Super-Adiabatic Particle Number: Vacuum Particle Production in Real Time

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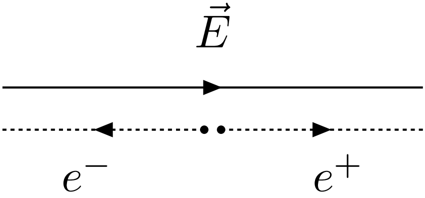
Schwinger Effect: Pair Production from Vacuum



Schwinger critical field: $2eE\frac{\hbar}{mc} \sim 2mc^2 \Rightarrow$

$$E_c \approx \frac{m^2c^3}{e\hbar} \approx 10^{16}\,\mathrm{V/cm} \qquad , \qquad I_c \approx 4 \times 10^{29}\,\mathrm{W/cm^2}$$

experimentally inaccessible scales

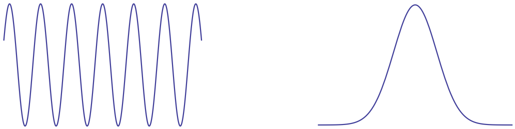
but, estimate based on constant-field approximation ...

Importance of Pulse Shape: Quantum Interference

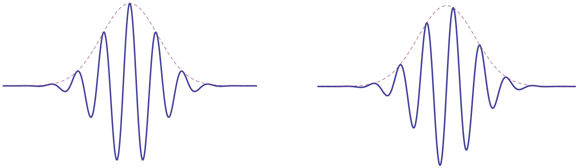
- realistic short laser pulses have temporal structure [envelope, carrier-phase, chirp, focussing, ...]
- particle production is extremely sensitive to these, due to quantum interference
- we can learn to take advantage of this, to enhance the particle production effect
- critical intensity can be lowered several orders of magnitude

Importance of Pulse Shape: Quantum Interference

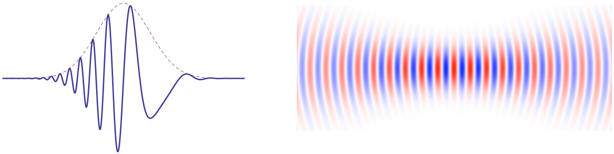
constant E field



monochromatic
or single pulse



pulse with
sub-cycle
structure;
carrier phase
effect



chirped pulse,
Gaussian beam, ...

Particle Number in Quantum Field Theory

- the Schwinger effect: non-perturbative production of electron-positron pairs when an external electric field is applied to the quantum electrodynamical (QED) vacuum
- Parker-Zeldovich effect: cosmological particle production due to expanding cosmologies
- Hawking radiation: particle production due to black holes and gravitational horizon effects
- Unruh radiation: particle number as seen by an accelerating observer

Particle Number in Quantum Field Theory

QED effective action: (Heisenberg/Euler, Feynman, Schwinger, Nambu, ...)

$$\langle 0_{\rm out} \,|\, 0_{\rm in} \rangle \equiv \exp\left(\frac{i}{\hbar}\,\Gamma[A]\right)$$

vacuum persistence probability:

$$|\langle 0_{\rm out} \,|\, 0_{\rm in} \rangle|^2 = \exp\left(-\frac{2}{\hbar}\,{\rm Im}\,\Gamma[A]\right) \approx 1 - \frac{2}{\hbar}\,{\rm Im}\,\Gamma[A]$$

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Fredholm determinant: (Feynman, Schwinger, Matthews/Salam, ...)

$$\Gamma[A] = \ln\,\det\left(i\,\not{D} + m + i\epsilon\right) \qquad , \qquad D_\mu \equiv \partial_\mu - \frac{ie}{\hbar c} A_\mu$$

simple computation for constant E field

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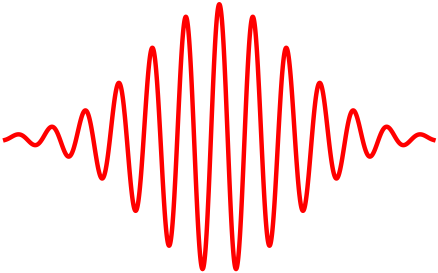
simple computation for constant E field
for realistic laser pulses we want more information:

- momentum distributions, spectra, ...
- optimization and quantum control
- backreaction

Particle Number in Quantum Field Theory

$|0\rangle_{\text{in}}$



$|0\rangle_{\text{out}}$

$t = -\infty$ intermediate time $t = +\infty$

Particle Number in Quantum Field Theory

local information?

$$\mathrm{Im}\,\Gamma = V_4\,\frac{E^2}{4\pi^3}e^{-\frac{m^2\pi}{E}} \quad ? \to ? \int d^4x\,\frac{E^2(x)}{4\pi^3}e^{-\frac{m^2\pi}{E(x)}} \quad ???$$

locally constant field approximation misses a lot of important physics: quantum interference

how to formulate this in general ?

perturbative effective field theory:

$$\Gamma_{\rm pert}[A] = \int d^4x\,\mathcal{L}_{\rm pert}\left(F, \partial F, \partial^2 F, ...\right)$$

non-perturbative effective field theory:

$$\mathrm{Im}\,\mathcal{L}_{\rm non-pert}(x) = ?$$

Puzzles with Particle Production

in addition to experimental challenges, vacuum particle production presents fundamental conceptual and computational problems

- can it be seen ?
- optimization and quantum control ?
- particle concept in time-dependent background fields ?
- backreaction ?
- cascades ?
- maximum electric field ?
- correlations and quantum noise ?
- entanglement entropy and mutual information ?

Mode Decomposition of Fields

spatial homogeneity $\quad \Rightarrow \quad \Phi_k(t) = a_k f_k(t) + b^\dagger_{-k} f^*_{-k}(t)$

mode Klein-Gordon equation:

$$\left(\frac{d^2}{dt^2} + \omega_k^2(t)\right) f_k(t) = 0$$

Schwinger effect:

$$\omega_k^2(t) = m^2 + k_\perp^2 + \left(k_\parallel - A_\parallel(t)\right)^2$$

Parker-Zeldovich effect: metric $ds^2 = -dt^2 + a^2(t)\, d\Sigma^2$

$$\omega_k^2(t) = H^2 \left(\gamma^2 + \left(\frac{2k+d-3}{2}\right)\left(\frac{2k+d-1}{2}\right) \text{sech}^2(Ht)\right)$$

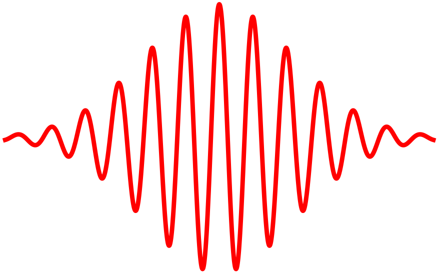
Adiabatic Particle Number

In a time-dependent background field there is no unique separation into positive and negative energy states with which to identify particles and anti-particles [Dirac, DeWitt, Parker].

