Light hadrons in 2+1 flavor lattice QCD

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American Physical Society & BNL

Modern Challenges for Lattice Field Theory Program Seminar

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Collaborators

MILC Collaboration (Jan 2004): E. Gregory, C. Aubin, R. Sugar, UMH, J. Hetrick, S. Gottlieb, C. Bernard, C. DeTar, J. Osborn, D. Toussaint

$+$ HPQCD & UKQCD Collaborations (for scale, m_s, $\hat{m}$, $m_s/\hat{m}$):
C. Davies, A. Gray, J. Hein, G. P. Lepage, Q. Mason, J. Shigemitsu, H. Trottier, M. Wingate

Outline

- Ensemble of Configurations
- Result Highlights/Summary
- Pseudoscalar sector
- Other meson masses
- Baryons
- Topology
- Other projects with MILC configurations
- Summery and Outlook

Ensemble of Configurations

To carry out a simulation we must select certain physical parameters:

- lattice spacing (a) or gauge coupling (β)
- grid size ($N_s^3 \times N_t$)
- quark masses ($m_{u,d} = m_l$, m_s)

To control systematic error we must

- take continuum limit
- take infinite volume limit
- extrapolate to physical light quark mass;
  we can work at physical s quark mass, or interpolate to it

Ensemble of Configurations

We also must choose an action and a simulation algorithm.

- The gauge action is a 1-loop improved Lüscher-Weisz action, with $\mathcal{O}(\alpha_s^2 a^2)$ discretization errors.
- The fermion action is a tree-level improved staggered action with a “fat” link to suppress taste violations of the staggered fermions. It has $\mathcal{O}(\alpha_s a^2)$ discretization errors.
- The algorithm is the Hybrid Molecular Dynamics R-algorithm, with the $det^{1/4}$ trick to eliminate the extra tastes.

Whether the $det^{1/4}$ trick induces non-localities in the interacting theory is an open question. Our results, so far, show no sign of a problem.

Configurations

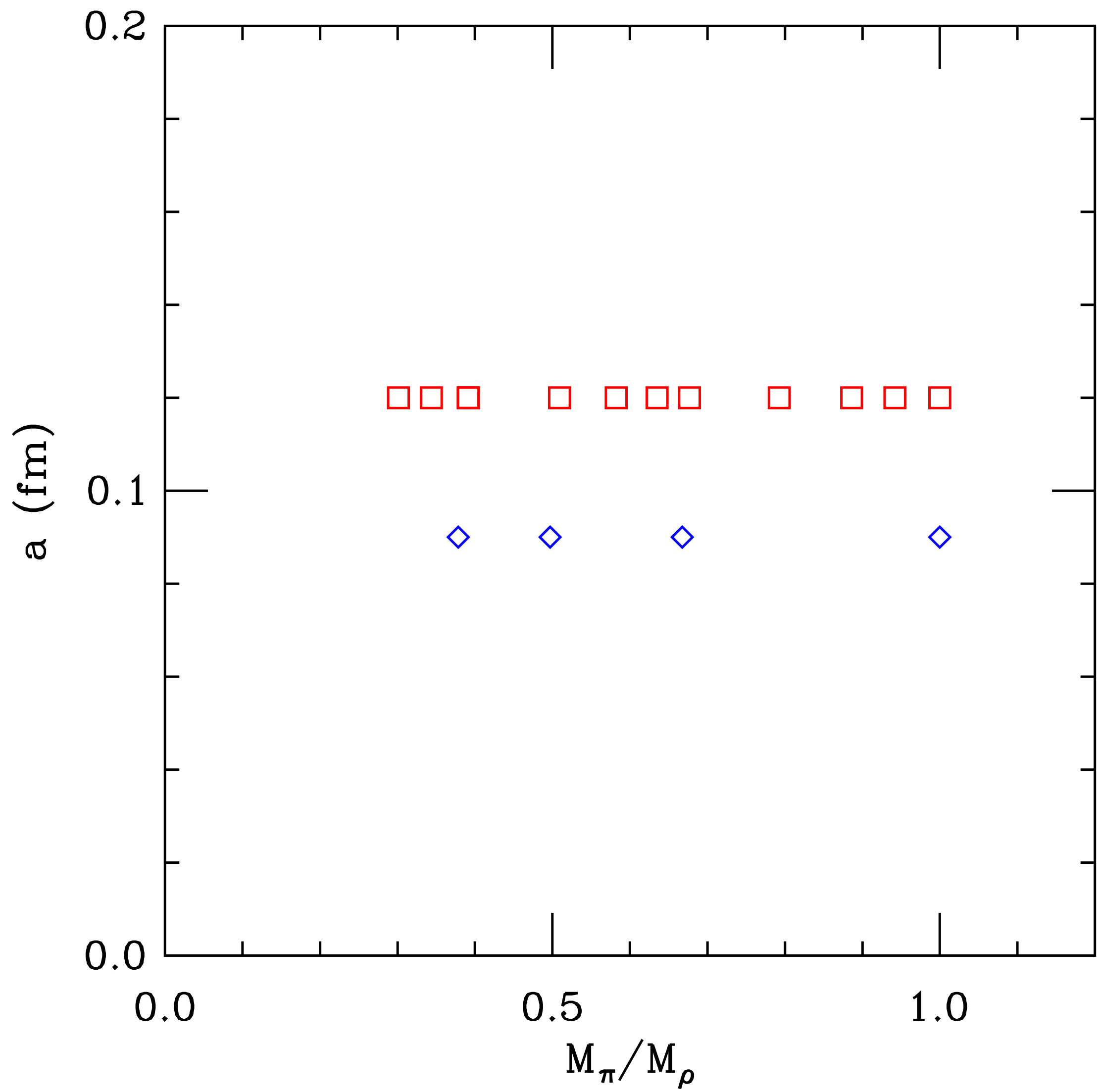
MILC has been generating three flavor configurations to allow control of these errors. Many configurations are available to others through NERSC Gauge Connection.
Some new configurations generated via SciDAC

$a = 0.09$ fm; $28^3 \times 96$		
$am_{u,d}$ / am_s	$10/g^2$	# config.
0.031 / 0.031	7.18	496
0.0124 / 0.031	7.11	527
0.0062 / 0.031	7.09	592
$a = 0.09$ fm; $40^3 \times 96$		
0.0031 / 0.031	7.08	≈ 100

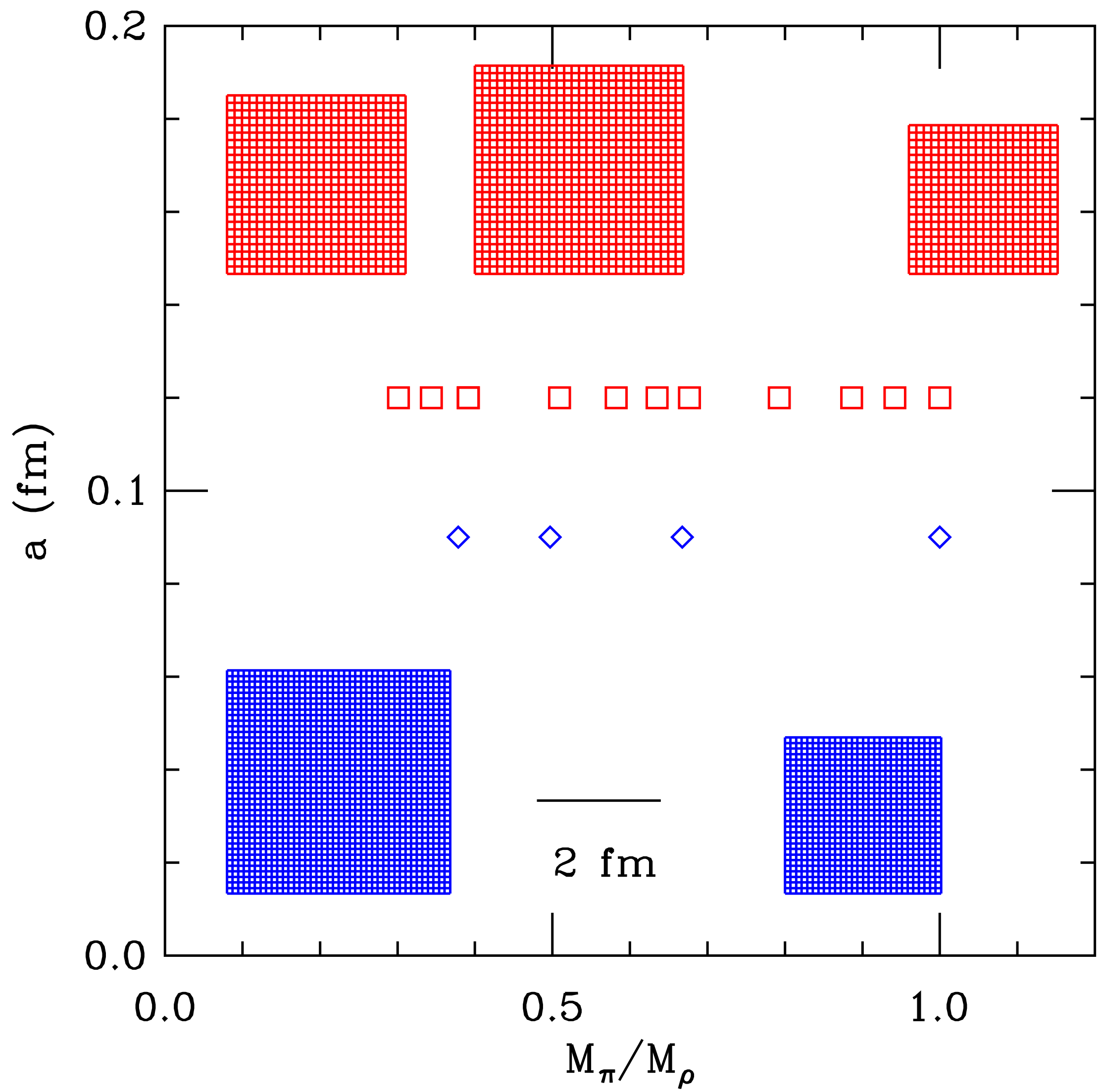
Configurations

$a = 0.12$ fm; $20^3 \times 64$		
$am_{u,d}$ / am_s	$10/g^2$	# config.
0.40 /0.40	7.35	332
0.20 /0.20	7.15	341
0.10 /0.10	6.96	339
0.05 /0.05	6.85	425
0.04 /0.05	6.83	351
0.03 /0.05	6.81	564
0.02 /0.05	6.79	484
0.01 /0.05	6.76	658
0.007/0.05	6.76	493
$a = 0.12$ fm; $24^3 \times 64$		
0.005/0.05	6.76	≈ 375

