String Formation in Gauge Theories

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Modern Challenges for Lattice Field Theory
March 28, 2005
KITP, Santa Barbara

Collaborators:

Jimmy Juge	Dublin
Francesca Maresca	Utah
Colin Morningstar	Carnegie Mellon
Mike Peardon	Dublin
James Drummond	Dublin

string counselor at UCSD: Ken Intriligator

Early work: Polyakov
Luscher
Polchinski, Strominger
Baker et al.
Michael
Teper
Gliozzi et al.
Hasenbusch, Pinn
JKM (old)
Munster
…

This talk: review on the excitation spectrum of the Dirichlet string and the closed string with unit winding (string-soliton)

Recent work in QCD and Z(2):

Juge, JK, Morningstar → fixed end spectrum with fine structure
HEP-LAT 0207004, PRL 90 (2003) 161601

Juge, JK, Maresca, Morningstar, Peardon → closed winding string with fine structure
HEP-LAT 0309180, Nucl.Phys.Proc.129:703-705,2004

Luscher, Weisz → ground state Casimir energy
JHEP 0207 (2002) 049, JHEP 0407:014,2004

Juge et al. and Caselle et al. → open-closed string duality

new work (first presented here) → Z(2) gauge model in 3 dimensions

OUTLINE

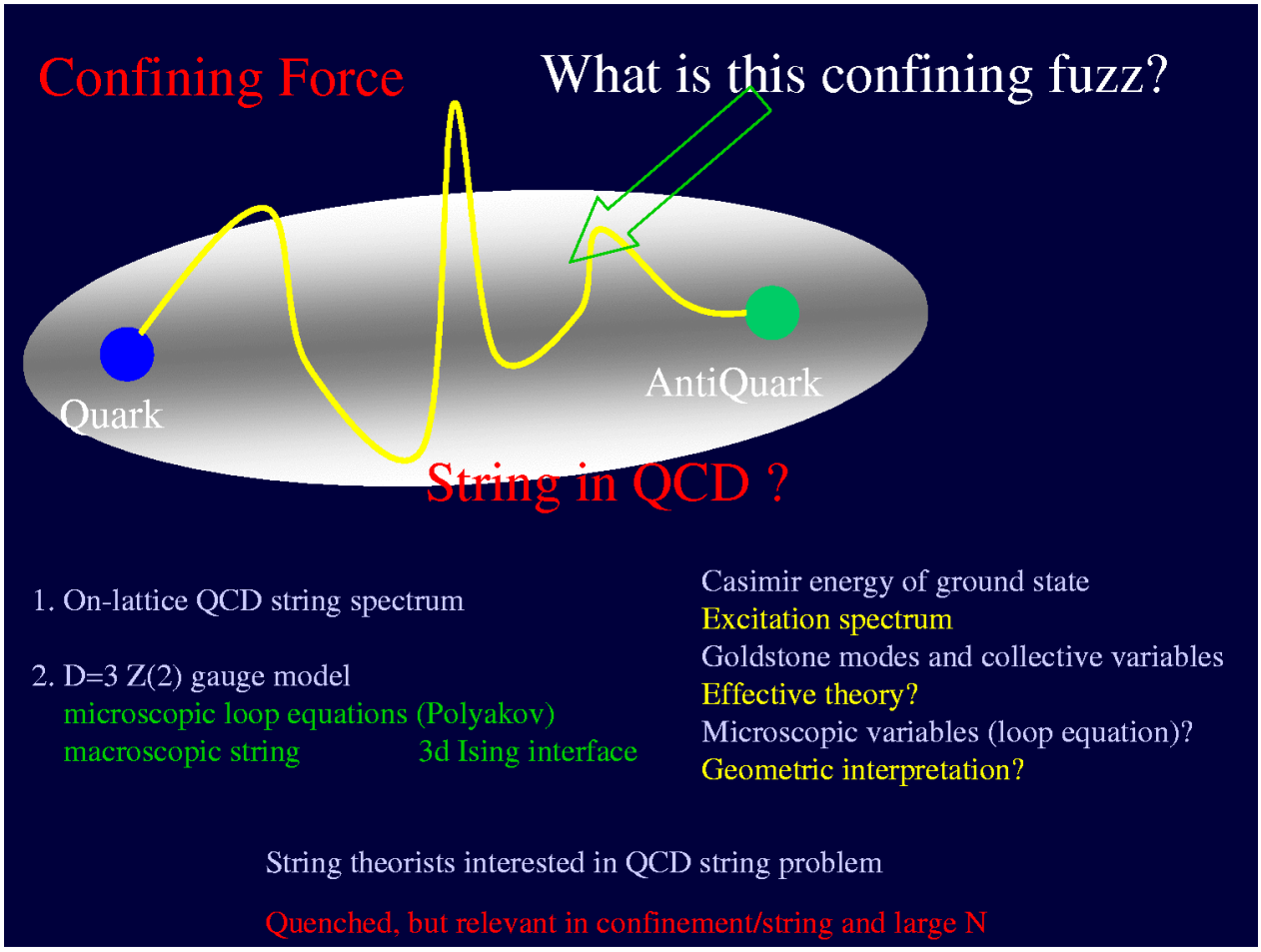
1. **String formation in field theory**
   - picture in space and time
   - main physical properties of the string
2. **Dirichlet Strings in D=4 and D=3 dimensions**
   - fixed end D=4 SU(3) QCD string spectrum
   - fixed end D=3 SU(2) QCD string and Z(2) string new results
3. **Dirichlet Casimir Energy**
   - origin of Casimir energy and effective string description
   - Luscher-Weisz results
   - Z(2)
   - paradox ?
   - 1+1 dimensional toy model insight from quantum mechanics
4. **Closed String (torelon) with unit winding**
   - D=4 SU(3) QCD spectrum new results
   - Closed string Casimir energy new results
   - D=3 Z(2) spectrum new results
5. **Conclusions**

Will not discuss: high spin Glueball spectrum } Teper and collaborators

Casimir scaling of the flux

`t Hooft flux quantization de Forcrand

low precision results on string spectrum



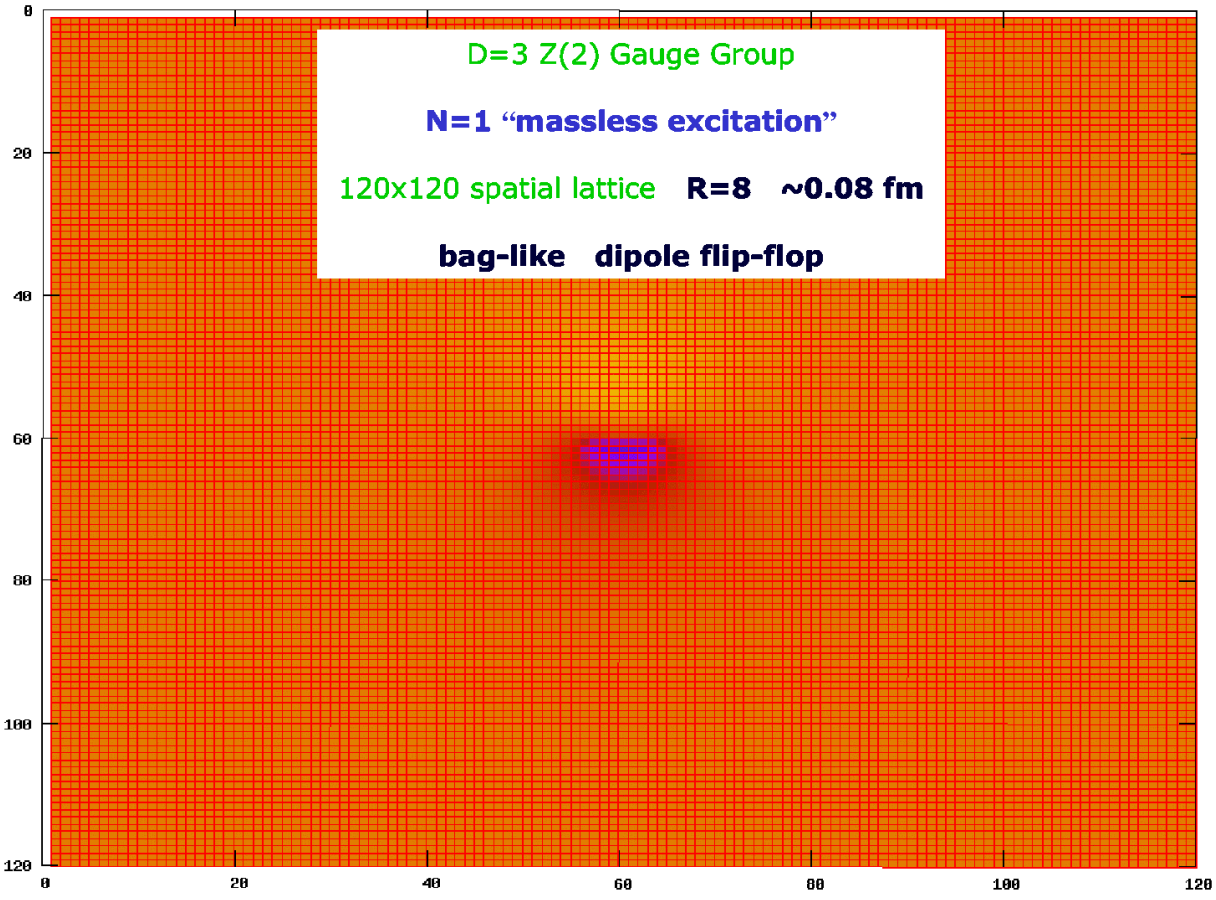
1. On-lattice QCD string spectrum

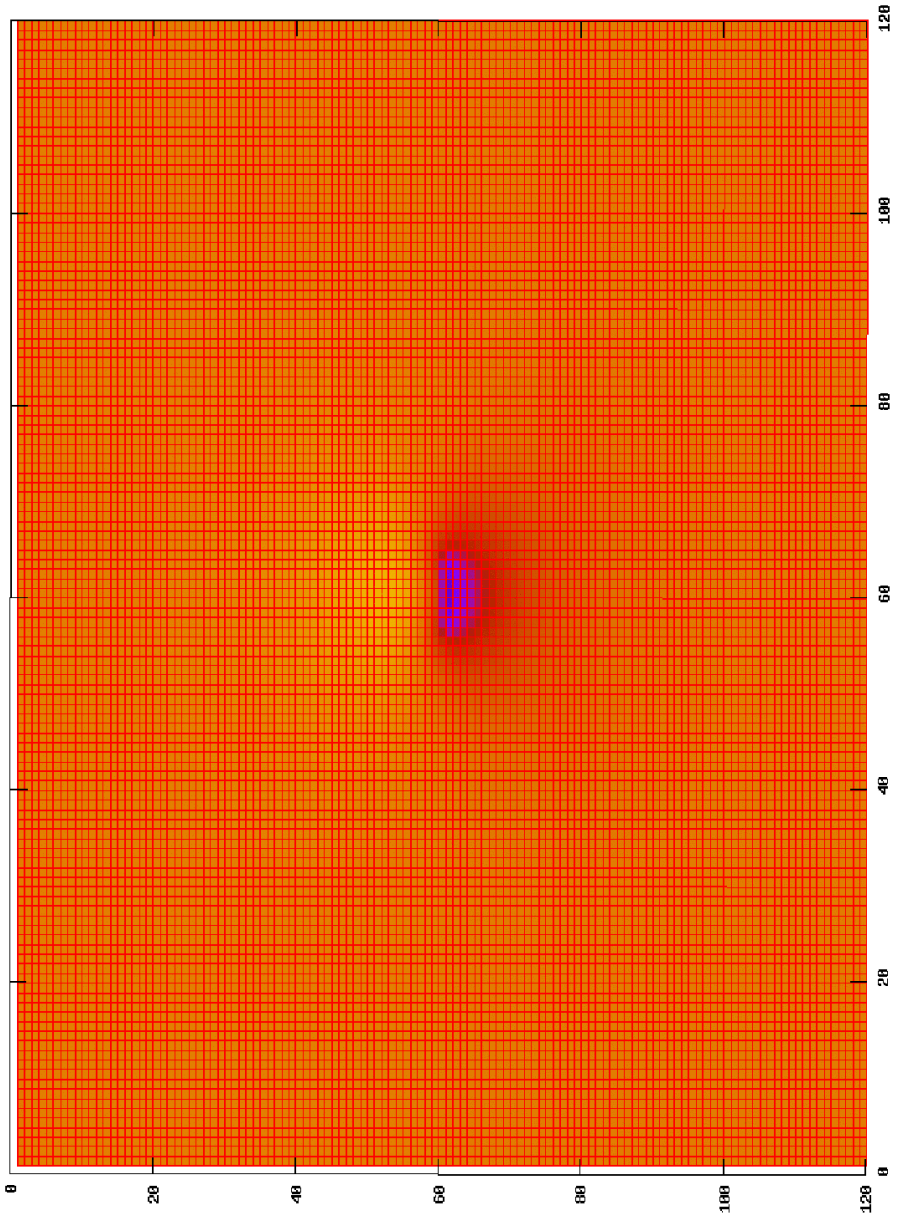
2. D=3 Z(2) gauge model
   microscopic loop equations (Polyakov)
   macroscopic string 3d Ising interface

Casimir energy of ground state
Excitation spectrum
Goldstone modes and collective variables
Effective theory?
Microscopic variables (loop equation)?
Geometric interpretation?