For a *slowly varying* time-dependent background we can define an adiabatic particle number with respect to a reference basis that reduces to ordinary positive and negative energy plane waves at intial and final times when the background is constant

Particle Number in Quantum Field Theory

$|0\rangle_{\text{in}}$



$|0\rangle_{\text{out}}$

$t = -\infty$

intermediate time

$t = +\infty$

Bogoliubov Transformation

Bogoliubov transformation to time-dependent creation/annihilation operators:

$$\begin{pmatrix} \tilde{a}_k(t) \\ \tilde{b}^\dagger_{-k}(t) \end{pmatrix} = \begin{pmatrix} \alpha_k(t) & \beta^*_k(t) \\ \beta_k(t) & \alpha^*_k(t) \end{pmatrix} \begin{pmatrix} a_k \\ b^\dagger_{-k} \end{pmatrix}$$

unitarity: $|\alpha_k(t)|^2 - |\beta_k(t)|^2 = 1$

$$\begin{aligned} \Phi_k(t) &= a_k f_k(t) + b^\dagger_{-k} f^*_{-k}(t) \\ &= \tilde{a}_k(t) \tilde{f}_k(t) + \tilde{b}^\dagger_{-k}(t) \tilde{f}^*_{-k}(t) \end{aligned}$$

$\tilde{f}_k(t)$: reference mode functions

$$f_k(t) = \alpha_k(t)\, \tilde{f}_k(t) + \beta_k(t)\, \tilde{f}^*_{-k}(t)$$

Bogoliubov Transformation & Adiabatic Particle Number

time-dependent adiabatic particle number:

$$\tilde{\mathcal{N}}_k(t) \equiv \langle 0|\, \tilde{a}_k^\dagger(t)\, \tilde{a}_k(t)\, |0\rangle = |\beta_k(t)|^2$$

vacuum: $a_k|0\rangle = 0 = b_{-k}|0\rangle$

total number of particles produced in mode k:

$$\tilde{N}_k \equiv \tilde{\mathcal{N}}_k(t = +\infty) = |\beta_k(t = +\infty)|^2$$

in this talk, consider the full time evolution of the adiabatic particle number $\tilde{\mathcal{N}}_k(t)$, as it evolves from an initial value of zero to some final asymptotic value $\tilde{N}_k \equiv \tilde{\mathcal{N}}_k(t = +\infty)$.

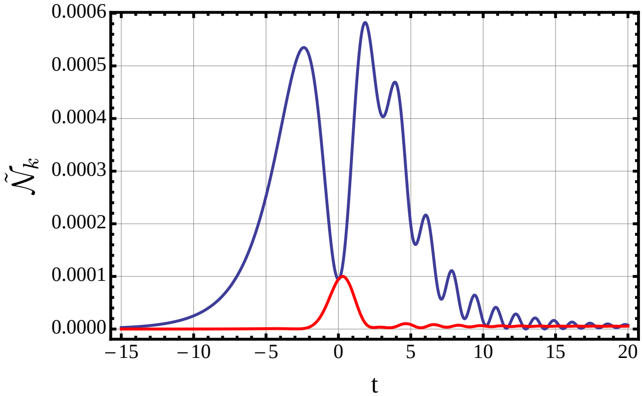
Bogoliubov Transformation & Adiabatic Particle Number

immediate problem:

time-dependent adiabatic particle number, $\tilde{\mathcal{N}}_k(t)$, depends on the reference mode functions and so is not unique, at intermediate times

even though the final value at $t = +\infty$ is unique

Basis Dependence of Adiabatic Particle Number

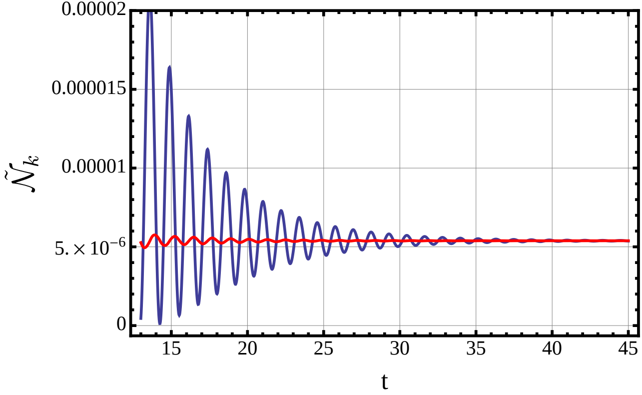


Schwinger effect: $E(t) = E\,\mathrm{sech}^2(at)$. Pulse parameters: $E = 0.25, a = 0.1, k_\perp = 0, k_\parallel = 0$

blue and red defined w.r.t. different reference mode functions

Basis Dependence of Adiabatic Particle Number

close-up at late times



blue & red defined w.r.t. different reference mode functions

same final value, but very different at intermediate times

Basis Dependence of Adiabatic Particle Number

reference functions: $f_k(t) = \alpha_k(t)\, \tilde{f}_k(t) + \beta_k(t)\, \tilde{f}^*_{-k}(t)$

$$\tilde{f}_k(t) \equiv \frac{1}{\sqrt{2\, W_k(t)}}\, e^{-i \int^t W_k}$$

$\tilde{f}_k(t)$ solves time-dependent oscillator equation if

$$W_k^2 = \omega_k^2 - \left[\frac{\ddot{W}_k}{2W_k} - \frac{3}{4} \left(\frac{\dot{W}_k}{W_k} \right)^2 \right]$$

leading order adiabatic exp$^{\mathrm{n}}$: $W_k(t) = \omega_k(t)$, cf. WKB

also need momentum field operators $\Pi_k(t) = \dot{\Phi}^\dagger_k(t)$

$$\dot{f}_k(t) = \left(-i\, W_k(t) + \frac{1}{2} V_k(t) \right) \alpha_k(t)\, \tilde{f}_k(t) + \left(i\, W_k(t) + \frac{1}{2} V_k(t) \right) \beta_k(t)\, \tilde{f}^*_{-k}(t)$$

Basis Dependence of Adiabatic Particle Number

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freedom in $W_k(t)$ & $V_k(t)$ encodes arbitrariness in defining positive and negative energy states at intermediate times

Basis Dependence of Adiabatic Particle Number

Conventional choices:

1. $W_k(t) = \omega_k(t)$ and $V_k(t) = 0 \rightarrow$ time evolution of Bogoliubov coefficients:

$$\begin{pmatrix} \dot{\alpha}_k(t) \\ \dot{\beta}_k(t) \end{pmatrix} = \frac{\dot{\omega}_k(t)}{2\omega_k(t)} \begin{pmatrix} 0 & e^{2i\int^t \omega_k} \\ e^{-2i\int^t \omega_k} & 0 \end{pmatrix} \begin{pmatrix} \alpha_k(t) \\ \beta_k(t) \end{pmatrix}$$

