# Conceptual problems in QED

## Part I: Polarization density of the vacuum

## (photon = classical field)

## Part II: Bare, physical particles, renormalization

## (photon = independent particle )

Rainer Grobe
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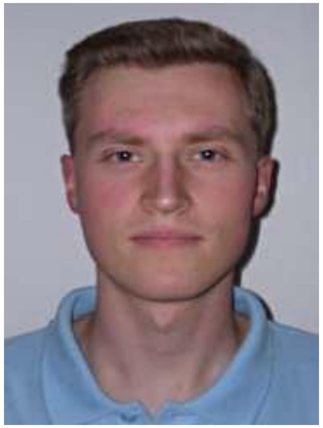
# Coworkers


Prof. Q. Charles Su


Dr. Sven Ahrens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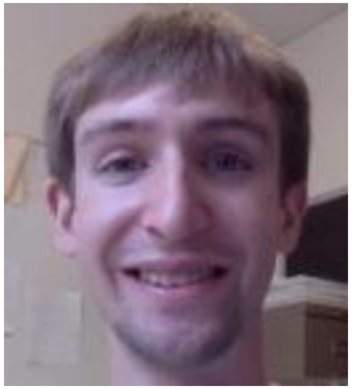
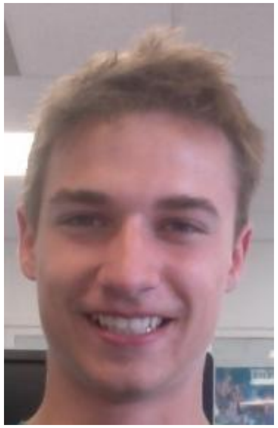
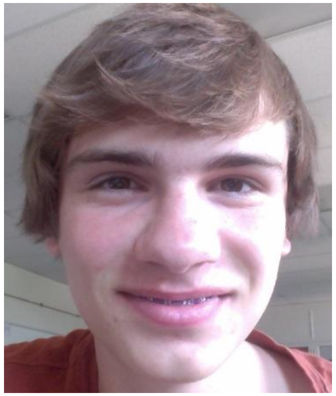

Andrew Steinacher

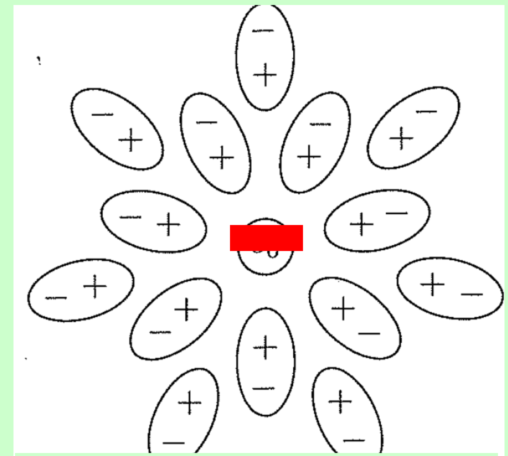

Jarrett Betke


Will Bauer

# Questions

Can we visualize the dynamics of QED interactions with space-time resolution?

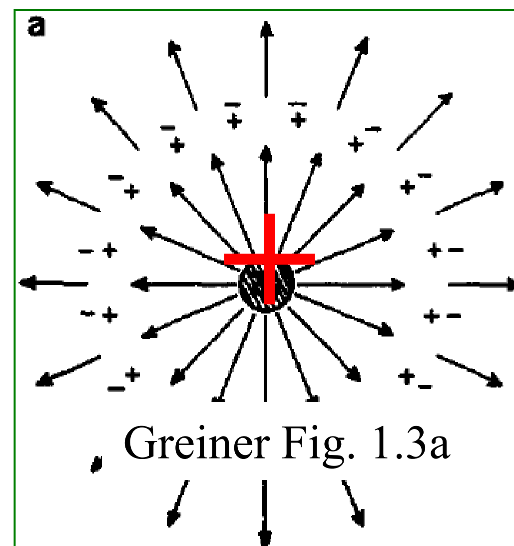
Is this picture correct?



Peskin Schroeder Fig. 7.8

Relationship between virtual and real particles?

Dynamics of virtual particles?



Greiner Fig. 1.3a

## Quantum mechanics:

**(1) know:** $\phi(x,t=0)$ and $h(t)$

**(2) solve:** $i\,\partial_t\,\phi(x,t) = h(t)\ \phi(x,t)$ for the initial state ONLY

**(3) compute:** observables $\langle\phi(t)|\ O\ |\phi(t)\rangle$

## Quantum field theory:

**(1) know:** $|\Phi(t=0)\rangle$ and $H(t)$

**(2) solve:** $i\,\partial_t\,\phi_E(x,t) = H(t)\ \phi_E(x,t)$ for **EACH** state $\phi_E(x)$ of **ENTIRE** Hilbert space

**(3) compute:** observables $\langle\Phi(t=0)|\ O(\text{ all } \phi_E(x,t)\ )\ |\Phi(t=0)\rangle$

# Charge density $\rho(z,t)$

$$\rho(z, t) = \langle \Phi(t=0) | - [\Psi^\dagger(z,t), \Psi(z,t)]/2 \; | \Phi(t=0) \rangle$$

charge operator

**example**: single electron $\quad | \Phi(t=0) \rangle = b_P^\dagger | vac \rangle$

$$\rho(z, t) = - \underbrace{|\phi_P(+;z,t)|^2}_{\text{electron's wave function (trivial)}} + \underbrace{\left(\Sigma_{E(+)} |\phi_E(+;z,t)|^2 - \Sigma_{E(-)} |\phi_E(-;z,t)|^2\right) /2}_{\text{vacuum's polarization density (not understood)}}$$

electron's wave function

(trivial)

vacuum's polarization density

(not understood)

$t=0: \quad h_0 \, \phi_E = E \, \phi_E$

$t>0: \quad i \, \partial_t \, \phi_E(t) = h \; \phi_E(t)$

# Quick overview

(1) Computational approach

- Steady state vacuum polarization $\rho(z)$
- Space-time evolution of $\rho(z,t)$
- Steady state and time averaged dynamics

(2) Analytical approaches

- Phenomenological model
- Decoupled Hamiltonians
- Perturbation theory

(3) Applications

- Coupling $\rho$ to Maxwell equation
- Relevance for pair-creation process
- Relationship to traditional work

## 2. Example: **$\rho(z)$ for the dressed vacuum state $|\,\mathrm{VAC}\,\rangle$**

$$\rho_{pol}(z) = \langle\, \mathbf{VAC} \,|\, -[\Psi^\dagger(z), \Psi(z)]/2 \;|\, \mathbf{VAC}\,\rangle$$

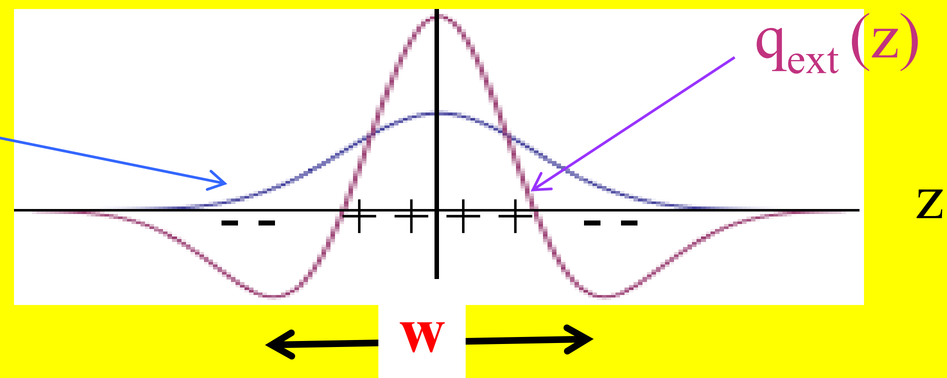
Dirac equation

$$[c\,\sigma_1\, p_z + mc^2\,\sigma_3 + \mathbf{V_{ext}\,(z)}]\,\Phi_E(z) = E\,\Phi_E(z)$$

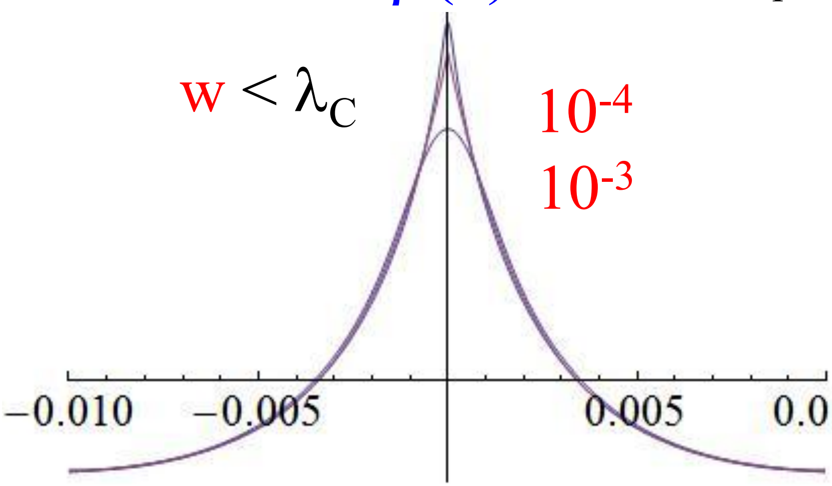
$$\rho_{pol}(z) = \left(\Sigma_{E(+)}\,|\Phi_E(+;z)|^2 - \Sigma_{E(-)}\,|\Phi_E(-;z)|^2\right)/2$$

# Width w of external potential $V_{ext}(z)$ determines $\rho(z)$

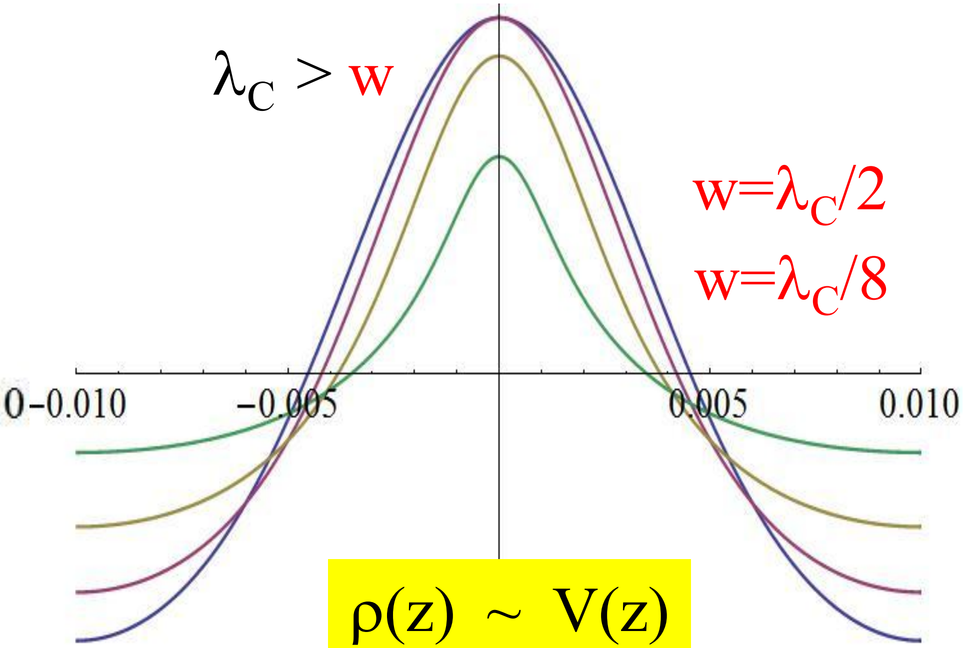
$V_{ext}(z) = V_0 \exp[-(z/w)^2]$




$\lambda_{Compton} = 1/c = 7\ 10^{-3}$




$\rho(z) \sim w \exp[-c|z|]$




$\rho(z) \sim V(z)$

# 3. Example: **Dynamics of the polarization density**

$$\rho_{pol}(z,t) = \langle \text{ bare vac } | -[\Psi^\dagger(z,t), \Psi(z,t)]/2 \; | \text{ bare vac} \rangle$$