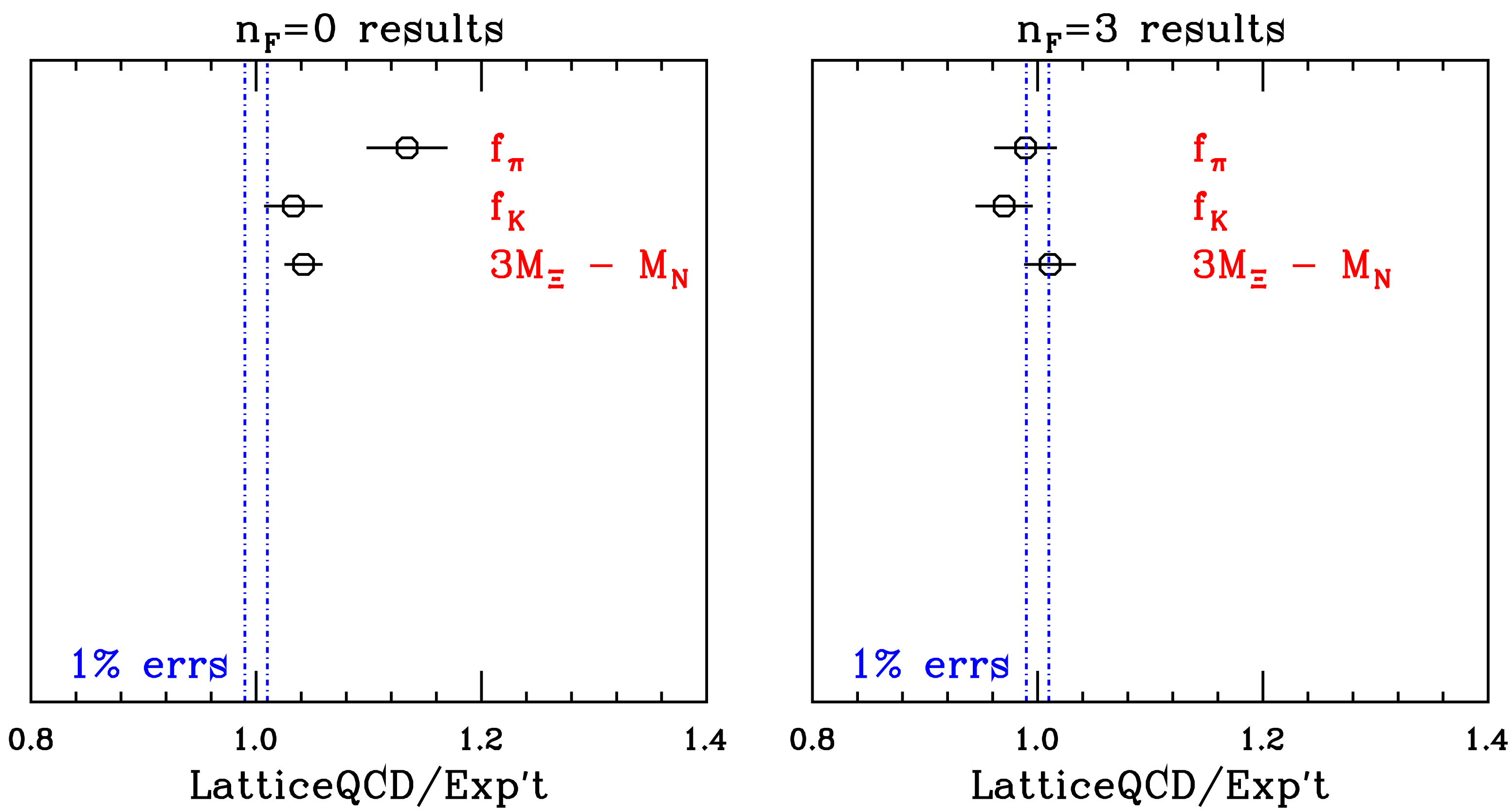
MILC Ensembles



MILC Ensembles

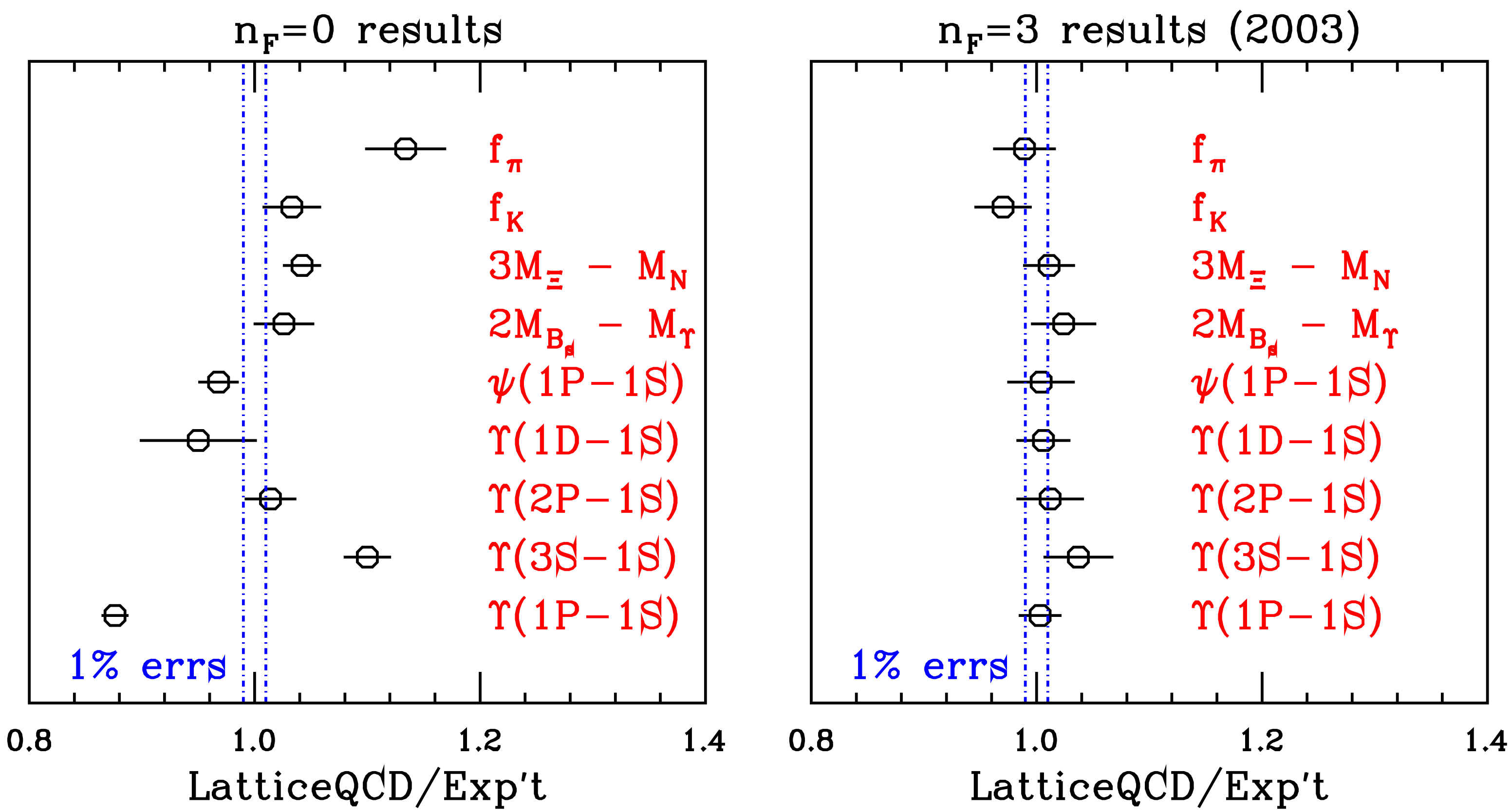


Ratio Plot



n_F=0 results
f_π
f_K
$3M_\Xi - M_N$
1% errs
0.8
1.0
1.2
1.4
LatticeQCD/Exp't
n_F=3 results
f_π
f_K
$3M_\Xi - M_N$
1% errs
0.8
1.0
1.2
1.4
LatticeQCD/Exp't

Ratio Plot

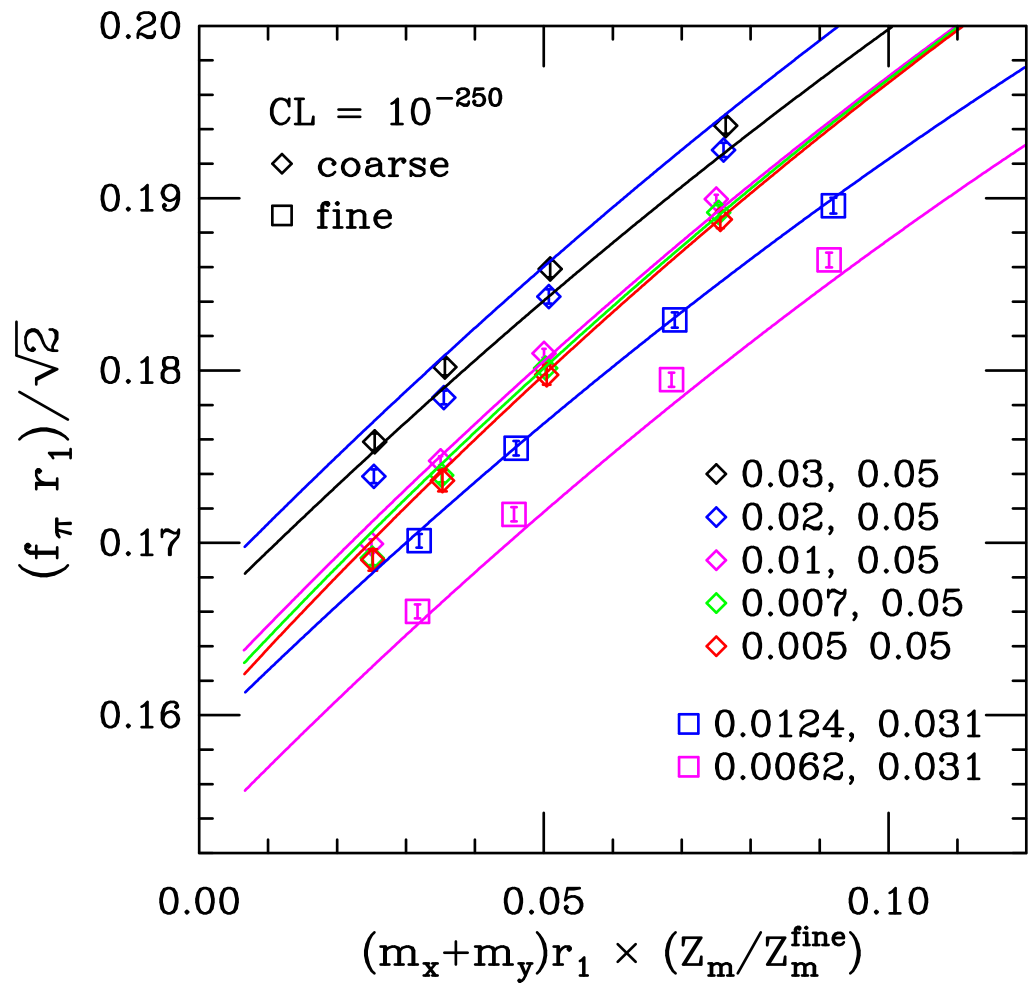


By sharing with FNAL, HPQCD and UKQCD

Pseudoscalar sector

Have precise measurements for mass and decay constants

- Continuum χPT fit to both f_π and m_π simultaneously
- Does not work: CL = 10^{-250}



Improved fits

Next show improved fit:

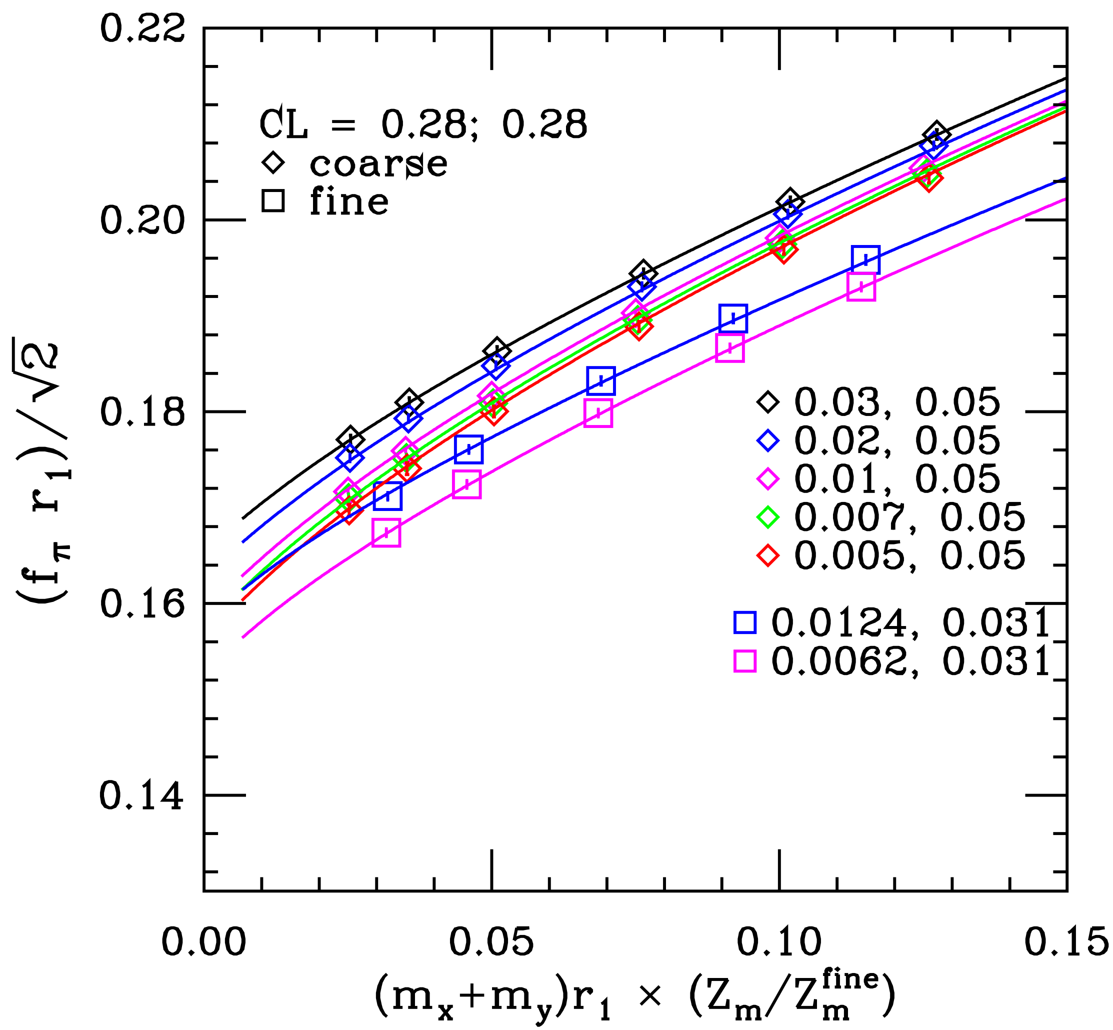
- Use SχPT (Aubin & Bernard), *i.e.* with taste violation effects; include NNLO corrections
- Fit coarse and fine lattices together
- Points plotted after finite volume correction

After fit, we:

- Extrapolate fit parameters to continuum
- Show difference between m_s' (simulation strange mass) and m_s (correct value)
- Details in hep-lat/0407028

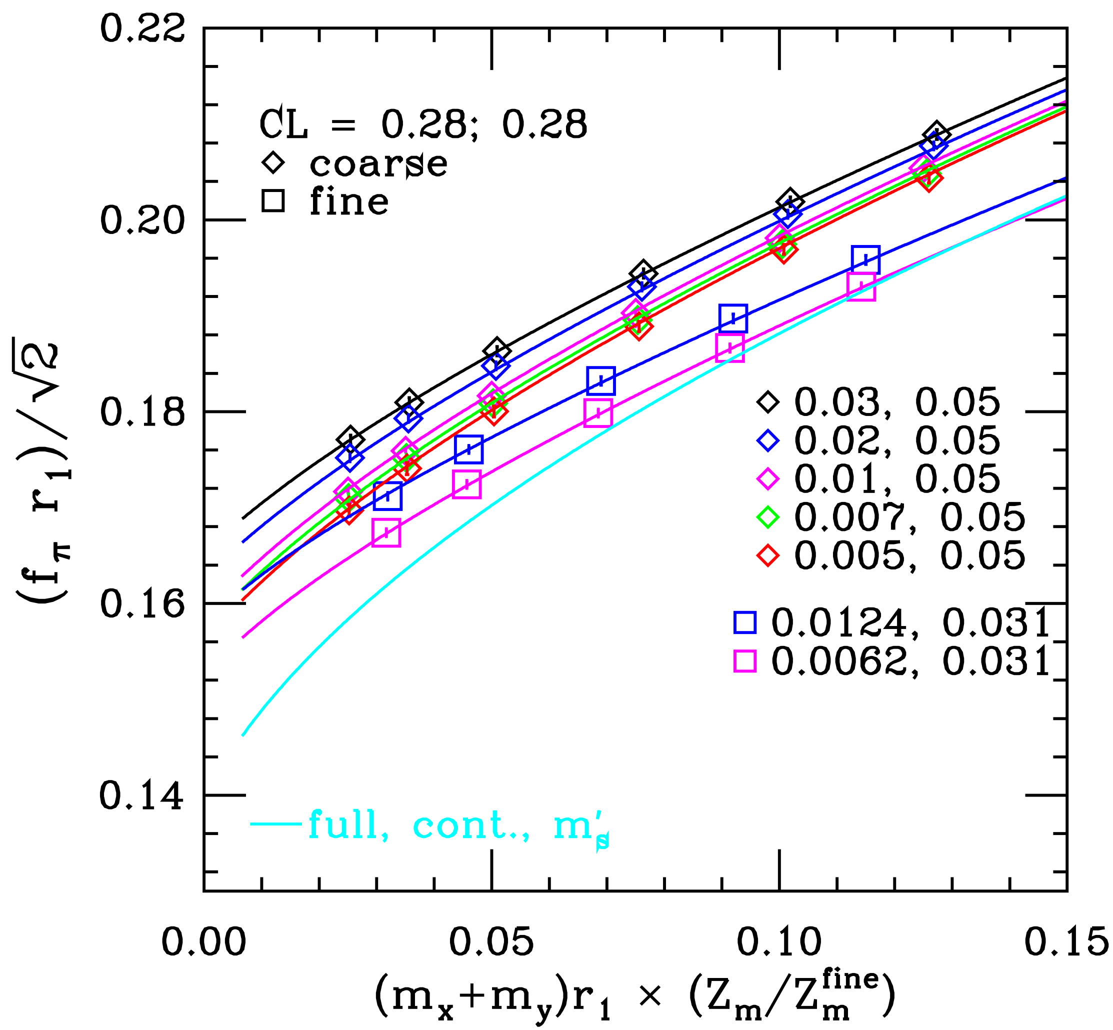
Fit of f_π

- Fit partially quenched f_π (and, simultaneously, m_π) with taste violations



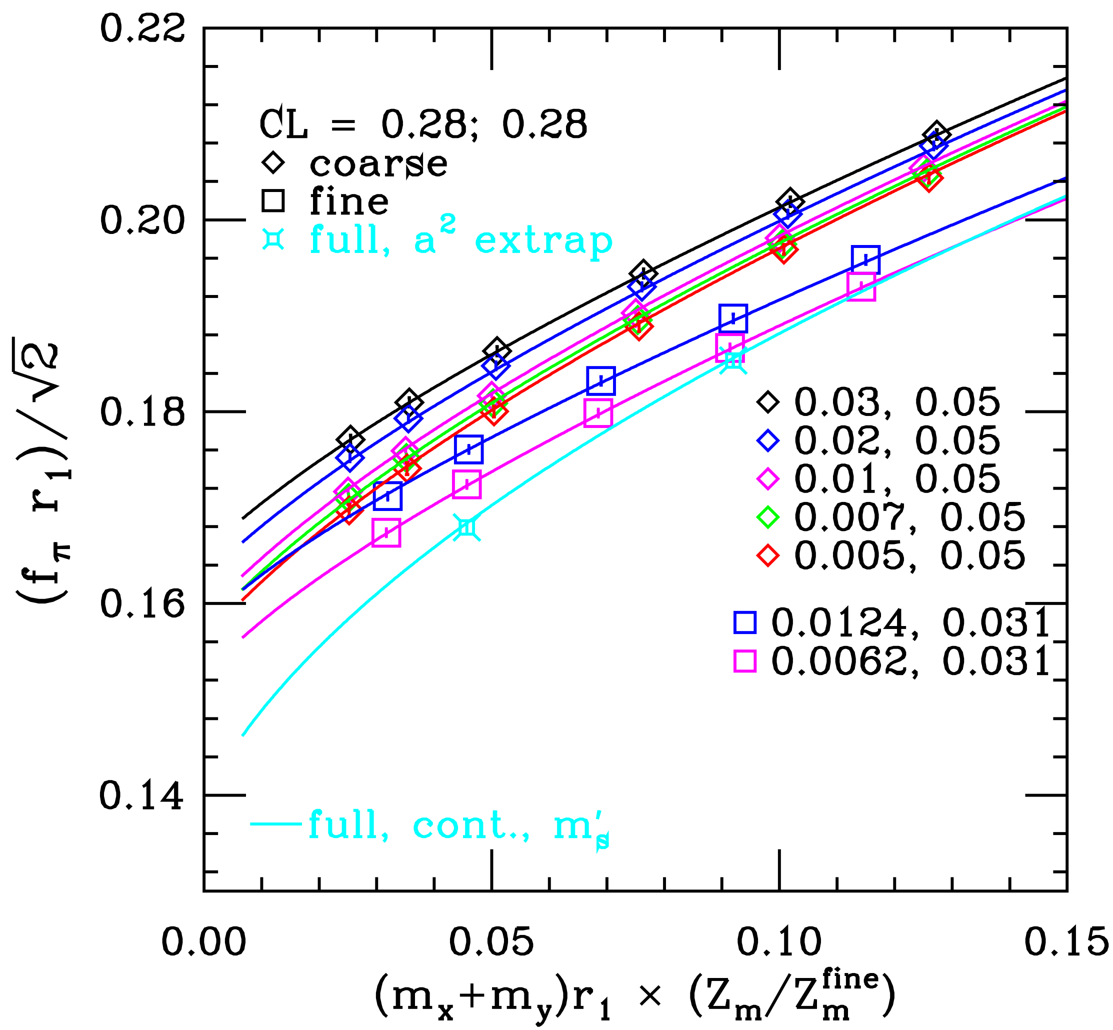
Fit of f_π

- Extrapolate fit params to continuum
- Go to "full QCD:" Set $\hat{m}'_{sea} = \hat{m}'_{val}$ and plot a function of $\hat{m}'_{val}$: ———