String theorists interested in QCD string problem

Quenched, but relevant in confinement/string and large N

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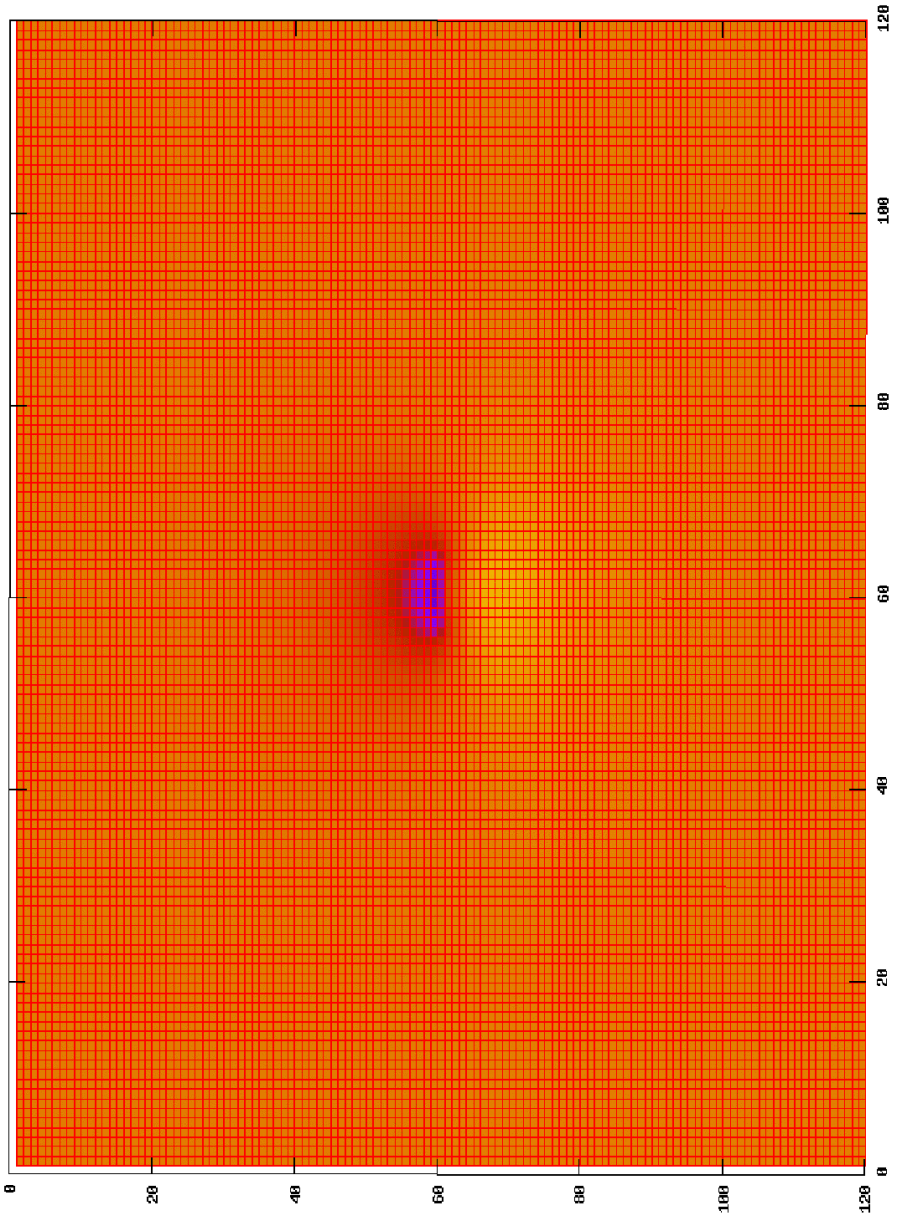
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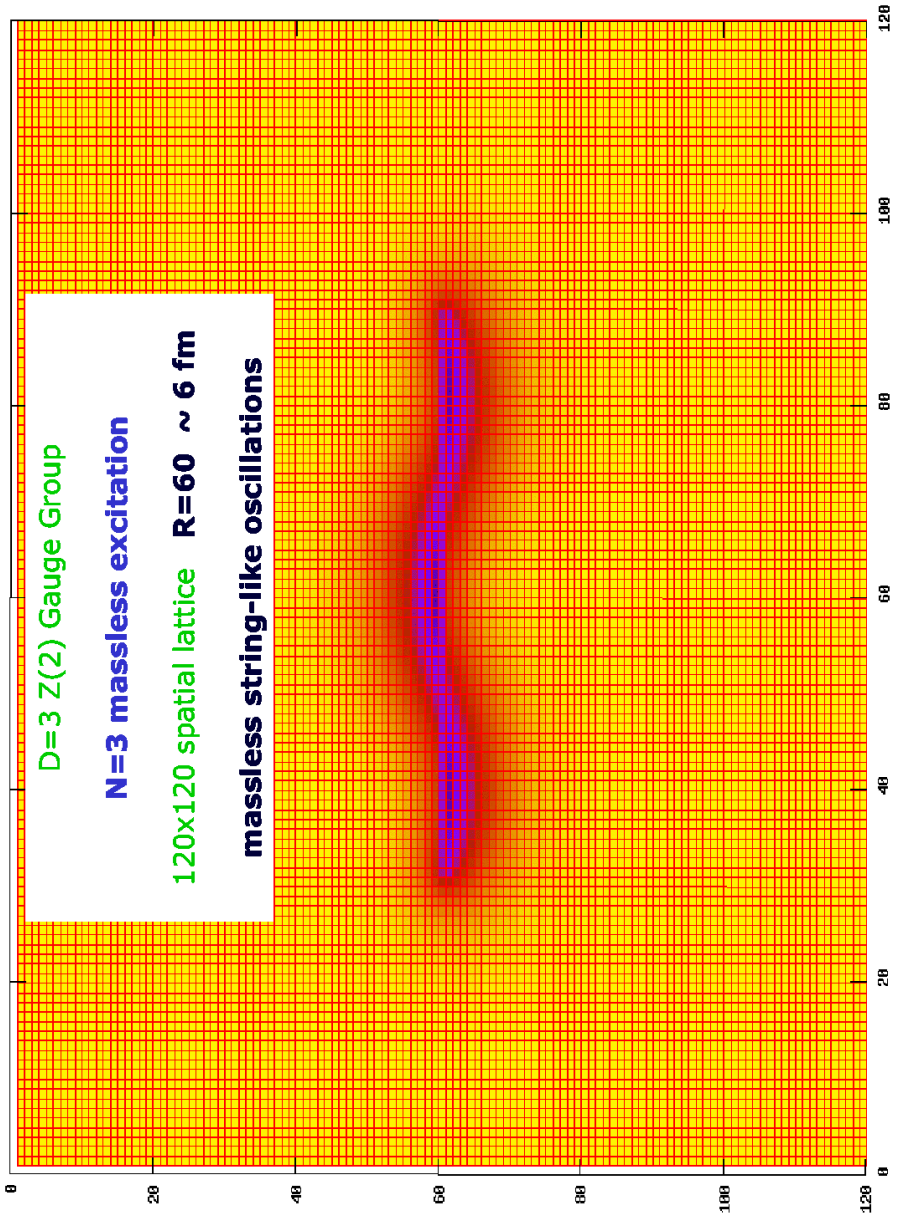
D=3 Z(2) Gauge Group
N=3 massless excitation
120x120 spatial lattice R=60 ~ 6 fm
massless string-like oscillations
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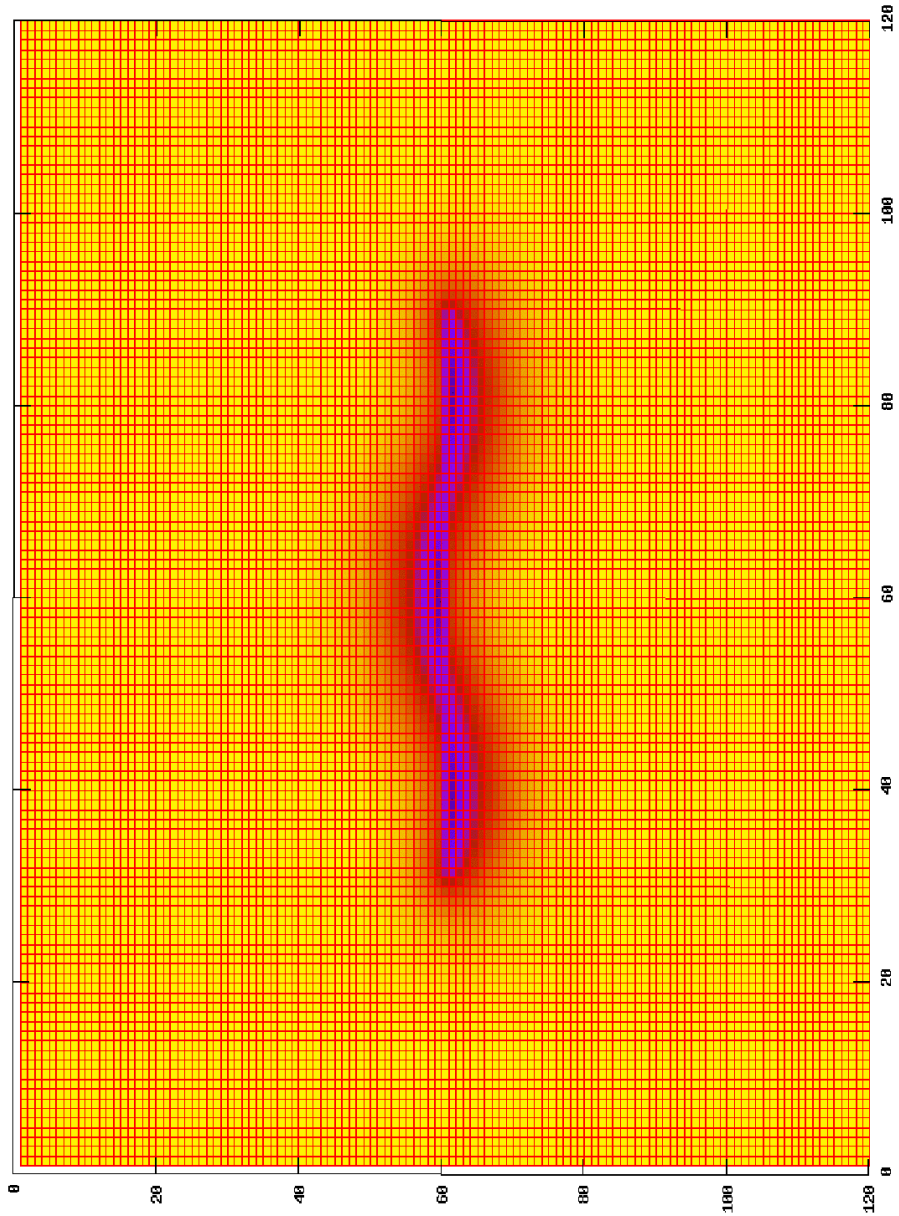
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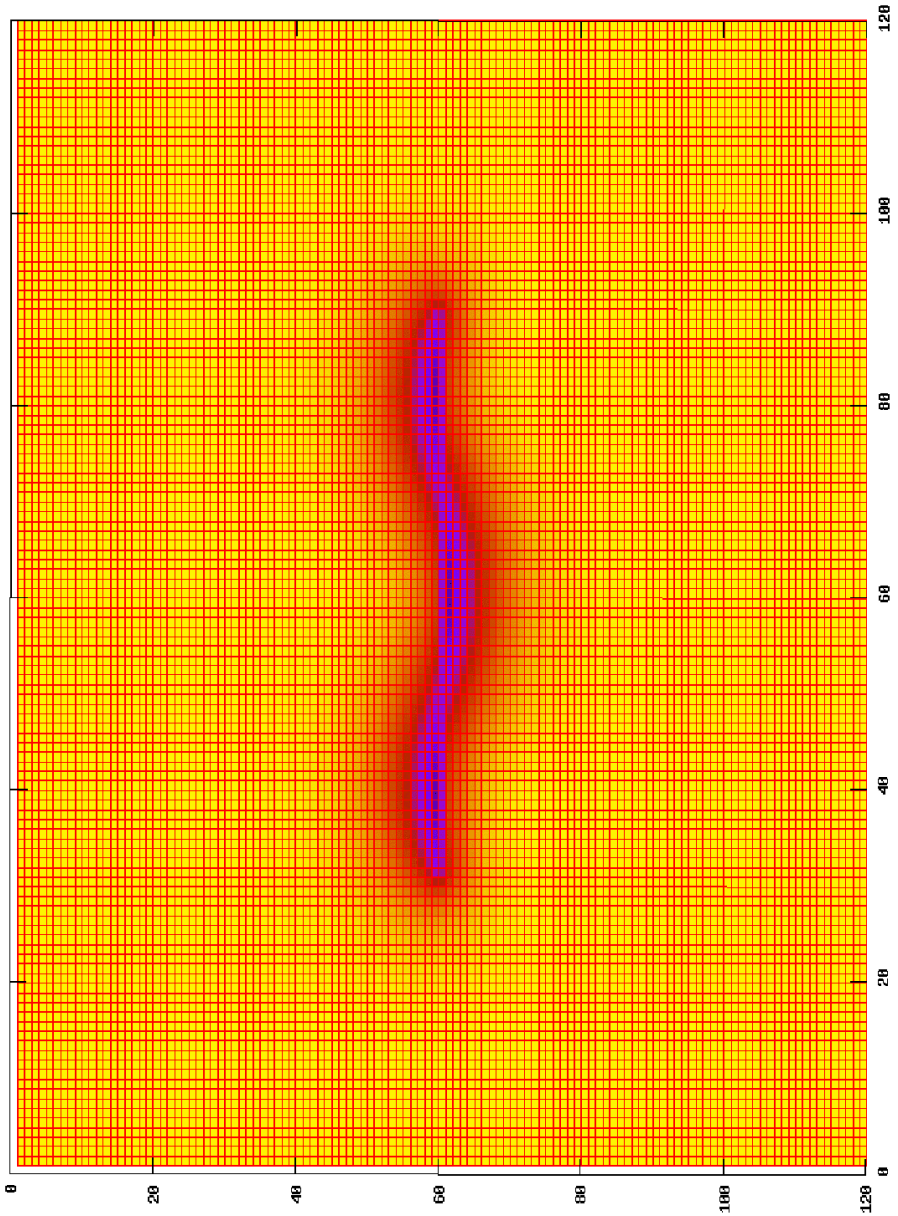
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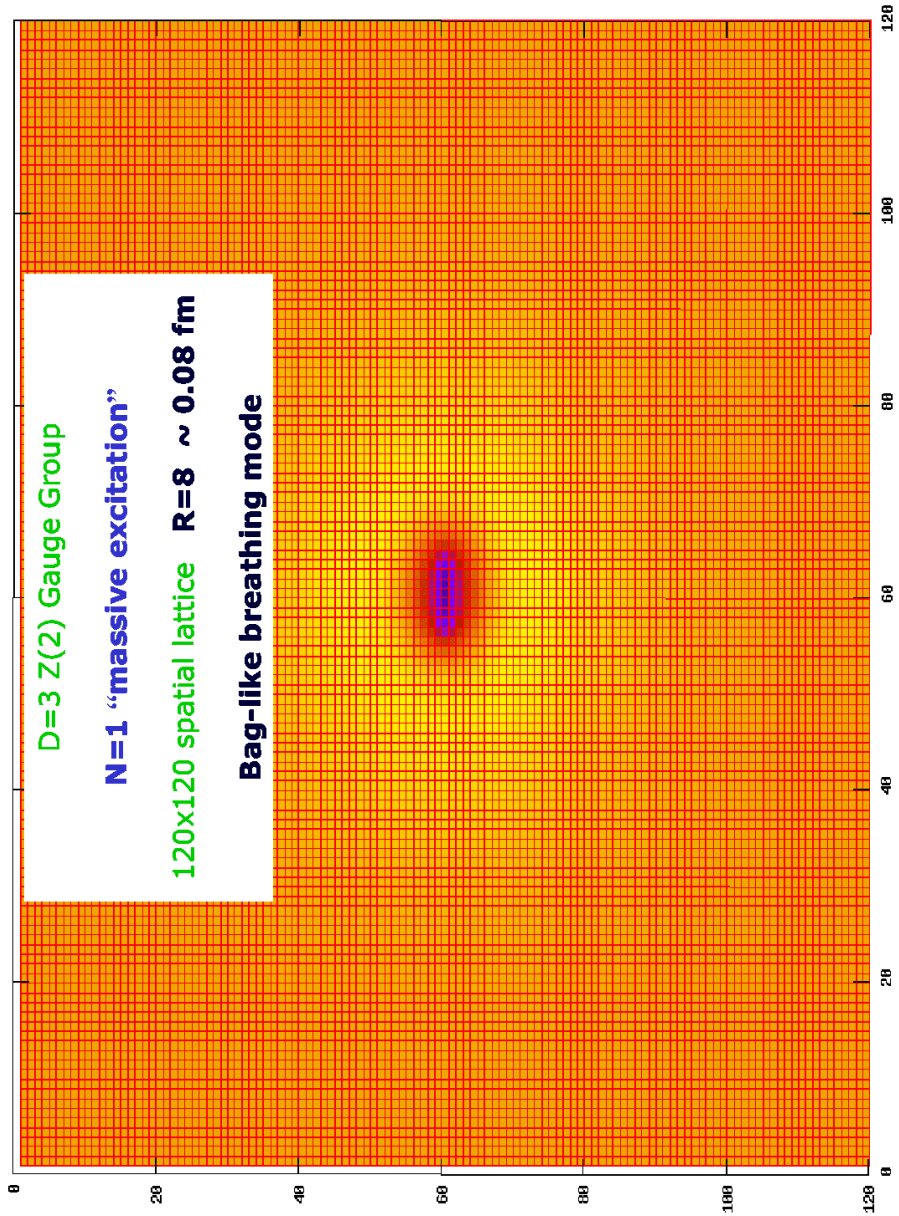
D=3 Z(2) Gauge Group