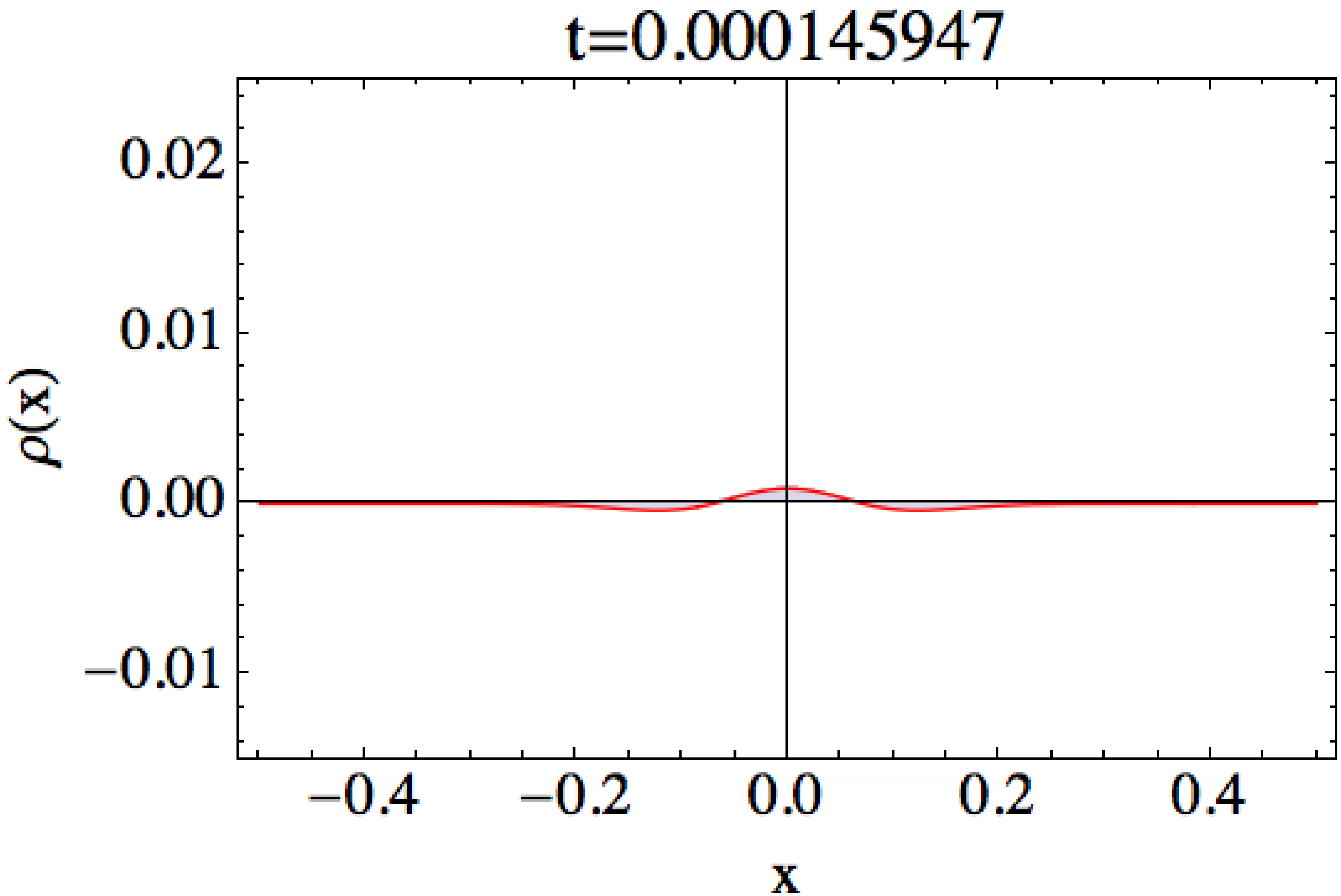
$$\rho_{pol}(z,t) = \left(\Sigma_{E(+)} |\phi_E(+;z,t)|^2 - \Sigma_{E(-)} |\phi_E(-;z,t)|^2\right) /2$$


time-dependent Dirac equation:

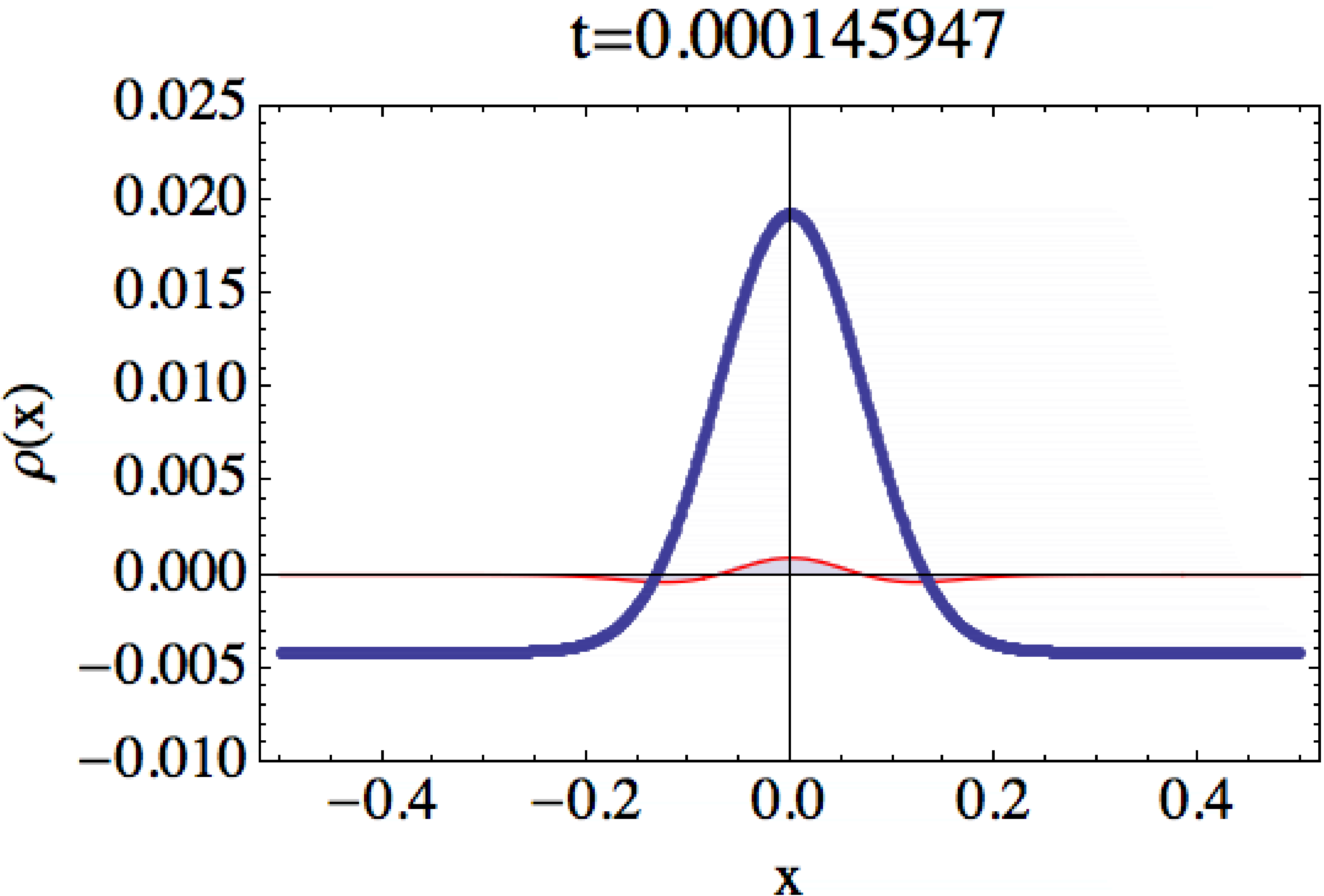
$$i\,\hbar\,\partial_t\,\phi_E(z,t) = [c\,\sigma_3\,p_z + mc^2\,\sigma_3 + V(z)]\,\phi_E(z,t)$$

# Temporal evolution of ρ(x,t)

$V_{ext}(x)$ ⟮ $\rho_{pol}(x,t)$



$$\rho_{steady}(x) = T^{-1} \int^{T} dt \; \rho(x,t)$$



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- Coupling ρ to Maxwell equation
- Relevance for pair-creation process
- Relationship to traditional work

Maxwell                    Dirac

$q_{ext}(z)$ ⟍ $V_{ext}(z)$ ⟍ $\rho_{pol}(z)$

# Phenomenological model for $\rho_{pol}(z,t)$

$$(\partial_{ct}{}^2 - \partial_z{}^2)\, \rho_{pol}(z,t) = 8\pi\, \chi\, q_{ext}(z) \qquad \chi = \alpha^3/(2\pi\, \lambda_C{}^2) \text{ (guess)}$$

## exact solution:

$$\rho_{pol}(z,t) = \chi\, [\, 2V_{ext}(z) - V_{ext}(z\text{-}ct) - V_{ext}(z+ct)\, ]$$

$$j_{pol}(z,t) = \chi\, c[\, V_{ext}(z+ct) - V_{ext}(z-ct)\, ]$$

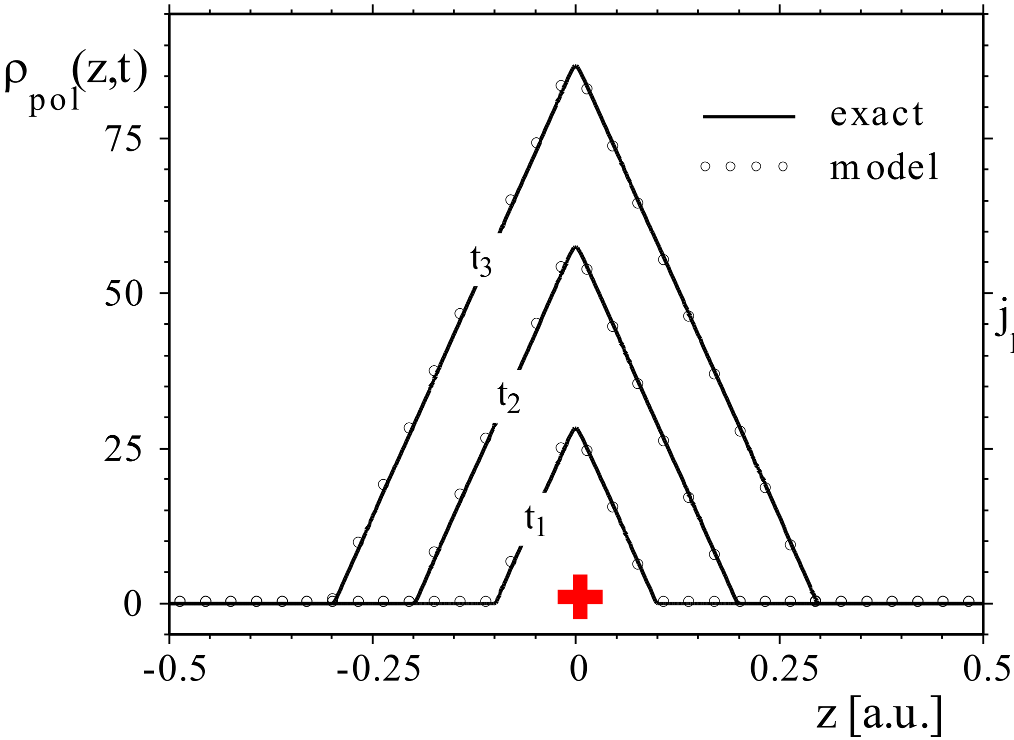
if width of $V_{ext} > \lambda_C$ => predictions for $\rho_{pol}(z, t)$ are highly accurate

