

Relativistic Strong Field Ionization and Compton Harmonics Generation

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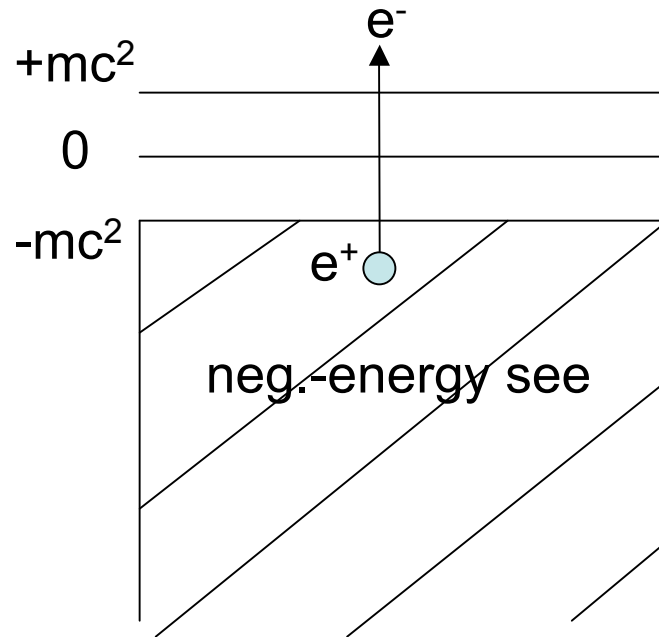
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Relativity and strong-field interaction:

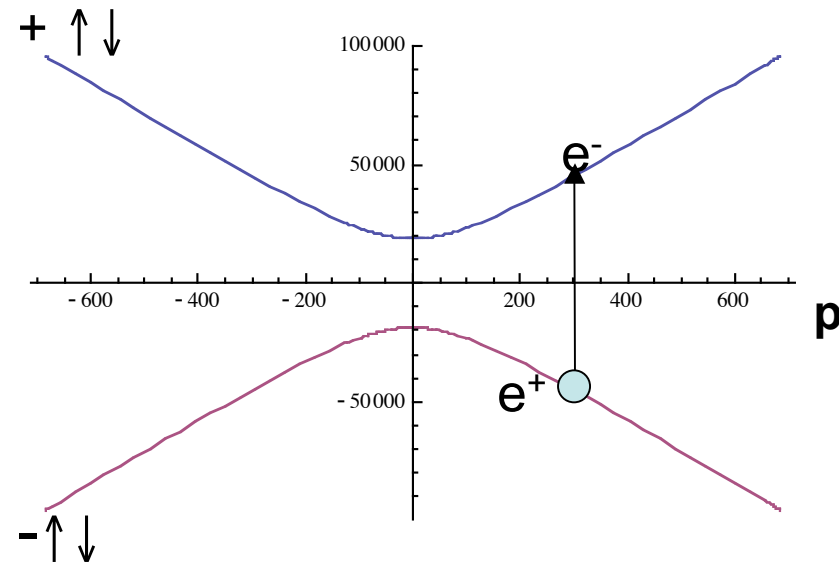
Dirac electron, vacuum, and negative energy states:

Picture A



(energy space)

Picture B



Dirac electron dispersion relation:

$$E(\mathbf{p}) = \pm (\mathbf{p}^2 c^2 + m^2 c^4)^{1/2}$$

(energy-momentum space)

Space-time approach to quantum electrodynamics

R.P. Feynman (1948-1949)

The Theory of Positrons

Phys. Rev. 76, 749 – Published 15 September 1949

R. P. Feynman

“The problem of the behavior of positrons and electrons in given external potentials, neglecting their mutual interaction, is analyzed by replacing the theory of *holes* by a *reinterpretation* of the solutions of the Dirac equation.

... and this solution contains *automatically all* the possibilities of virtual (and real) pair formation and annihilation together with the ordinary scattering processes, including the correct relative signs of the various terms.

... For such a particle the *amplitude for transition* from an initial to a final state is analyzed to any order in the potential by considering it to undergo a *sequence* of such scatterings.

...*Equivalence* to the second quantization theory of holes is proved in an appendix.”

Space-time approach to relativistic quantum dynamics

R.P. Feynman (1948-1949):

Feynman Free-electron Propagator:

satisfies Stueckelberg-Feynman boundary condition:

$$\begin{aligned} G_f(x, x') &= -i \frac{m}{E} \theta(x_0 - x'_0) \sum_{s=1,2} \left[\int \frac{d^3 p}{(2\pi)^3} \phi_p^{(s)}(x) \bar{\phi}_p^{(s)}(x') \right] \\ &\quad + i \frac{m}{E} \theta(x'_0 - x_0) \sum_{s=3,4} \left[\int \frac{d^3 p}{(2\pi)^3} \phi_p^{(s)}(x) \bar{\phi}_p^{(s)}(x') \right] \\ &= \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x - x')} \frac{\not{p}_N + m}{p_N^2 - m^2 + i0} \quad (N=0) \end{aligned}$$

$$\bar{\phi}_p(x) = \phi_p^\dagger(x) \gamma_0 \quad (\text{Dirac free-electron spinors})$$

Dirac Equation of Laser Atom Interaction

$$i\hbar \frac{\partial}{\partial t} \Psi(t) = H_D(t) \Psi(t)$$

$$H_D(\mathbf{x}, t) = c \boldsymbol{\alpha} \cdot \left(-i\hbar \frac{\partial}{\partial \mathbf{x}} - \frac{e_0}{c} \mathbf{A}(\mathbf{x}, t) \right) + V(\mathbf{x}) + \beta m_0 c^2$$

$$\mathbf{A}(\mathbf{x}, t) = A_0(x) (\boldsymbol{\epsilon}_1 \cos(\xi/2) \cos(\omega t - \boldsymbol{\kappa} \cdot \mathbf{x}) - \boldsymbol{\epsilon}_2 \sin(\xi/2) \sin(\omega t - \boldsymbol{\kappa} \cdot \mathbf{x}))$$

Dirac -Feynman 4-vector notation and Feynman slash notation:

$$x \equiv (x_0, \mathbf{x}), x_0 = ct, |\mathbf{x}| = r; p \equiv (p_0, \mathbf{p}), p_0 = m_0 c, a \cdot b = a_\mu b_\mu = a_0 b_0 - \mathbf{a} \cdot \mathbf{b}, \not{a} = \gamma_\mu a_\mu, \bar{u} = u^\dagger \gamma_0, \\ \not{\partial} \equiv \gamma_\mu \frac{\partial}{\partial x_\mu} \text{ and } \not{\mathcal{A}} \equiv \gamma_\mu \mathcal{A}_\mu$$

“wave-number scale”: $e \equiv \frac{e_0}{\hbar c}$ and $m \equiv \frac{m_0 c}{\hbar}$

$$(i\gamma_\mu \frac{\partial}{\partial x_\mu} - e\gamma_\mu \mathcal{A}_\mu(x)) \Psi(t) = m \Psi(t)$$

covariant forms:

$$(i \not{\partial} - e \not{\mathcal{A}}(x) - m) \Psi(t) = 0$$

Volkov solution of Dirac equation with e.m. field:

Volkov states:

$$\phi_p(x) = \sqrt{\frac{m}{E}} e^{-ip \cdot x - i f_p(x)} \left(1 + \frac{e \not{\kappa} \not{A}(x)}{2 \kappa \cdot p} \right) \hat{u}_p$$

$$f_p(x) = \frac{1}{2 \kappa \cdot p} \int^{\kappa \cdot x} [(2eA(\zeta) \cdot p - e^2 A^2(\zeta))] d\zeta,$$

e.g. constant amp. field:

$$f_p(x) = -a_p \sin(u + \chi_p) + b_p \sin(2u) + \lambda_p u$$

$$\hat{u}_p = \frac{\not{p} + m}{\sqrt{2m(m + E)}},$$

$$u_p^{(s)} = \hat{u}_p w^{(s)} \quad (\text{Dirac spinors})$$

$$w^{(1=\uparrow)} = (1000)^\dagger$$

$$w^{(2=\downarrow)} = (0100)^\dagger$$

$$w^{(3=\uparrow)} = (0010)^\dagger$$

$$w^{(4=\downarrow)} = (0001)^\dagger$$

Volkov-spinors: (provide a *complete* set of “+ -” energy states)

$$\phi_p^{(s)}(x) = \sum_{n=-\infty}^{\infty} \sqrt{\frac{m}{E}} e^{-i(p+\lambda_p \kappa - n\kappa) \cdot x} Q_n(p) u_p^{(s)}$$

$$\hat{u}_p = \frac{\not{p} + m}{\sqrt{2m(m+E)}}$$

$$u_p^{(s)} = \hat{u}_p w^{(s)}$$

$$Q_n(p) = \left(J_n + \frac{eA_0}{4\kappa \cdot p} \not{\kappa} (\not{\xi} J_{n-1} + \not{\xi}^* J_{n+1}) \right)$$

(Dirac spinors)

$$(J_n \equiv J_n(a_p, b_p, \chi_p) = \sum_{m=-\infty}^{\infty} J_{n+2m}(a_p) J_m(b_p) e^{(n+2m)\chi_p})$$

Note the appearance of “field dressed 4-momentum” $q=(q_0, \mathbf{q})$, in place of the “bare” 4-momentum $p=(p_0, \mathbf{p})$ in the absence of the field:

$$q = p + (\lambda_p - n) \kappa,$$

i.e.

