Enhancement of Schwinger's effect: analytical and numerical aspects (and maybe some qubits...)

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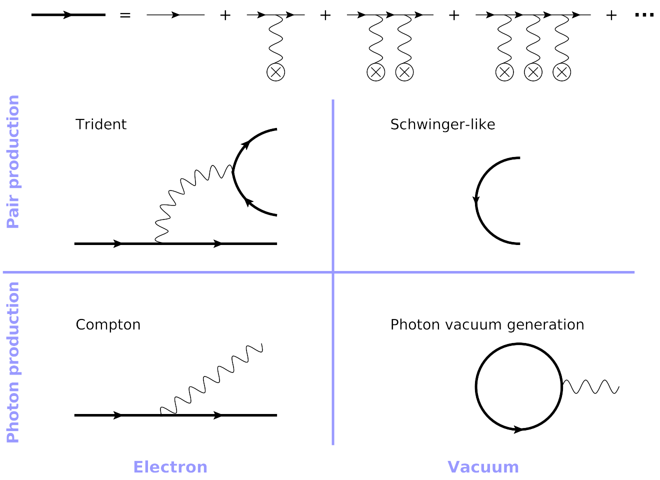
KITP, August 1st 2014

Outline

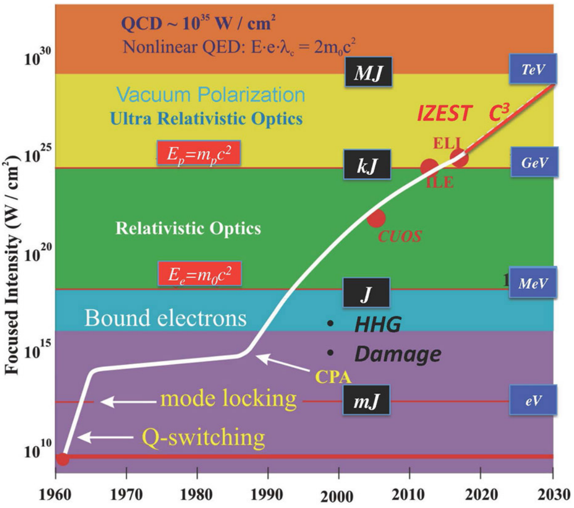
1. QED processes in high intensity lasers
2. Pair production in multi-center systems
   - Model description
   - Pair production in inhomogeneous field
   - Numerical results for pair production
     - Position of resonances and pair production
     - Total Rate: REPP and ECEPP
3. Numerical solution of the Dirac equation
   - Numerical method
   - Numerical results
4. Schwinger pair production in a tightly focused configuration
5. Conclusion

QED processes

QED processes in high intensity lasers

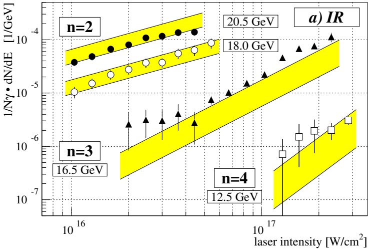


Laser intensity regimes



Some key results of SLAC E-144

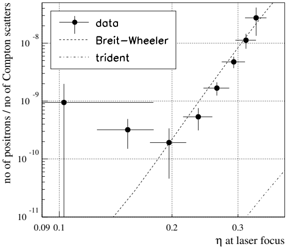
Compton scattering



- Conclusion:
  - Detection of Compton Scattering
  - Good fit with simulation results

Bula, C., et al. "Observation of nonlinear effects in Compton scattering." Physical Review Letters 76.17 (1996): 3116.

Electron-positron production



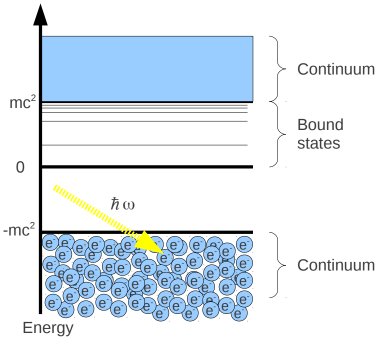
- Conclusion:
  - Detection of Breit-Wheeler process
  - NO detection of trident process

Burke, D. L., et al. "Positron production in multiphoton light-by-light scattering". Physical Review Letters 79.9 (1997): 1626.

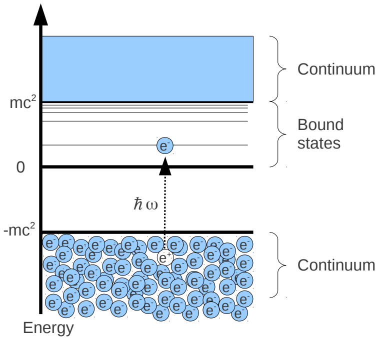
Pair production in multi-center systems

Pair creation

Photon Excitation



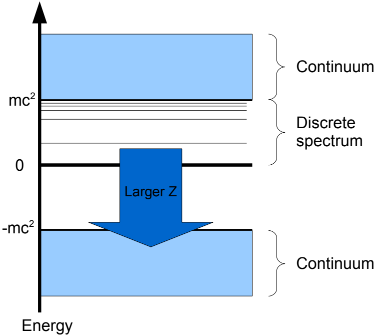
Hole created = positron



What if we modify the external field?