2. $W_k(t) = \omega_k(t)$ and $V_k(t) = -\frac{\dot{W}_k(t)}{W_k(t)} \rightarrow$ time evolution of Bogoliubov coefficients:

$$\begin{pmatrix} \dot{\alpha}_k(t) \\ \dot{\beta}_k(t) \end{pmatrix} = \frac{1}{4i} \left(\frac{3}{2} \frac{\dot{\omega}_k^2(t)}{\omega_k^3(t)} - \frac{\ddot{\omega}_k(t)}{\omega_k^2(t)} \right) \begin{pmatrix} 1 & e^{2i\int^t \omega_k} \\ -e^{-2i\int^t \omega_k} & -1 \end{pmatrix} \begin{pmatrix} \alpha_k(t) \\ \beta_k(t) \end{pmatrix}$$

Computation for Time Dependent Electric Fields

- in a chosen basis, evolve Bogoliubov coefficients $\alpha_k(t)$ and $\beta_k(t)$ from $t = -\infty$ to $t = +\infty$, with $\beta_k(-\infty) = 0$

$$\begin{pmatrix} \alpha_k(-\infty) \\ \beta_k(-\infty) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \longrightarrow \qquad \begin{pmatrix} \alpha_k(+\infty) \\ \beta_k(+\infty) \end{pmatrix}$$

- total asymptotic particle number: $\tilde{N}_k = |\beta_k(t = +\infty)|^2$
- do for each momentum mode k, momentum of the produced particles along the electric field direction
- useful analogy: $\frac{|\beta_k(t=+\infty)|^2}{|\alpha_k(t=+\infty)|^2}$ is the reflection probability for over-the-barrier scattering for the "Schrödinger equation" $\left(-\frac{d^2}{dt^2} - (k_\parallel - A_\parallel(t))^2\right) f_k(t) = (m^2 + k_\perp^2)\, f_k(t)$

Importance of Quantum Interference

- realistic short laser pulses have temporal structure [envelope, carrier-phase, chirp, ...]
- particle production is extremely sensitive to these, due to quantum interference
- we can learn to take advantage of this, to enhance the particle production effect

Particle Number in Quantum Field Theory

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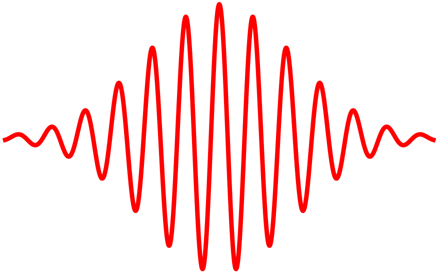


$|0\rangle_{\text{out}}$

$t = -\infty$

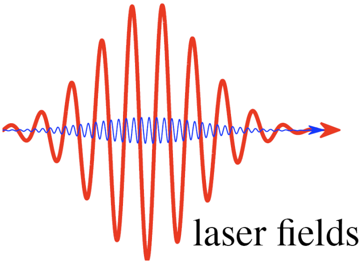
intermediate time

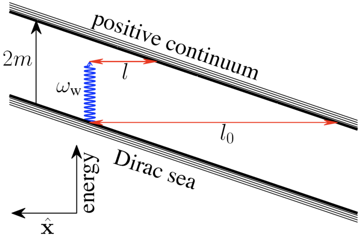
$t = +\infty$

Dynamical Schwinger Mechanism

[Schützhold, GD, Gies, PRL 2008; Di Piazza, Lötstedt, Milstein, Keitel, PRL 2009]

superimpose strong slow field (optical laser) with fast weak pulse (X-ray laser): leads to exponential enhancement



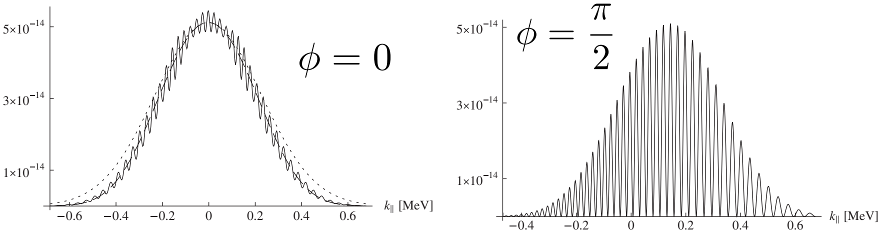


lowers Schwinger critical field by several orders of magnitude

Quantum Interference: Carrier Phase

carrier phase effect [Hebenstreit, Alkofer, GD, Gies, PRL 2009]

$$E(t) = E_0 \exp\left(-\frac{t^2}{\tau^2}\right) \cos\left(\omega\, t + \phi\right)$$

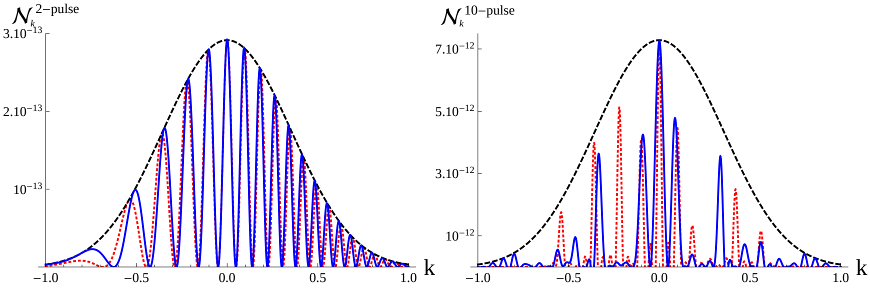


time-domain analogue of multiple-slit interference, from vacuum

sensitive measure of sub-cycle pulse structure

Quantum Interference: Ramsey Effect

alternating-sign pulse sequence [Akkermans, GD, PRL 2012]



N-pulse sequence $\Rightarrow$ N^2 enhancement in certain modes

time-domain analogue of N-slit interference

measured in tunnel junctions (Gabelli/Reulet, 2012)

Quantum Interference: Complex WKB

$$\left(-\frac{d^2}{dt^2} - (k_\parallel - A_\parallel(t))^2\right) f_k(t) = (m^2 + k_\perp^2)\, f_k(t)$$

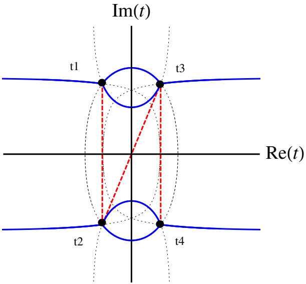
- over-the-barrier scattering: turning points only in the complex plane
- pulses with sub-structure have multiple sets of turning points
- WKB integral between complex conjugate turning points $\rightarrow$ magnitude of creation event
- WKB integral between neighboring turning points $\rightarrow$ interference phases

