

Generation of keV harmonics and ultrashort waveforms driven by midinfrared lasers

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CAMPUS DE EXCELENCIA INTERNACIONAL

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People involved

'Generation of keV harmonics and ultrashort waveforms driven by midinfrared lasers'



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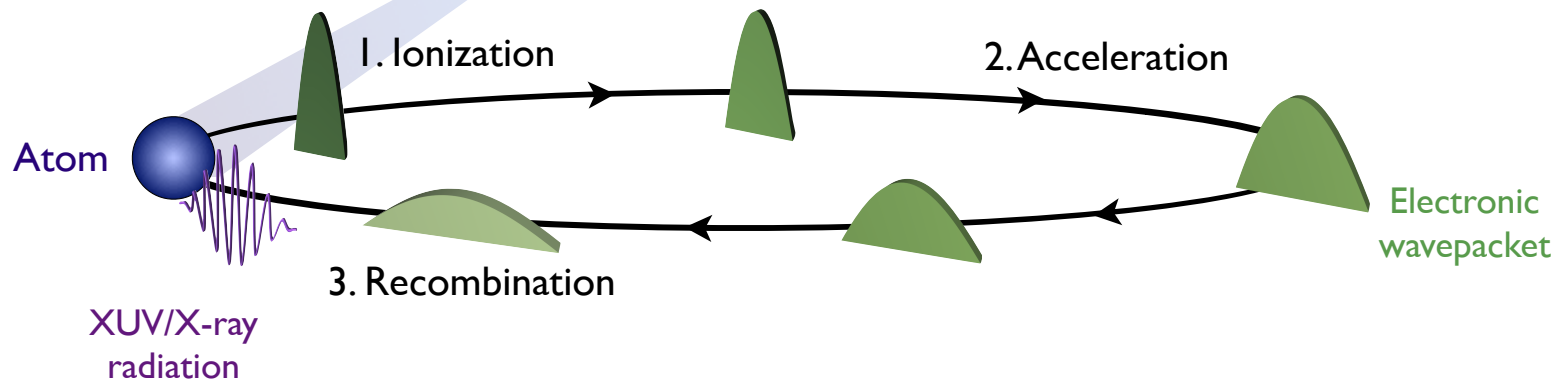
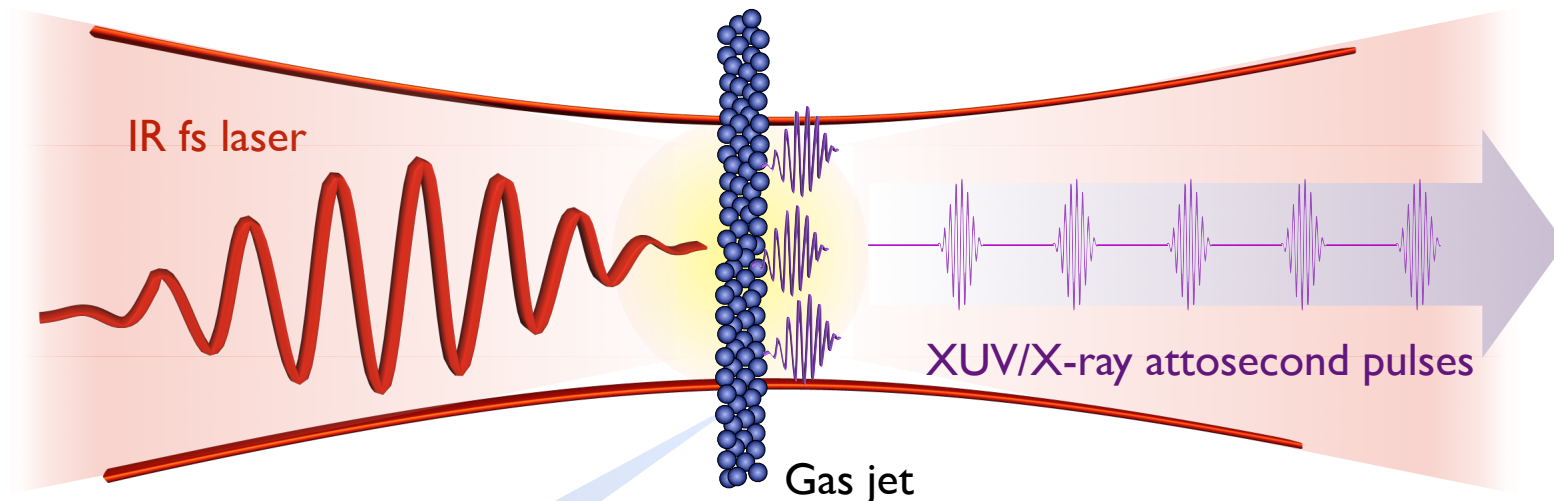
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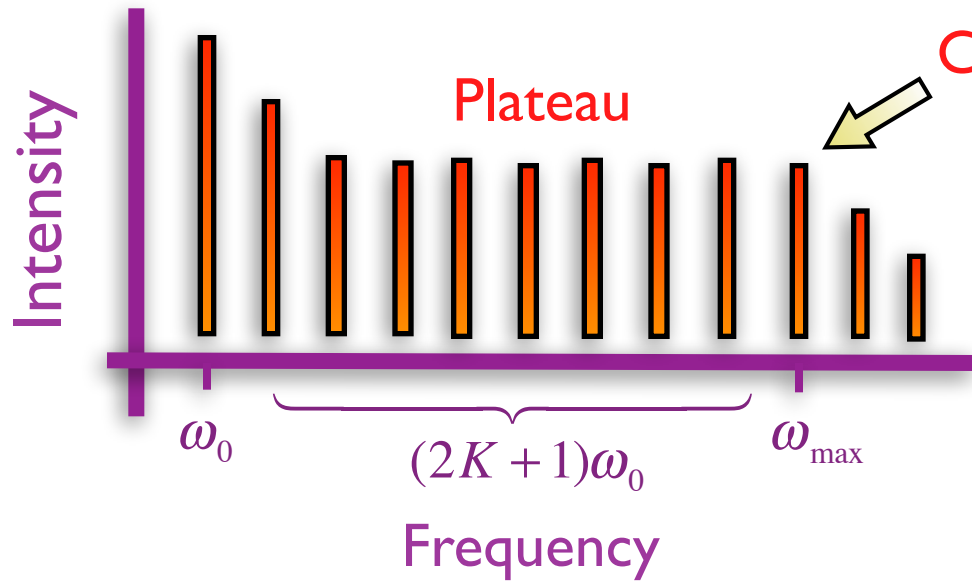
supported by DOE, NSF,
Janus supercomputer (Univ. Colorado)

High-order Harmonic Generation (HHG)



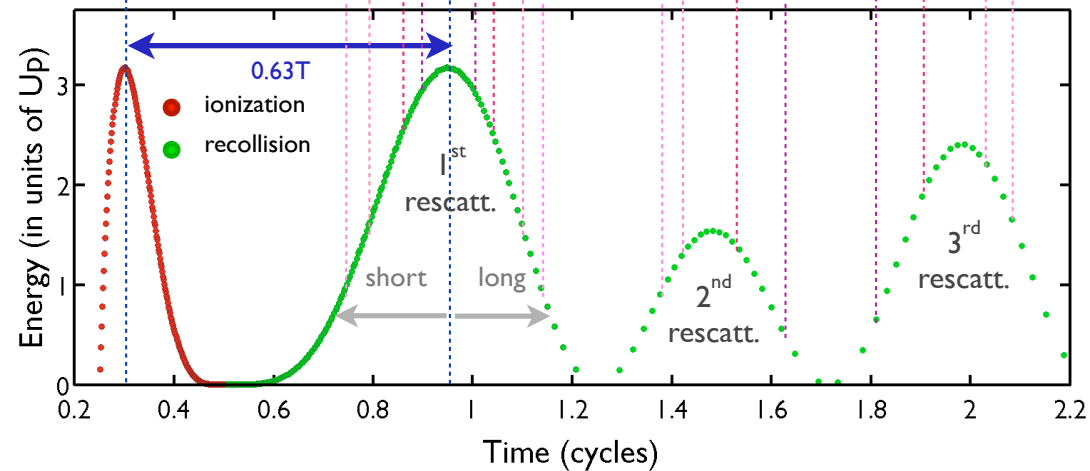
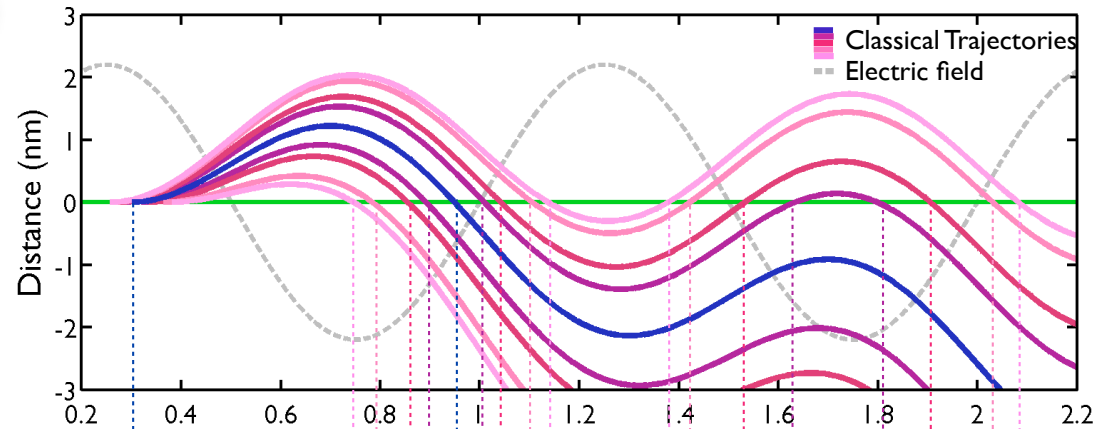
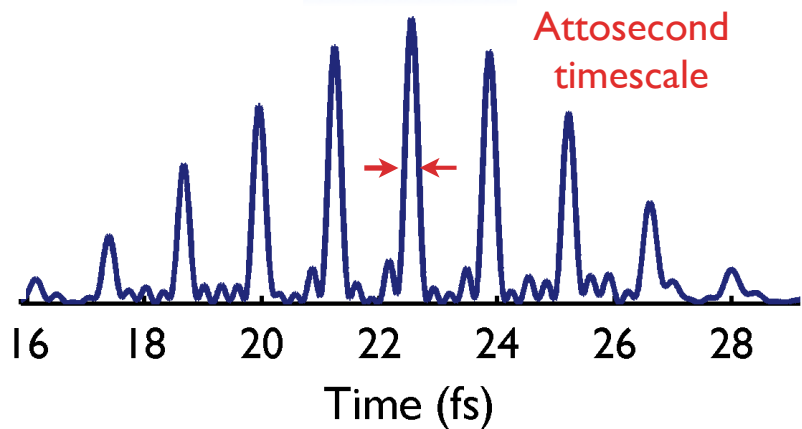
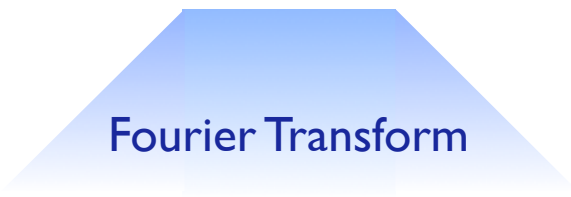
Corkum, PRL 71, 1994 (1993)
Schafer et al. PRL 70 1599 (1993)

High-order Harmonic Generation



$$\hbar\omega_{\max} \approx I_p + 3.17U_p \propto I\lambda^2$$

$$U_p = \frac{E_0^2}{4\omega_0^2}$$



Macroscopic Phase-matching

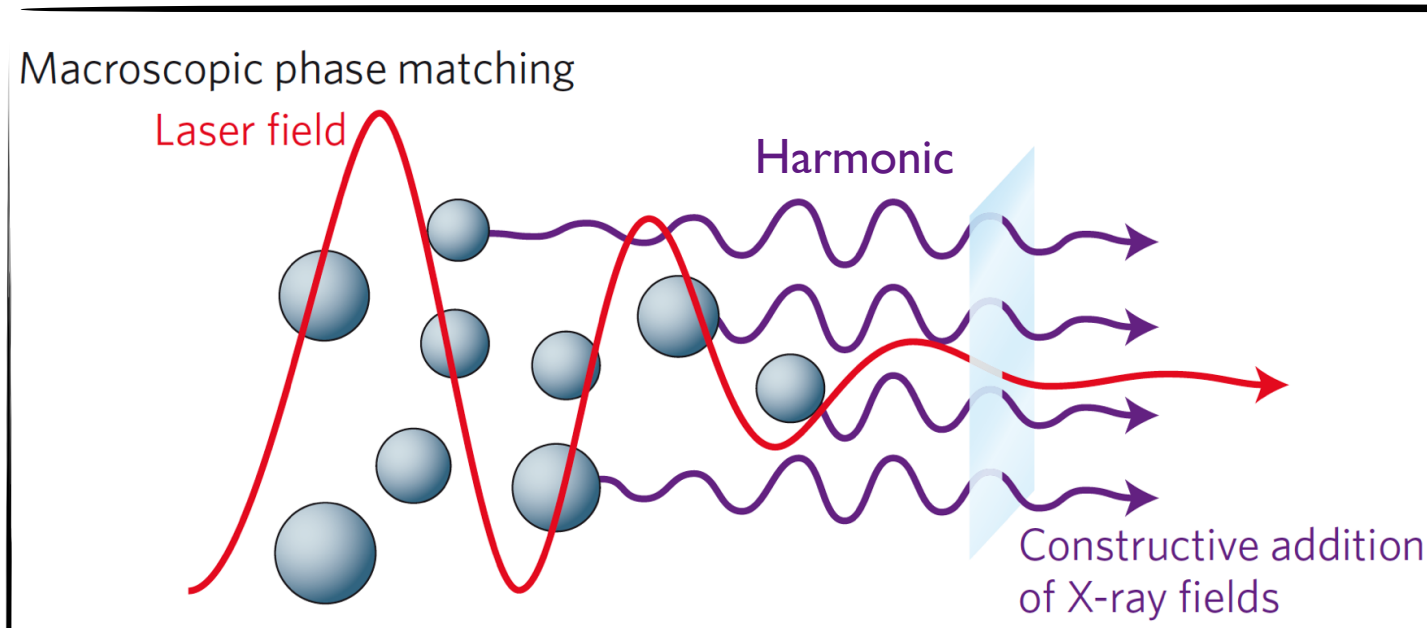


Figure courtesy of
T. Popmintchev

Phase mismatch function: $\Delta k_q = k_q - qk_1$

Perfect
phase-matching

$$\Delta k_q = 0$$

P. Salières, A. L'Huillier, and M. Lewenstein, PRL, 74, 3776 (1995)

P. Balcou, P. Salières, A. L'Huillier, and M. Lewenstein, PRA, 55, 3204 (1996)

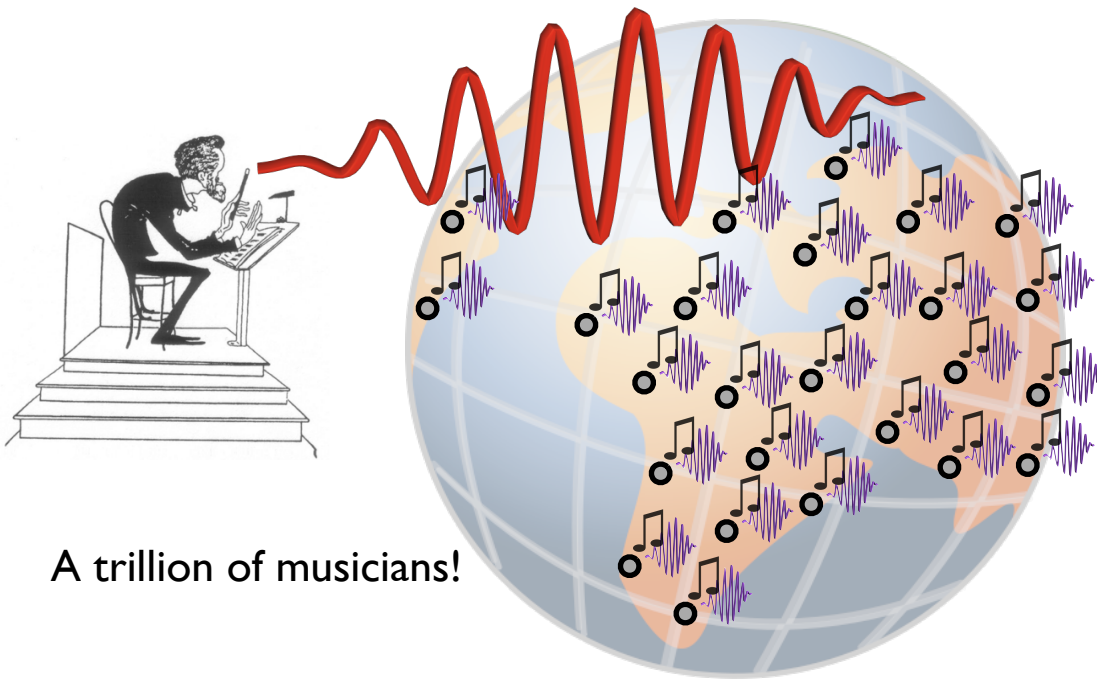
A. Rundquist et al, Science 280, 1412 (1998)

M. B. Gaarde, J. L. Tate, and K.J. Schafer J. Phys. B: At. Mol. Opt. Phys. 41 (2008)

T. Popmintchev et al, PNAS 106, 10516 (2009)

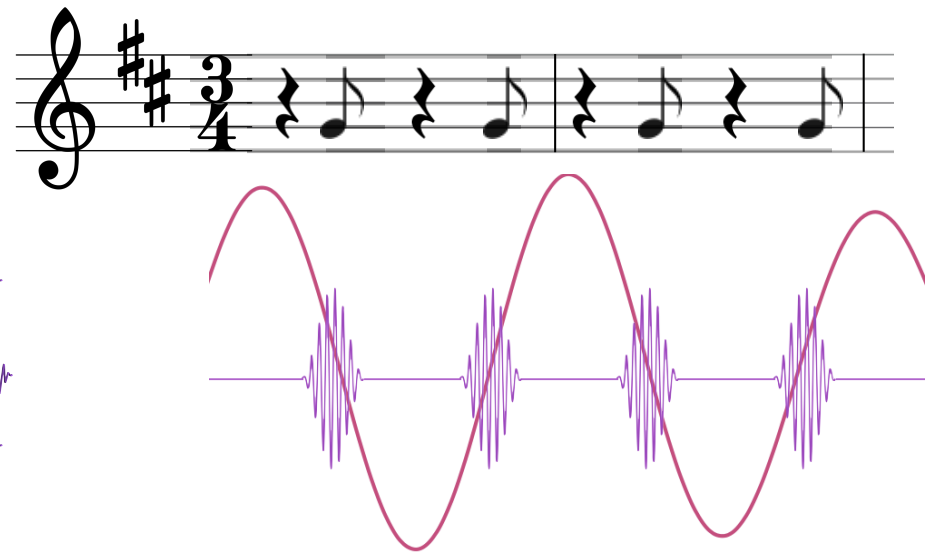
The harmonic orchestra

Laser as the orchestra director...



A trillion of musicians!

... and each atom of the gas as a musician, emitting **attosecond pulses** synchronized with the rhythm dictated by the **laser**



As a result, a *melody of attosecond pulses* is emitted coherently.

Outline

'Generation of keV harmonics and ultrashort waveforms driven by midinfrared lasers'

- **Microscopic HHG: single-atom**
 - Method: SFA+
 - Multiple rescatterings to produce zeptosecond waveforms
- **Macroscopic HHG: phase-matching**
 - Method: Discrete Dipole Approximation
 - Time-gated phase-matching: isolating X-ray attosecond pulses
- **Conclusions**

Microscopic HHG

Single-atom



Quantum description of HHG

Harmonic emission spectrum is proportional to the power spectrum of dipole acceleration

$$I(\omega) \propto |\vec{a}_d(\omega)|^2$$

$$\vec{a}_d(\omega) = \int \vec{a}_d(t) e^{i\omega t} dt$$

$$\vec{a}_d(t) = \frac{1}{m} \langle \psi(t) | \vec{\nabla} V + \vec{E} | \psi(t) \rangle$$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$
$$H = \frac{\vec{p}^2}{2m} + V + e\vec{E} \cdot \vec{r}$$

Exact solution: Time Dependent Schrödinger Equation (TDSE) for $\psi(t)$

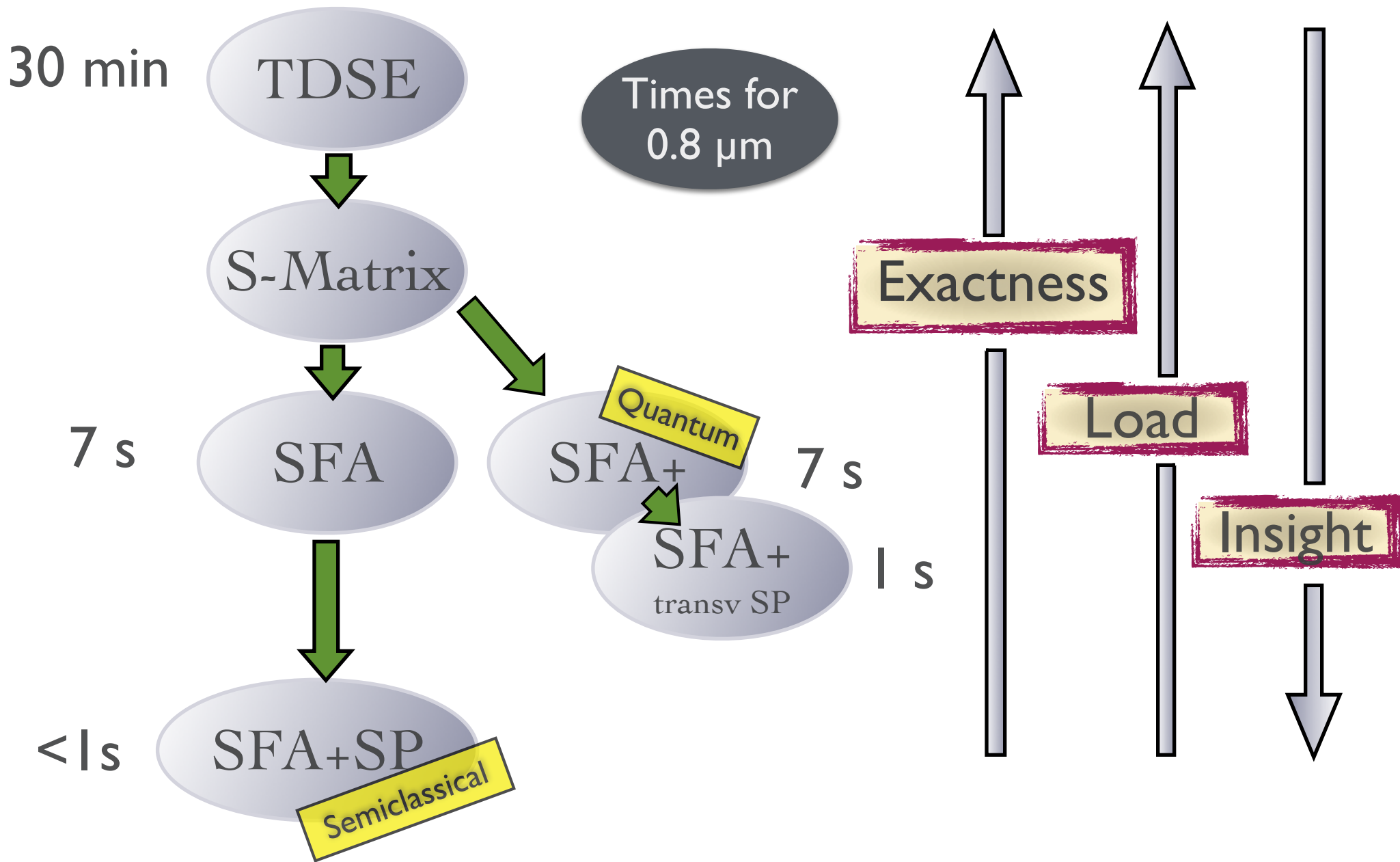
TDSE is very challenging when:

1. Considering **phase-matching** (involves lot of single-atom calculations)
2. **Long wavelengths** (huge spatio-temporal grid)