# $\rho_{pol}(z,t)$ and $j_{pol}(z,t)$ for an external point charge ✚

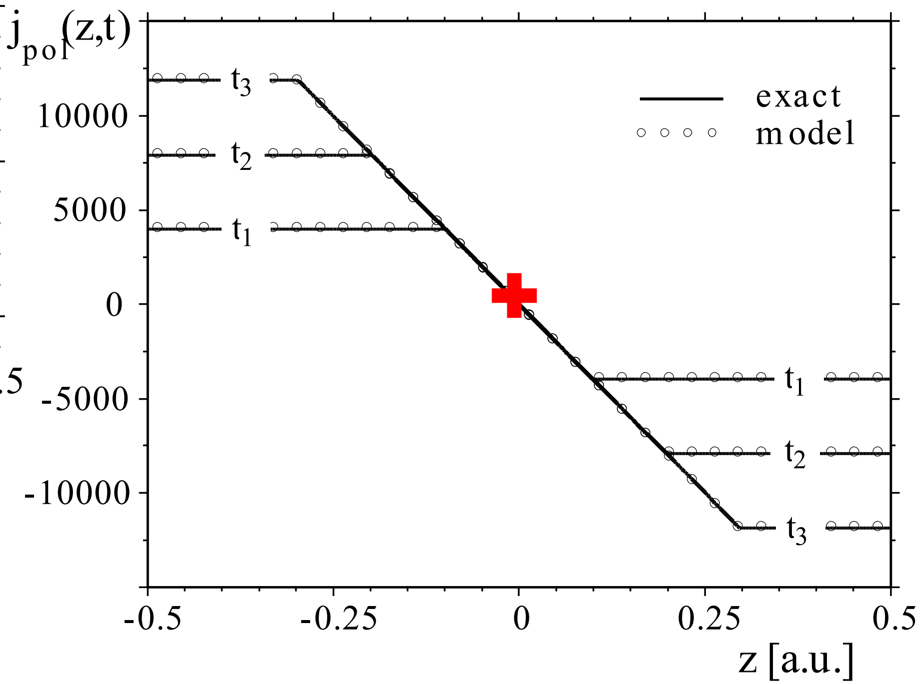
$$q_{ext}(z) = q\ \delta(z)$$

$$V_{ext}(z) = -2\pi\ q\ |z|$$

$$\rho(z,t) = \chi\ [2V(z) - V(z-ct) - V(z+ct)]$$




$$j(z,t) = \chi\ c[V(z+ct) - V(z-ct)]$$




- charge conservation ✓
- $\lim_{L,\, t\to\infty}\ \rho(z=0,t) \to \infty$ ✓
- j(z) grows everywhere ✓

# Decoupled Hamiltonian model

$H(+) = [m^2c^4+c^2p^2]^{1/2} + V_{ext}(z)$ => bound states

$H(-) = [m^2c^4+c^2p^2]^{1/2} - V_{ext}(z)$ => scattering states

$$\rho_{pol}(z, t) = \left(\Sigma_{E(+)} |\phi_E(+;z,t)|^2 - \Sigma_{E(-)} |\phi_E(-;z,t)|^2\right) /2$$

predictions for $\rho_{pol}(z, t)$ are highly accurate

=> transitions between positive and negative Dirac states irrelevant

# Traditional perturbation theory

$$|\Psi_E^{(1)}\rangle = |\phi_E\rangle + \Sigma_{E'} \langle\phi_{E\square}|V_{ext}|\phi_{E\square}\rangle \,/\, (E-E')\, |\phi_{E\square}\rangle + \dots$$

$$|\Psi_E^{(1)}\rangle = |\phi_E\rangle - \Sigma_{E'} \langle\phi_{E\square}|V_{ext}|\phi_{E\square}\rangle \,/\, (E-E')\, |\phi_{E\square}\rangle + \dots$$

$$\rho_{pol}(z, t) = \left(\Sigma_E |\Psi_E^{(1)}(z)|^2 - \Sigma_E |\Psi_E^{(1)}(z)|^2\right) /2$$

predictions for $\rho_{pol}(z, t)$ are highly accurate

=> perturbative approach applicable also to 2 and 3D ?

# Intermediate summary

$$(\partial_{ct}^2 - \partial_z^2)\, \rho_{pol}(z,t) = 8\pi\, \chi\, q_{ext}(z) \quad \text{with} \quad \chi = \alpha^3/(2\pi\, \lambda_C^2)$$

4 independent approaches:
(steady, dynamics, phenom, decoupled hamiltonian)
predict:

**massless virtual positive particles**

**accumulate around positive charges**

=> Is the energy conserved?

=> What if real particles are created in addition?

=> Consistent with traditional QED methods?

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# Coupled Dirac-Maxwell equation

$$\rho_{pol}(z, t) = \left(\Sigma_{E(+)} |\phi_E(+;z,t)|^2 - \Sigma_{E(-)} |\phi_E(-;z,t)|^2\right) /2$$

Dirac equation:

$$i\, \partial_t\, \phi_E(z,t) = \left[c\, \sigma_1\, [p_z - A(z,t)/c] + mc^2\, \sigma_3 + V(z,t)\right] \phi_E(z,t)$$

Maxwell equation:

$$[\partial_{ct}{}^2 - \partial_z{}^2]\, V(z,t) = 4\pi\, \rho(z,t)$$

$$[\partial_{ct}{}^2 - \partial_z{}^2]\, A(z,t) = 4\pi\, j(z,t)/c$$

# Energy conservation

$$E_{tot} = E_{mat}(t) + E_{int}(t) + E_{field}(t)$$

$$E_{mat}(t) = \int dz \, \langle \, \Psi^\dagger(z,t) \, \{c \, \sigma_1 \, p + \sigma_3 \, mc^2\} \, \Psi(z,t) \, \rangle$$

$$E_{int}(t) = q \int dz \, \langle \, \Psi^\dagger(z,t) \, \{V(z,t) - \sigma_1 \, A(z,t)\} \, \Psi(z,t) \, \rangle$$

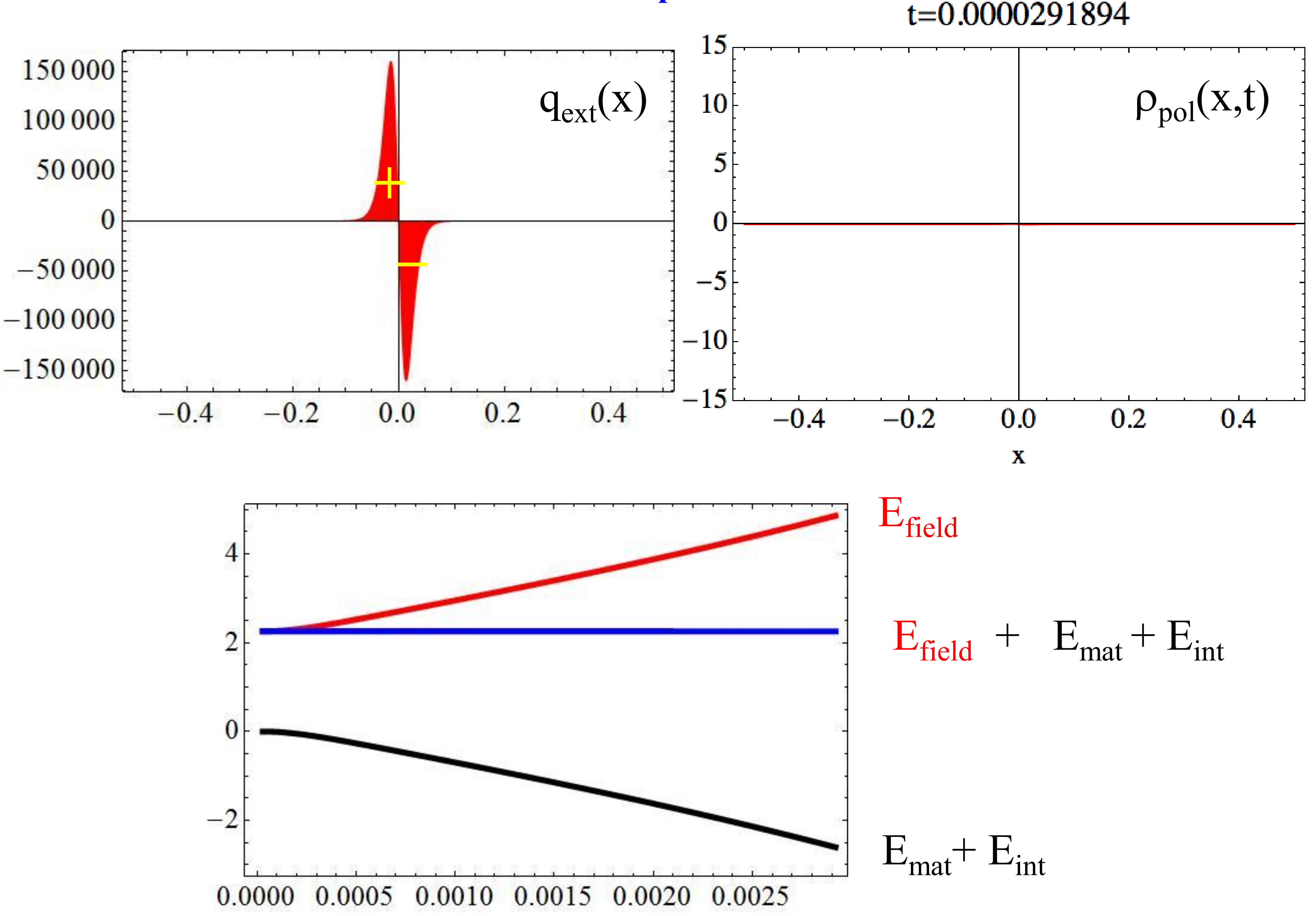
$$E_{field}(t) = (8\pi)^{-1} \int dz \, \{ \, [\partial_{ct} A(z,t)]^2 - [\partial_z V(z,t)]^2 \, \}$$

Temporal gauge:

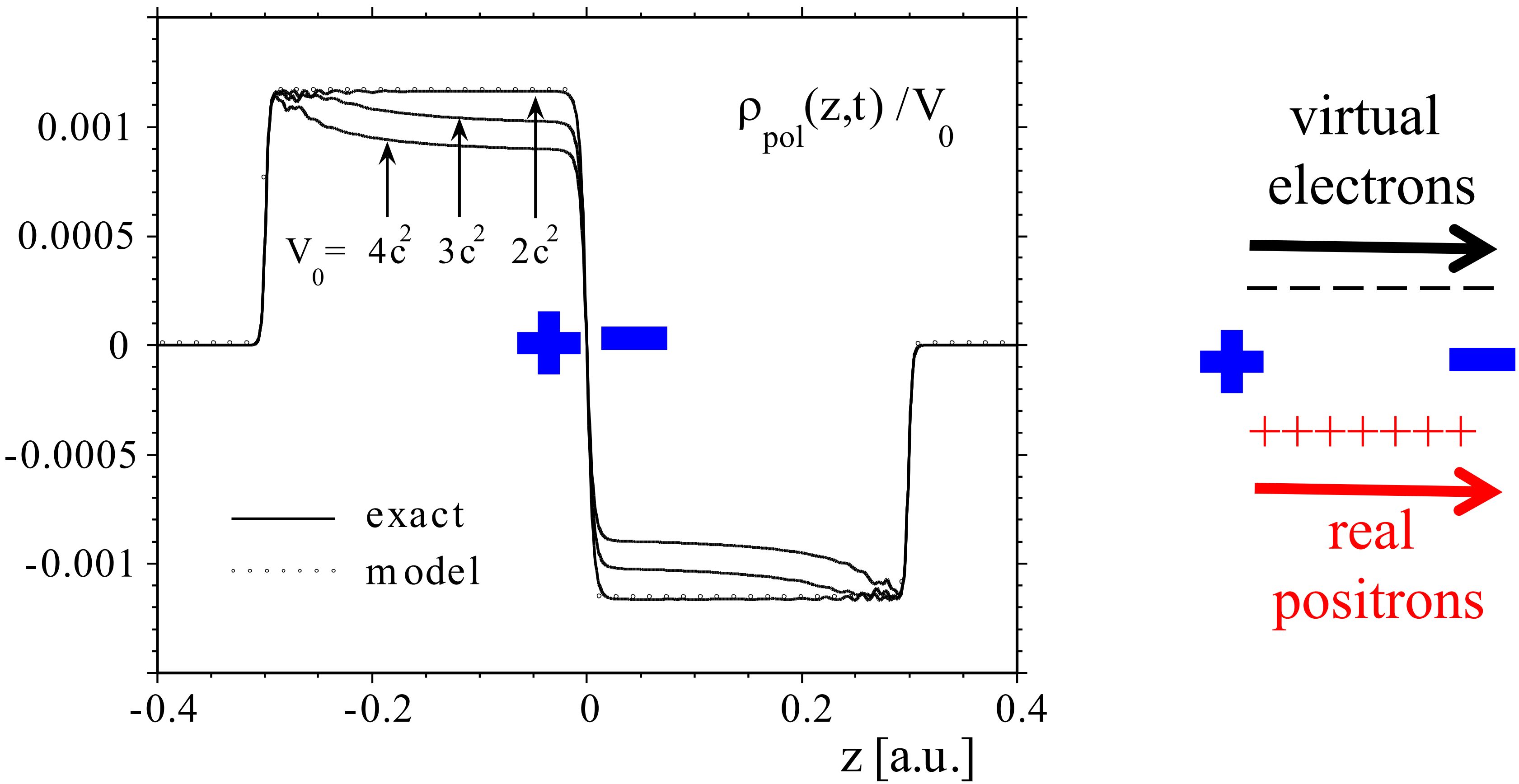
$$E_{int}(t) = -q \int dz \, \langle \, \Psi^\dagger(z,t) \, \sigma_1 \, A(z,t) \, \Psi(z,t) \, \rangle$$

$$E_{field}(t) = (8\pi)^{-1} \int dz \, E^2(z,t)$$

# Energy is conserved despite $q_{pol} \to \infty$



# Pair creation regime: $\rho_{pol} = \rho_{vac} + \rho_{e\text{-}e+}$




real particles **reduce** the total charge density
induced and real particles obey **opposite "force laws"**

# More quantitatively: mass density of real particles

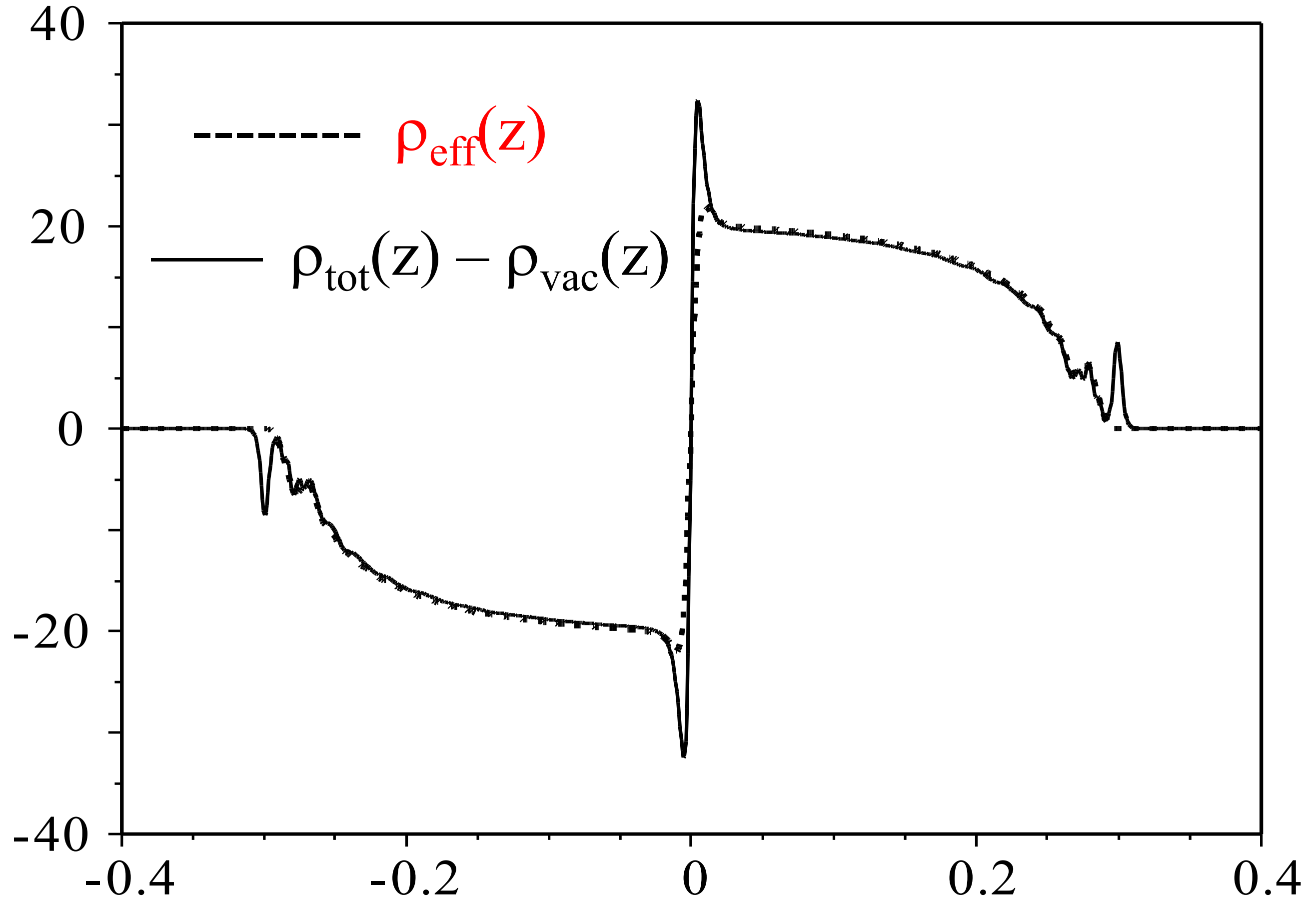
$$\Psi(z,t) \equiv \Psi(e^-) + C\,\Psi(e^+)$$

$$m(e^-; z, t) \equiv \langle \text{vac} \,|\, \Psi^\dagger(e^-)\Psi(e^-) \,|\, \text{vac} \rangle$$

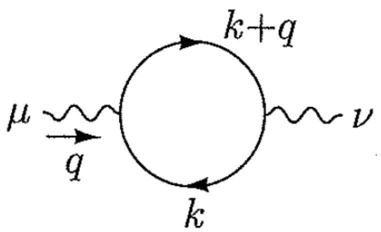
$$m(e^+; z, t) \equiv \langle \text{vac} \,|\, \Psi^\dagger(e^+)\Psi(e^+) \,|\, \text{vac} \rangle$$

effective charge density

$$\rho_{eff}(z) \equiv m(e^+; z) - m(e^-; z)$$



# Traditional pert. QED approach:




$$q_{ext}(\mathbf{r}) = e\,\delta(z)$$

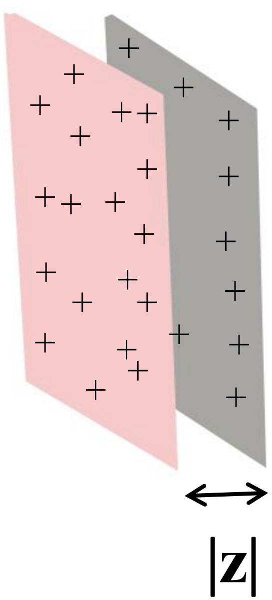
Peskin-Schroeder Eq. 7.93

$$\Pi_2 = 4\alpha\left[k^{-2} - 4c^4 k^{-3}(4c^4-k^2)^{-1/2}\arctan[k(4c^4-k^2)^{-1/2}]\right]$$

necessary: regularization (P-V or dim.) and charge renormalization

$$E_{int}(z) = (2\pi)^{-1}\int dk\,\exp(ikz)\,e^2 k^{-2}\,[1-\Pi_2]^{-1}$$

$$\cong \underbrace{-\,2\pi\, e^2\,|z|}_{\text{Coulomb}}\;\underbrace{-\,\alpha\, e^2\, 4\pi/3\,|z|^3}_{\text{vac. pol. correction}} + \ldots$$



# Traditional pert. QED approach:

$$E_{int}(z) \cong \underbrace{-\,2\pi\, e^2\ |z|}_{\text{Coulomb}} \ \underbrace{-\ \alpha\, e^2\, 4\pi/3\ \ |z|^3}_{\text{vac. pol. correction}} \ + \ldots$$

# Possible connection to our approach:

$$-\,\partial^2_z\ V_{ext}(z) \ = \ 4\pi\ q_{ext}(z) \qquad => \ V_{ext}(z) = -\,2\pi\ e\ |z| \quad ✓$$

$$-\,\partial^2_z\ \rho_{pol}(z) \ = \ 8\pi\ \chi\ q_{ext}(z) \qquad => \ \rho_{pol}(z) \ = \ -\,4\pi\ e\ \chi\ |z|$$

$$+\,\partial^2_z\ V_{pol}(z) \ = \ 4\pi\ \rho_{pol}(z) \qquad => \ V_{pol}(z) \ = \ -\,8\pi^2/3\ e\ \chi\ |z|^3$$

$$= \ -\,\alpha\ e\ 4\pi/3\ \ |z|^3 \quad ✓$$

regularization and charge renormalization NOT necessary

# (Too) many open questions....

- $q_{ext}(z) = \delta(z)$ => infinite plane: $\lim_{t\to\infty} \rho(z,t) \to \infty$
- 1D $\neq$ 3D with spatial symmetry (relativity)
- implications for 2D and 3D: $\Box \rho_{pol} = 8\pi\, \chi\, q_{ext}(r)$ ??
- more contact with traditional methods
- experimental implications
- .....


Q.Z. Lv, J. Betke, W. Bauer, Q. Su and R. Grobe, **Phys. Rev. Lett.** (in preparation)

A. Steinacher, J. Betke, S. Ahrens, Q. Su and R. Grobe, **Phys. Rev. A** 89, 062016 (2014).

A. Steinacher, R. Wagner, Q. Su and R. Grobe, **Phys. Rev. A** 89, 032119 (2014).

# Conceptual problems in QED

## Part I: Polarization density of the vacuum

## (photon = classical field)

## Part II: Bare, physical particles, renormalization

## (photon = independent particle )

# 5 drawbacks of the S matrix

(1) $T \rightarrow \infty$ is built in

(2) $d\sigma/d\omega$ is rate based

(3) usually only perturbative

(4) no spatial information

(5) black box approach

What happens inside the interaction zone?