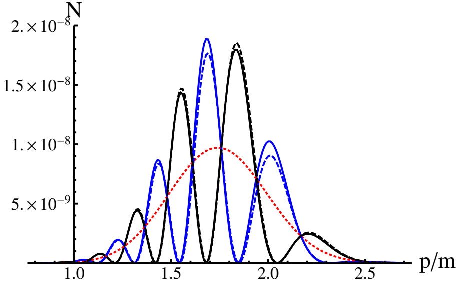
$$\beta_k(\infty) \approx \sum_{t_p} \left(e^{-2i\int_{-\infty}^{Re(t_p)} \omega_{\mathbf{k}}(t)\, dt}\right) \left(e^{-2i\int_{Re(t_p)}^{Re(t_p)+i\, Im(t_p)} \omega_{\mathbf{k}}(t)\, dt}\right)$$

Quantum Interference: Stokes Phenomenon

particle production requires global information to connect
$t = -\infty$ to $t = +\infty$

Stokes lines and anti-Stokes lines separate different regions of asymptotic behavior

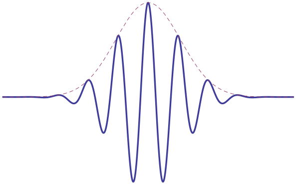


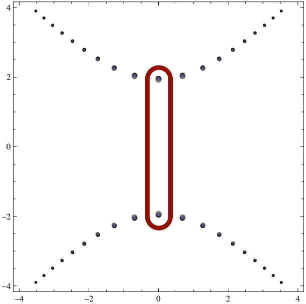


particle production = Stokes phenomenon, the appearance/disappearance of exponentially small/large terms as Stokes lines are crossed [Dumlu, GD, PRL 2010]

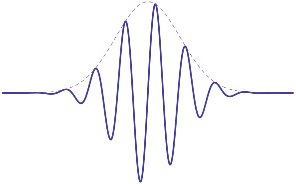
Quantum Interference: Complex WKB

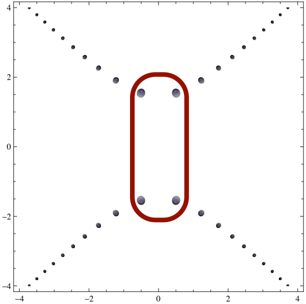
$$E(t) = E_0\, e^{-t^2/\tau^2} \cos(\omega\, t)$$





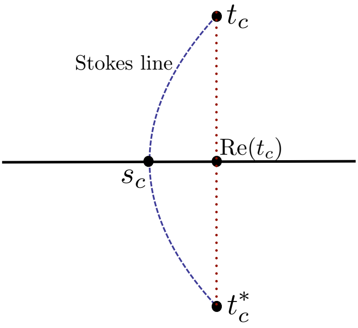
$$E(t) = E_0\, e^{-t^2/\tau^2} \sin(\omega\, t)$$





Adiabatic Particle Number and Quantum Interference

- what effect does quantum interference have on adiabatic particle number ?
- what is happening near the turning points ?



The Adiabatic Expansion: recall $\tilde{f} = \frac{e^{-i\int^t W_k}}{\sqrt{2W_k(t)}}$

expand $W_k^2 = \omega_k^2 - \left[\frac{\ddot{W}_k}{2W_k} - \frac{3}{4}\left(\frac{\dot{W}_k}{W_k}\right)^2\right]$ in derivatives of $\omega_k(t)$:

1. Leading order: $W_k^{(0)}(t) = \omega_k(t)$
2. Next-to-leading order: $W_k^{(1)}(t) = \omega_k(t) - \frac{1}{4}\left(\frac{\ddot{\omega}_k}{\omega_k^2} - \frac{3}{2}\frac{\dot{\omega}_k^2}{\omega_k^3}\right)$
3. Next-to-next-to-leading order:

$$\begin{aligned} W_k^{(2)}(t) &= \omega_k(t) - \frac{1}{4}\left(\frac{\ddot{\omega}_k}{\omega_k^2} - \frac{3}{2}\frac{\dot{\omega}_k^2}{\omega_k^3}\right) \\ &\quad - \frac{1}{8}\left(\frac{13}{4}\frac{\ddot{\omega}_k^2}{\omega_k^5} - \frac{99}{4}\frac{\dot{\omega}_k^2\ddot{\omega}_k}{\omega_k^6} + 5\frac{\dot{\omega}_k\dddot{\omega}_k}{\omega_k^5} + \frac{1}{2}\frac{\overset{(4)}{\omega}_k}{\omega_k^4} + \frac{297}{16}\frac{\dot{\omega}_k^4}{\omega_k^7}\right) \end{aligned}$$

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a complicated mess !!! ... looks hopeless

Super-Adiabatic Basis

a closer look at the adiabatic expansion:

- adiabatic expansion is divergent (asymptotic)
- expressions for $W_k^{(j)}(t)$ in terms of $\omega_k(t)$ rapidly become more and more complicated
- situation look uninteresting and hopeless at high orders
- Dingle found remarkable universal large-order behavior

Super-Adiabatic Basis

Define the "singulant" variable

$$F_k(t) = 2i \int_{t_c}^{t} \omega_k(t')\, dt'$$

expand

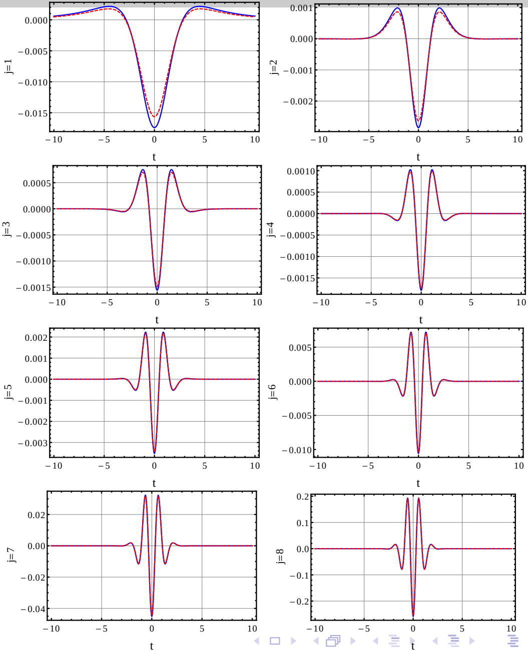
$$W_k(t) = \omega_k(t) \sum_{l=0}^{\infty} \varphi_k^{(2l)}(t)$$

simple and universal large-order behavior:

$$\varphi_k^{(2l+2)}(t) \sim -\frac{2}{\pi} \frac{(2l+1)!}{[F_k(t)]^{2l+2}} \qquad , \quad l \gg 1$$

Super-Adiabatic Basis

$W_k^{(0)}$: 1 term
$W_k^{(1)}$: 3 terms
$W_k^{(2)}$: 8 terms
$W_k^{(3)}$: 19 terms
$W_k^{(4)}$: 41 terms
$W_k^{(5)}$: 83 terms
$W_k^{(6)}$: 160 terms
$W_k^{(7)}$: 295 terms
$W_k^{(8)}$: 526 terms
$W_k^{(9)}$: 911 terms
$W_k^{(10)}$: 1538 terms
$W_k^{(11)}$: 2540 terms