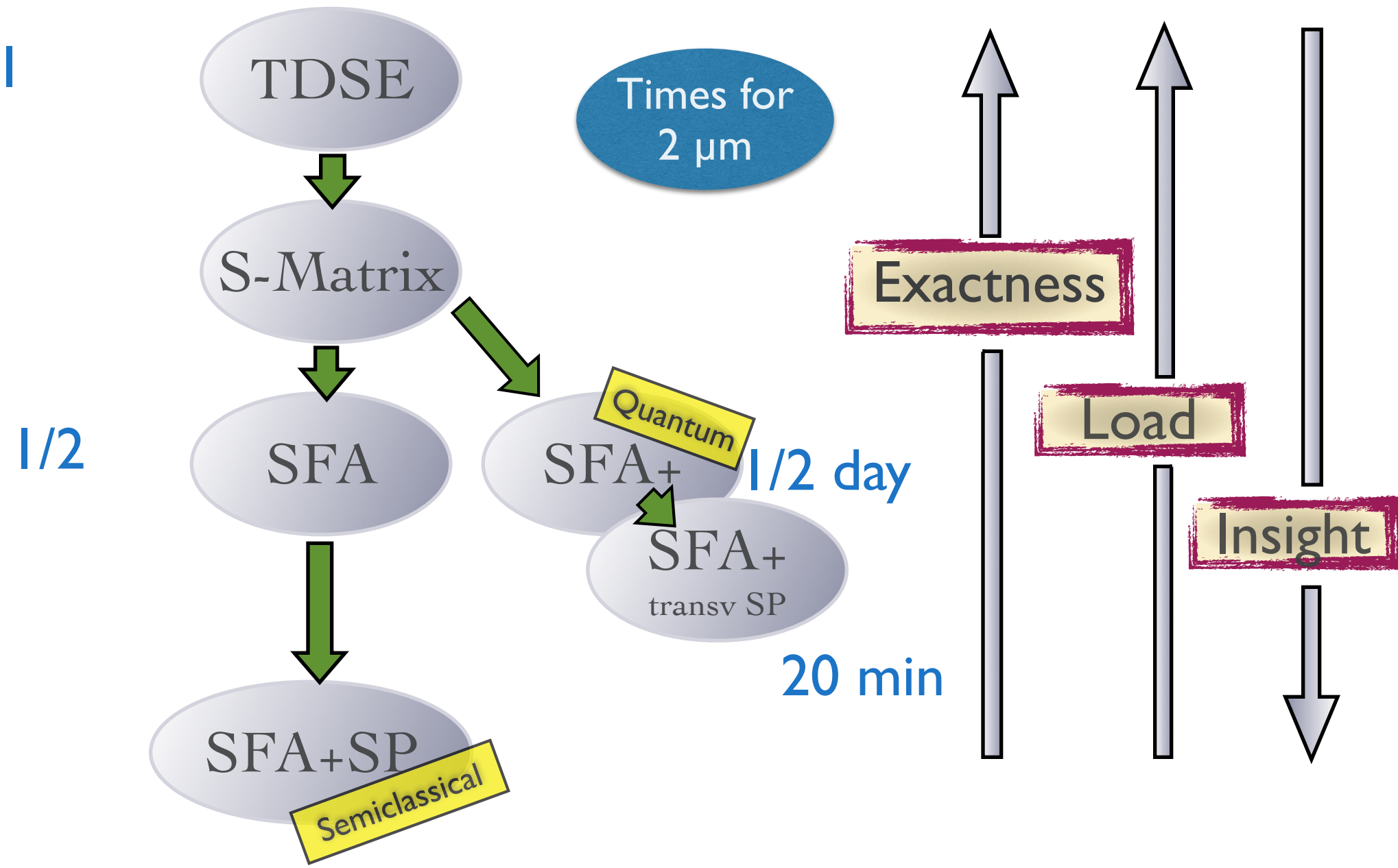
We have to use approximations:

- ✓ The standard way: **SFA** and saddle point M. Lewenstein et al. PRA 49, 2117(1994)
- ✓ Our choice: **SFA+** J.A. Pérez-Hernández, et al, Opt. Express 17, 9891 (2009)

Strategies for computing HHG



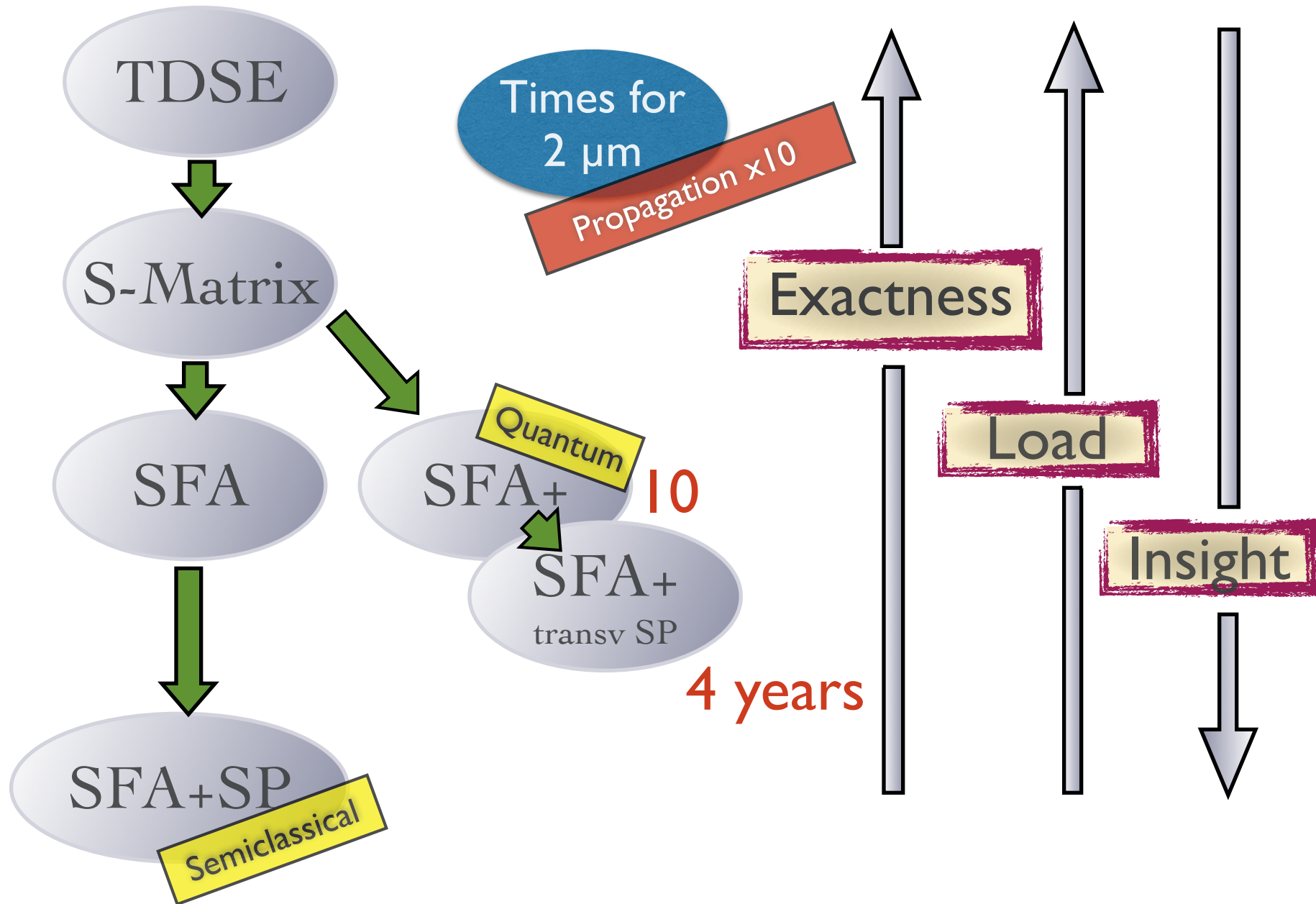
Strategies for computing HHG



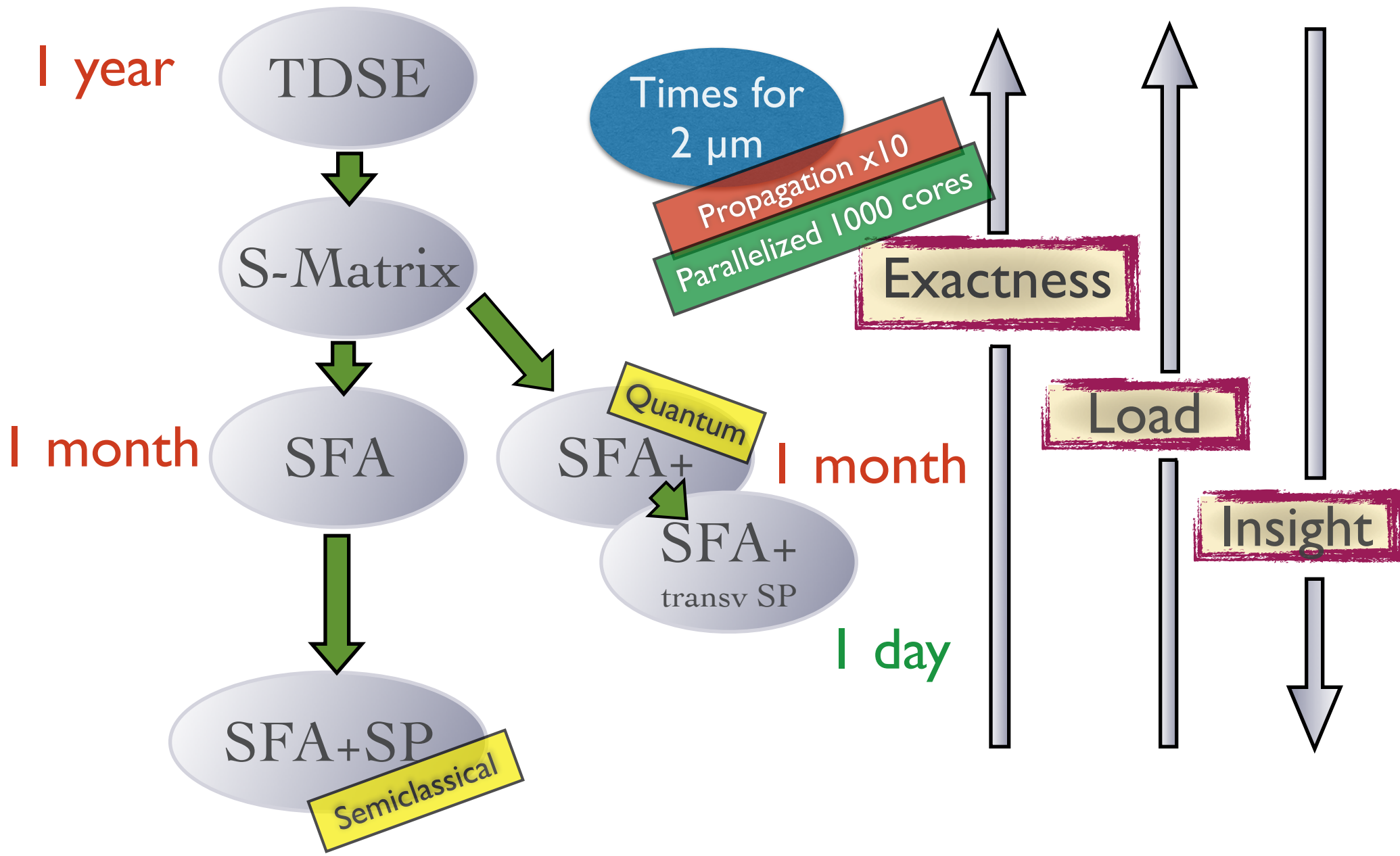
Strategies for computing HHG

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10



Strategies for computing HHG



Principles of SFA+

Strong Field Approximation (SFA)

1. Considering the electrons promoted to the continuum to have no possibility to recombine.
2. Neglecting the the bound-state excitations (field-dressing).
3. Considering the electrons as free particles in the continuum.

$$a(\mathbf{k}, t) = -\frac{i}{\hbar} C_F \int_{t_0}^t dt_1 \langle \phi_0 | \hat{a} | \mathbf{k} \rangle e^{-i\frac{1}{\hbar} S(\mathbf{k}, t, t_1)} V_i(\mathbf{k}, t_1) \langle \mathbf{k} | r^{-n} | \phi_0 \rangle$$

Transition back to ground state Transition into the continuum at time t_1

Evolution as a free electron

$$S(\mathbf{k}, t, t_1) = \int_{t_1}^t [\epsilon(\mathbf{k}, \tau) - \epsilon_0] d\tau$$

$$V_i(t) = -(q/mc)A(t)p_z + q^2/(2mc^2)A^2(t)$$

$$\epsilon(\mathbf{k}, \tau) = \hbar^2 k^2 / 2m - (\hbar q / mc)A(\tau)k_z + q^2 A^2(\tau) / (2mc^2)$$

Extended Field Approximation (SFA+)

To include the possibility of atomic bound-state excitations (field-dressing).

$$a(t) = a_b(t) + a_d(t) + c.c.$$

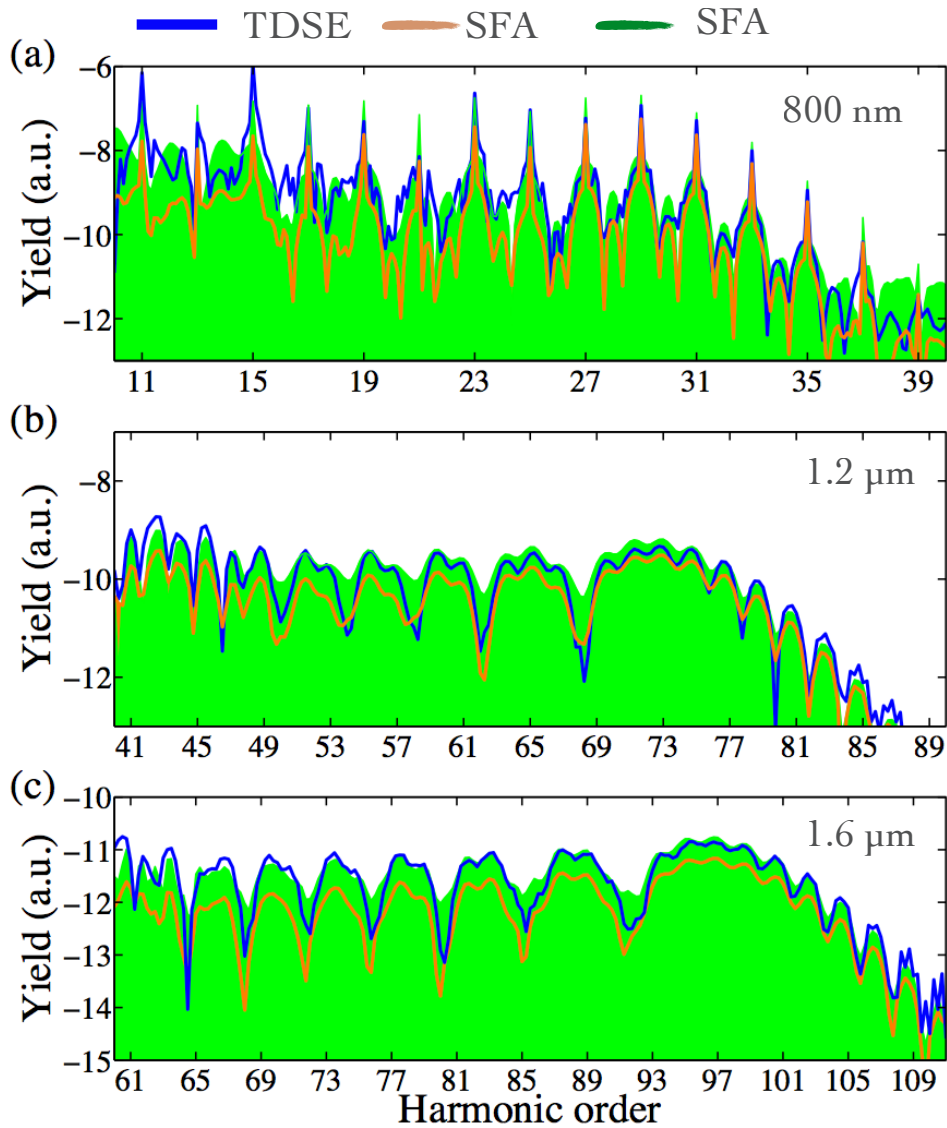
$$a_d(\mathbf{k}, t) \simeq - \left[1 + \frac{k^2/2m - \epsilon_0}{\Delta_s} \right] a_b(\mathbf{k}, t)$$

$$\Delta_s = \langle V_i(t) \rangle = (1/\delta t_s) \int_{t-\delta t_s}^t [-(q\hbar/mc)A(\tau)k_z + (q^2/2mc^2)A^2(\tau)] d\tau$$

J.A. Pérez-Hernández, L. Roso and L. Plaja, Opt. Express 17, 9891 (2009)

J.A. Pérez-Hernández, C. Hernández-García, J. Ramos, E. Conejero Jarque, L. Plaja, and L. Roso, Springer Series in Chemical Physics Vol. 100 (Springer, New York, 2011), Chap. 7, p. 145

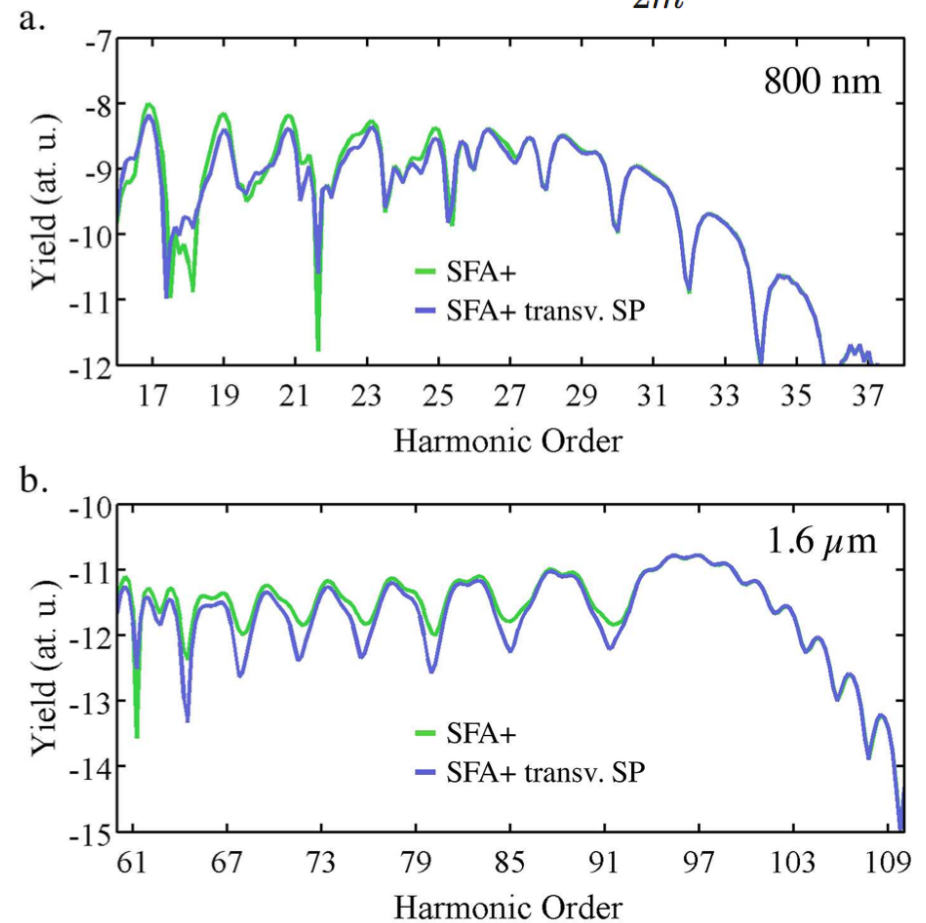
Principles of SFA+



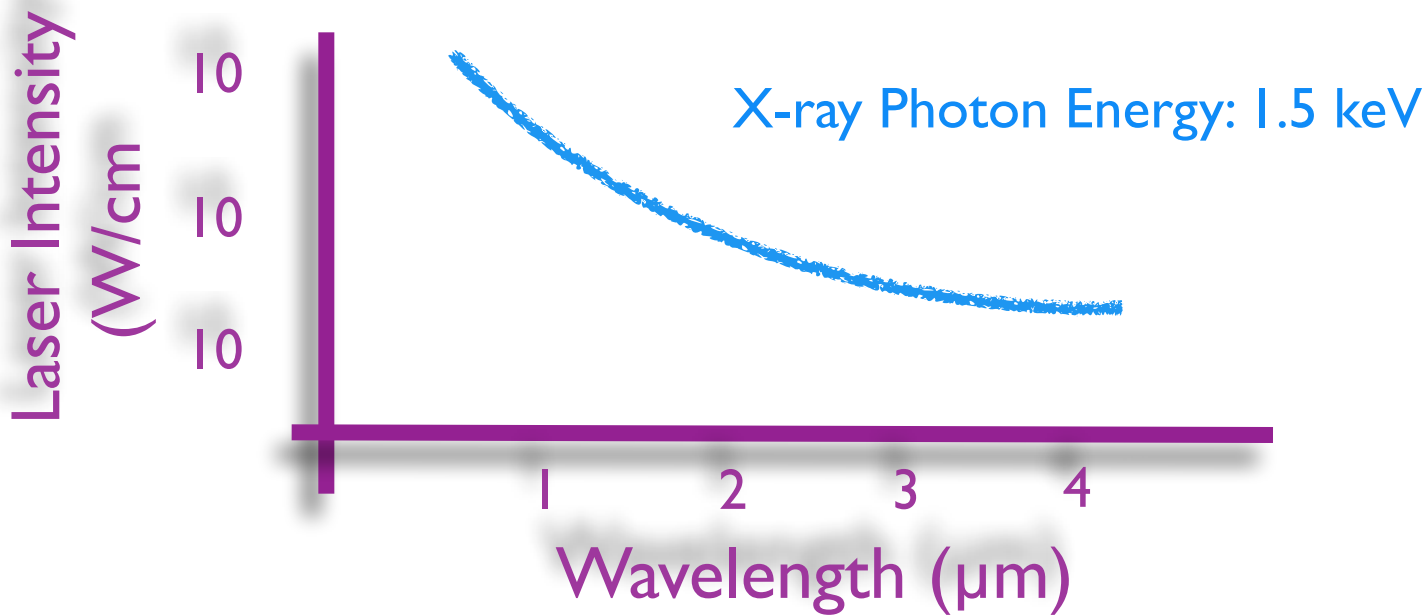
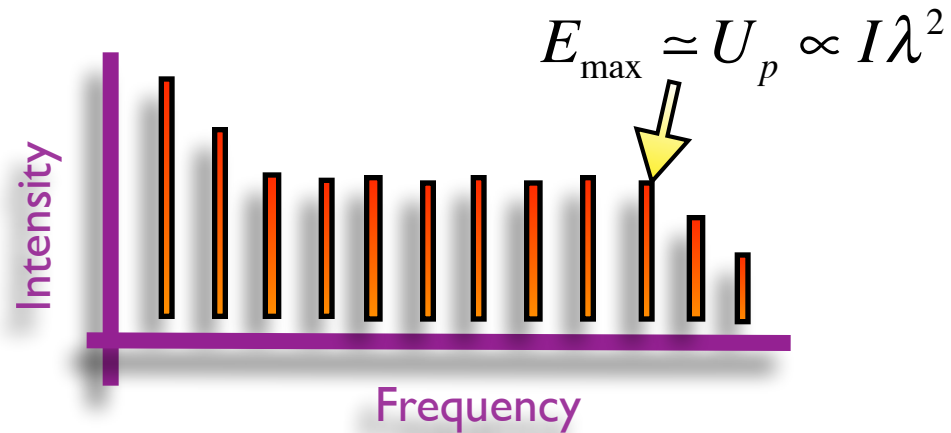
SFA+ with transversal saddle-point

$$\nabla_{k_\rho} S(\mathbf{k}, t, t_1) = 0$$

$$S(\mathbf{k}, t, t_1) \simeq S(k_z, k_\rho^{st}, t, t_1) + \frac{\hbar^2}{2m} (t - t_1) k_\rho^2$$

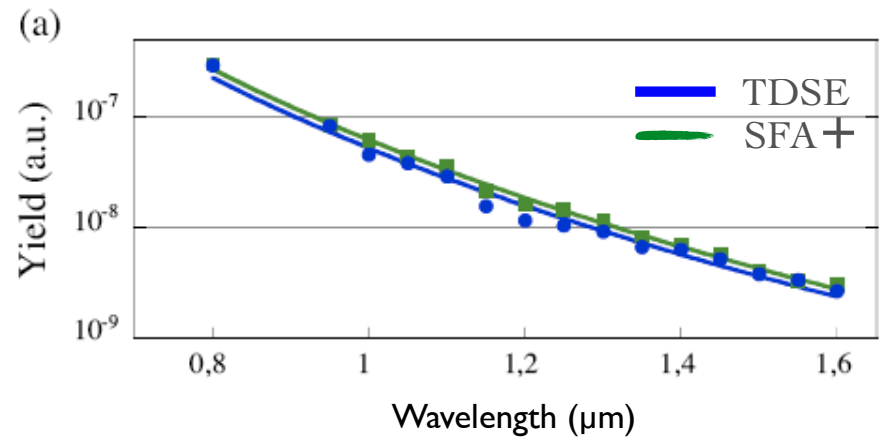
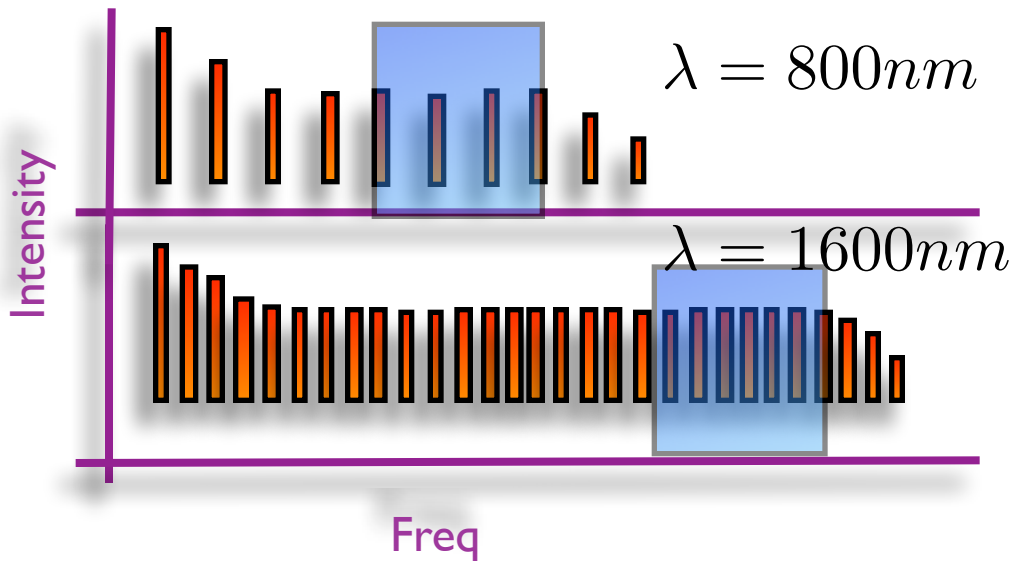


