From Υ Spectroscopy to B Meson Decay Constants

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Contents

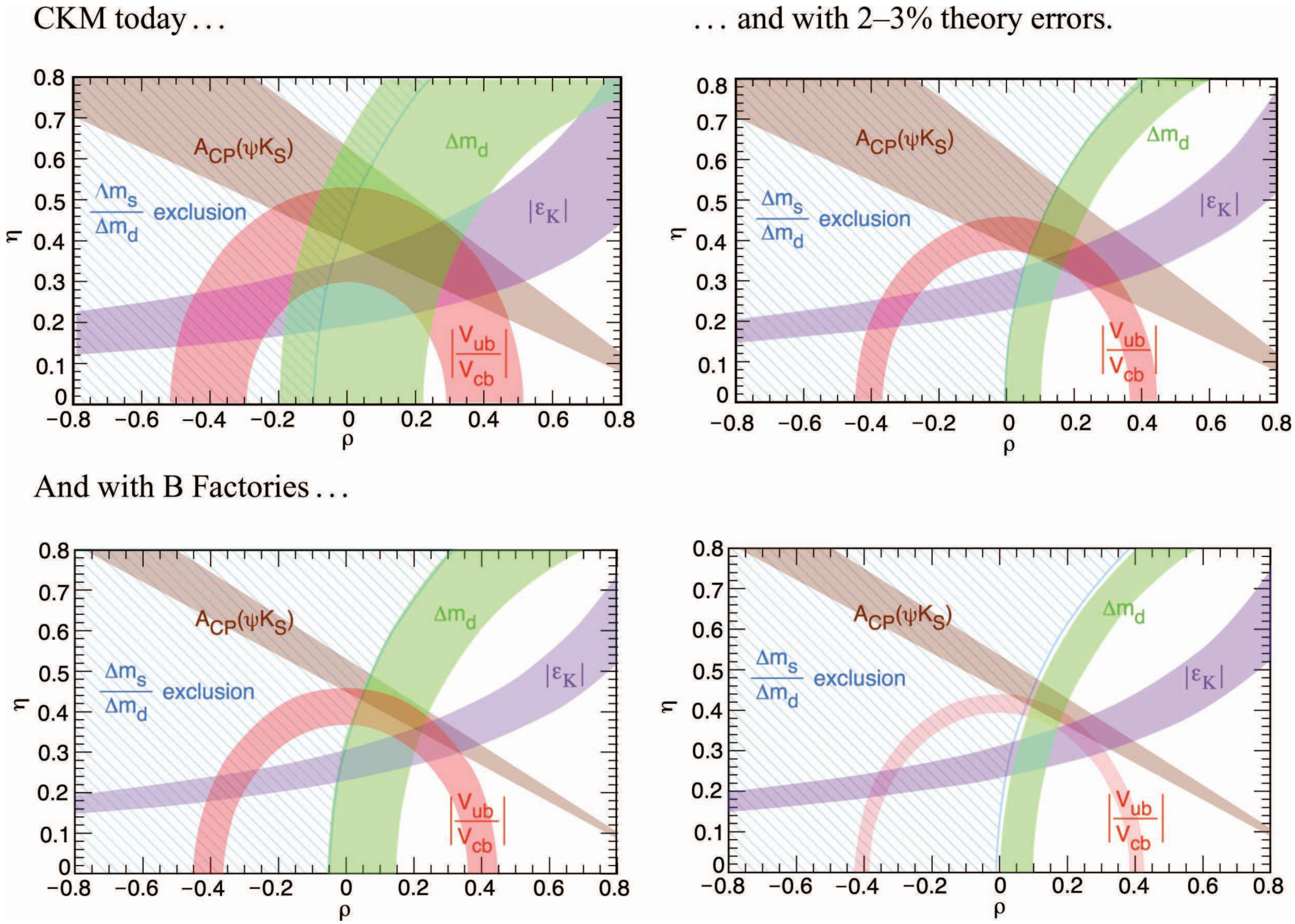
- **Υ Spectrum**

  Calculate spectrum to fix m_b and a^{-1} and compare with experiment to give confidence in b quark methods.

- **B decays**

  Need precision calculation of f_B and B_B in order to pin down CKM parameters. In particular need combination $\frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}}$. Aim is to reduce theory errors to a few percent otherwise will dominate uncertainties from experiment.

Motivation



C. Bernard, SCIDAC Meeting March 2004;

CLEO-c

MILC Ensembles

- MILC collaboration have used improved staggered formulation to generate the first ensembles of configurations which include $2+1$ flavors of dynamical quarks.
- 2 = u, d degenerate with masses down to $m_s/8$.
  1 = s (can ignore heavy c, b, t dynamical qs.)
  2 values of lattice spacing, $a \approx$ 0.12fm and 0.08fm.
- Fix 5 free parameters of QCD (bare $m_u = m_d$, m_s, m_c, m_b, and $a \equiv \alpha_s$) using
  $m_\pi, m_K, m_{D_s}, m_\Upsilon$ and $\Delta E_\Upsilon(2S-1S)$.
- These are 'gold-plated' quantities (e.g. stable hadron masses). Compute other 'gold-plated' quantities as a test of (lattice) QCD.

Heavy Quarks on the Lattice

- b quarks typically have low velocity v within the meson.
- Can use effective theory Lattice NRQCD - expansion of Lagrangian in powers of v^2 - to get high statistics (calculation of the quark Green's functions can be done using a simplistic evolution equation instead of having to solve a boundary value Dirac problem).

Υ System

- Upsilon (Υ) meson is bound state of b and $\bar{b}$ Radial quantum number $n = 1, 2, 3...$ Orbital angular momentum quantum number $L = S, P, D, F$ corresponding to $0, 1, 2, 3$ respectively.
  e.g. $\Upsilon(2S)$ has $n = 2$, $L = 0$.
- The stable states are very precisely known experimentally.
- Calcs have the advantage that valence quarks are not the same as the light dynamical quarks. Results indicate how the dynamical quarks feed in to meson properties.
- Can systematically improve Lagrangian order by order in v^2 and match to continuum - these results use $\mathcal{O}(v^4)$ Lagrangian with tree level tadpole improved matching.

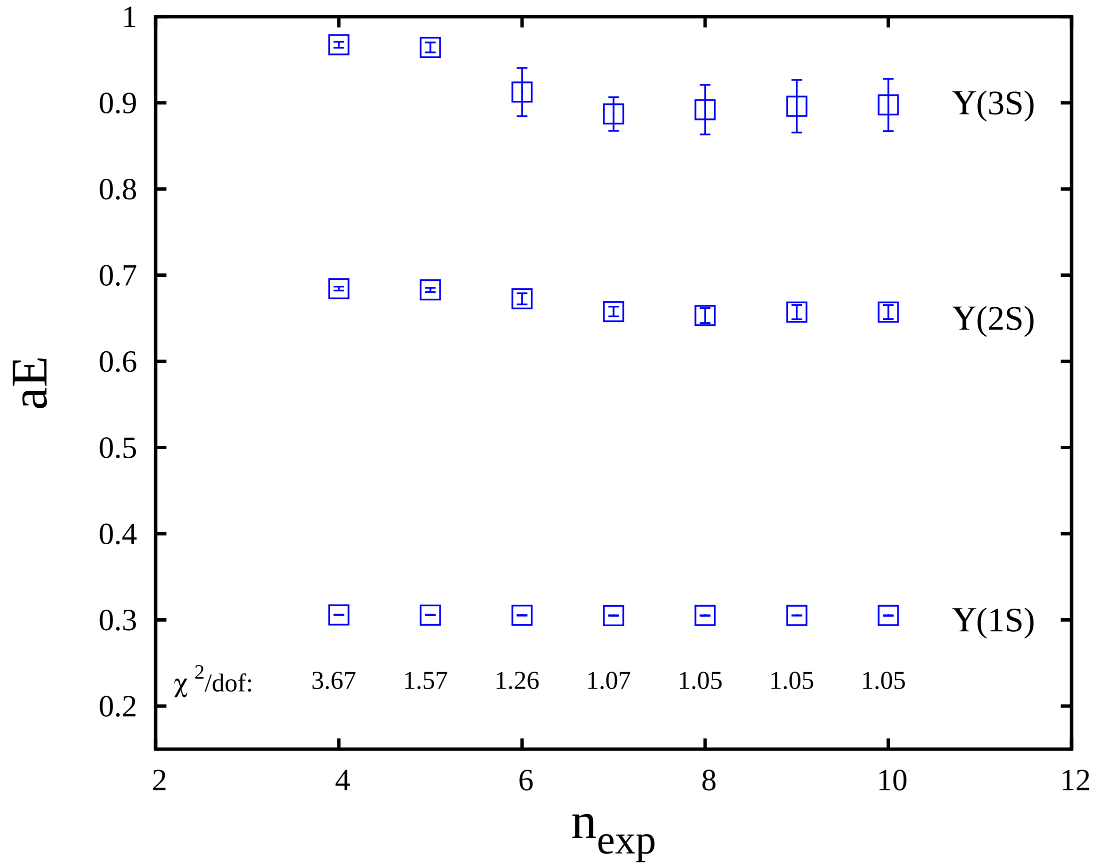
Lattice	n_f	β	am_l, am_s	u_{0L}	aM_b^0	n_{conf}	n_{orig}
$20^3 \times 64$ coarse	0	8.0	-	0.856	2.8	210	16
	2	7.2	0.02,-	0.845	2.8	210	16
	2+1	6.76	0.01,0.05	0.836	2.8	210	16
	2+1	6.79	0.02,0.05	0.837	2.8	210	16
	2+1	6.81	0.03,0.05	0.8378	2.8	210	16
	3	6.85	0.05,0.05	0.8391	2.8	210	8
$28^3 \times 96$ fine	0	8.4	-	0.8652	1.95	210	16
	2+1	7.09	0.0062, 0.031	0.8461	1.95	159	16,12*
	2+1	7.11	0.0124, 0.031	0.855	1.95	210	16