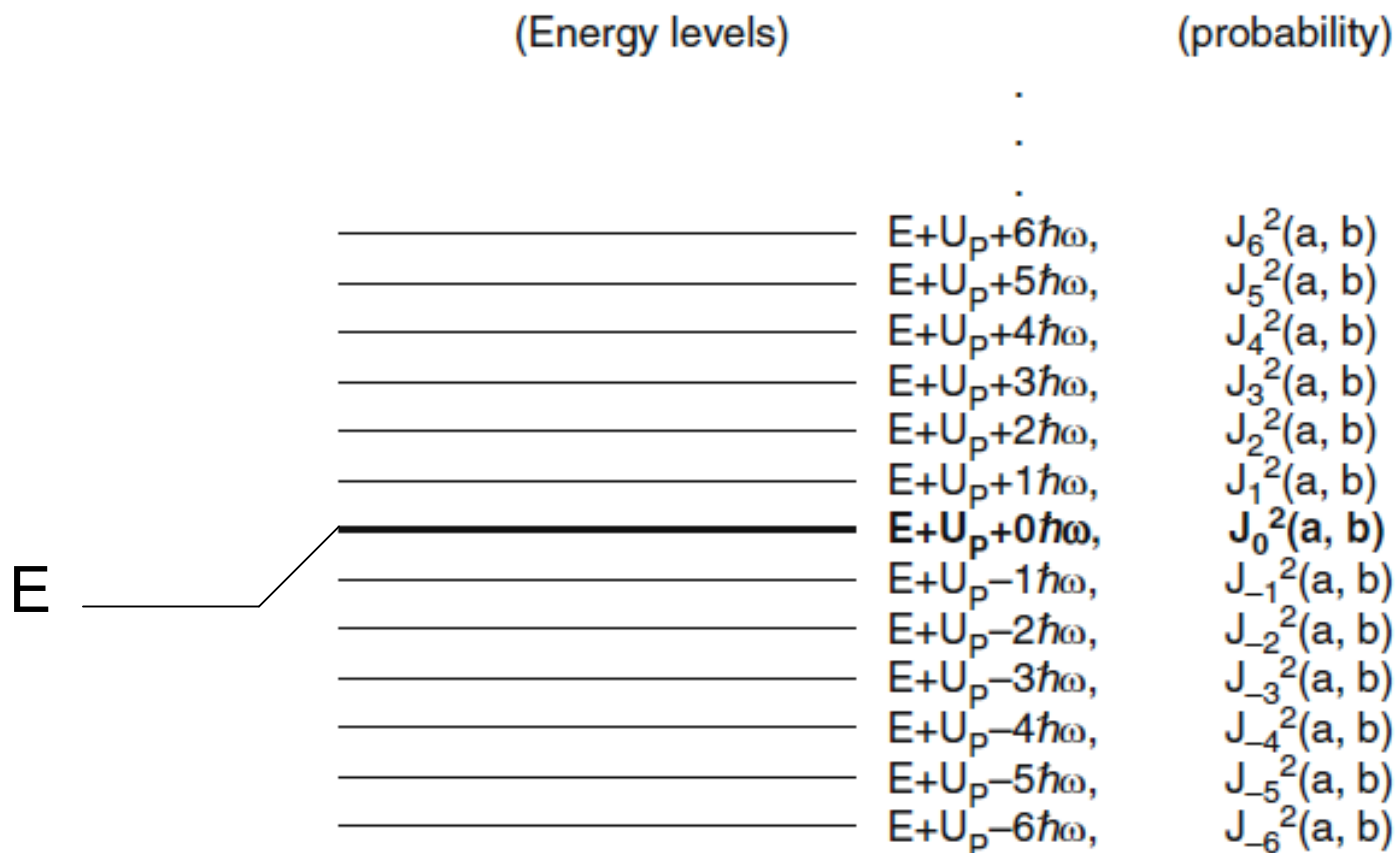
$$q_0 = p_0 + (\lambda_p - n) \kappa_0, \quad \mathbf{q} = \mathbf{p} + (\lambda_p - n) \boldsymbol{\kappa}$$

$$\lambda_p = \frac{(eA_0)^2}{4\kappa \cdot p}$$

$$\kappa \cdot p = \kappa_0 p_0 - \boldsymbol{\kappa} \cdot \mathbf{p}$$

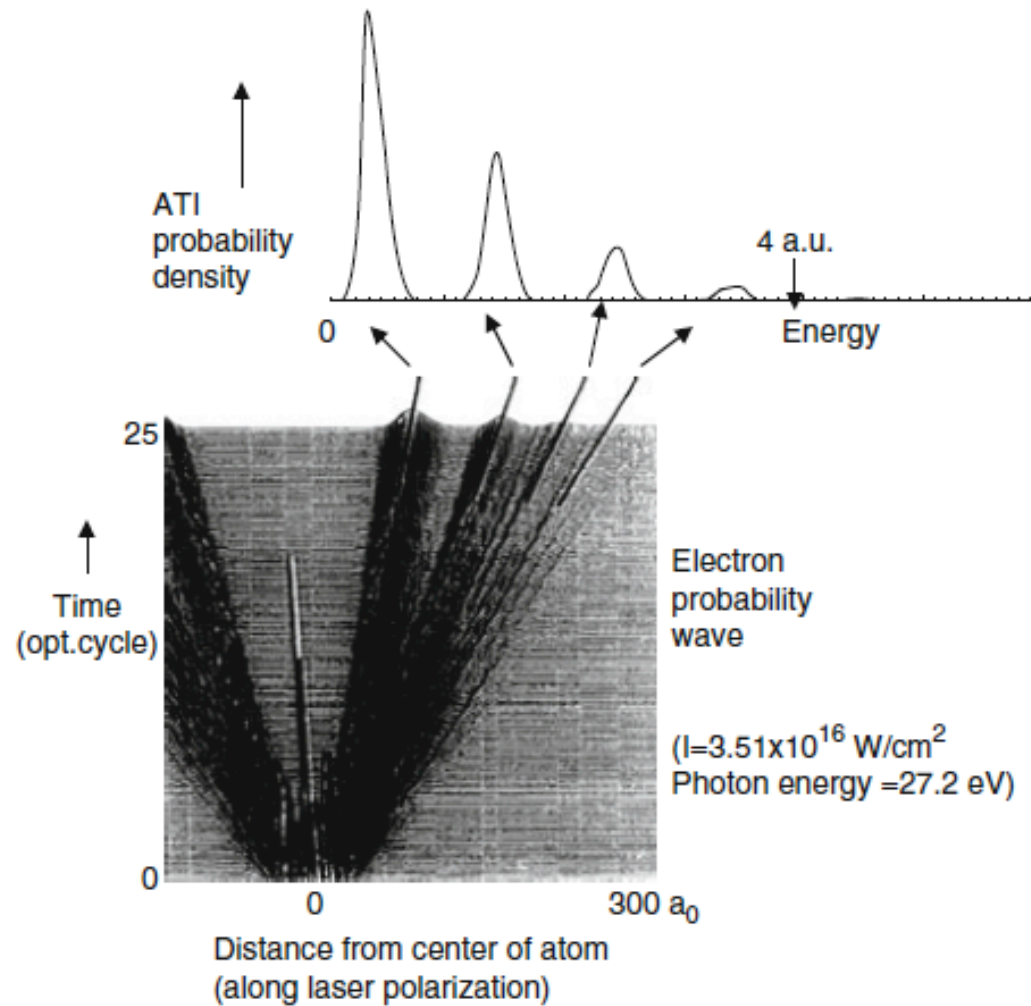
Volkov spectrum:

Distribution of Energy of a Free Electron in a Laser Field



$$\sum_n J_n^2(a, b, c) = 1$$

Transformation of virtual Volkov spectrum to real ATI spectrum:



Schwengelbeck and Faisal,
Phys. Rev. A **50**, 632 (1994)

Volkov-Feynman Propagator

$$(i \not{\partial} - e \not{A}(x) - m)G_f(x, x') = \delta^4(x - x'), \quad A(x) = (0, \mathbf{A}(x))$$

$$G_f(x, x') = -i \frac{m}{E} \theta(x_0 - x'_0) \sum_{s=1,2} \left[\int \frac{d^3 p}{(2\pi)^3} \phi_p^{(s)}(x) \bar{\phi}_p^{(s)}(x') \right] \\ + i \frac{m}{E} \theta(x'_0 - x_0) \sum_{s=3,4} \left[\int \frac{d^3 p}{(2\pi)^3} \phi_p^{(s)}(x) \bar{\phi}_p^{(s)}(x') \right]$$

e.g. for a constant/slowly varying envelope field (slow = $d/dt a(t) \ll \omega$, $a(t)$ =envelope):

$$G_F(x, x') = \sum_{n=-\infty}^{\infty} e^{in\kappa \cdot x} \sum_{N=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-x')} Q_{n-N}(p) \frac{\not{p}_N + m}{p_N^2 - m^2 + i0} Q_{-N}^\dagger(p)$$

(It reduces to Feynman's free electron propagator for zero field, $a(t)=0$)

Dirac equation: $(i \not{\partial} - e \not{A}(x) - m)\Psi(x) = 0$

One converts it to integral form (Lippmann-Schwinger-Dyson Equation)

$$\Psi(x) = \phi^{(0)}(x) + \int d^4x' G(x, x') V_i(x') \Psi(x')$$

State-function (in form of an iteration series):

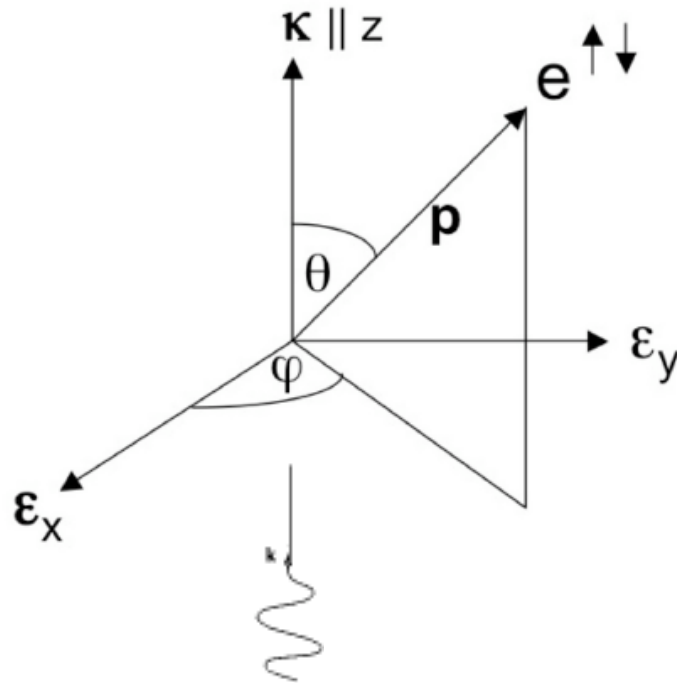
$$\begin{aligned} \Psi(x) = & \phi^{(0)}(x) + \int d^4x' G_f(x, x') V_i(x') \phi^{(0)}(x') \\ & + \int \int d^4x'' d^4x' G_f(x, x'') V_f(x'') G_f(x'', x') V_i(x') \phi^{(0)}(x') + \dots \end{aligned}$$

Transition amplitude:

$$\begin{aligned} \mathcal{A}_{fi} = & -i \int d^4x \bar{\phi}_p(x) e \not{A}(x) \phi^{(0)}(x) - i \int \int d^4x d^4x' \bar{\phi}_p(x) \gamma_0 U(x) G_f(x, x') e \not{A}(x') \phi^{(0)}(x') \\ & - i \dots \end{aligned}$$

Strong-field relativistic ionization of an inner-shell
(hydrogenic atomic / ionic core e.g. H or U^{91+})

Spin and ionization in an intense circularly polarized field:



A covariant form of the ground state (arbitrary Z)

$$\phi_{1s}^{(s)}(\mathbf{r}) = N_{1s} r^{\gamma'-1} e^{-p_B r} \not{n}_\mu(\hat{\mathbf{r}}) w^{\uparrow, \downarrow} \quad (s = 1, 2).$$