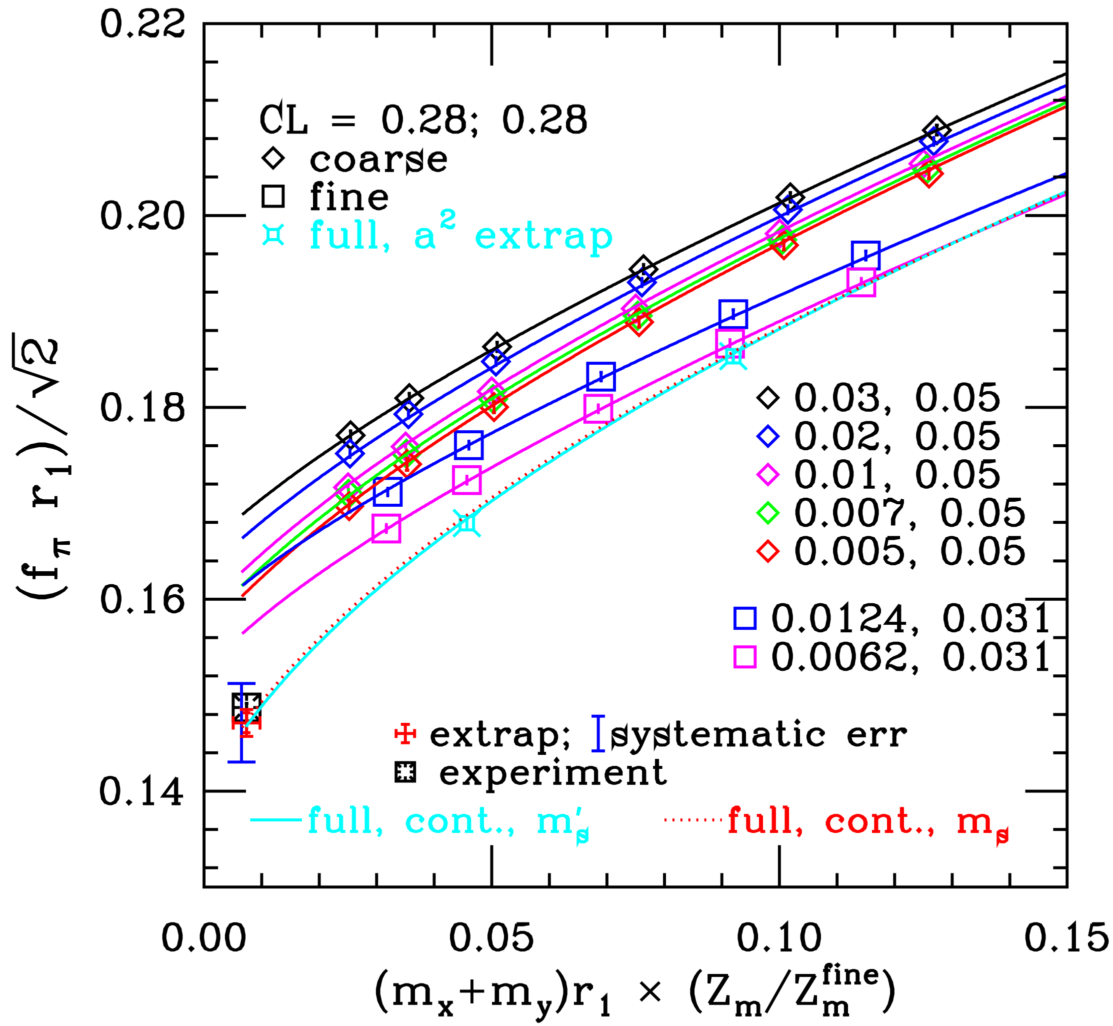
Fit of f_π

- Consistency check: extrapolate points with sea masses = valence masses to continuum at fixed quark mass