Table 1: Parameters and details of MILC configurations used for Υ correlator calculations. *For 92 of the 159 configurations, data from 4 of the 16 origins was rejected due to corruption on the last timeslice of the configuration.

Fitting

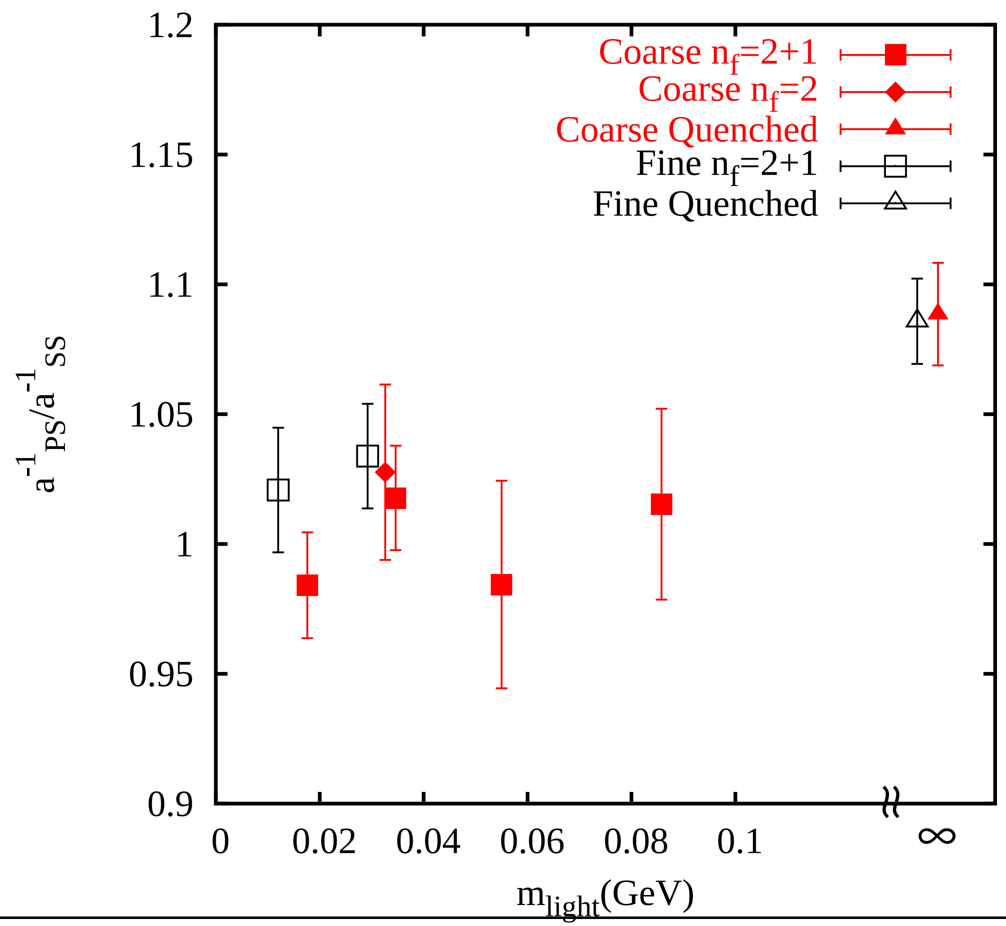
- Υ correlator fit function $G(t) = \sum_{j=0}^{n_{exp}-1} C_j e^{-E_j t}$

  Use *Bayesian Fitting* (augment χ^2 with a Bayesian term) to allow whole range of data to be fitted to large n_{exp}



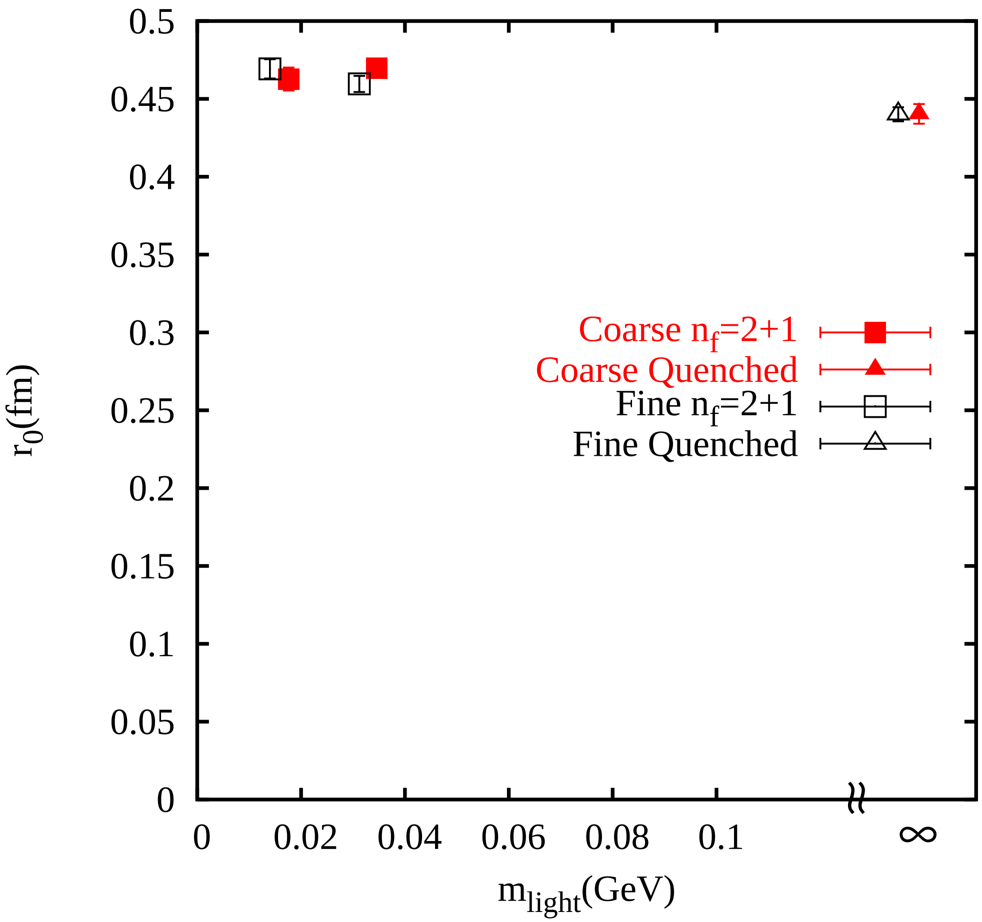
Lattice Spacing Results

- Compare a determinations from orbital $({}^1P_1 - {}^3S_1)$ (PS) and radial $(2^3S_1 - 1^3S_1)$ (SS) energy splittings - gives good insight about dynamical content of configurations being used.