Obtaining X-rays from HHG



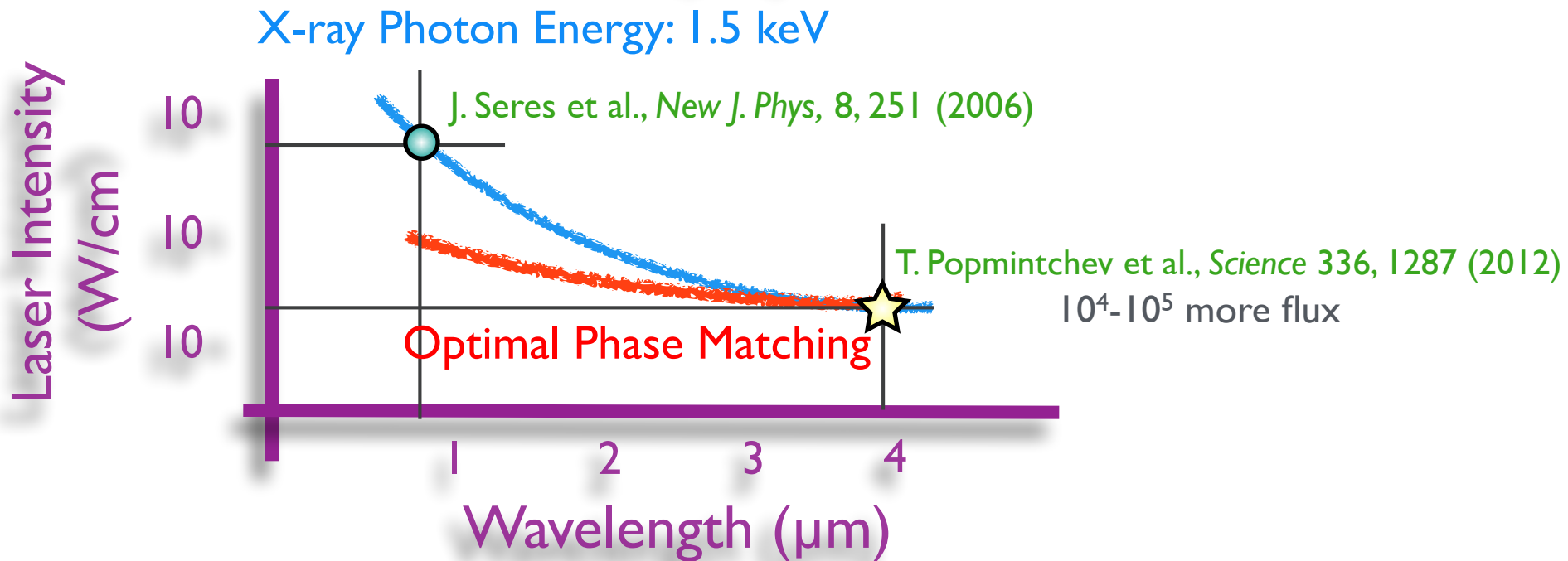
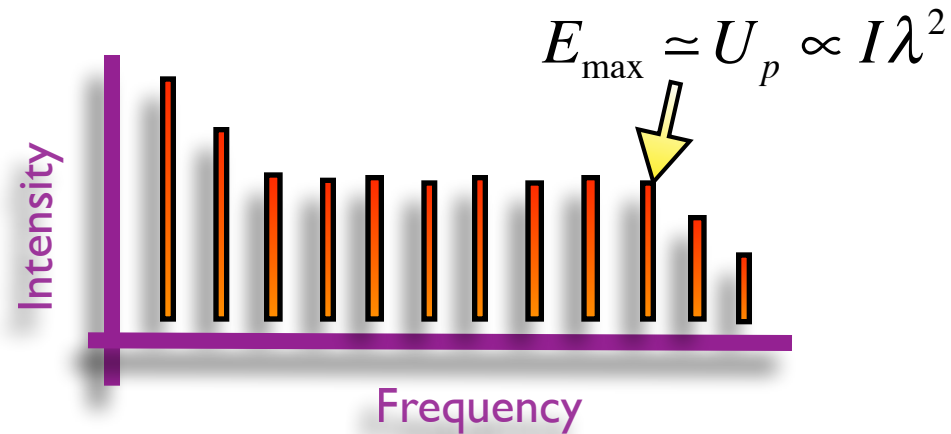
Unfavorable yield scaling law with wavelength

HHG yield scaling law $\lambda^{-5} - \lambda^{-6}$



- J. L. Tate, T. Augustine, H. G. Muller, P. Salières, P. Agostini, and L. F. DiMauro, PRL 98, 013901 (2007)
K. Schiessl, K.L. Ishikawa, E. Person, and J. Burgdörfer, PRL 99, 253903 (2007)
M.V. Frolov, N.L. Manakov, and A. Starace, PRL 100, 173001 (2008)
J.A. Pérez-Hernández, L. Roso and L. Plaja, Opt. Express 17, 9891 (2009)

Obtaining X-rays from HHG



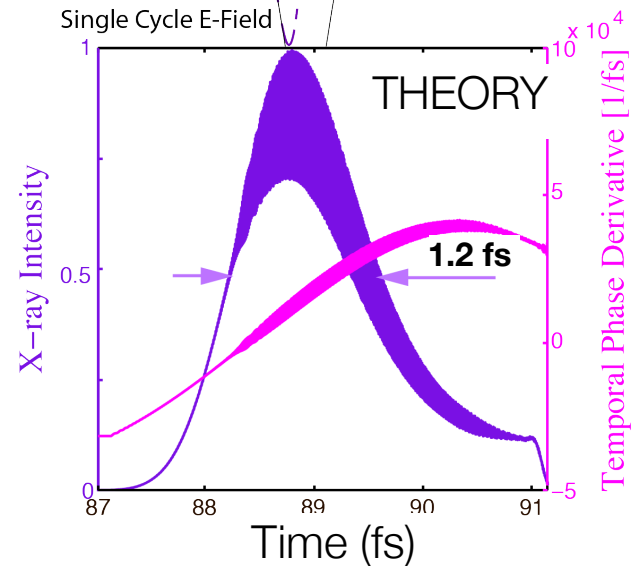
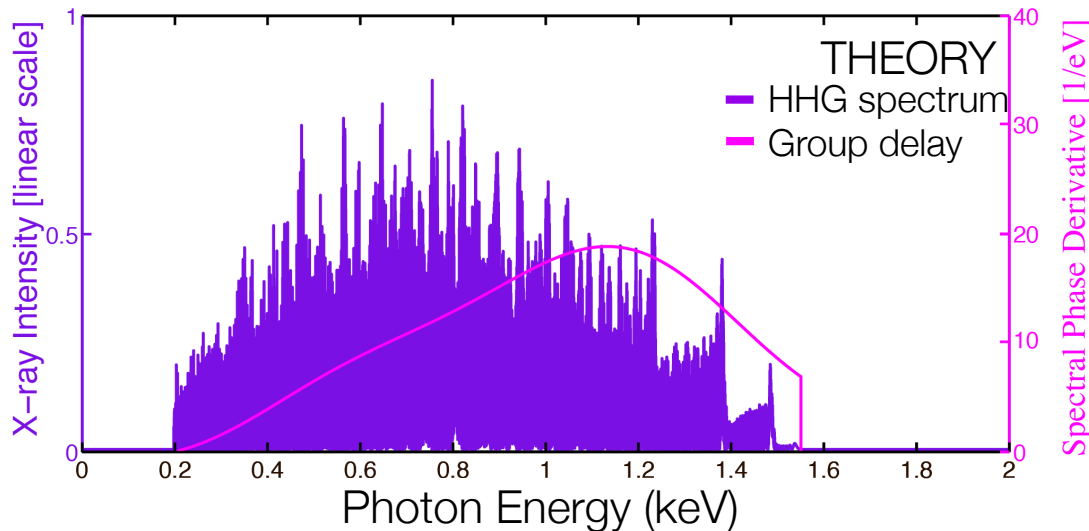
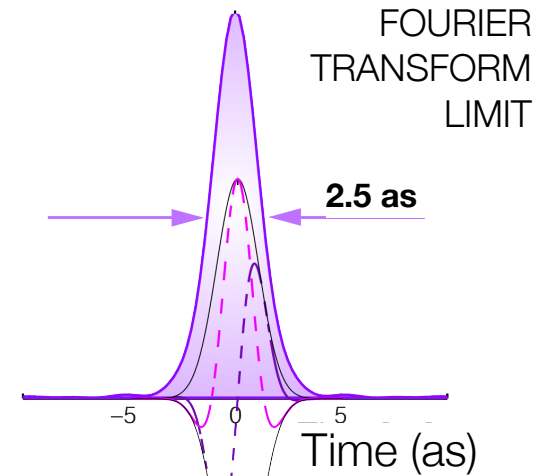
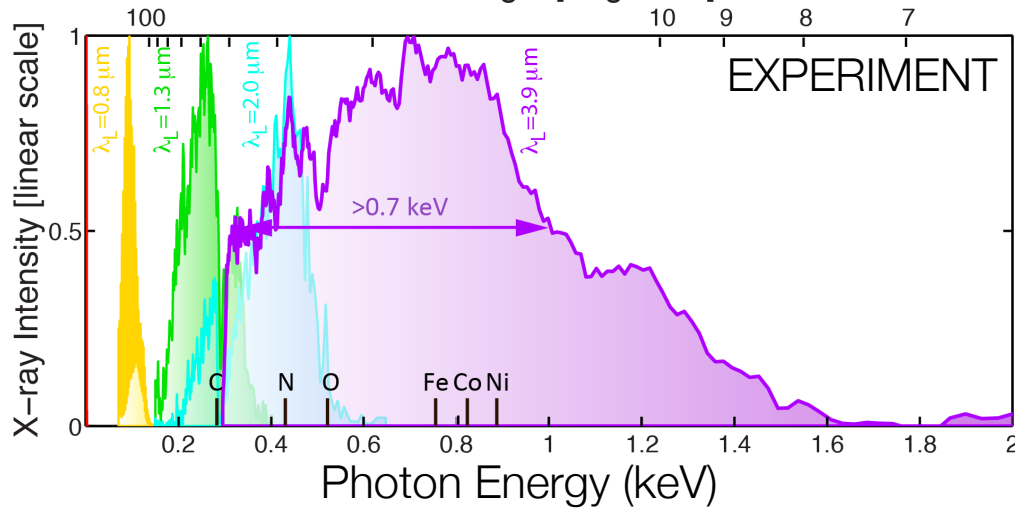
Phase-matched X-rays



Bright Coherent Ultrahigh Harmonics in the keV X-ray Regime from Mid-Infrared Femtosecond Lasers
 Tenio Popmintchev *et al.*
Science **336**, 1287 (2012);
 DOI: 10.1126/science.1218497



Wavelength [Angstrom]

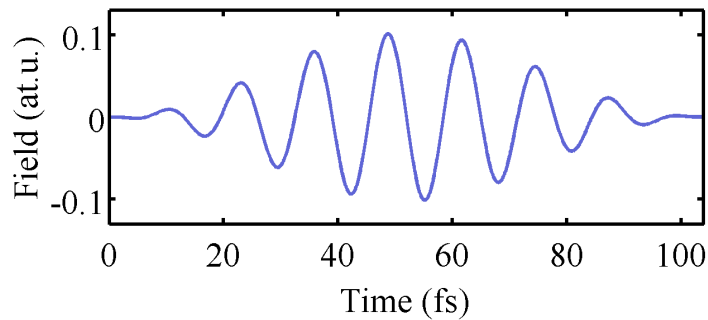


Single atom calculations at 4 μm

$$\lambda = 3.9 \mu\text{m}$$

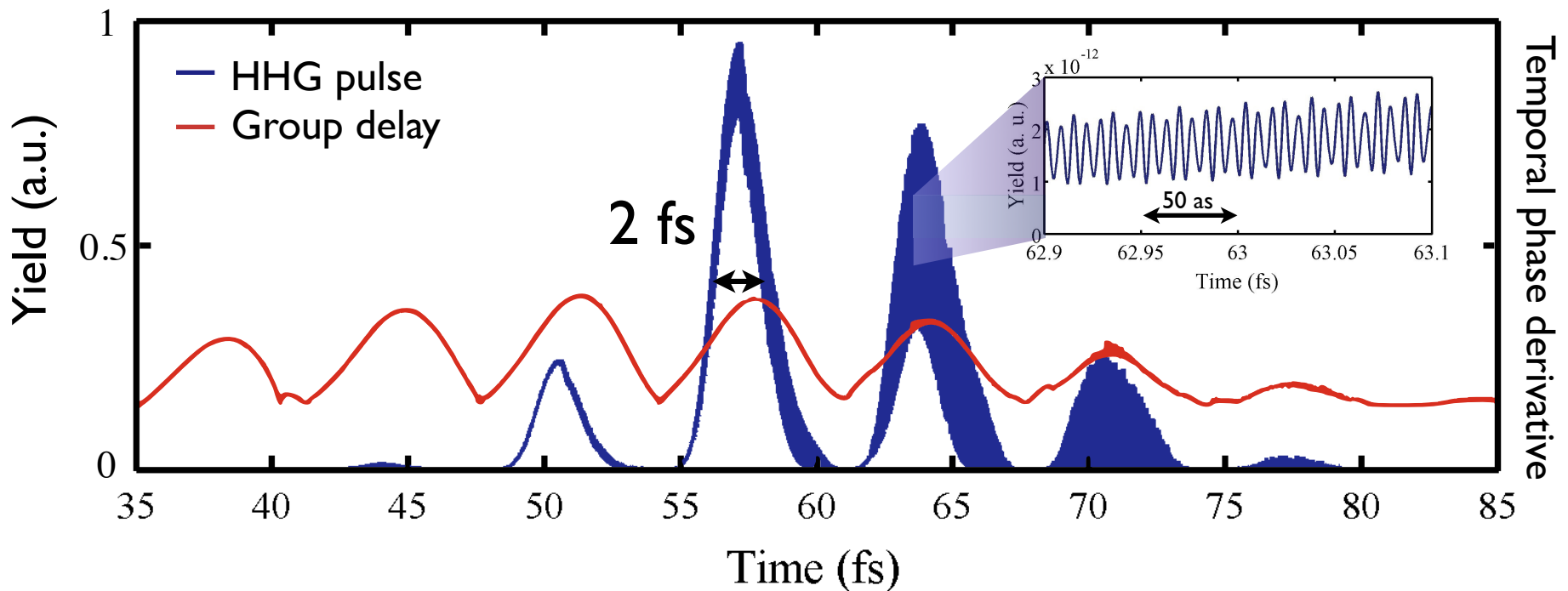
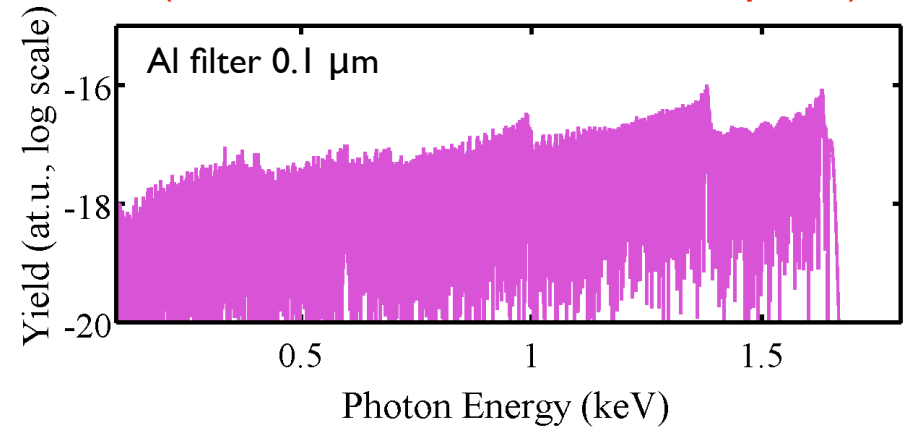
Laser field: $I = 3.6 \times 10^{14} \text{ W/cm}^2$

3 cycles (38 fs)

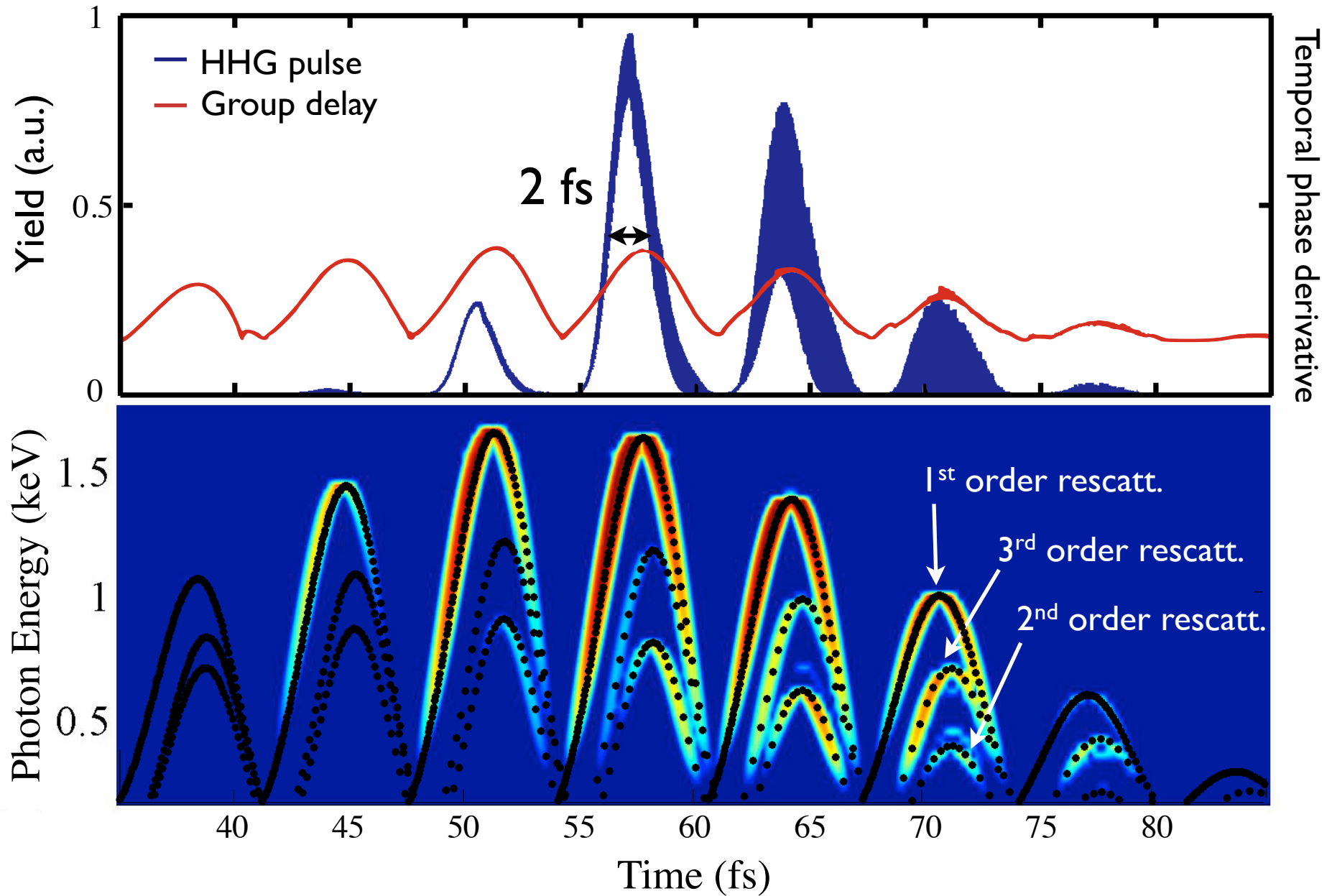


HHG spectrum

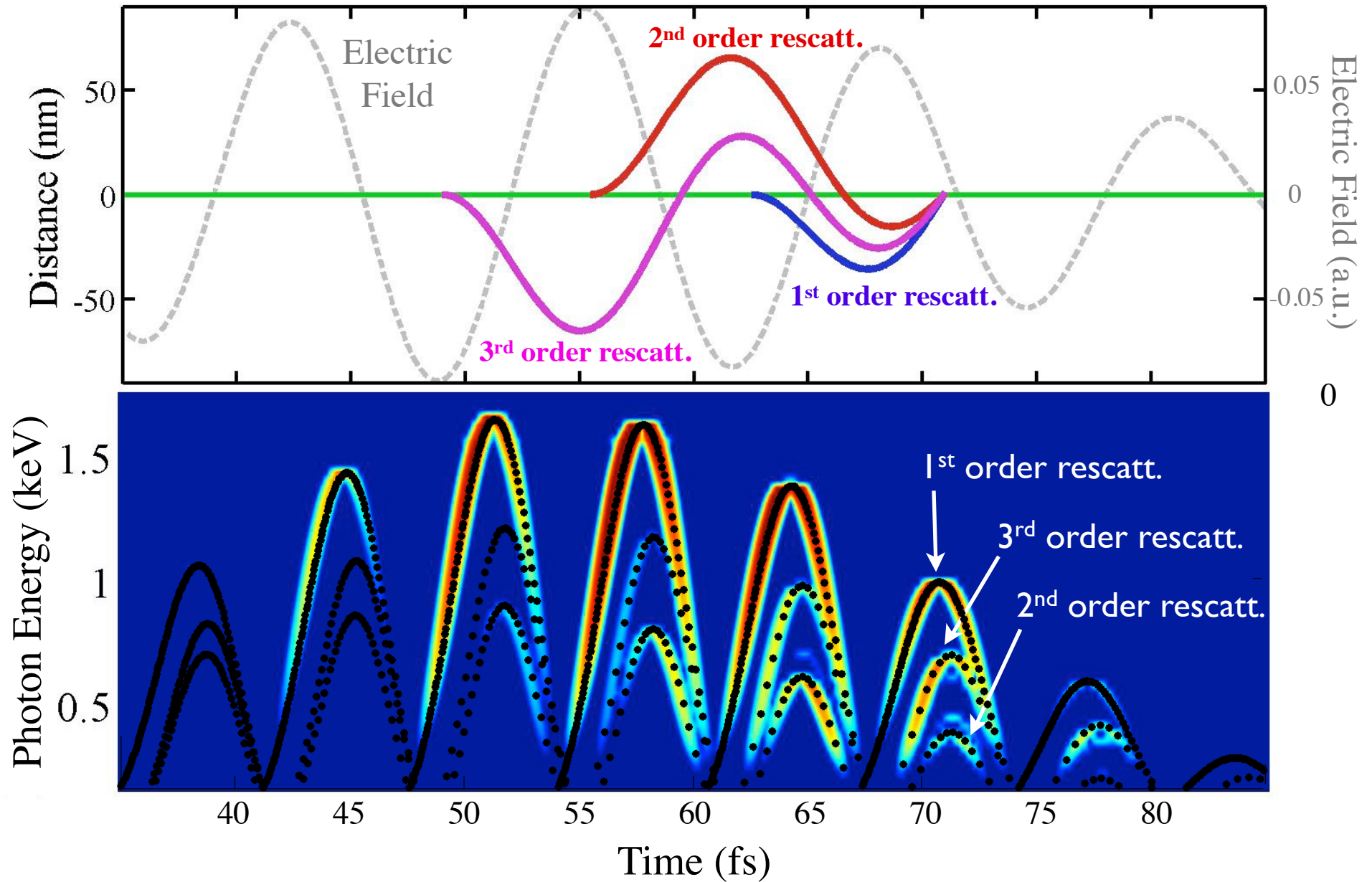
(SFA+ and transversal saddle point)