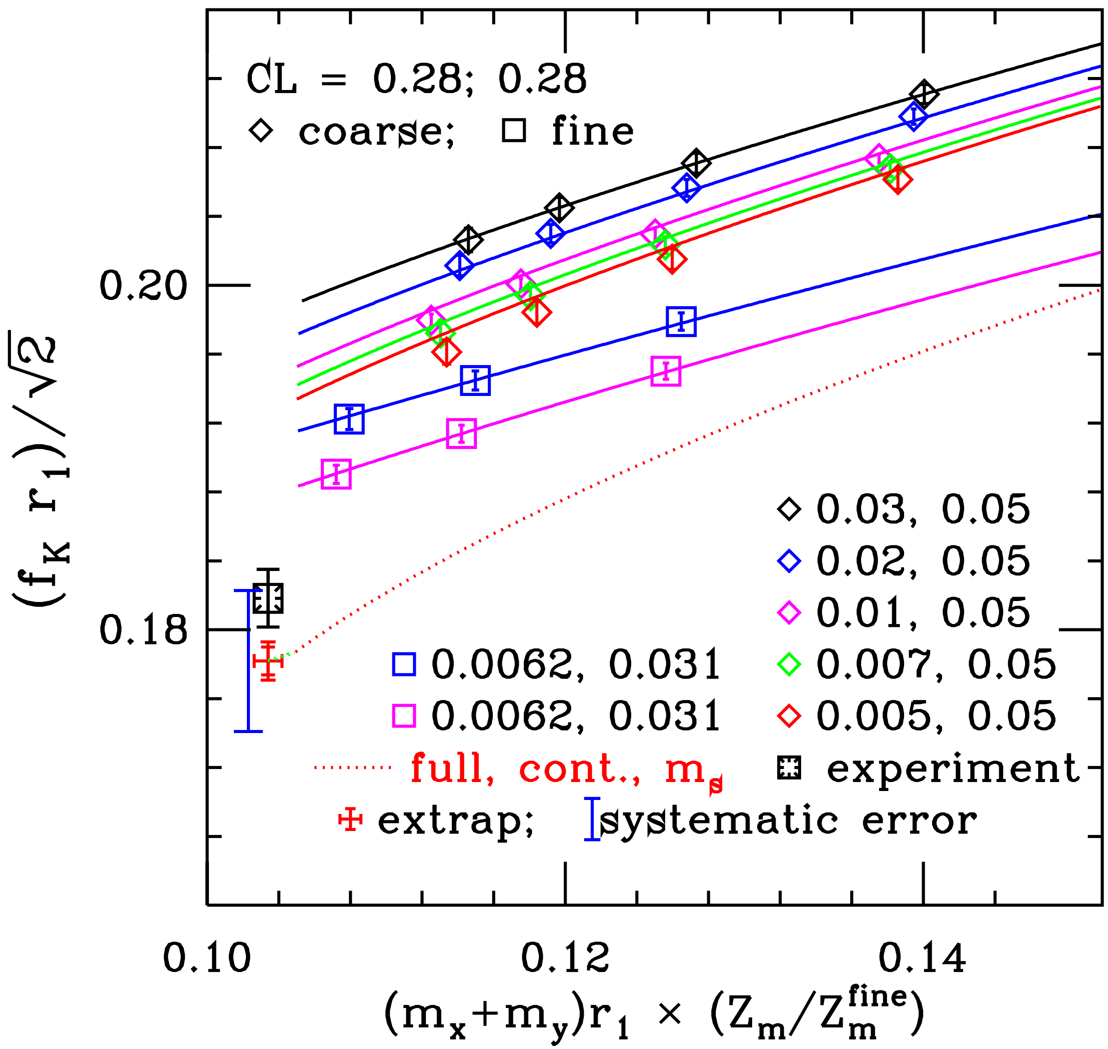
Fit of f_π

- Correct from simulation strange mass, m_s', to correct value, m_s

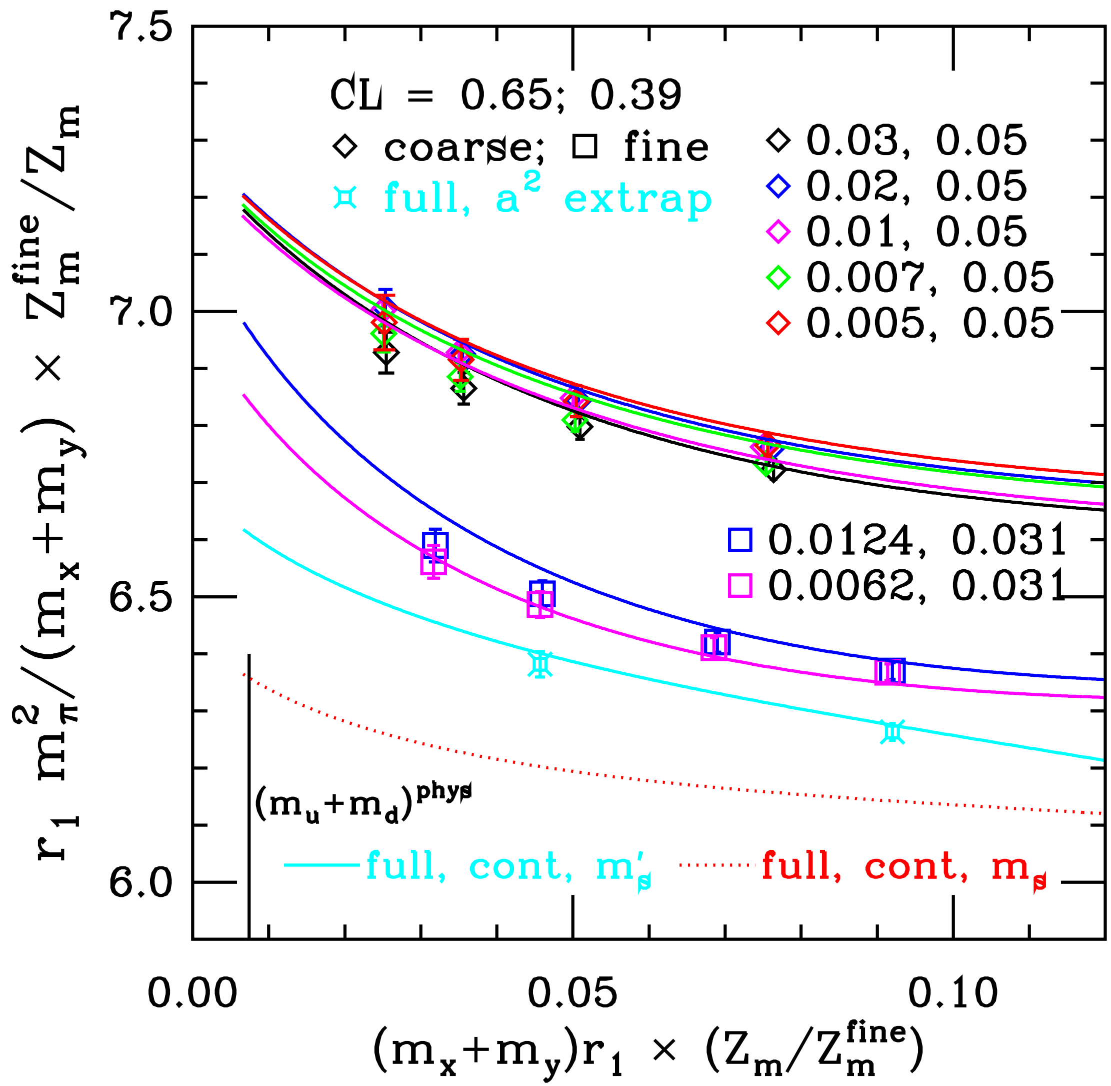


Fit of f_K

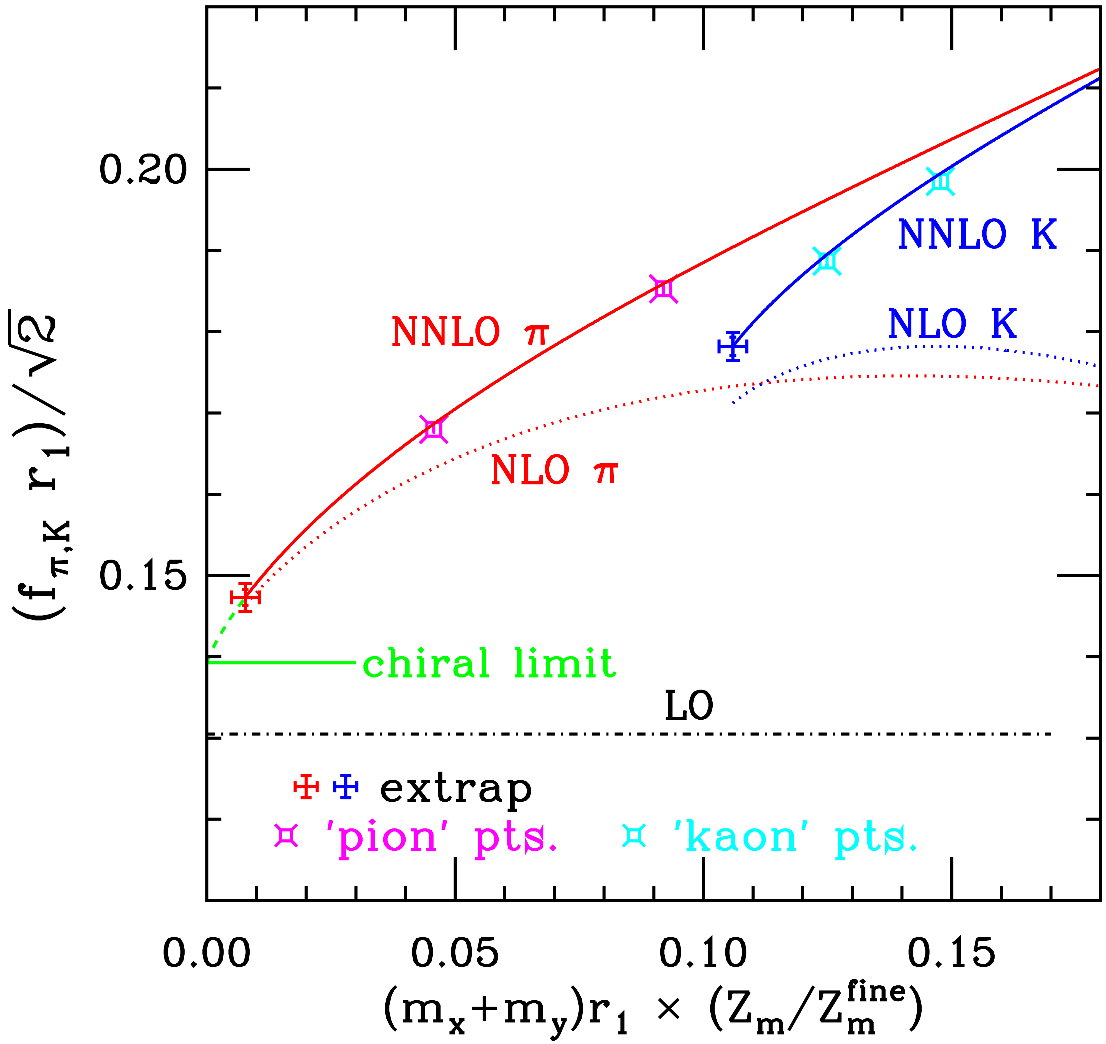
- Similar procedure for f_K.
- But note that f_K is the decay constant of K^+
- Here we need to extrapolate light valence quark to m_u, but light sea quark to $\hat{m}$



Fit of $m_\pi^2/(m_x+m_y)$



Convergence of $SU(3)_L \times SU(3)_R$ χPT



Light Quark Masses

To find quark masses, must extrapolate to the physical meson masses. Electromagnetic and isospin-violating effects are important

- Experimental masses:
  $m_{\pi^0}^{\rm expt}, m_{\pi^+}^{\rm expt}, m_{K^0}^{\rm expt}, m_{K^+}^{\rm expt}$
- Masses with EM effects turned off:
  $m_{\pi^0}^{\rm QCD}, m_{\pi^+}^{\rm QCD}, m_{K^0}^{\rm QCD}, m_{K^+}^{\rm QCD}$
- Masses with EM effects turned off and $m_u = m_d = \hat{m}$:
  $m_{\hat{\pi}}, m_{\hat{K}}$

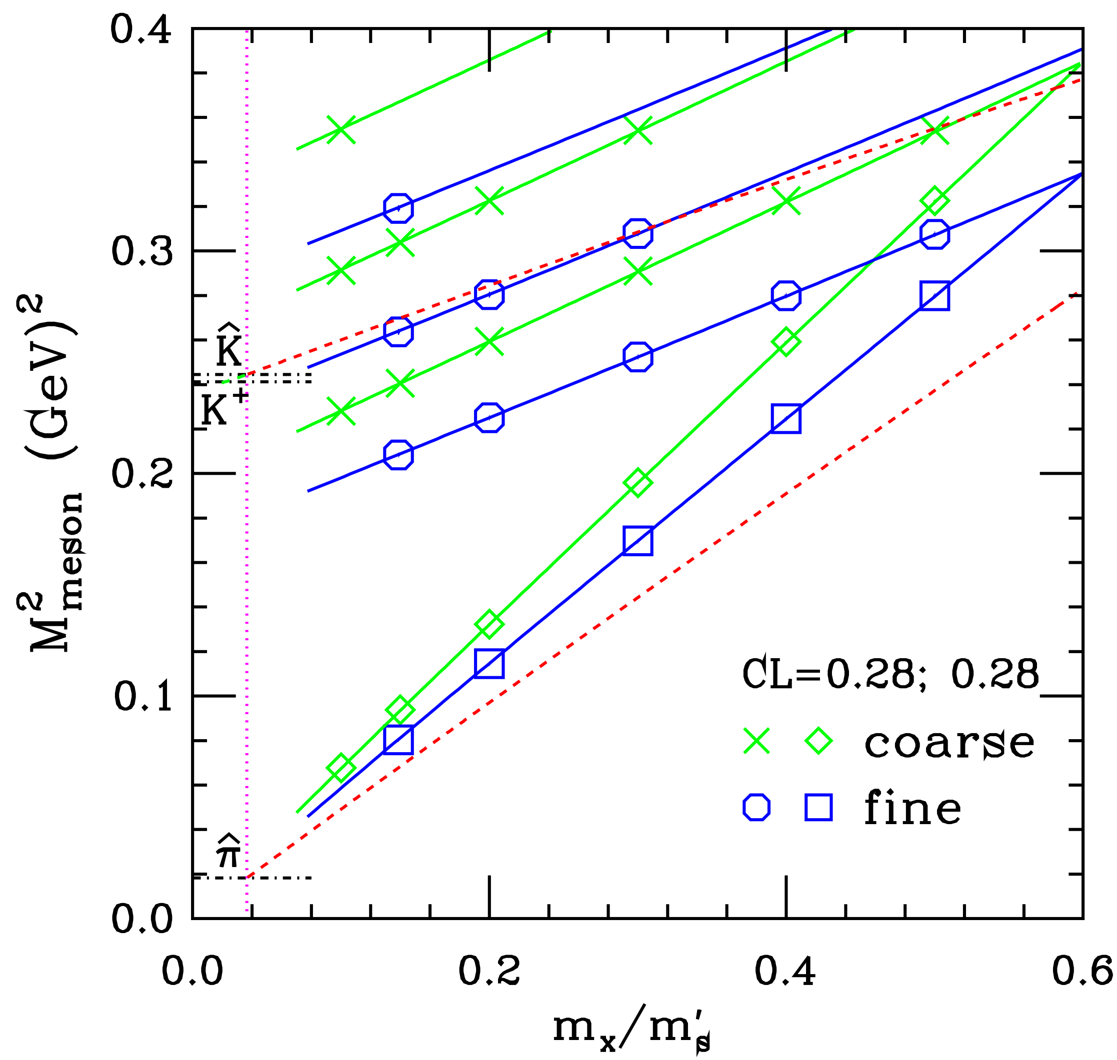
EM & Isospin Violation

$$
\begin{aligned}
m_{\hat{\pi}}^2 &\approx (m_{\pi^0}^{\mathrm{QCD}})^2 \approx (m_{\pi^0}^{\mathrm{expt}})^2 \\
m_{\hat{K}}^2 &\approx \frac{(m_{K^0}^{\mathrm{QCD}})^2 + (m_{K^+}^{\mathrm{QCD}})^2}{2} \\
(m_{K^0}^{\mathrm{QCD}})^2 &\approx (m_{K^0}^{\mathrm{expt}})^2 \\
(m_{K^+}^{\mathrm{QCD}})^2 &\approx (m_{K^+}^{\mathrm{expt}})^2 - (1 + \Delta_E)\left((m_{\pi^+}^{\mathrm{expt}})^2 - (m_{\pi^0}^{\mathrm{expt}})^2\right)
\end{aligned}
$$

- $\Delta_E = 0$ is “Dashen’s theorem.”
- Continuum suggests: $\Delta_E \approx 1$.
- We use $0 < \Delta_E < 2$