N=1 "massive excitation"
120x120 spatial lattice R=8 ~ 0.08 fm
Bag-like breathing mode
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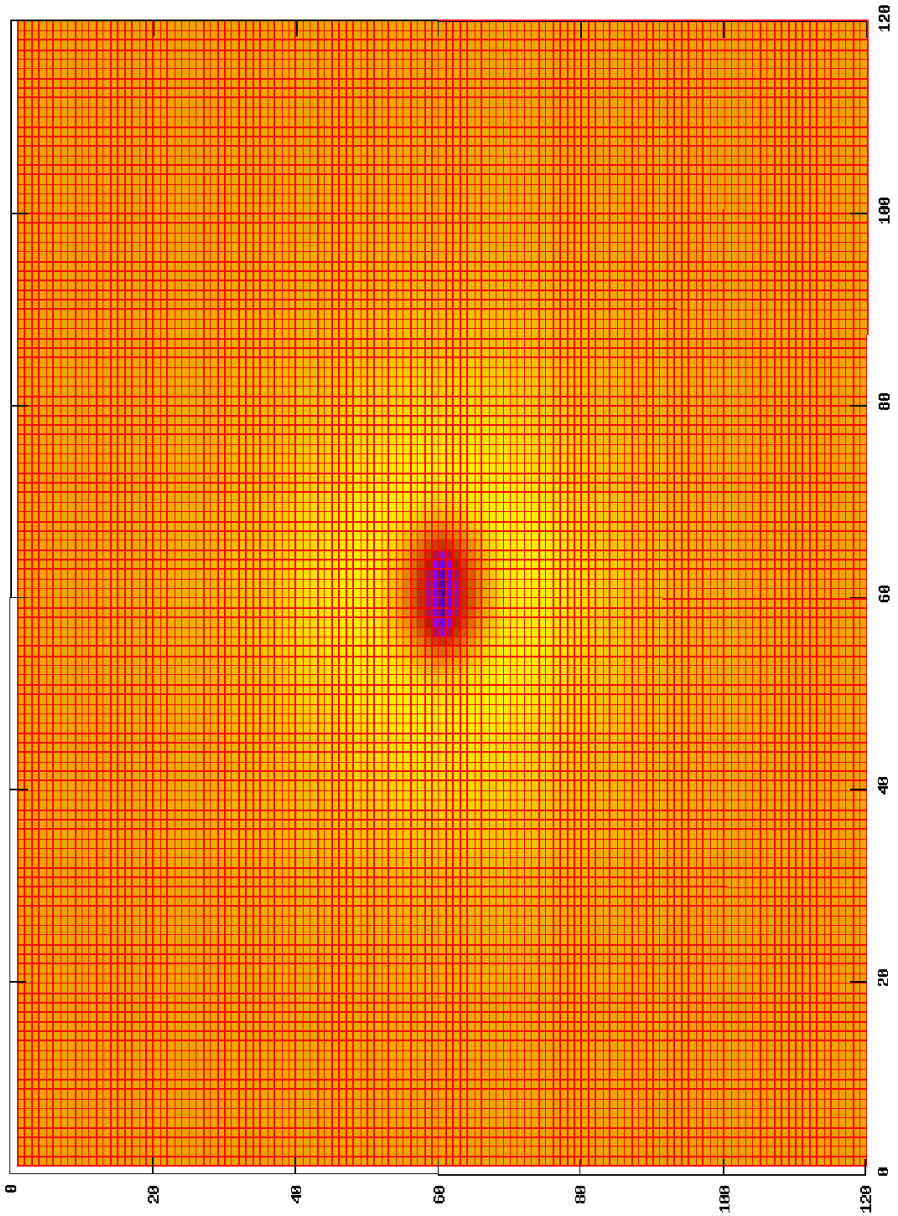
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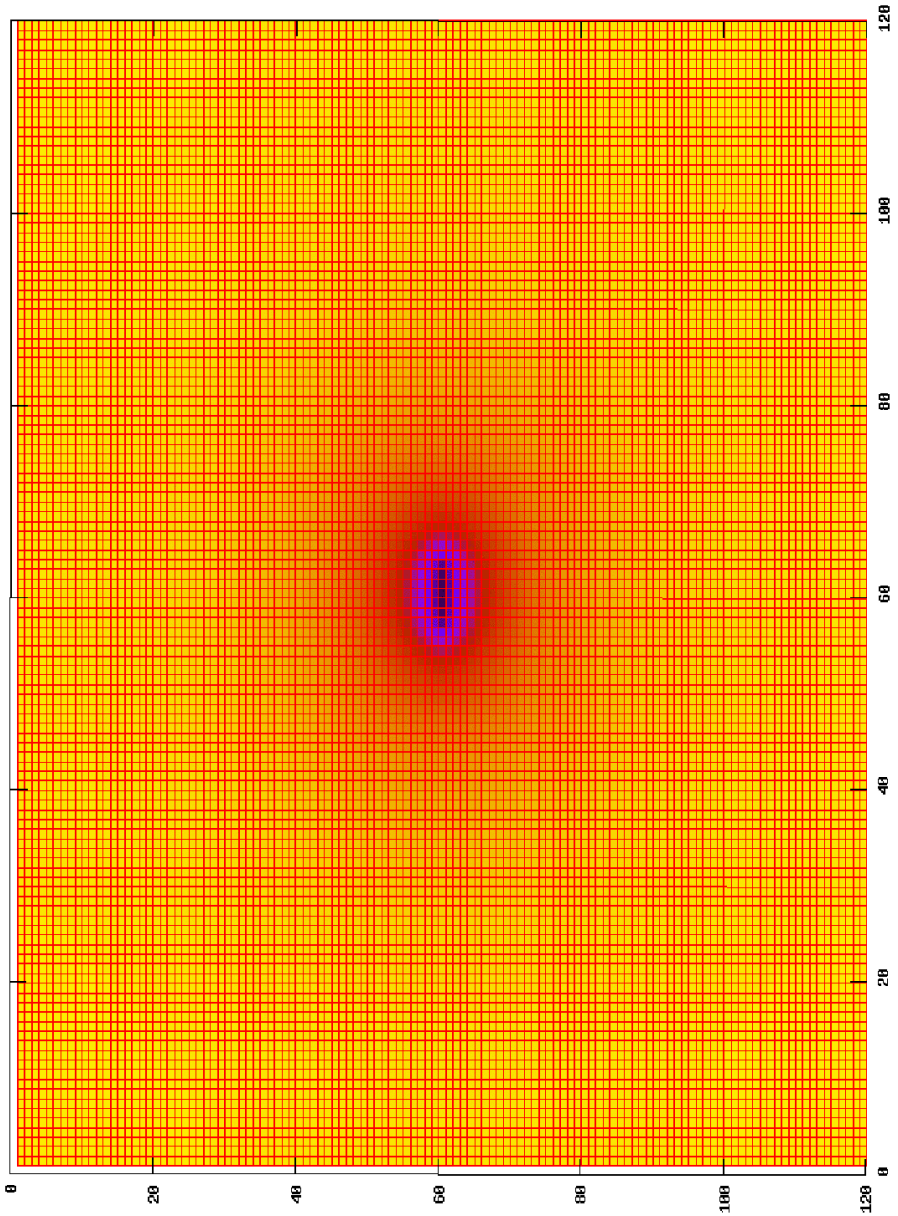
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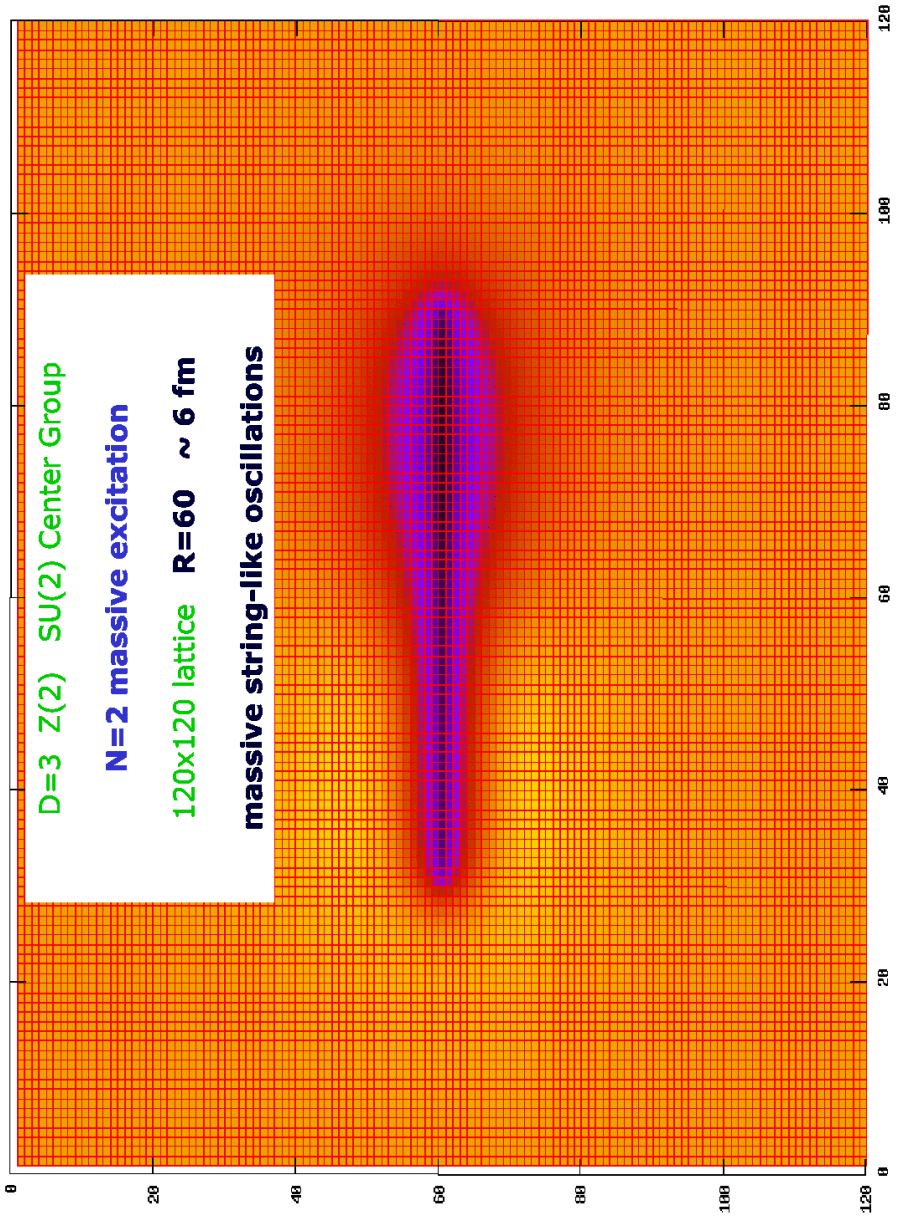
D=3 Z(2) SU(2) Center Group
N=2 massive excitation
120x120 lattice R=60 ~ 6 fm
massive string-like oscillations
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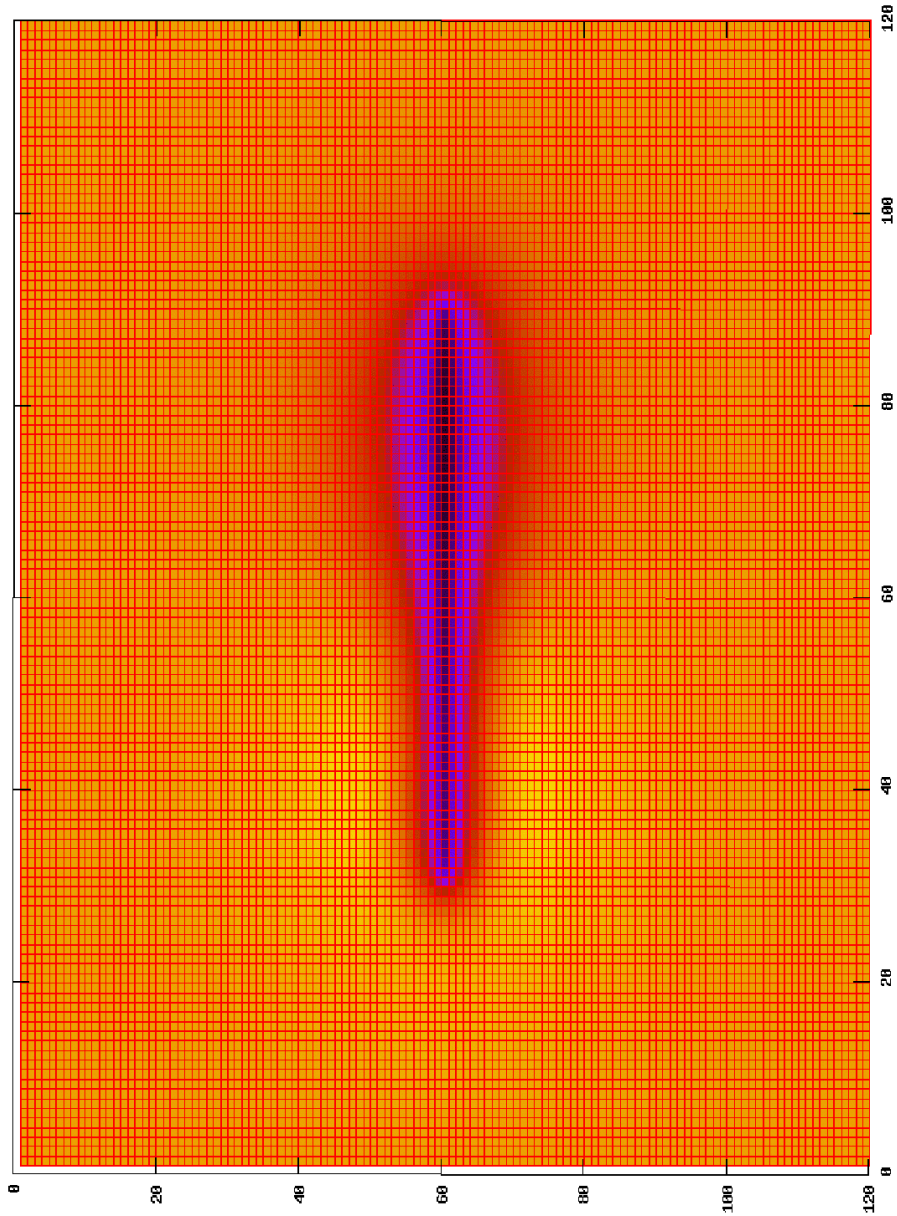
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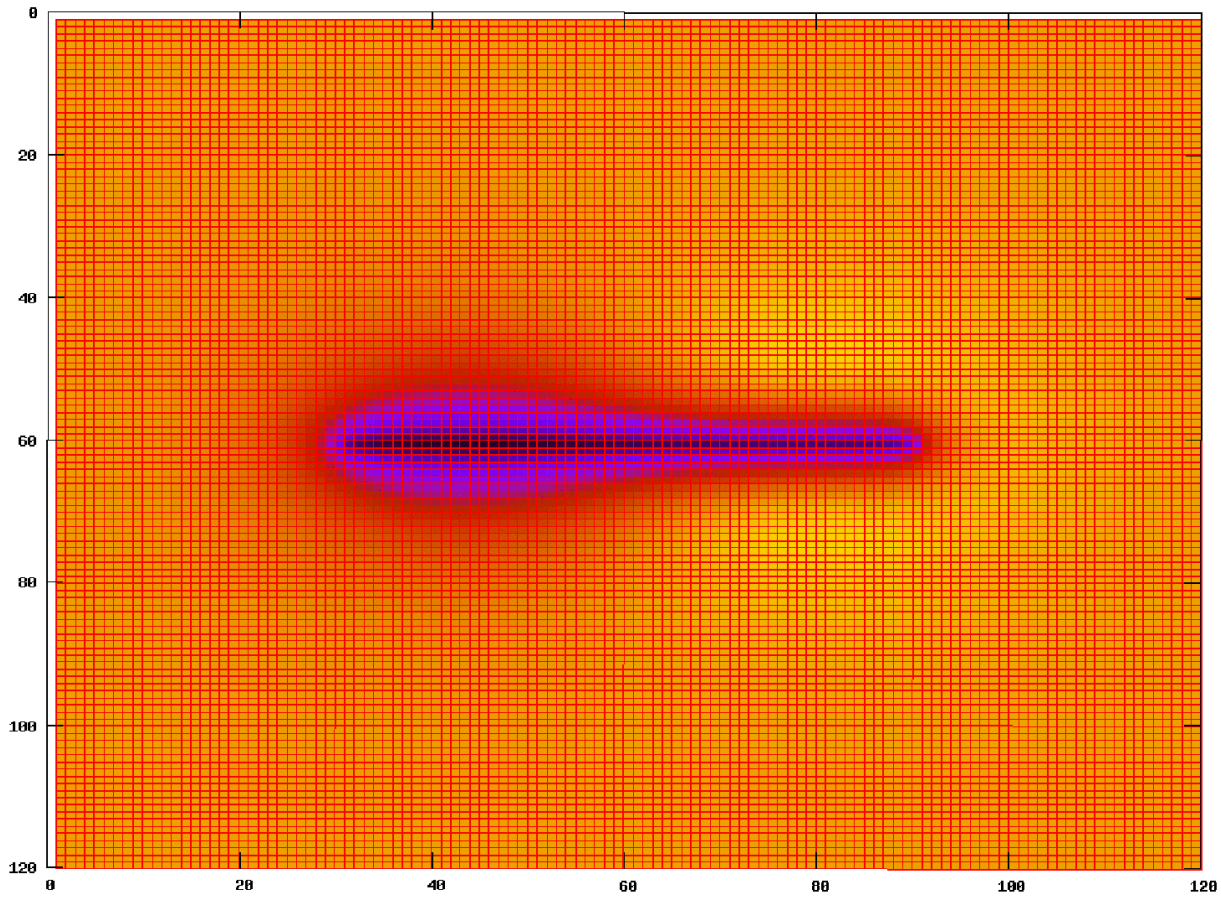
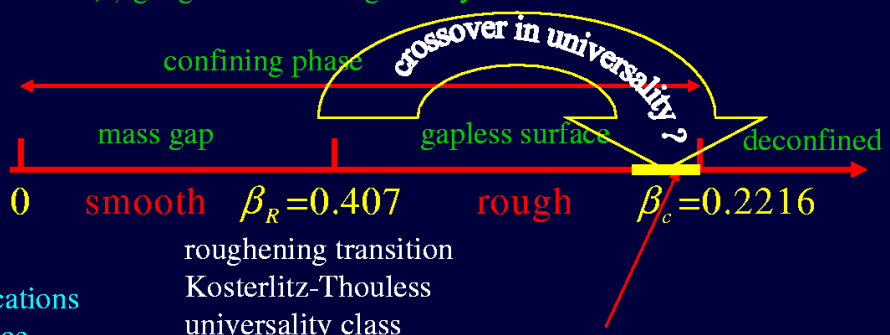


