

Signatures of radiation reaction in laser – electron - beam and laser - plasma interactions

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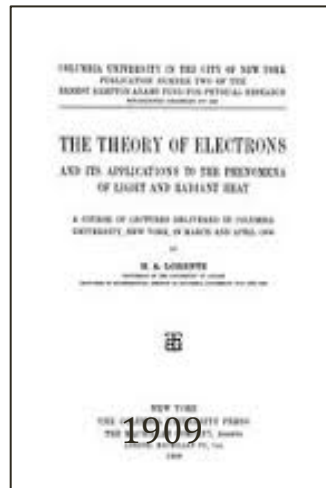
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für Kernphysik

Why the problem of radiation reaction becomes important now?

Abraham



Lorentz



Classical theory of radiating electrons

By P. A. M. DIRAC, F.R.S., *St John's College, Cambridge*

(Received 15 March 1938)

THE CLASSICAL THEORY OF FIELDS

Fourth Revised English Edition

L. D. LANDAU AND E. M. LIFSHITZ
Institute for Physical Problems, Academy of Sciences of the U.S.S.R.

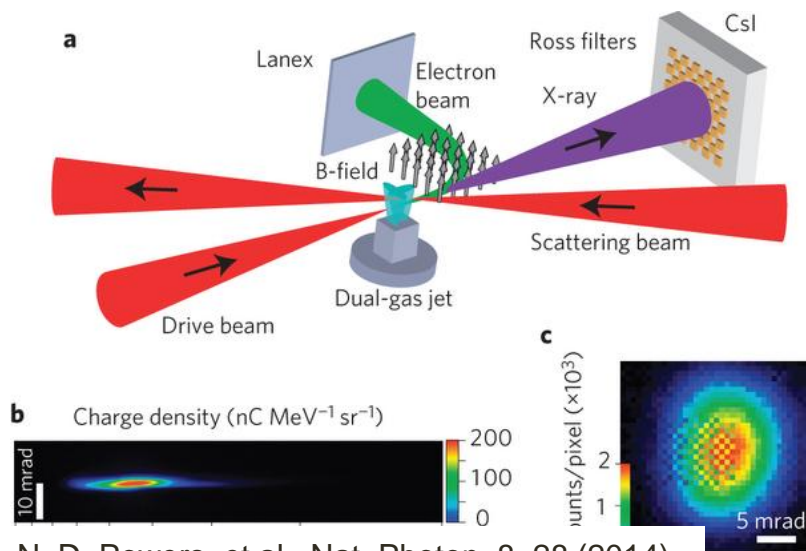
THE INFLUENCE OF RADIATION DAMPING ON THE SCATTERING OF LIGHT AND MESONS BY FREE PARTICLES. I

By W. HEITLER

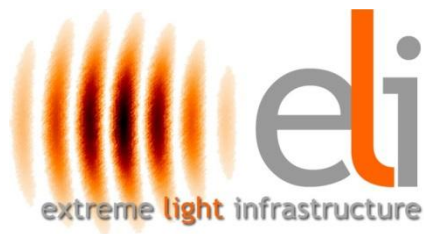
Communicated by A. H. WILSON

Received 16 February 1941

$$I \sim 10^{22} \text{ W/cm}^2$$



N. D. Powers, et al. Nat. Photon. 8, 28 (2014)



$$I \rightarrow 10^{25} - 10^{26} \text{ W/cm}^2$$

Energy loss channel in laser-plasma interaction:

PHYSICAL REVIEW E 86, 036401 (2012)

Modeling of radiation losses in ultrahigh power laser-matter interaction

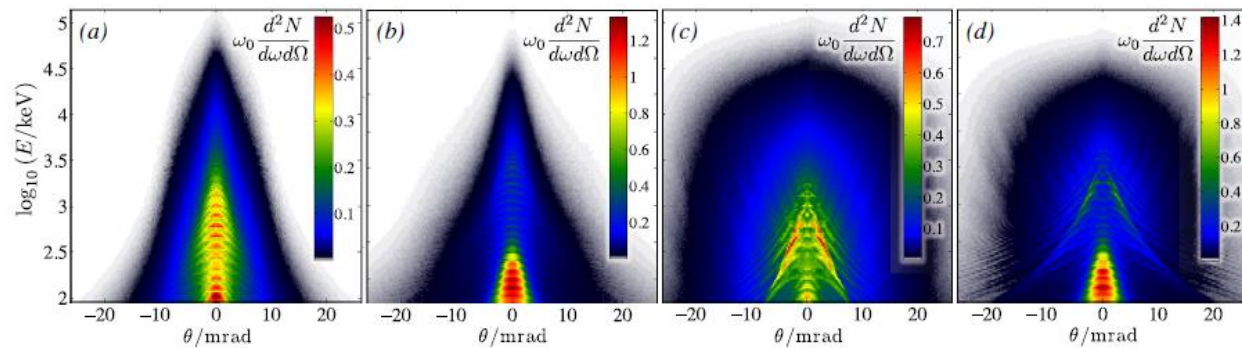
R. Capdessus,^{*} E. d'Humières, and V. T. Tikhonchuk

University Bordeaux, CNRS, CEA, CELIA, UMR 5107, F-33400 Talence, France

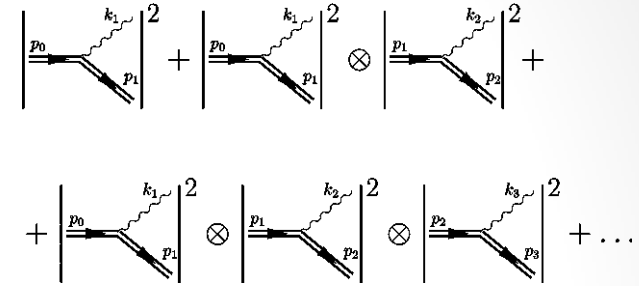
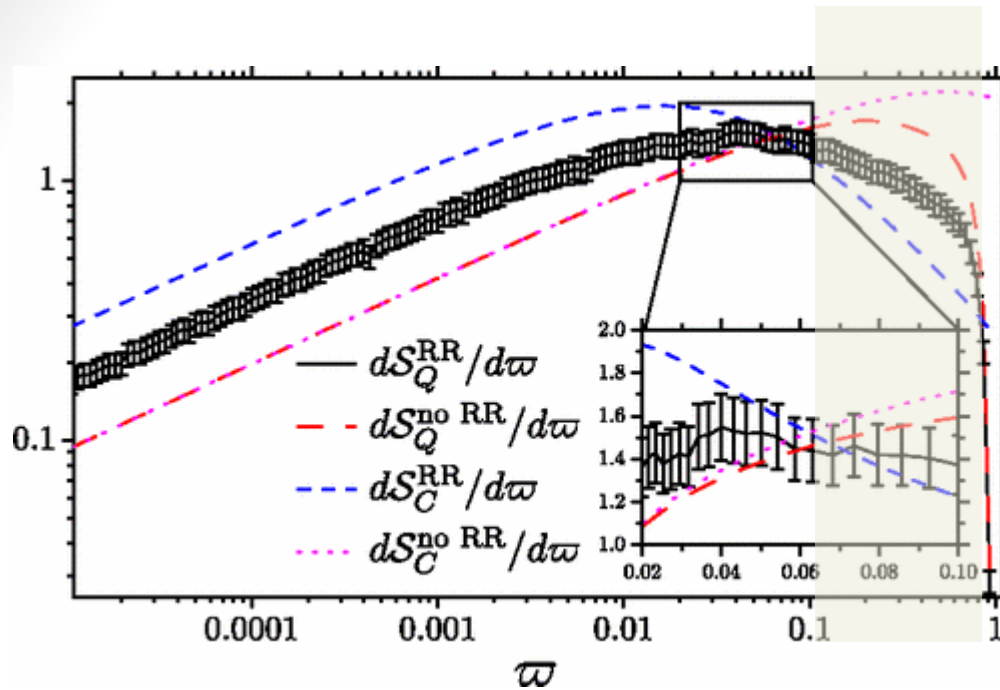
PHYSICAL REVIEW X 2, 041004 (2012)