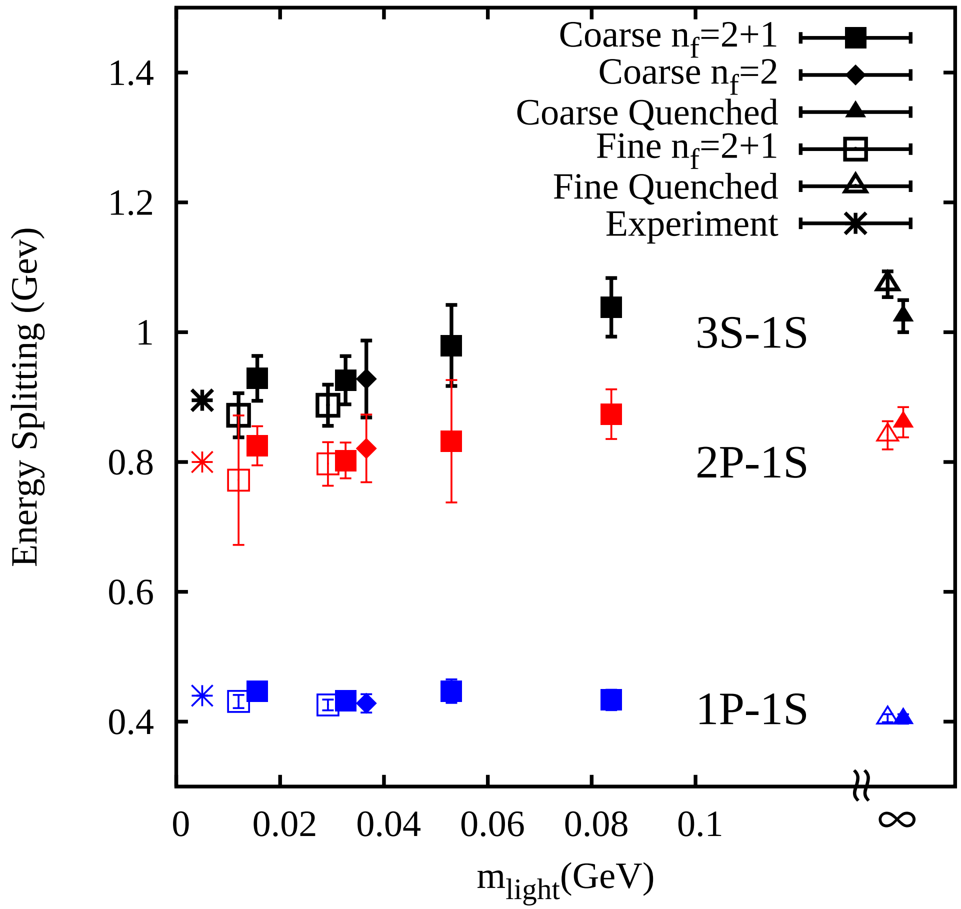
- a_{PS}^{-1} consistent with a_{SS}^{-1} on most chiral ensembles. This indicates realistic dynamical quark content.

r0 Determination



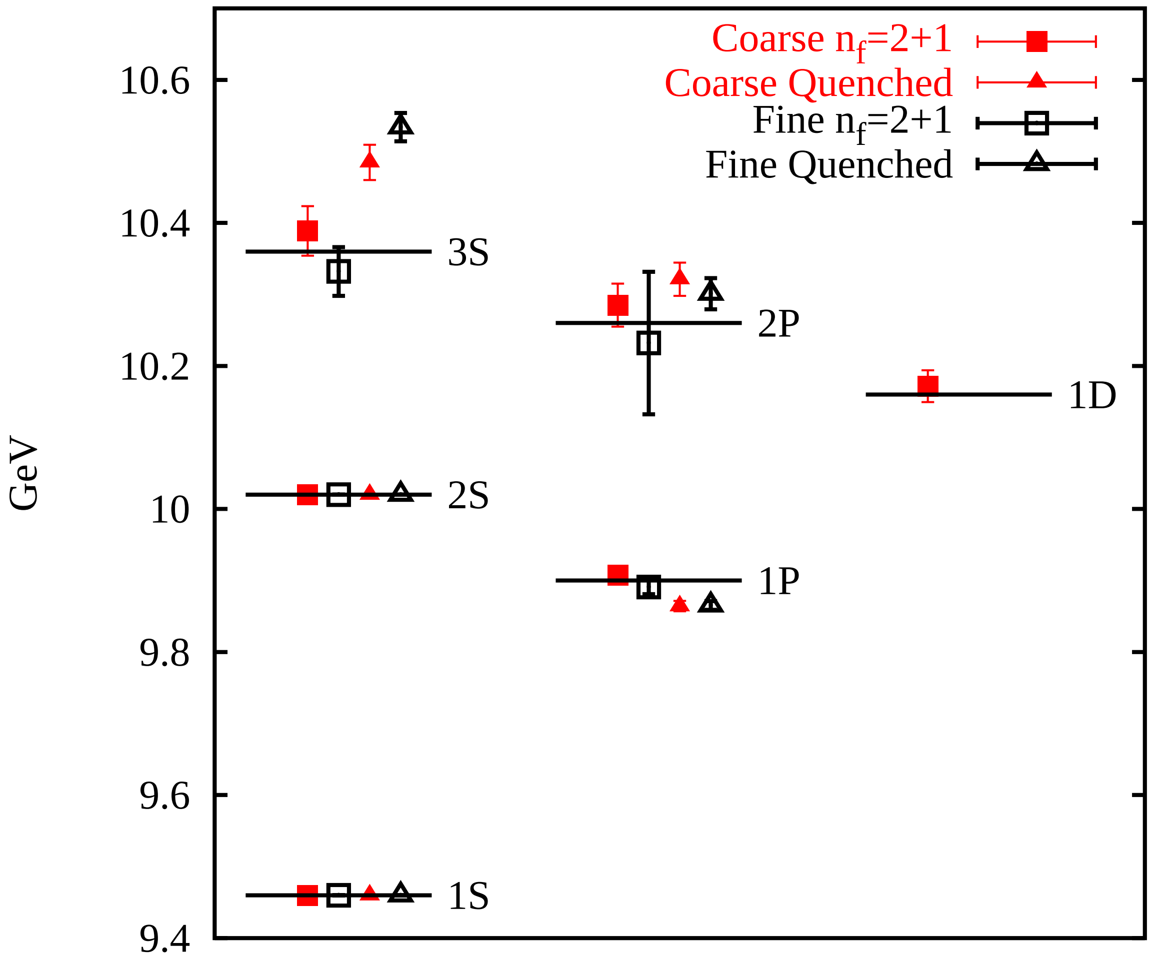
- MILC measure r_0/a, but physical val of r_0 not well known. Use our a^{-1}_{SS} to determine r_0.

Upsilon Spectrum



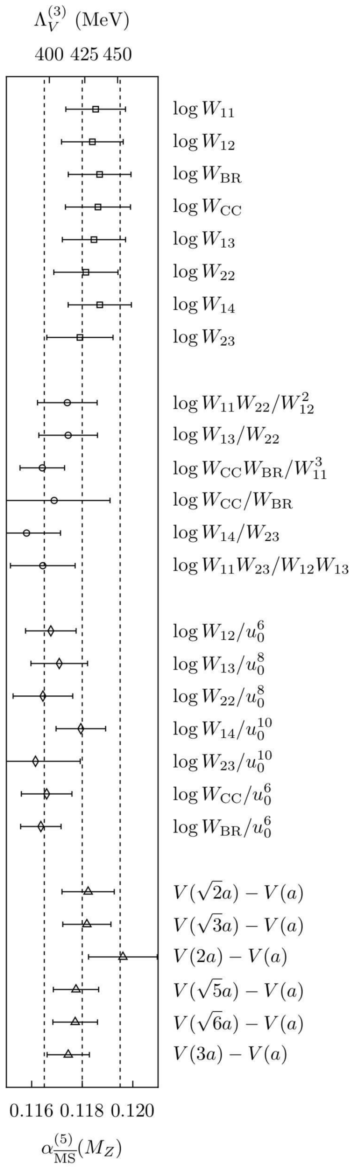
- $2S - 1S$ splitting used to fix a.
- Splittings become independent of dynamical u/d quark mass for the region we are working in.