The challenge:

study QED interactions with space-time resolution

- ⊙ construct Hamiltonian H
- ⊙ evolve $|\Psi(t=0)\rangle$ to $|\Psi(t)\rangle$

  by solving $i\hbar\, \partial/\partial t\, |\Psi(t)\rangle = H\, |\Psi(t)\rangle$

- ⊙ convert $|\Psi(t)\rangle$ into observables


The problems:

- ☹ Hilbert space is gigantic
- ☹ Hamiltonian is “wrong” and requires serious repair
- ☹ correct physical operators are unknown

# **Electron – positron – photon** interactions

$b_p{}^{□}$ $d_p{}^{□}$ $a_p{}^{□}$

$H_0 = \Sigma + dp\ E_p\, b_p{}^{□} b_p + \Sigma + dp\ E_p\, d_p{}^{□} d_p + \Sigma + dp\ \omega_p\, a_p{}^{□} a_p$

$H_{int} = \Sigma + + dp\ dk \ldots.$ 8 basic "processes"

Photon annihilation:


Photon creation:

# Three Hamiltonians of quantum field theory

$$H_{bare} = b^\dagger b + a^\dagger a + b^\dagger b\, a^\dagger + \ldots$$

- ⊙ wrong energies
- ⊙ bad operators $H\ b^\dagger|0\rangle \neq E\ b^\dagger|0\rangle$

$$H_{renorm} = b^\dagger b + a^\dagger a + \infty\, b^\dagger b\, a^\dagger + \ldots$$

- ⊙ correct energies ✓
- ⊙ bad operators

$$H_{dressed} = B^\dagger B + A^\dagger A + B^\dagger B^\dagger BB + \ldots ?? \ldots$$

- ⊙ correct energies ✓
- ⊙ good operators ✓

# Overview

Repair work I: find $H_{renom}$

☺ compute the physical mass (numerical renormalization)

Dynamics in terms of bare particles

① vacuum

② single particle

③ two-particles (e-$\gamma$ and e-e scattering)

Repair work II: find $H_{dressed}$

☹ construct physical operators

# Bare mass m ≠ physical mass M

$H_0 \; = \; +dp \; e_p \; b_p{}^{\Box} b_p \quad + \quad +dk \; \omega_k \; a_k{}^{\Box} a_k$

$H_{int} \; = \; ++dp \; dk \; g(p,k) \; b_{p+k}{}^{\Box} b_p \; (a_k + a_{-k}{}^{\Box}) + \ldots$

where $e_p = [m^2c^4 + c^2p^2]^{1/2}$

m = bare mass

---

Measurement: physical mass of an electron is 1kg (= M)

---

The problem: eigenvalue of H is # and ***not*** m and ***not*** 1kg

$(H_0 + H_{int}) \, |P\rangle \; = \; E_P \; |P\rangle$

$E_P = [\#^2c^4 + c^2P^2]^{1/2}$

---

The goal: choose m such that # ***is*** the physical mass M

## **Assume** bare mass m and **compute** physical mass M

$H = +dp\ e_p b_p{}^{□} b_p\ +\ +dk\ \omega_k a_k{}^{□} a_k\ +\ ++dp\ dk\ g(p,k)\ b_{p+k}{}^{□} b_p\ (a_k + a_{-k}{}^{□})\ +$ ...

(1) use $e_p = \sqrt{(m^2c^4+c^2p^2)}$ with *trial value: m =1 kg*

(2) diagonalize H to determine eigenvalue $E_P = \sqrt{(M^2c^4+c^2p^2)}$

leading to mass M = 0.7 kg

repeat (1) and (2) with different trial bare mass m

until we obtain desired mass M=1 kg

Complication:

eigenvalue $E_P$ depends on maximum momentum $\Lambda$

if $\Lambda$ --> □ then M --> – □ (but we want M=1kg)

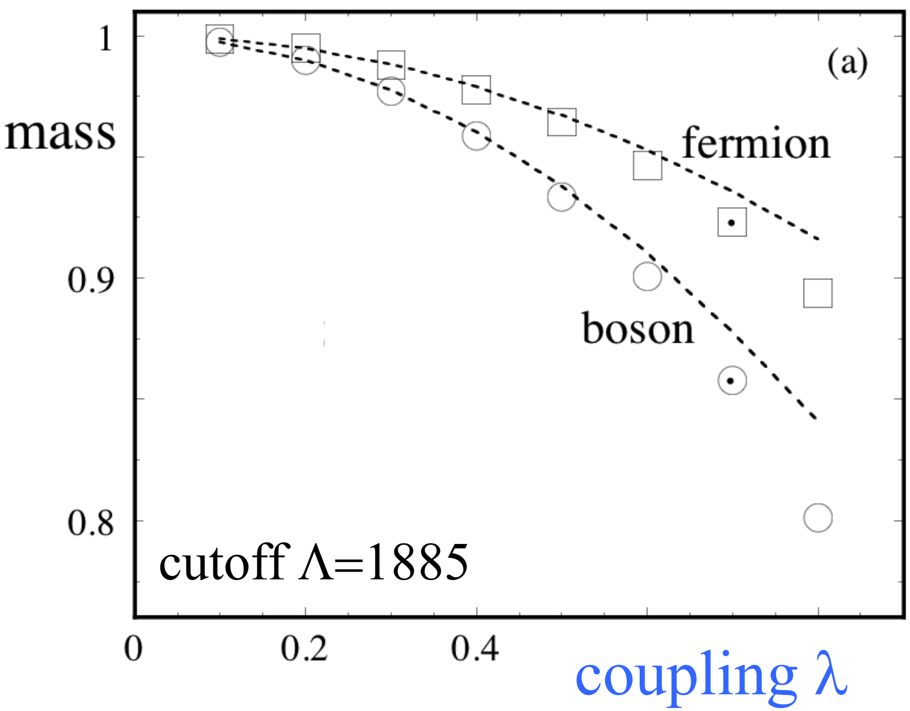
Solution:

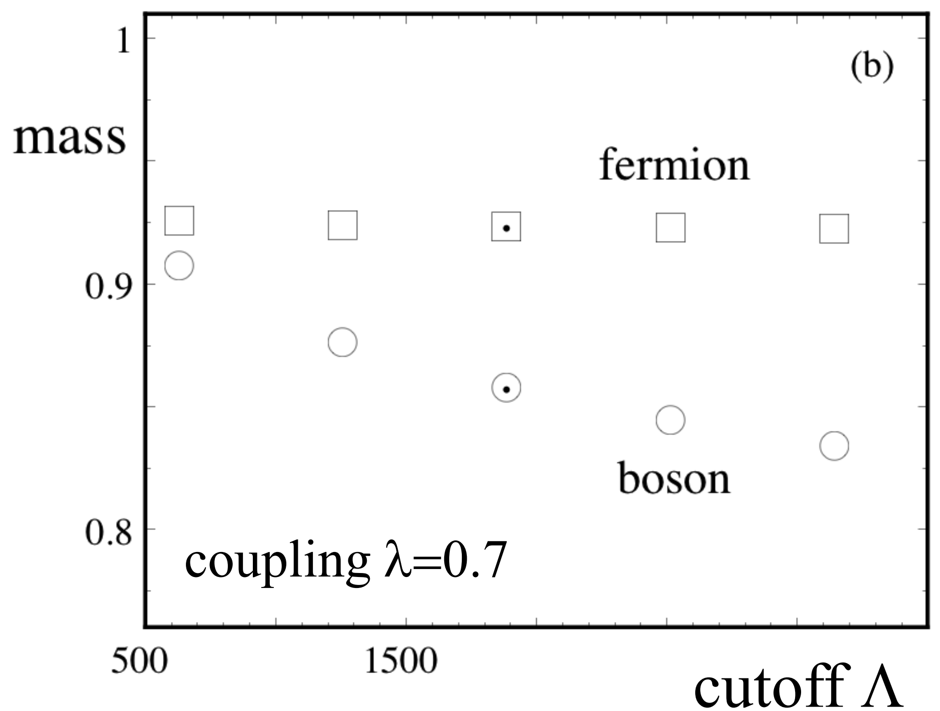
if $\Lambda$ --> □ then m --> □ (to keep M=1kg)

QED with photon mass≠0 & spin=0 => scalar Yukawa

$$H_{bare} = H_0 + \lambda + dx \;\; \psi(x)^{\square}\gamma^0\, \psi(x)\, \phi(x)$$

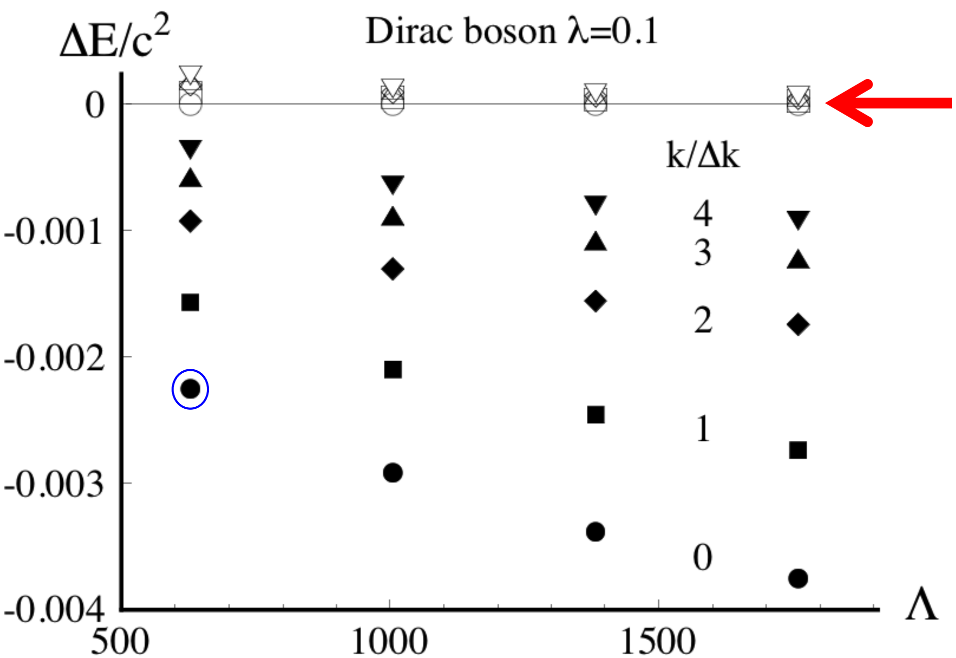
# Lowest energy eigenvalue

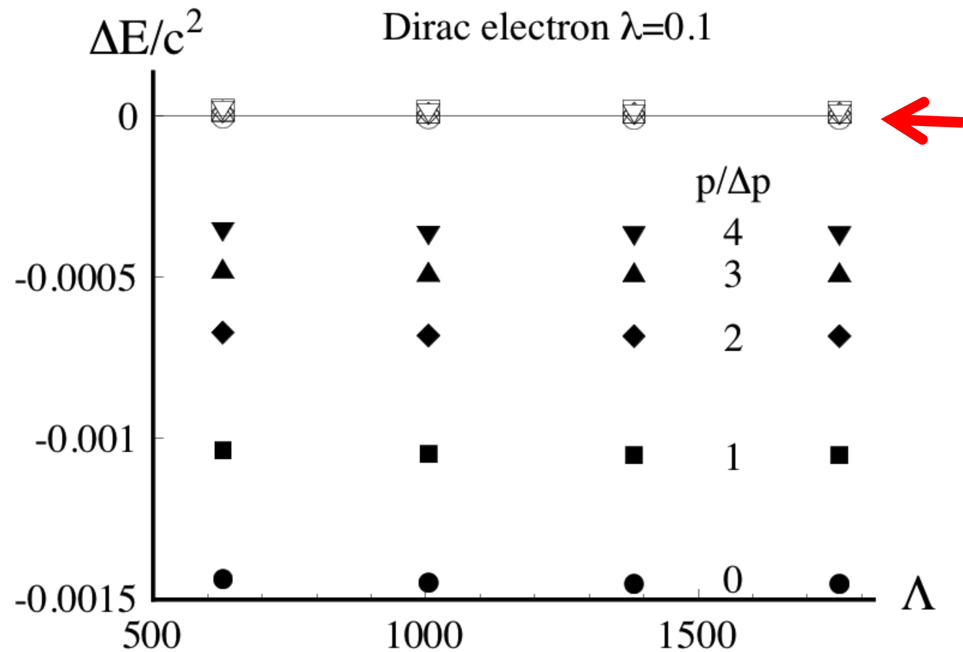


# Renormalization of one-particle energies

$$\Delta E(\Lambda) := E_{num}(m_e, m_\gamma, \Lambda) - \sqrt{(M_{phys}{}^2 c^4 + c^2 p^2)}$$