4-vector $n_\mu = (n_0, \mathbf{n}) \equiv (1, i\beta' \hat{\mathbf{r}})$

constants

$$N_{1s} = (2p_B)^{\gamma'+\frac{1}{2}} \left(\frac{1+\gamma'}{8\pi\Gamma(1+2\gamma')} \right)^{\frac{1}{2}}$$

$$p_B = mZ\alpha$$

$$\gamma' = \sqrt{1 - (Z\alpha)^2},$$

$$\beta' = (1 - \gamma')/(Z\alpha)$$

Transition matrix

$$T_{s \rightarrow s'}^{(n)}(\mathbf{q}) = \frac{eA_0}{2} N(E) \bar{u}_p^{(s')} \beta_n^* \tilde{\phi}_{1s}^{(s)}(\mathbf{q}), \quad s, s' = (u, d)$$

$$B_n = (B_n^0, \mathbf{B}_n)$$

$$B_n^0 = \frac{eA_0 \kappa_0}{4\kappa \cdot p} (2J_n + \cos \xi (J_{n+2} + J_{n-2}))$$

$$\mathbf{B}_n = \boldsymbol{\epsilon}(\xi) J_{n-1} + \boldsymbol{\epsilon}^*(\xi) J_{n+1} + \hat{\mathbf{\kappa}} B_n^0$$

$$\boldsymbol{\epsilon}(\xi) = \boldsymbol{\epsilon}_1 \cos(\xi/2) + i\boldsymbol{\epsilon}_2 \sin(\xi/2)$$

$$\kappa \cdot p = \kappa_0 p_0 - \boldsymbol{\kappa} \cdot \mathbf{p}.$$

$$N(E) = \sqrt{\frac{m}{E}}$$

Reduced transition matrix

$$t_{s \rightarrow s'}^{(n)}(\mathbf{q}) = \bar{u}_p^{(s')} \beta_n^* \not{w}^{(s)}$$

spin-specific transition amplitudes (t-matrix elements)

$$t_{u \rightarrow u}^{(n)} = B_n^{0*} (m_1 + m_2 g(q) (\hat{\mathbf{p}} \cdot \hat{\mathbf{q}} + i(\hat{\mathbf{p}} \times \hat{\mathbf{q}})_z)) + \mathbf{B}_n^* \cdot (m_2 \hat{\mathbf{p}} + m_1 g(q) \hat{\mathbf{q}}) - i(\mathbf{B}_n^* \times (m_2 \hat{\mathbf{p}} - m_1 g(q) \hat{\mathbf{q}}))_z$$

$$t_{u \rightarrow d}^{(n)} = m_2 g(q) B_n^{0*} (i(\hat{\mathbf{p}} \times \hat{\mathbf{q}})_x - (\hat{\mathbf{p}} \times \hat{\mathbf{q}})_y) - i(\mathbf{B}_n^* \times (m_2 \hat{\mathbf{p}} - m_1 g(q) \hat{\mathbf{q}}))_x + (\mathbf{B}_n^* \times (m_2 \hat{\mathbf{p}} - m_1 g(q) \hat{\mathbf{q}}))_y$$

$$t_{d \rightarrow u}^{(n)} = m_2 g(q) B_n^{0*} (i(\hat{\mathbf{p}} \times \hat{\mathbf{q}})_x + (\hat{\mathbf{p}} \times \hat{\mathbf{q}})_y) - i(\mathbf{B}_n^* \times (m_2 \hat{\mathbf{p}} - m_1 g(q) \hat{\mathbf{q}}))_x - (\mathbf{B}_n^* \times (m_2 \hat{\mathbf{p}} - m_1 g(q) \hat{\mathbf{q}}))_y$$

$$t_{d \rightarrow d}^{(n)} = B_n^{0*} (m_1 + m_2 g(q) (\hat{\mathbf{p}} \cdot \hat{\mathbf{q}} - i(\hat{\mathbf{p}} \times \hat{\mathbf{q}})_z)) + \mathbf{B}_n^* \cdot (m_2 \hat{\mathbf{p}} + m_1 g(q) \hat{\mathbf{q}}) + i(\mathbf{B}_n^* \times (m_2 \hat{\mathbf{k}} - m_1 g(q) \hat{\mathbf{q}}))_z.$$

“dressed momentum” $\mathbf{q} = \mathbf{p} + (\lambda_p - n) \boldsymbol{\kappa}$

$$\lambda_p = \frac{(eA_0)^2}{4\boldsymbol{\kappa} \cdot \mathbf{p}}$$

$$\boldsymbol{\kappa} \cdot \mathbf{p} = \kappa_0 p_0 - \boldsymbol{\kappa} \cdot \mathbf{p}$$

$$g(q) = \beta' \left[\frac{p_B}{q} - \frac{\gamma' + 1}{\gamma'} \right]$$

Spin selected Differential ionization probability per unit time:

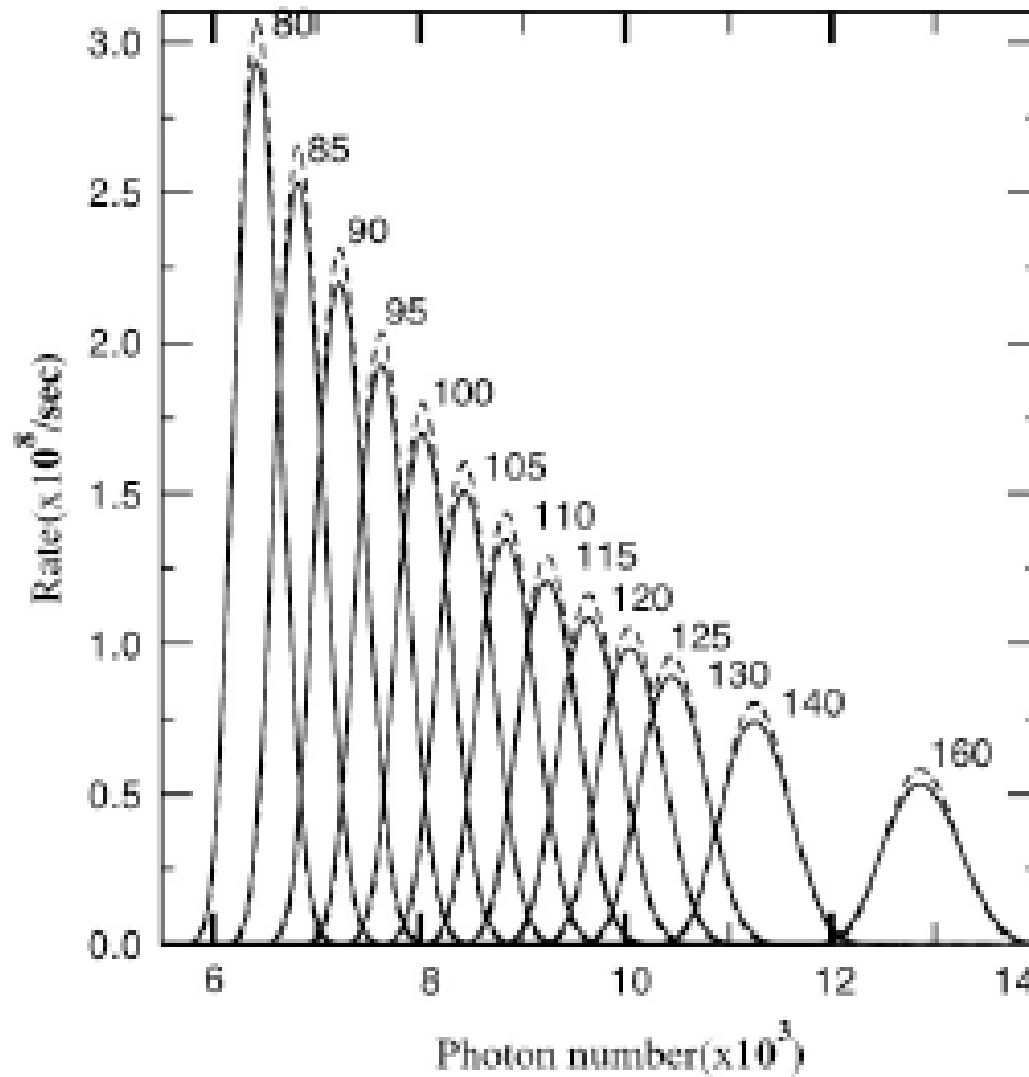
$$dW_{s \rightarrow s'}^{(n)} = \left(\frac{eA_0}{2} N(E) N_{1s} c_0(q) \right)^2 \left| t_{s \rightarrow s'}^{(n)}(\mathbf{q}) \right|^2 \\ \times (2\pi c) \delta(p_0 + p_0^B + \lambda_p \kappa_0 - n\kappa_0) \frac{1}{(2\pi)^3} d^3 p$$

energy conservation: $n\omega = \epsilon_B + \epsilon_{kin} + \lambda_p \omega$

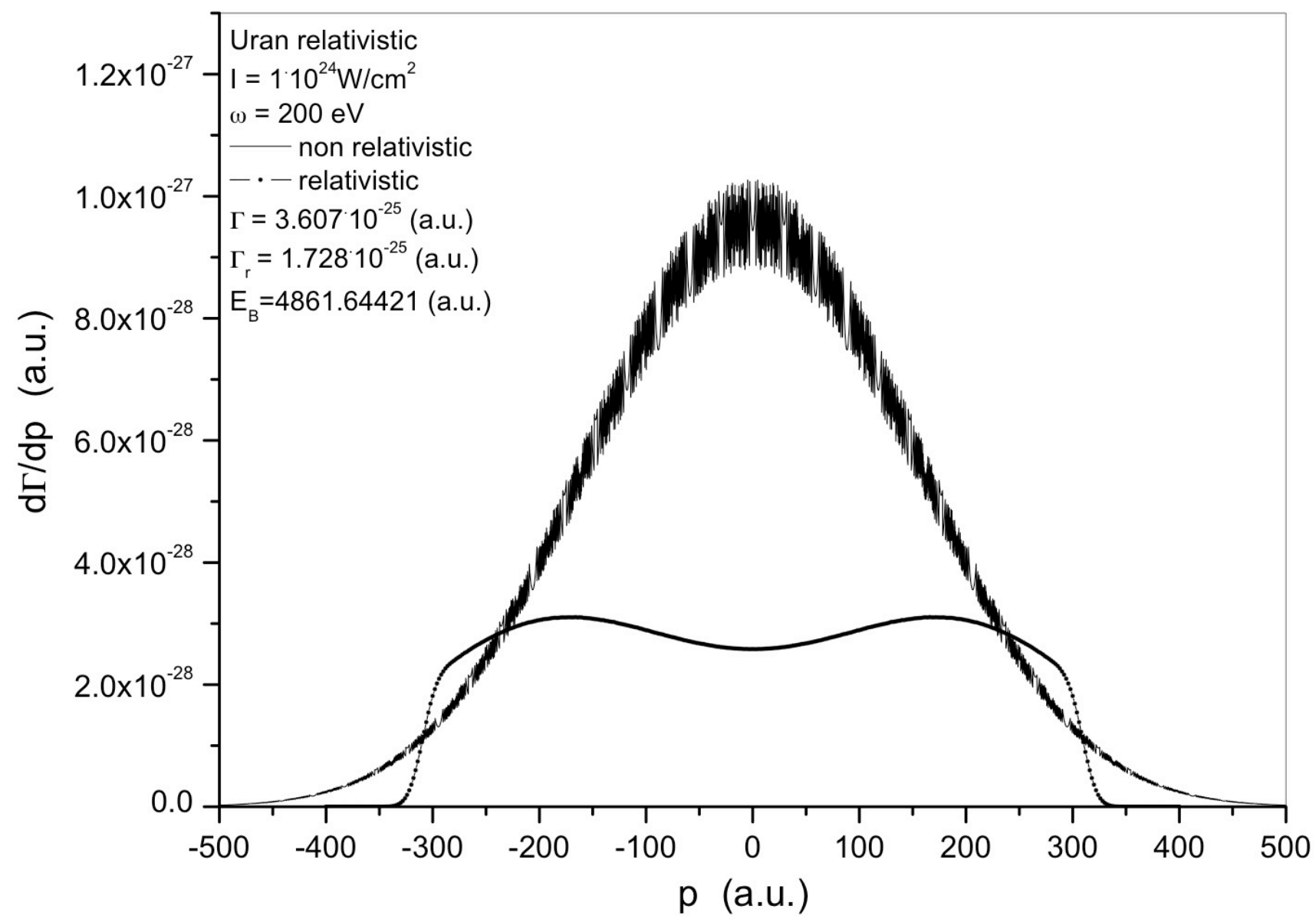
binding energy $\epsilon_B = m_0 c^2 \left(1 - \sqrt{1 - \left(\frac{p_B}{m} \right)^2} \right)$