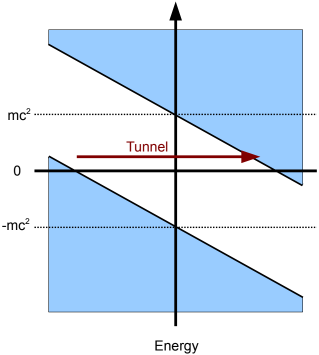
Mechanisms for non-perturbative pair production

“Plunging” in the Dirac sea



Relativistic Heavy Ion Collisions
(GSI, APEX, RHIC, CERN)

Schwinger’s process

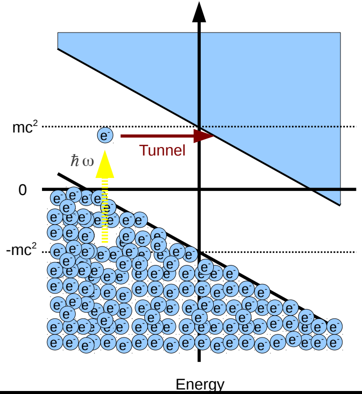


$I \approx 10^{26} - 10^{29}$ W/cm^2

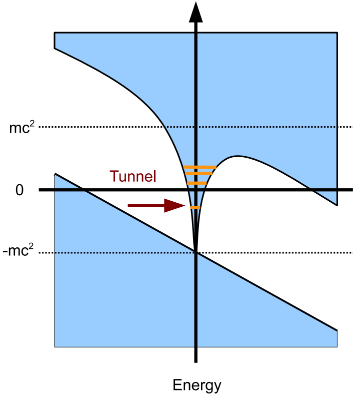
Combination of different mechanisms

Dynamically assisted Schwinger's mechanism

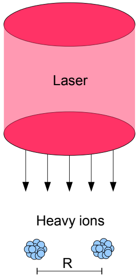
R Schutzhold, H Gies, G Dunne



Single nucleus in a constant electric field



Two-center systems (diatomic “molecule”)



- $R \approx 10.$ a.u.
- $mc^2 \approx 18769.$ a.u.
- $E_g^{U^{91+}} \approx 13908.$ a.u.

$I \approx 1.5 \times 10^{24}$ W/cm^2

Questions?

- Can we use effects from non-relativistic ionization of molecules to enhance pair production (CREI)
- Stark effect at large inter-nuclei distance:

$$\Delta E_{\mathrm{Stark}} \approx \pm \frac{FR}{2} \approx 2mc^2$$

Simple model description

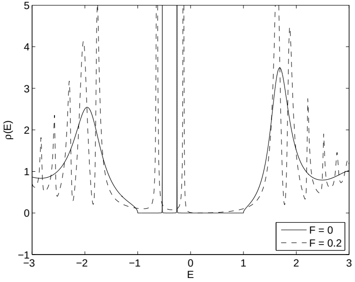
- (Very) Simple toy model
  1. 1-D model
  2. Nuclei potential: delta function wells

$$V_{\text{nucl.}}(s) = -g \sum_{i=1}^{N_{\text{nuc}}} \delta(x - R_i)$$

$$g \approx 0.8 = \text{U}^{91+}$$

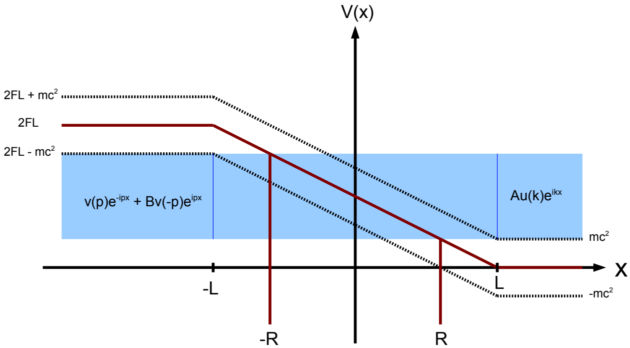
  3. Laser electric field: static (adiabatic limit) $V_{\text{field}}(x) = -Fx$
- Relativistic wave equation: time-independent Dirac equation

Spectral density



Position of resonances → WTK

Pair production = Transmission-reflection problem



$$\frac{d\langle n\rangle}{dtdE} = \frac{1}{2\pi}|A(E)|^2, \; E \in \Omega_{\mathrm{Klein}}$$

$$\frac{d\langle n\rangle}{dt} = \frac{1}{2\pi}\int_{\Omega_{\mathrm{Klein}}} dE\,|A(E)|^2$$

Solving the Dirac equation: numerical approach

The solution of the Dirac equation can be written formally as

$$\begin{aligned}\psi(x_f) &= \mathcal{P}\exp\left\{\frac{1}{c}\int_{x_i}^{x_f} dy\left[-\sigma_y mc^2 - i\sigma_z\left[F(y-L)+E\right]\right]\right\}\psi(x_i)\\ &= U(x_i,x_f)\psi(x_i)\end{aligned}$$