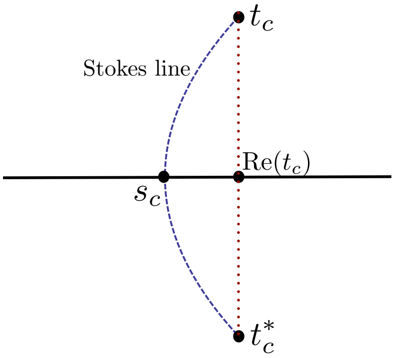
Super-Adiabatic Basis

Berry: universal time dependence of Bogoliubov coefficient near turning point:

$$\beta_k(t) \approx \frac{i}{2}\,\mathrm{Erfc}\left(-\sigma_k(t)\right)\, e^{-F_k^{(0)}}$$

$$F_k^{(0)} = \mathrm{Im}\int_{t_c}^{t_c^*} \omega_k(t)\, dt$$

$$\sigma_k(t) \approx \frac{2\,\omega_k(s_c)\,(t - s_c)}{\sqrt{2\,F_k(s_c)}}$$



Super-Adiabatic Particle Number

super-adiabatic particle number:

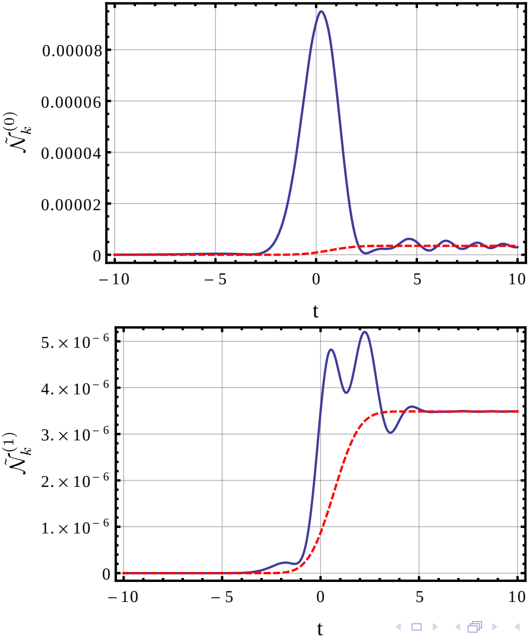
$$\tilde{\mathcal{N}}_k(t) \approx \frac{1}{4} \left| \mathrm{Erfc}\left(-\sigma_k(t)\right) e^{-F_k^{(0)}} \right|^2$$

recall: optimal order of truncation of an asymptotic expansion depends on the value of the expansion parameter

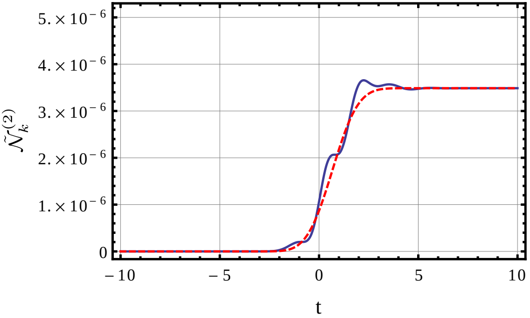
optimal truncation of adiabatic expansion at order j:

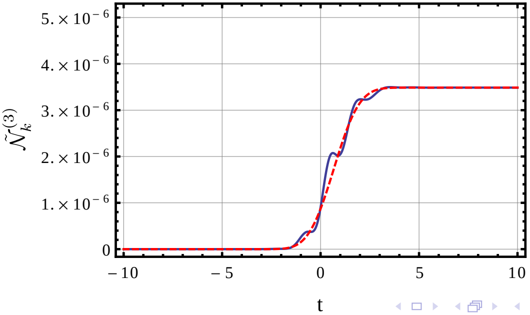
$$j \approx \mathrm{Int}\left[\frac{1}{2}\left(\left|F_k^{(0)}\right| - 1\right)\right]$$

Super-Adiabatic Particle Number: examples

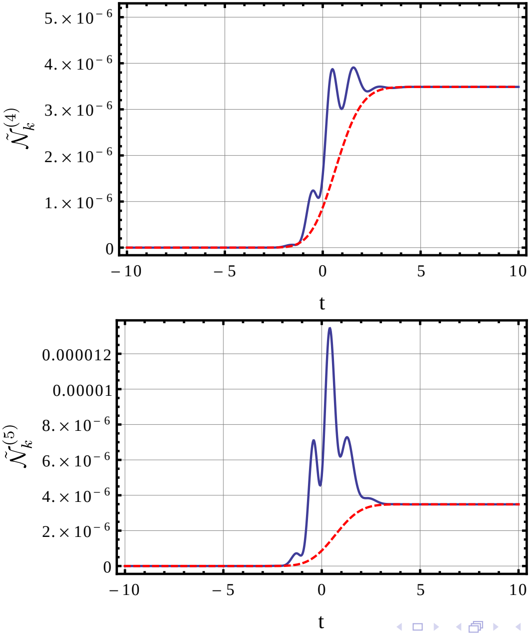


Super-Adiabatic Particle Number: examples

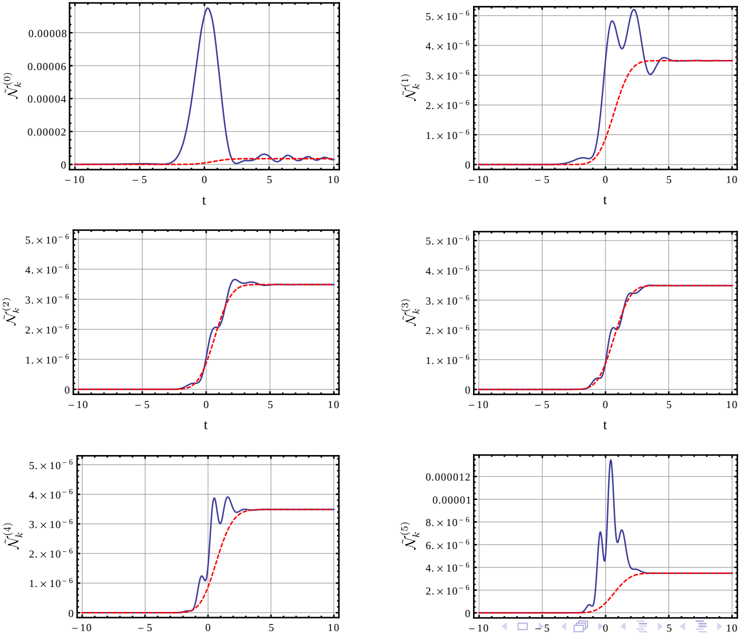




Super-Adiabatic Particle Number: examples



Super-Adiabatic Particle Number: examples

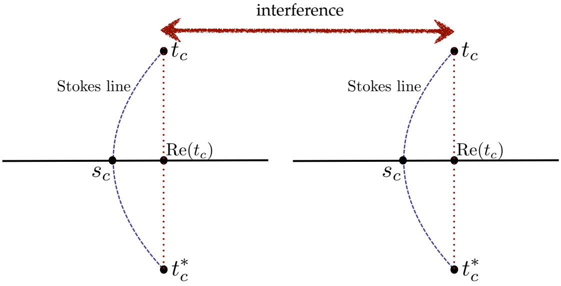


Super-Adiabatic Particle Number

realistic laser pulses have temporal sub-structure, which means multiple pairs of complex turning points