Upsilon Spectrum

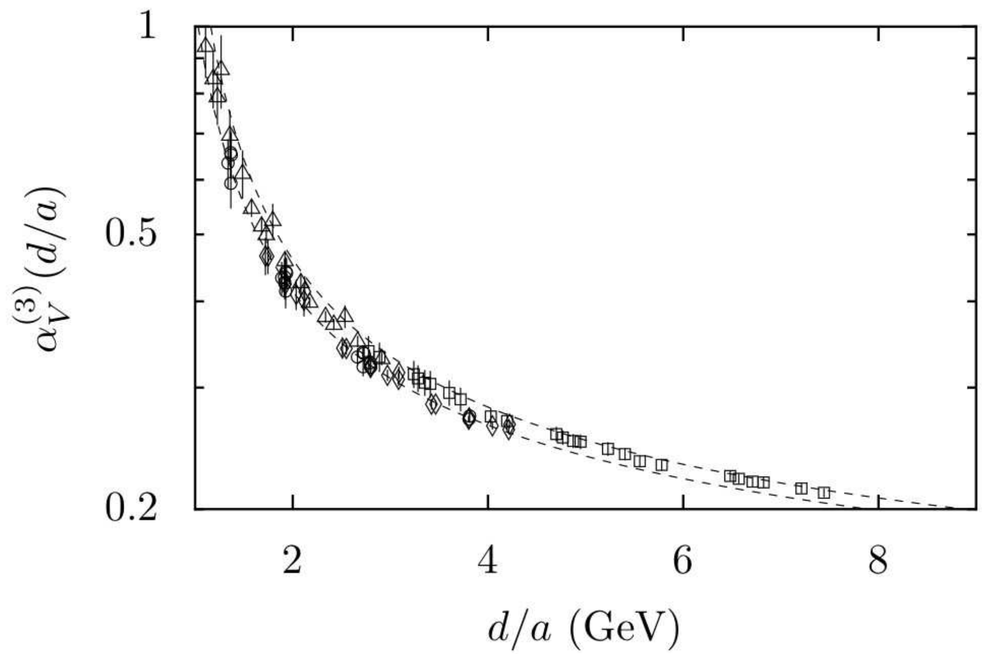


- Quenched values do not agree with experiment
- Complete agreement to experiment with unquenching
- $1D$ State only recently discovered experimentally (Cleo, Summer 2002).

α_s

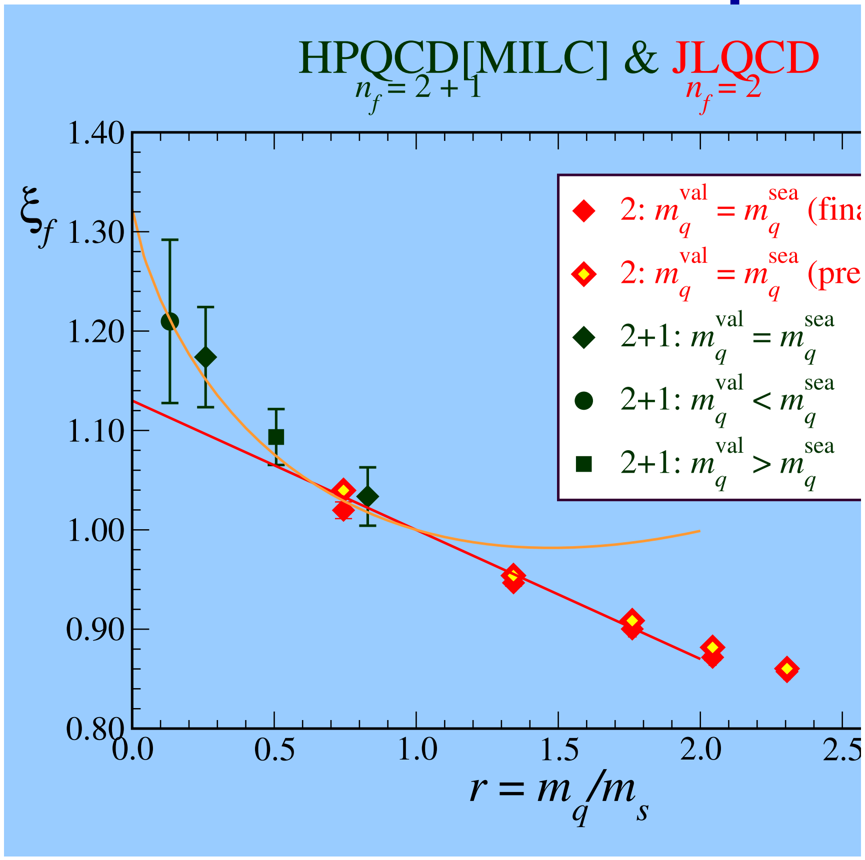


$\Lambda_V^{(3)}$ (MeV)
400 425 450
$\log W_{11}$
$\log W_{12}$
$\log W_{\rm BR}$
$\log W_{\rm CC}$
$\log W_{13}$
$\log W_{22}$
$\log W_{14}$
$\log W_{23}$
$\log W_{11}W_{22}/W_{12}^2$
$\log W_{13}/W_{22}$
$\log W_{\rm CC}W_{\rm BR}/W_{11}^3$
$\log W_{\rm CC}/W_{\rm BR}$
$\log W_{14}/W_{23}$
$\log W_{11}W_{23}/W_{12}W_{13}$
$\log W_{12}/u_0^6$
$\log W_{13}/u_0^8$
$\log W_{22}/u_0^8$
$\log W_{14}/u_0^{10}$
$\log W_{23}/u_0^{10}$
$\log W_{\rm CC}/u_0^6$
$\log W_{\rm BR}/u_0^6$
$V(\sqrt{2}a) - V(a)$
$V(\sqrt{3}a) - V(a)$
$V(2a) - V(a)$
$V(\sqrt{5}a) - V(a)$
$V(\sqrt{6}a) - V(a)$
$V(3a) - V(a)$
0.116 0.118 0.120
$\alpha_{\overline{\rm MS}}^{(5)}(M_Z)$



$\alpha_V^{(3)}(d/a)$
1
0.5
0.2
2
4
6
8
d/a (GeV)

B Leptonic Decays



M. Wingate, A. Kronfeld review Lattice 2003

- $$\xi_f = \frac{f_{B_s}\sqrt{m_{B_s}}}{f_B\sqrt{m_B}}$$
- Hint of chiral logs but stat. errors large. Aim is to reduce stat. errors.
- We have used smearing to succesfully do this.

Simulation details

- MILC $2+1$ flavour dynamical configs. For light quarks use asqtad action.
- For heavy b quarks use standard tadpole improved Lattice NRQCD action correct through $1/(am_b)^2$ at $am_b = 2.8$.
- a^{-1} and m_b fixed by Υ,
  $m_{u,d}$ and m_s fixed by π and K.
  Hence **no adjustable parameters**.

$am_f \equiv am_{u/d}$	a^{-1}(GeV)	am_q	n_{conf}	n_{src}
Coarse				
0.01	1.596	0.005	568	4
		0.01	568	2
		0.02	568	2
		0.04	568	1
0.02	1.605	0.02	486	2
		0.04	486	1
Fine				
0.0062	2.258	0.0062	465	4
		0.031	472	4

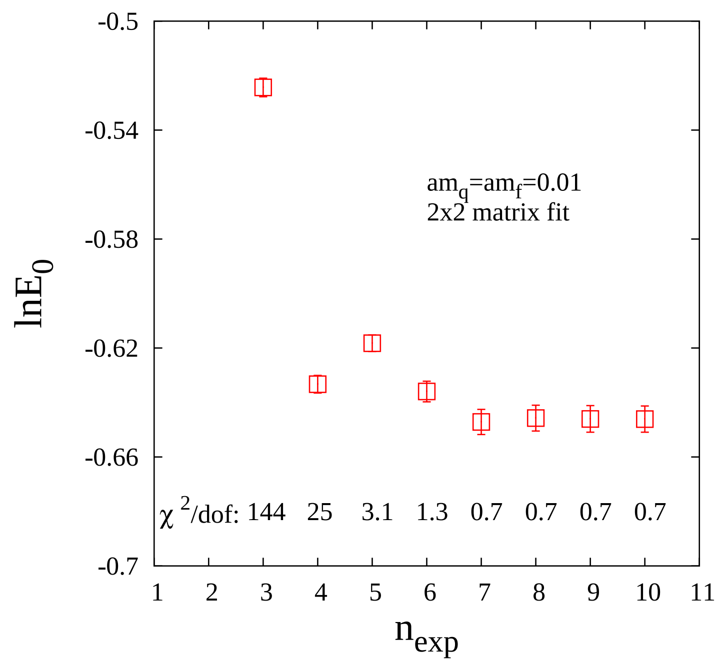
Note some points partially quenched.