- Massless Goldstone modes?
- Local derivative expansion for their interactions?
  from fine structure in the spectrum
- Massive excitations?
- Breathing modes in effective Lagrangian?
- String properties ? Bosonic, NG, rigid, …?

Wilson Surface of 3d Z(2) Gauge Model

Similar picture expected in QCD

$\overline{\beta} = -\frac{1}{2}\ln(\tanh\beta)$ Z(2) gauge ⟷ Ising duality



roughening transition
Kosterlitz-Thouless
universality class

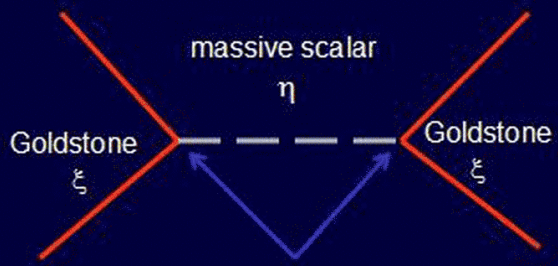
role of skrew dislocations
in Wilson surface

critical region
continuum limit (QCD)
effective ϕ^4 field theory

Semiclassical Loop Expansion
Soliton Quantization (string)

In rough phase (close to bulk critical point)

Most important step in deriving correction terms in effective action
of Goldstone modes in Z(2) D=3 gauge model:



$\partial_\mu\xi\partial_\mu\xi\eta$

$\sim \partial_\mu\xi\partial_\mu\xi\frac{1}{q^2+M^2}\partial_\mu\xi\partial_\mu\xi$

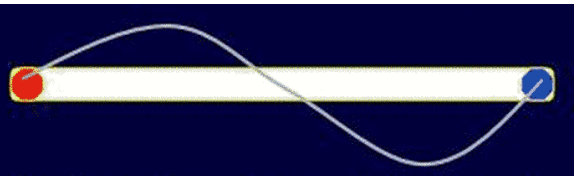
when $q = \frac{\pi}{R}n << M$

$n << R\cdot M/\pi$

resonance
non-local terms

$$S_2 = \frac{1}{4}c_2\int_0^T d\tau\int_0^R d\sigma\left\{\frac{1}{2}(\partial_a\xi\partial_a\xi)(\partial_b\xi\partial_b\xi)\right\}+\cdots$$

$$S_{eff} = \frac{1}{2\pi\alpha'}\int_0^T d\tau \int_0^R d\sigma \left\{\frac{1}{2}\partial_a\xi\partial_a\xi + \cdots\right\}$$



ξ is the D-2 dimensional displacement vector (collective string variables)

Massless Goldstone field ⇔ collective string coordinate

Small wavelengths unstable! => glueball emission

$$S_1 = \frac{1}{4}b\int_0^T d\tau \left\{(\partial_1\xi\partial_1\xi)_{\sigma=0} + (\partial_1\xi\partial_1\xi)_{\sigma=R}\right\}$$

Boundary operators set to zero in open-closed string duality

$$V(R) = \sigma R + \mu - \frac{\pi}{24R}(d-2)\left(1+\frac{b}{R}\right)$$

$$\Delta E = \frac{\pi}{R}\left(1+\frac{b}{R}\right)$$

$$S_2 = \int_0^T d\tau \int_0^R d\sigma \left\{\frac{c_2}{2}(\partial_a\xi\partial_a\xi)(\partial_b\xi\partial_b\xi) + \frac{c_3}{2}(\partial_a\xi\partial_b\xi)(\partial_a\xi\partial_b\xi) + \ldots\right\}$$

higher dimensional ops $O(1/R^3)$

c_3 term is not independent in D=3

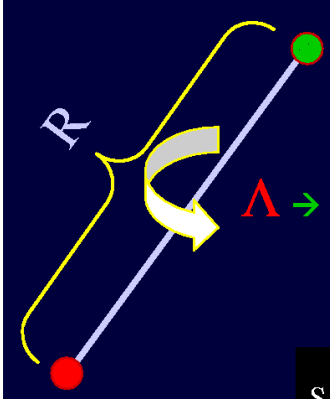
2. **Dirichlet Strings in D=4 and D=3 dimensions**
   - fixed end D=4 SU(3) QCD string spectrum
   - fixed end D=3 SU(2) QCD string and Z(2) string new results

3. **Dirichlet Casimir Energy**
   - origin of Casimir energy and effective string description
   - Luscher-Weisz results
   - Z(2)
   - paradox ?
   - 1+1 dimensional toy model insight from quantum mechanics

4. **Closed String (torelon) with unit winding**
   - D=4 SU(3) QCD spectrum new results
   - Closed string Casimir energy new results
   - D=3 Z(2) spectrum new results

5. **Conclusions**

SU(3) D=4



Opposite fixed color source (antiquark)

Λ → angular momentum projected along quark-antiquark axis

Fixed color source (quark)

S states (Λ =0) Σ_g^{+-}

P states (Λ =1) Π^{+-}

D states (Λ =2) Δ^{+-}

⋮

Three exact quantum numbers characterize gluon excitations:

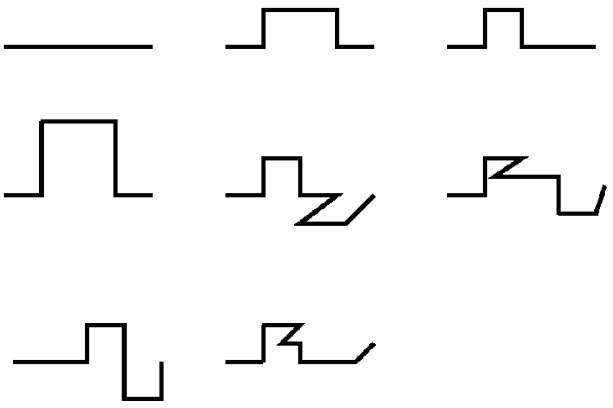
Λ^{+-} **Angular momentum with chirality**

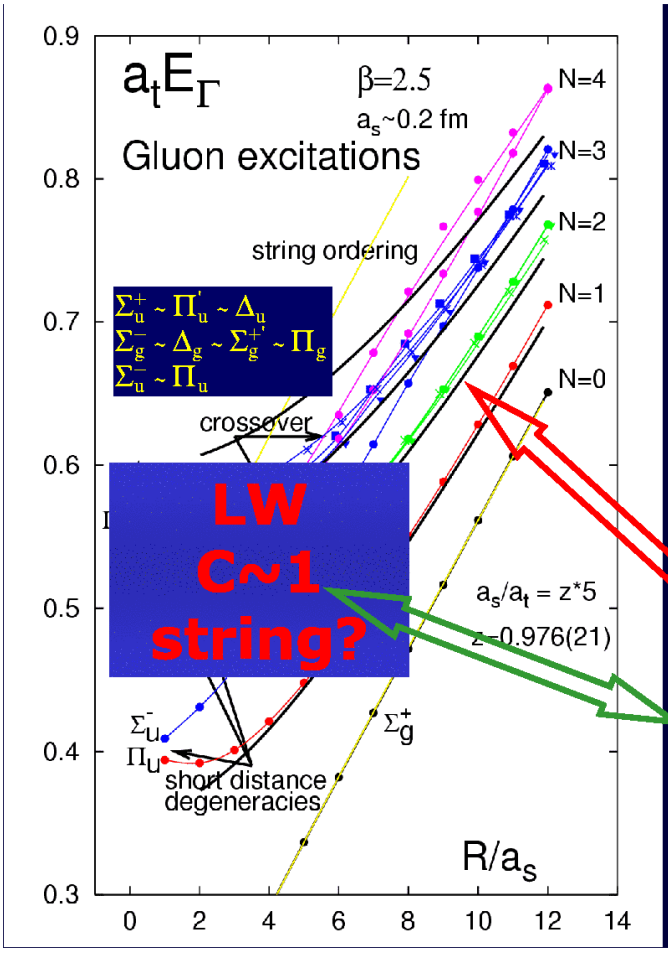
$+-$ **Chirality, or reflection symmetry for $\Lambda = 0$**

CP **g** (gerade) **CP even**
u (ungerade) **CP odd**

Gluon excitations are projected out with generalized Wilson loop operators on time sclices

the spatial straight line is replaced by linear combinations of twisted paths





Three length scales in energy spectrum:

R < 0.5 fm
Short distance QCD
Bag-like nearly spherical symmetry
OPE (Soto et al.)

R ~ 0.5 fm – 1.5 fm
Crossover (model sensitive)

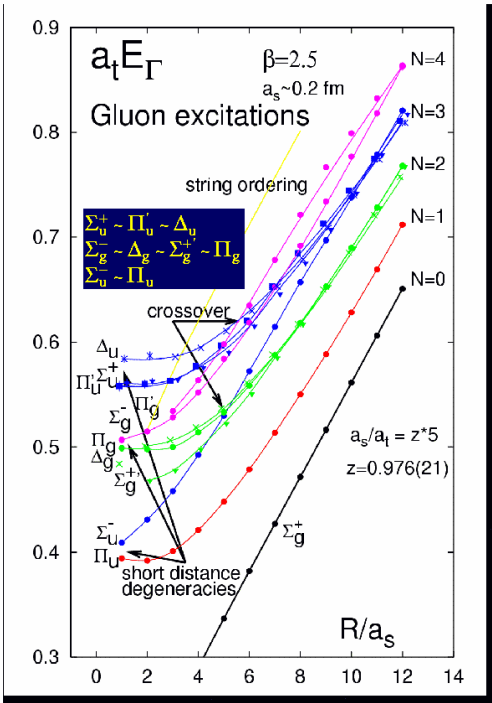
R ~ 2 fm - 3 fm
Onset of string ordering

Nambu-Goto levels
in black → Fine structure

Casimir energy puzzle:

seen around R ~ 0.5 fm ?

1. **Very few stable modes**
2. **Non string-like distortions**



Short distance region

R<0.5fm approximate spherical symmetry
Bag-like "non-string" picture

Multipole operator product expansion
of $\vec{A}(\vec{r},t)$ and $\vec{J}(\vec{r},t)$:

$$H_{Cb} = H_{gauge} + \frac{g^2 Q\cdot\bar{Q}}{4\pi|\vec{r}_1-\vec{r}_2|} - g^3(Q\times\bar{Q})_a \int d^3\vec{r}\,\vec{A}_a(\vec{r},t)\vec{J}(\vec{r},t)$$

$$4\pi\vec{J}(\vec{r}) = \left(\frac{1}{|\vec{r}-\vec{r}_1|} - \frac{1}{|\vec{r}_1-\vec{r}_2|}\right)\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}_2|} - \frac{1}{|\vec{r}_1-\vec{r}_2|}\right)$$

$$Q\cdot\bar{Q} = \begin{cases} -\frac{4}{3} & \text{singlet} \\ +\frac{1}{6} & \text{octet} \end{cases}$$

Crossover transition region
difficult to interpret
model dependent

D=3 SU(2) and Z(2) will be presented together

Wilson line of Z(2) gauge group becomes a

seam of flipped links in dual (Ising) representation

Multispin coding of Ising bits in Monte Carlo

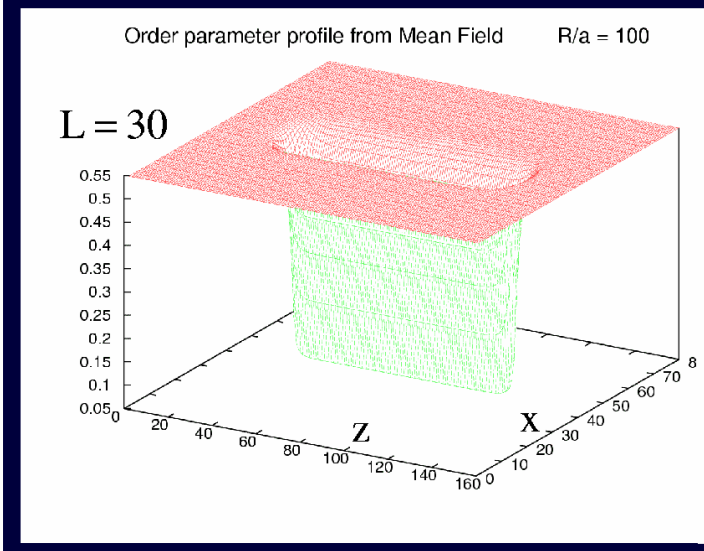
Parallel * Parallel

cluster algorithms

Effective Schrodinger equation
based on fluctuation matrix of string soliton

$$M = -\nabla^2 + U''(\phi_{soliton})$$

V_{eff}



-in long flux limit spectrum is expected to factorize

- translational zero mode of soliton

- Goldstone spectrum

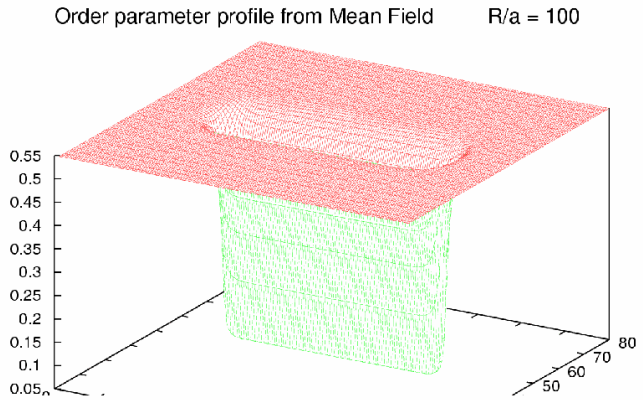
$\phi(z) \cdot \exp(iqx)$

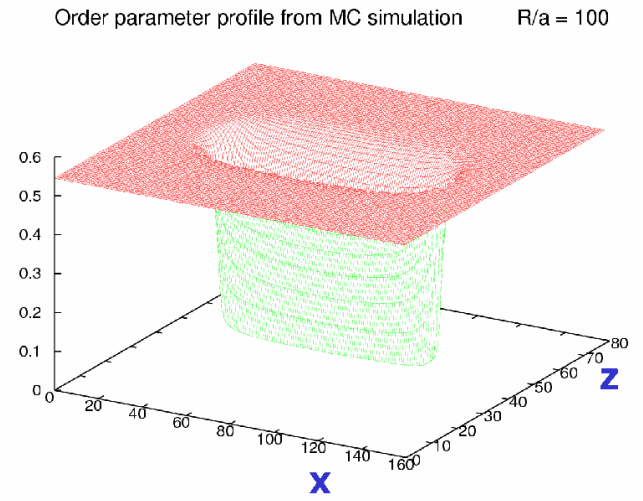
zero energy bound state

quantized momenta of Goldstone modes in box of length L

$$q = \frac{\pi}{L} n, \quad n = 1,2,3...$$

shape and end effects distort!