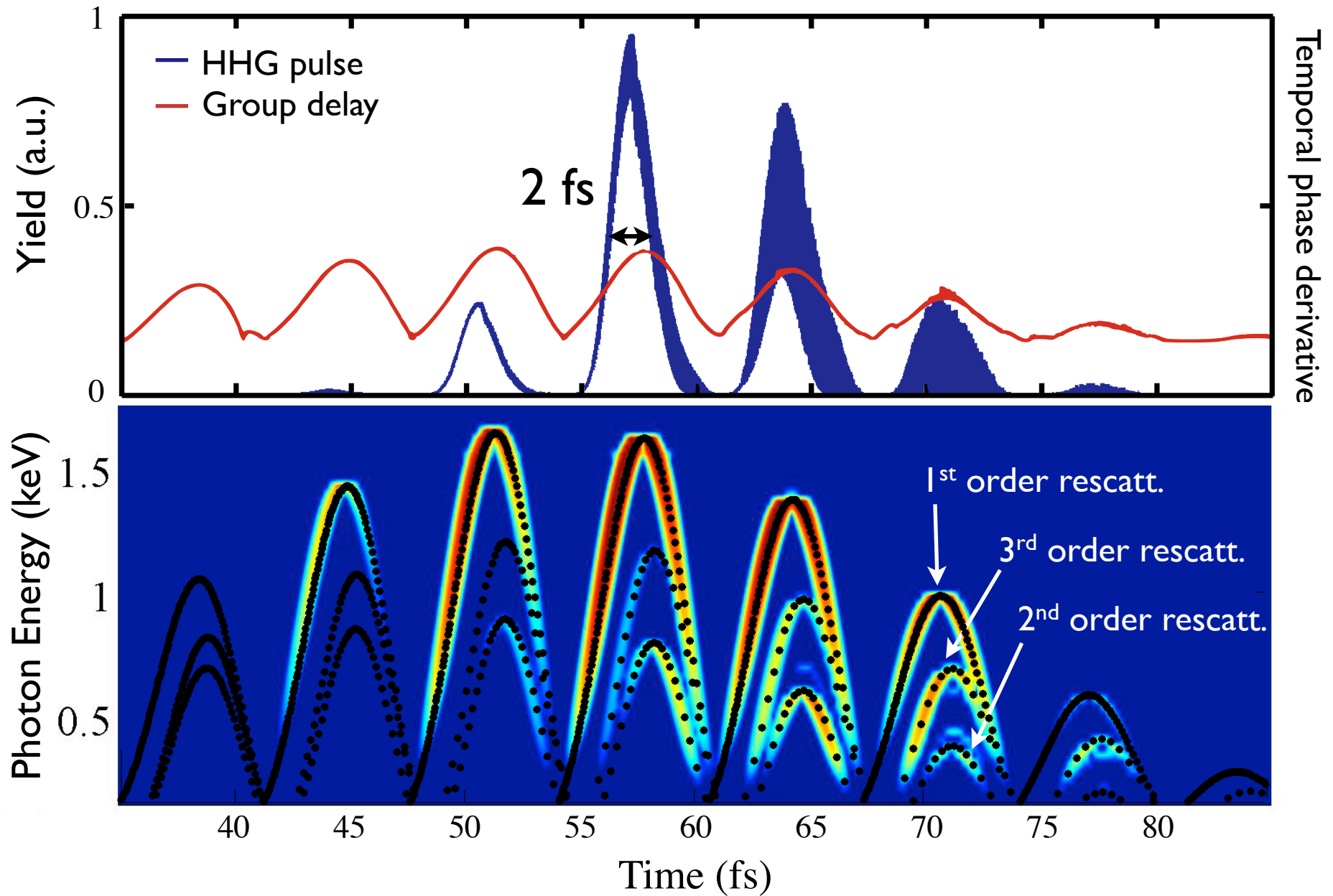
High-order rescattering effects



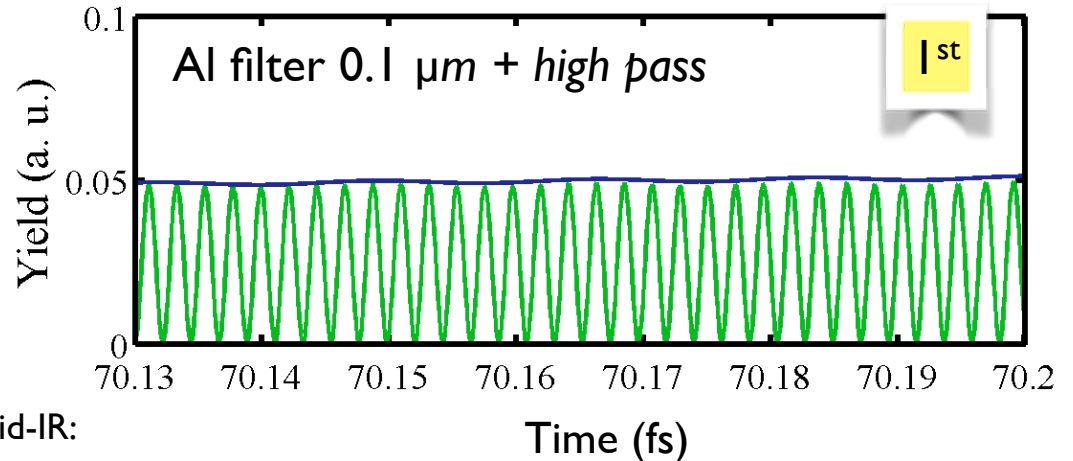
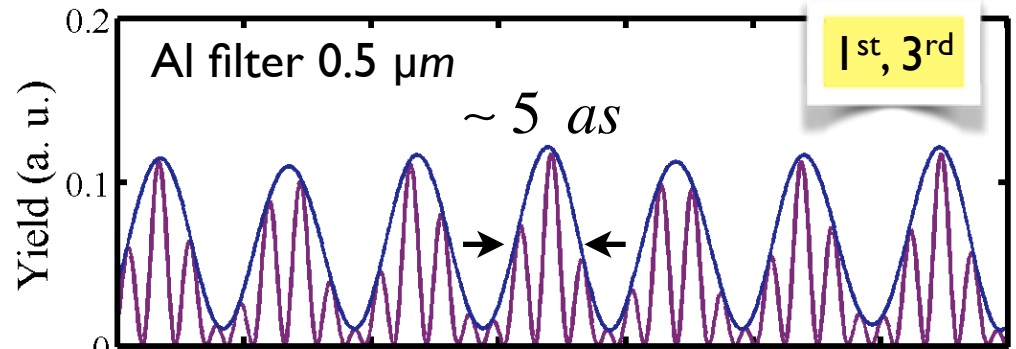
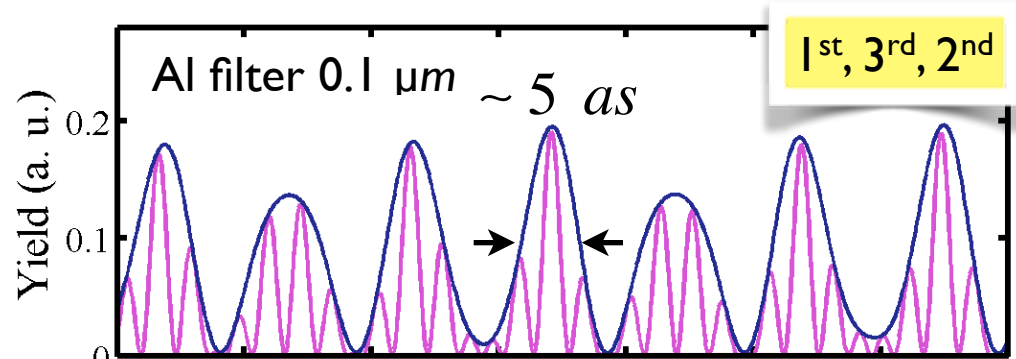
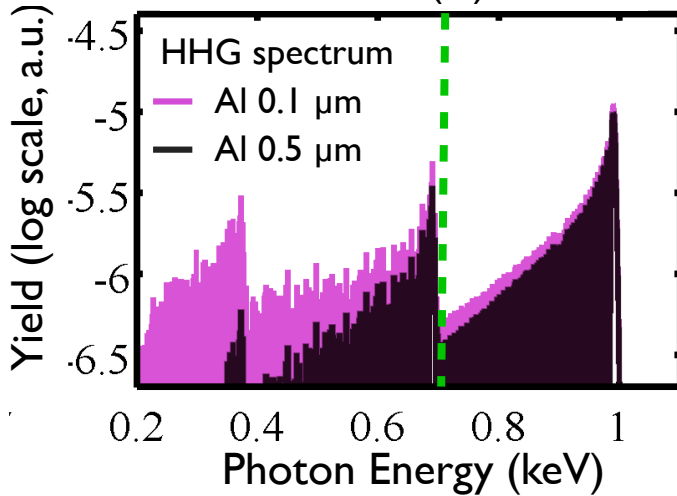
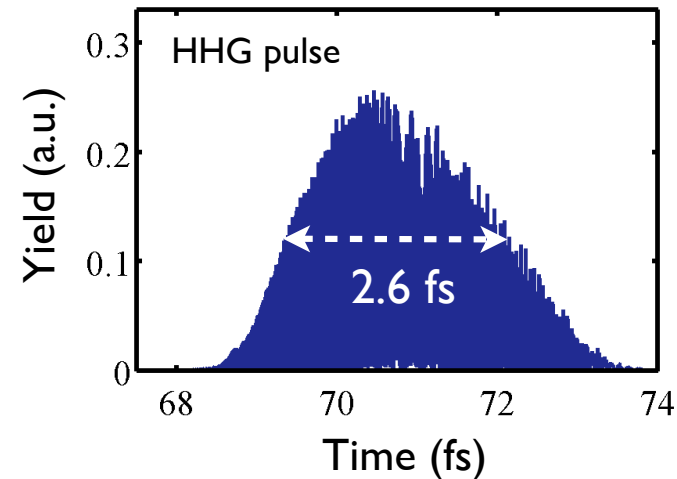
High-order rescattering effects



High-order rescattering effects



Attosecond pulse trains in the keV



C. Hernández-García, J.A. Pérez-Hernández, T. Popmintchev, M. Murnane, H. Kapteyn, A. Jaron-Becker, A. Becker, and L. Plaja, PRL 111, 033002 (2013)

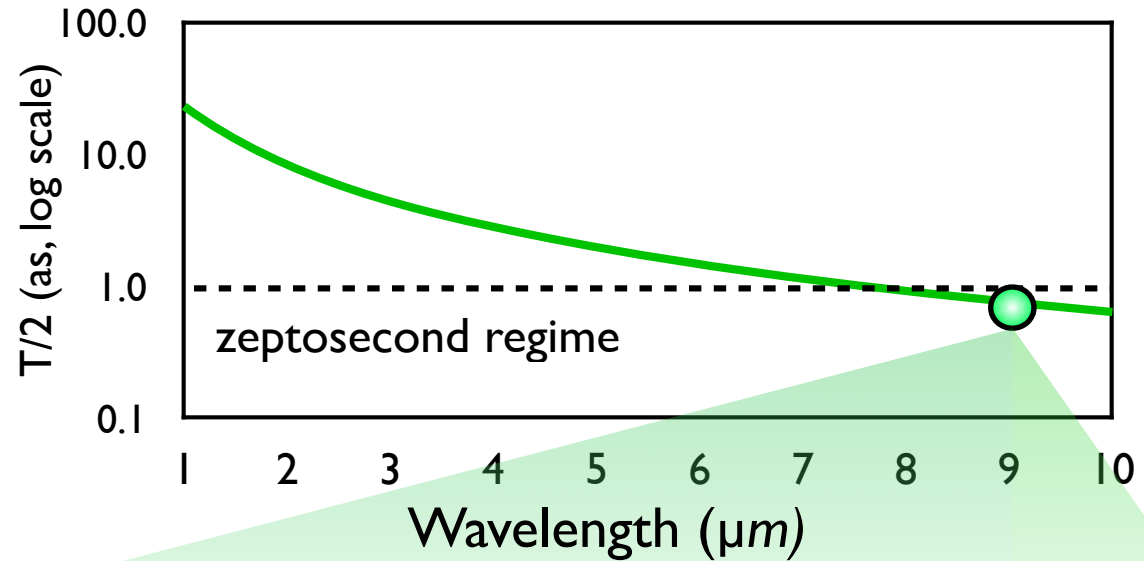
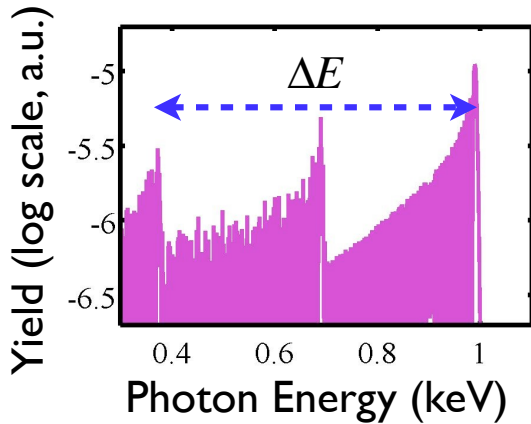
Previous works involving multiple rescatterings using mid-IR:

J. L. Tate, et al. PRL 98, 013901 (2007)

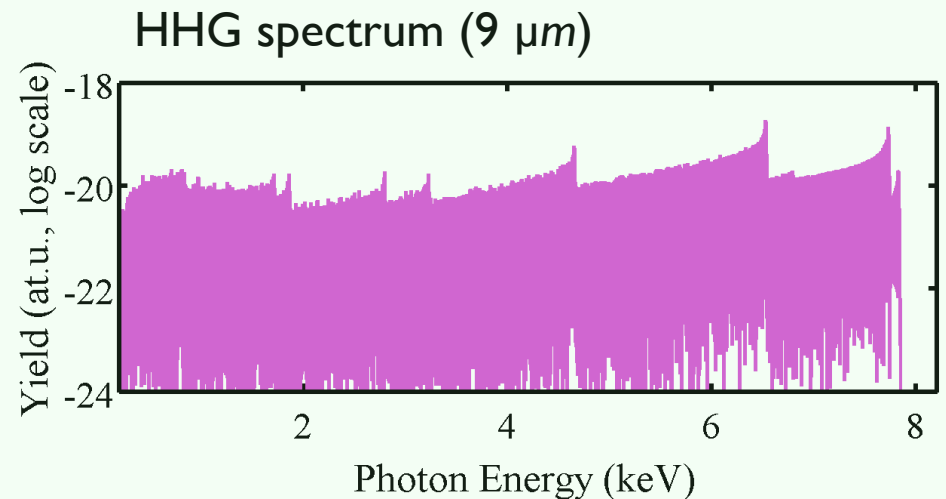
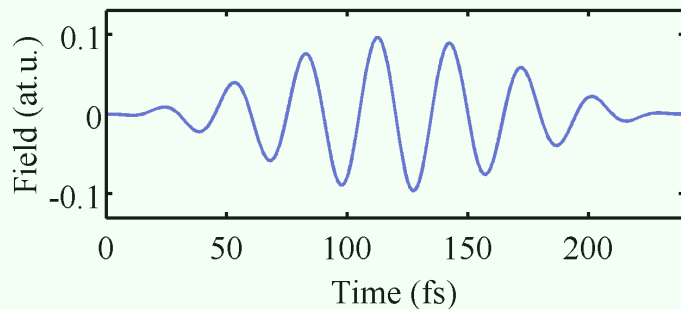
D. Hickstein et al. PRL 109, 073004 (2012)

Towards zeptosecond pulse trains

$$T = \frac{2\pi}{\Delta E}$$

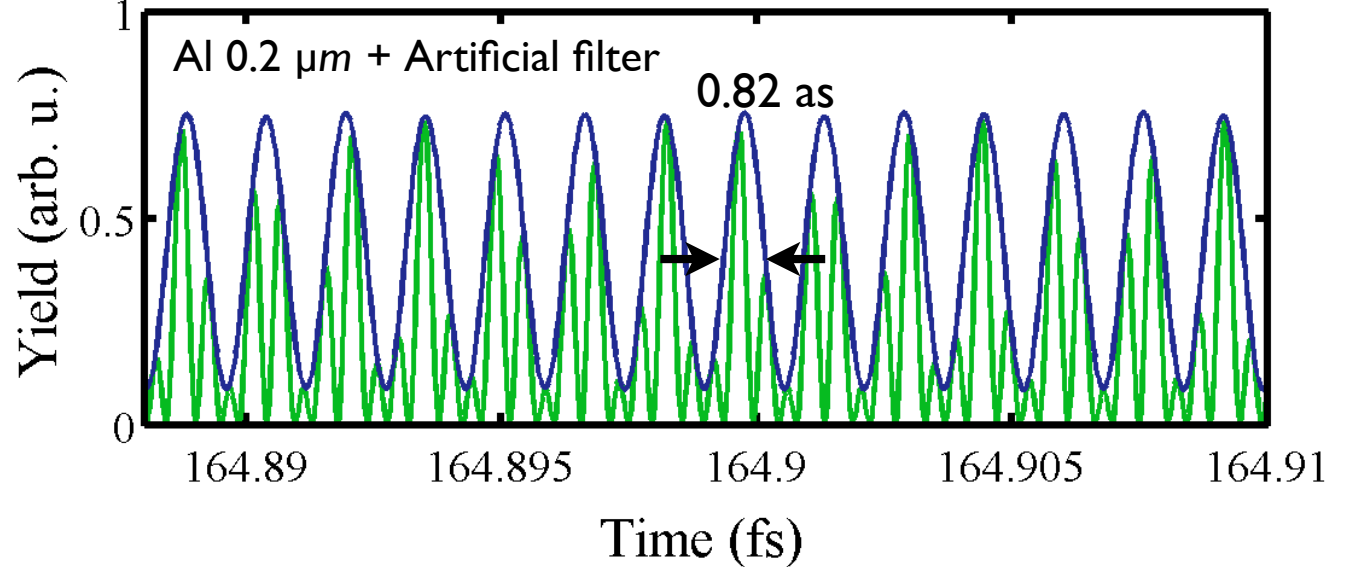
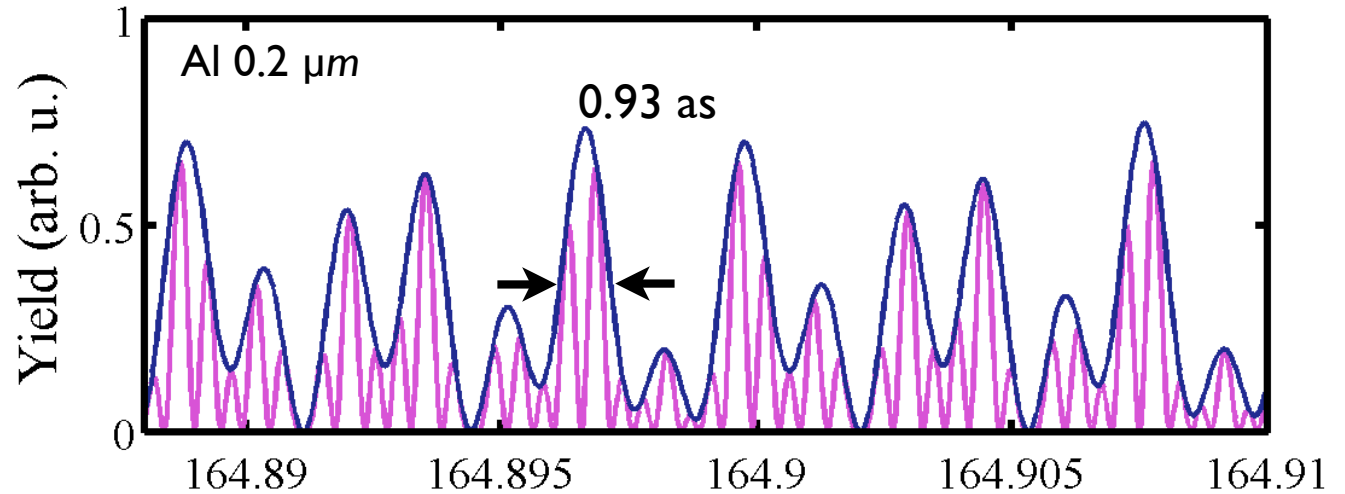
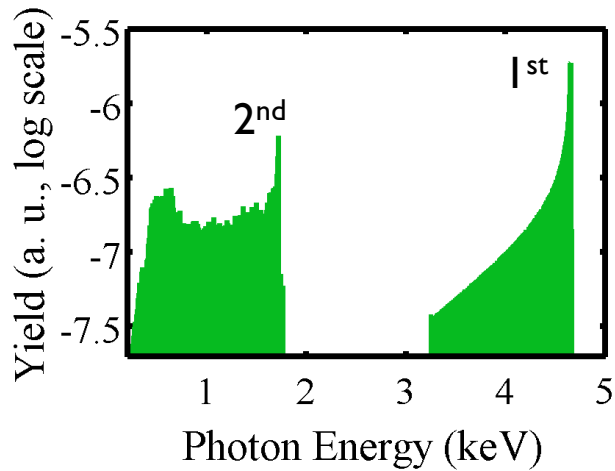
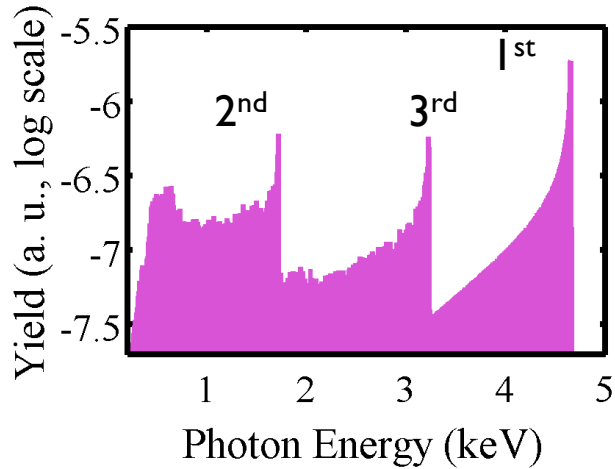


$\lambda = 9 \mu\text{m}$
Laser field: $I = 3.16 \times 10^{14} \text{ W/cm}^2$
3 cycles (90 fs)



Towards zeptosecond pulse trains

$$\lambda = 9 \mu\text{m}$$

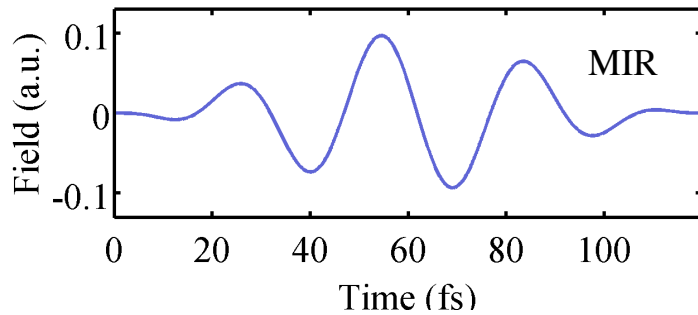


Zeptosecond waveforms

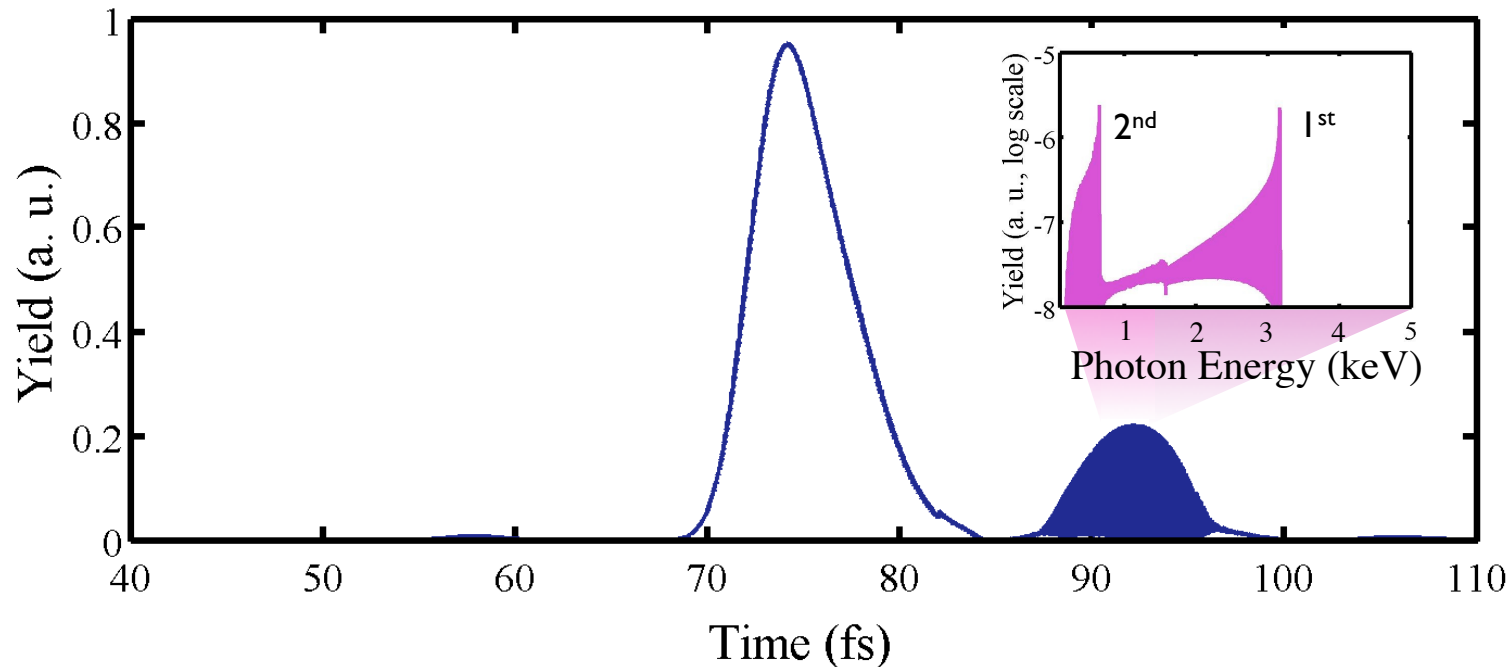
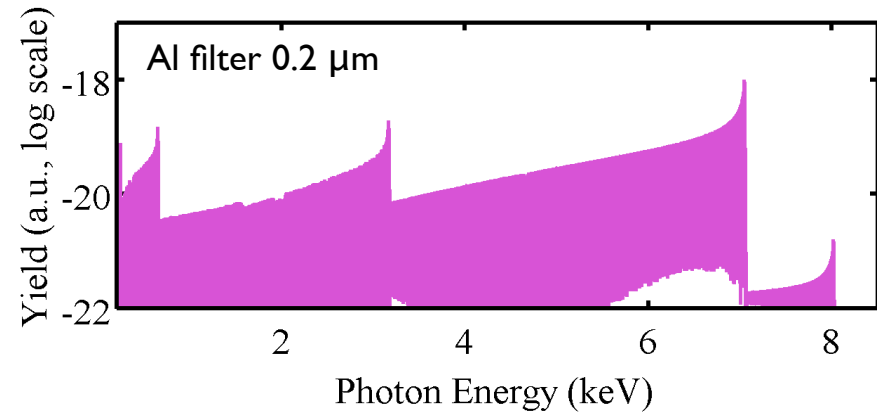
$$\lambda = 9 \mu\text{m}$$

Laser field: $I = 3.4 \times 10^{14} \text{ W/cm}^2$

1.44 cycles (43 fs)

