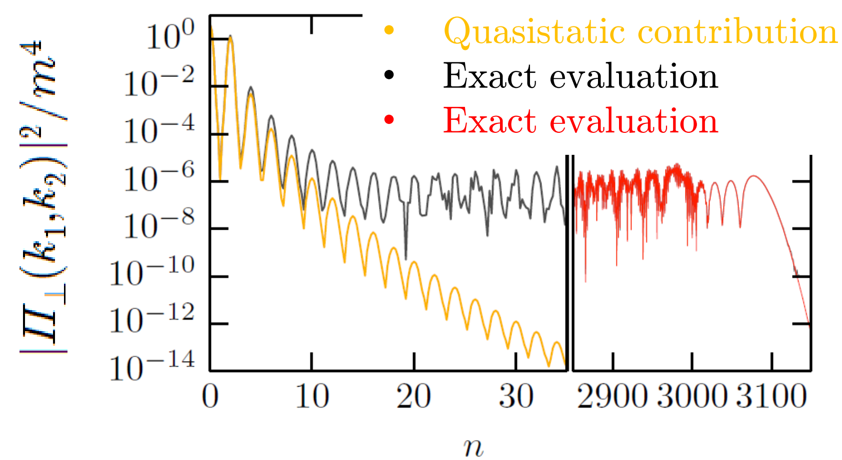
- The saddle-point condition in ρ reads

$$(\phi_2-\phi_1)v(\phi_1)=\int_{\phi_1}^{\phi_2}d\phi\, v(\phi)$$

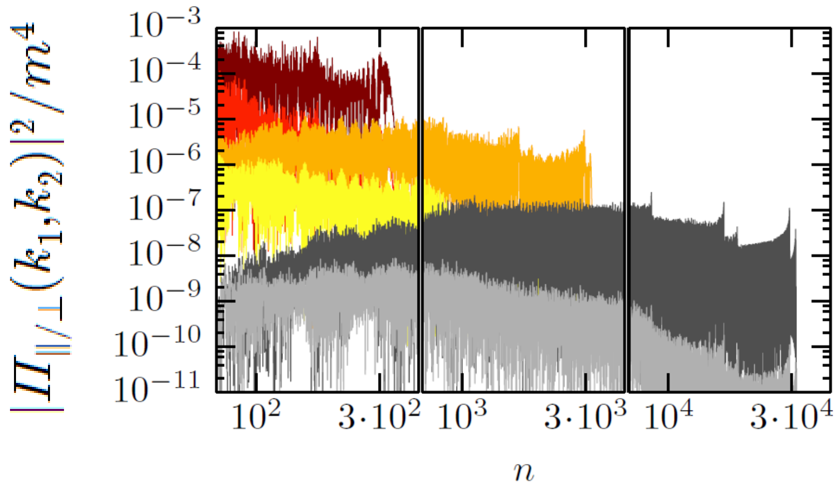
  and it coincides with the condition for the recollision of a classical electron and positron created at the same point each with momentum $\boldsymbol{p}=\boldsymbol{k}_1/2$
- The saddle points are in the region $\rho=(\phi_2-\phi_1)/2\sim2\pi$ and the phase at the saddle points is $\Phi(\rho,\phi_2)\sim\xi^3/\chi\sim n\gg1$
- The electron and the positron propagate for about a cycle in the laser field and absorb a large number of laser photons
- Multiple saddle points correspond to multiple recollisions



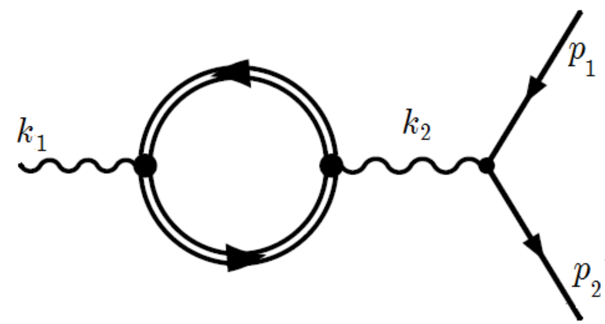
- Are the internal electron and positron real? At $\chi(\phi_1)\ll 1$ the amplitude contains the same factor $\exp[-4/3\chi(\phi_1)]$ as in the amplitude of real pair photoproduction (at the phase point ϕ_1)
- The recollision contribution in $|\Pi(k_1,k_2)|^2$ is suppressed by a factor ξ^{-6} with respect to the quasistatic contribution: 1) wave-packet spreading 2) required tuning in the initial electron/positron momenta in order the recollision to occur (longitudinal spreading $\lesssim\alpha\lambda_C$)
- The maximal energy that the pair can absorb from the laser field: $\omega_M\approx 3.17\xi^3\omega_L/\chi$ exactly corresponds to $2\times 3.17\ U_p$ for an electron-positron pair created with both particles initially at rest
- Numerical example: $\xi=10$, $\chi=1$, 5-cycle, $\sin^2$-laser pulse
- Head-on laser-photon collision
- $\Pi_{\parallel/\perp}(k_1,k_2)$=polarization operator for incoming photon polarized along/perpendicularly to the laser polarization



- Numerical example: $\chi=1$, 5-cycle, $\sin^2$-laser pulse
- $\xi=10^{2/3}\approx 4.6$ ($\perp$ and $\parallel$), $\xi=10$ ($\perp$ and $\parallel$), and $\xi=10^{4/3}\approx 21.5$ ($\perp$ and $\parallel$)
- Height $\sim \xi^{-6}$, cut-off $\sim \xi^{3}$



- The final photon (${k_2}^{\mu}={k_1}^{\mu}+n{k_L}^{\mu}$) cannot propagate in vacuum [${k_2}^2=2n(k_1k_L)\neq 0$] and one can use it, for example, to produce a muon-antimuon pair ($m_\mu\approx 200m$)
- At $\chi=1$, it is $\chi_\mu\approx 1/200^2$ and the direct muon-antimuon pair production is exponentially suppressed ($\sim e^{-8/3\chi_\mu}$)



- Threshold on the number of absorbed photons: $n_0=2{m_\mu}^2/(k_1k_L)$ implies that $\xi\gtrsim m_\mu/m\approx 200$ (recall that $n\sim\xi^3/\chi$)
- Recollision-assisted probability per photon exponentially larger than the direct one but still very small ($\sim 10^{-20}$)
- Application to two-photon emission (no threshold in ξ)

Conclusions

- Recollision processes play a fundamental role in atomic physics
- A rigorous description of recollision processes in strong-field QED in intense lasers has been derived based on the polarization operator
- Analytical and numerical results show analogies between atomic and strong-field QED photon spectra resulting from recollision processes:

  1. Harmonic yield in the plateau: $\sim\xi^{-6}$
  2. Plateau extension: $\sim\xi^{3}/\chi$

- Also in strong-field QED the extension of the plateau can be interpreted classically