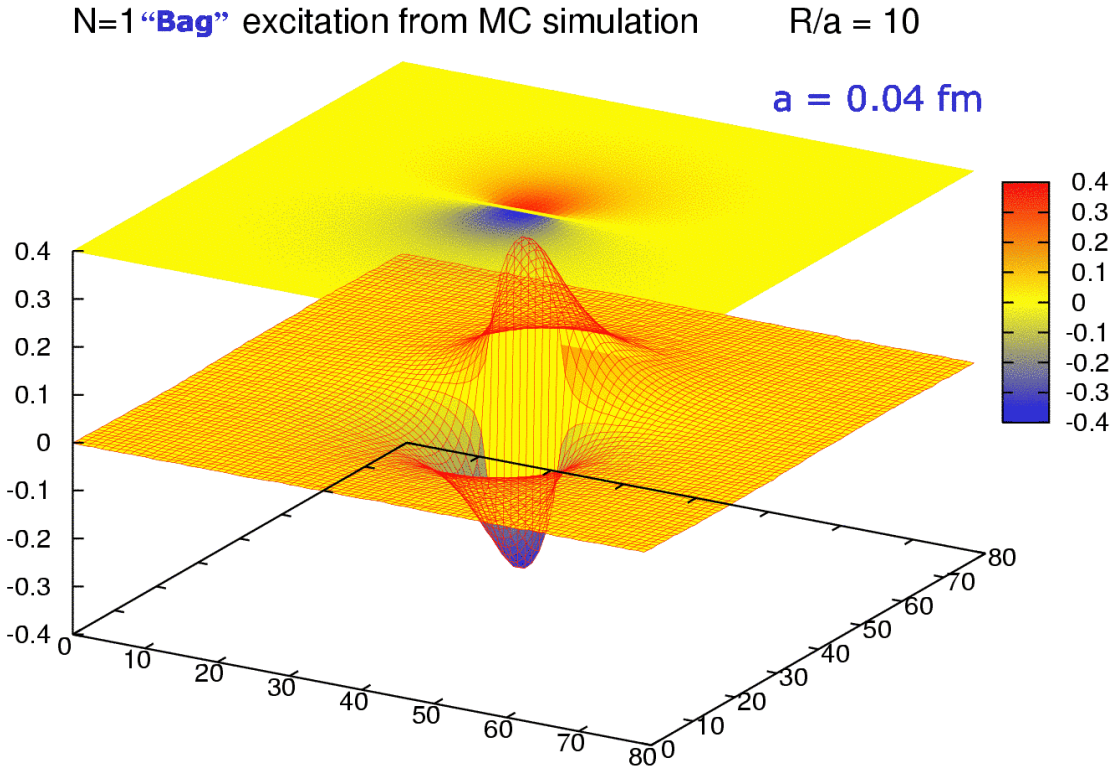
good analytic/numerical handle on the Z(2) model

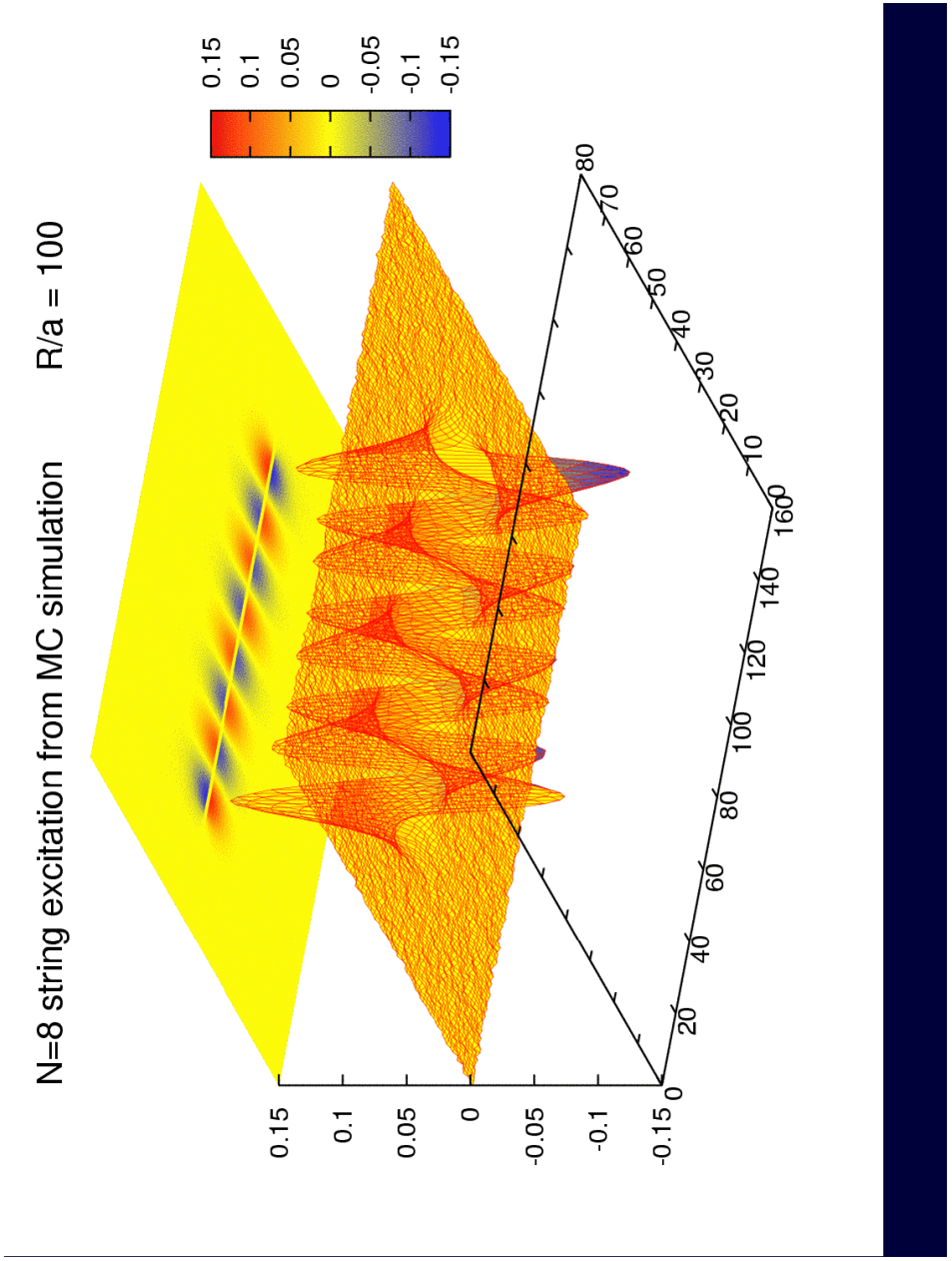
in addition to MC

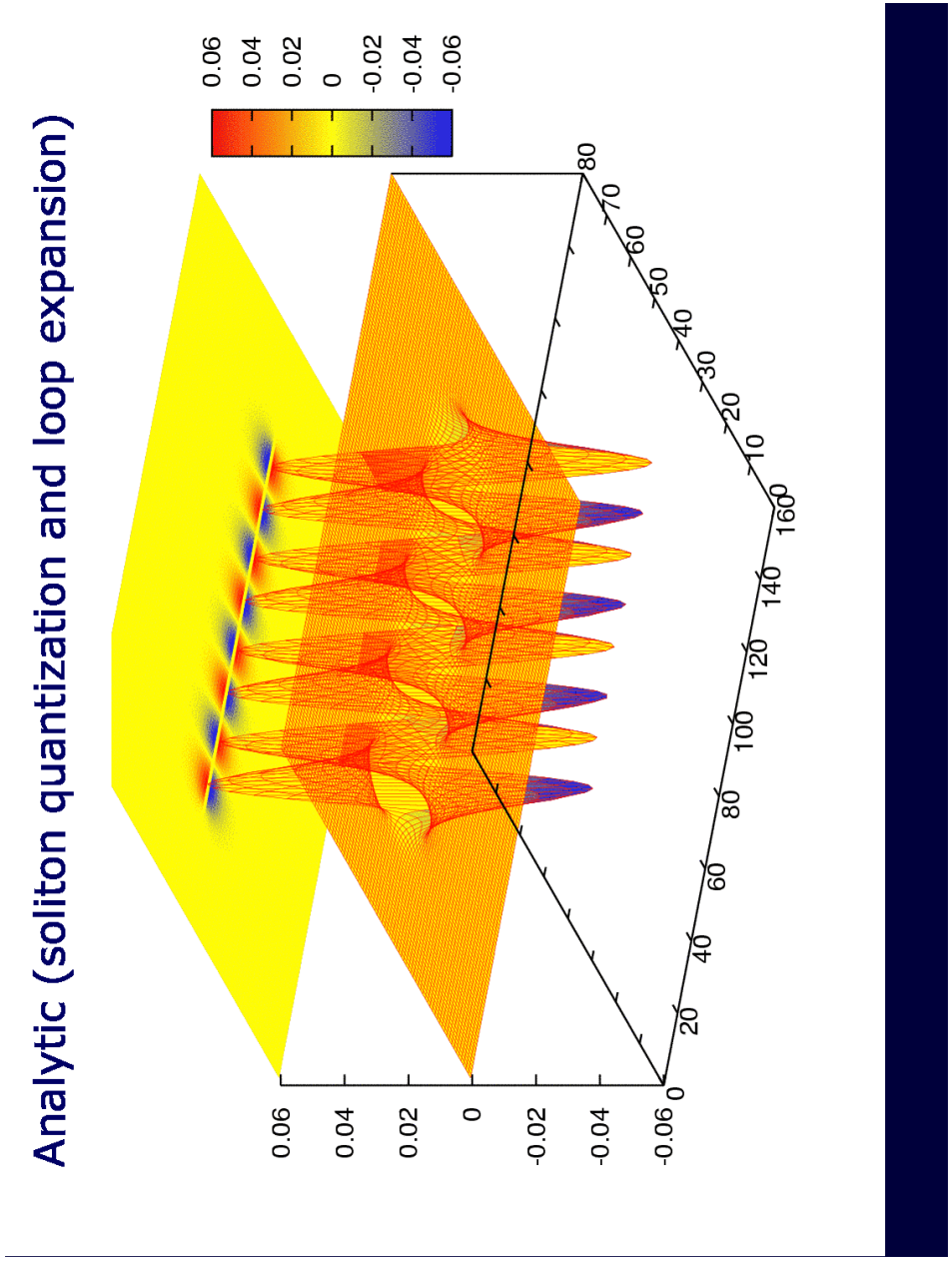
Effective Schrodinger potential

$P_x = +-1$ and $P_z = +-1$

two symmetry quantum numbers




N=8 string excitation from MC simulation
R/a = 100
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0
20
40
60
80
100
120
140
160
0
10
20
30
40
50
60
70
80


Analytic (soliton quantization and loop expansion)
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0
20
40
60
80
100
120
140
160
0
10
20
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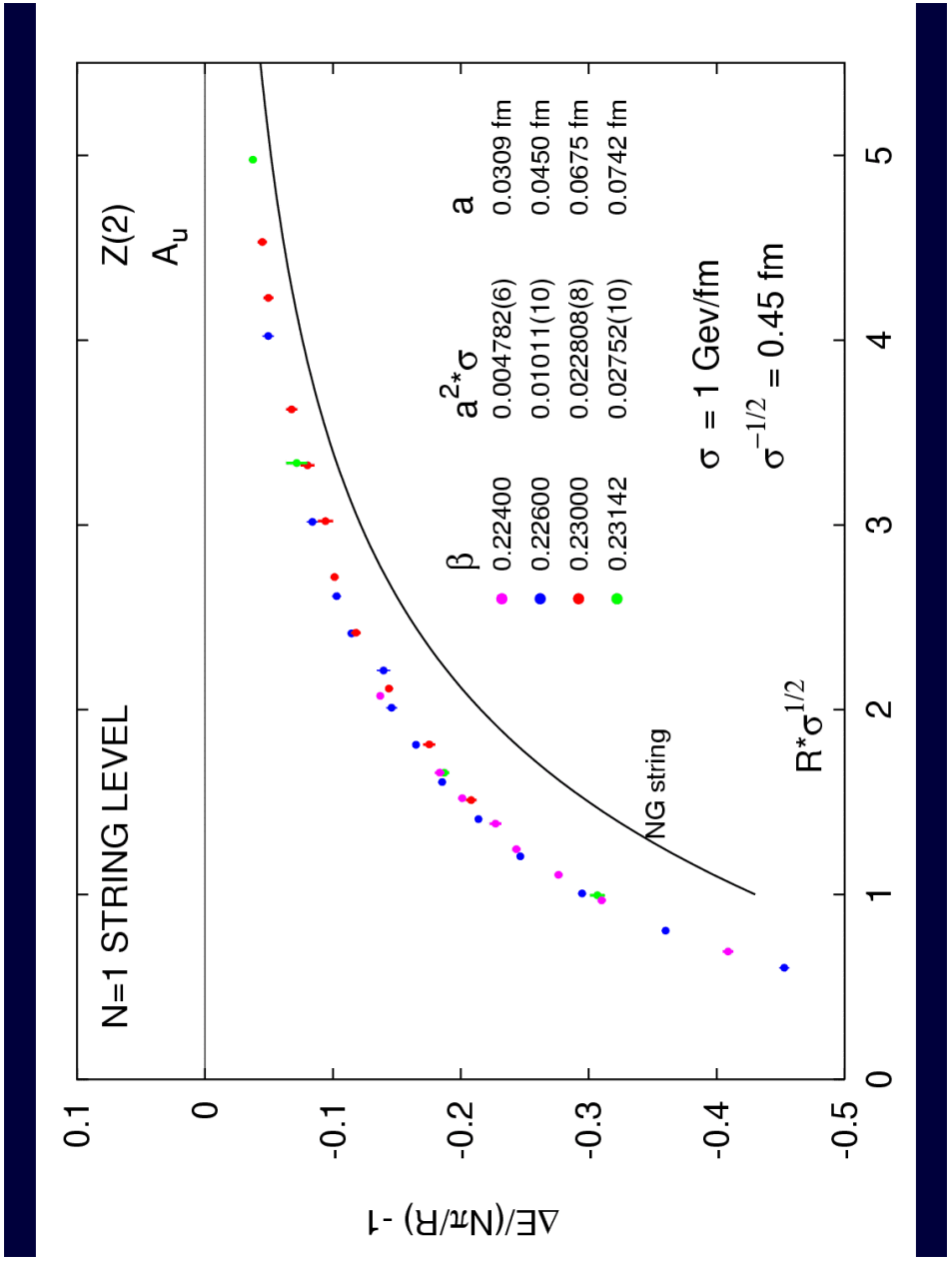
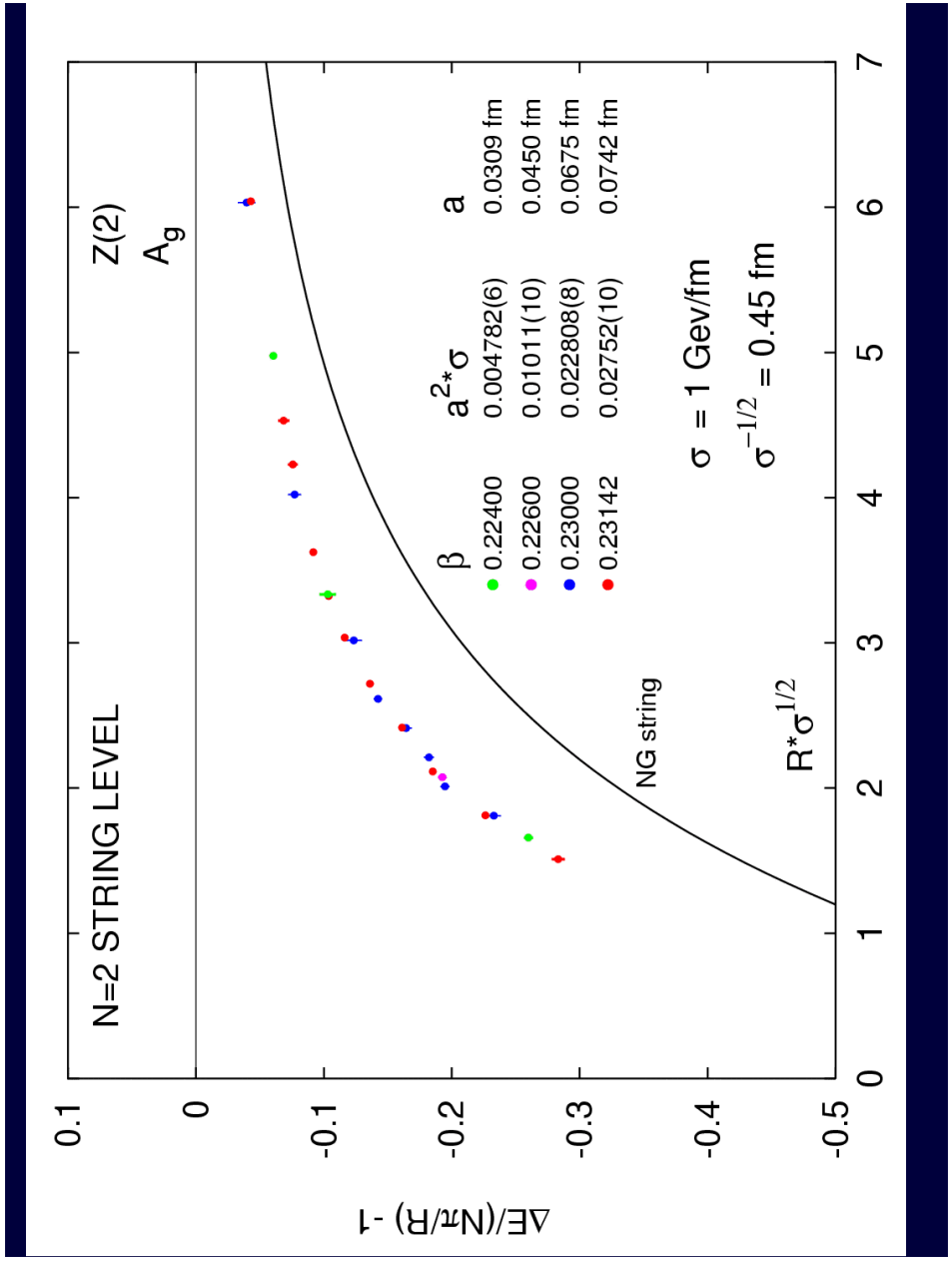
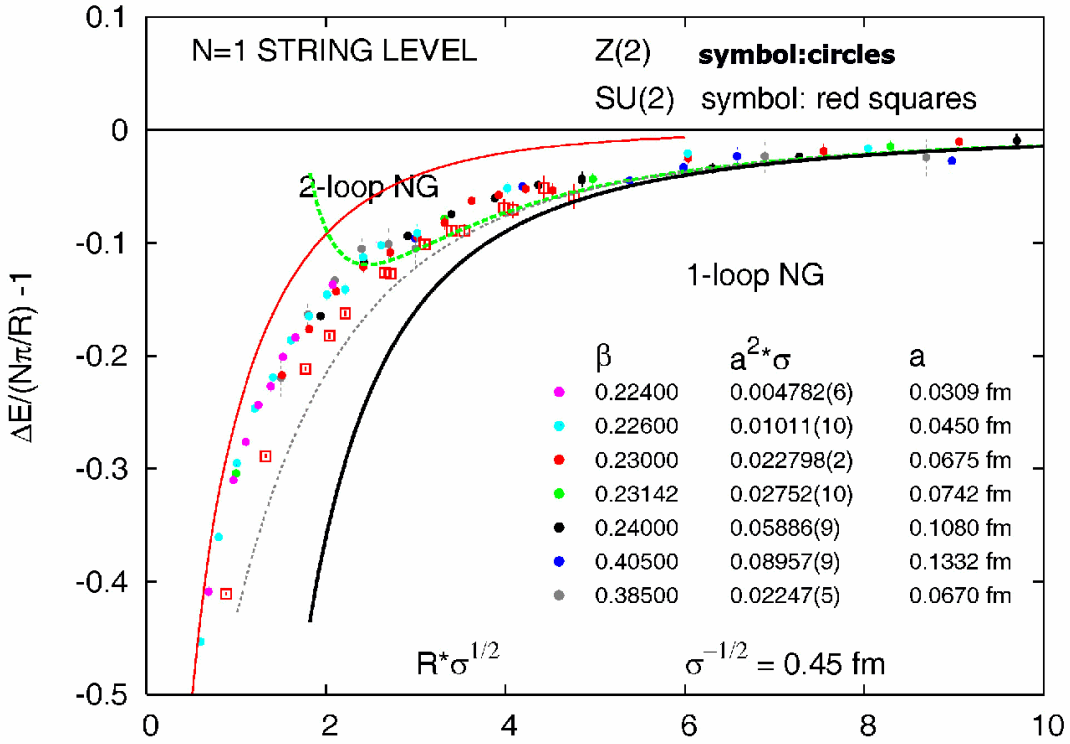
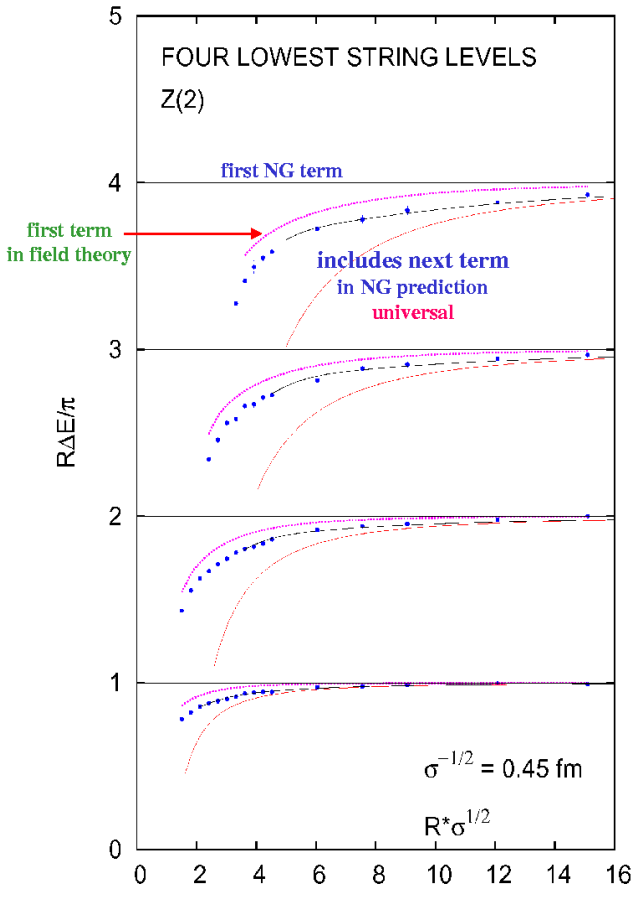