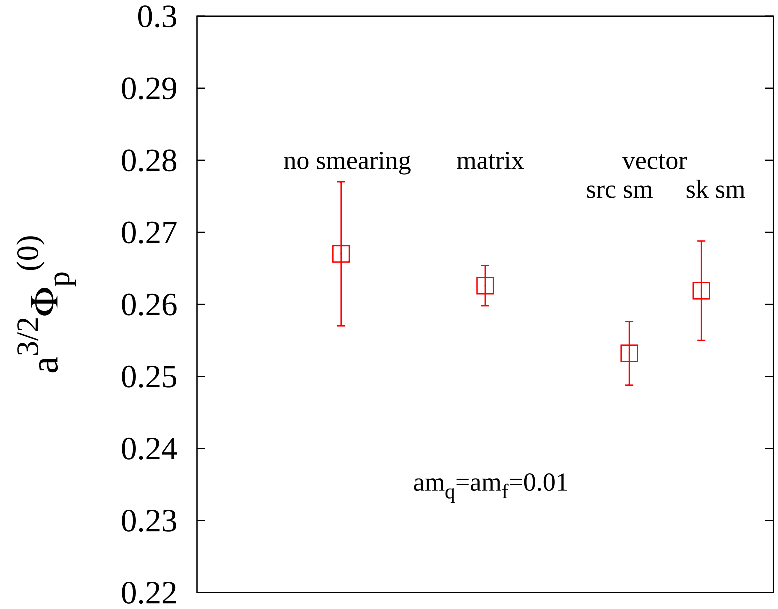
$am_q = 0.04$ corresponds to B_s.

Smearing & Fitting

- Smear heavy quark at source and sink. Use ground state hydrogenic style wavefunctions as have been used for Υ.
- Find optimal radius: that which minimises fit errors while maintaining reasonable $\chi^2/$dof
- Do Bayesian multi exponential fits. Compare single correlator fits to simultaneus *vector* 2×1 (source or sink smearing only) and *matrix* 2×2 (source and sink smearing) fits.
- Fit function is $G(t) = \sum_{j=0}^{n_{exp}-1} C^{(0j)} (-1)^{jt} e^{-m_j t}$
- Extract $\Phi^{(0)} = f_B^{(0)} \sqrt{m_B} = 2\sqrt{C^{(00)}}$. Similarly get next order in $1/m_b$ parts: $C^{(10)}, \Phi^{(1)}$ and combine through 1-loop

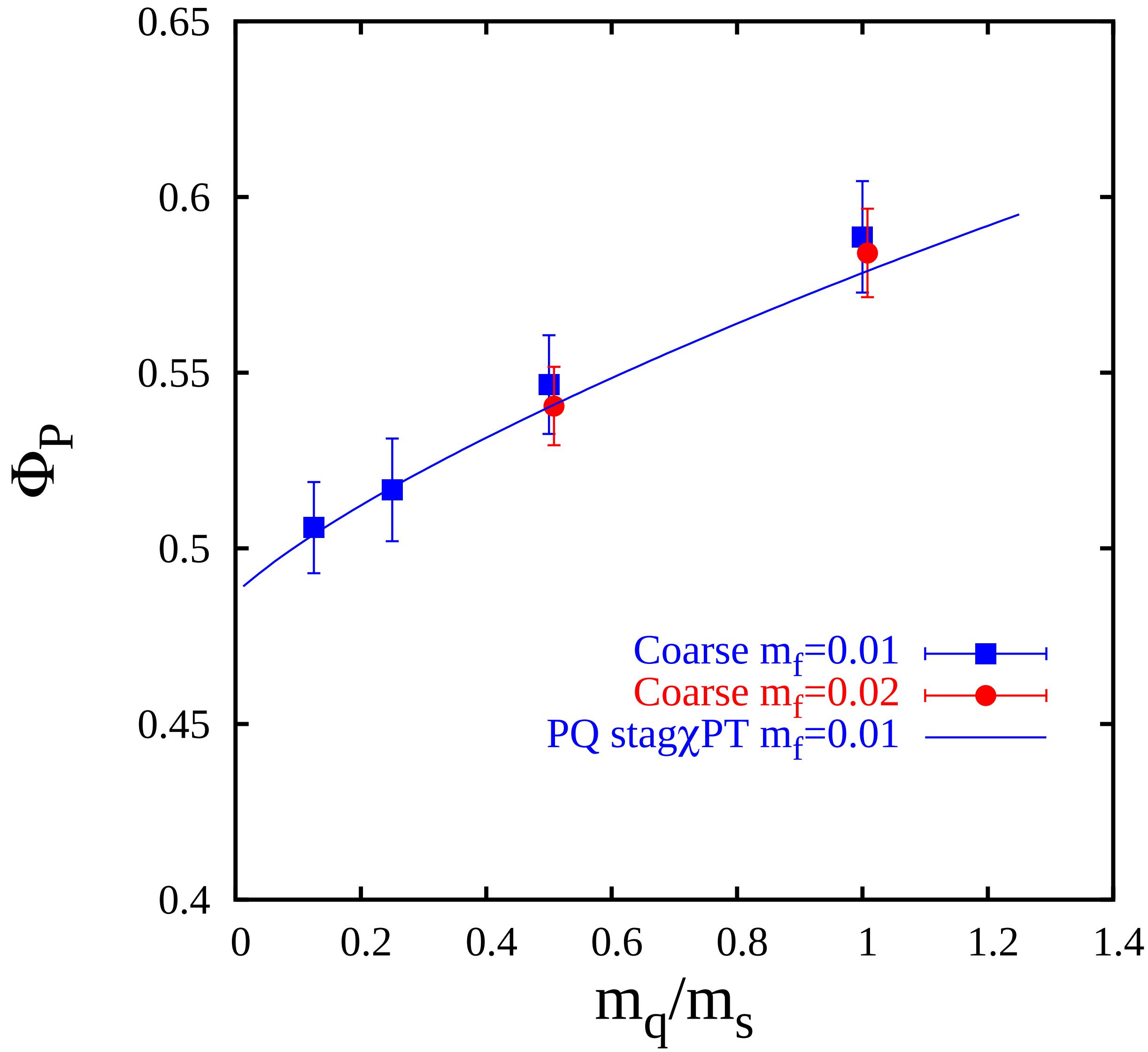
Smearing Results





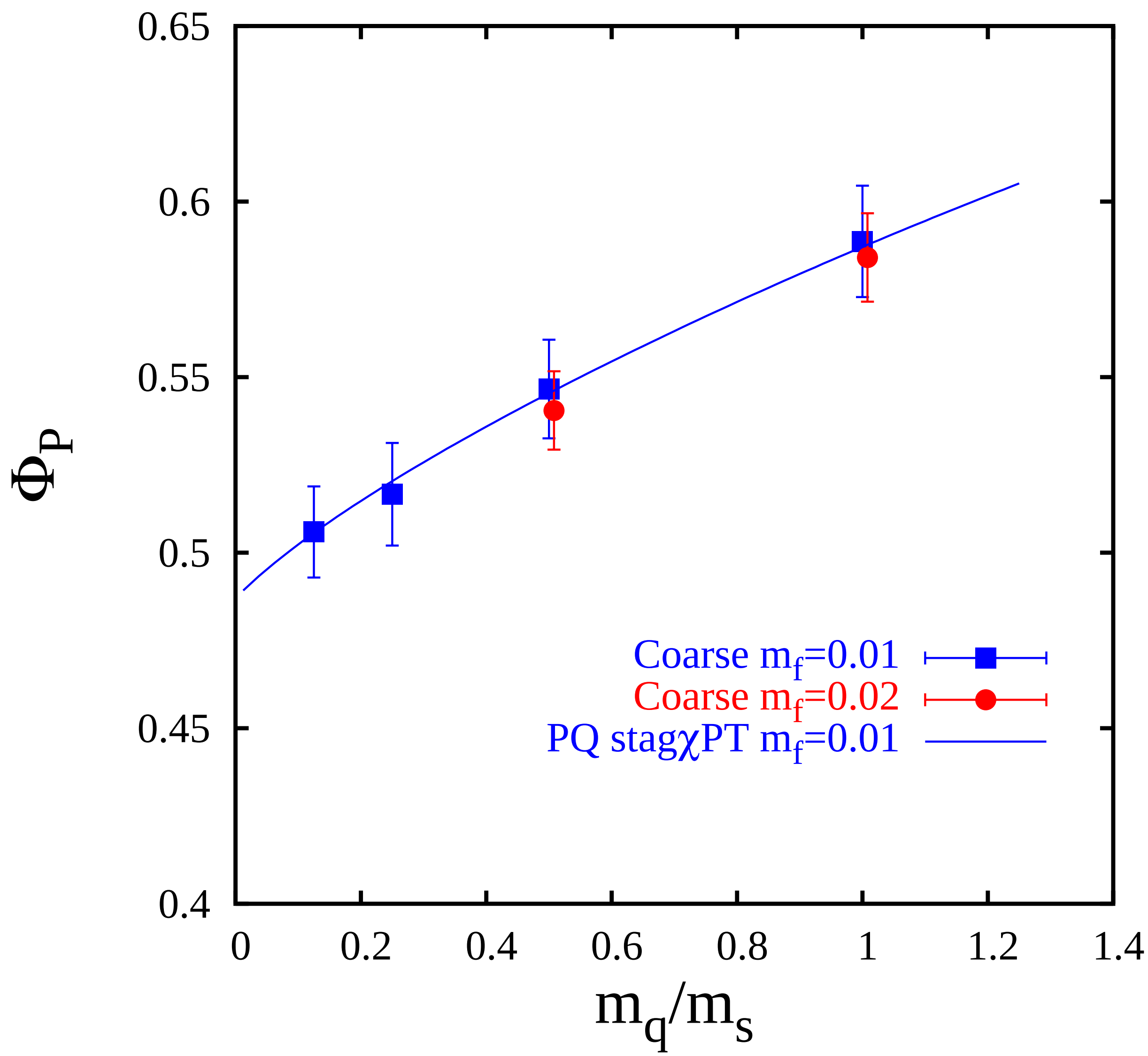
- Fit good for $n_{exp} \geq 7$
- Matrix fit substantially reduces stat. errors.