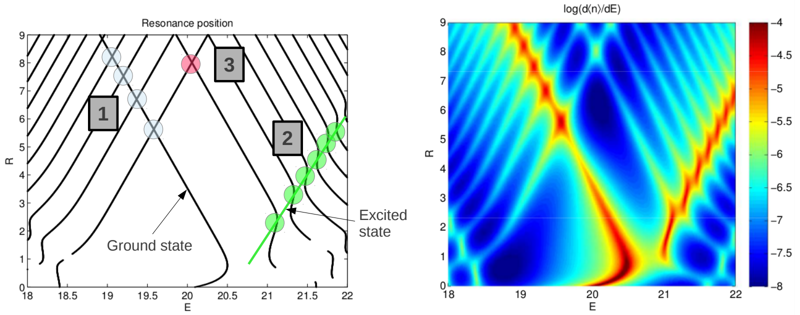
Approximated by

$$\begin{aligned}\psi(-L) &= \tilde{U}(-L,x_n)\tilde{U}(x_n,x_{n-1})\cdots\tilde{U}(x_{j_{N_p}+1},x_{j_{N_p}})G^{-1}\tilde{U}(x_{j_{N_p}},x_{j_{N_p}-1})\\ &\quad \cdots\tilde{U}(x_{j_1+1},x_{j_1})G^{-1}\tilde{U}(x_{j_1},x_{j_1-1})\cdots\tilde{U}(x_1,L)\psi(L)+O(\delta x^3)\end{aligned}$$

Much more efficient than the analytical approach!

At least, vs Maple and Mathematica implementation of parabolic cylinder functions and for $F < 1.0$

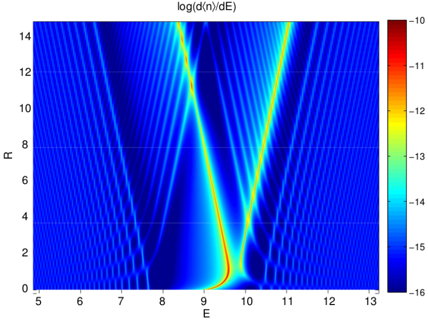
Position of resonances and pair production: $g = 0.8$ (Uranium), $F = 0.2 \times 10^{18}$ V/m $\rightarrow I = 2.5 \times 10^{27}$ W/cm^2

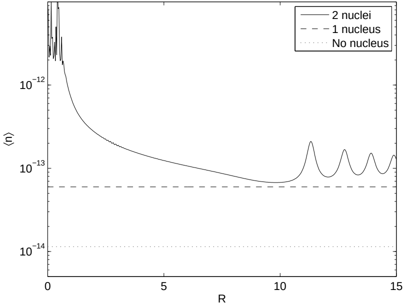


1. Channel 1: ground state crosses with negative energy resonances
2. Channel 2: excited state goes through avoided crossing with positive energy resonances
3. Channel 3: negative energy states cross with positive energy states

Total rate: $d\langle n\rangle/dt$

For $g = 0.8$ (Uranium),
$F = 0.09 \times 10^{18}$ V/m $\rightarrow I = 8.1 \times 10^{26}$ W/cm^2,
$L = 100.0 \times 0.38$ pm

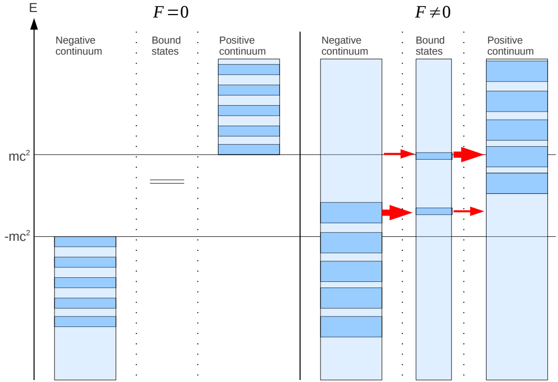




- REPP at large R, dominated by the ground state crossings
- ECEPP at small R

F. Fillion-Gourdeau *et al*, Phys. Rev. Lett. 110, 013002 (2013)

REPP: at LARGE interatomic distance

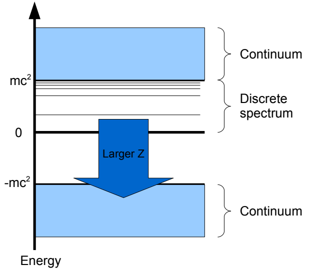


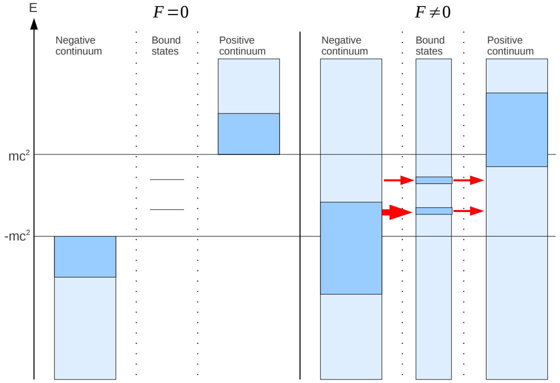
F. Fillion-Gourdeau *et al*, Phys. Rev. Lett. 110, 013002 (2013)