Strong Radiation-Damping Effects in a Gamma-Ray Source Generated by the Interaction of a High-Intensity Laser with a Wakefield-Accelerated Electron Beam

A. G. R. Thomas,^{1,2} C. P. Ridgers,³ S. S. Bulanov,⁴ B. J. Griffin,² and S. P. D. Mangles⁵



Quantum radiation dominated regime in Compton scattering



Typical modification of spectrum in the quantum regime

$\varepsilon=1$ GeV

$\omega=1.5$ eV

$I=5 \times 10^{22}$ W/cm²

$\xi=154$; $\chi=1.8$; $R_Q=1$

Emission of 16 photons

Contribution of more photons 2%

Number of photons 10 keV-1MeV:
 $(N_0 - N_{RR})/N_0 \sim 40\%$

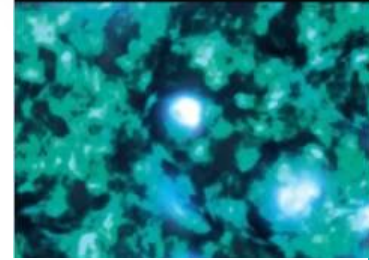
Di Piazza, KZH, Keitel PRL 105, 220403 (2010)

Outline

- Radiation reaction. Radiation Dominated Regime
- Multiphoton Compton scattering
- Properties of the radiation spectra in dependence of the driving laser pulse duration
- Radiation reaction impact on plasma instabilities
- Conclusion

- How to distinguish trident pair production from the cascade

Radiation Dominated Regime



$$m\dot{\mathbf{v}} = e\mathbf{E} + \frac{e}{c}[\mathbf{v}\mathbf{H}] + \frac{2e^2}{3c^3}\ddot{\mathbf{v}} \longrightarrow mc\frac{du^i}{ds} = \frac{e}{c}F^{ik}u_k + g^i$$

$$g^i = \frac{2e^2}{3c}\frac{d^2u^i}{ds^2}$$

$$\mathbf{f} = \frac{2e^3}{3mc^3}\dot{\mathbf{E}} + \frac{2e^4}{3m^2c^4}[\mathbf{E}\mathbf{H}] \longrightarrow g^i = \frac{2e^3}{3mc^3}\frac{\partial F^{ik}}{\partial x^l}u_k u^l - \frac{2e^4}{3m^2c^5}F^{il}F_{kl}u^k + \frac{2e^4}{3m^2c^5}(F_{kl}u^l)(F^{km}u_m)u^i$$

$$\lambda \gg r_0 \quad r_0 = \frac{e^2}{mc^2} \quad f_x = -\frac{2e^4}{3m^2c^4}\frac{(E_y - H_z)^2 + (E_z + H_y)^2}{1 - v^2/c^2}$$

$$\alpha\chi \ll 1 \quad \chi = \frac{E}{E_{cr}} \quad \alpha\chi \ll 1 \quad \chi = \frac{E'}{E_{cr}} = \frac{\gamma E}{E_{cr}}$$

$$E_{cr} = m^2/e \quad \frac{F_R}{F_L} = \frac{\alpha\gamma^2 E}{E_{cr}} = \alpha\chi\gamma \quad F_R \sim F_L$$

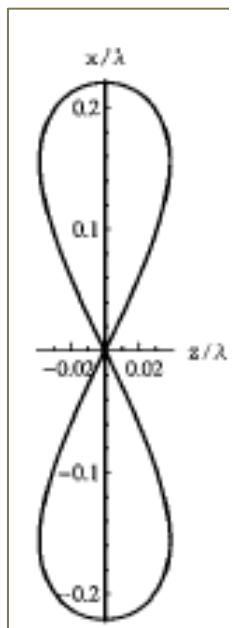
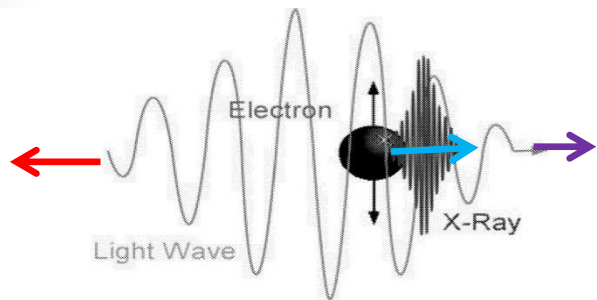
In the relativistic domain a regime is possible when radiation reaction force is not perturbation in the Lab frame

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Multiphoton Compton scattering



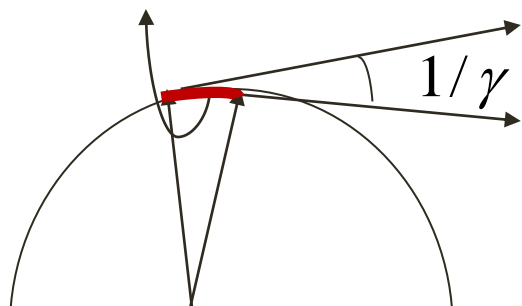
$$F \sim \frac{v}{c} \times BA$$

$$\omega' = n\omega'_L$$

$$\xi = \frac{eE_0}{mc\omega_L}$$

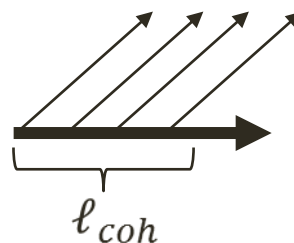
$$n \sim \xi^3$$

$$\ell_f = R\vartheta \sim \lambda_L/\gamma$$



$$\gamma \sim \xi$$

$$R \sim \lambda_L$$



$$|\omega t - kr| < \pi$$

$$r \approx vt; t \approx \frac{l}{v}$$

$$\ell_{coh} = \frac{\pi v}{\omega - kv} \sim \frac{2\pi v}{\omega} \gamma^2$$

$$\ell_f \sim \ell_{coh}$$

$$\frac{\lambda_L}{\gamma} \sim \frac{2\pi v}{\omega} \gamma^2$$

$$\omega \sim \gamma^3 \omega_L \sim \xi^3 \omega_L$$

$$\frac{\omega}{\varepsilon} \sim \frac{\omega_L \xi^2}{m} \sim \frac{\omega_L}{m} \xi \gamma \approx \chi$$

$$\mathbf{p}_\perp + e\mathbf{A} = const$$

$$p_\perp = m\xi$$

$$\gamma = \sqrt{1 + p_\perp^2/m^2} \sim \xi$$

$$I \sim 10^{22} W/cm^2$$

MeV γ - rays

Quantum effects in multiphoton Compton scattering

$$\chi \sim \frac{\omega}{m}$$

$$\chi = \frac{e}{m^3} \sqrt{(F_{\mu\nu} p^\nu)^2} = \frac{E'}{E_S}$$

$$E_{cr} = \frac{m^2 c^3}{e \hbar}$$

$$I_{cr} = 2.3 \cdot 10^{29} \frac{W}{cm^2}$$

$\chi \geq 1$ Quantum regime: emitted photon recoil is significant

$\chi \ll 1$ Classical regime

$$E' \approx 2\gamma E$$

$$\chi \sim 2 \frac{\omega_L}{m} \xi \gamma \sim 4 \cdot 10^{-6} \xi \gamma$$

$$\xi = \frac{e E_0}{m c \omega_L}$$

$$\chi \sim 1 : \xi \sim 100, \gamma \sim 10^3$$

Radiation Dominated Regime

$$\Delta\varepsilon^{(T)}_{rad} \sim \varepsilon$$

The characteristic emitted photon energy:

$$\omega_c \sim m\gamma\chi$$

The probability of a photon emission on a coherence length:

$$\alpha$$

Phase interval for a coherence length:

$$1/\xi$$

Number of coherence lengths on a laser period :

$$\xi$$

Number of emitted photons during a laser period :

$$N_{ph} \sim \alpha\xi$$

The electron radiative energy loss during a laser period:

$$\Delta\varepsilon^{(T)}_{rad} \sim \alpha\xi\chi m\gamma$$

Radiation Dominated Regime (RDR):

$$R \equiv \frac{\Delta\varepsilon^{(T)}_{rad}}{\varepsilon} \sim \alpha\xi\chi \geq 1$$