N=1 STRING LEVEL
Z(2)
A_u
β
$a^2*\sigma$
a
0.22400 0.004782(6) 0.0309 fm
0.22600 0.01011(10) 0.0450 fm
0.23000 0.022808(8) 0.0675 fm
0.23142 0.02752(10) 0.0742 fm
σ = 1 Gev/fm
$\sigma^{-1/2}$ = 0.45 fm
NG string
$R*\sigma^{1/2}$
ΔE/(Nπ/R) -1


N=2 STRING LEVEL
Z(2)
A_g
β
$a^2*\sigma$
a
0.22400 0.004782(6) 0.0309 fm
0.22600 0.01011(10) 0.0450 fm
0.23000 0.022808(8) 0.0675 fm
0.23142 0.02752(10) 0.0742 fm
σ = 1 Gev/fm
$\sigma^{-1/2}$ = 0.45 fm
NG string
$R*\sigma^{1/2}$
ΔE/(Nπ/R) -1

SU(2) and center Z(2) exhibit nearly universal behavior





Summary of main results on the spectrum of the fixed end Z(2) string

$x = \sqrt{\sigma} R$ dimensionless scale variable

$$E_N/\sqrt{\sigma} = x\sqrt{1-\frac{D-2}{12x^2}\pi+\frac{2\pi N}{x^2}}$$ NG

Expand energy gaps for large x

$$\frac{R(E_N-E_0)}{\pi} = N+\frac{a(N)}{x^2}+\frac{b(N)}{x^4}+\ldots$$

First correction to asymptotic spectrum appears to be universal

Higher corrections code new physics like string rigidity, etc.

Similar expansion for string-soliton with unit winding

Data for R < 4 fermi prefers field theory description which incorporates end effects naturally

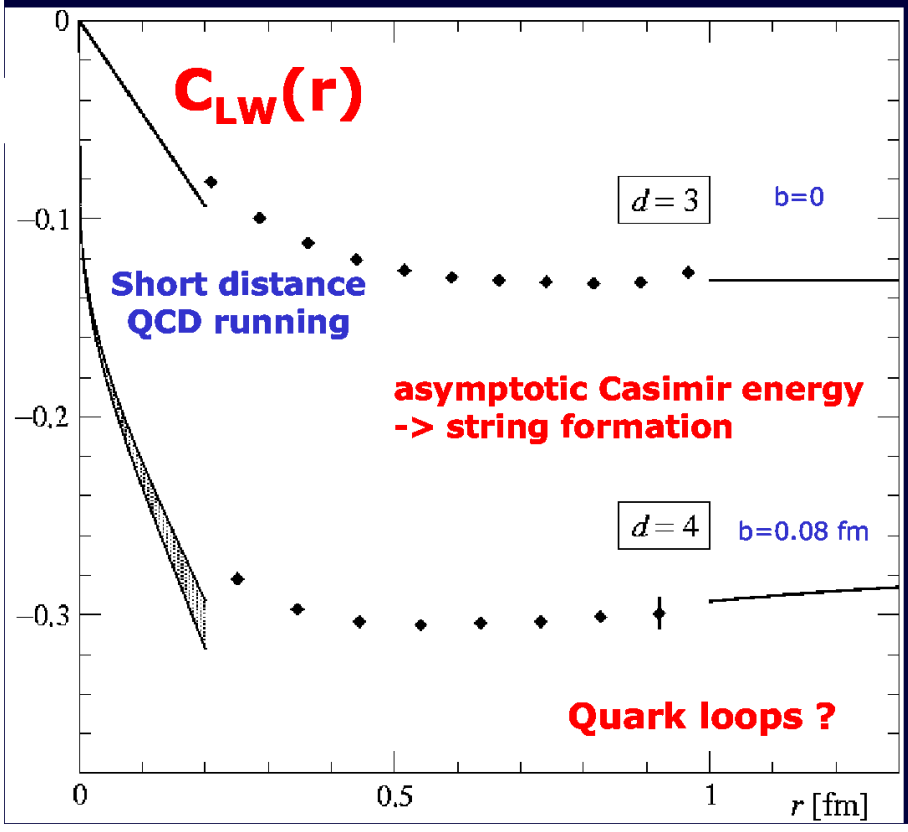
3. **Dirichlet Casimir Energy**
   - origin of Casimir energy and effective string description
   - Luscher-Weisz results
   - Z(2)
   - paradox ?
   - 1+1 dimensional toy model insight from quantum mechanics

4. **Closed String (torelon) with unit winding**
   - D=4 SU(3) QCD spectrum new results
   - Closed string Casimir energy new results
   - D=3 Z(2) spectrum new results

5. **Conclusions**

Luscher-Weisz Casimir Energy

SU(3)



$V(r) = \sigma r + \text{const} - \pi(d-2)/24r$

$F(r) = V'(r)$

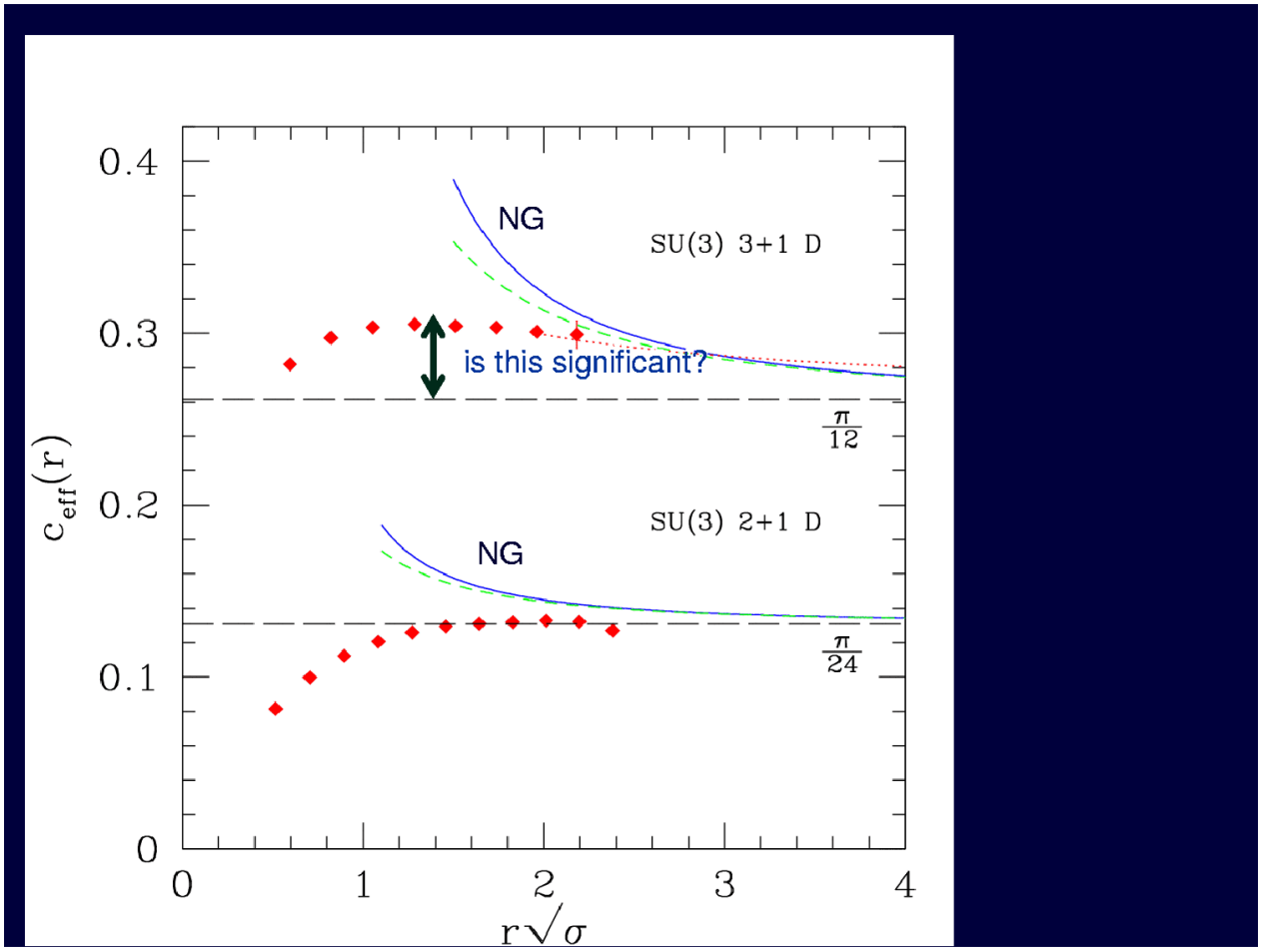
$$C_{LW}(r) = \frac{1}{2} r^3 F'(r)$$

$-\pi(d-2)/24$

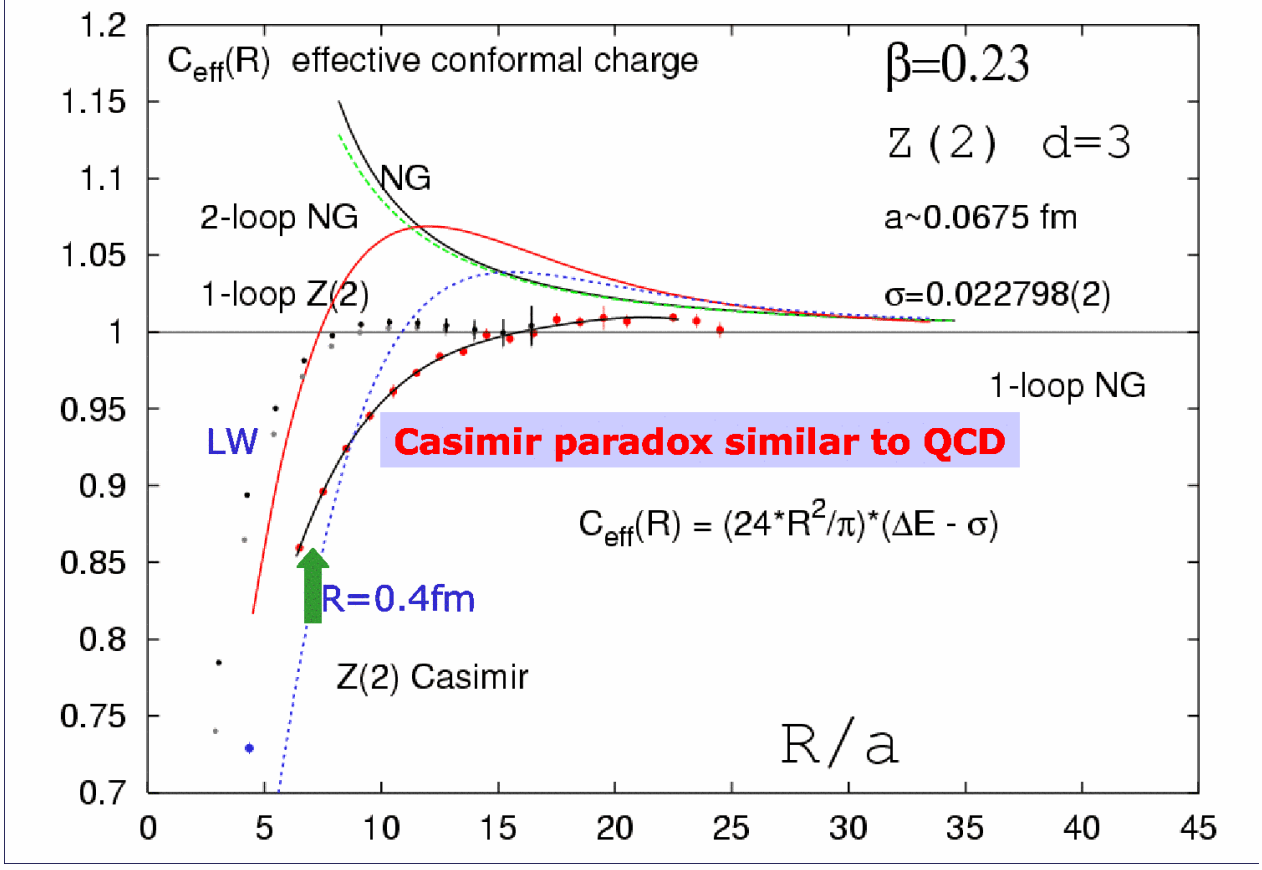
asymptotic r → infinity

Evidence for string formation in QCD?

Loop equations
ADS string theory



Z(2) Casimir energy



Two basic questions:

- Why the precocious onset of $C_{eff} \sim 1$?
- Where does the central charge C=1 reside?

  On a geometric string?

  Or distributed between massless Goldstone modes and the bulk?

  Answer to second question will determine whether early onset of $C_{eff} \sim 1$ is a true signal of string formation, or just an accident

$$\frac{1}{2}\sum_{n=1}^{\infty} n = -\frac{\pi}{24}$$

smart enough for string theory?