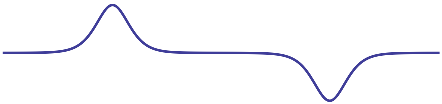
Super-Adiabatic Particle Number

super-adiabatic particle number with multiple turning point pairs:

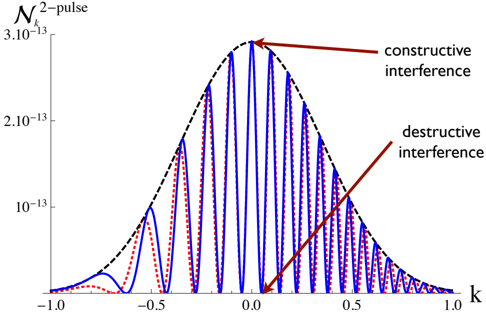


$$\tilde{\mathcal{N}}_k(t) \approx \frac{1}{4}\left|\sum_{t_p} \exp\left(2i\theta_k^{(p)}\right)\exp\left(-F_{k,t_p}^{(0)}\right)\mathrm{Erfc}\left(-\sigma_k^{(p)}(t)\right)\right|^2$$

Super-Adiabatic Particle Number: examples

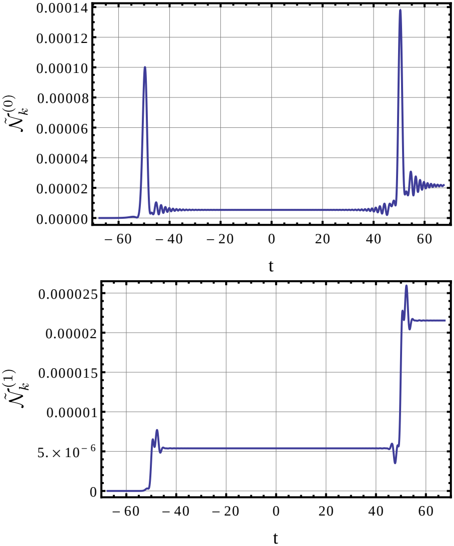


two alternating-sign pulses ⇒ constructive or destructive interference for different k modes



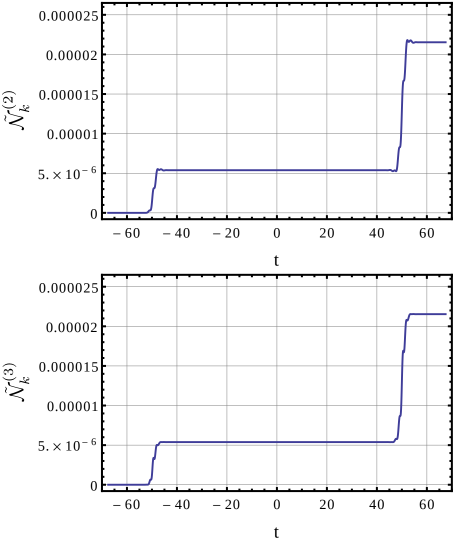
Super-Adiabatic Particle Number: examples

two alternating-sign pulses, with constructive interference:



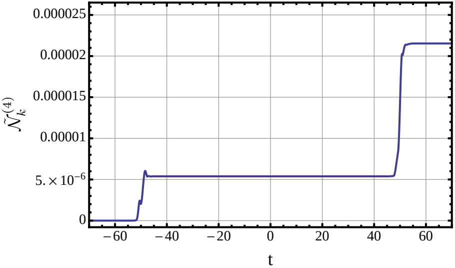
Super-Adiabatic Particle Number: examples

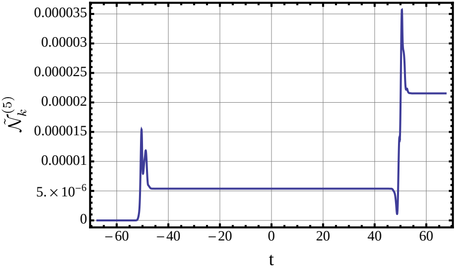
two alternating-sign pulses, with constructive interference:



Super-Adiabatic Particle Number: examples

two alternating-sign pulses, with constructive interference:





Super-Adiabatic Particle Number: examples

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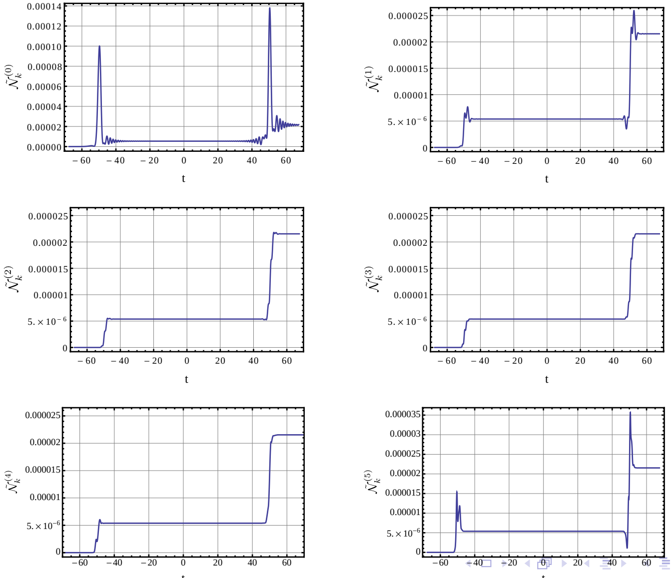
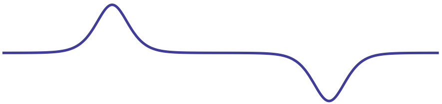
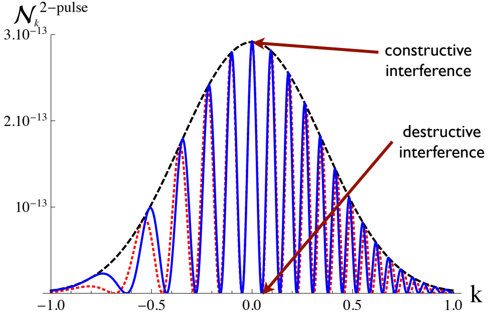