Classical RDR:

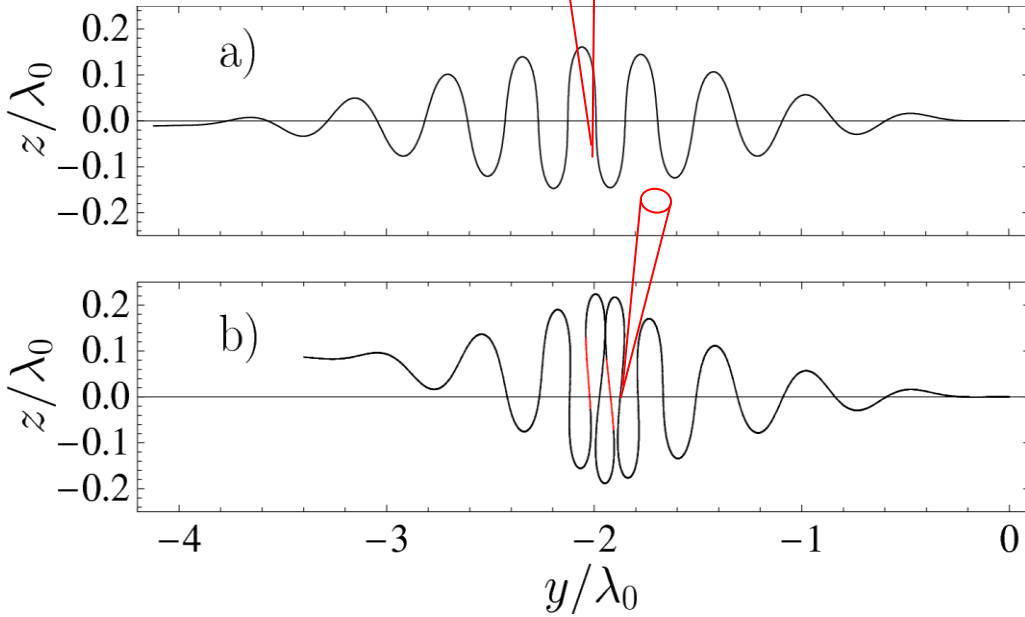
$$\chi \ll 1, R \geq 1 \rightarrow \xi \geq 10^3 \\ I \sim 10^{24} \text{ W/cm}^2$$

Radiation dominated dynamics in Thomson scattering

$$R \geq \frac{4\gamma_0^2 - \xi^2}{2\xi^2}$$

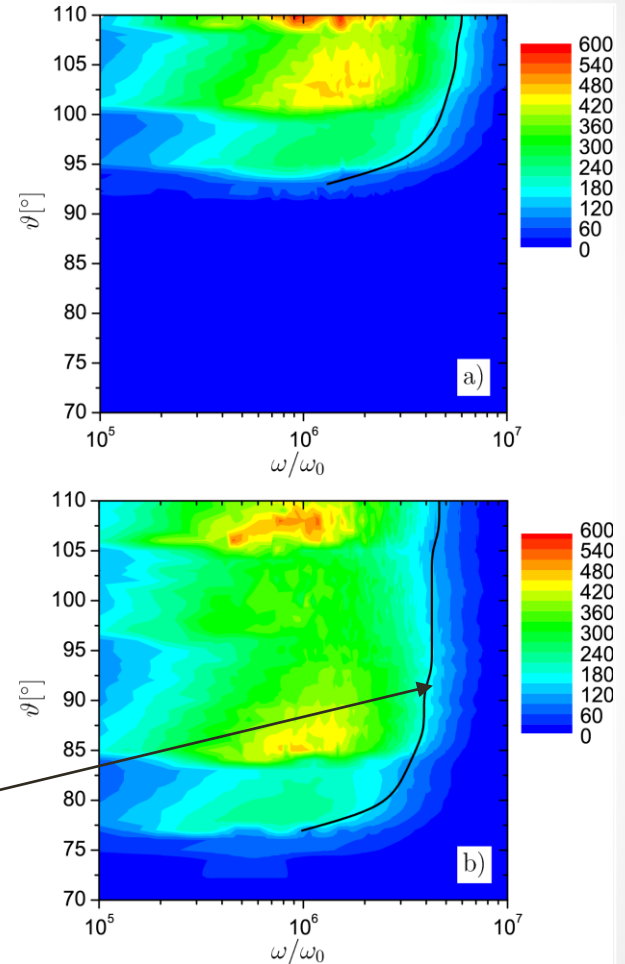
$$\gamma_d \sim \frac{\xi}{2} \sim \gamma$$

Electron trajectory



$$\pi\gamma^2 c / \omega_c \approx \rho / \gamma$$

Angle resolved radiation spectra

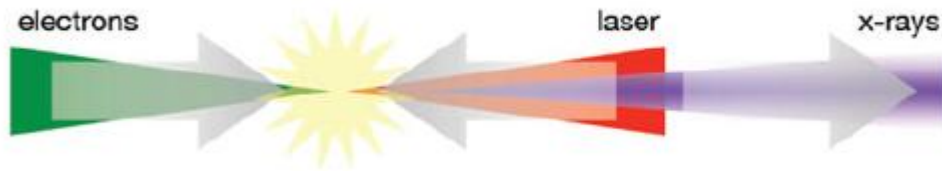


$$\gamma=80 \quad a_0=150 \quad I \sim 5 \times 10^{22} \text{ W/cm}^2$$

10^9 electrons in beam
 10^4 photons per pulse below 90°
 1% of electrons contribute

Di Piazza, KZH, Keitel, PRL 102, 254802 (2009)

Robust signature of quantum radiation reaction



Electron: energy 500 MeV ($\gamma=1000$),
density 10^{17}cm^{-3} (10^7 electrons, $3\times 6\ \mu\text{m}$)

Laser pulse: intensity $7\times 10^{22}\text{W/cm}^2$ ($\xi=230$),
pulse duration 0.5-40 cycles
beam waist size $10\lambda/4\lambda$

$$\chi=0.6 R \approx 1$$

Quantum RDR:

$$R = \alpha \xi \chi \geq 1$$

$$\chi \approx 2\gamma \xi \omega_L / m \sim 1$$

For a focused ultrashort laser pulse, the approximate solution of Maxwell equations should use two parameters on equal footing.

- Diffraction parameter $(k_0 w_0)^{-1}$ [1]
- Temporal parameter $(\omega_0 \tau_0)^{-1}$ [2]

$$\mathbf{A} = (E_0/k_0)[\hat{x}\psi(r, \eta) + i\hat{y}\psi(r, \eta)e^{i\pi/2}]e^{i\eta}$$

with $\psi = f(1 + i\eta/s^2)\exp[i\phi_0 - f\rho^2 - \eta^2/(2s^2)]$, $f = i/(i + \nu/z_r)$,
 $\nu = z + \eta/(2k_0)$, $\eta = \omega_0 t - k_0 z$, $\rho = r/w_0$, $r = \sqrt{x^2 + y^2}$,
 $s = \omega_0 \tau_0 / 2 \sqrt{2 \log 2}$, $z_r = k_0 w_0^2 / 2$ is the Rayleigh length, ϕ_0 is the constant phase, E_0 is the laser amplitude, w_0 , ω_0 , k_0 , and τ_0 is the waist radius, frequency, wave vector, and pulse duration of the laser

Classical equation of motion with quantum radiation

Electron dynamics in the laser field is classical, but radiation is quantum mechanical.