kinetic energy $\epsilon_{Kin} = m_0 c^2 \left(\sqrt{1 + \left(\frac{p}{m} \right)^2} - 1 \right)$

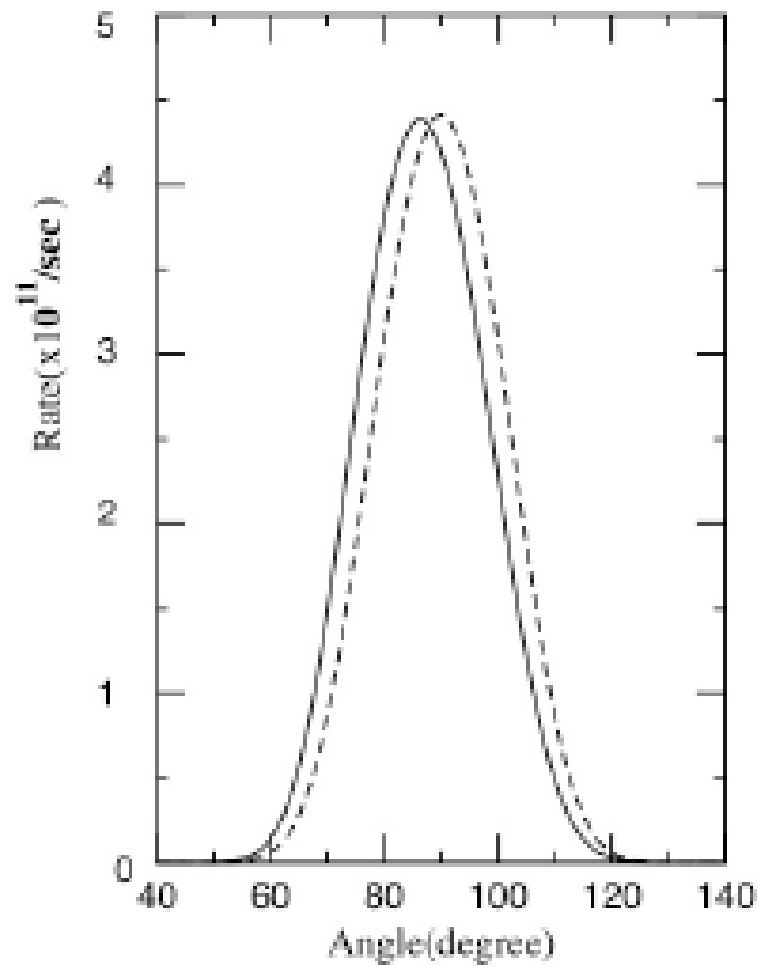
Intensity dependent energy distributions



$\omega = 5 \text{ eV}$
 $I = (80 \text{ to } 160)$
 $3.51 \times 10^{16} \text{ W/cm}^2$



Angular distribution



$$\omega = 200 \text{ eV}$$

$$I = 10^{21} \text{ W/cm}^2$$

... non-relativistic

— relativistic

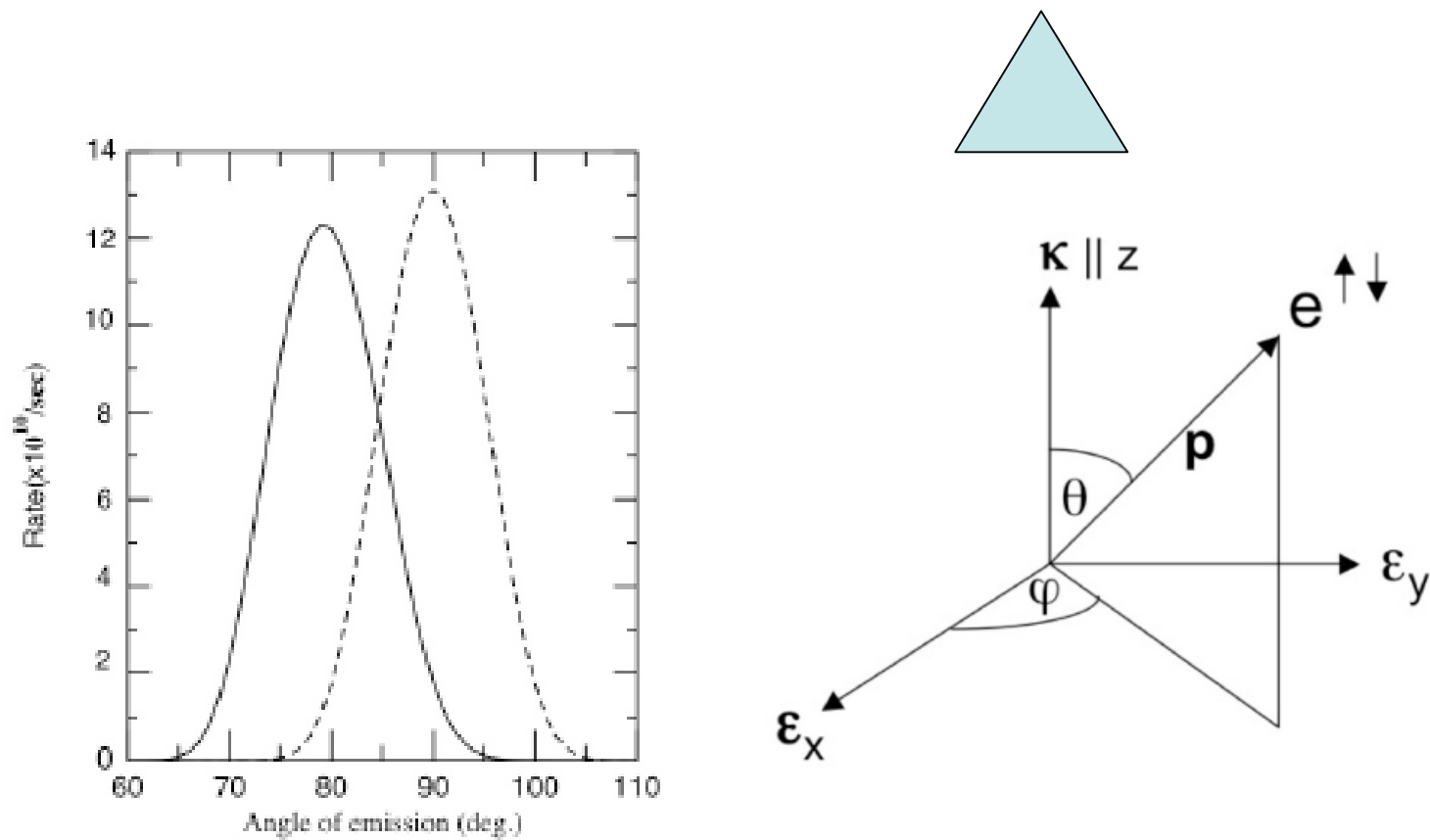


Fig. 6 Ionization rate: comparison of angular distributions at $\omega = 200$ eV, $I = 10^{22}$ W/cm²: intense-field relativistic Dirac (solid line), non-relativistic Schrödinger case (dashed line). Note the focusing effect toward the smaller angle.

“Dressed momentum”

$$\mathbf{q} = \mathbf{p} + (\lambda_p - n) \mathbf{\kappa}$$

$$q_x = p_x, \quad q_y = p_y, \\ q_z = p_z + (\lambda_p - n) \kappa$$

1. shift in propagation direction z and
2. no shift in transverse direction)
3. $\Delta p_z = (\lambda_p - n) \kappa$