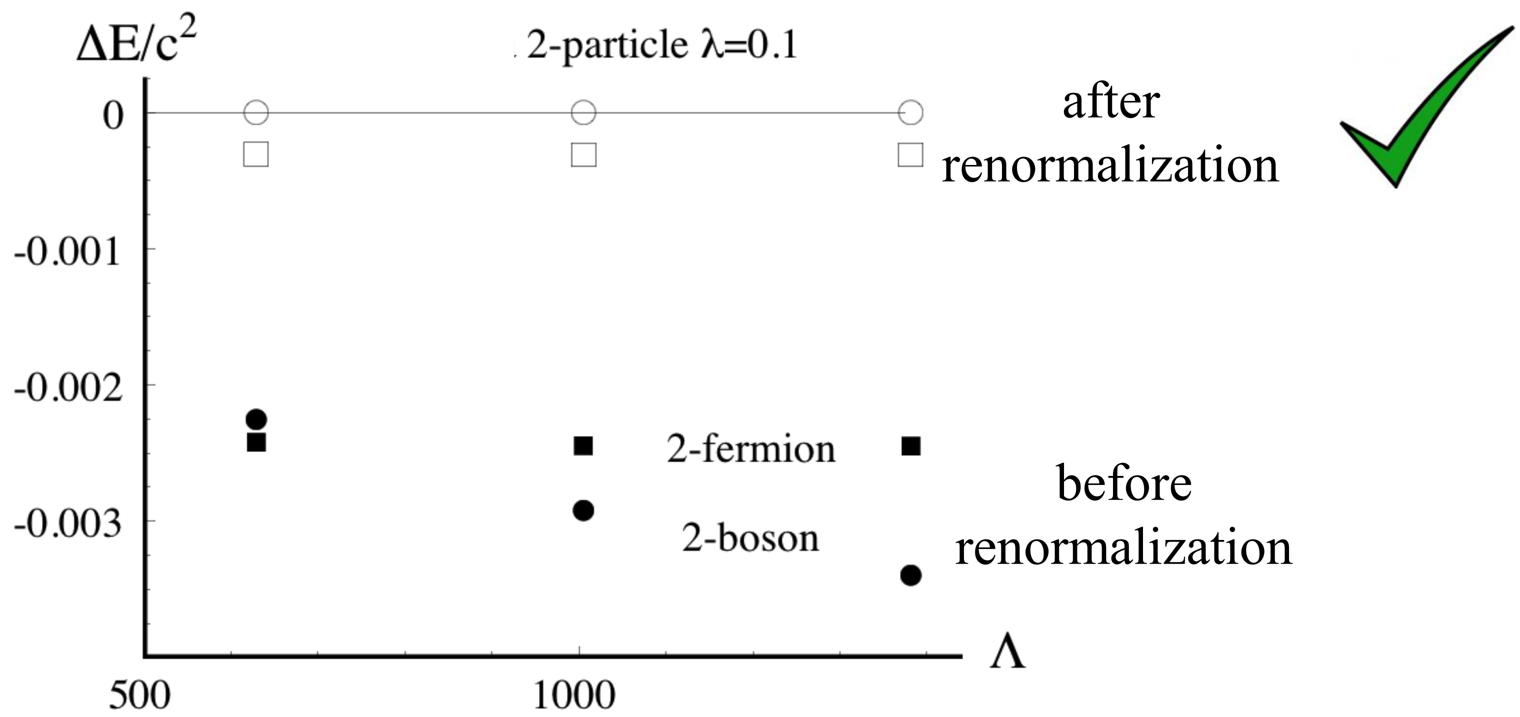




☺ find $(m_e, m_\gamma)$ for p=0 state to get $M_{phys}c^2$

☺ $(m_e, m_\gamma)$ works for all other p to get $\sqrt{(M_{phys}{}^2 c^4 + c^2 p^2)}$

# Renormalization of two-particle masses

$$\Delta E(\Lambda) = E_{num}(m_e, m_\gamma, \Lambda) - 2M_{phys}\, c^2$$




- ☺ $(m_e, m_\gamma)$ can repair entire spectrum
- ☺ 2-fermion bound state energy can now be analyzed

Repair work I: find $H_{renom}$

☺ non-perturbative exact numerical renormalization

Dynamics in terms of bare particles

① vacuum

② single particle

③ two-particles (e-γ and e-e scattering)

Repair work II: find $H_{dressed}$

☹ construct physical operators

# The vacuum contains "**virtual**" particles

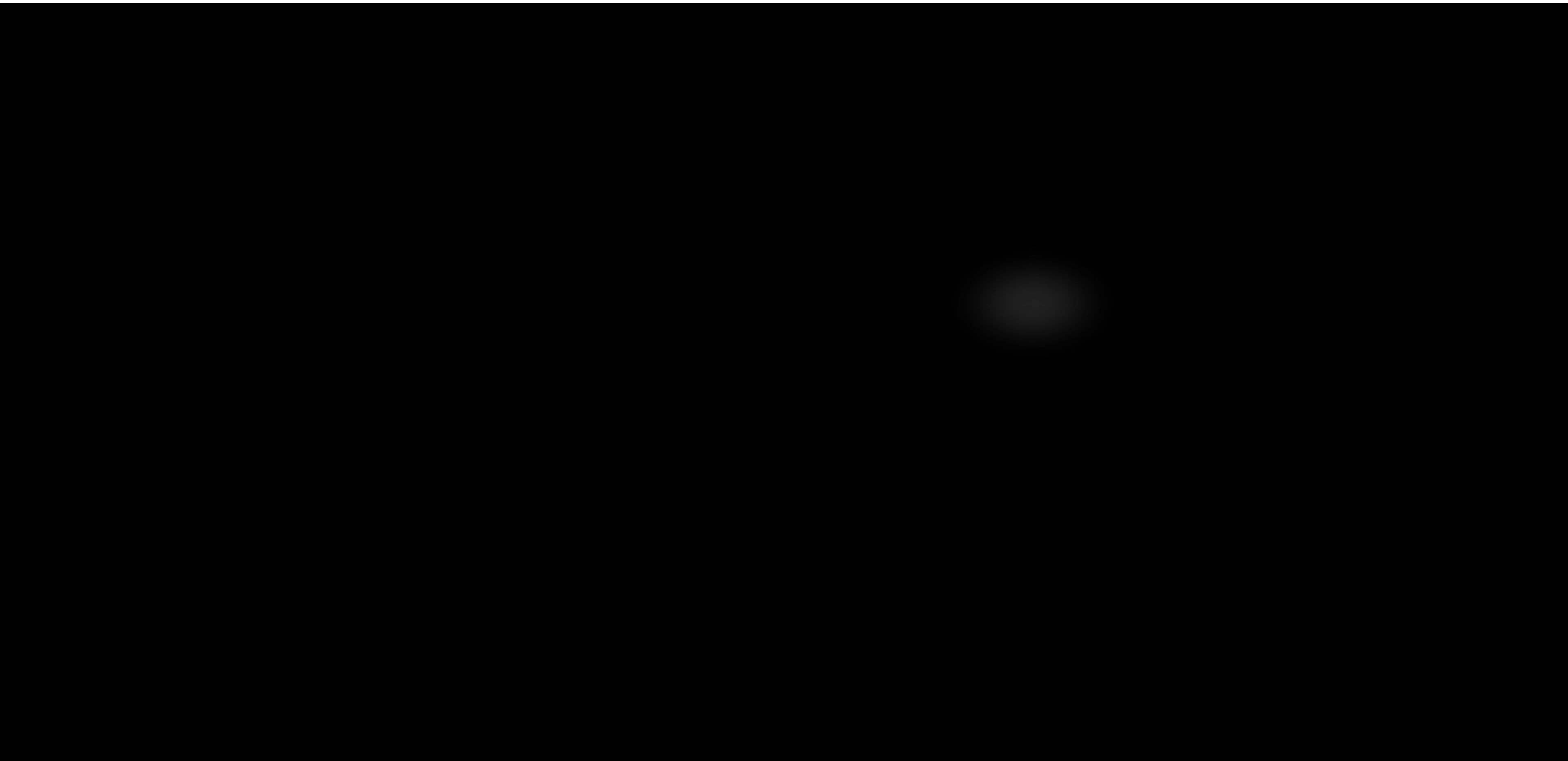
$(H_0+\mathbf{V})\,|VAC\rangle = E_{VAC}\;|VAC\rangle$

with LOWEST energy

$H_0\,|0\rangle = 0\;\;|0\rangle$

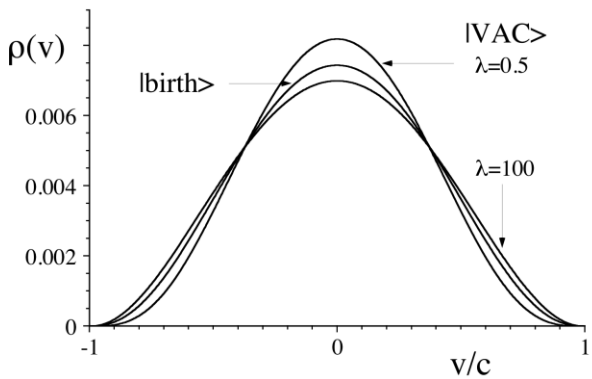
no particles, no interaction

state **with** particles $|VAC\rangle$ can have less energy than $|0\rangle$



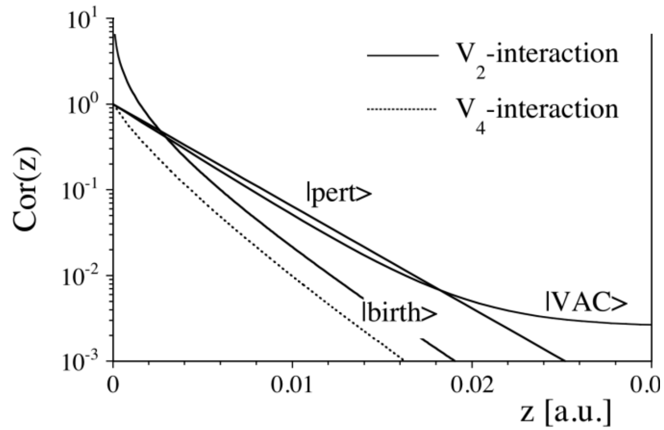
# I    Properties of virtual particles in |VAC⟩