Φ Results



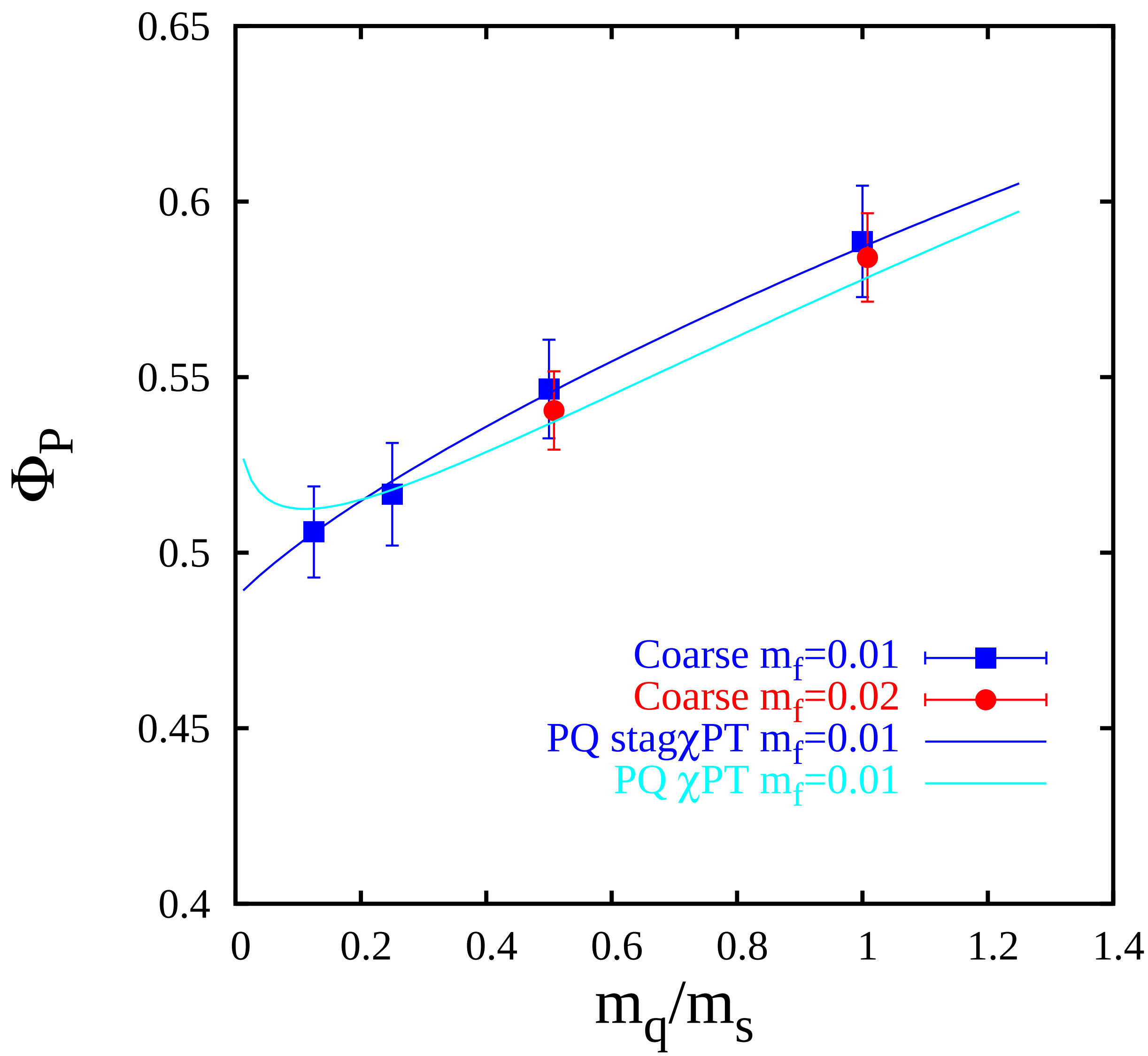
- χPT by C. Bernard
- Chiral Fit to all coarse points.

Φ Results



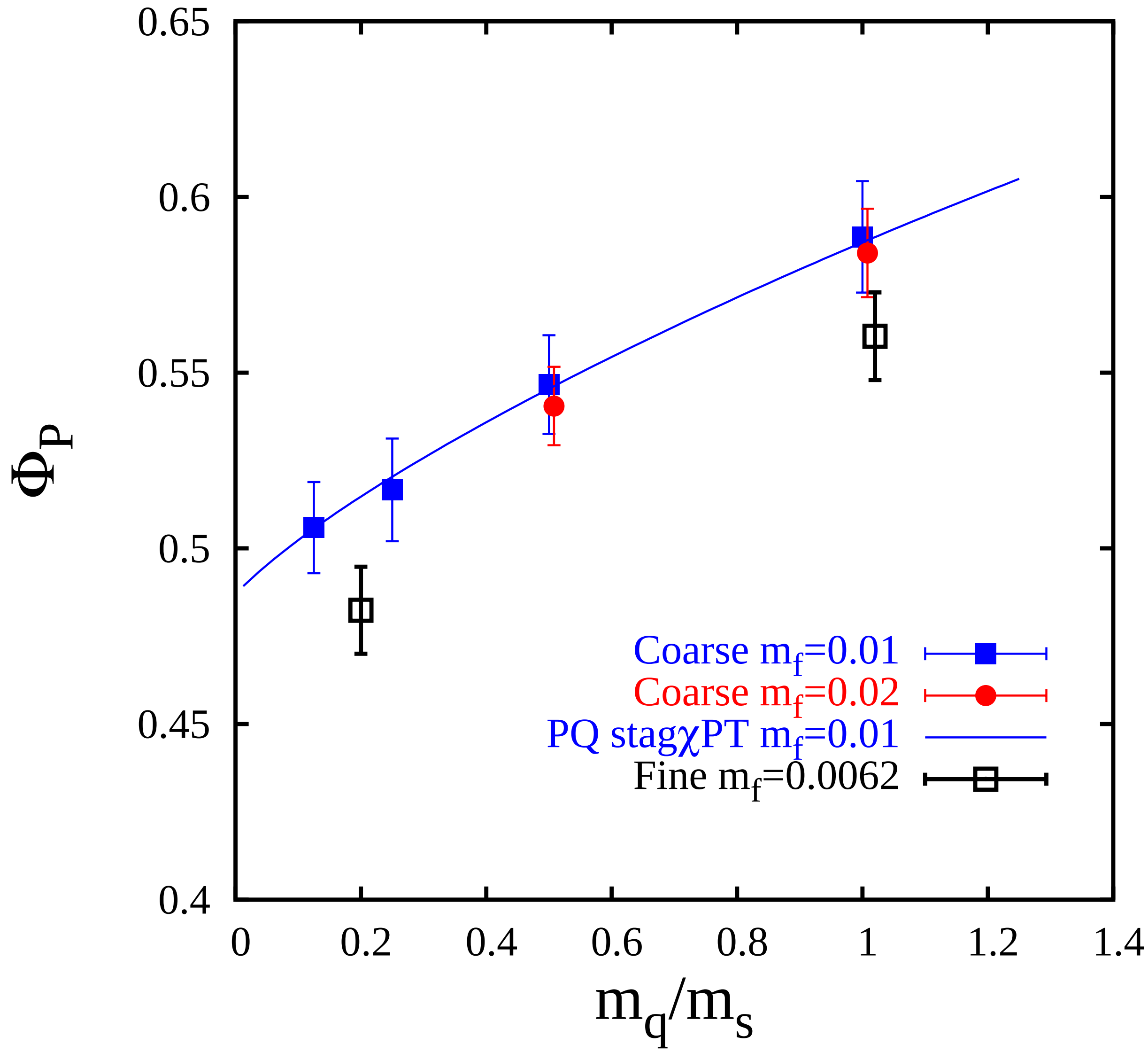
- Chiral Fit to 4 most chiral coarse points.