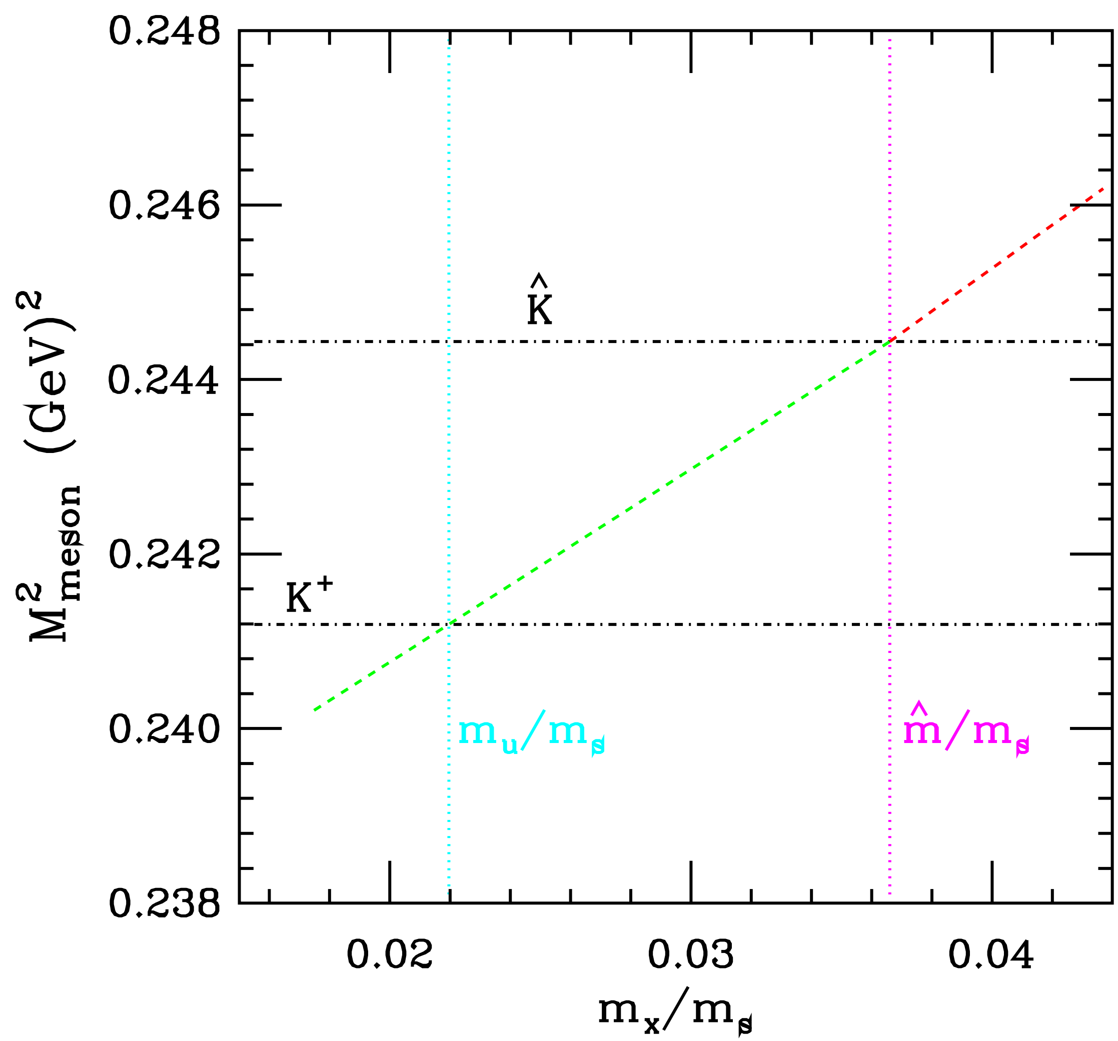
Finding $\hat{m}$, m_s

- Subset of data with fits
- Red lines are continuum extrapolated full QCD fits with m_s adjusted so that both $\hat{\pi}$ and $\hat{K}$ are fit



Finding m_u

- Next estimate m_u by extrapolating in quark mass to K^+ mass
- Below $\hat{m}$ only valence mass changes
- There is a small isospin violation because for sea quarks $m_u = m_d = \hat{m}$



Quark mass results

We find

$$m_u/m_d = 0.43(0)(2)(8)\ ,$$

where the errors are statistical (rounded down to 0), lattice systematics, and a conservative estimate of EM effects.

Using instead a phenomenological result of Bijnens and Prades, $\Delta_E = 0.84 \pm 0.25$, we would obtain

$$m_u/m_d = 0.44(0)(1)(2)\ .$$

Quark mass results

In collaboration with the HPQCD and UKQCD groups, using a one-loop mass renormalization constant, we find:

$$
\begin{aligned}
m_s^{\overline{\mathsf{MS}}} &= 76(0)(3)(7)(0)\ \mathrm{MeV}\ , \\
\hat{m}^{\overline{\mathsf{MS}}} &= 2.8(0)(1)(3)(0)\ \mathrm{MeV}\ , \\
m_s/\hat{m} &= 27.4(1)(4)(0)(1)\ ,
\end{aligned}
$$

where the errors are from statistics, simulation, perturbation theory, and electromagnetic effects, respectively. The renormalization scale of the masses is $2\,\mathrm{GeV}$.
With m_u/m_d from above, then:

$$
\begin{aligned}
m_u^{\overline{\mathsf{MS}}} &= 1.7(0)(1)(2)(2)\ \mathrm{MeV}\ , \\
m_d^{\overline{\mathsf{MS}}} &= 3.9(0)(1)(4)(2)\ \mathrm{MeV}\ .
\end{aligned}
$$

Results for light decay constants

We find:

$$
\begin{aligned}
f_\pi &= 129.5 \pm 0.9 \pm 3.5 \text{ MeV} \ , \\
f_K &= 156.6 \pm 1.0 \pm 3.6 \text{ MeV} \ , \\
f_K/f_\pi &= 1.210(4)(13) \ .
\end{aligned}
$$

Experiments:
$f_\pi = 130.7 \pm 0.4\,\text{MeV}$, $f_K = 159.8 \pm 1.5\,\text{MeV}$, $f_K/f_\pi = 1.223(12)$.

- Using our $f_K/f_\pi \Rightarrow V_{us} = 0.2219(26)$
- Unitarity: $|V_{ud}|^2 + |V_{us}|^2 = 0.9979(15)$
- PDG value: $V_{us} = 0.2196(26)$
- Recent KTeV: $V_{us} = 0.2252(8)(21)$

Results: Low Energy Constants

Also get (in units of 10^{-3}, at chiral scale m_η):

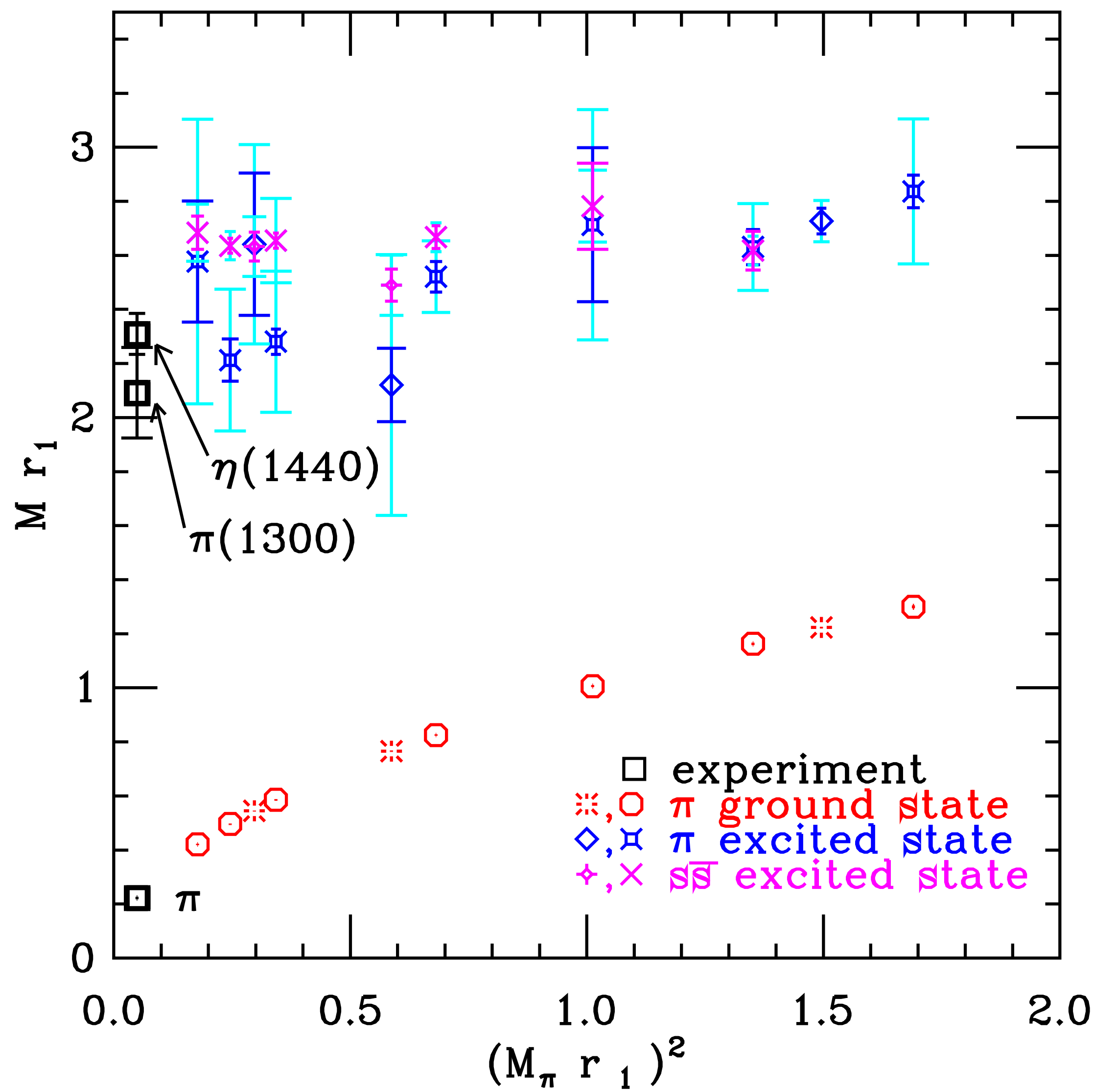
$$\begin{aligned} 2L_6 - L_4 &= 0.5(2)(4)\ , \\ 2L_8 - L_5 &= -0.2(1)(2)\ , \\ L_4 &= 0.2(3)(3)\ , \\ L_5 &= 1.9(3)(3)\ . \end{aligned}$$

- Consistent with "conventional results" summarized, *e.g.*, in Cohen, Kaplan, & Nelson, JHEP 9911, 027 (1999): $L_5 = 2.2(5)$, $L_6 = 0.0(3)$, $L_4 = 0.0(5)$.
- Our result for $2L_8 - L_5$ is far from range that would allow $m_u = 0$, $-3.4 \le 2L_8 - L_5 \le -1.8$ (Kaplan & Manohar; Cohen, Kaplan & Nelson) – but see critique by Creutz.
- Consistent with (but not independent of) direct determination of m_u.