$\langle \hat{a}^\dagger(p)\hat{a}(p)\rangle$



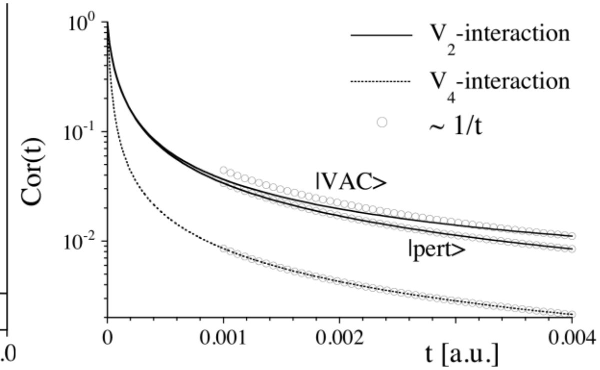

velocity:
mainly at rest

$\langle \hat{a}^\dagger(0)\hat{a}^\dagger(0)\hat{a}(z)\hat{a}(z)\rangle$




position:
on top of each other

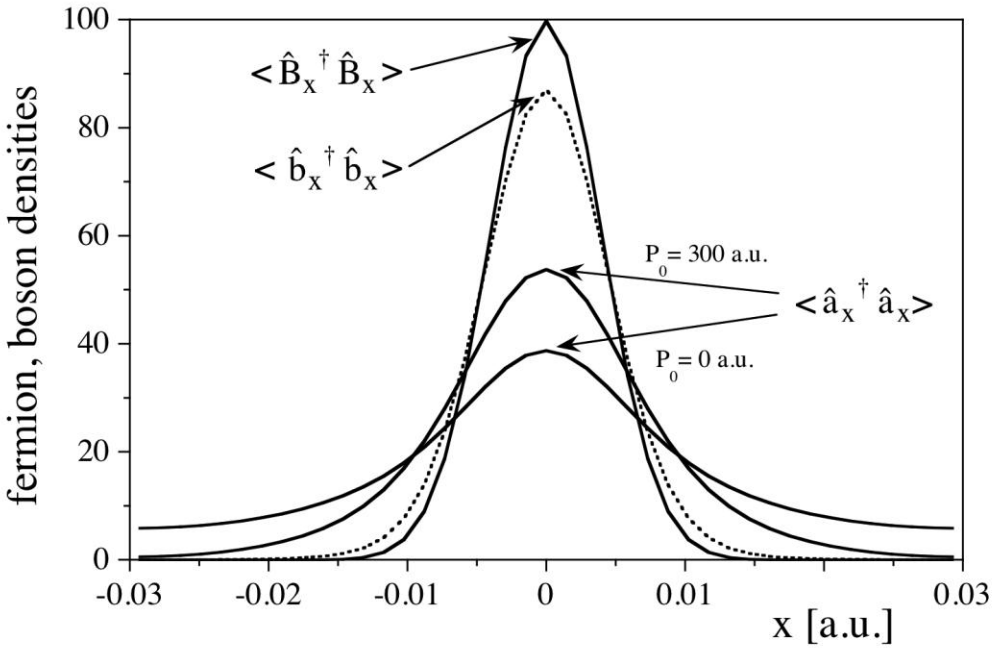
$\langle \hat{a}^\dagger(t{=}0)\hat{a}^\dagger(t{=}0)\hat{a}(t)\hat{a}(t)\rangle$




life time:
non-exponential

model in terms of an ensemble of classical particles?

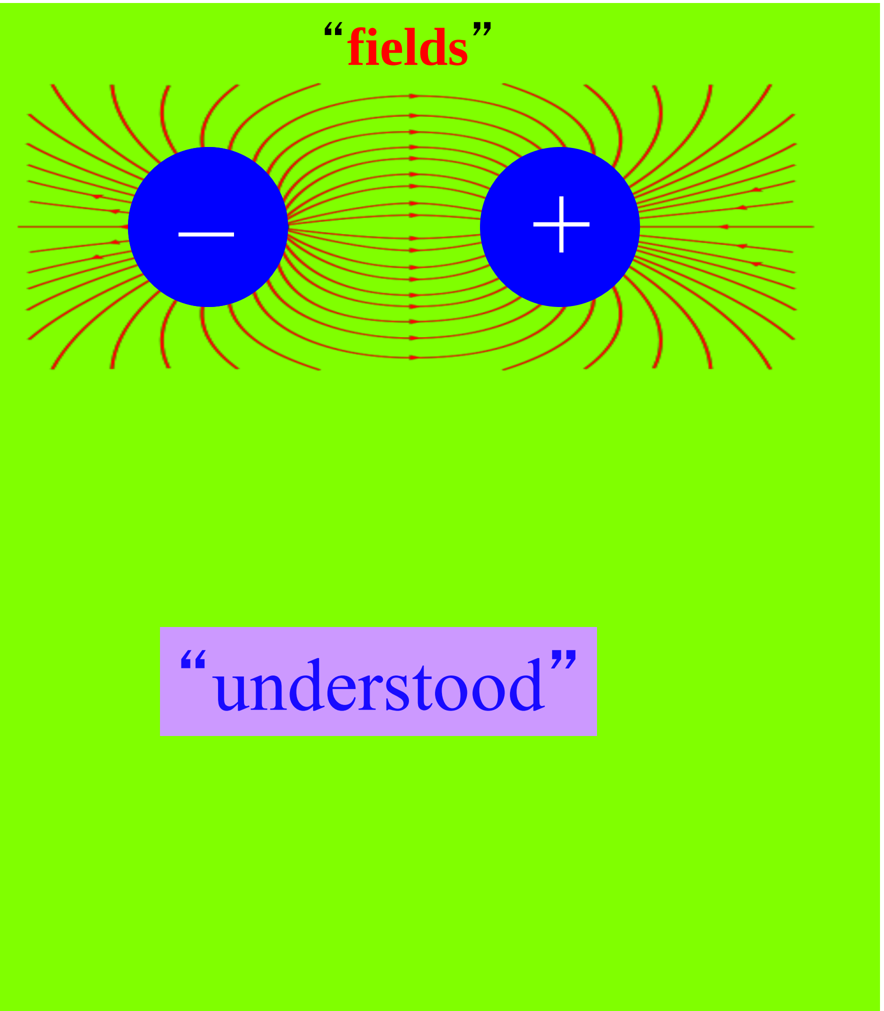
# II Properties of virtual particles in single particle state

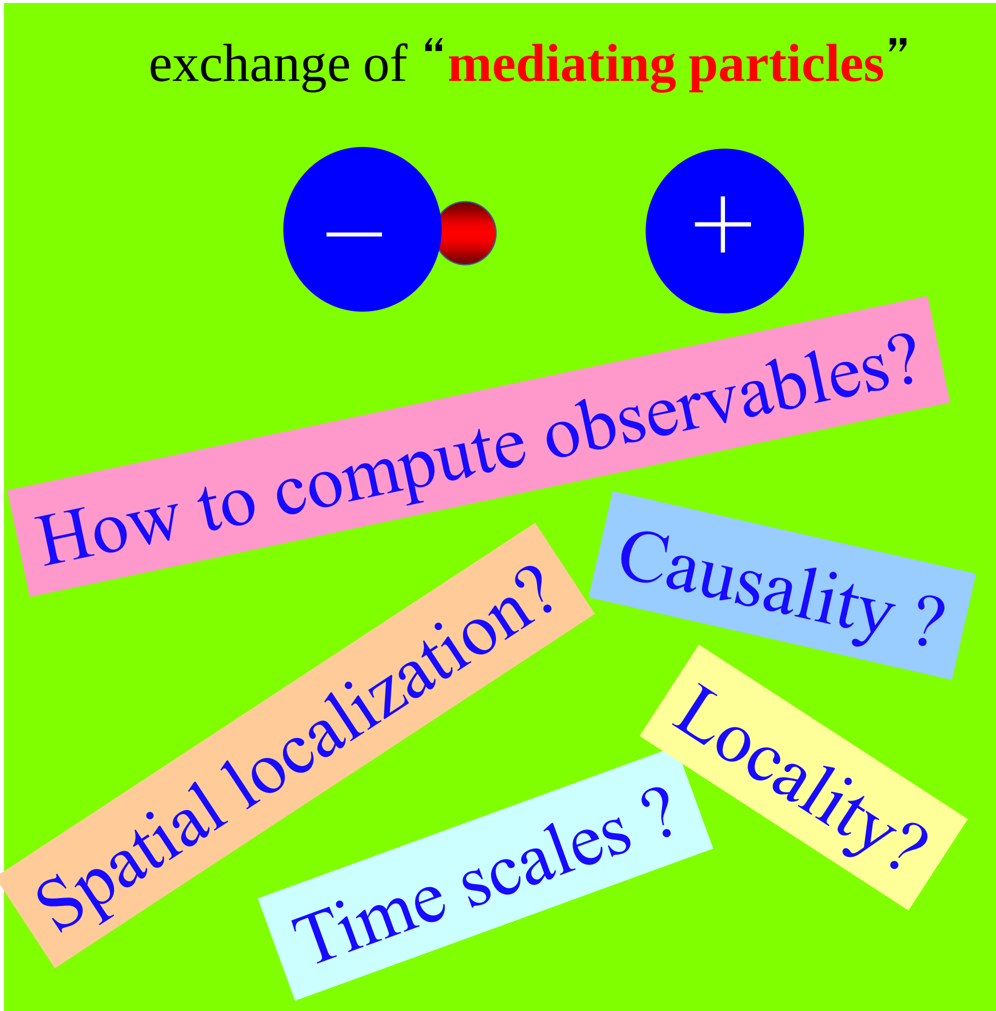



- ☺ impact of mass renormalization on dynamics
- ☺ bare photons = electric field around charge
- ☺ electric field depends on velocity

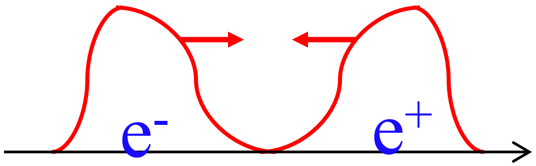
# Interactions between particles: “forces”

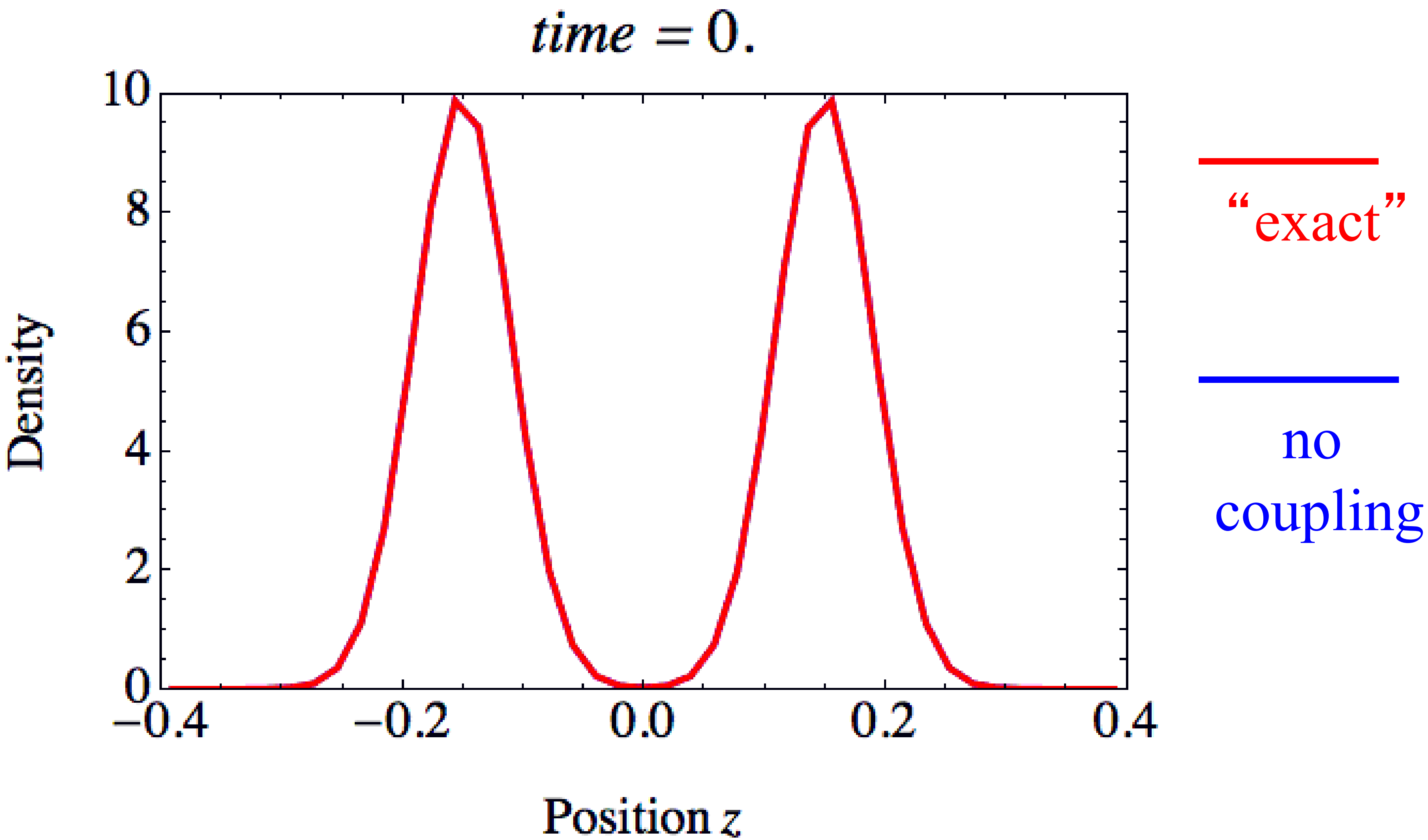
two charges attract through ...