$\xi \gg 1, \ell_{coh} \ll \lambda_L$ Radiation is determined by the electron local characteristics

The emitted radiation is calculated quantum mechanically, and the differential probability per unit phase interval is [1]

$$\frac{dW_{fi}}{d\eta d\tilde{\omega}} = \frac{\alpha\chi m^2 [\int_{\tilde{\omega}_r}^{\infty} K_{5/3}(x) dx + \tilde{\omega}\tilde{\omega}_r \chi^2 K_{2/3}(\omega_r)]}{\sqrt{3}\pi(k_0 \cdot p_i)},$$

$\tilde{\omega}_r = \tilde{\omega}/\rho_0$, with the recoil parameter $\rho_0 = 1 - \chi\tilde{\omega}$ and $\tilde{\omega} = \omega'/(\gamma\chi)$. If $\tilde{\omega}_r \gtrsim 1$, $\frac{dW_{fi}}{d\eta d\tilde{\omega}}$ is very small. Thus, $\tilde{\omega}_r = \tilde{\omega}/\rho_0 = 1$, the cut-off frequency

A. I. Nikishov, V. I. Ritus, JETP 1964

$$\frac{dp^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} p_\nu + \frac{dp_R^\mu}{d\tau}$$

Classical equation of motion with quantum radiation

Electron dynamics in the laser field is classical, the radiation is quantum mechanical.

$$\frac{dp^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} p_\nu + \frac{dp_R^\mu}{d\tau}$$

$$p + nk = p' + k' \quad n = \frac{p \cdot k'}{p \cdot k - k \cdot k'}$$

$$\Delta p = nk - k' \quad \frac{dp_R^\mu}{d\tau} \approx \frac{\Delta p_R^\mu}{\Delta\tau}$$

$$\frac{k'}{d\tau} \rightarrow \int k' \frac{k \cdot p}{d\phi} \frac{dW}{d^3\mathbf{k}'} d^3\mathbf{k}' \approx \int \frac{d\wp}{d\omega'}(\phi) d\omega' \quad \phi = \omega t - kz$$

$$\frac{nk}{d\tau} \rightarrow k \int \frac{p \cdot p}{p \cdot k - k \cdot k'} \frac{d\wp}{d\omega'} d\omega' = \frac{2r_0}{3} \frac{\wp}{\wp_c} F^{\mu\nu} F_{\nu\sigma} p^\sigma$$

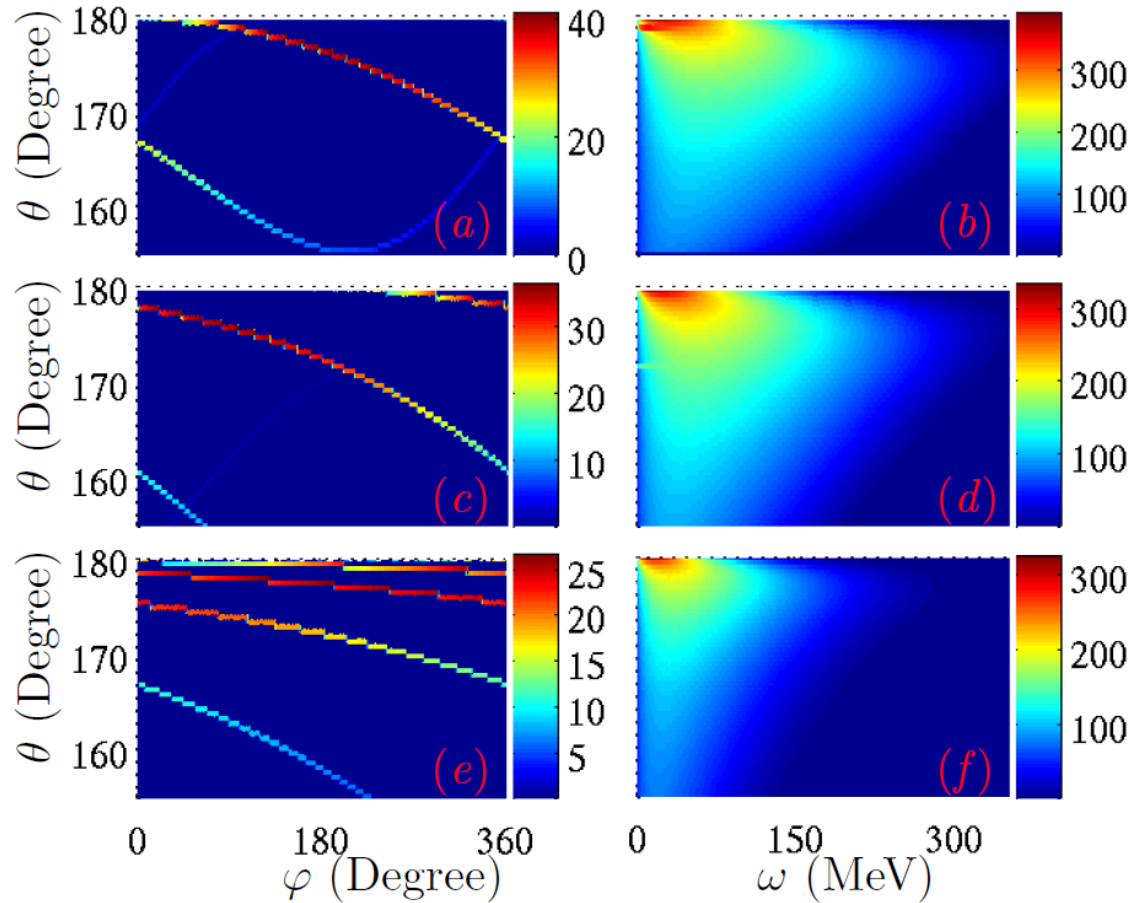
$$\frac{dp^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} p_\nu - \frac{p}{m} \wp + \frac{2r_0}{3} \frac{\wp}{\wp_c} F^{\mu\nu} F_{\nu\sigma} p^\sigma \quad \wp_c = 2\alpha\omega^2\xi^2$$

Radiation angle resolved spectra

Quantum RDR:

$$R = \alpha\chi\xi \geq 1$$

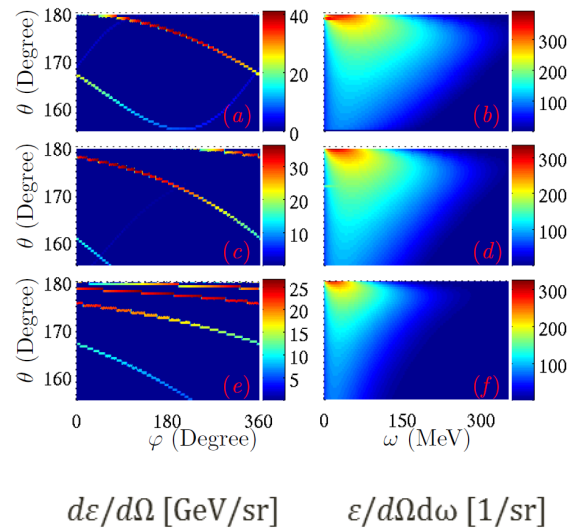
$$\chi \approx 2\gamma\xi\omega/m \sim 1$$