Mechanism: **CREI**

T Seideman, MY Ivanov, PB Corkum , Phys. Rev. Lett. 75, 2819 (1995)
T. Zuo and A. D. Bandrauk , Phys. Rev. A. 52, R2511 (1995)

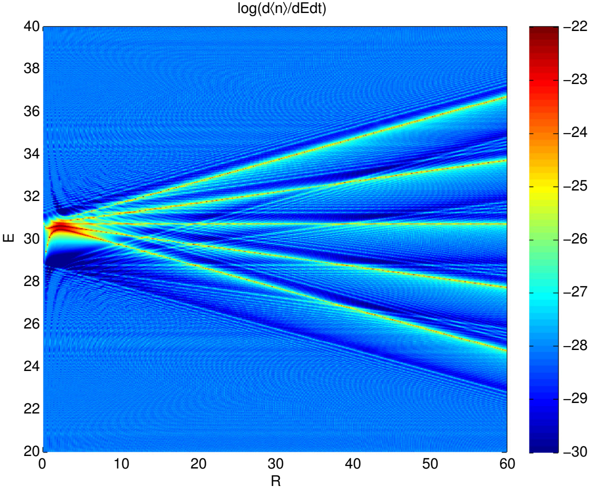
ECEPP: at SMALL interatomic distance



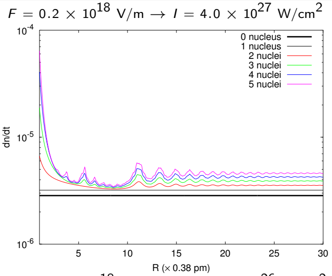


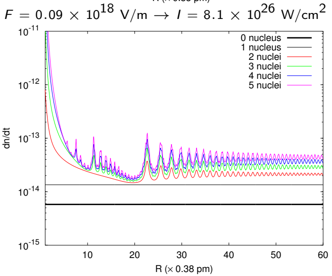
Many-center case: 5 nuclei

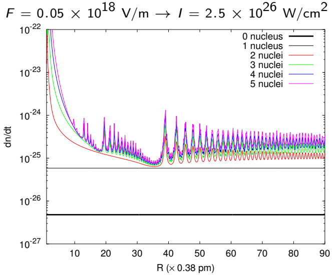
$$F = 0.05 \times 10^{18}\ \mathrm{V/m} \rightarrow I = 2.5 \times 10^{26}\ \mathrm{W/cm^2}$$



Total rate: variation with electric field strength ($g = 0.8$)







- Relative enhancement increases
- REPP occurs at larger R
- Exponential suppression of the rate

Numerical solution of the Dirac equation

Dirac Equation

- Relativistic wave equation
- Describes spin-1/2 particles (electrons, quarks)
- In non-covariant notation, it is given by:

$$i\partial_t\psi(x) = \left[-ic\boldsymbol{\alpha}\cdot\nabla - e\boldsymbol{\alpha}\cdot\mathbf{A}(x) + \beta mc^2 + V(x)\right]\psi(x)$$

where $\psi(x) \in L^2(\mathbb{R}^3) \otimes \mathbb{C}^4$

$$\alpha_i := \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix} \quad , \quad \beta := \begin{bmatrix} \mathbb{I}_2 & 0 \\ 0 & -\mathbb{I}_2 \end{bmatrix}.$$

The σ_i are the usual 2×2 Pauli matrices defined as

$$\sigma_x := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad , \quad \sigma_y := \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \text{and} \quad \sigma_z := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

- **A**, V are the potentials of the external field (minimal coupling prescription)

Numerical challenge

1. Computation time:
   - Time step is small: $\delta t < 1/mc^2$
   - Typical time scale of macroscopic field is large
   - Many initial states to consider (for pair production calculations)
2. Coupled system of equations
3. Spectrum is NOT bounded from below

Numerical method: time discretization

- Formal solution of the Dirac equation:

$$\psi(t_{n+1}) = T\exp\left[-i\int_{t_n}^{t_{n+1}} H(t)dt\right]\psi(t_n), \\ = e^{-i\Delta t(H(t_n)+\mathcal{T})}\psi(t_n)$$

- Split-operator method + Alternate Direction Iteration (ADI):

$$\hat{H}_1 = -ic\alpha_x\partial_x, \\ \hat{H}_2 = -ic\alpha_y\partial_y, \\ \hat{H}_3 = -ic\alpha_z\partial_z, \\ \hat{H}_4 = -e\boldsymbol{\alpha}\cdot\mathbf{A}(t,\mathbf{x}) + \beta mc^2 + e\mathbb{I}_4 V(t,\mathbf{x}).$$

- Scheme (first order):

$$\psi(t_{n+1}) = e^{-i\Delta t\mathcal{T}}e^{-i\Delta t\hat{H}_4(t_n)}e^{-i\Delta t\hat{H}_3}e^{-i\Delta t\hat{H}_2}e^{-i\Delta t\hat{H}_1}\psi(t_n) + O(\Delta t^2)$$

Treat each exponential independently!