This is NOT a paradox

We turn to the D=1+1 lattice for learning how to do the sum:

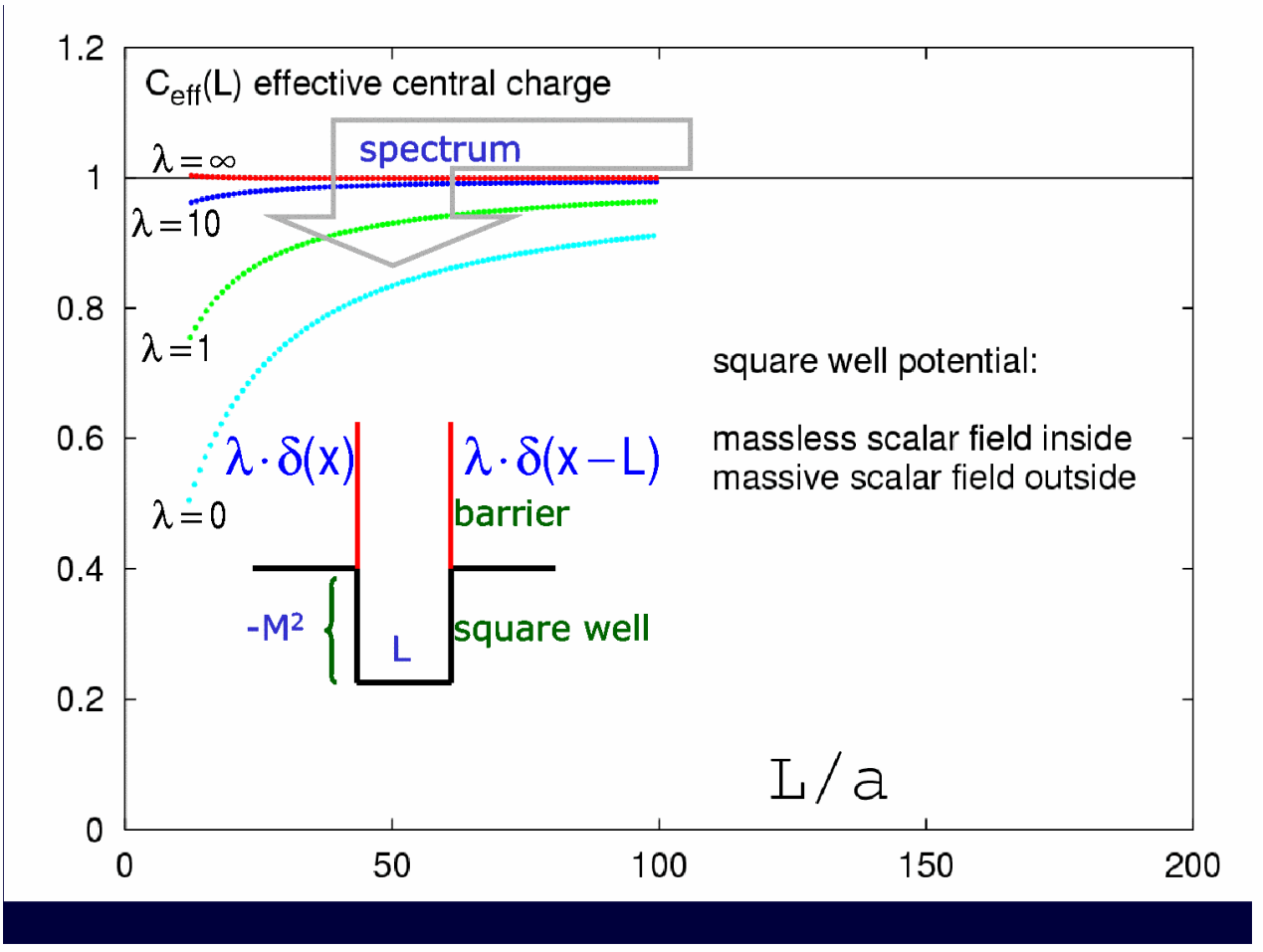
$$E_{reg}(L,a) = \frac{2L}{\pi a^2} - \frac{1}{2a} \boxed{- \frac{\pi}{24L}} + O(a^2)$$

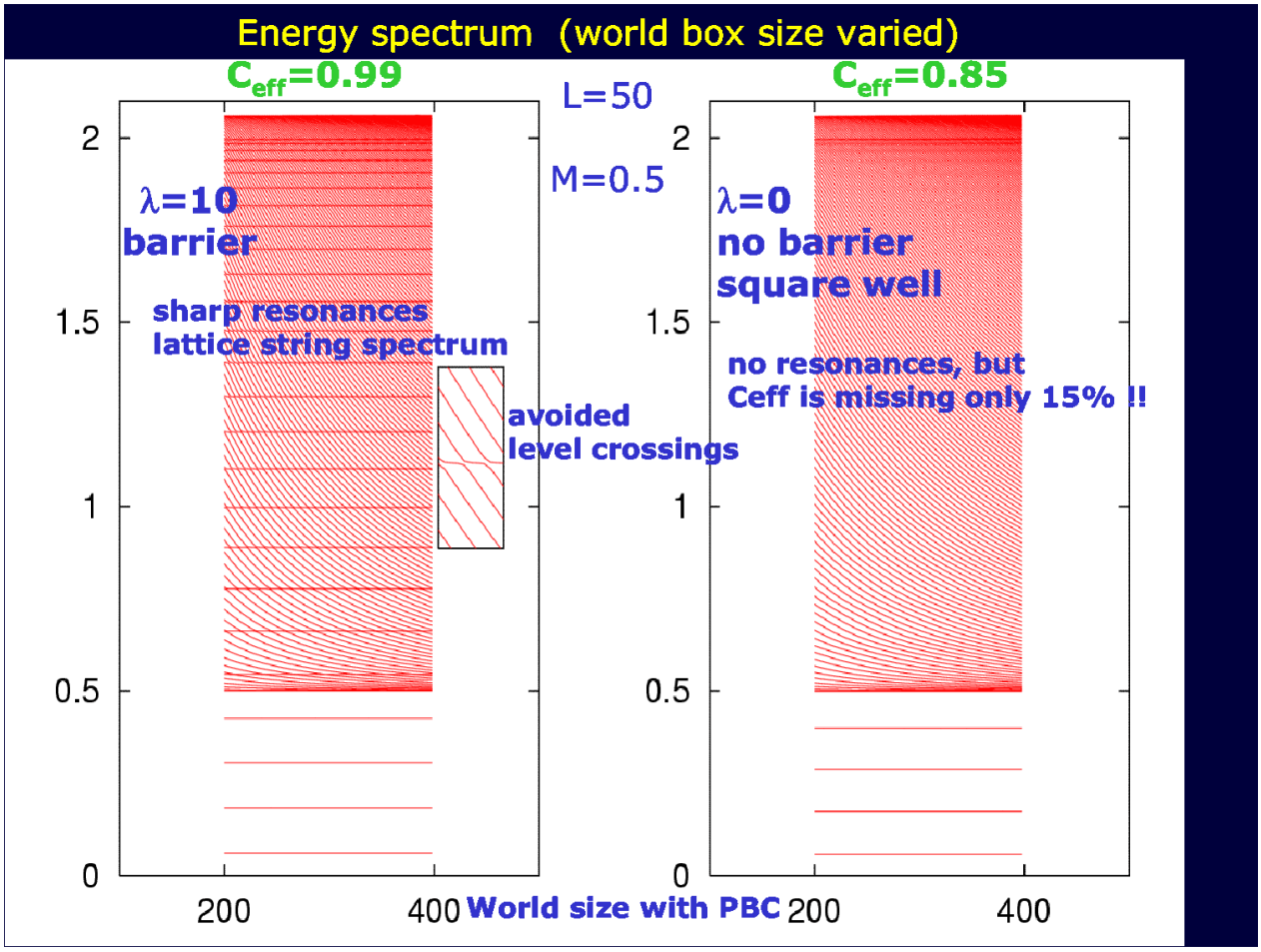
$$\frac{1}{2}\sum_{n=1}^{3} n = -\frac{\pi}{24}$$

This IS a paradox

How to eliminate problematic end effects?

→ **string-solitons with unit winding**





4. Closed String (torelon) with unit winding

- D=4 SU(3) QCD spectrum new results
- Closed string Casimir energy new results
- D=3 Z(2) spectrum new results

4. **Closed String (torelon) with unit winding**

- D=4 SU(3) QCD spectrum new results
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5. **Conclusions**

Relativistic excitation energies of D=3 string soliton

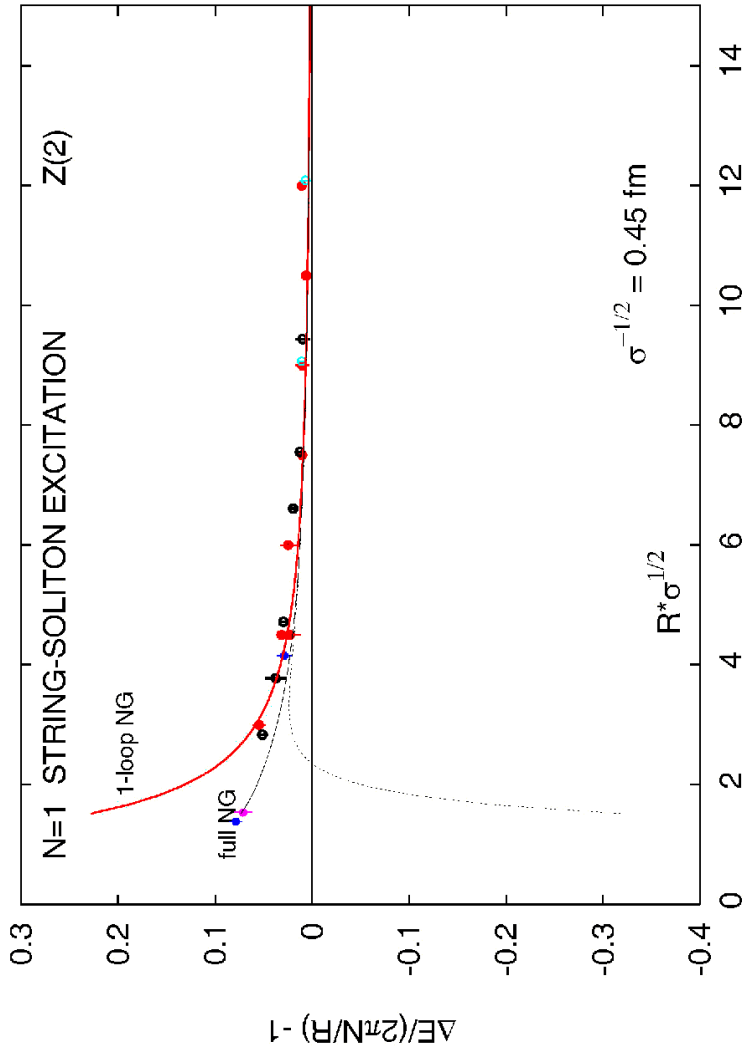
$$E^2_{N+\bar{N}} = R^2\sigma^2\{1-\frac{\pi}{3\sigma R^2}+\frac{4\pi}{\sigma R^2}(N+\bar{N})+\frac{4\pi^2 n^2}{(\sigma R^2)^2}\}$$

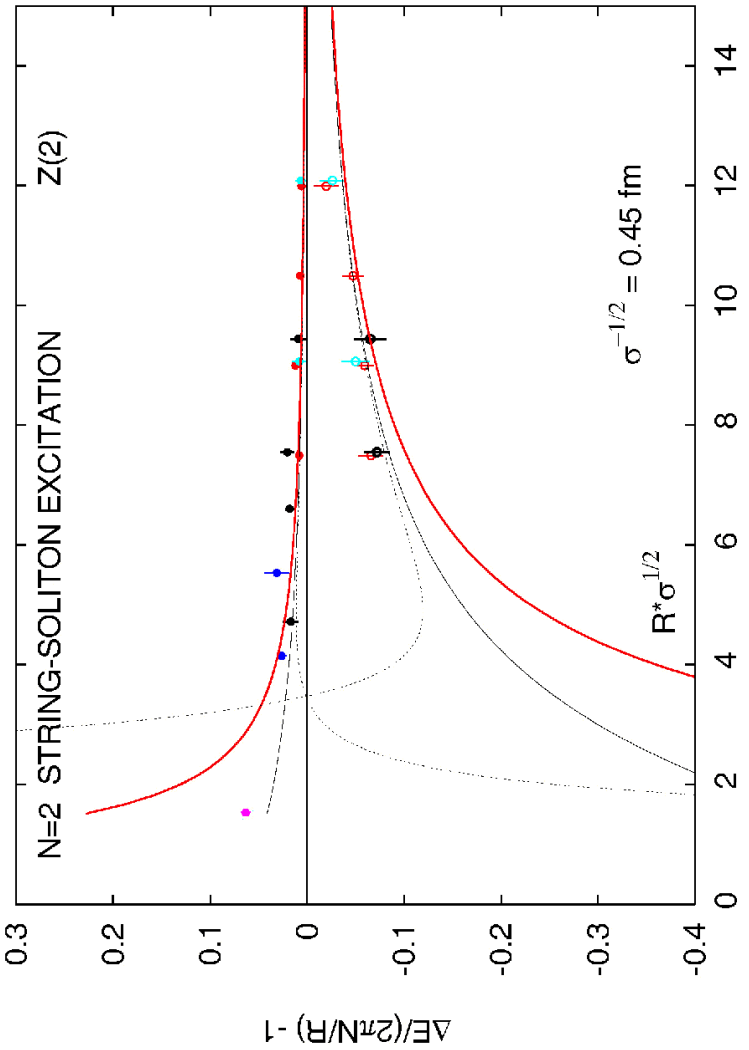
$$n = N-\bar{N}$$

$$p = \frac{2\pi}{R}n$$

Exact in NG

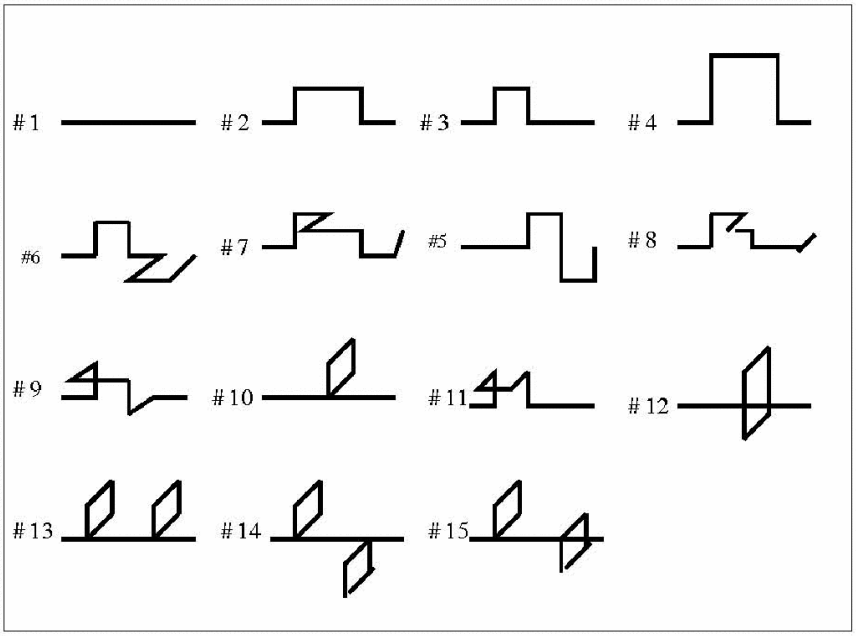
$O(R^{-4})$ corrections in Polchinski-Strominger


N=1 STRING-SOLITON EXCITATION
Z(2)
1-loop NG
full NG
$\sigma^{-1/2} = 0.45$ fm
$R*\sigma^{1/2}$
$\Delta E/(2\pi N/R) - 1$
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
2
4
6
8
10
12
14


N=2 STRING-SOLITON EXCITATION
Z(2)
$\sigma^{-1/2} = 0.45$ fm
$R*\sigma^{1/2}$
$\Delta E/(2\pi N/R) - 1$
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
2
4
6
8
10
12
14

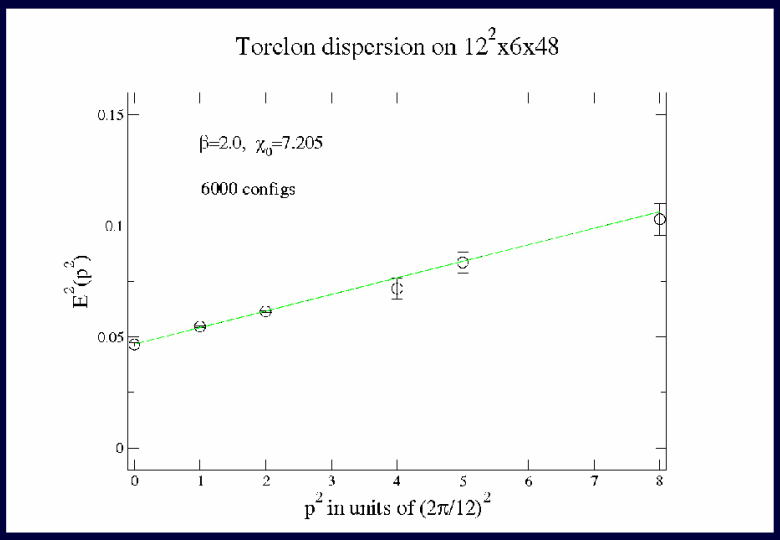
Francesca Maresca, PhD thesis, Dublin, 2004