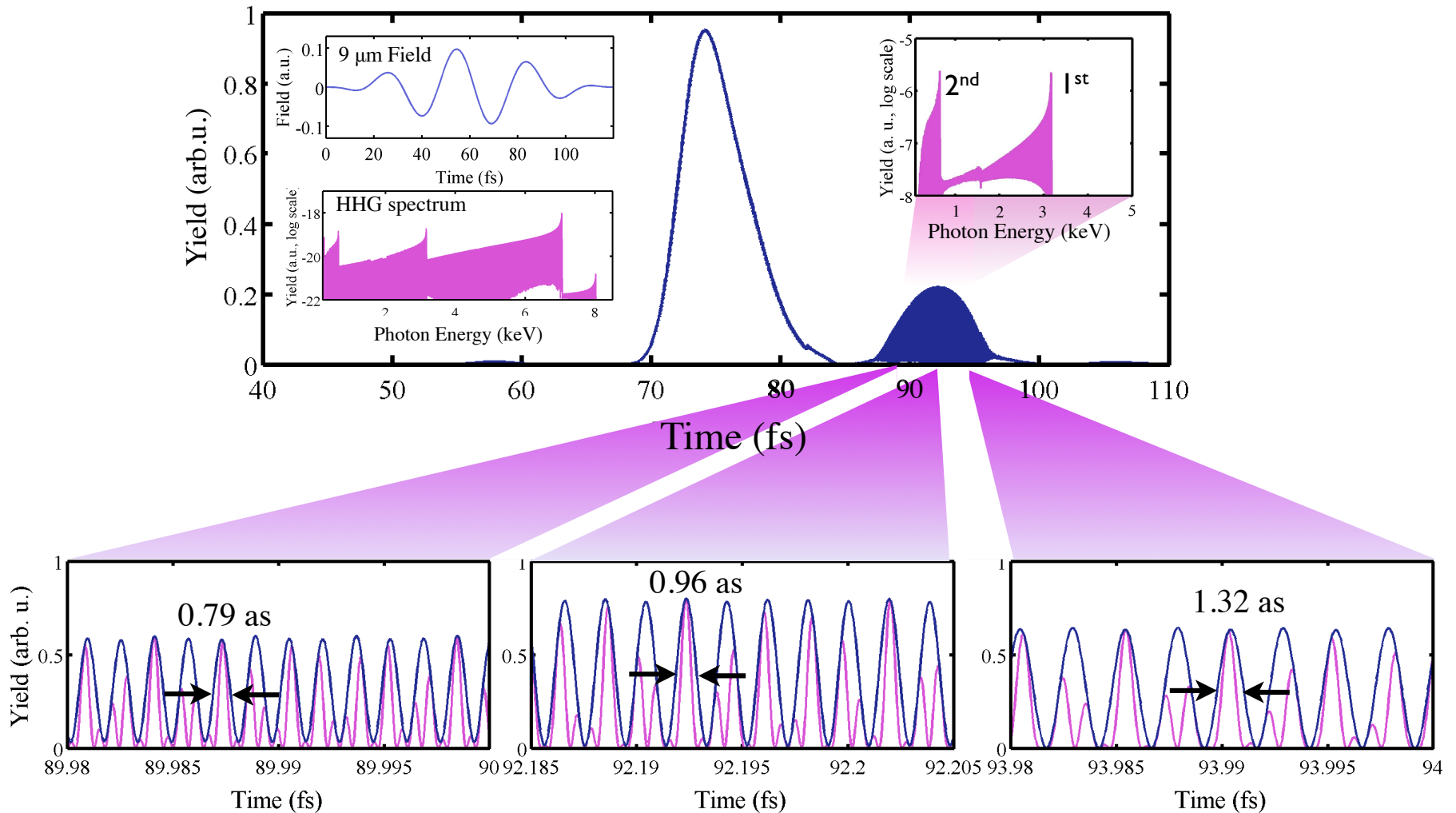


HHG spectrum



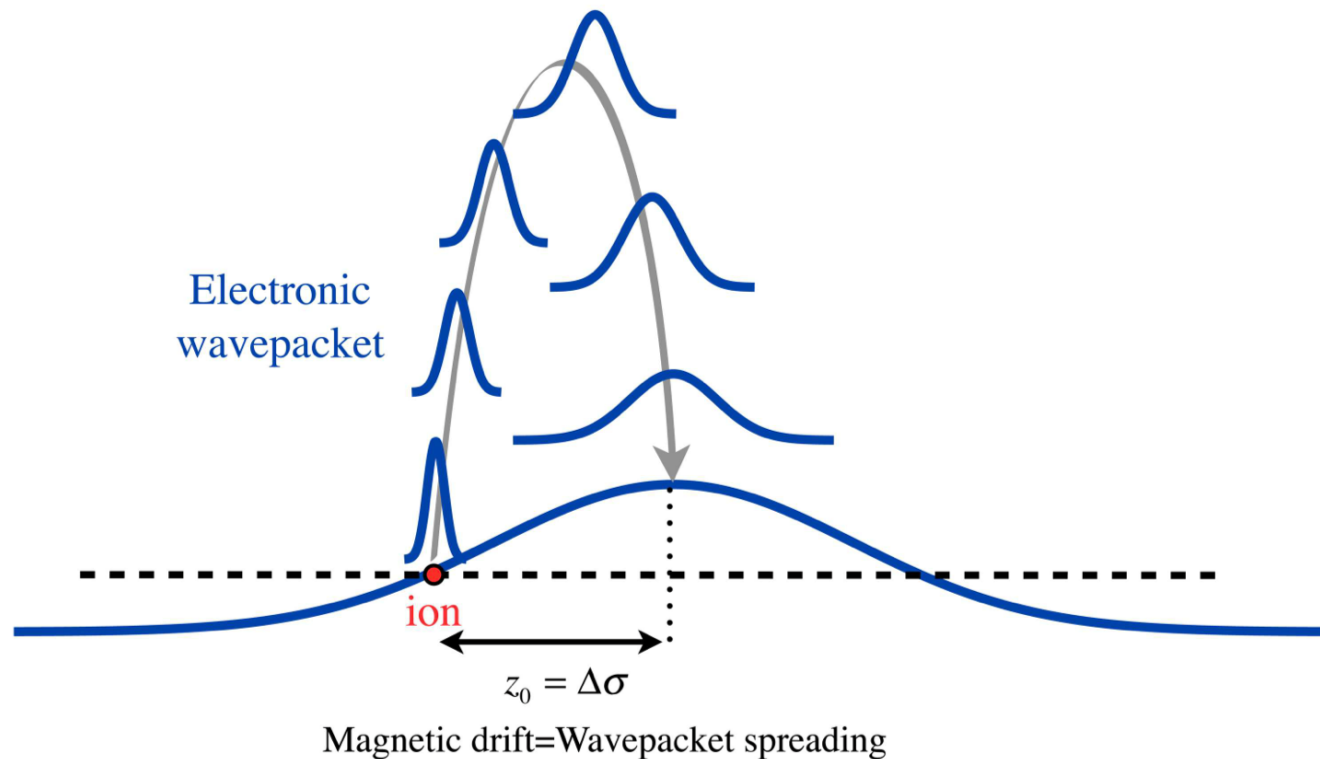
Zeptosecond waveforms

$$\lambda = 9 \mu m$$



Why not even longer wavelengths?

The magnetic drift mitigates the harmonic yield



The yield would drop by an order of magnitude at 10 μm

Macroscopic HHG

Phase-matching effects



Macroscopic Phase-matching

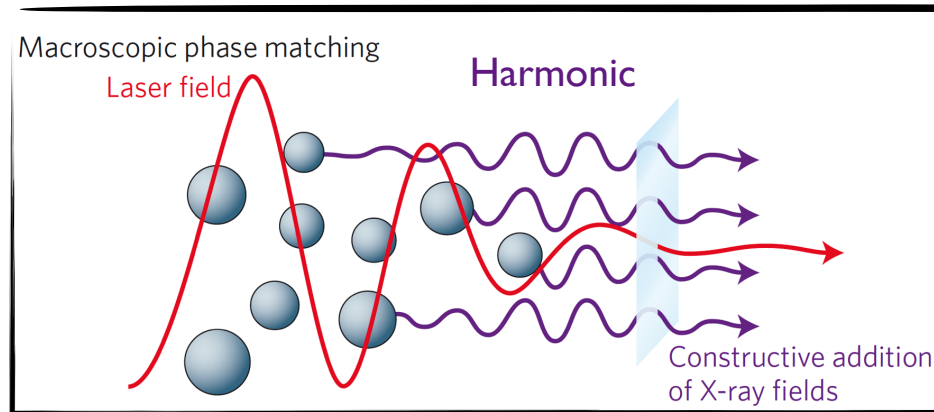


Figure courtesy of
T. Popmintchev

Electric field of the q th-order harmonic:

$$E_q(\mathbf{r}) \propto \left[|d_q^j(\mathbf{r})| e^{i\phi_q^j(\mathbf{r})} \right] e^{i(\mathbf{r}_d - \mathbf{r})\mathbf{k}_q}$$

$|d_q^j(\mathbf{r})|$ spectral amplitude of the single-atom dipole, and
 $\phi_q^j(\mathbf{r})$ its phase

Phase mismatch: $\Delta\mathbf{k}_q = \mathbf{k}_q - \nabla\phi_q^j = k_q\mathbf{e}_z - \frac{\partial\phi_q^j}{\partial z}\mathbf{e}_z - \frac{\partial\phi_q^j}{\partial\rho}\mathbf{e}_\rho$

Perfect
phase-matching
 $\Delta k_q = 0$

$$\Delta k_q^{\parallel} = k_q - \frac{\partial\phi_q^j}{\partial z} \approx q\frac{\omega}{c} [n_r(q\omega_0) - n_r(\omega_0)] + (q-1)\frac{\partial\zeta(z)}{\partial z} + \alpha_q^j \frac{\partial I(z)}{\partial z}$$

Longitudinal PM

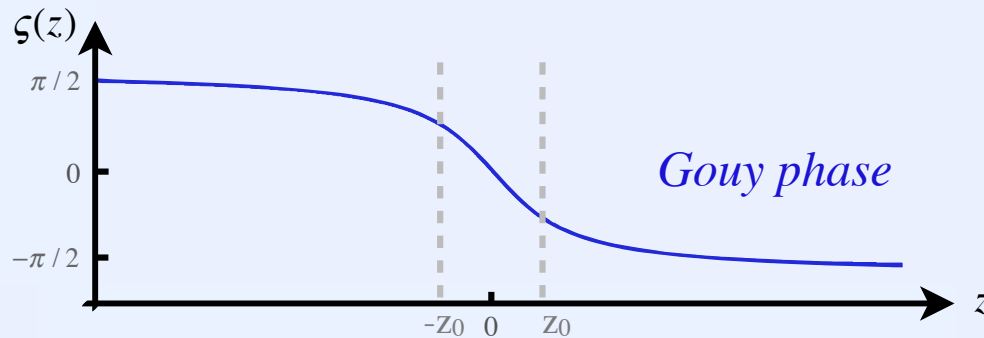
$$\Delta k_q^{\perp} = -\frac{\partial\phi_q^j}{\partial\rho} \approx -q\frac{\omega}{c} n_r(\omega) \frac{\rho}{R(z)} + \alpha_q^j \frac{\partial I(z)}{\partial z}$$

Transversal PM

Contributions to phase-matching

$$\Delta k_q = k_q - qk_1 \approx \Delta k_q^{\text{geom}} + \Delta k_q^{\text{int}} + \Delta k_q^f + \Delta k_q^b$$

Geometrical: Gouy + angle of detection



$$\Delta k_q^{\text{geom}} \approx \frac{q-1}{z_0} - \frac{q\omega_0}{2c} \theta^2$$

Intrinsic phase $\phi_q^i = \frac{1}{\hbar} S(\vec{k}^{st}, t^{st,i}, t_{ion}^{st,i}) + \omega t^{st,i}$ $\Delta k_q^{\text{int}} \approx \alpha_q^i \frac{\partial I(z)}{\partial z}$

Free electrons and neutrals $n_r \approx 1 + 2\pi(\chi_f + \chi_b)$

$$\Delta k_q^{f,b} = \frac{4\pi^2}{\lambda_q} \left[\chi_{f,b}(\lambda_q) - \chi_{f,b}(\lambda_0) \right] \approx \frac{4\pi^2}{\lambda_q} \chi_{f,b}(\lambda_0)$$

Computing Harmonic Propagation

1) The standard way: to solve the Wave equation

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial}{\partial t^2} \vec{E} = \frac{4\pi}{c^2} \frac{\partial}{\partial t} \vec{J}$$

A. L'Huillier et al, J. Opt. Soc. Am. B 7, 527 (1990).
A. L'Huillier, et al, Phys. Rev. A 46, 2778 (1992).

- ✓ Slowly Varying Envelope Approximation: $\partial^2/\partial t^2 \ll \omega \partial/\partial t$
- ✓ Paraxial Approximation: $\partial^2/\partial z^2 \ll k \partial/\partial z$

2) **Our choice:** to use the formal solution of the Maxwell's equations for an elementary radiator:

$$\vec{E}_i(\vec{r}, t) = -\frac{1}{c^2} \int d\vec{r}' \frac{1}{|\vec{r} - \vec{r}'|} \left[\frac{\partial}{\partial t'} \vec{J}(\vec{r}', t') \right]_{ret}$$

without Slowly Varying Envelope and Paraxial Approximations.

C. Hernández-García, et al, Phys. Rev. A 82, 0033432 (2010)

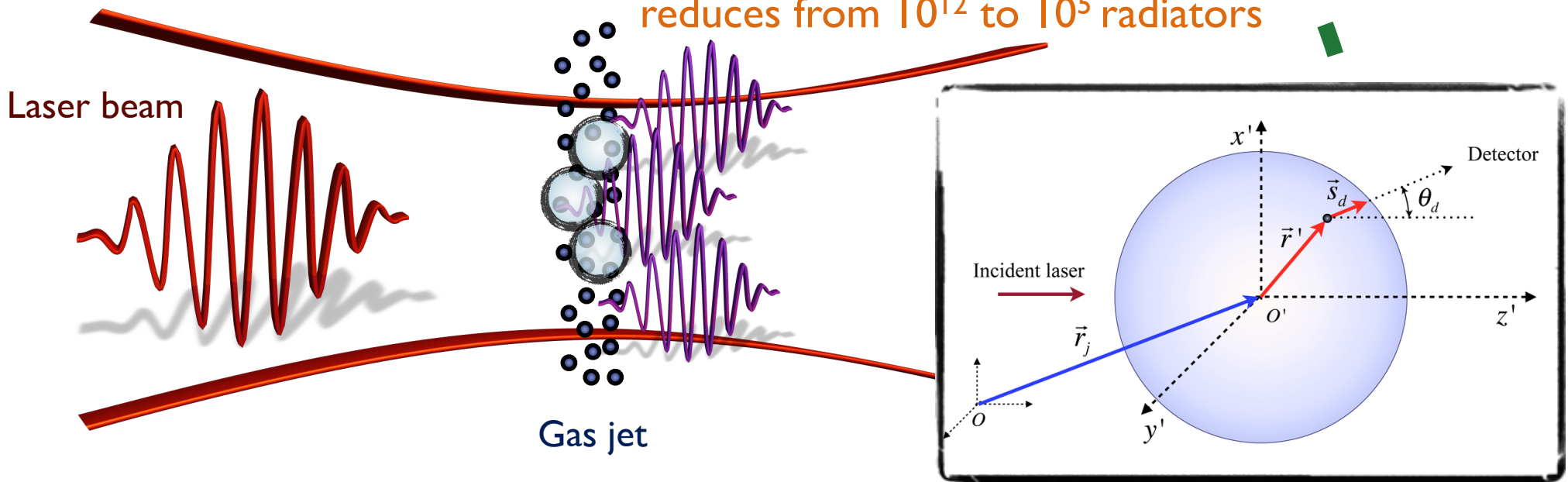
Sum of elementary radiators

We decompose the integral into a discrete sum of elementary contributions:

$$\vec{E}_i^j(\vec{r}, t) = -\frac{1}{c^2} \sum_{j=1}^N \frac{q_j}{|\vec{r} - \vec{r}_j(0)|} \vec{s}_d \times \left(\vec{s}_d \times \vec{a}_j \left(t - |\vec{r} - \vec{r}_j(0)|/c \right) \right)$$

Discrete Dipole Approximation
reduces from 10^{12} to 10^5 radiators

Detectors



C. Hernández-García, J.A. Pérez-Hernández, J. Ramos, E. Conejero Jarque, L. Roso, and L. Plaja, Phys. Rev. A 82, 0033432 (2010)

$$\vec{E}_{i,j}(\vec{r}_d, \omega) \propto -N(\vec{r}_j) \vec{s}_d \times \left(\vec{s}_d \times \vec{a}_j(\vec{r}_j, \omega) e^{-i\frac{\omega}{c}|\vec{r}_d - \vec{r}_j|} F(\theta_d, \omega) \right)$$

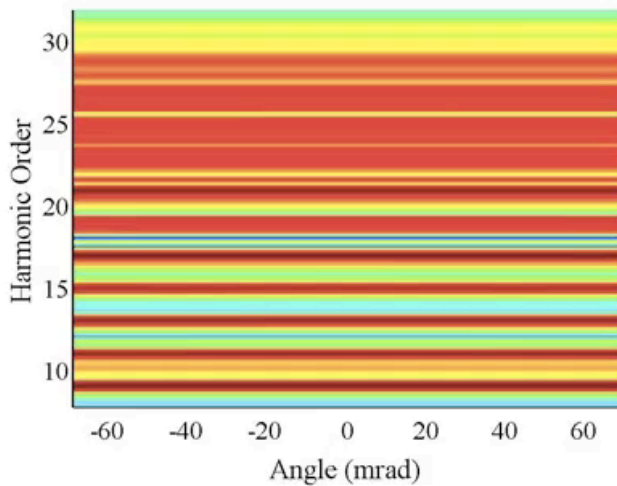
DDA - Convergence

Without DDA

← With DDA →

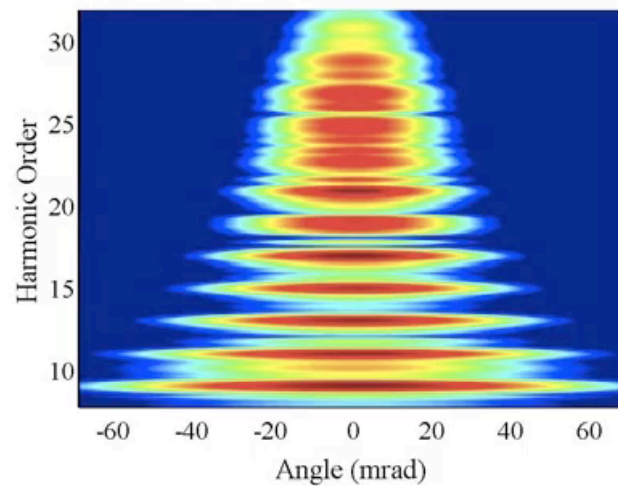
Point Atoms

1 spheres



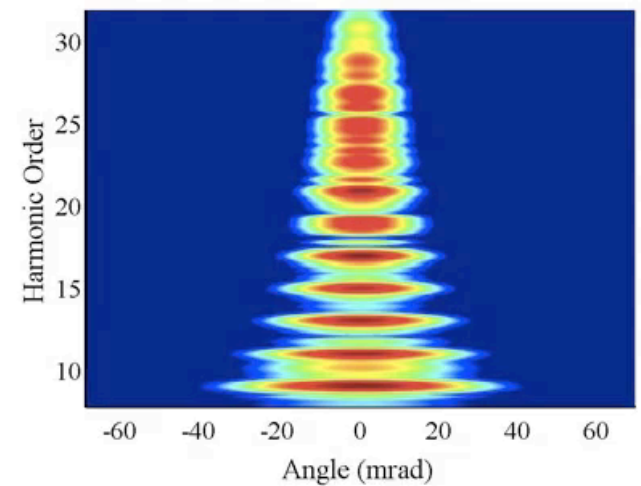
1 μm Spheres

1 spheres

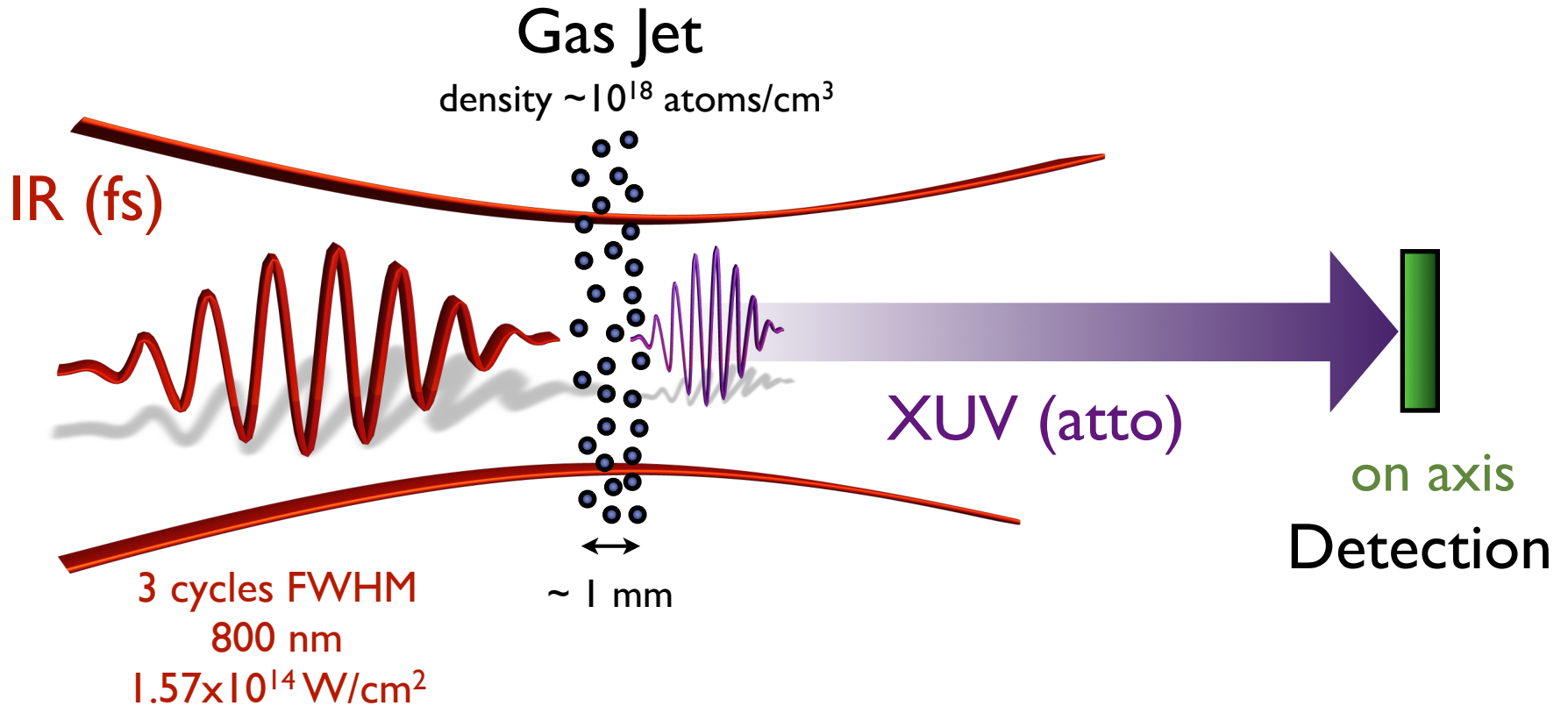


2 μm Spheres

1 spheres



On-axis phase-matching in a gas jet

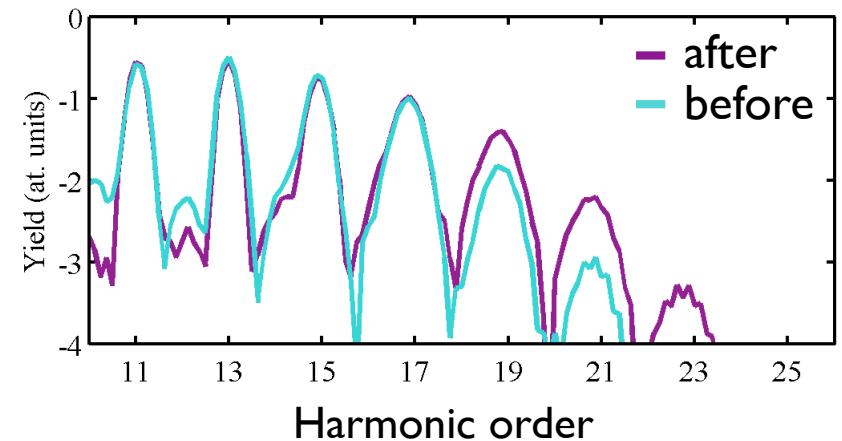
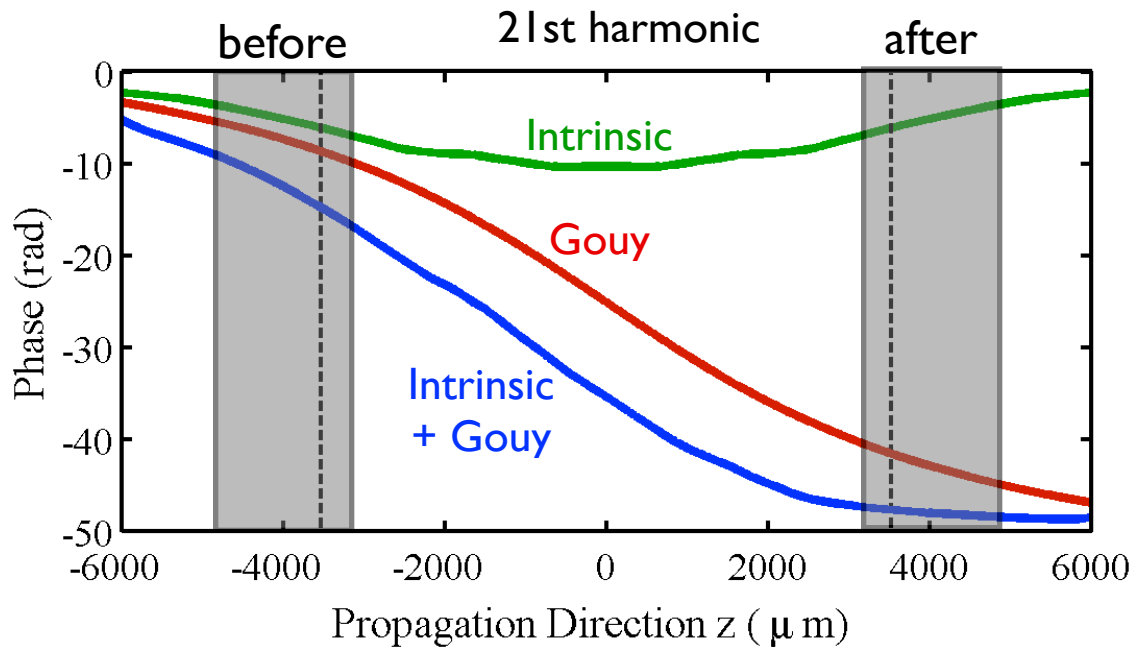


$$\Delta k_q \approx \Delta k_q^{Gouy} + \Delta k_q^{int} + \Delta k_q^f + \Delta k_q^b \approx \frac{q-1}{z_0} + \alpha_q^i \frac{\partial I(z)}{\partial z} + q \frac{e^2 n_f \lambda_0}{mc^2}$$