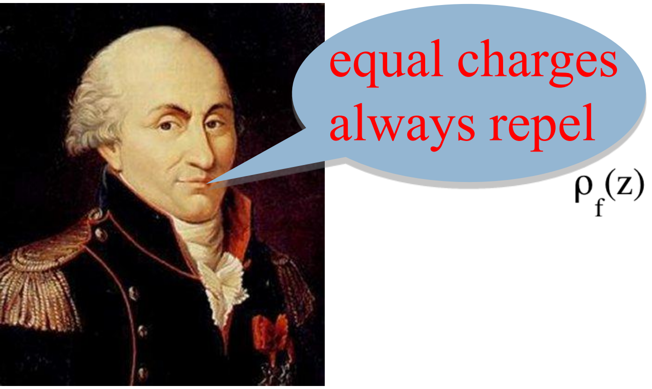



# III Impact of virtual particles on forces

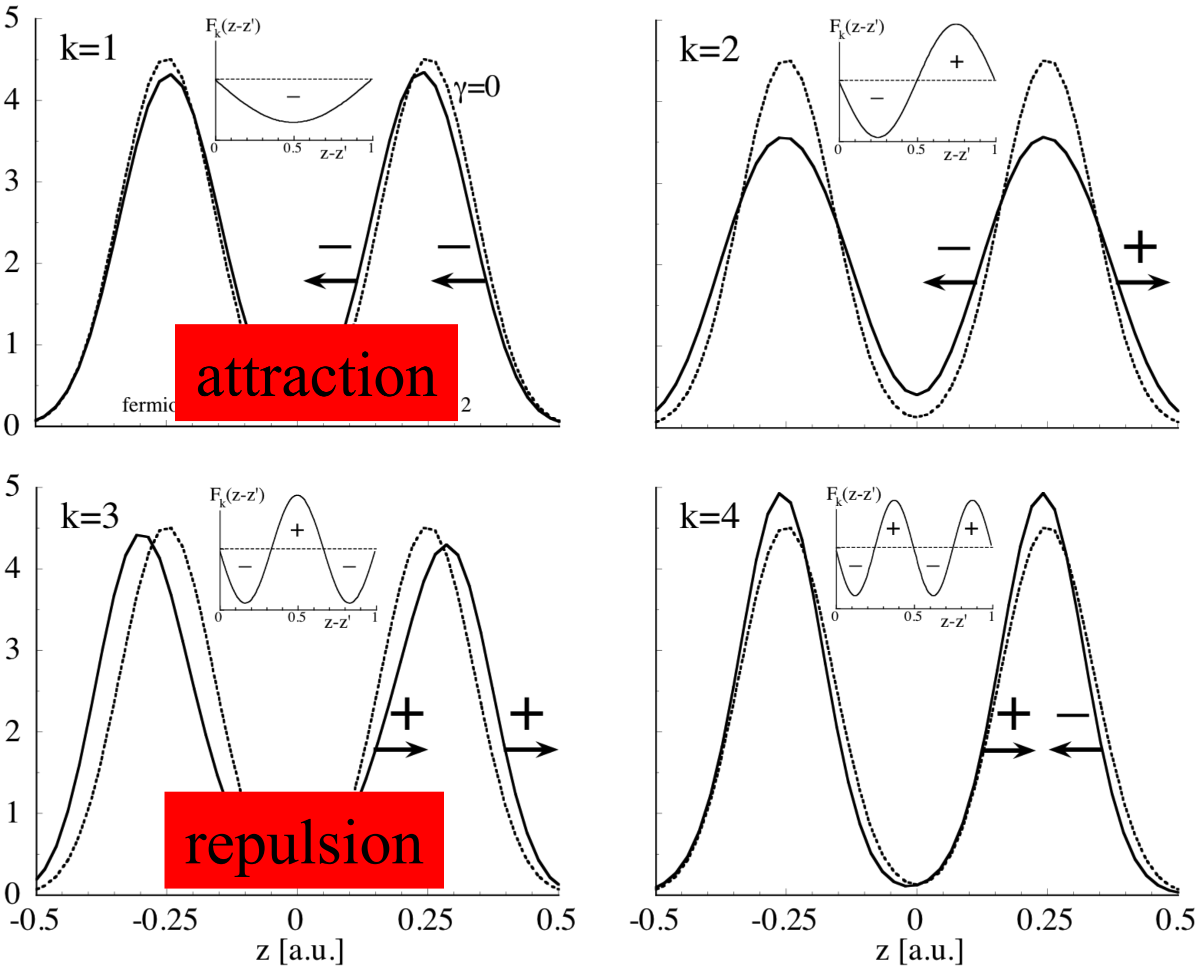


# Change Coulomb law by manipulating virtual photons








R.W. , **M. Ware** et al., ***Phys. Rev. Lett.*** 106, 023601 (2011)

# Overview

Repair work I: find $H_{renom}$

☺ compute the physical mass (numerical renormalization)

Dynamics in terms of bare particles

① vacuum

② single particle

③ two-particles (e-$\gamma$ and e-e scattering)

Repair work II: find $H_{dressed}$

☹ construct physical operators

# Beautiful special case:
# Greenberg-Schweber model

$$H_{bare} = \mathbf{E}_{bare} \ \Sigma_p \ b_p^\dagger b_p + \ \Sigma_k \ \omega_k a_k^\dagger a_k \ - \lambda^2 \ \Sigma_p \ \Sigma_k \ (2\omega_k)^{-1} \ b_{p+k}^\dagger b_p \ (a_k + a_{-k}^\dagger)$$

$$\mathbf{B_p} \ = U^\dagger b_p U$$

$$\text{with } U \ \alpha \ \exp\left[\lambda \ \Sigma_p \ \Sigma_k \ (2\omega_k^3)^{-1/2} \ b_{p+k}^\dagger b_p \ (a_k^\dagger - a_{-k}) \ \right]$$

$$\mathbf{A_k} \ = U^\dagger a_k U$$

$$H_{dress} = \mathbf{E}_{phys} \ \Sigma_p \ B_p^\dagger B_p \ + \Sigma_k \ \omega_k A_k^\dagger A_k \ - \lambda^2 \ \Sigma_p \Sigma_q \Sigma_k (2\omega_k^2)^{-1} \ B_{p+k}^\dagger B_q^\dagger B_p B_{q+k}$$

$$H_{bare} = \mathbf{E}_{bare}\ \Sigma_p\, b_p{}^{\square} b_p + \Sigma_k\, \omega_k a_k{}^{\square} a_k - \lambda^2\, \Sigma_p\, \Sigma_k\, (2\omega_k)^{-1}\, b_{p+k}{}^{\square} b_p\, (a_k + a_{-k}{}^{\square})$$

$$H_{dress} = \mathbf{E}_{phys}\, \Sigma_p\, B_p{}^{\square} B_p + \Sigma_k\, \omega_k A_k{}^{\square} A_k - \lambda^2\, \Sigma_p \Sigma_q \Sigma_k (2\omega_k{}^2)^{-1}\, B_{p+k}{}^{\square} B_q{}^{\square} B_p B_{q+k}$$

□ physical energy $E_{phys} = E_{bare} - \lambda^2 \Sigma_k (2\omega_k{}^2)^{-1}$

□ $B_P{}^{\square}\, |0\rangle$ is eigenstate of H, as $H\ B_P{}^{\square}\, |0\rangle = E_P\ B_P{}^{\square}\, |0\rangle$

□ no force intermediating virtual photons (no $e^-$-$\gamma$ interaction)

□ new $e^-$-$e^-$ interaction: $e^-(q+k) + e^-(p) \rightarrow e^-(q) + e^-(p+k)$

# The construction of the dressed particle Hamiltonian

$H_{dressed} = + \, dp \; e_p \; B_p{}^{\square} B_p + +dp \; e_p \; D_p{}^{\square} D_p \;+\; +dp \; \omega_p A_p{}^{\square} A_p \;+\;$ V

$V = \;+++++\; \alpha(p,q,p',q') \; B_p{}^{\square} \, B_q{}^{\square} \, B_{p'} \; B_{q'}$ ($e^-$-$e^-$ interaction)

$+ \;++++++\; \beta(p,q,p',q') \; B_p{}^{\square} \, D_q{}^{\square} \, B_{p'} \; D_{q'}$ ($e^-$-$e^+$ interaction)

$+ \;+++++++\; \gamma(p,q,p',q',q'') \; B_p{}^{\square} \, B_q{}^{\square} \, B_{p'} \; B_{q'} \; A_{q''}$ ($e^-$-$\gamma$ interaction )

$+ \ldots$

(1) use $H_{renorm}$ to compute scattering matrix S

(2) find $\alpha$, $\beta$, $\gamma$ etc. to match S

# Example: dressed particle Hamiltonian for scalar Yukawa system

$$H_{bare} = H_0 + \lambda + dx \ \psi(x)^{\square}\gamma^0 \psi(x)\, \phi(x)$$

$$V_{e\text{--}e\text{-}} = +++++\ \alpha(p,q,p',q')\ B_p^{\square} B_q^{\square} B_{p'} B_{q'}$$

$$\alpha(p,q,p',q') \sim \delta(p+q-p'-q') / [(q-q')^2 + M^2c^2]$$

- $+dx \exp[i(q-q')x]\ \alpha(p,q,p',q') \sim \exp(-M|x|)$
- direct interpretation possible

# Summary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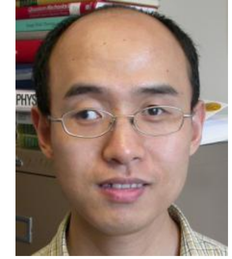
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C. Gerry


Matt


Emily


Kara


Nic


Kevin

- Goal: visualization of QED processes
- Main tool: computational quantum field theory
- First progress: $H_{bare}$ --> $H_{renorm}$
- Early stage progress: $H_{renorm}$ --> $H_{dressed}$
- many conceptual and computational challenges ...