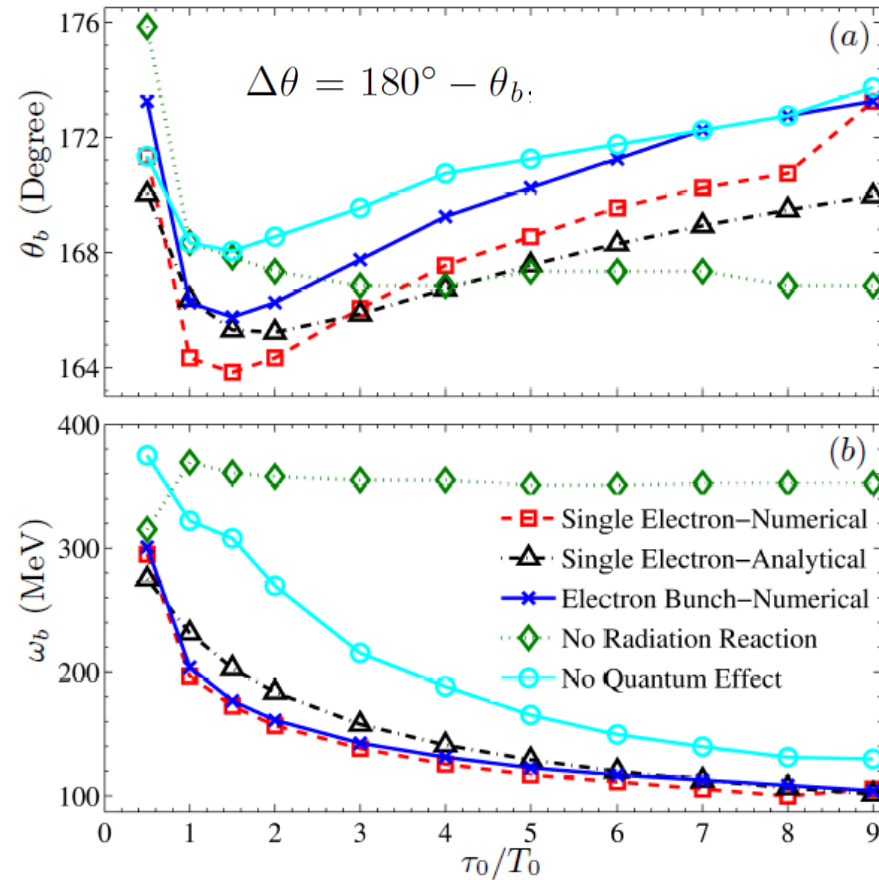
For each θ there is φ , where the radiation energy is maximal.

Spectra of electron radiation in laser pulses of various durations: the left column displays $d\epsilon/d\Omega$ [GeV/sr] and the right $d\epsilon/d\Omega d\omega$ [1/sr] for 1, 1.5 and 5 cycle pulses $\lambda = 1 \mu\text{m}$, $w_0 = 10\lambda$, $\xi = 230$ and $\gamma = 1000$.

Robust signatures of quantum radiation reaction for nonlinear Compton scattering in focused ultrashort laser pulses



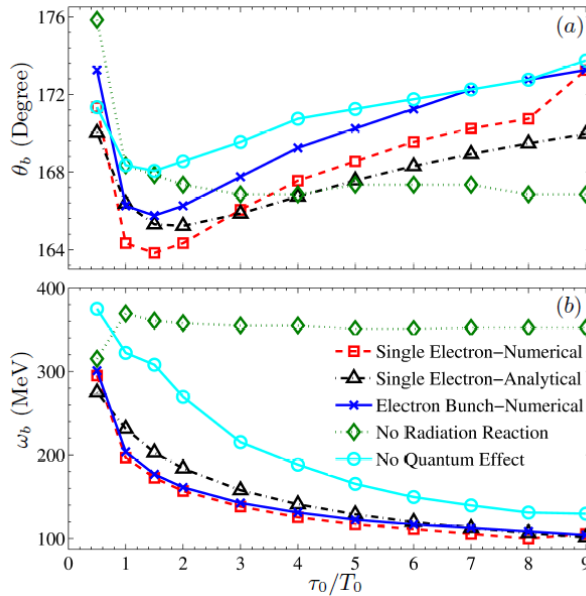
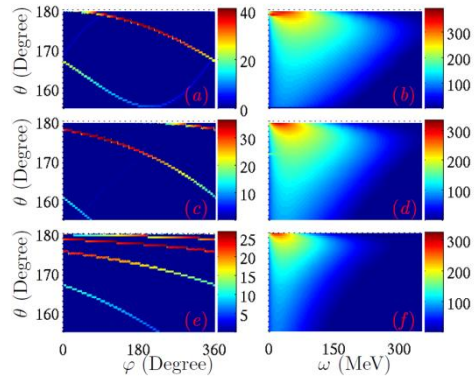
Quantum regime
 $R = \alpha\xi\chi \geq 1$
 $\chi \approx 2\gamma\xi\omega/m \sim 1$



$$\xi \leq \gamma \leq 20\xi \quad \Rightarrow \quad R \sim 1 \Rightarrow \chi \sim 1$$

$$w_0 > 4\lambda \quad w_e < w_0/2 \quad \chi \ll 1 \Rightarrow R \ll 1$$

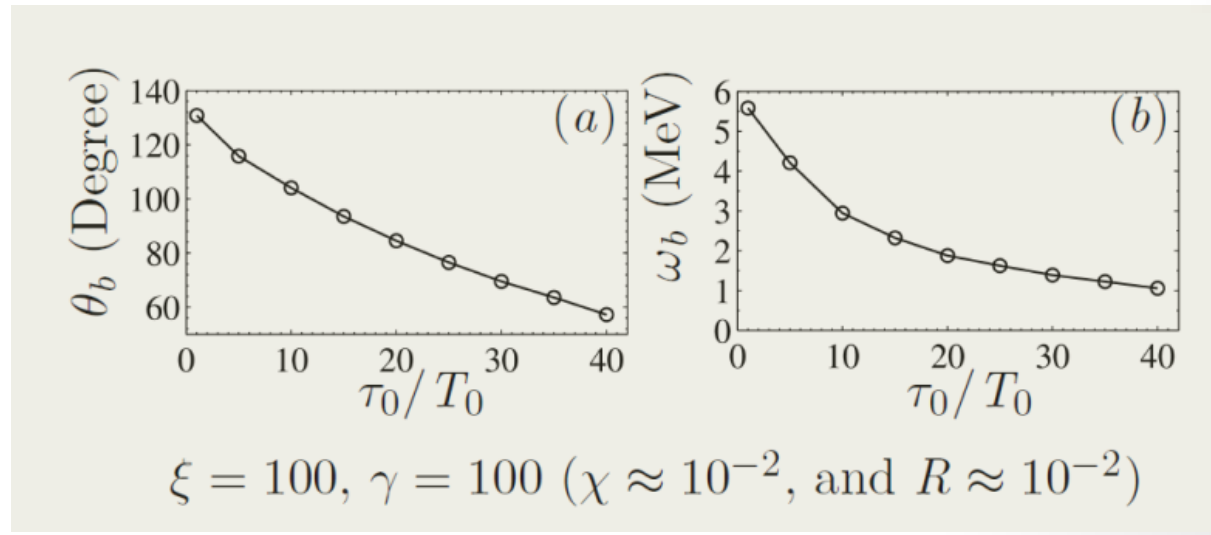
Robust signatures of quantum radiation reaction for nonlinear Compton scattering in focused ultrashort laser pulses



Quantum regime
 $R = \alpha\xi\chi \geq 1$
 $\chi \approx 2\gamma\xi\omega/m \sim 1$

The RR signatures in the classical RR regime:

$R \ll 1$
 $\chi \ll 1$



Explanation of spectral features in RDR

$$\frac{d\varepsilon}{d\phi} \sim \frac{\Delta\varepsilon}{\Delta\phi_{coh}}$$

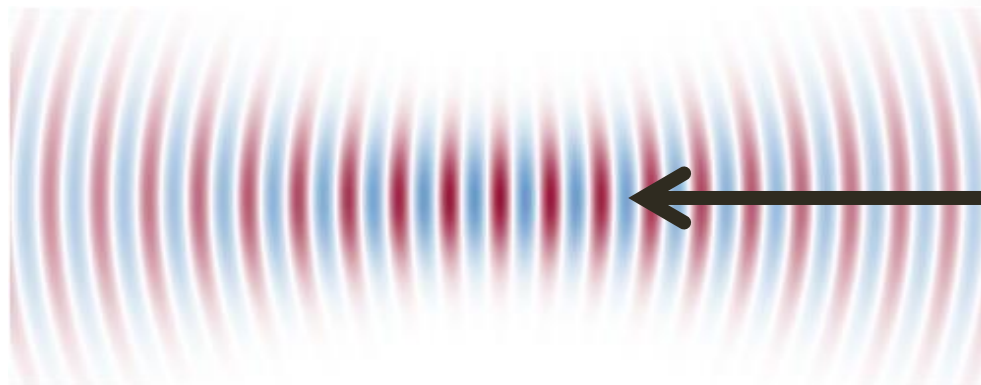
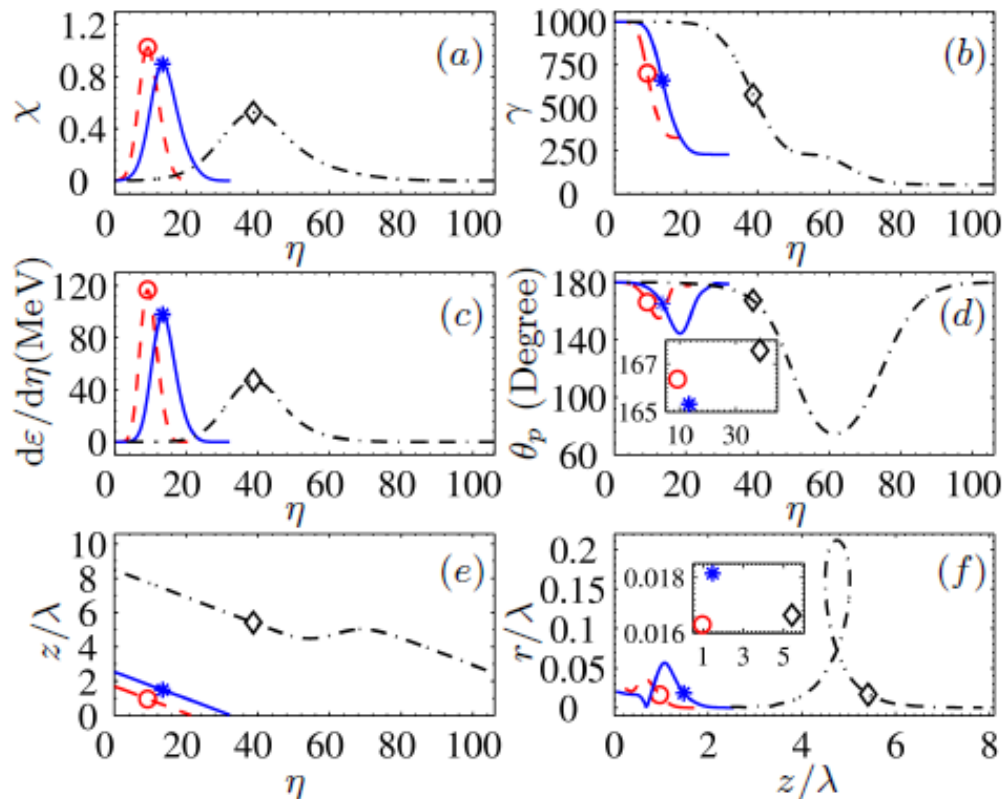
$$\Delta\varepsilon \sim \alpha\omega_c \quad \omega_c \sim \frac{m\chi\gamma}{1+\chi}$$

$$\Delta\phi_{coh} \sim \frac{2\pi}{\xi} \quad \chi \approx 2 \frac{\omega_L}{m} \xi\gamma$$

$$\frac{d\varepsilon}{d\phi} \sim \frac{\alpha\omega_c\xi}{2\pi} \propto \chi^2$$

$$\chi \propto \xi\gamma$$

$$p_{\perp m} \sim m\xi(\phi_m) \quad \theta_b \sim p_{\perp m}/p_{\parallel m}$$



Impact of radiation reaction on Raman scattering of an ultraintense laser pulse in plasma

Nonlinear mixing of Raman sidebands due to radiation reaction

Radiation reaction force induced nonlinear mixing of Raman sidebands of an ultraintense laser pulse in plasma

$$\text{Raman scattering: } \omega_0 = \omega_s \pm \omega_p \quad \mathbf{k}_0 = \mathbf{k}_s \pm \mathbf{k}_p$$

Radiation losses: additional source of free energy for perturbations to grow in plasma

Phase shift of the electron current due to radiation reaction is responsible for enhancement of FRS

Radiation reaction is treated classically perturbatively via Landau-Lifshitz equation

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{p} = -e \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) - \frac{2e^4}{3m_e^2 c^5} \gamma^2 \mathbf{v} \left[\left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right)^2 - \left(\frac{\mathbf{v}}{c} \cdot \mathbf{E} \right)^2 \right],$$

Laser field is circularly polarized: unperturbed equation - Akhiezer-Polovin solution

$$\mathbf{p}_\perp = e\mathbf{A}/c,$$

$$\partial v_z / \partial t = e \nabla_z \phi / (m_e \gamma_0) - e^2 \nabla_z |A|^2 / (2m_e^2 \gamma_0^2 c^2), \quad \nabla_\perp |A_0|^2 = 0,$$

Radiation reaction force induced nonlinear mixing of Raman sidebands

Stokes and anti-Stokes waves:

$$\mathbf{A} = [\mathbf{A}_0 e^{i\psi_0} + \delta\mathbf{A}_+ e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} e^{i\psi_+} + \delta\mathbf{A}_-^* e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} e^{-i\psi_-^*}] / 2 + \text{c.c.}$$

$$\psi_+ = (k_z + k_0)z - (\omega + \omega_0)t \text{ and } \psi_-^* = (k_z - k_0)z - (\omega^* - \omega_0)t$$

Beating Stokes (anti-Stokes) wave with the pump laser:

$$\delta\tilde{n} = (e^2 k_z^2 / 2m_e^2 \gamma_0^2 c^2 D_e) (\hat{A}_0^* \delta A_+ + A_0 \delta A_-), \quad D_e = \omega^2 - \omega_p^2$$

$$p_z \ll p_\perp$$

$$\frac{\partial}{\partial t} \left(\mathbf{p}_\perp - \frac{e}{c} \mathbf{A} \right) = -\frac{e\mu\omega_0}{c} \mathbf{A} \gamma |\mathbf{A}|^2 (1 - 2\beta_z), \quad \mu = 2e^4 \omega_0 / 3m_e^3 c^7,$$

$$\mu \gamma |\mathbf{A}|^2 \ll 1$$

$$\mathbf{p}_\perp = [\mathbf{p}_0 e^{i\psi_0} + \mathbf{p}_+ e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} e^{i\psi_+} + \mathbf{p}_-^* e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} e^{-i\psi_-^*}] / 2 + \text{c.c.}$$

$$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = \frac{\omega_p^2}{\gamma c^2} \left(1 + \frac{\delta n}{n_0} \right) \frac{c}{e} \mathbf{p}_\perp.$$

$e^{i\psi_0}$:

$$\omega_0^2 = k_0^2 c^2 + \omega_p^2 (1 - i\mu |A_0|^2 \gamma_0 / 2)$$

damping of the pump laser field

$$\omega_0 = \omega_{0r} - i\delta\omega_0, \quad \delta\omega_0 \ll \omega_{0r} \quad \delta\omega_0 = \omega_p^2 \varepsilon \gamma_0 a_0^2 / 2\omega_{0r}$$

$$\varepsilon = r_e \omega_{0r} / 3c, \quad r_e = e^2 / m_e c^2$$

Radiation reaction force induced nonlinear mixing of Raman sidebands

Stokes and anti-Stokes waves:

$$\mathbf{A} = [\mathbf{A}_0 e^{i\psi_0} + \delta\mathbf{A}_+ e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} e^{i\psi_+} + \delta\mathbf{A}_-^* e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} e^{-i\psi_-^*}] / 2 + \text{c.c.}$$

$$\psi_+ = (k_z + k_0)z - (\omega + \omega_0)t \text{ and } \psi_-^* = (k_z - k_0)z - (\omega^* - \omega_0)t$$

$$e^{i\psi_\pm} e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}:$$

$$\left(\frac{R_+}{D_+} + \frac{R_-}{D_-} \right) = 1,$$

$$\varepsilon = 0$$

$$R_+ = R_- \equiv R.$$

$$D_\pm = (\omega \pm \omega_0)^2 - \omega_p^2 \left(1 - \frac{i\varepsilon a_0^2 \gamma_0 \omega_0}{\omega \pm \omega_0} \right) - k_\perp^2 c^2 - (k_z \pm k_0)^2 c^2,$$

$$R_\pm = \frac{\omega_p^2 a_0^2}{4\gamma_0^3} \left[\frac{k_z^2 c^2}{D_e} \left(1 \mp i\varepsilon a_0^2 \gamma_0 + \frac{2i\varepsilon a_0^2 \gamma_0}{k_z c} \frac{\omega \omega_0}{\omega \pm \omega_0} \right) - \left(1 \mp i\varepsilon a_0^2 \gamma_0 \frac{\omega}{\omega \pm \omega_0} + 4i\varepsilon \gamma_0^3 \frac{\omega_0}{\omega \pm \omega_0} \right) \right].$$

The coupling of Stokes and anti-Stokes is modified due to radiation reaction: phase-shift of the current.