Φ Results



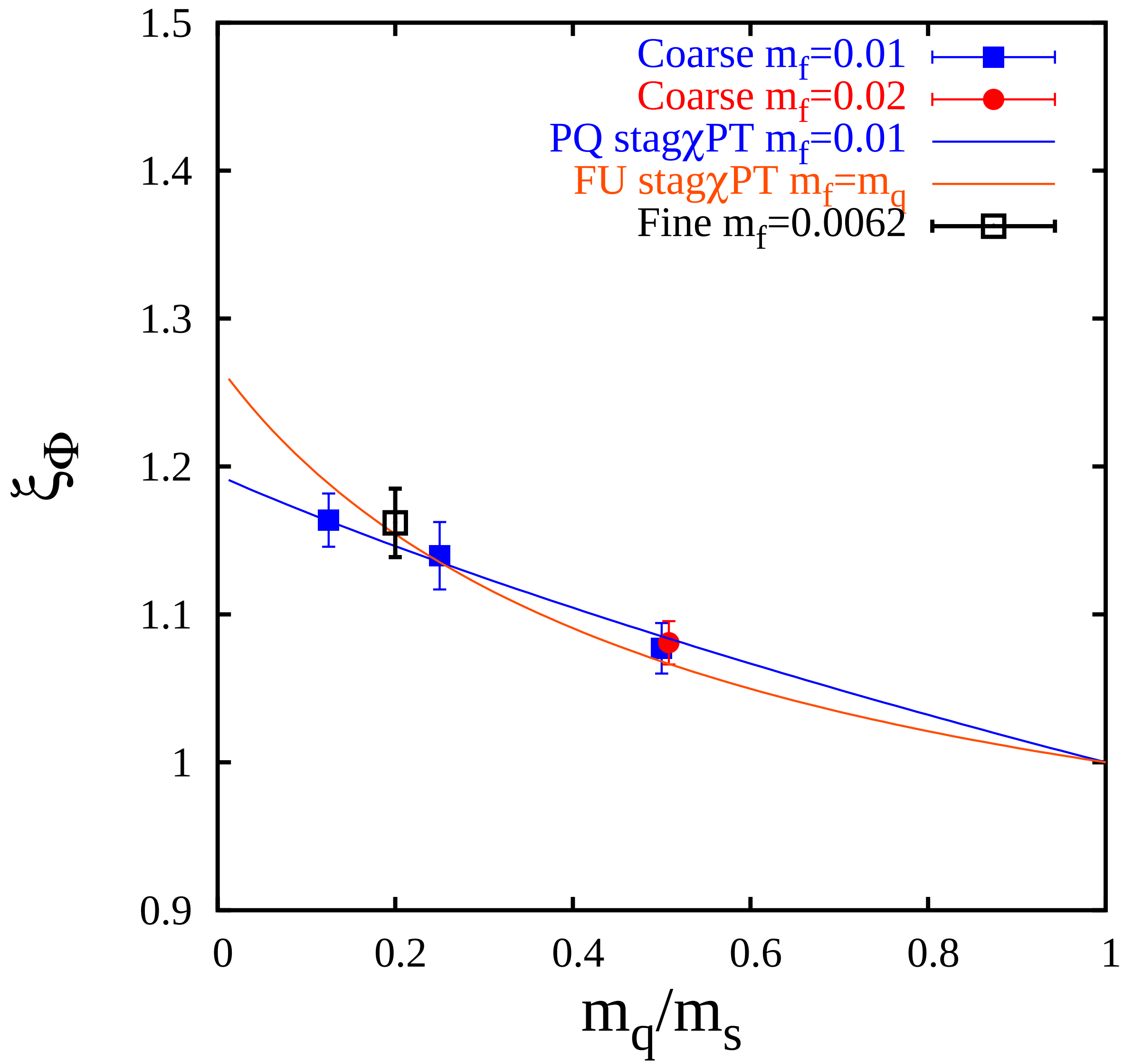
- Include fit with a^2 terms turned off and staggered energy splittings set to 0.

Φ Results



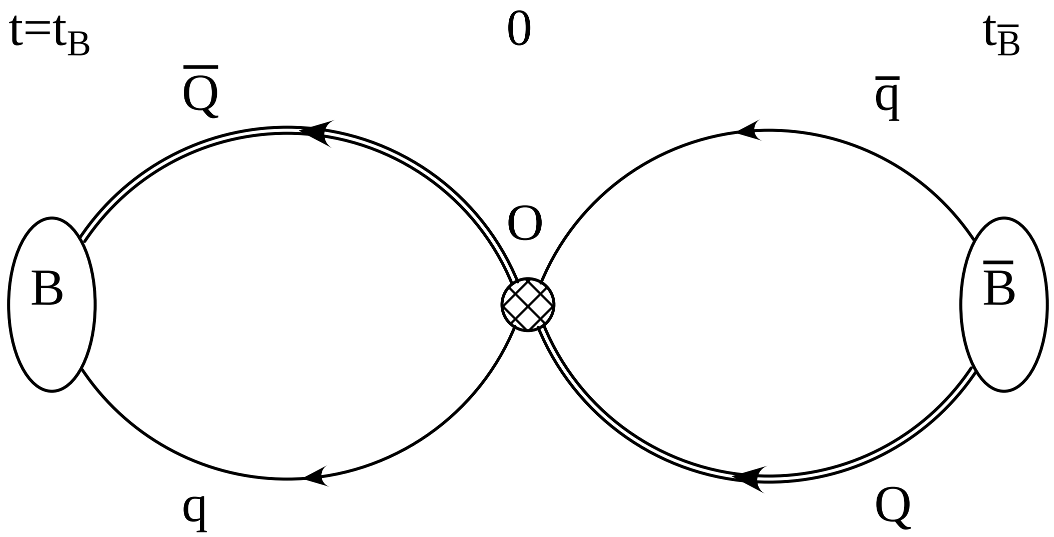
- Fine points NOT yet included in fit.

ξ Results



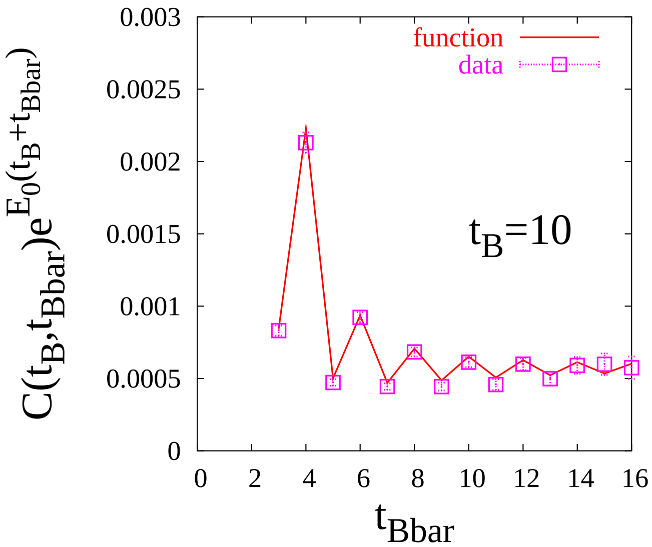
- $\xi_\Phi = \frac{\Phi_{B_s}}{\Phi_B} \equiv \frac{f_{B_s}\sqrt{m_{B_s}}}{f_B\sqrt{m_B}}$
- Systematics cancel in ratio.