$$4. \quad \lambda_p = (eA)^2 / (4 \kappa \cdot p)$$

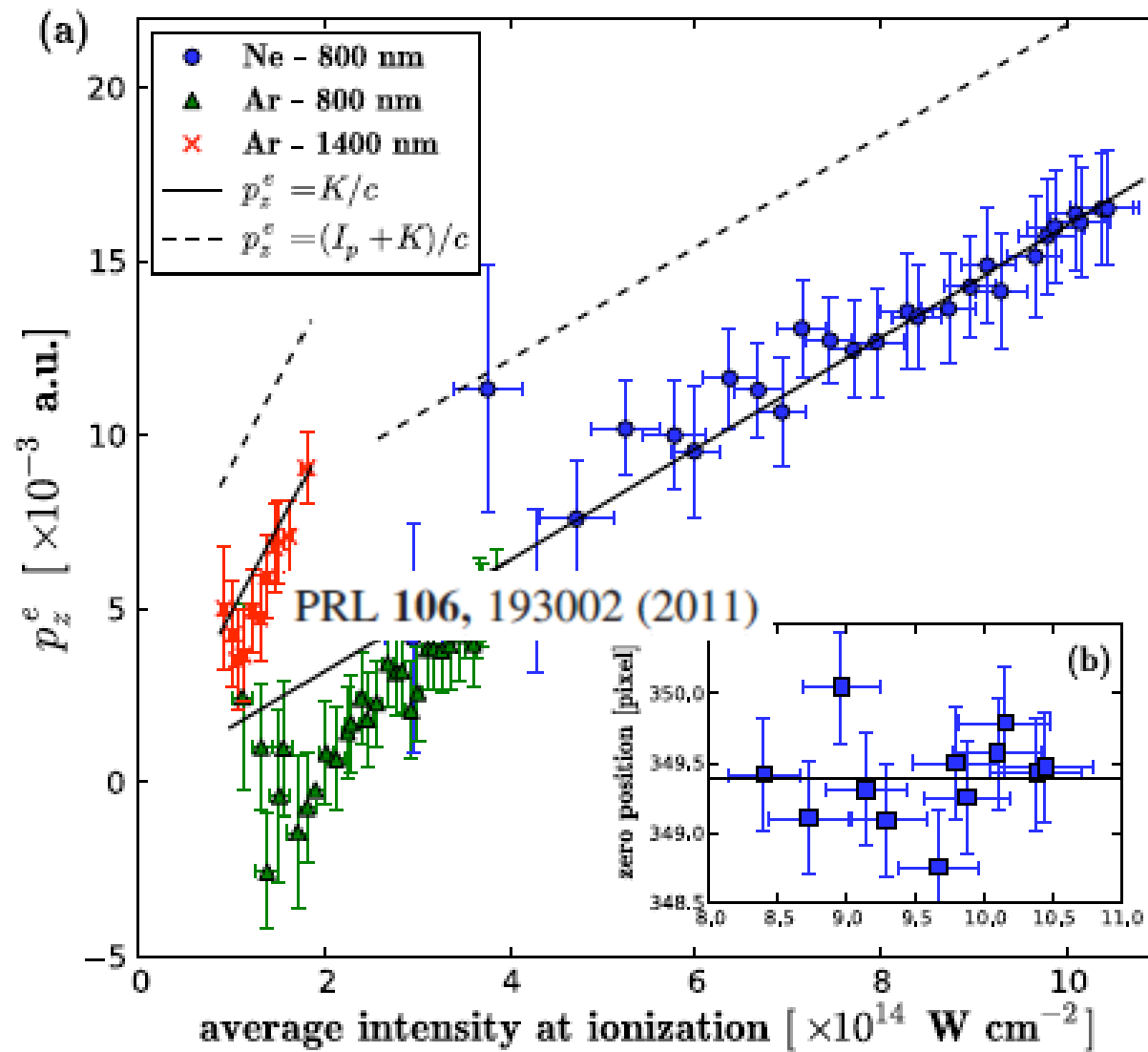
$$= U_p / \omega' (1 + o(v_z/c)^2);$$

with:

$$\omega' = \omega (1 - v_z/c)$$

(frequency shift in motion's direction)

“ ... intensity at all angles. The “peak values” are seen to occur not on the plane of polarization ($\theta = 90^\circ$) but at a somewhat smaller angle from it. It is also seen to move farther away from the polarization plane with increasing intensity [20]. This behavior is because of the change of the electron momentum in an intense field caused by the combined effect of retardation and field intensity. This can be seen from the expression of the “dressed” momentum \vec{q} of the electron in the field [see below Eq. (8)] where the extra term that adds to \vec{p} depends on $n\vec{\kappa}$ and on intensity via $(A_0/c)^2 = I/\omega^2$ (a.u.) in $\lambda_p \vec{\kappa}$. In Fig. 1 for ... ”



Smeenk et al. PRL 106, 193002 (2011)

Spin:

Spin-flip probability

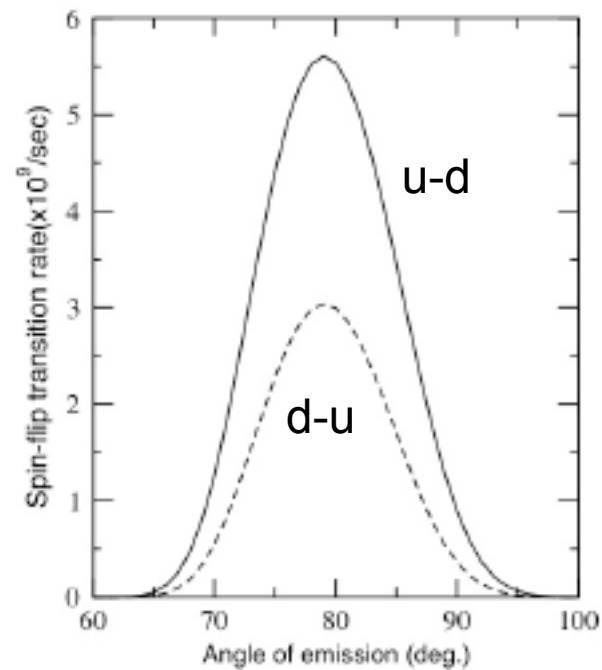


Fig. 12 Spin-flip ionization rate: angular distributions for $\omega = 200$ eV, $I = 3.5 \times 10^{22}$ W/cm²: intense-field relativistic: \uparrow to \downarrow (solid line), \downarrow to \uparrow (dashed line)

Spin-asymmetry parameter:

$$\langle A \rangle (\theta) = \left(\frac{dW^{up}}{d\Omega} - \frac{dW^{down}}{d\Omega} \right) / \left(\frac{dW^{up}}{d\Omega} + \frac{dW^{down}}{d\Omega} \right)$$

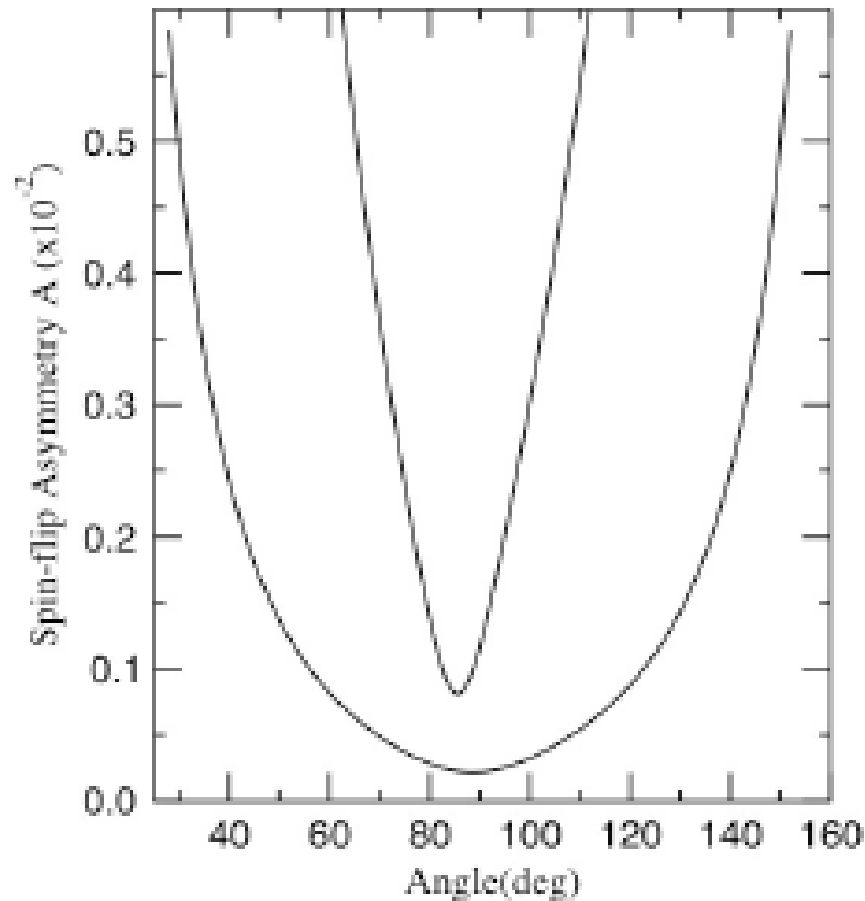
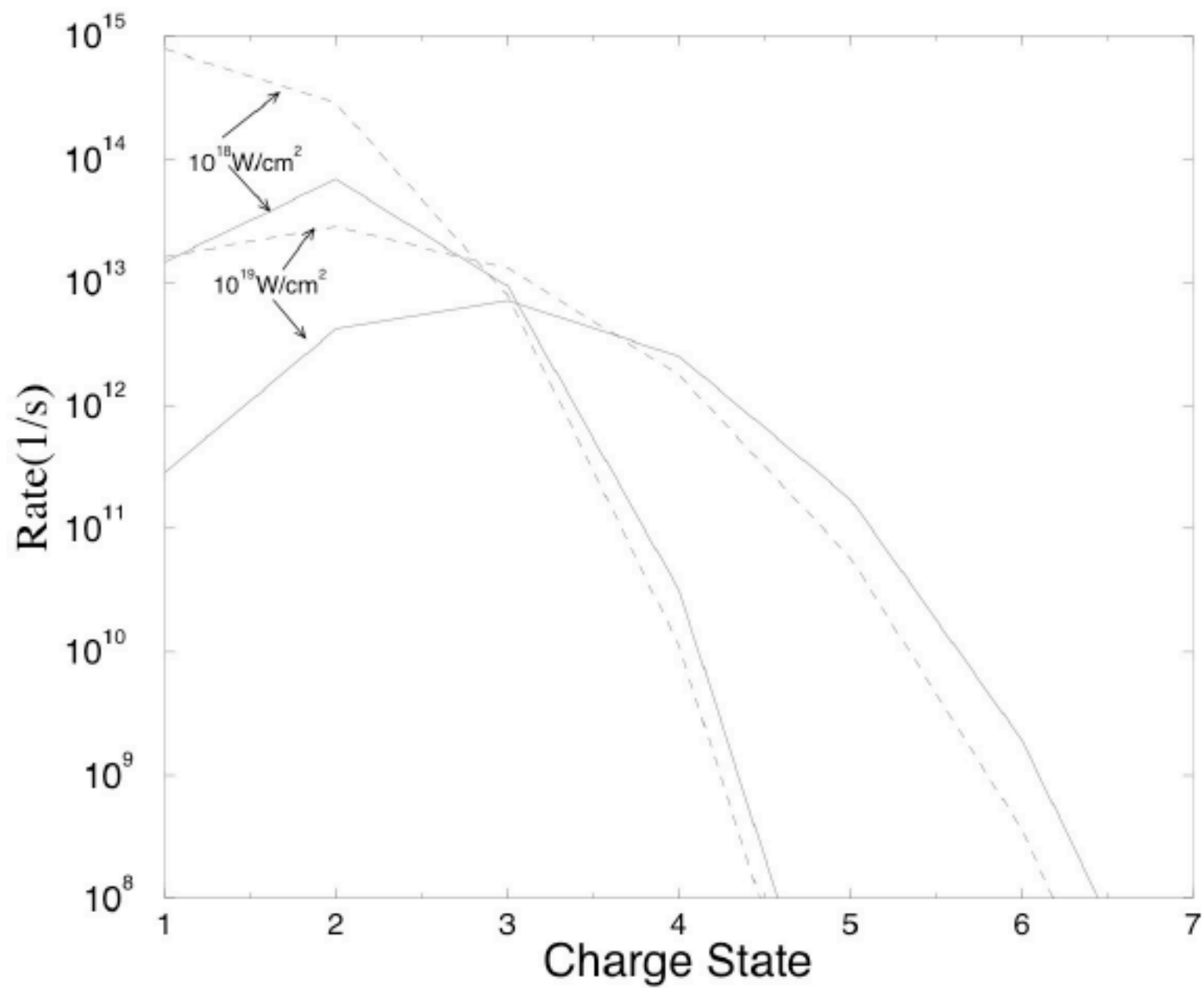


Fig. 14 Spin-flip asymmetry parameter $A(\theta)$ vs. angle of electron emission: $\omega = 1.55 \text{ eV}$, $I = 10^{16} \text{ W/cm}^2$ (outer curve) and $I = 10^{17} \text{ W/cm}^2$ (inner curve)



External control of spin current currents:

Transformation symmetry of the spin amplitudes:

changing helicity right circular $\xi = \pi/2$ to left circular $\xi = -\pi/2$,
we get:

$$\begin{aligned}t^{(n)}_{u \rightarrow u} &\rightarrow t^{(n)*}_{d \rightarrow d}, \\t^{(n)}_{d \rightarrow d} &\rightarrow t^{(n)*}_{u \rightarrow u}, \\t^{(n)}_{u \rightarrow d} &\rightarrow -t^{(n)*}_{d \rightarrow u}, \\t^{(n)}_{d \rightarrow u} &\rightarrow -t^{(n)*}_{u \rightarrow d}.\end{aligned}$$

Therefore, we may change the light polarization from left to the right circular and control preferentially the spin flip direction from. This effect survives for weak laser field strengths and hence could be also of interest in spintronics.