On-axis phase-matching in a gas jet

$$\Delta k_q \approx \Delta k_q^{Gouy} + \Delta k_q^{\text{int}} + \Delta k_q^f + \cancel{\Delta k_q^p} \approx \frac{q-1}{z_0} + \alpha_q^i \frac{\partial I(z)}{\partial z} + q \frac{e^2 n_f \lambda_0}{mc^2}$$

before the focus	>0	>0
after the focus	>0	<0

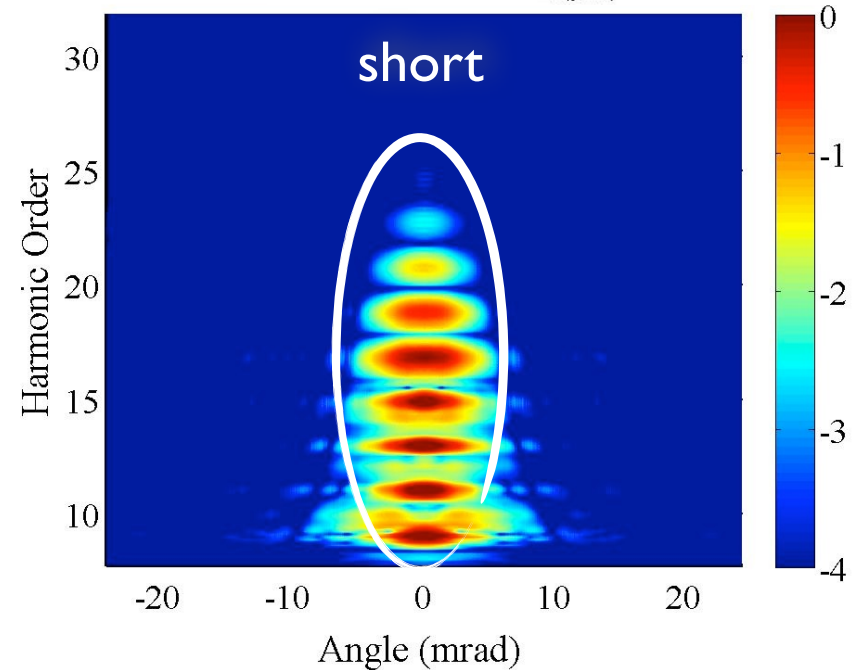
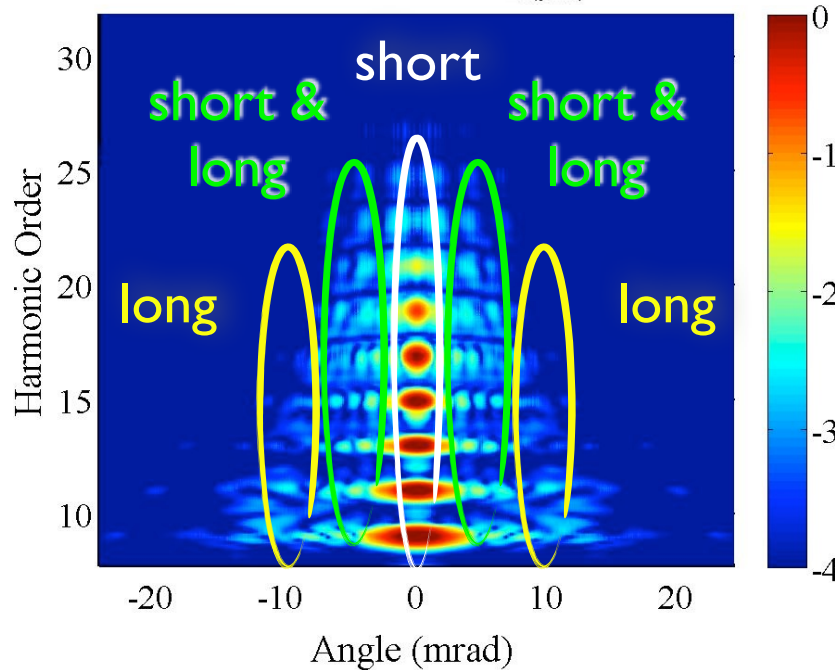
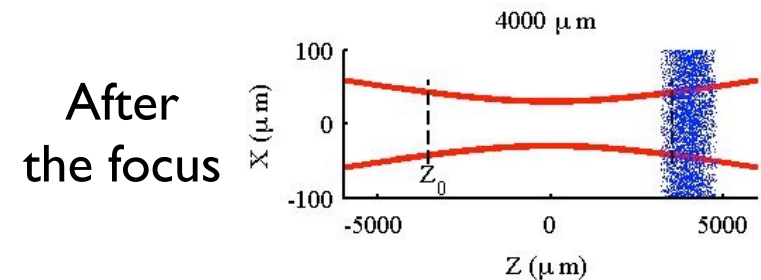
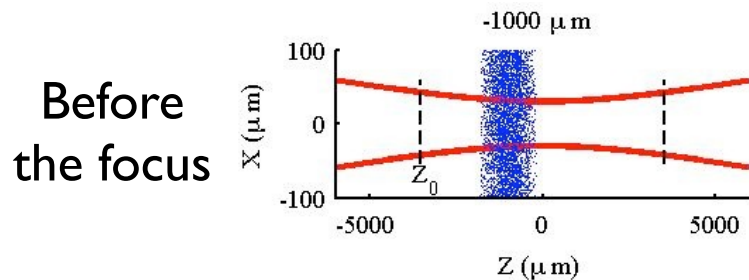


Already demonstrated:

- P. Salières, A. L'Huillier, and M. Lewenstein, PRL, 74, 3776 (1995)
- P. Antoine, A. L'Huillier, and M. Lewenstein, PRL, 77, 1234 (1996)
- M. B. Gaarde, J. L. Tate, and K. J. Schafer J. Phys. B: At. Mol. Opt. Phys. 41 (2008)

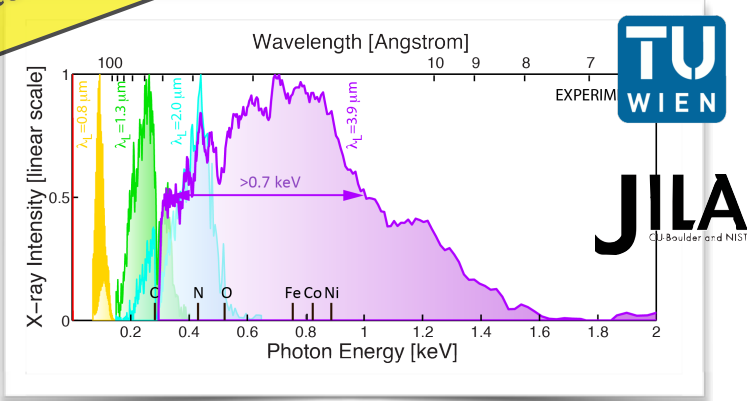
Off-axis phase-matching in a gas jet

$$\Delta k_q \approx \Delta k_q^{Gouy} + \Delta k_q^{\text{int}} + \Delta k_q^f + \Delta k_q^{\text{angle}} \approx \frac{q-1}{z_0} + \alpha_q^i \frac{\partial I(z)}{\partial z} + q \frac{e^2 n_f \lambda_0}{mc^2} - \frac{q\omega_0}{2c} \theta^2$$



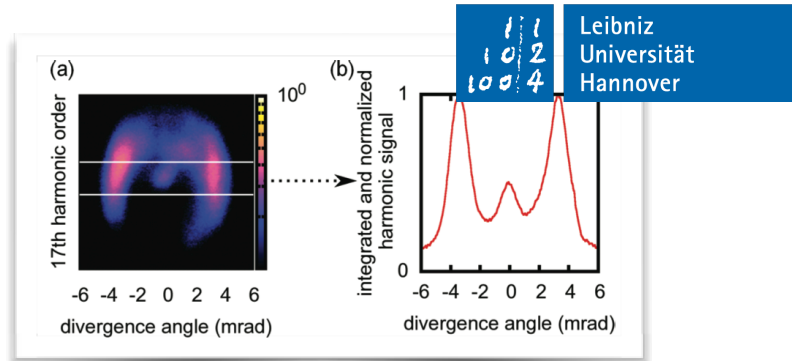
Deliverables of our Phase-matching code

Experimental comparisons



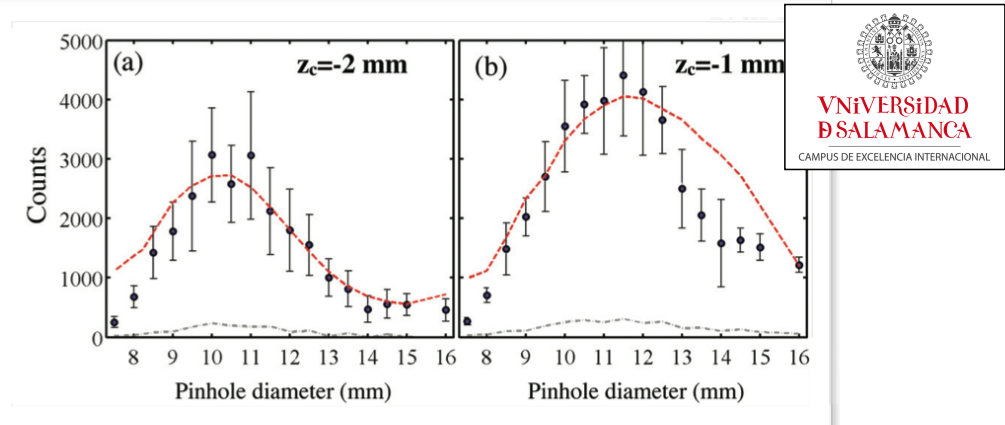
1.6 keV HHG X-rays

T. Popmintchev, et al *Science* 336, 1287 (2012)



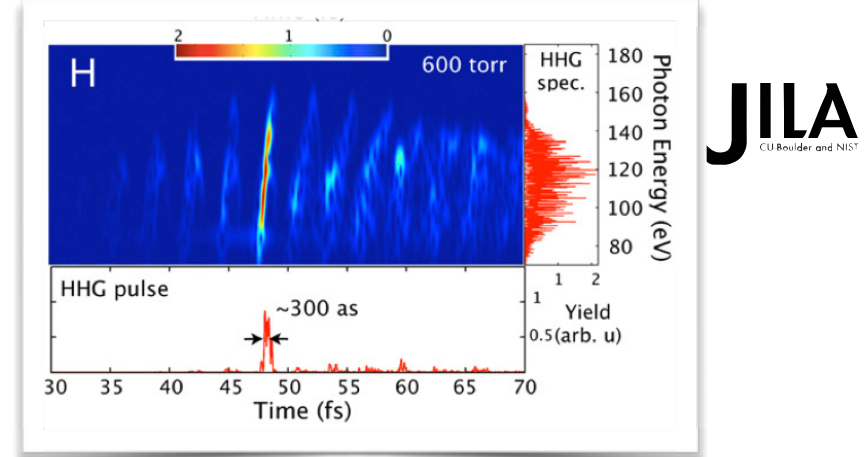
XUV harmonics in gas cell

M. Kretschmar, et al *Phys. Rev.A* 88, 013805 (2013)



Role of transversal phase-matching

C. Hernández-García, et al *Phys. Rev.A* 88, 043848 (2013)



Isolated X-ray attosecond pulses

M.-C. Chen, et al *PNAS* 111, E2361-E2367 (2014)

XUV harmonics from few-cycle pulses

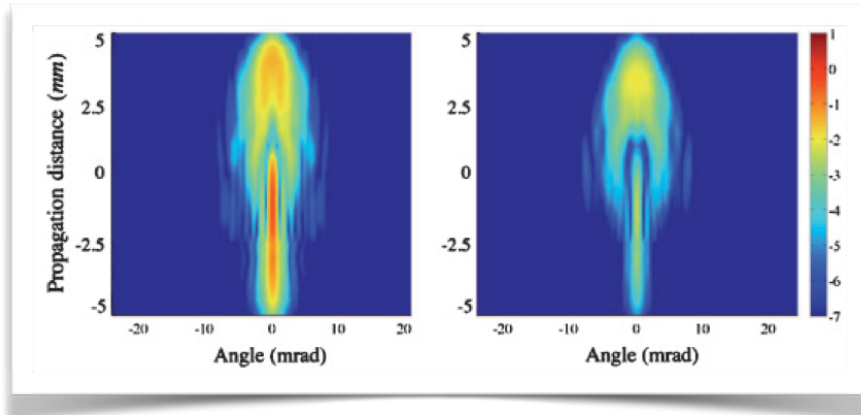
W. Holgado et. al. (in preparation) (2014)

U. PORTO

UNIVERSIDAD DE SALAMANCA
CAMPUS DE EXCELENCIA INTERNACIONAL

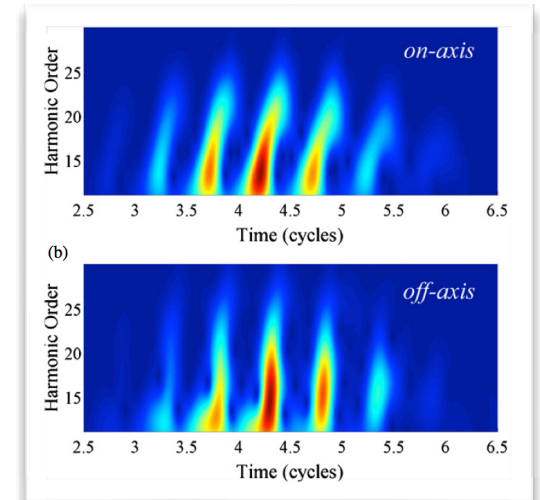
Deliverables of our
Phase-matching code

New theoretical findings



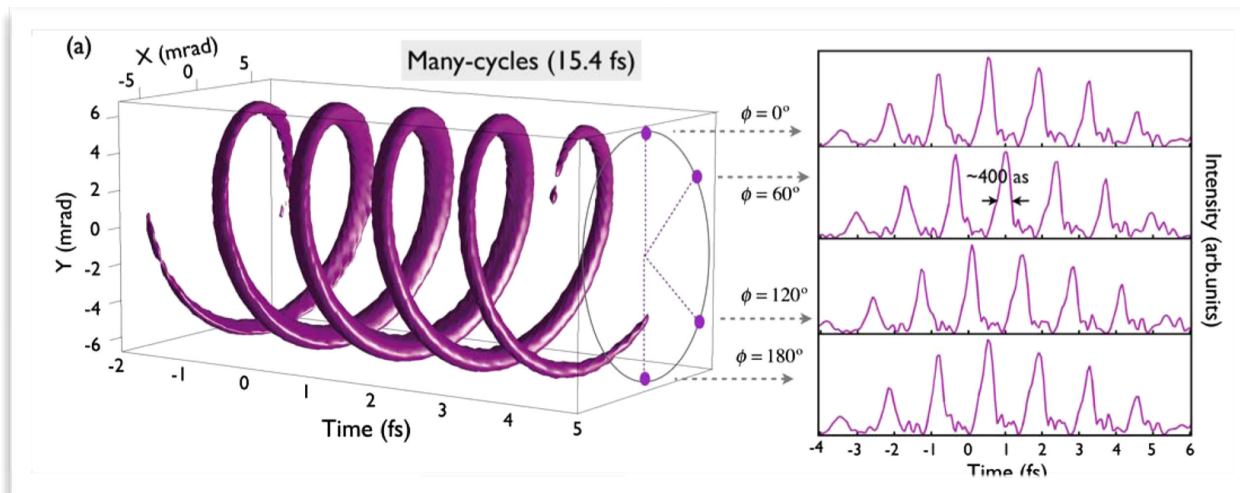
Phase-matching conditions along the propagation direction

C. Hernández-García, et al, *Phys. Rev,A* 82, 0033432 (2010)



Off-axis compensation of
the atto-chirp

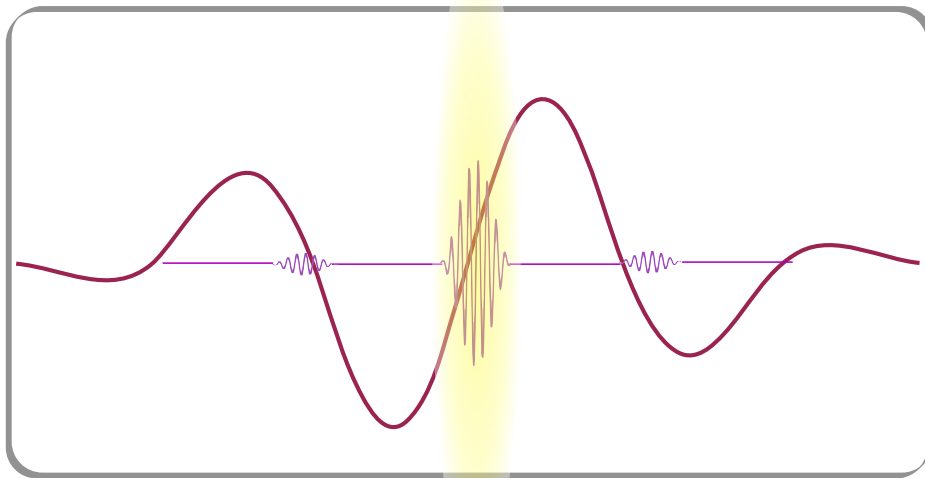
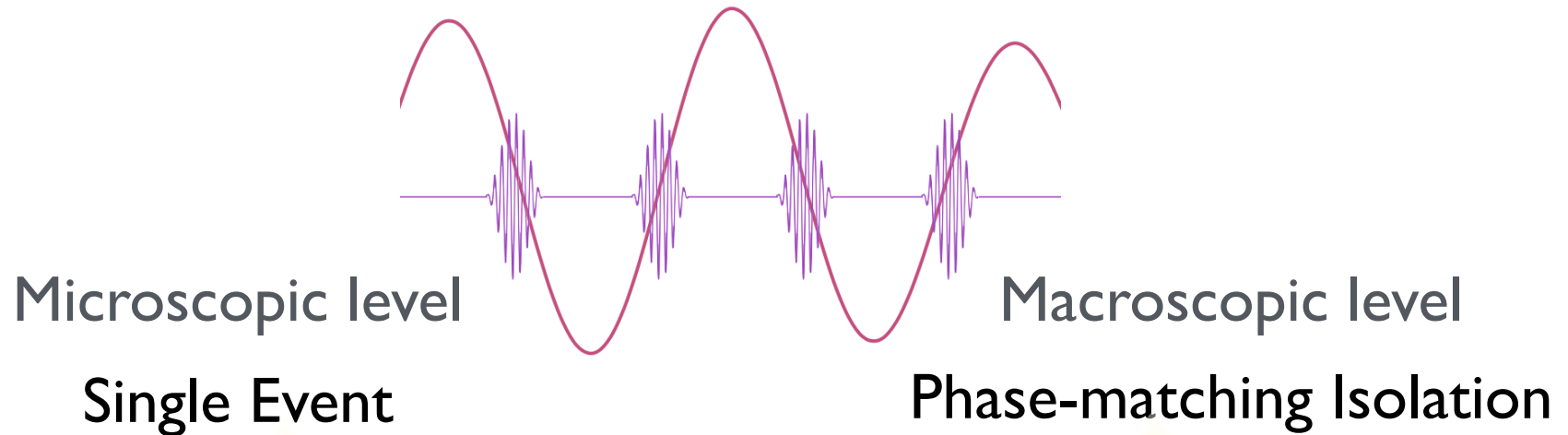
C. Hernández-García, and L. Plaja,
J. Phys. B:At. Mol. Opt. Phys. 45,
074021 (2012)



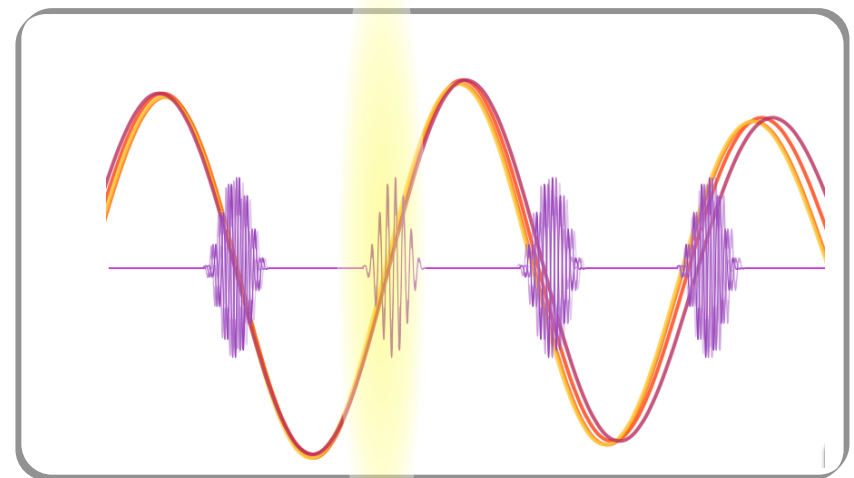
XUV vortices: conservation of orbital angular momentum (OAM) in HHG

C. Hernández-García, A. Picón, J. San Román, and L. Plaja, *Phys. Rev. Lett.* 111, 083602 (2013)

Isolating an attosecond pulse

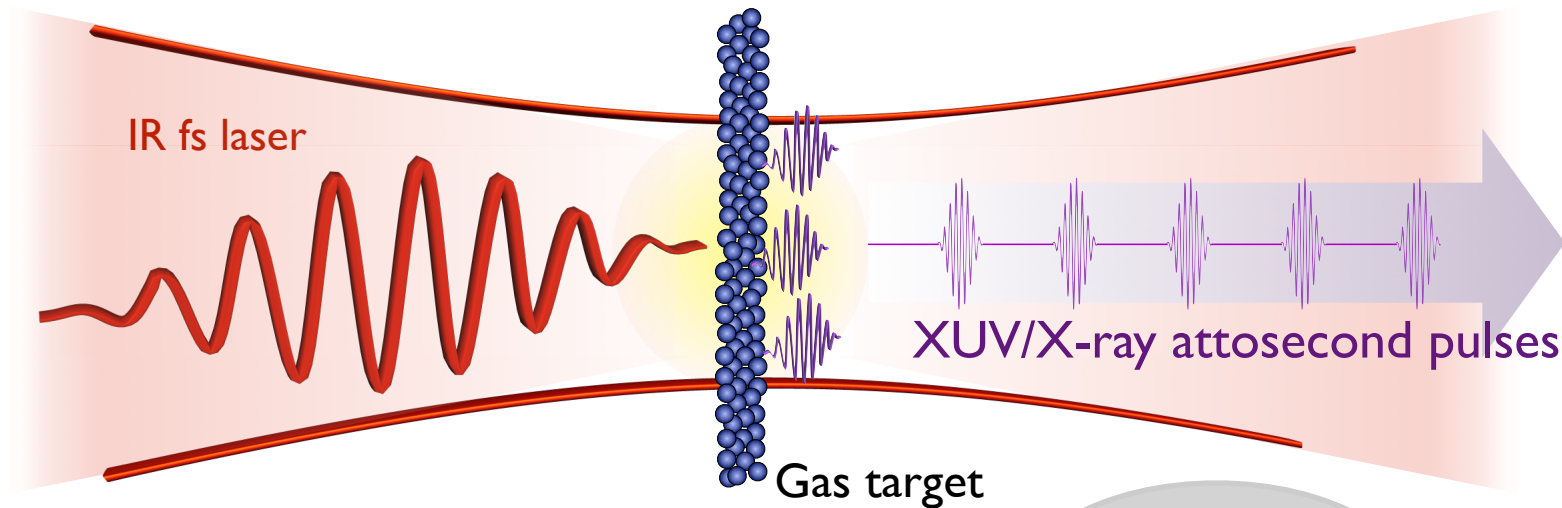


E. Goulielmakis et al., *Science* **320**, 1614–1617 (2008).
B. Shan et al., *J. Mod. Opt.* **52**, 277 (2005).
I. J. Sola et al., *Nature Physics* **2**, 319–322 (2006).
G. Sansone et al., *Science* **314**, 443–446 (2006).
E. J. Takahashi, et al., *Phys. Rev. Lett.* **104**, 233901 (2010).
Zhao et al. *Opt. Lett.* **37**, 3891 (2012)



M. J. Abel et al., *Chem Phys* **366**, 9–14 (2009)
I. Thomann et al., *Opt. Express* **17**, 4611 (2009)
F. Ferrari, F. Calegari, M. Lucchini, C. Vozzi, S. Stagira, G. Sansone, and M. Nisoli, *Nature Photonics* **4**, 875–879 (2010)
M.-C. Chen, C. Mancuso, C. Hernández-García, F. Dollar, B. Galloway, D. Popmintchev, B. Langdon, A. Auger, P.-C. Huang, B. C. Walker, L. Plaja, A. Jaron-Becker, A. Becker, M. M. Murnane, H. C. Kapteyn, T. Popmintchev, *PNAS* **111** (23), E2361–E2367 (2014)

Optimal phase-matching



Phase-mismatch: $\Delta k_q = k_q - qk_1$

Perfect
phase-matching
 $\Delta k_q = 0$

$$\Delta k_q \approx \cancel{\Delta k_q^{geom}} + \cancel{\Delta k_q^{int}} + \Delta k_q^f + \Delta k_q^b$$