Numerical method: space discretization

- Introduce rotation operators $(S_a)_{a=x,y,z}$:

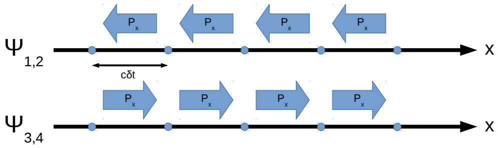
$$\psi(t_{n+1}) = Q_c(t_n,\delta t)\left[S_z P_z(\delta t) S_z^{-1}\right]\left[S_y P_y(\delta t) S_y^{-1}\right]\left[S_x P_x(\delta t) S_x^{-1}\right] \times\psi(t_n) + O(\delta t^2)$$

- Sequence of unitary operators:
  - Rotation operator: $S_a := \frac{1}{\sqrt{2}}(\beta + \alpha_a)$
  - Translation operator: $P_a(\Delta t) := e^{-c\Delta t \beta \partial_a}$
  - Local operator: Q_c

- Choosing space lattice:

$$c\delta t = N\delta x$$

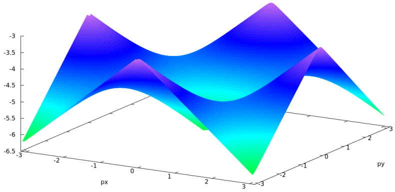
with $N = \frac{1}{2}, 1, \frac{3}{2}, 2, \cdots$



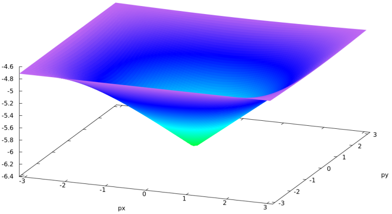
Dispersion relation: Fermion doubling

$$E = \pm|p|$$

n = 1



n = 1/2: staggered mesh



Zeroes of the dispersion relation are poles of the fermion propagator

No fermion doubling for $N = \dfrac{1}{2}$

Numerical method features

Pros

- No fermion doubling
- Computational complexity: $O(N)$
- Easily/Efficiently parallelized
- Conservative (L^2-norm is conserved)
- Adaptable:
  - 1-D, 2-D and 3-D
  - Many external potentials (coupled to Maxwell's equation solver)
  - Many initial states
- Extended to higher order splitting (order 2 and 3)

Cons

- Alternate direction iteration
- Relation between the time and space increments:
  - Small time steps required

$$\delta t \ll \frac{1}{mc^2}$$

  - Small δx
- Slow to find bound states (Feit-Fleck method)

Implementation on a quantum computer: quantum walk

$$\psi(t_{n+1}) = Q_c(t_n, \delta t)\left[S_z P_z(\delta t) S_z^{-1}\right]\left[S_y P_y(\delta t) S_y^{-1}\right]\left[S_x P_x(\delta t) S_x^{-1}\right]\psi(t_n)$$

Quantum register

A bunch of two-level systems (qubits)!

$$|\psi\rangle = \sum_{s_1=0}^{1} \cdots \sum_{s_m=0}^{1} \alpha_{s_1,\cdots,s_m} \bigotimes_{l=1}^{m} |s_l\rangle,$$

Mapping of the discretized wave function on qubits:

$$\psi^n_{i,j,k} \to \alpha_{s_1,\cdots s_m}$$

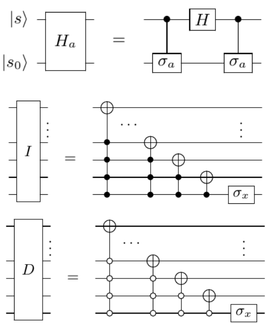
Quantum gates

Unitary operations on qubits

Mapping of unitary operators on quantum gates

$S_a \to$

$P_a \to$



“Efficient” implementation on a quantum computer

- Computational complexity (preliminary results):
  - Classical computer: $O(N)$
  - Quantum computer: $O(\log_2^2(N))$

Exponential speedup

- Feasibility: ???

"Efficient" implementation on a quantum computer

- Computational complexity (preliminary results):
  - Classical computer: $O(N)$
  - Quantum computer: $O(\log_2^2(N))$

Exponential speedup

- Feasibility:

PRL **96,** 170501 (2006) PHYSICAL REVIEW LETTERS week ending 5 MAY 2006

Benchmarking Quantum Control Methods on a 12-Qubit System

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(Received 17 December 2005; published 1 May 2006)

In this Letter, we present an experimental benchmark of operational control methods in quantum information processors extended up to 12 qubits. We implement universal control of this large Hilbert space using two complementary approaches and discuss their accuracy and scalability. Despite decoherence, we were able to reach a 12-coherence state (or a 12-qubit pseudopure cat state) and decode it into an 11 qubit plus one qutrit pseudopure state using liquid state nuclear magnetic resonance quantum information processors.