Nuclear Excitation/Disintegration reaction

Wavefunction of “Atomic Electron + Nucleus” System in a Laser Field

$$H(t) = H_N + H_{Vol}(t) + V_{e-N}$$

$$H_i = H_N + H_a$$

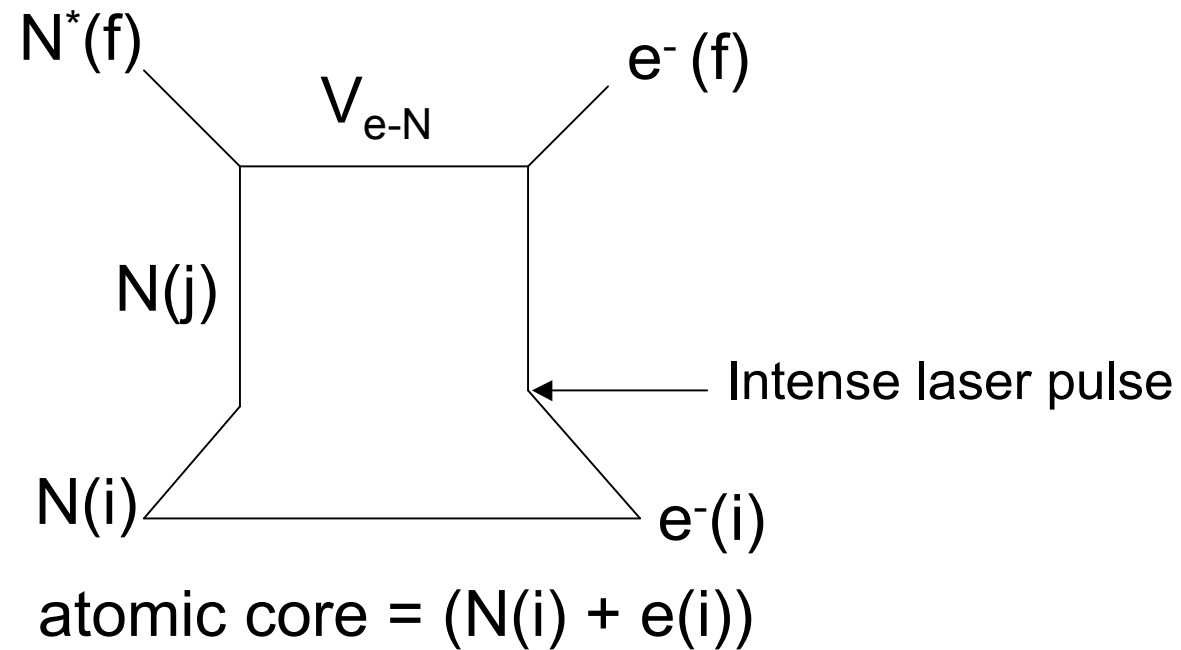
$$H_{Vol}(t) = c\boldsymbol{\alpha} \cdot (\mathbf{p}_{op} - \frac{e_0}{c}\mathbf{A}(u)) + m_0c^2$$

$$V_i = V_{e-L} = -e\boldsymbol{\alpha} \cdot \mathbf{A}(\mathbf{r}, t)$$

Nucleus \otimes Volkov propagator

$$G_0(x, X; x', X') = G_{Vol}(x; x') \sum_j \psi_j(X) \psi_j^*(X')$$

“electron-bridge” mechanism of laser excitation of atomic nucleus:

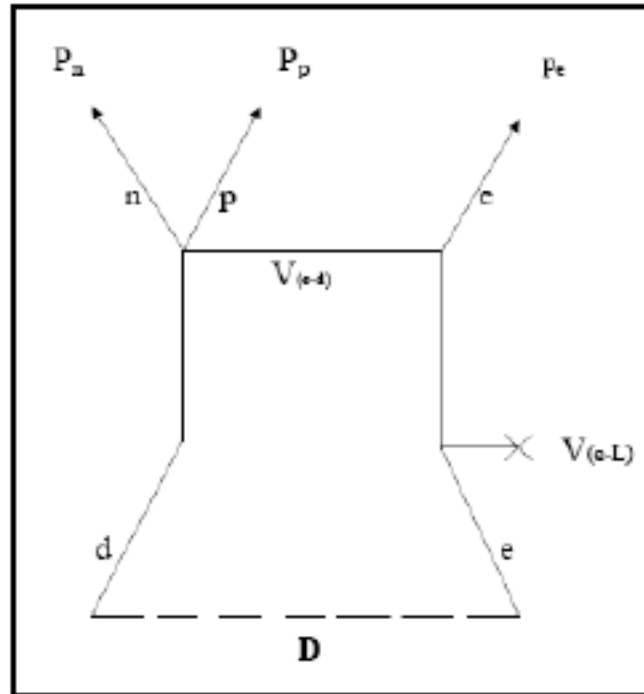


$$\mathcal{A}_{dis} = \mathcal{A}_{(1)} + \mathcal{A}_{(e-bridge)} + \dots$$

$$\mathcal{A}_{(1)} = \int dx_0 \langle \psi_f^*(X) \bar{\phi}_{p_f}(x) e^{iA(x)} \Phi_i(x, X) \rangle$$

$$\mathcal{A}_{(e-bridge)} = \int dx_0 \int dx'_0 \langle \langle \psi_f^*(X) \bar{\phi}_{p_f}(x) \gamma_0 V_0(x, X) G_0(x, X; x', X') e^{iA(x')} \Phi_i(x', X') \rangle \rangle$$

Nuclear disintegration reaction



“e-bridge” mechanism

Deuteron disintegration reaction

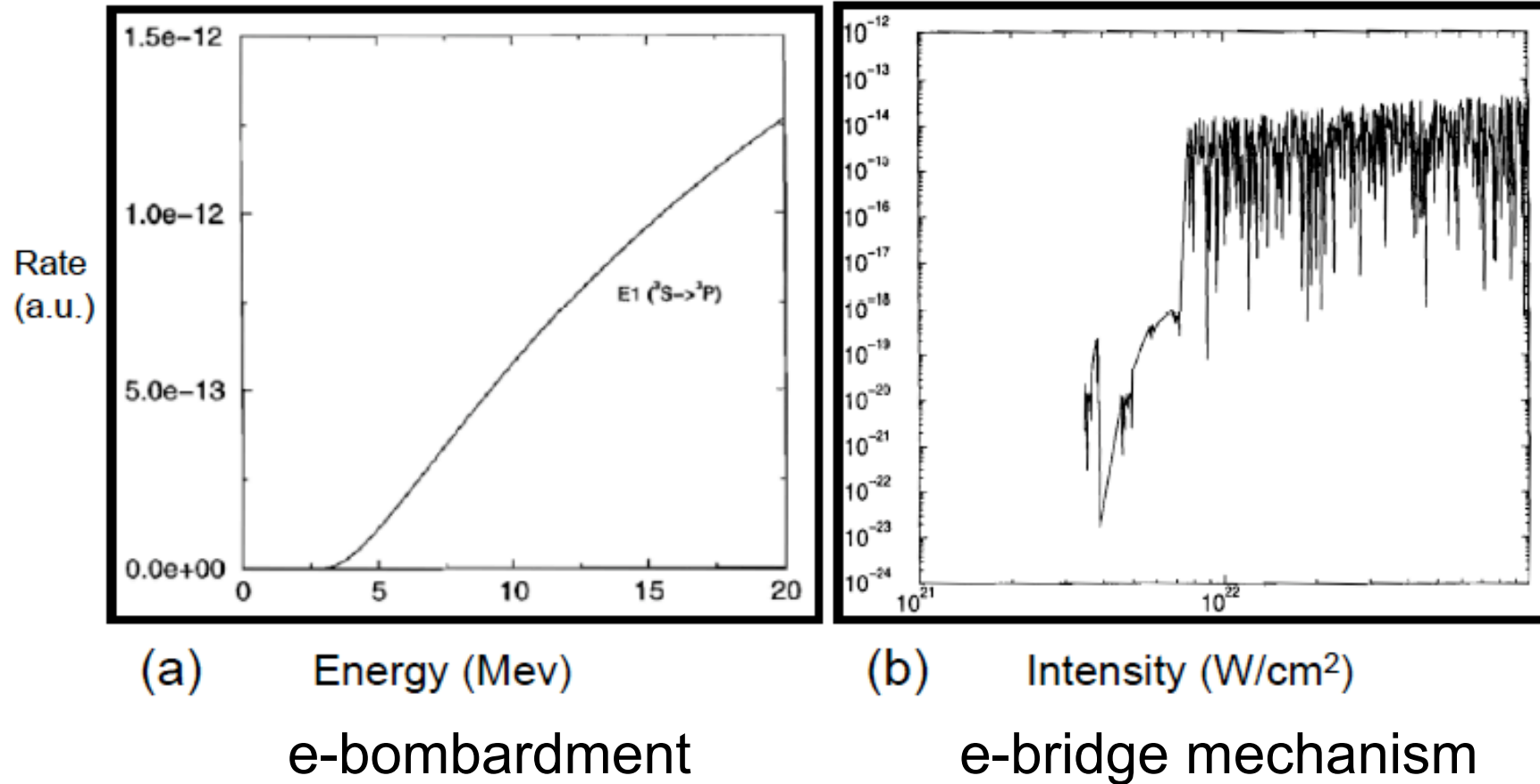


FIGURE 2. (a) Calculated electron impact disintegration rate of deuteron vs. incident electron energy. (b) Rate of laser-induced disintegration of deuterium nucleus (deuteron) via "e-Bridge" mechanism vs. incident laser intensity, at 800 nm.