Optimal phase-matching: $|\Delta k_q^f| = |\Delta k_q^b| \rightarrow \Delta k_q = 0$

Optimal phase-matching

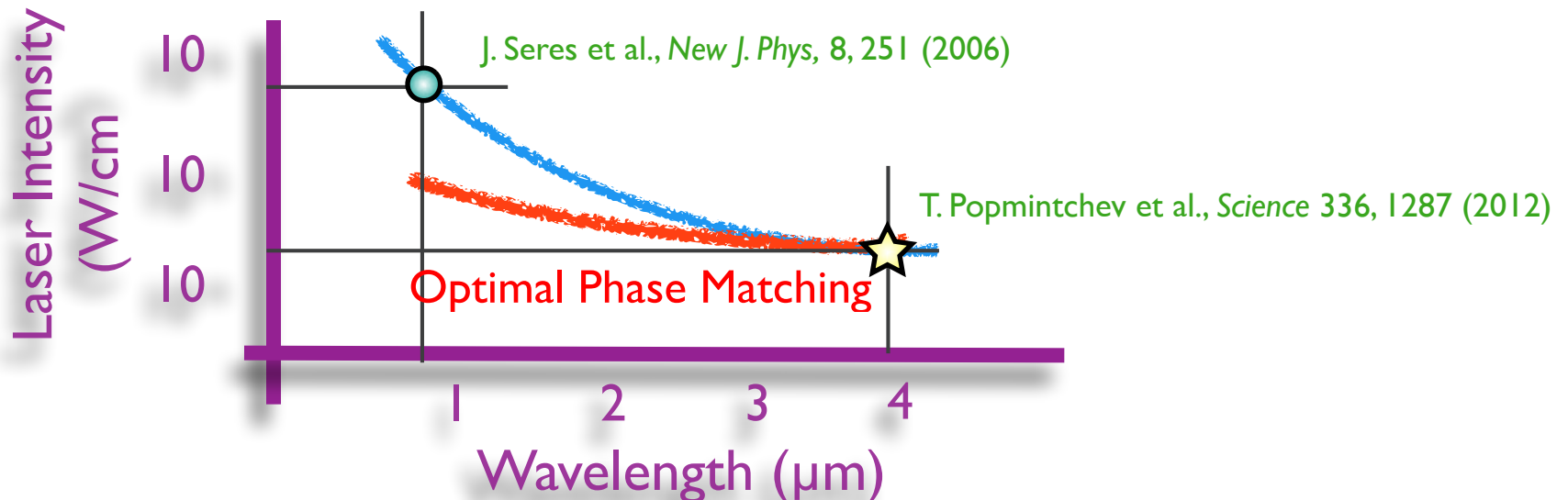
$$\Delta k_q = k_q - qk_1 \approx \cancel{\Delta k_q^{geom}} + \cancel{\Delta k_q^{int}} + \Delta k_q^f + \Delta k_q^b$$

$$\Delta k_q \approx -\frac{4\pi^2}{\lambda_q} [\chi_f(\lambda_0) + \chi_b(\lambda_0)] \approx q \left[\frac{P_f n_0 e^2 \lambda_0}{mc^2} - \frac{4\pi^2 (1 - P_f) n_0 \chi_b^0}{\lambda_0 \eta_0} \right]$$

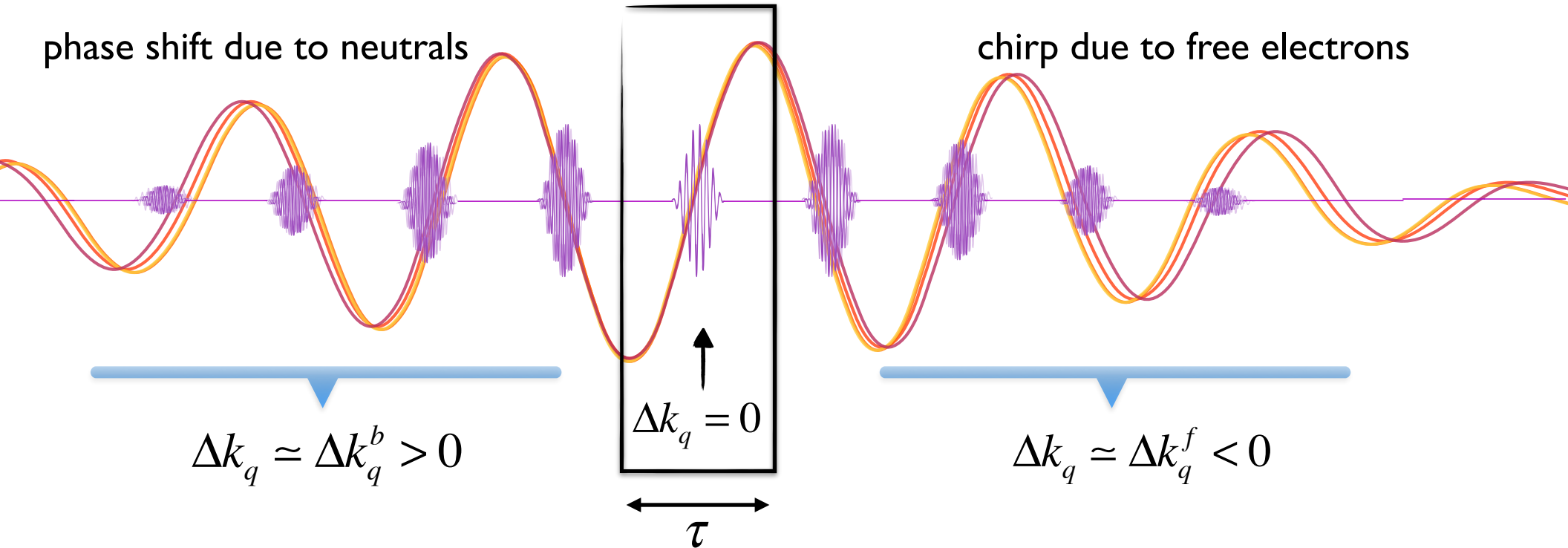
$$\Delta k_q = 0 \rightarrow P_f^{opt} \approx \frac{\chi_b^0}{\eta_0} \left(\frac{e^2 \lambda_0}{4\pi mc^2} - \frac{\chi_b^0}{\eta_0} \right)^{-1}$$

Optimal ionized population,
 $P_f^{opt} \rightarrow (\lambda_0, I)$

X-ray Photon Energy: 1.5 keV



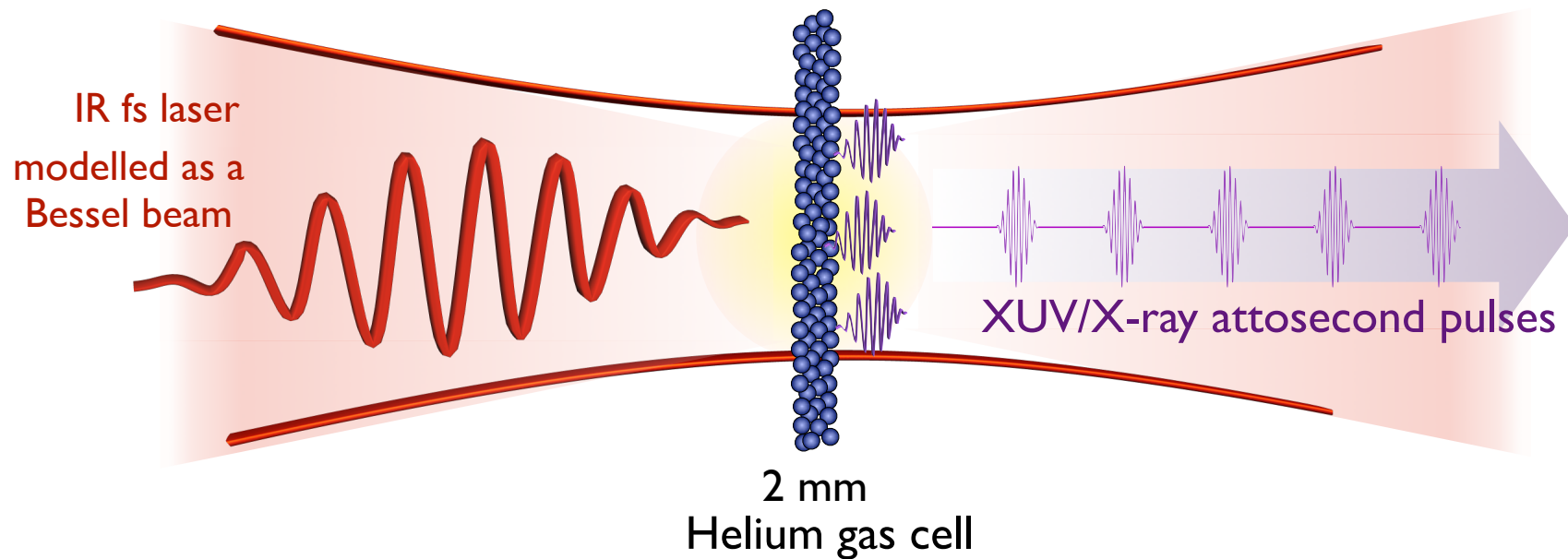
Time gated phase-matching



Size of the phase-matching window

$$\frac{1}{\tau} \propto \frac{\partial \Delta k_q}{\partial t} \propto \lambda_0^{1.5} P$$

3D simulations



0.8 μm Intensities given by
2.0 μm phase-matching
conditions

Theoretical Method:

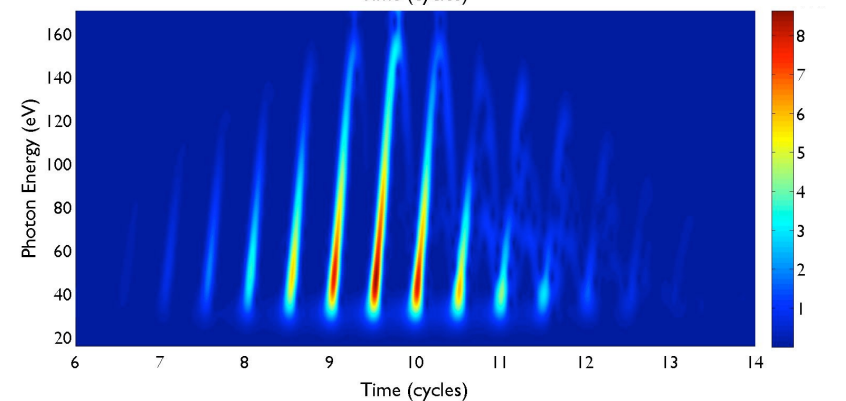
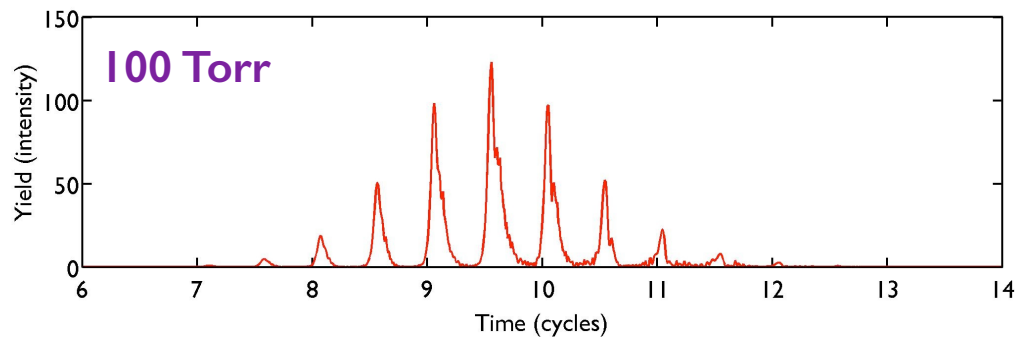
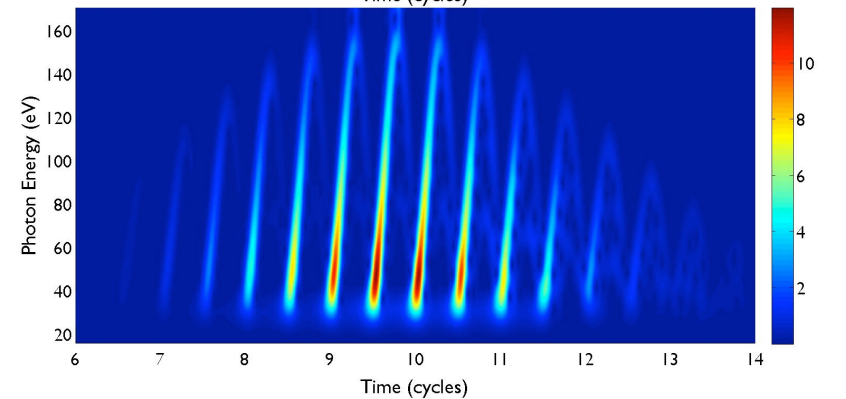
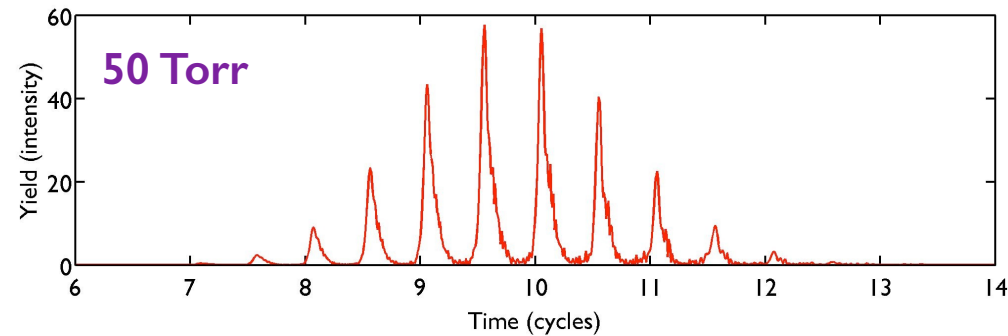
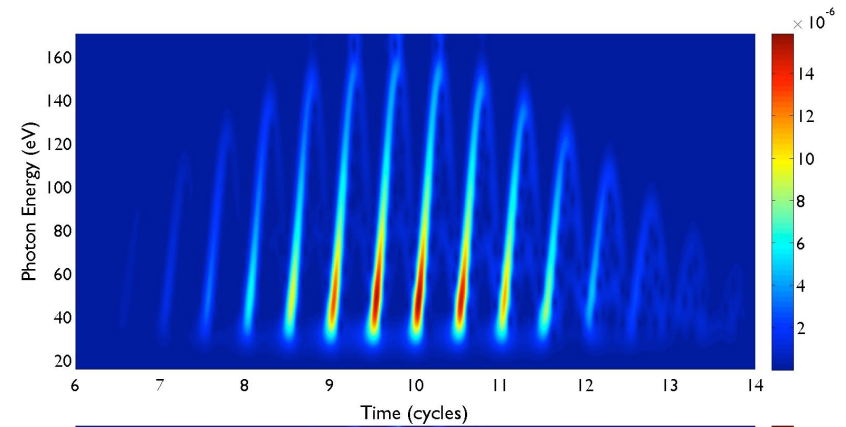
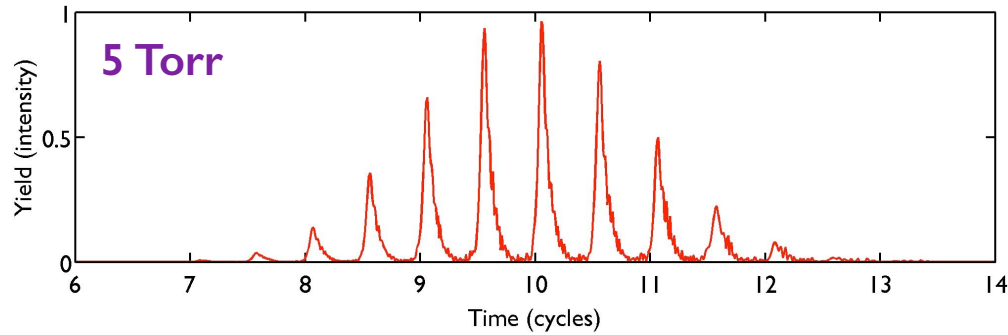
- High-Harmonic Generation: SFA+
- Propagation: Discrete Dipole Approximation using Maxwell equations

C. Hernández-García, et al. *Phys. Rev.A* 82, 033432 (2010)

3D simulations: 800 nm, 5.8 cycles

2 mm He gas cell, peak intensity $6.6 \times 10^{14} \text{ Wcm}^{-2}$

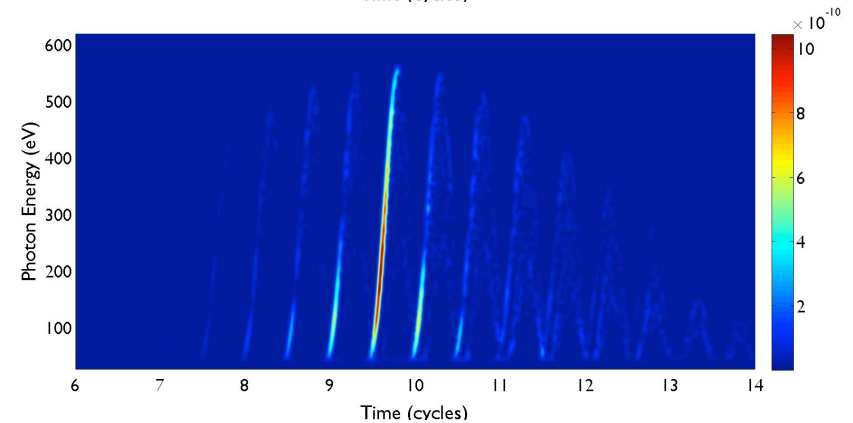
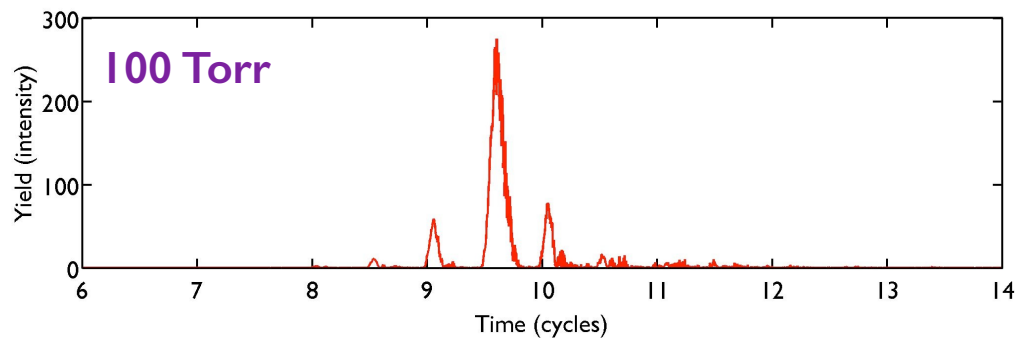
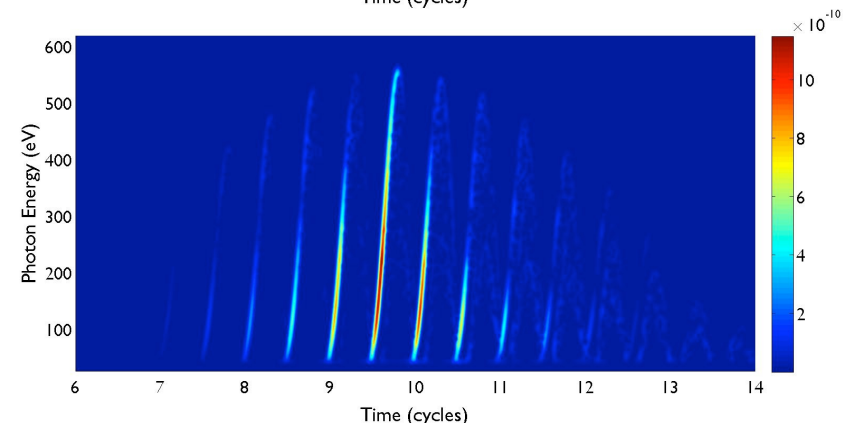
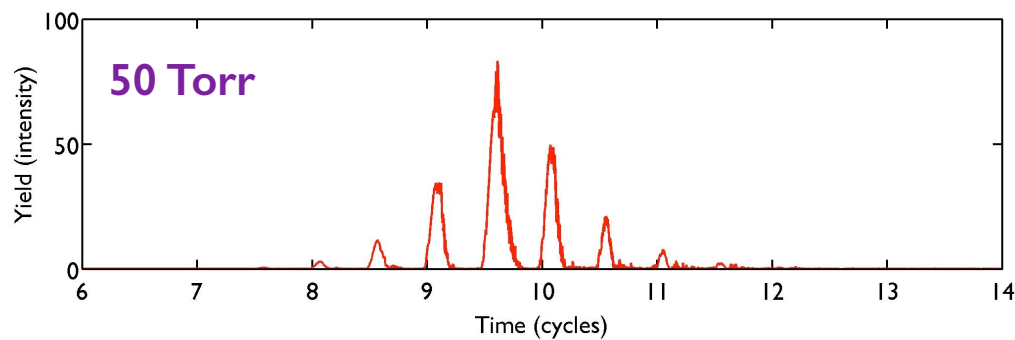
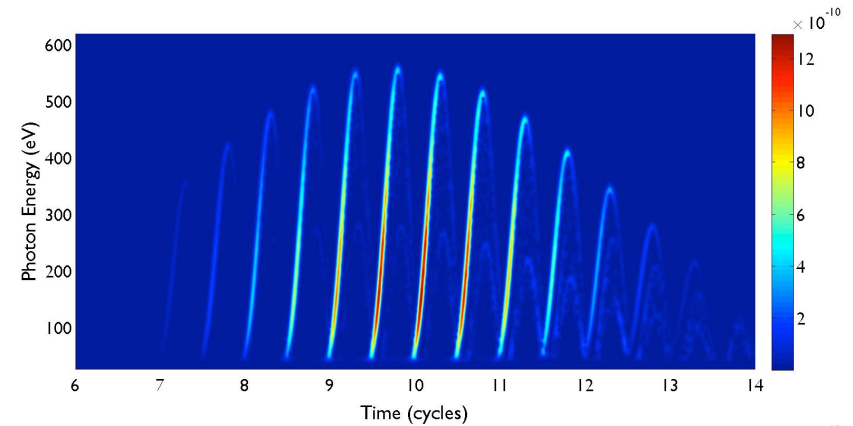
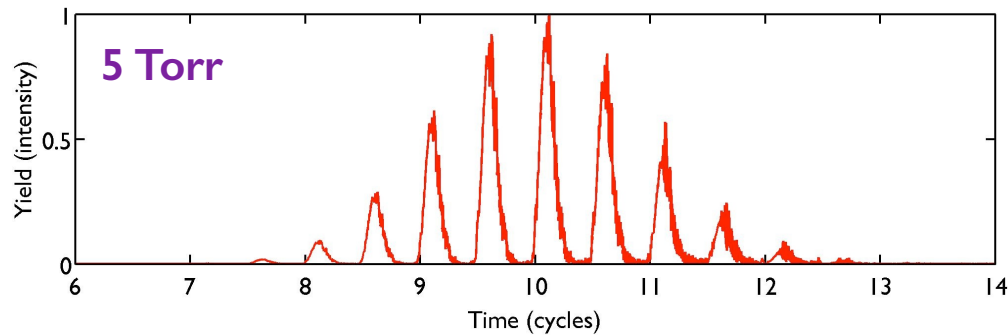
$$\frac{\partial \Delta k_q}{\partial t} \propto \lambda_0^{1.5} P$$