Radiation reaction force induced nonlinear mixing of Raman sidebands

Both Stokes and anti-Stokes are resonant and coupled:

$$\Gamma_{\text{FRS}} = -\frac{\omega_p^2 \varepsilon a_0^2}{2\omega_{0r}} \pm \frac{\omega_p^2 a_0}{\sqrt{8}\gamma_0^2 \omega_{0r}} \cos(\theta/2)$$

$$\times \sqrt{(1 + 2\varepsilon^2 a_0^2 \gamma_0^4)^2 + \varepsilon^2 a_0^4 \gamma_0^2 \left(\frac{\omega_{0r}}{\omega_p'}\right)^2},$$

$$\theta = \tan^{-1}\left(\frac{-\varepsilon a_0^2 \gamma_0 (\omega_{0r}/\omega_p')}{(1 + 2\varepsilon^2 a_0^2 \gamma_0^4)}\right).$$

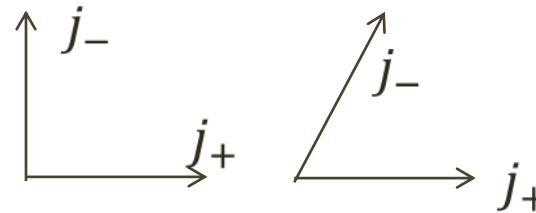
$$D_{\pm} \approx (\omega \pm \omega_{0r})^2 - \omega_p'^2 - (k_z \pm k_0)^2 c^2 = 0,$$

$$\omega = \omega_p' + i\Gamma_{\text{FRS}}$$

Radiation reaction enhances the growth rate when:

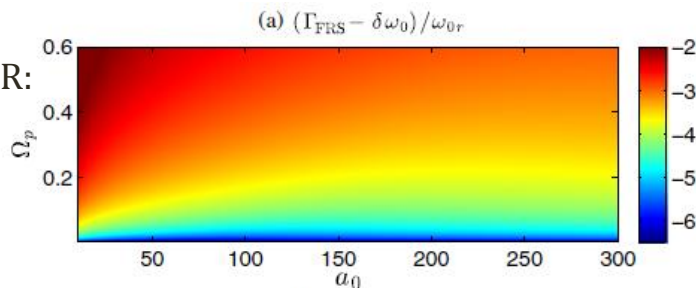
$$\Omega_p \equiv \omega_p/\omega_{0r} \ll 1 \quad a_0 \gg 1$$

Enhancement is due to mixing Stokes and anti-Stokes modes mediated by RR due to the phase shift between the currents

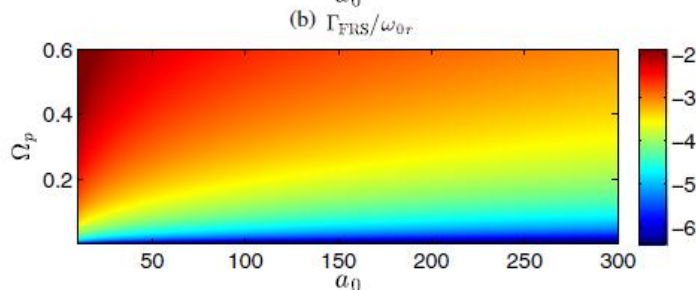


$$R_{\pm} = \frac{\omega_p^2 a_0^2}{4\gamma_0^3} \left[\frac{k_z^2 c^2}{D_e} \left(1 \mp i\varepsilon a_0^2 \gamma_0 + \frac{2i\varepsilon a_0^2 \gamma_0}{k_z c} \frac{\omega \omega_0}{\omega \pm \omega_0} \right) - \left(1 \mp i\varepsilon a_0^2 \gamma_0 \frac{\omega}{\omega \pm \omega_0} + 4i\varepsilon \gamma_0^3 \frac{\omega_0}{\omega \pm \omega_0} \right) \right].$$

With RR:



no RR:

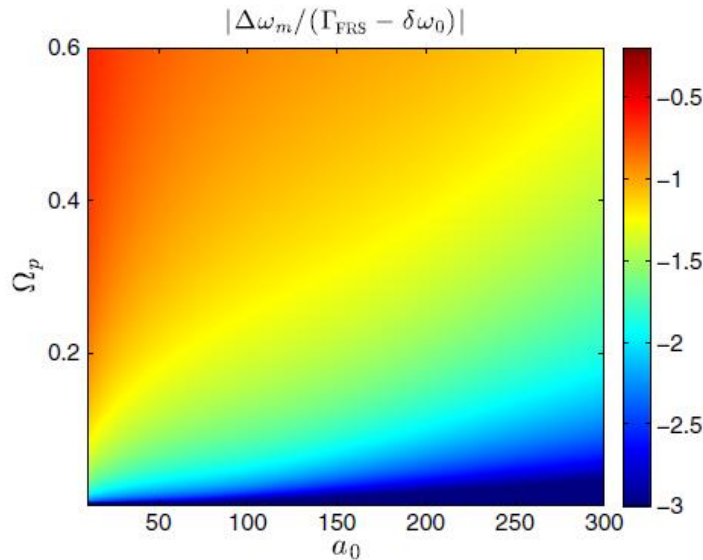


Radiation reaction force induced nonlinear mixing of Raman sidebands

FRS: condition when both modes are excited $D_{\pm} \approx (\omega \pm \omega_{0r})^2 - \omega_p'^2 - (k_z \pm k_0)^2 c^2 = 0$

Frequency mismatch should be smaller than the growth rate:

$$\Delta\omega_m = \omega_p' + \omega_{0r} - [\omega_p'^2 + c^2(k_p' + k_0)^2 + D_+]^{1/2} < \Gamma_{\text{FRS}} - \delta\omega_0$$



The growth rate enhancement disappeared when only one mode is available.

BRS: anti-stokes wave is not resonant mode of plasma

$$\Gamma_{\text{BRS}} = \frac{\sqrt{3}}{2} \left(\frac{\omega_{0r}}{2\omega_p} \right)^{1/3} \frac{\omega_p a_0^{2/3}}{(1 + a_0^2/2)^{1/2}} \left(1 + \frac{\epsilon a_0^2 \gamma_0}{3\sqrt{3}} \right).$$

The growth rate enhancement is not essential.