Excited State 0^{-+} Masses

In the pseudoscalar sector we have good enough statistics to try to extract exited states

Details in
hep-lat/0402030

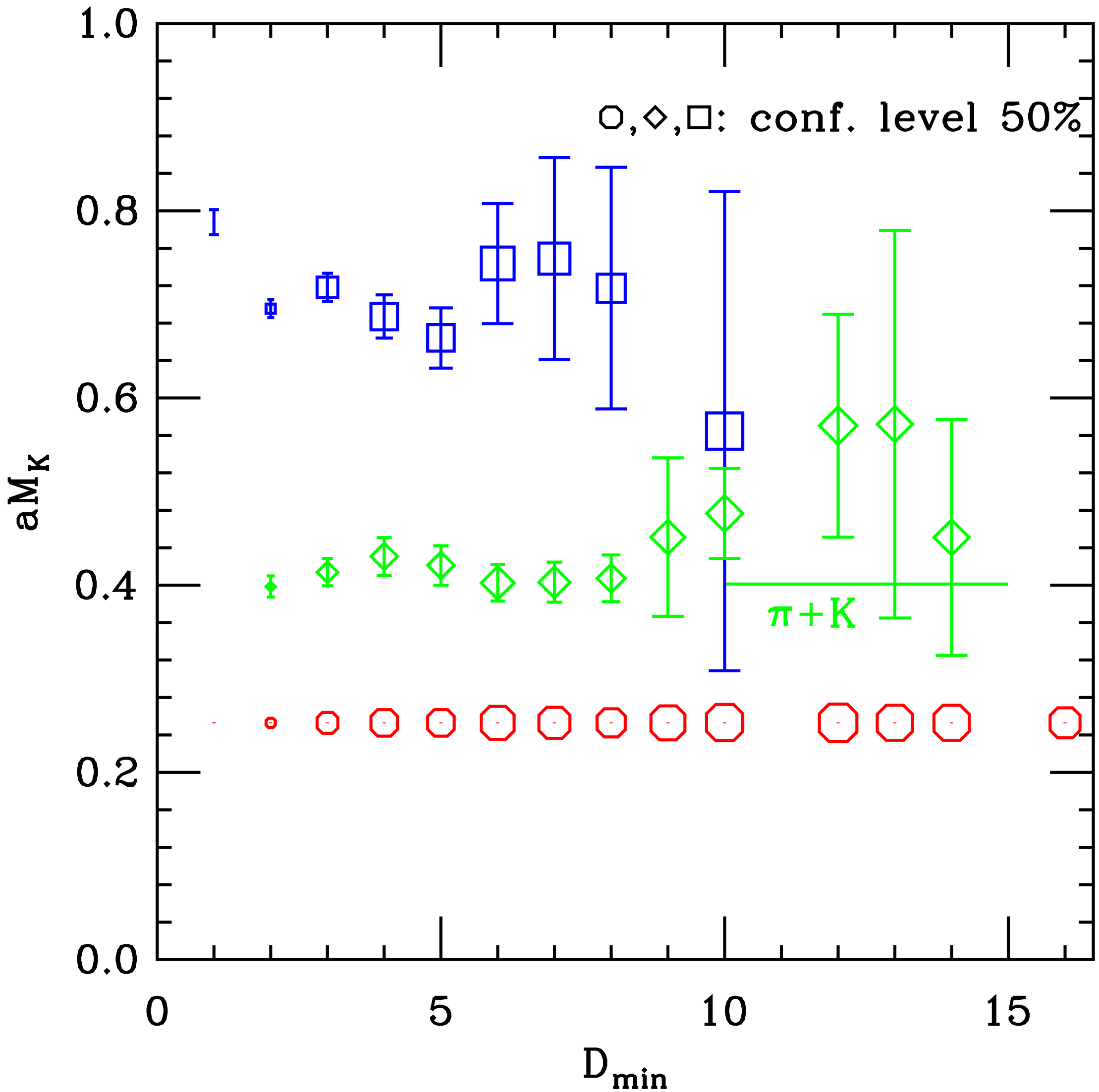


K excited fit, dynamical

Example of three-state fits to kaon from three-flavor run, $10/g^2 = 7.09$ and $am_{l/s} = 0.0062/0.031$

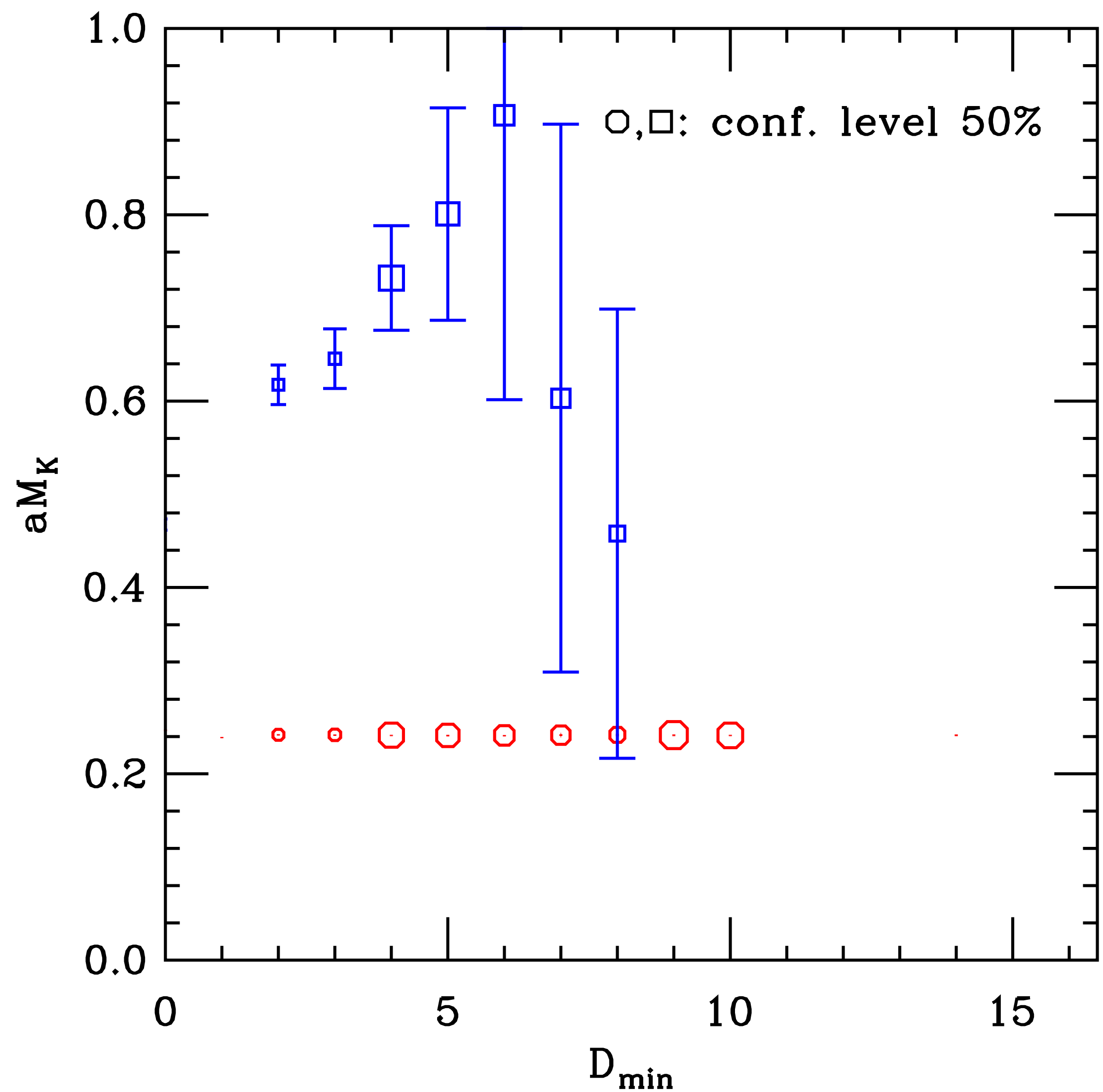
Diamond is opposite parity

Square is excited state

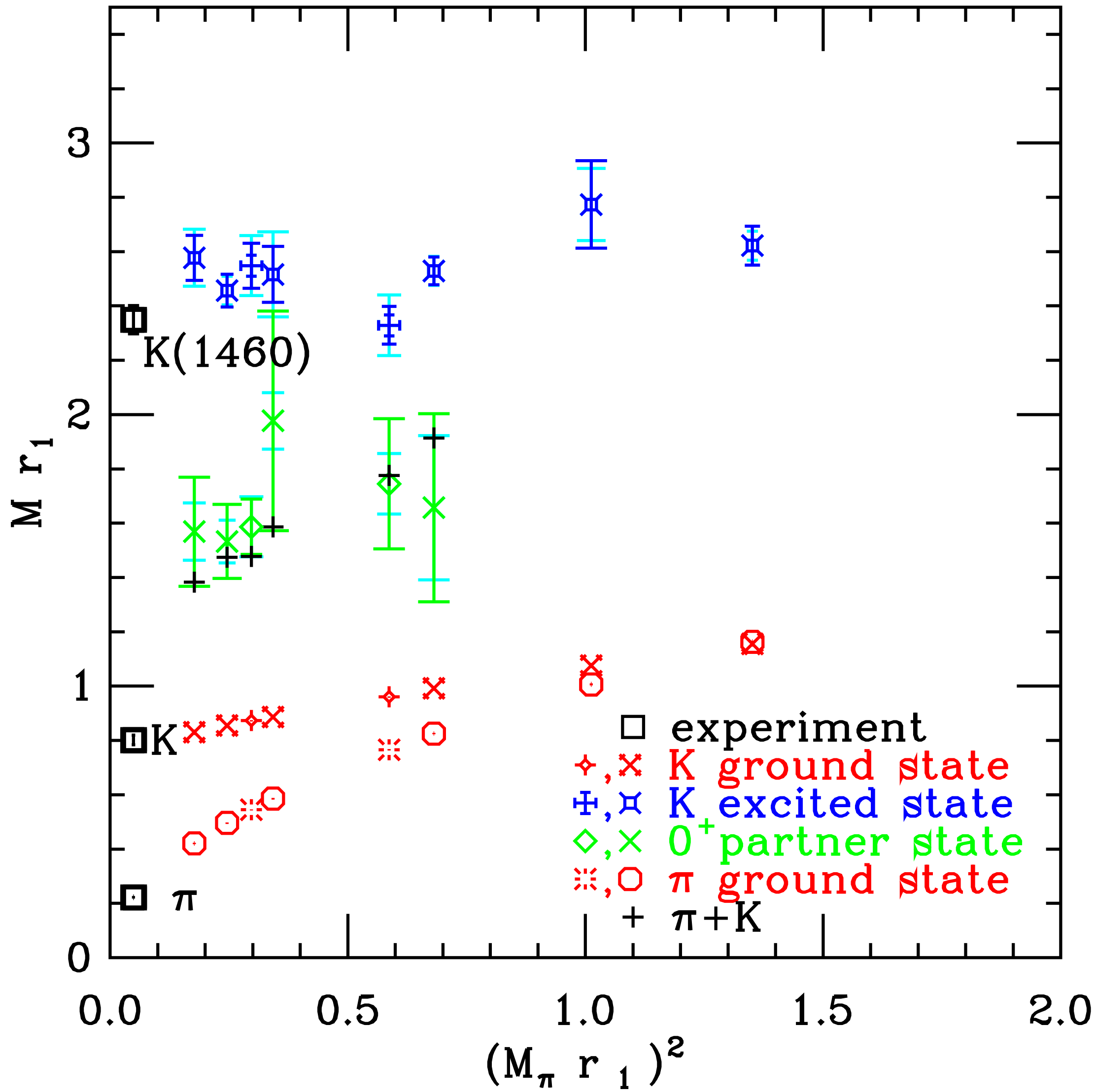


K excited fit, quenched

Note that the $\pi + K$ opposite parity state is absent in an otherwise matched quenched simulation, $10/g^2 = 8.40$ and $am_{l/s} = 0.0062/0.031$