High Compton harmonics generation from free electrons
(e.g. electrons prepared by pre-ionization)

a “super intense” field parameter:

$$\begin{aligned} q &= \sqrt{\frac{2e^2 \langle A^2 \rangle}{(mc^2)^2}} \\ &= \sqrt{\frac{2r_0 I \lambda^2}{\pi mc^3}} \\ &= \sqrt{\frac{4U_p}{mc^2}} \\ &= \frac{ea}{mc^2} . \end{aligned}$$

Exact Solution of Relativistic Classical Motion of e in a Light Pulse:

(ref. Y. Salamin and F.H.M. Faisal, PRA, 54, 4365 (1996); ibid 55, 3964 (1997))

An arbitrary vector potential

$$\mathbf{A}(\eta) = f(\eta) \mathbf{a}(\eta), \text{ where } \eta = \omega t - \mathbf{k} \cdot \mathbf{r},$$

Hamilton-Jacoby Equation of motion

$$\left(\frac{\partial S}{\partial t} \right)^2 = c^2 \left(\nabla S + \frac{e}{c} \mathbf{A} \right)^2 + (mc^2)^2.$$

$$S(\mathbf{r}, t) = \boldsymbol{\alpha} \cdot \mathbf{r} + \beta c t + F(\eta)$$

A general solution:

$$F(\eta) = \frac{1}{2} (\boldsymbol{\alpha} \cdot \mathbf{k} + \beta k)^{-1} \int_{\eta_0}^{\eta} [\alpha^2 - \beta^2 + (mc)^2 + 2(e/c) \boldsymbol{\alpha} \cdot \mathbf{A} + (e/c)^2 A^2] d\eta$$

Initial conditions:

$$\mathbf{A}=0, \mathbf{P}_{can} = \gamma_0 m \mathbf{v}_0, \text{ and } E = \gamma_0 m c^2,$$

$$\gamma_0 = (1 - \beta_0^2)^{-1/2} \text{ and } \beta_0 = \mathbf{v}_0 / c$$

$$\alpha = \gamma_0 m \mathbf{v}_0 \quad \beta = -\gamma_0 m c$$

solution: momentum

$$\mathbf{p}(\eta) = E(\eta) = \gamma_0 m c^2 \left[1 + \frac{\frac{1}{2} \left(\frac{e \mathbf{A}(\eta)}{\gamma_0 m c^2} \right)^2 + \left(\frac{e \mathbf{A}(\eta)}{\gamma_0 m c^2} \right) \cdot \left(\frac{\mathbf{v}_0}{c} \right)}{\left(1 - \frac{\mathbf{k} \cdot \mathbf{v}_0}{k c} \right)} \right] \cdot \left(\frac{\mathbf{v}_0}{c} \right)$$

solution: energy

$$E(\eta) = \gamma_0 m c^2 \left[1 + \frac{\frac{1}{2} \left(\frac{e \mathbf{A}(\eta)}{\gamma_0 m c^2} \right)^2 + \left(\frac{e \mathbf{A}(\eta)}{\gamma_0 m c^2} \right) \cdot \left(\frac{\mathbf{v}_0}{c} \right)}{\left(1 - \frac{\mathbf{k} \cdot \mathbf{v}_0}{k c} \right)} \right]$$

solution: trajectory

$$\mathbf{r}(\eta) = \mathbf{r}_0 + \frac{1}{k} \int_{\eta_0}^{\eta} \left[\frac{\gamma_0 m \mathbf{v}_0 + \frac{e}{c} \mathbf{A}(\eta')}{\gamma_0 m c \left(1 - \frac{\mathbf{k} \cdot \mathbf{v}_0}{k c} \right)} \right] d\eta' + \frac{\mathbf{k}}{k^2} \int_{\eta_0}^{\eta} \left[\frac{\frac{1}{2} \left(\frac{e \mathbf{A}(\eta')}{\gamma_0 m c^2} \right)^2 + \left(\frac{e \mathbf{A}(\eta')}{\gamma_0 m c^2} \right) \cdot \left(\frac{\mathbf{v}_0}{c} \right)}{\left(1 - \frac{\mathbf{k} \cdot \mathbf{v}_0}{k c} \right)^2} \right] d\eta'$$

solution: (scaled) velocity $\boldsymbol{\beta} \equiv \mathbf{v}/c$

$$\boldsymbol{\beta}(\eta) = \frac{\left\{ \boldsymbol{\beta}_0 + \frac{e}{\gamma_0 m c^2} \mathbf{A}(\eta) + \hat{\mathbf{k}} \left[\frac{\frac{1}{2} \left(\frac{e \mathbf{A}(\eta)}{\gamma_0 m c^2} \right)^2 + \left(\frac{e \mathbf{A}(\eta)}{\gamma_0 m c^2} \right) \cdot \boldsymbol{\beta}_0}{1 - \hat{\mathbf{k}} \cdot \boldsymbol{\beta}_0} \right] \right\}}{\left\{ 1 + \left[\frac{\frac{1}{2} \left(\frac{e \mathbf{A}(\eta)}{\gamma_0 m c^2} \right)^2 + \left(\frac{e \mathbf{A}(\eta)}{\gamma_0 m c^2} \right) \cdot \boldsymbol{\beta}_0}{1 - \hat{\mathbf{k}} \cdot \boldsymbol{\beta}_0} \right] \right\}}$$

High Compton Harmonics Generation:

Fully relativistic classical electrodynamics yields spectrum of radiation emitted from accelerated charge:

$$\frac{d^2E}{d\Omega d\omega} = \frac{(e\omega)^2}{4\pi^2 c^3} \times \left| \int_{-\infty}^{\infty} \mathbf{n} \times [\mathbf{n} \times \mathbf{v}(t)] \exp \left\{ i\omega \left[t - \frac{\mathbf{n} \cdot \mathbf{r}(t)}{c} \right] \right\} dt \right|^2$$

e.g. J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1976)

Re-express it for convenience of analysis:

$$\frac{d^2 E}{d\Omega d\omega} = \frac{(e\omega)^2}{4\pi^2 c^3} [|\mathbf{K}|^2 - |\mathbf{n} \cdot \mathbf{K}|^2]$$

$$\mathbf{K} = \int_{-\infty}^{\infty} \frac{d\mathbf{r}}{d\eta} \exp \left\{ i \frac{\omega}{\omega_0} \left[\eta + \frac{\omega_0}{c} [z - \hat{\mathbf{n}} \cdot \mathbf{r}(\eta)] \right] \right\} d\eta$$

$\mathbf{K} = (K_x, K_y, K_z)$ are along $\mathbf{r}(\eta) = x(\eta), y(\eta), z(\eta)$)

Compton harmonic spectrum

$$\begin{aligned}
 \frac{dP^{(n)}}{d\Omega} = & \frac{(e\omega_0)^2}{2\pi c} \frac{n^2}{[1 - a_1 \sin\theta \cos\phi + 2a_3 \sin^2(\theta/2)]^4} \left\{ J_n^2(n\Theta) \left[(1 - \sin^2\theta \cos^2\phi) \left(a_1 + \frac{b_1}{\Theta} \sin\zeta \right)^2 + (1 - \sin^2\theta \sin^2\phi) \right. \right. \\
 & \times \left(\frac{b_2}{\Theta} \cos\zeta \right)^2 + \sin^2\theta \left(a_3 + \frac{b_3}{\Theta} \sin\zeta \right)^2 - 2 \sin^2\theta \sin\phi \cos\phi \left(a_1 + \frac{b_1}{\Theta} \sin\zeta \right) \left(\frac{b_2}{\Theta} \cos\zeta \right) - 2 \sin\theta \cos\theta \cos\phi \left(a_1 + \frac{b_1}{\Theta} \sin\zeta \right) \\
 & \times \left(a_3 + \frac{b_3}{\Theta} \sin\zeta \right) - 2 \sin\theta \cos\theta \sin\phi \left(\frac{b_2}{\Theta} \cos\zeta \right) \left(a_3 + \frac{b_3}{\Theta} \sin\zeta \right) \left. \right] + J_n'^2(n\Theta) [(1 - \sin^2\theta \cos^2\phi)(b_1 \cos\zeta)^2 + (1 \\
 & - \sin^2\theta \sin^2\phi)(b_2 \sin\zeta)^2 + \sin^2\theta(b_3 \cos\zeta)^2 + 2b_1 b_2 \sin^2\theta \sin\phi \cos\phi \sin\zeta \cos\zeta - 2b_1 b_3 \sin\theta \cos\theta \cos\phi \cos^2\zeta \\
 & + 2b_2 b_3 \sin\theta \cos\theta \sin\phi \cos\zeta \sin\zeta] \left. \right\}. \tag{33}
 \end{aligned}$$

$$a_1 = \frac{\beta_0 \sin\theta_0}{1 - \beta_0 \cos\theta_0},$$

$$b_1 = b_2 = \frac{(q/\gamma_0 \sqrt{2})}{1 - \beta_0 \cos\theta_0},$$

$$a_3 = \frac{\beta_0 \cos\theta_0}{1 - \beta_0 \cos\theta_0} + \frac{(q/2\gamma_0)^2}{(1 - \beta_0 \cos\theta_0)^2},$$

$$b_3 = \frac{(q/\gamma_0 \sqrt{2})\beta_0 \sin\theta_0}{(1 - \beta_0 \cos\theta_0)^2},$$

Frequency of harmonics in the laboratory frame:

depends on initial e^- velocity, and direction of harmonic emission observed:

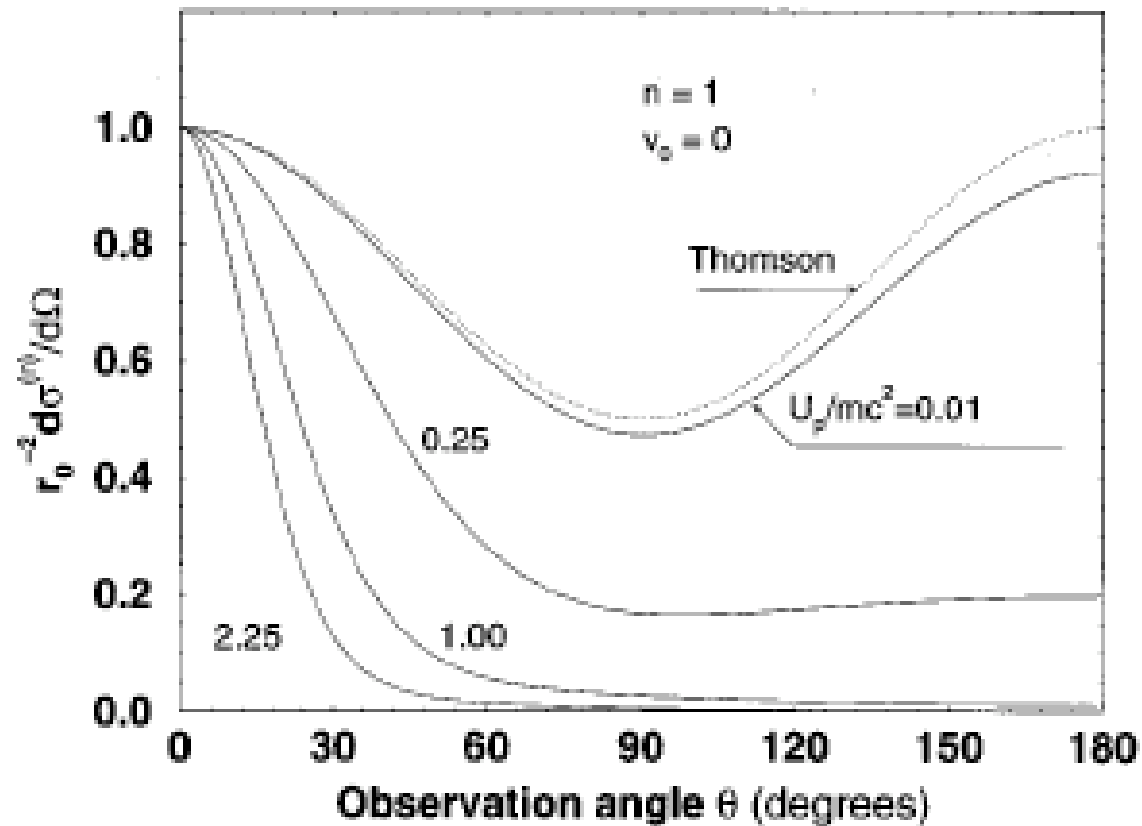
$$\omega_L^{(n)} = n \omega_L^{(0)} \frac{(1 - \beta_D \cos \xi)}{[1 - \beta_D (\sin \theta \cos \phi \sin \xi + \cos \theta \cos \xi)]}.$$

$$\beta_D = \frac{[\beta_0 \sin \theta_0 (1 - \beta_0 \cos \theta_0)] e_x + [\beta_0 \cos \theta_0 + (q/2\gamma_0)^2] e_z}{(1 - \beta_0 \cos \theta_0) + (q/2\gamma_0)^2}.$$

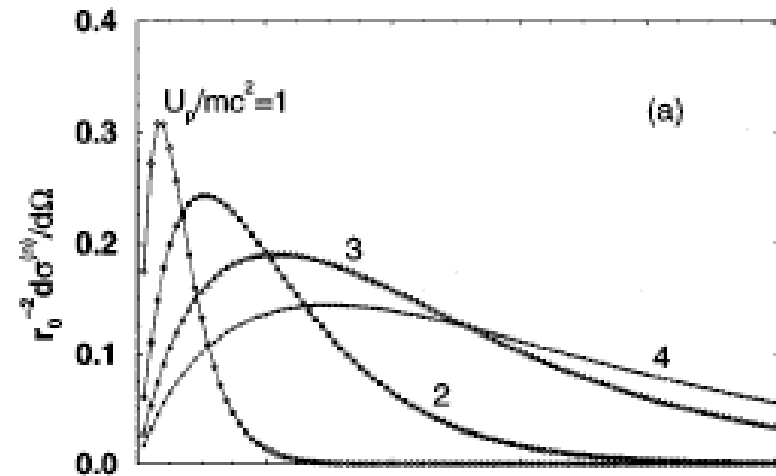
$$\tan \xi = \frac{\beta_0 \sin \theta_0 (1 - \beta_0 \cos \theta_0)}{\beta_0 \cos \theta_0 + (q/2\gamma_0)^2}$$

Non-linear Thomson Scattering:

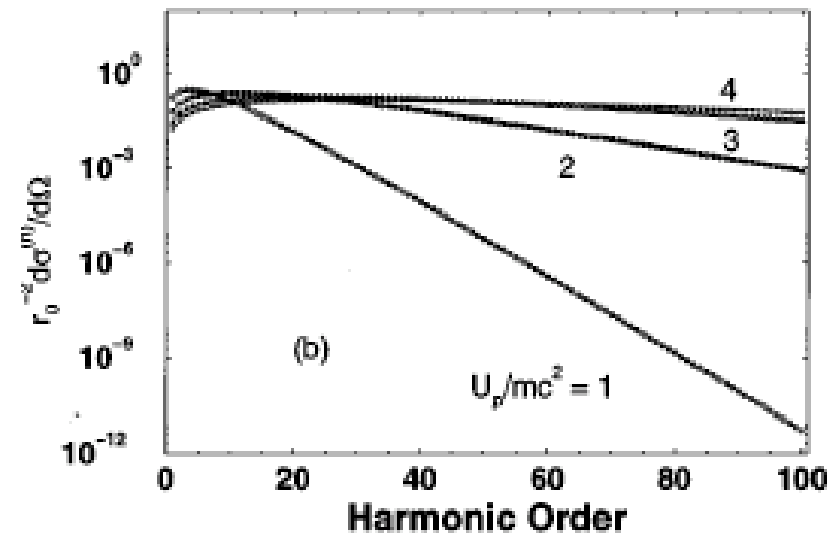
Departure from Thomson scattering angular distribution:
 $(1 + \cos^2\theta)/2$



High Compton Harmonics Emission (observation angle $\theta=40$ Deg.)

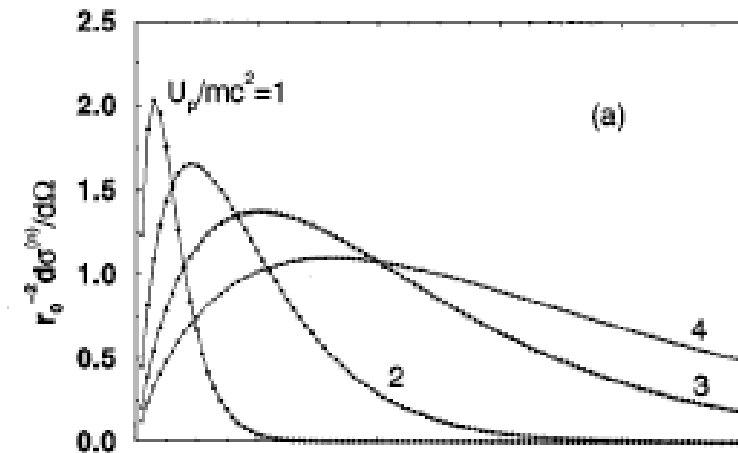


(linear scale)

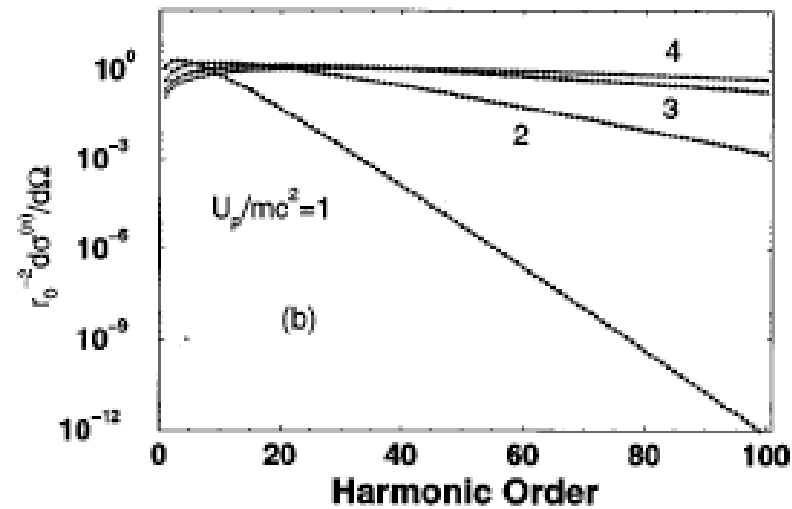


(semi-log scale)

High Compton Harmonics Emission (observation angle $\theta=13$ Deg.)

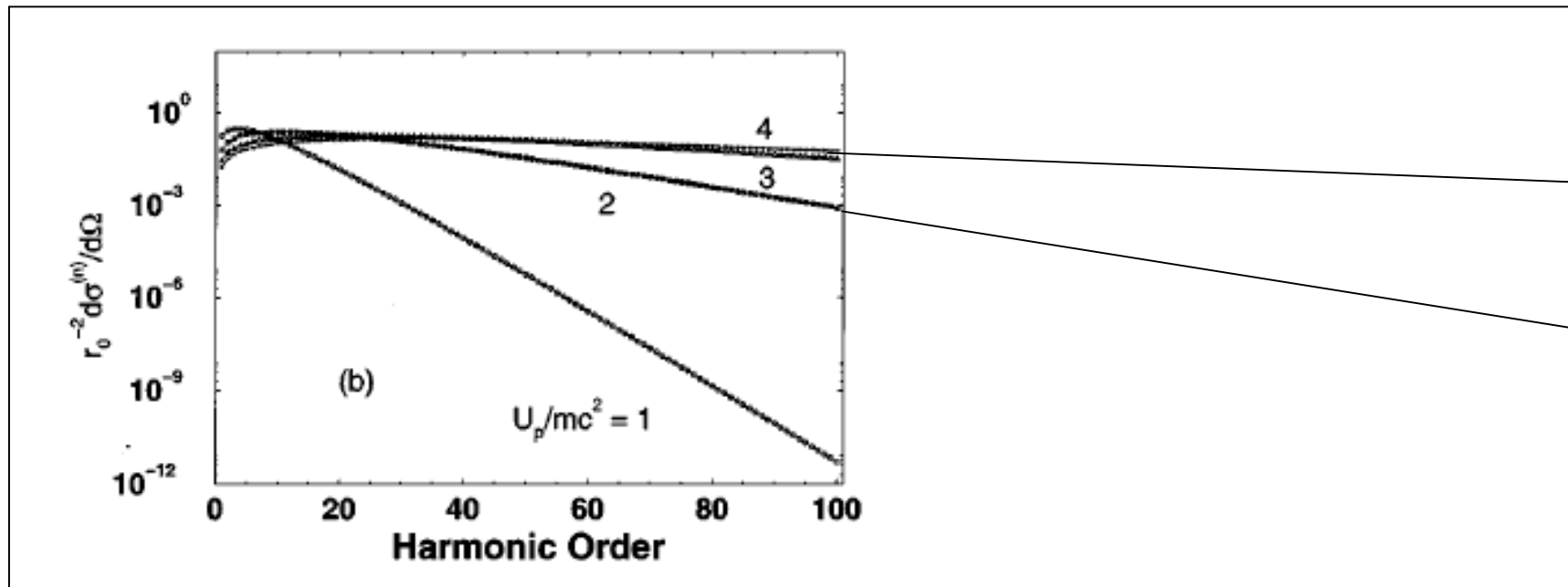


(linear scale)



(semi-log scale)

Extrapolation in semi-log scale shows evidence of “very high” order Compton harmonics for $U_p/mc^2 > 1$.



Thank you!