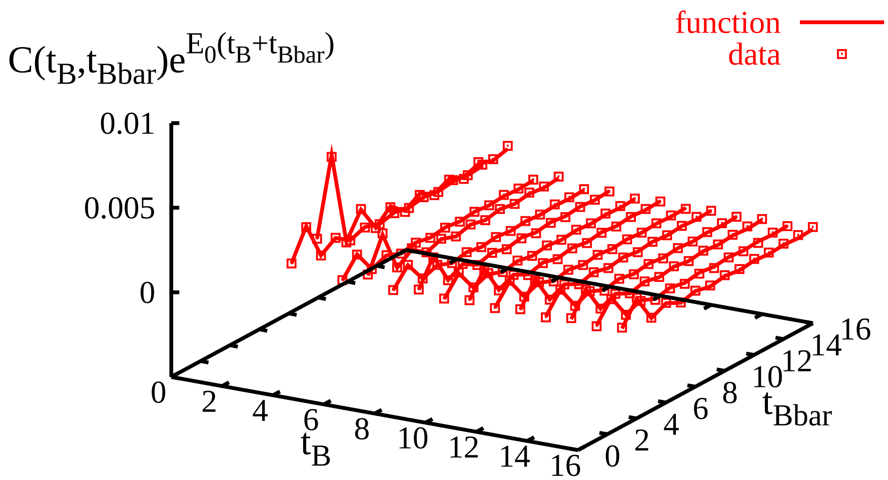
$B - \bar{B}$ Mixing



- Continuum $\langle O_L \rangle^{\overline{MS}}$ has contribution from lattice $\langle O_L \rangle_{lat}$ and $\langle O_S \rangle_{lat}$ at 1-loop:
  $O_L = [\,\overline{\psi}_Q\, \gamma^\mu (1-\gamma_5)\, \psi_q\,]\,[\,\overline{\psi}_{\overline{Q}}\, \gamma_\mu (1-\gamma_5)\, \psi_q\,]$
  $O_S = [\,\overline{\psi}_Q\, (1-\gamma_5)\, \psi_q\,]\,[\,\overline{\psi}_{\overline{Q}}\, (1-\gamma_5)\, \psi_q\,]$
- Same simulation params as B leptonic decay but only so far with $m_f = 0.01$, $m_q = 0.04$ (i.e B_s), and only leading order in $1/m_b$

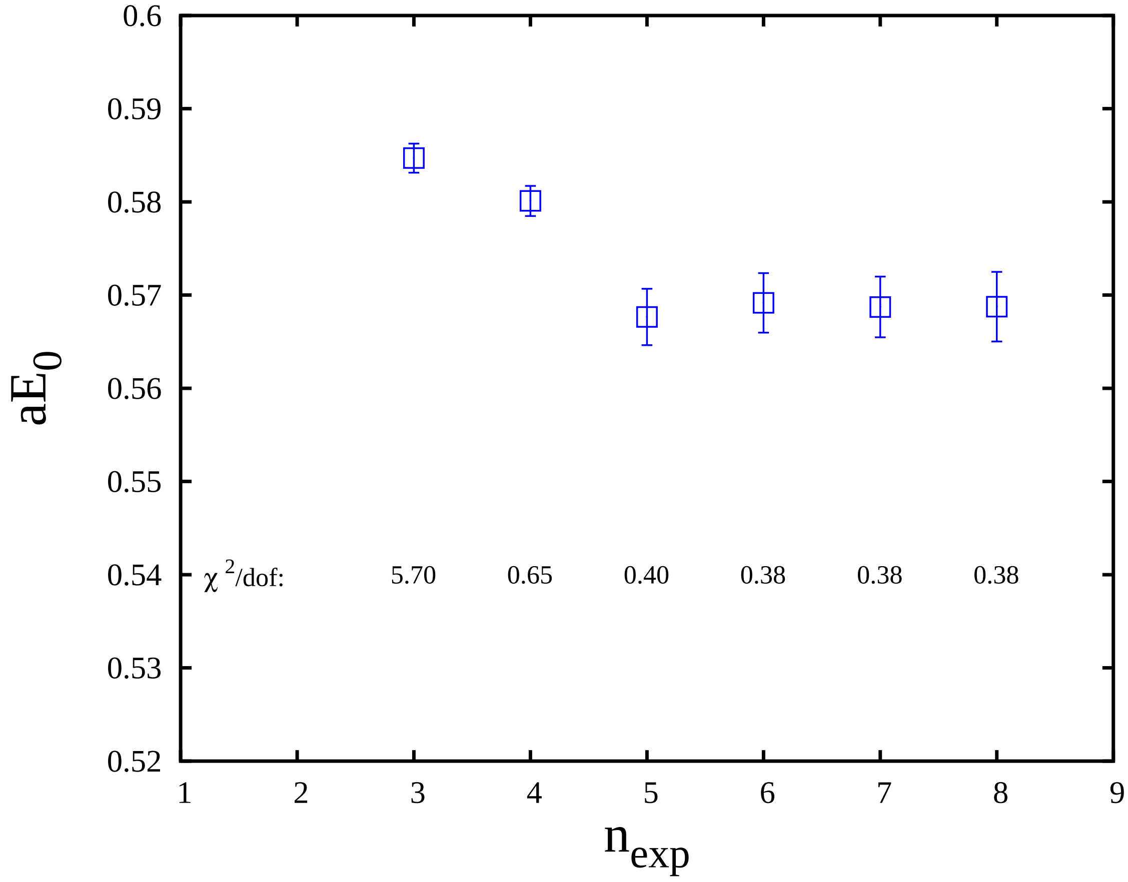
Fitting





- Corr has form $C(t_B, t_{\bar{B}}) = \sum_{j,k=0}^{n_{exp}-1} A_{jk}(-1)^{jt_B}e^{-m_j t_B}(-1)^{kt_{\bar{B}}}e^{-m_k t_{\bar{B}}}$
- Again Bayesian fitting used.

Fitting


0.6
0.59
0.58
0.57
0.56
0.55
0.54
0.53
0.52
aE0
χ 2/dof:
5.70
0.65
0.40
0.38
0.38
0.38
1
2
3
4
5
6
7
8
9
nexp

$f_B\sqrt{B_B}$

- $a^6\langle O_L\rangle^{\overline{MS}} = [1+\rho_{LL}\,\alpha_s]\,\langle O_L\rangle_{lat} \quad + \quad \rho_{LS}\,\alpha_s\,\langle O_S\rangle_{lat}$
  $O_L = [\,\overline{\psi}_Q\,\gamma^\mu(1-\gamma_5)\,\psi_q\,]\,[\,\overline{\psi}_{\overline{Q}}\,\gamma_\mu(1-\gamma_5)\,\psi_q\,]$
  $O_S = [\,\overline{\psi}_Q\,(1-\gamma_5)\,\psi_q\,]\,[\,\overline{\psi}_{\overline{Q}}\,(1-\gamma_5)\,\psi_q\,]$
- ρ_{LS}, ρ_{LL} calculated pertubatively.
- In terms of 3-pnt (A_{00}) and B 2-pnt (ξ_{BB}) correlator groundstate amplitudes,
  $\frac{A_{00}^{(OL,S)}}{\xi_{BB}}2 = \frac{1}{2M_B a^3}\,\langle O_{L,S}\rangle_{lat}$
- Note: we fit directly to 3-point without first taking ratio over 2-point. Don't need to wait for plateau - fit at low t where error is still small by including ex states.
- B_B is defined through $\langle O_L\rangle^{\overline{MS}} = \frac{8}{3}f_B^2 M_B^2 B_B$
- prelim result $f_{B_s}\sqrt{B_{B_s}(m_b)} = 0.244(15)(32)$ GeV
  errors are fitting (fits still prelim) and systematic (Λ_{QCD}/m_b, α_s^2 etc.).
  In process of doing $1/m_b$ corrections.

Conclusions

- Υ spectroscopy results provide m_b and a^{-1} for B decay and mixing calcs, as well as confidence that methods work.
- Have implemented smearing in B simulations to substantially reduce statistical errors of parameters needed for f_B.
- Chiral fits look promising. Need more fully unquenched points at light quark mass.
- Successful fit done to $B - \bar{B}$ mixing correlator looks good. Now include $1/m_b$ corrections and repeat with different $m_{q,f}$.