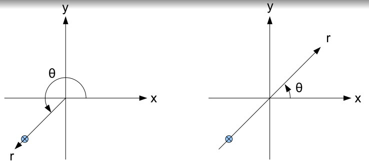
$2^{12} = 4096$ lattice points $\rightarrow$ "Hilbert space is a big place" - Carlton Caves

Dirac equation in Cylindrical coordinates

- Cylindrical coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$

- Azimuthal symmetry: $A^{\mu}(r,z)$



Separation of variables:

$$\Psi(\mathbf{x},t) = \begin{bmatrix} \psi_1(t,r,z)e^{i\mu_1\theta} \\ \psi_2(t,r,z)e^{i\mu_2\theta} \\ \psi_3(t,r,z)e^{i\mu_1\theta} \\ \psi_4(t,r,z)e^{i\mu_2\theta} \end{bmatrix}$$

Angular momentum

$$\mu_{1,2} := j_z \mp 1/2$$
$$j_z = \ldots, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \ldots$$

Boundary condition ($r=0$):

$$\psi_1(r) = (-1)^{|\mu_1|}\psi_1(-r),$$
$$\psi_2(r) = (-1)^{|\mu_2|}\psi_2(-r),$$
$$\psi_3(r) = (-1)^{|\mu_1|}\psi_3(-r),$$
$$\psi_4(r) = (-1)^{|\mu_2|}\psi_4(-r).$$

$$i\partial_t\psi(t,r,z) = \left\{\alpha_x\left[-ic\partial_r - ic\frac{1}{2r} - eA_r(t,r,z)\right] + \alpha_y\left[c\frac{j_z}{r} - eA_\theta(t,r,z)\right] + \alpha_z\left[-ic\partial_z - eA_z(t,r,z)\right] + \beta mc^2 + eV(t,r,z)\right\}\psi(t,r,z)$$

Numerical method: time discretization

- Splitting (first order):

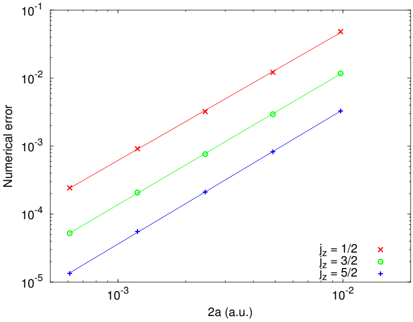
$$
\begin{aligned}
i\partial_t \psi^{(1)}(t) &= \left[-ic\alpha_x \partial_r - \boxed{ic\alpha_x \frac{1}{2r} + c\alpha_y \frac{j_z}{r}} \right] \psi^{(1)}(t) \\
i\partial_t \psi^{(2)}(t) &= -ic\alpha_z \partial_z \psi^{(2)}(t) \\
i\partial_t \psi^{(3)}(t) &= \left[\beta mc^2 + e\mathbb{I}_4 V(t) - e\alpha_x A_r(t) - e\alpha_y A_\theta(t) - e\alpha_z A_z(t) \right] \\
&\quad \times \psi^{(3)}(t),
\end{aligned}
$$

- Second order: Strang-like splitting
- Similar to the Cartesian case, BUT
  1. 2nd and 3rd step: same as Cartesian case $\rightarrow \delta z = c\delta t$
  2. 1st step: no advection solution + singular terms $(1/r)$

Strategy:

- Poisson's integral solution of the wave equation
- Time staggered mesh

Order of convergence



$$E_{\ell^2}(t) := ||\psi_{\mathrm{exact}}(t) - \psi(t)||_{\ell^2}$$

- Initial state

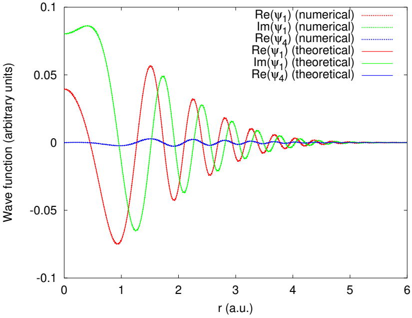
$$\psi(t=0,r) = \mathcal{N} \begin{bmatrix} r^{|\mu_1|} \\ 0 \\ 0 \\ 0 \end{bmatrix} e^{-\frac{r^2}{4\Delta^2}}$$

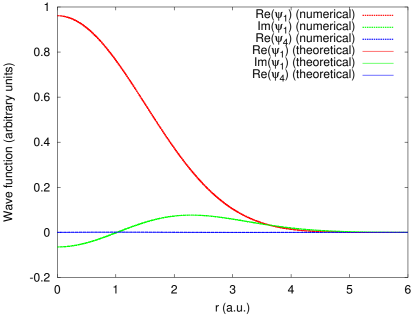
- Order of convergence

j_z	Order of convergence (q)
$\frac{1}{2}$	1.8999
$\frac{3}{2}$	1.9435
$\frac{5}{2}$	1.9767

- Similar to Cartesian case ~ 2

2-D Gaussian wave packet





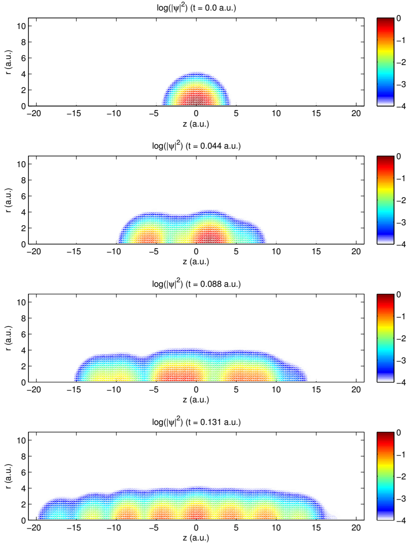
- $\Delta = 0.1$ a.u.
- $t_f = 0.143$ a.u.