3D simulations: 2.0 μm , 5.8 cycles

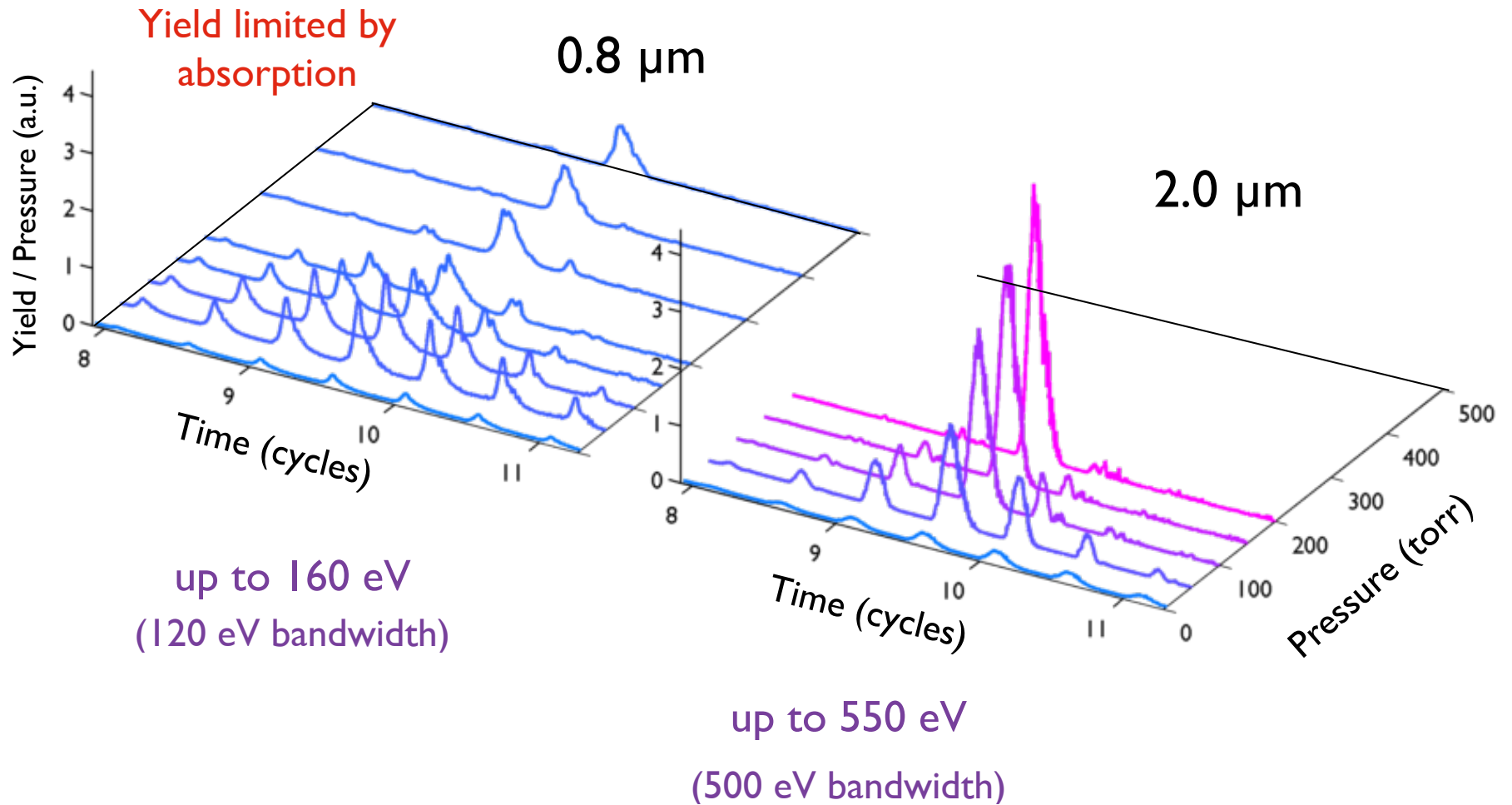
2 mm He gas cell, peak intensity $4.4 \times 10^{14} \text{ Wcm}^{-2}$

$$\frac{\partial \Delta k_q}{\partial t} \propto \lambda_0^{1.5} P$$

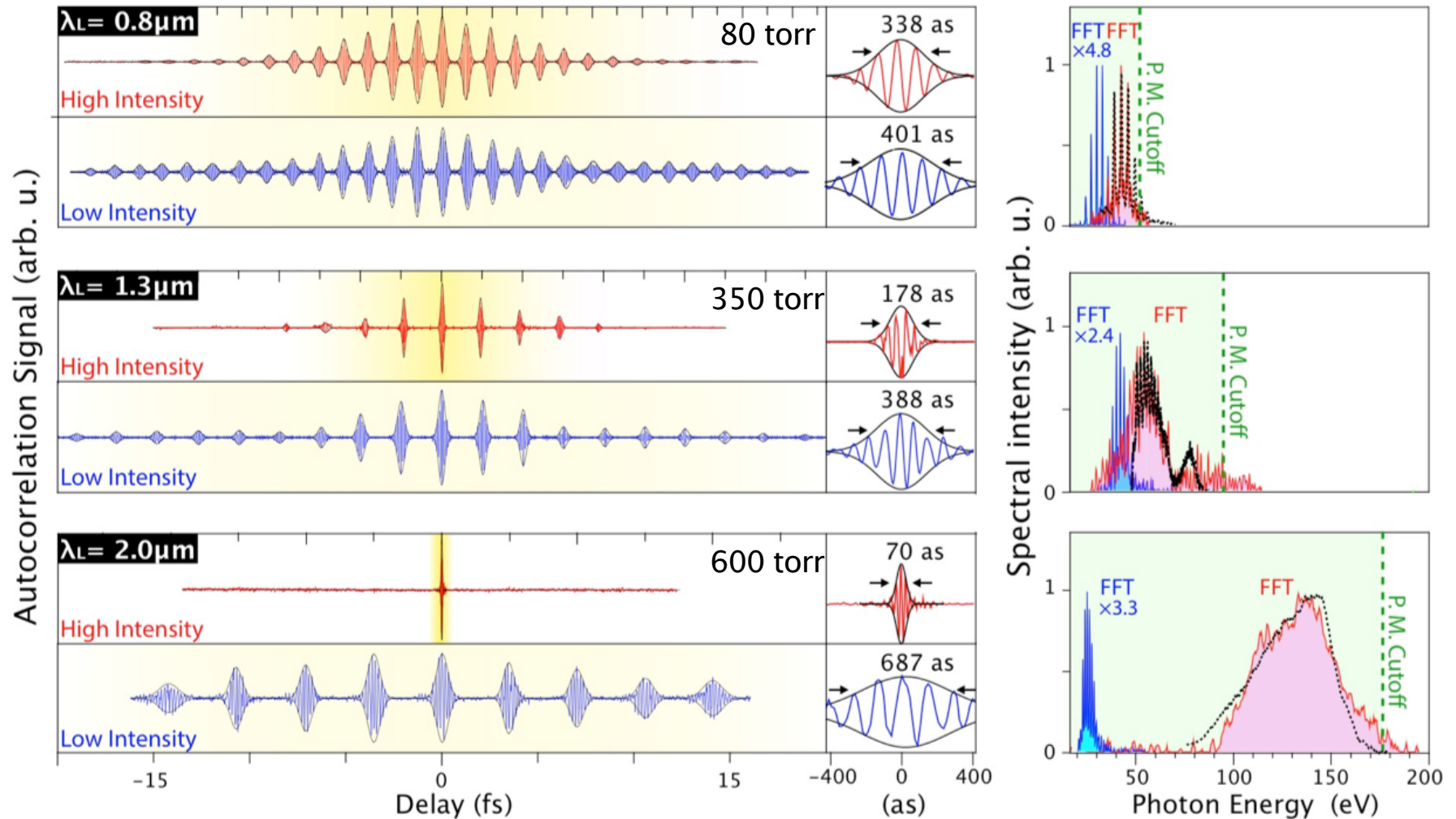


Phase-matching with multi-cycle pulses

2 mm He gas cell

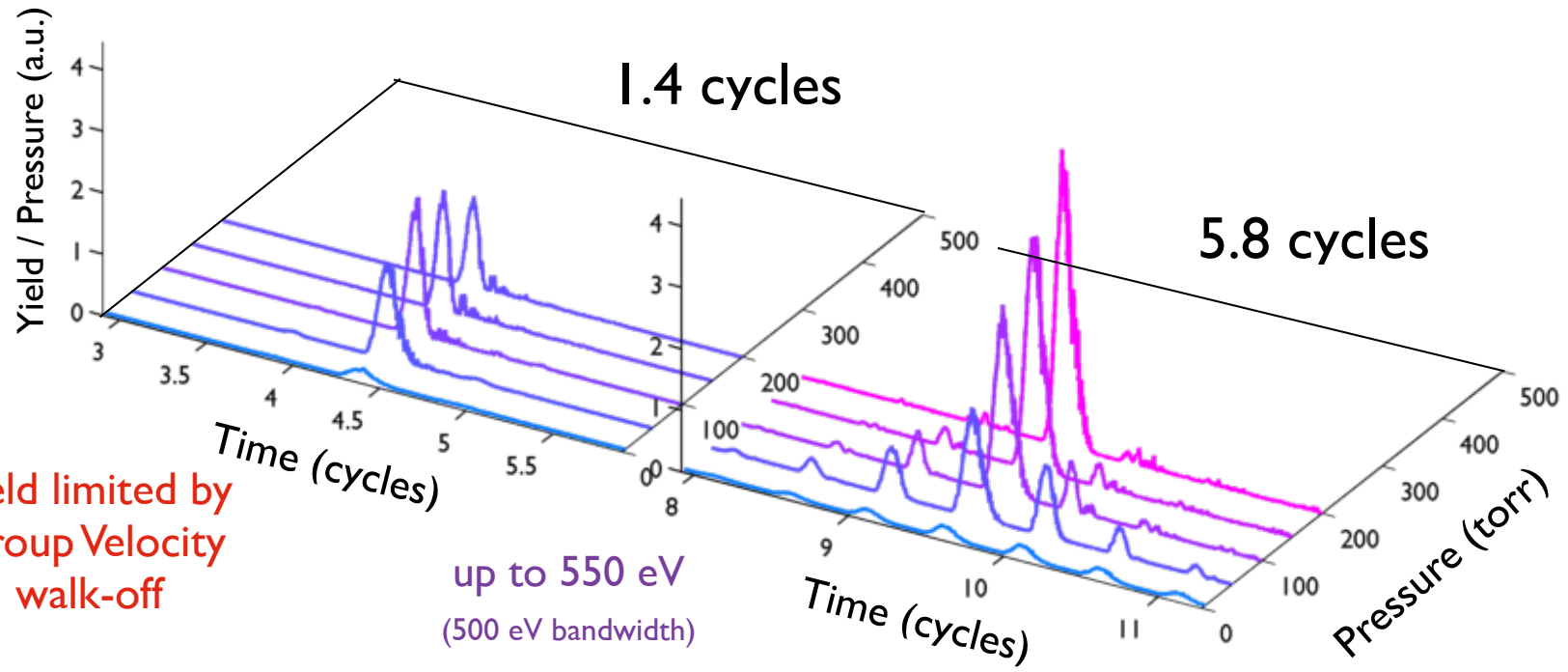
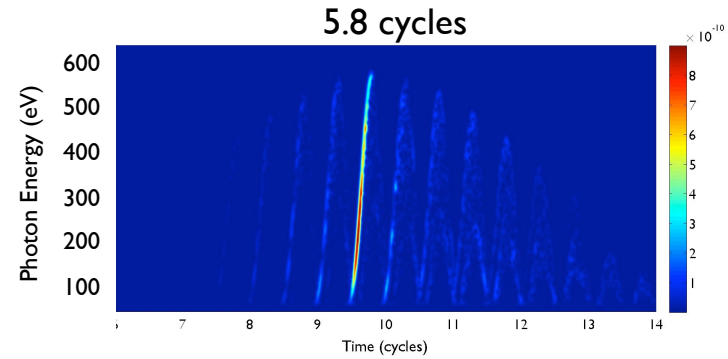
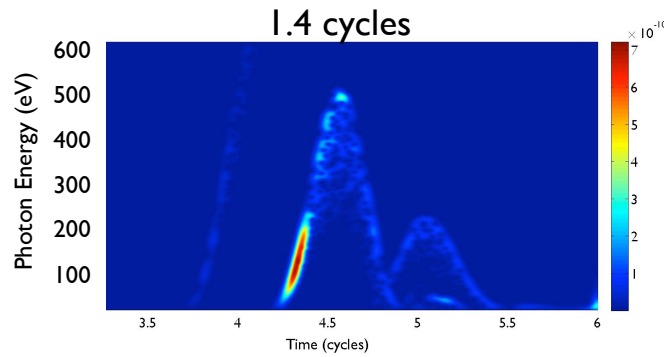


Experimental results



M.-C. Chen, C. Mancuso, C. Hernández-García, F. Dollar, B. Galloway, D. Popmintchev, B. Langdon, A. Auger, P.-C. Huang, B.C. Walker, L. Plaja, A. Jaron-Becker, A. Becker, M. M. Murnane, H. C. Kapteyn, T. Popmintchev, *PNAS* 111 (23), E2361-E2367 (2014)

Few-cycle vs Multi-cycle at 2 μm

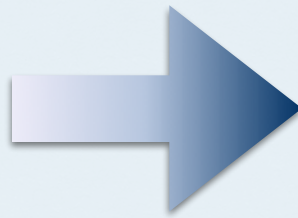
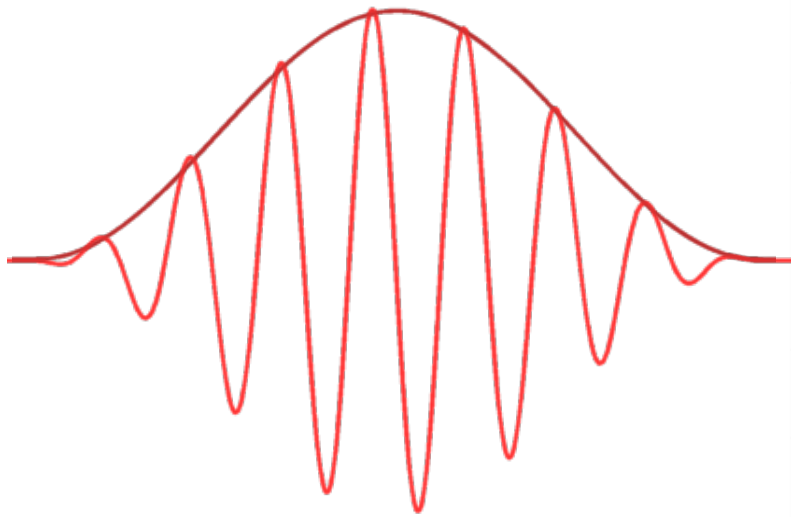


Yield limited by
Group Velocity
walk-off

up to 550 eV
(500 eV bandwidth)

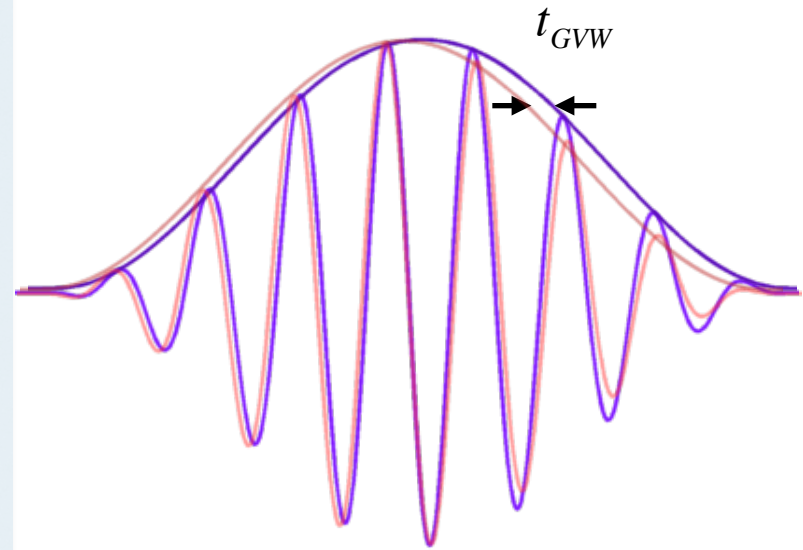
Group velocity walk-off

Electric Field before entering the target



Target: gas medium

Electric Field after the target



Group velocity: $\frac{1}{v_g} - \frac{1}{v_{ph}} = \frac{\omega}{c} \frac{\partial n(\omega)}{\partial \omega}$

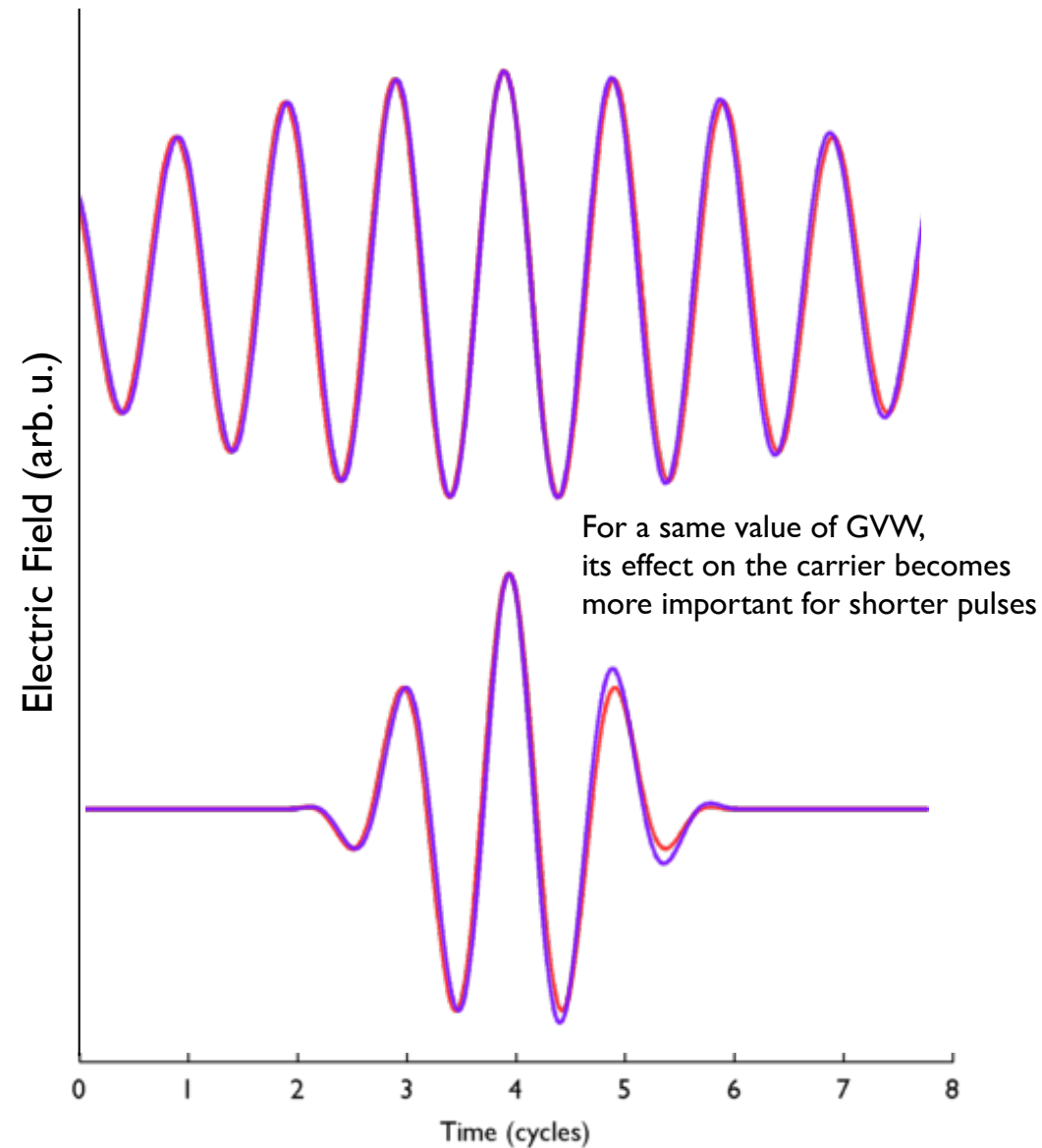
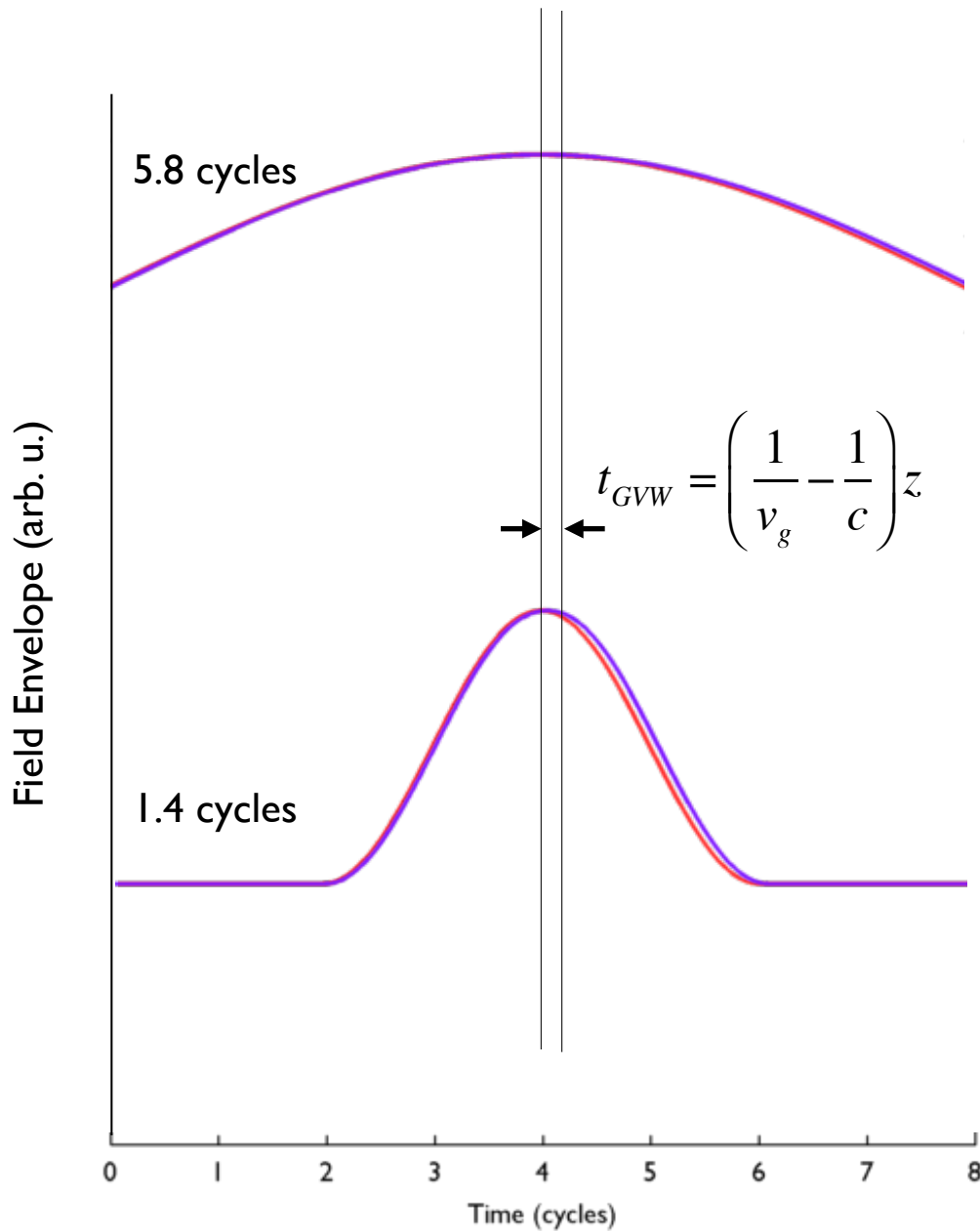
Temporal delay due to group velocity walk-off $t_{GVW} = \left(\frac{1}{v_g} - \frac{1}{c} \right) z$

Group velocity matching:

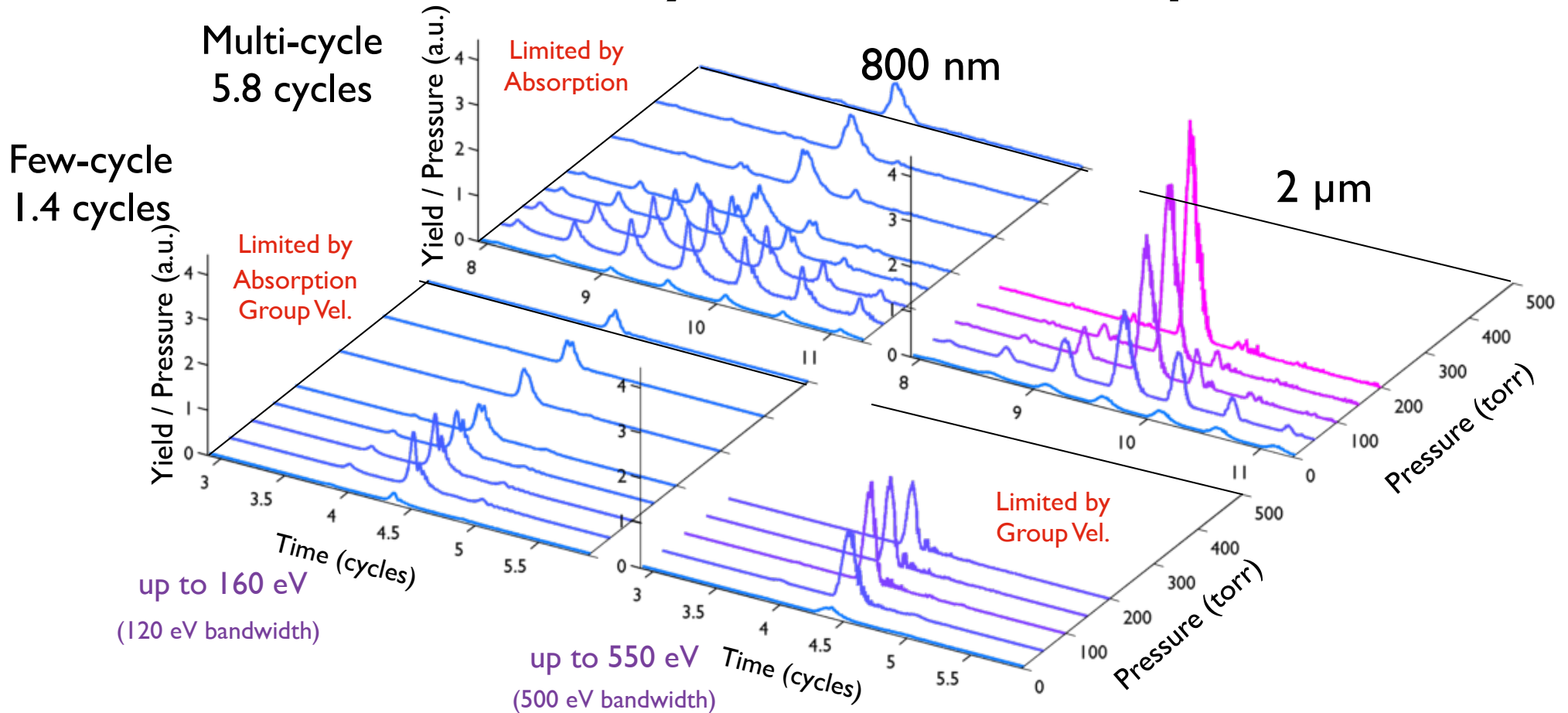
Fundamental field travels with $v_g(\omega_0)$

Harmonic field travels with $v_g(q\omega_0) \approx c$

Effect of the group velocity walk-off on the carrier



Isolated X-ray attosecond pulses



- We derive a route for **extending isolated attosecond pulses to the x-ray regime** by using mid-IR wavelengths.
- **Multi-cycle laser pulses** show better yield scaling due to group velocity walk-off.

Conclusions

- We propose a route for producing zeptosecond waveforms in the keV regime, driving HHG by mid-IR laser pulses.
- We demonstrate the generation of isolated attosecond pulses in the X-ray regime using mid-IR multi-cycle laser pulses.