Super-Adiabatic Particle Number: examples

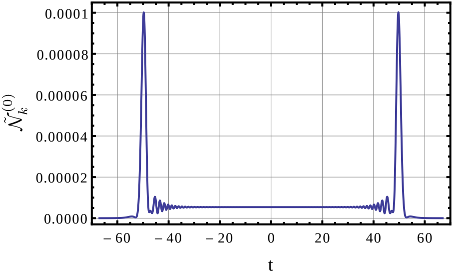


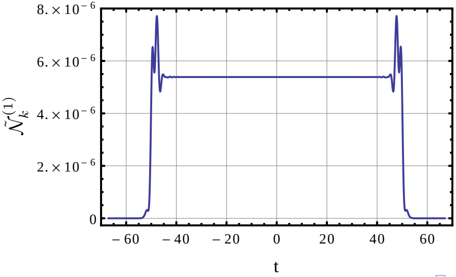
two alternating-sign pulses ⇒ constructive or destructive interference for different k modes



Super-Adiabatic Particle Number: examples

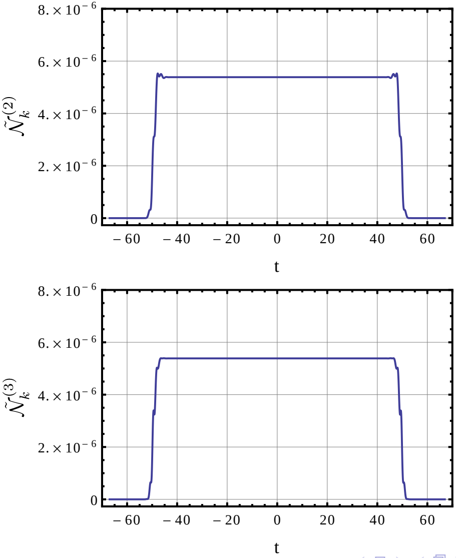
two alternating-sign pulses, with destructive interference:





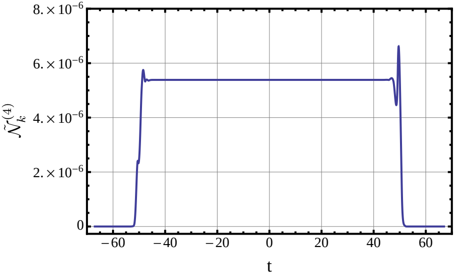
Super-Adiabatic Particle Number: examples

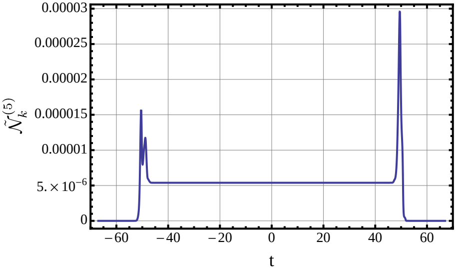
two alternating-sign pulses, with destructive interference:



Super-Adiabatic Particle Number: examples

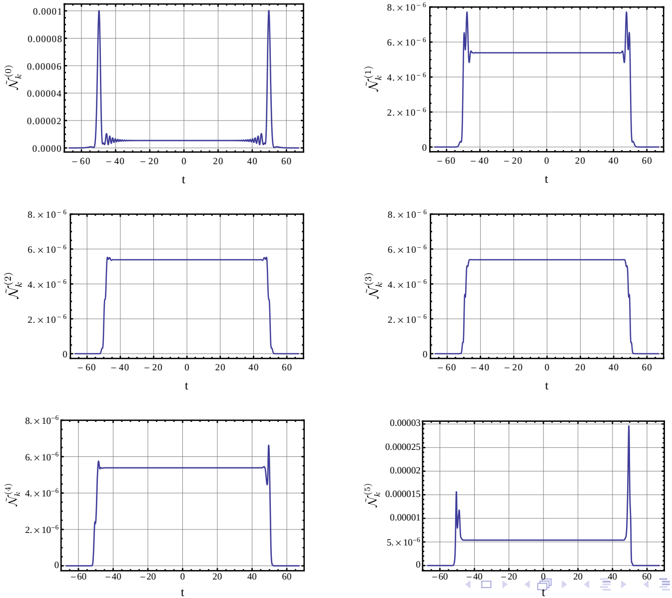
two alternating-sign pulses, with destructive interference:





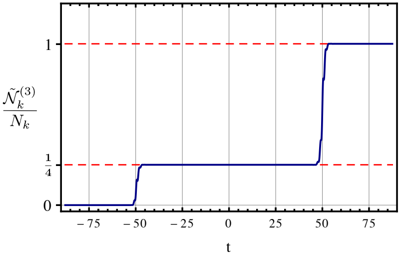
Super-Adiabatic Particle Number: examples

two alternating-sign pulses, with destructive interference:

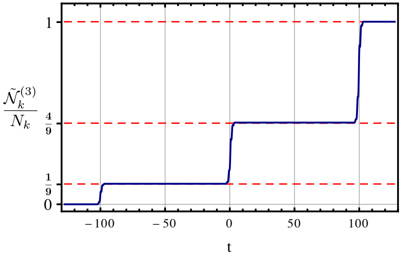


Super-Adiabatic Particle Number: examples

alternating-sign pulses with coherent constructive interference:



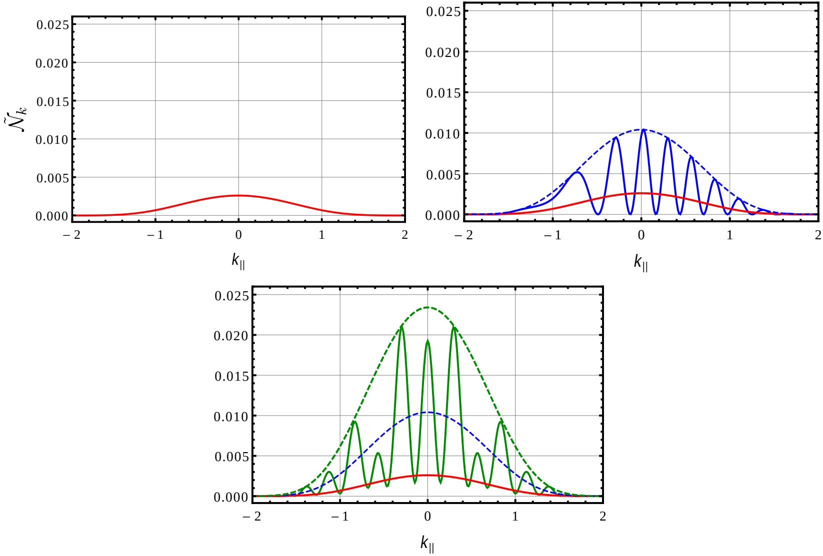
$1 : 4$



$1 : 4 : 9$

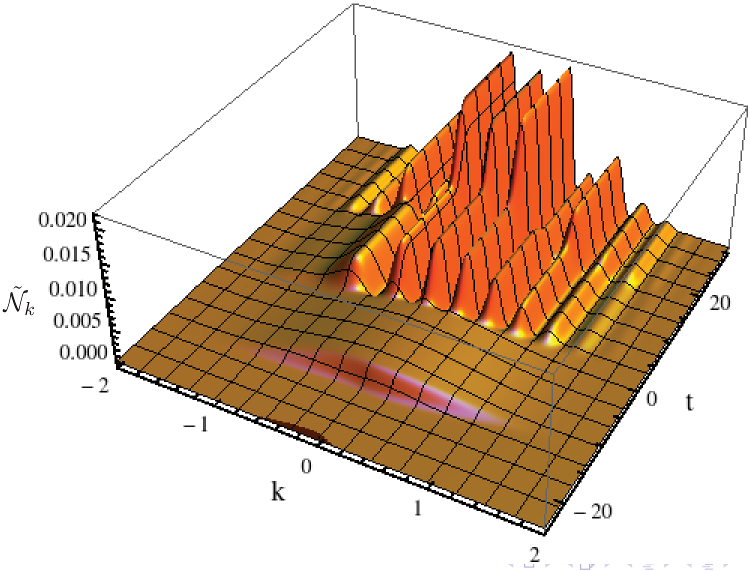
Super-Adiabatic Particle Number: examples

momentum spectrum for alternating-sign pulses:

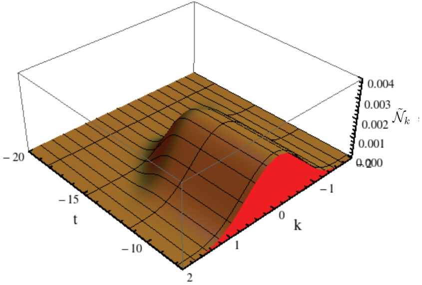


Super-Adiabatic Particle Number: examples

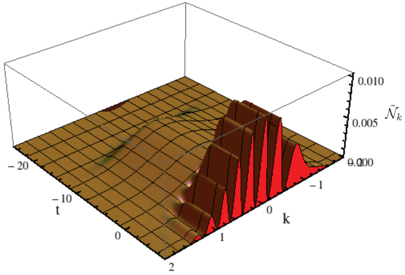
momentum spectrum for alternating-sign pulses:



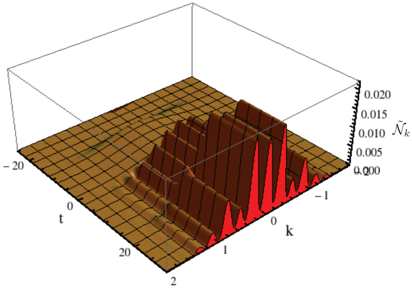
Super-Adiabatic Particle Number: examples



$t = -7.5$



$t = +7.5$



$t = +30$

Super-Adiabatic Particle Number

$$\tilde{\mathcal{N}}_k(t) \approx \frac{1}{4} \left| \sum_{t_p} \exp\left(2i\theta_k^{(p)}\right) \exp\left(-F_{k,t_p}^{(0)}\right) \mathrm{Erfc}\left(-\sigma_k^{(p)}(t)\right) \right|^2$$

- super-adiabatic particle number is smooth in time, and in momentum
- sensible definition because of universality (Dingle/Berry)

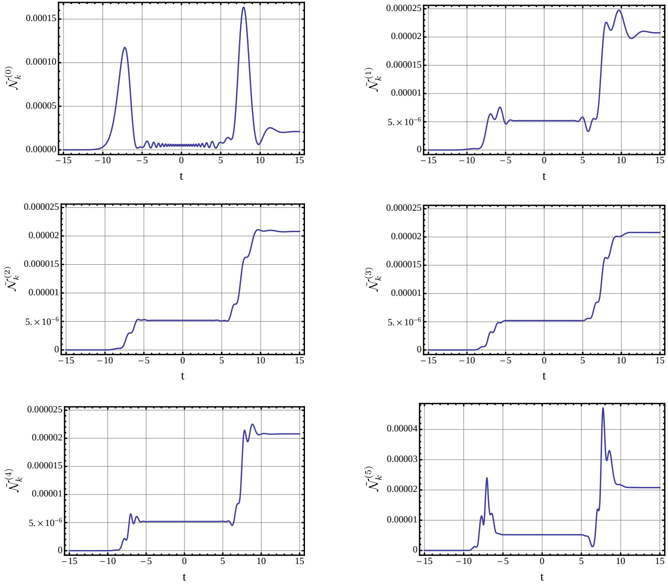
Super-Adiabatic Particle Number: de Sitter space

cosmological particle production: particle number in an expanding/contracting universe

- particle production in 4 dimensional spacetime (Mottola)
- particle production in even dimensional dS spacetime, but no particle production in odd dimensional dS spacetime (Bousso et al)

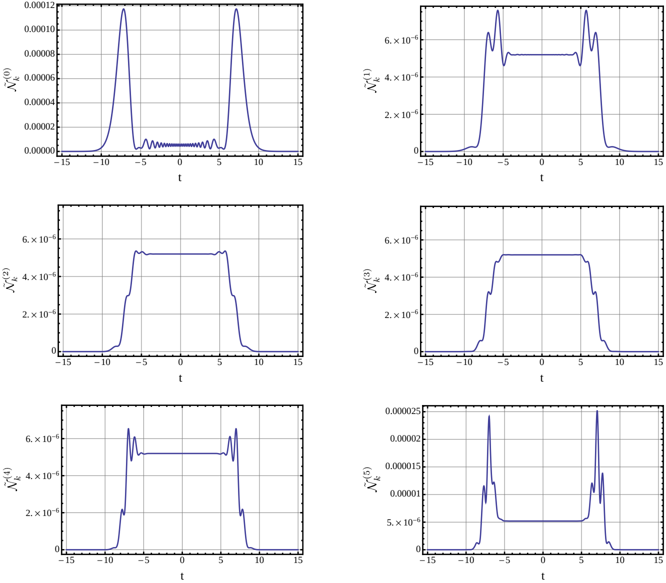
question: physical reason for this difference ?

Super-Adiabatic Particle Number: 4d de Sitter space



cf. two-pulse coherent-constructive Schwinger effect

Super-Adiabatic Particle Number: 3d de Sitter space



cf. two-pulse coherent-destructive Schwinger effect

Conclusions

- adiabatic particle number is basis dependent at non-asymptotic times
- preferred super-adiabatic basis: optimal truncation of adiabatic expansion
- super-adiabatic particle number is smooth across "particle creation events"
- super-adiabatic particle number reveals quantum interference
- difference between dS_3 and dS_4 is quantum interference
- future: implications for back-reaction and quantum noise
- future: analogue systems: time-dependent Bogoliubov transformation problems