- $\Delta = 1.0$ a.u.
- $t_f = 0.570$ a.u.

$$\psi_1(t,r,z) = 2\mathcal{N}\Delta^2 \int_0^\infty dp p e^{-\Delta^2 p^2} J_0(pr) \left[\cos(Et) - i\frac{mc^2}{E}\sin(Et)\right]$$

$$\psi_4(t,r,z) = 2\mathcal{N}\Delta^2 e^{i\theta} \int_0^\infty dp p J_1(pr) \frac{cp}{E}\sin(Et) e^{-\Delta^2 p^2}$$

3-D Gaussian wave packet

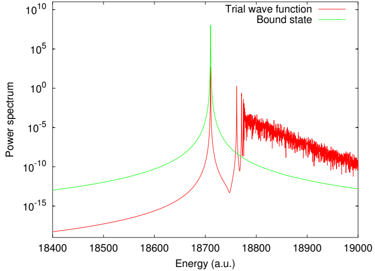


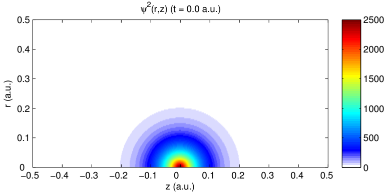
- Counterpropagating laser field:

$$E(t) = E_0 f(t) \cos(\omega t)$$

- Parameters:
  - $\omega = 100$ a.u.
  - $E_0 = 3.65 \times 10^6$ a.u.
  - 2 cycles
- Hint of pair production

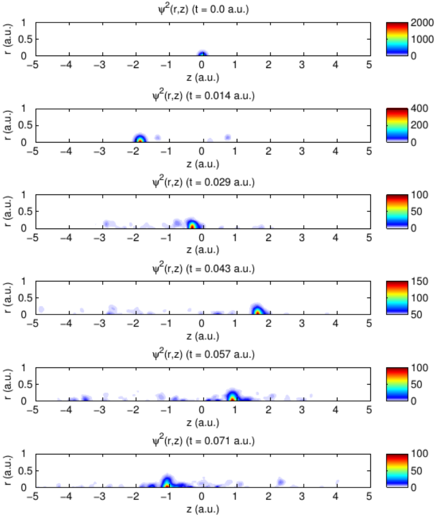
Initial state of hydrogen-like atom





- Feit-Fleck spectral method
  - Allows to compute eigenstates from time evolution of trial state
  - Main ingredient: FFT of auto-correlation function
  - Energy resolution increases with simulation time
- Coulomb-like potential: $Z = 10$
- Simulation:
  - $t_f = 47.5$ a.u. $\rightarrow \delta E \sim 0.13$ a.u.
  - $E_{\text{ground}} \approx 18710.3$ a.u.
    (Analytically: $E_{\text{ground}} \approx 18729.9$ a.u.)

Time evolution of the ground state in a laser field



- Coulomb-like potential: $Z = 10$
- Counterpropagating laser field:

$$E(t) = E_0 f(t) \cos(\omega t)$$

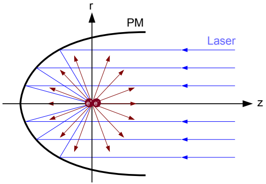
- Parameters:
  - $\omega = 100$ a.u.
  - $E_0 = 3.65 \times 10^5$ a.u.
  - 2 cycles
- Electron is fully ionized
- Electron is driven by the field

Schwinger pair production in a tightly focused configuration

Towards the calculation in “realistic” conditions

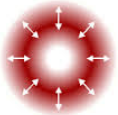
- Question:
  “How many pairs can be produced in a “realistic” experimental setup?”
- *e*-dipole pulse: optimized pulses (Marklund *et al.*)
- Experimental setup:

High numerical aperture parabolic mirror



High field strength

Radially polarized incident beams



Focused on a smaller focal spot
+
B-field is zero at the focal spot

Pair production rate

- Schwinger's formula ($l_f \sim l_C \ll \lambda$) Narozhny, Fedotov, 2014:

$$\frac{dN}{dV} = \frac{\alpha}{\pi} \int_{-\infty}^{\infty} \mathcal{E}(t,\mathbf{x})\mathcal{H}(t,\mathbf{x}) \coth\left(\pi \frac{\mathcal{H}(t,\mathbf{x})}{\mathcal{E}(t,\mathbf{x})}\right) \exp\left(-\frac{\pi}{\mathcal{E}(t,\mathbf{x})}\right),$$

where

$$\begin{aligned} \mathcal{E}(t,\mathbf{x}) &= \sqrt{\sqrt{\mathcal{F}^2(t,\mathbf{x}) + \mathcal{G}^2(t,\mathbf{x})} + \mathcal{F}(t,\mathbf{x})}, \\ \mathcal{H}(t,\mathbf{x}) &= \sqrt{\sqrt{\mathcal{F}^2(t,\mathbf{x}) + \mathcal{G}^2(t,\mathbf{x})} - \mathcal{F}(t,\mathbf{x})}. \end{aligned}$$