Conclusion

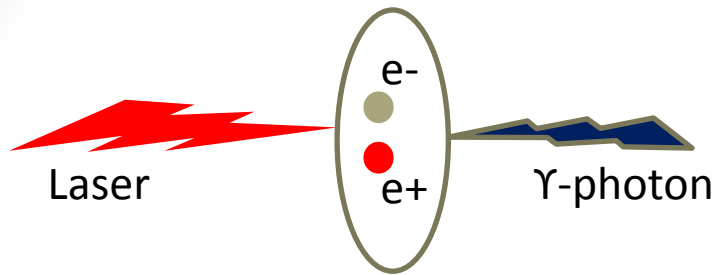
- We have identified signatures of quantum RDR in dependence of both the angular spread and the spectral bandwidth of the Compton radiation spectra on the laser pulse duration, which are distinct from those in the classical RR regime. They are robust and observable in a broad range of electron and laser beam parameters.
- Due to an interplay between laser beam focusing and quantum RR effects the angular spread of the main photon emission region has a maximum at an intermediate pulse duration and decreases along the further increase of the pulse duration.
- The spectral bandwidths of the radiation in the quantum and classical regimes both monotonously decrease when the laser pulse duration is increased, but the former is by orders of magnitude larger due to much stronger RR effects.
- The radiation reaction force strongly enhances the growth of the FRS only when both the Stokes and the anti-Stokes modes are the resonant modes of the plasma. This is a signature of RR in the spectra of low-energy optical photons.
- The enhancement is due to the radiation-reaction-force-induced nonlinear mixing of the anti-Stokes and the Stokes modes.

Comment:

on the competition between the Breit-Wheeler and trident processes for the electron-positron pair production in laser fields

Breit-Wheeler vs Trident process

Momentum distribution



Threshold: c.m. frame

$$n\omega' = \Omega' = m_*$$

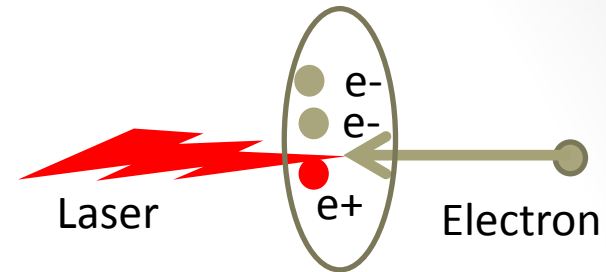
$$2\gamma n\omega = \frac{\Omega}{2\gamma} = m_*$$

$$\mathbf{q}'_+ = \mathbf{q}'_- = 0$$

$$\mathbf{q}_+ = \gamma(\mathbf{q}'_+ - \beta\epsilon')$$

$$\epsilon = \gamma(\epsilon' - \beta p'_z) = \gamma m_*$$

$$\mathbf{q}_+ = \mathbf{q}_- = -\mathbf{n}\Omega/2 \approx -\mathbf{n}\epsilon_e/2$$



c.m. frame

$$n\omega' = q'_e$$

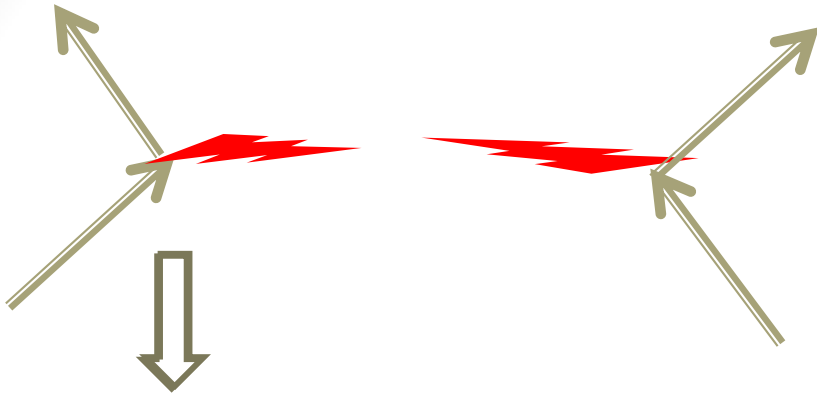
$$n\omega' + \epsilon'_e = 3m_*$$

$$q'_e = \frac{4m_*}{3}, \epsilon'_e = \frac{5m_*}{3}$$

$$\epsilon_e = \gamma(\epsilon'_e + \beta q'_e) \approx 3\gamma m_*$$

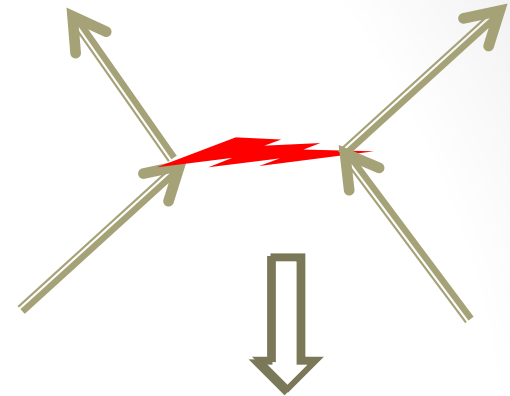
$$\mathbf{q}_+ = \mathbf{q}_- = -\mathbf{n}\gamma m_* = -\mathbf{n}\epsilon_e/3$$

Breit-Wheeler vs Trident process



$$\frac{r_e^2}{\gamma} J_L \tau \sim \alpha \xi N_L$$

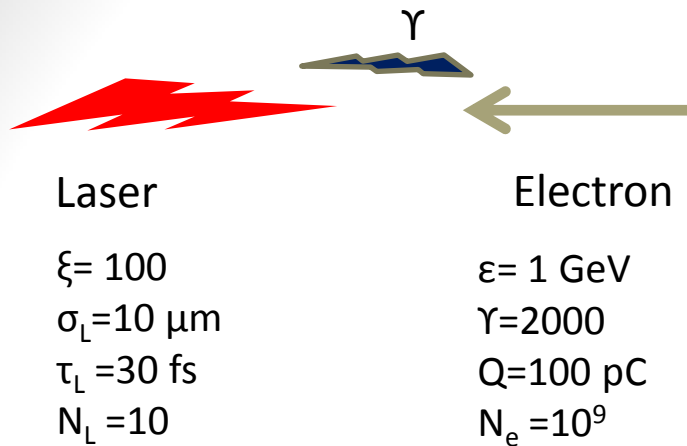
$$(\alpha \xi N_L)^2$$



$$\alpha \frac{r_e^2}{\gamma} J_L \tau \sim \alpha^2 \xi N_L$$

$$\frac{BW}{T} \sim \xi N_L$$

Multiphoton Compton scattering



$\xi = 100$
 $\sigma_L = 10 \mu\text{m}$
 $\tau_L = 30 \text{ fs}$
 $N_L = 10$

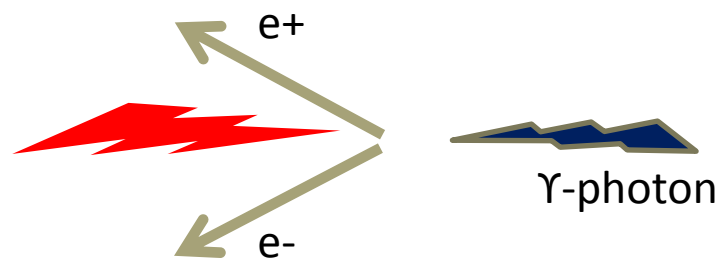
$\epsilon = 1 \text{ GeV}$
 $\gamma = 2000$
 $Q = 100 \text{ pC}$
 $N_e = 10^9$

$\xi \gg 1$ Multiphoton processes

$\chi_e < 1$:

$$N_\gamma \approx 0.6 \alpha \xi N_L N_e$$

Breit-Wheeler process



$$\chi_\gamma < 1: N_{BW} \approx \frac{9}{32} \frac{\alpha m^2}{\Omega} \left(\frac{\chi_\gamma}{2\pi} \right)^{3/2} e^{-\frac{8}{3\chi_\gamma}} N_\gamma \tau$$

Ritus, 1966

$$\chi_\gamma = 2 \frac{\Omega}{m} \frac{\omega_L}{m} a_0; \quad \chi_\gamma \approx \chi_e, \text{ if } \chi_e \sim 1$$

$$N_{C+BW} \approx \frac{9}{16} (\alpha \xi N_L)^2 N_e \sqrt{\frac{\chi_\gamma}{2\pi}} e^{-\frac{8}{3\chi_\gamma}}$$

$$\xi = 100; \gamma = 2000 \quad N_{C-BW} = 10^7$$