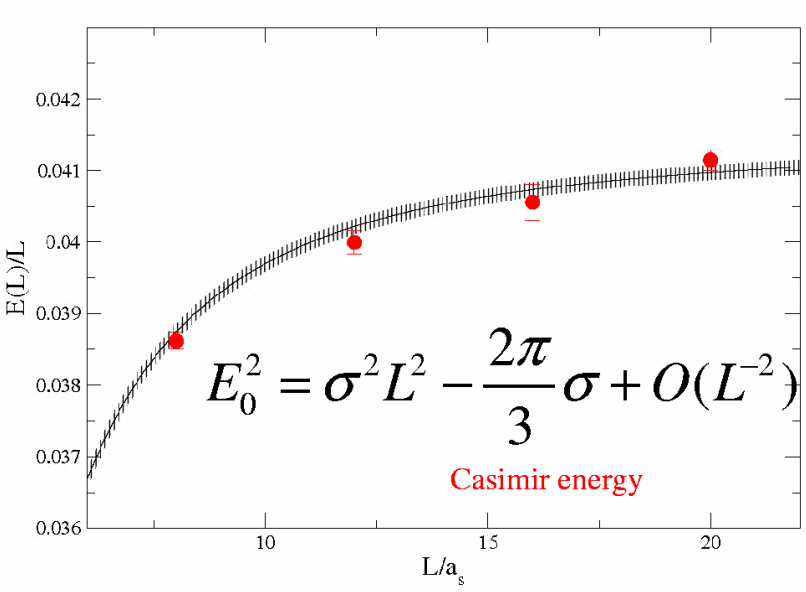
15 basic torelon operators translated and fuzzed in large correlation matrices



Point group notation for string states

Level	Type	State	QCD $R(p_z)$
E=0	t_0	$\|0\rangle$	$A_{1g}(0)$
E=1	$1 \times t_1$	$(a_1^{d\dagger} + a_1^{s\dagger}, a_1^{d\dagger} - a_1^{s\dagger})\|0\rangle$	$E(1)$
E=2	$2 \times t_1$	$(a_1^{d\dagger}\tilde{a}_1^{d\dagger} + a_1^{s\dagger}\tilde{a}_1^{s\dagger})\|0\rangle$	$B_{1g}(0)$
		$(a_1^{d\dagger}\tilde{a}_1^{d\dagger} - a_1^{s\dagger}\tilde{a}_1^{s\dagger})\|0\rangle$	$B_{2g}(0)$
		$(a_1^{s\dagger}\tilde{a}_1^{d\dagger} - a_1^{d\dagger}\tilde{a}_1^{s\dagger})\|0\rangle$	$A_{2u}(0)$
		$(a_1^{s\dagger}\tilde{a}_1^{d\dagger} + a_1^{d\dagger}\tilde{a}_1^{s\dagger})\|0\rangle$	$A_{1g}^*(0)$
		$a_1^{s\dagger}a_1^{d\dagger}\|0\rangle$	$A_1(2)$
	$1 \times t_1^2$	$((a_1^{d\dagger})^2 + (a_1^{s\dagger})^2)\|0\rangle$	$B_1(2)$
		$((a_1^{d\dagger})^2 - (a_1^{s\dagger})^2)\|0\rangle$	$B_2(2)$
	$1 \times t_2$	$(a_2^{d\dagger} + a_2^{s\dagger}, a_2^{d\dagger} - a_2^{s\dagger})\|0\rangle$	$E(2)$
E=3^1	$1 \times t_1 + 1 \times t_2$	$(\tilde{a}_1^{d\dagger}a_2^{d\dagger} + \tilde{a}_1^{s\dagger}a_2^{s\dagger})\|0\rangle$	$B_1(1)$
		$(\tilde{a}_1^{d\dagger}a_2^{d\dagger} - \tilde{a}_1^{s\dagger}a_2^{s\dagger})\|0\rangle$	$B_1(1)$
		$(\tilde{a}_1^{s\dagger}a_2^{d\dagger} + \tilde{a}_1^{d\dagger}a_2^{s\dagger})\|0\rangle$	$A_1(1)$
		$(\tilde{a}_1^{s\dagger}a_2^{d\dagger} - \tilde{a}_1^{d\dagger}a_2^{s\dagger})\|0\rangle$	$A_2(1)$



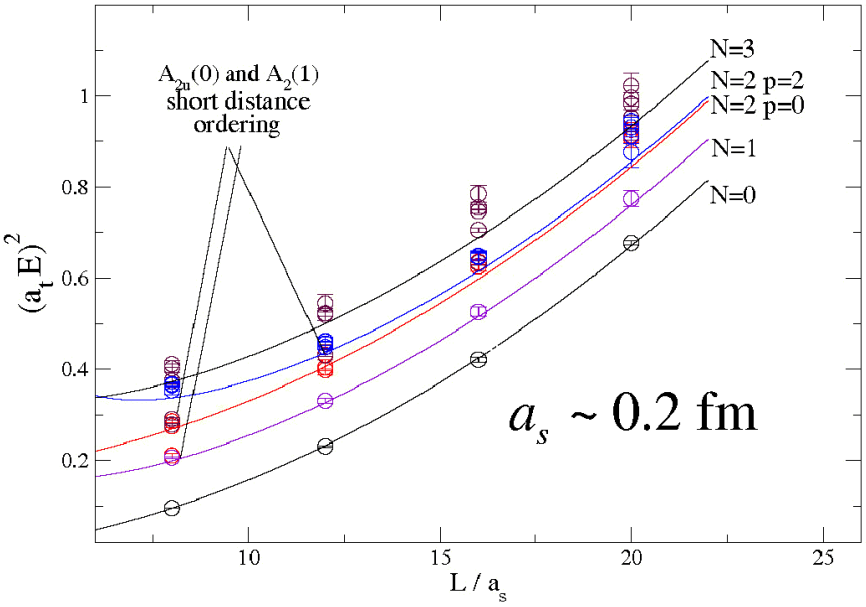


Exact in NG string no $O(L^{-2})$ correction

Polchinski-Strominger expect corrections
even without rigidity term

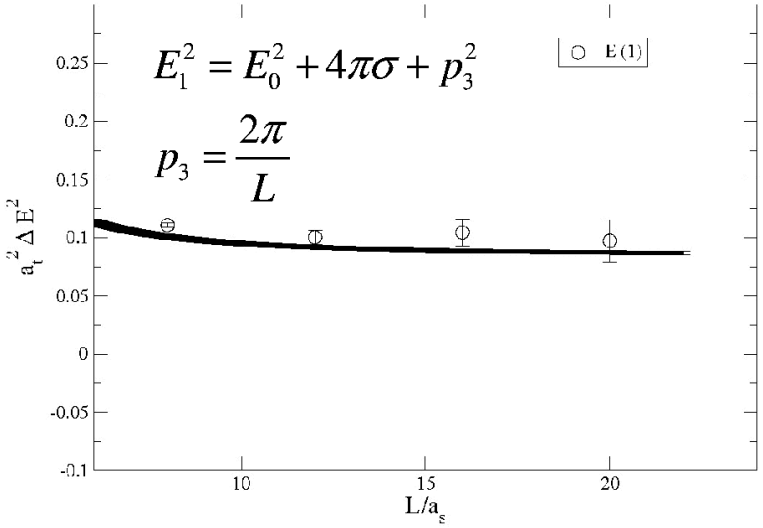
Crossover from short distance behavior to string level ordering

Torelon excitations



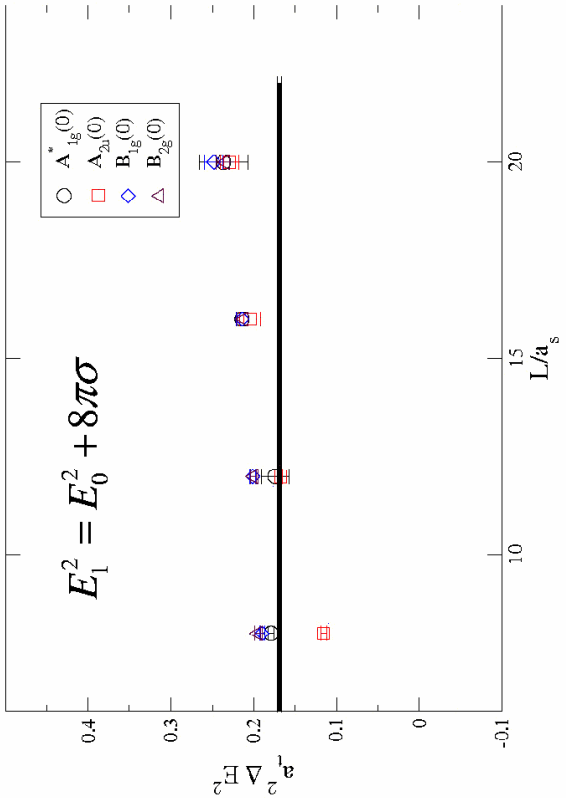
Expected string behavior

N=1 string level



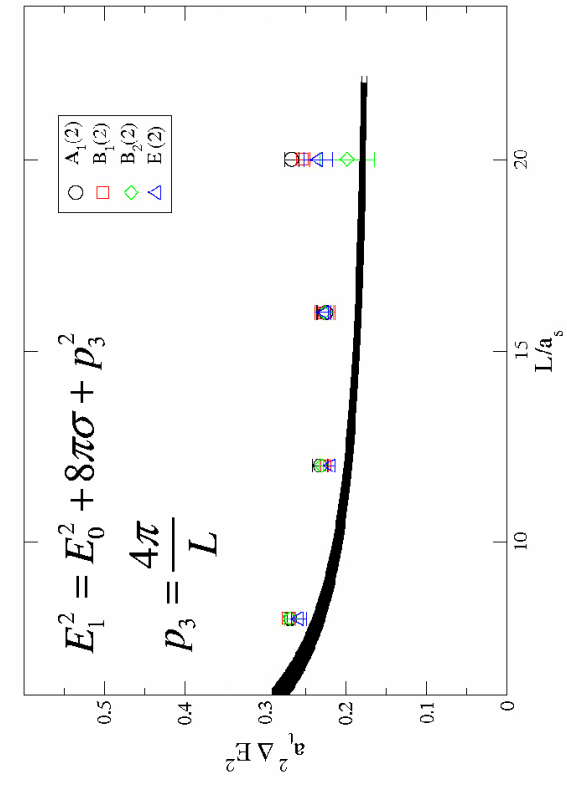
Large L?

N=2 p=0 string level

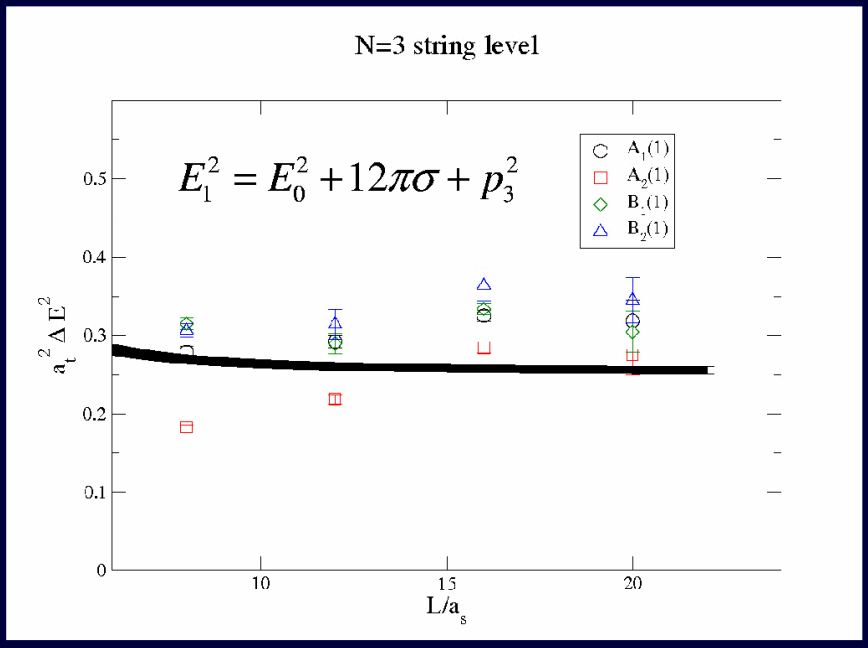


Large L?

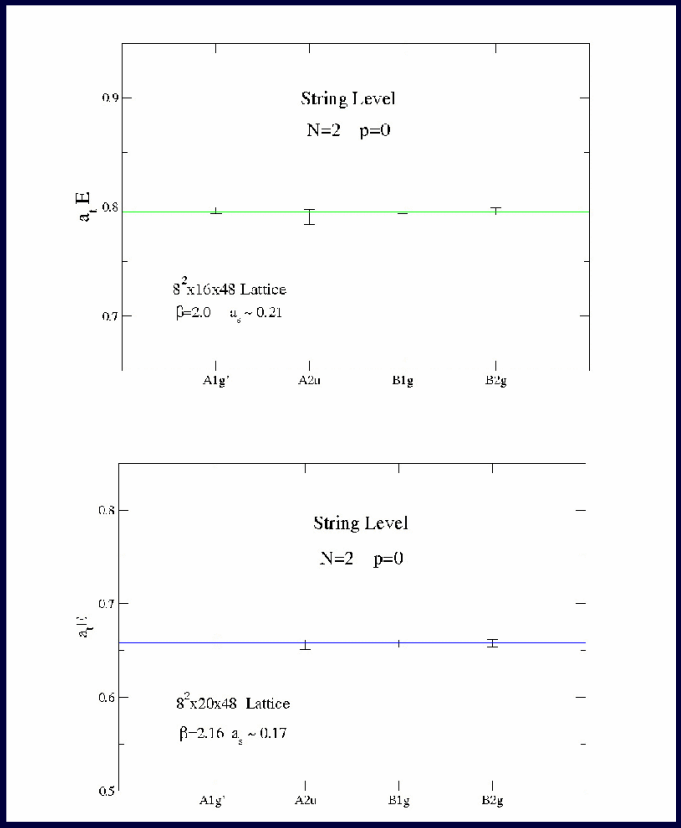
N=2 p=2 string level

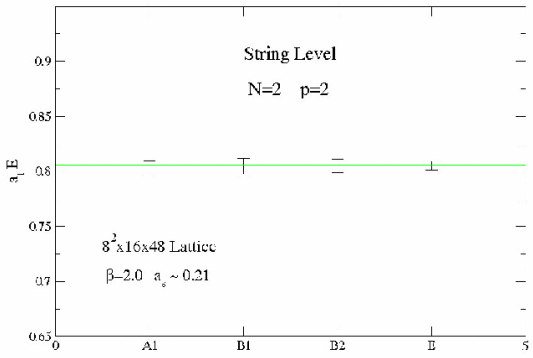


Fine structure not Nambu-Goto Large L?

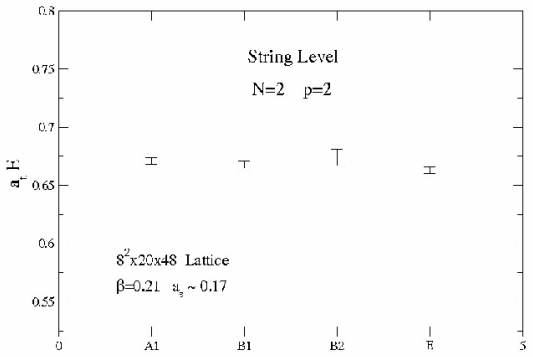


Perfect string degeneracies





Perfect string degeneracies



QCD String check list

- Massless Goldstone modes ✓
- Local derivative expansion for their interactions ?
  from fine structure in the spectrum ✓
- Massive excitations
- Breathing modes in effective Lagrangian
- String properties ? Bosonic, NG, rigid, …

} ?

WITHIN REACH of

LATTICE GAUGE THEORY

Conclusions:

1. Fine structure in QCD string spectrum
   Progress on string-soliton spectrum

neither was seen before

2. Casimir energy paradox: low energy Goldstone modes → geometric string theory?

3. Is bulk behavior and related resonance spectrum the clue? Origin of central charge?

4. Effective low-energy string theory?
   Universality class of QCD string ?