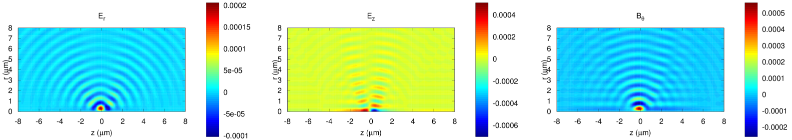
$\mathcal{F}$ and $\mathcal{G}$ are the Lorentz invariants:

$$\begin{aligned} \mathcal{F}(t,\mathbf{x}) &= \frac{\mathbf{E}^2(t,\mathbf{x}) - \mathbf{B}^2(t,\mathbf{x})}{2}, \\ \mathcal{G}(t,\mathbf{x}) &= \mathbf{E}(t,\mathbf{x}) \cdot \mathbf{B}(t,\mathbf{x}), \end{aligned}$$

Computing the field at the focus

Compute the field at the focal spot: Stratton-Chu vectorial diffraction

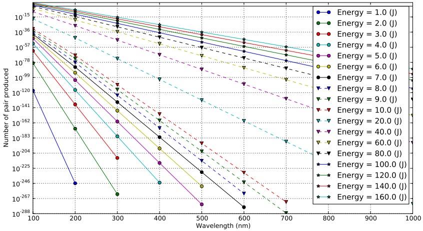
$$
\begin{aligned}
\mathbf{E}_{\rm ext}(\mathbf{r},t) &= \frac{1}{4\pi}\int_{\mathcal{S}}\left\{ik(\hat{\mathbf{n}}\times\mathbf{B}_{\mathcal{S}})G + (\hat{\mathbf{n}}\times\mathbf{E}_{\mathcal{S}})\times\nabla_{\mathcal{S}}G + (\hat{\mathbf{n}}\cdot\mathbf{E}_{\mathcal{S}})\nabla_{\mathcal{S}}G\right\}d\mathcal{S} \\
&\quad + \frac{1}{4\pi ik}\oint_{\partial\mathcal{S}}(\nabla_{\mathcal{S}}G)\mathbf{B}_{\mathcal{S}}\cdot d\boldsymbol{\ell}, \\
\mathbf{B}_{\rm ext}(\mathbf{r},t) &= \frac{1}{4\pi}\int_{\mathcal{S}}\left\{ik(\mathbf{E}_{\mathcal{S}}\times\hat{\mathbf{n}})G + (\hat{\mathbf{n}}\times\mathbf{B}_{\mathcal{S}})\times\nabla_{\mathcal{S}}G + (\hat{\mathbf{n}}\cdot\mathbf{B}_{\mathcal{S}})\nabla_{\mathcal{S}}G\right\}d\mathcal{S} \\
&\quad - \frac{1}{4\pi ik}\oint_{\partial\mathcal{S}}(\nabla_{\mathcal{S}}G)\mathbf{E}_{\mathcal{S}}\cdot d\boldsymbol{\ell},
\end{aligned}
$$

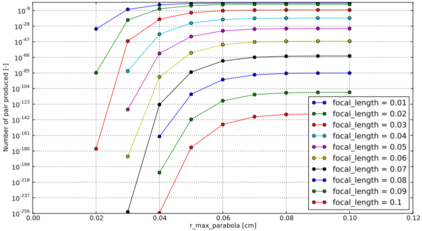


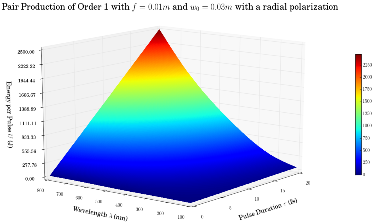
Preliminary results (by C. Murphy)

Experimental parameters:

- Focal length
- Frequency
- Aperture size
- Beam width
- Laser pulse energy and duration







Conclusion

Conclusion

- Schwinger pair production in a multi-center system
  - Position of resonances

    F. Fillion-Gourdeau *et al*, 2012 J. Phys. A: Math. Theor. 45 215304
  - Two mechanisms that enhance pair production rate:
    1. At large R: REPP
    2. At small R: ECEPP

    F. Fillion-Gourdeau *et al*, Phys. Rev. Lett. 110, 013002 (2013)

    F. Fillion-Gourdeau *et al*, 2013 J. Phys. B: At. Mol. Opt. Phys. 46 175002
- Numerical methods for the Dirac equation
  - Split-operator method
  - Efficient implementation on a quantum computer

    F. Fillion-Gourdeau *et al*, Computer Physics Communications, Volume 183, Issue 7, July 2012, 1403-1415

    F. Fillion-Gourdeau *et al*, J. Comp. Phys., Volume 272, September 2014, Pages 559-587

    F. Fillion-Gourdeau *et al*, Phys. Rev. A 85, 022506 (2012)
- Schwinger pair production in a realistic scenario
- In the future...
  - Use numerical methods to study more realistic systems (3D, time-dependent)
  - Study other QED processes