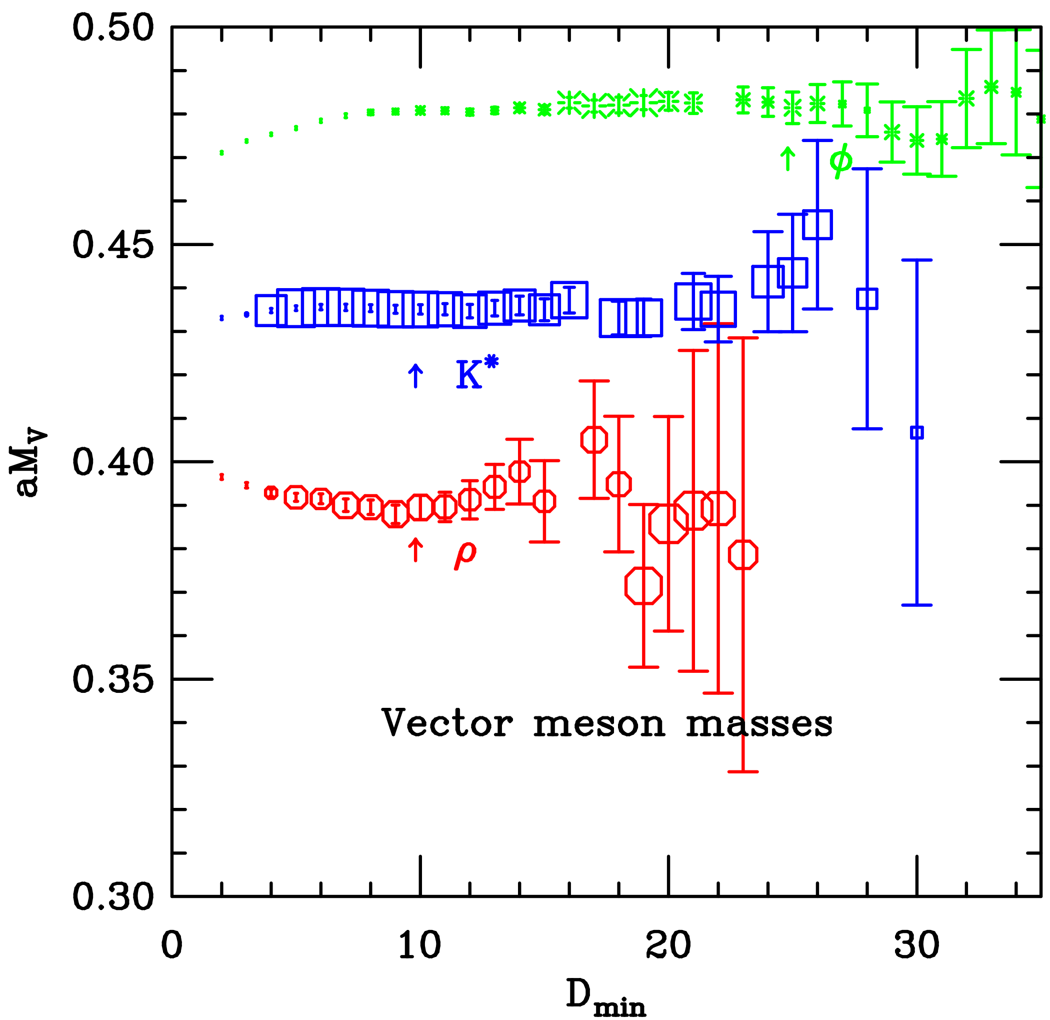


K and excited K state

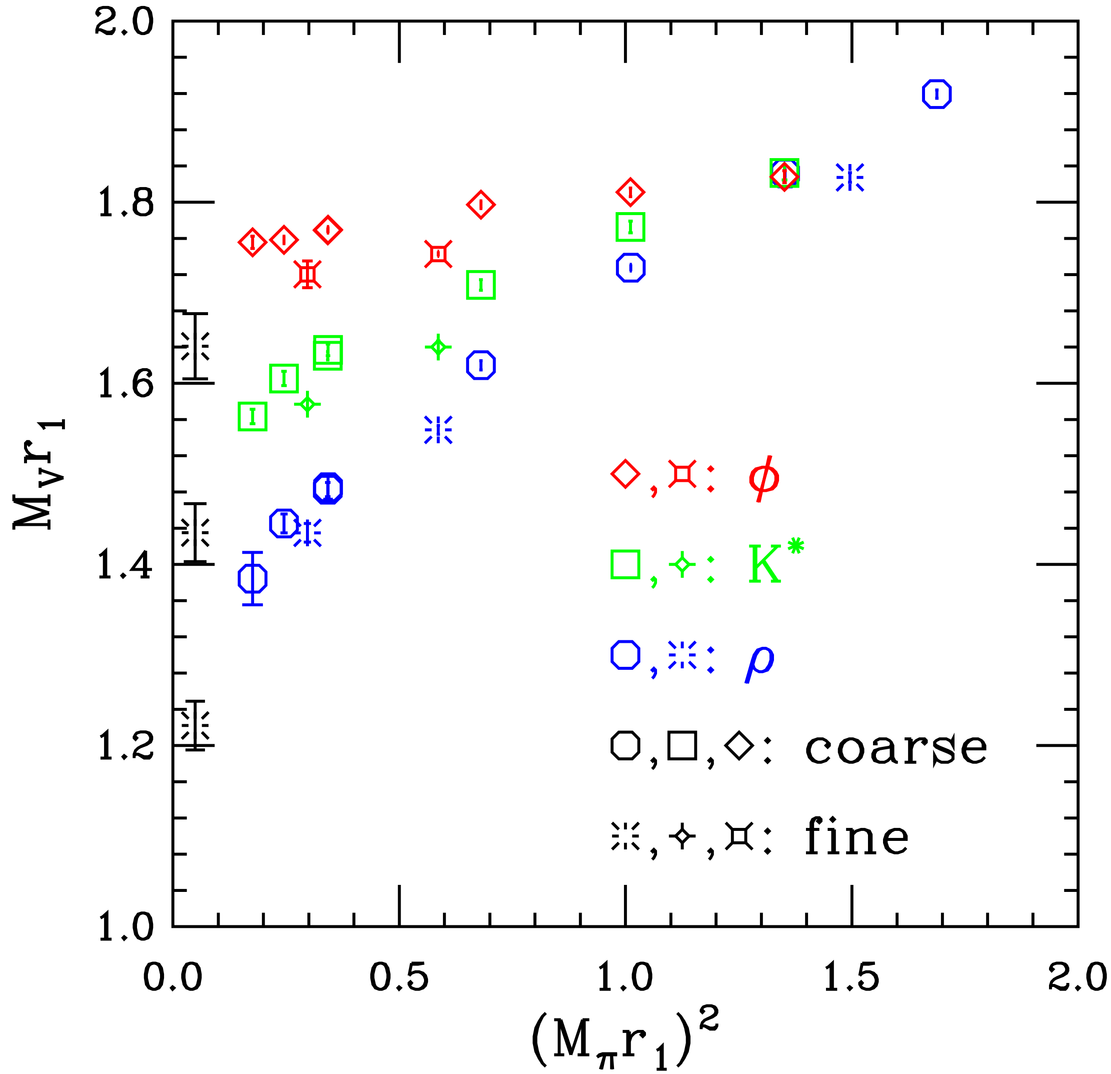


Vector meson fits

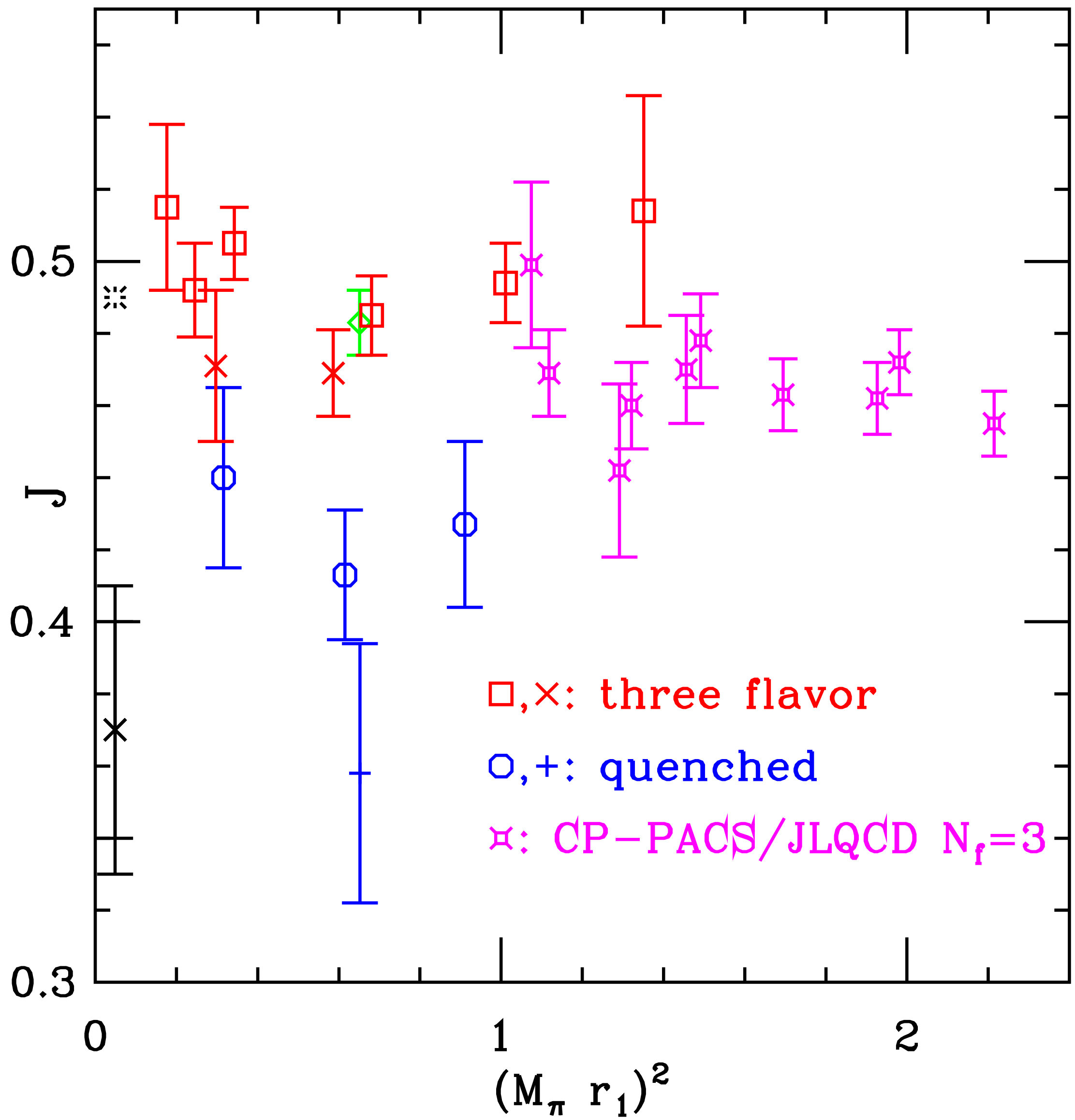
Vector meson mass fits: $10/g^2 = 7.09$ and $am_{l/s} = 0.0062/0.031$



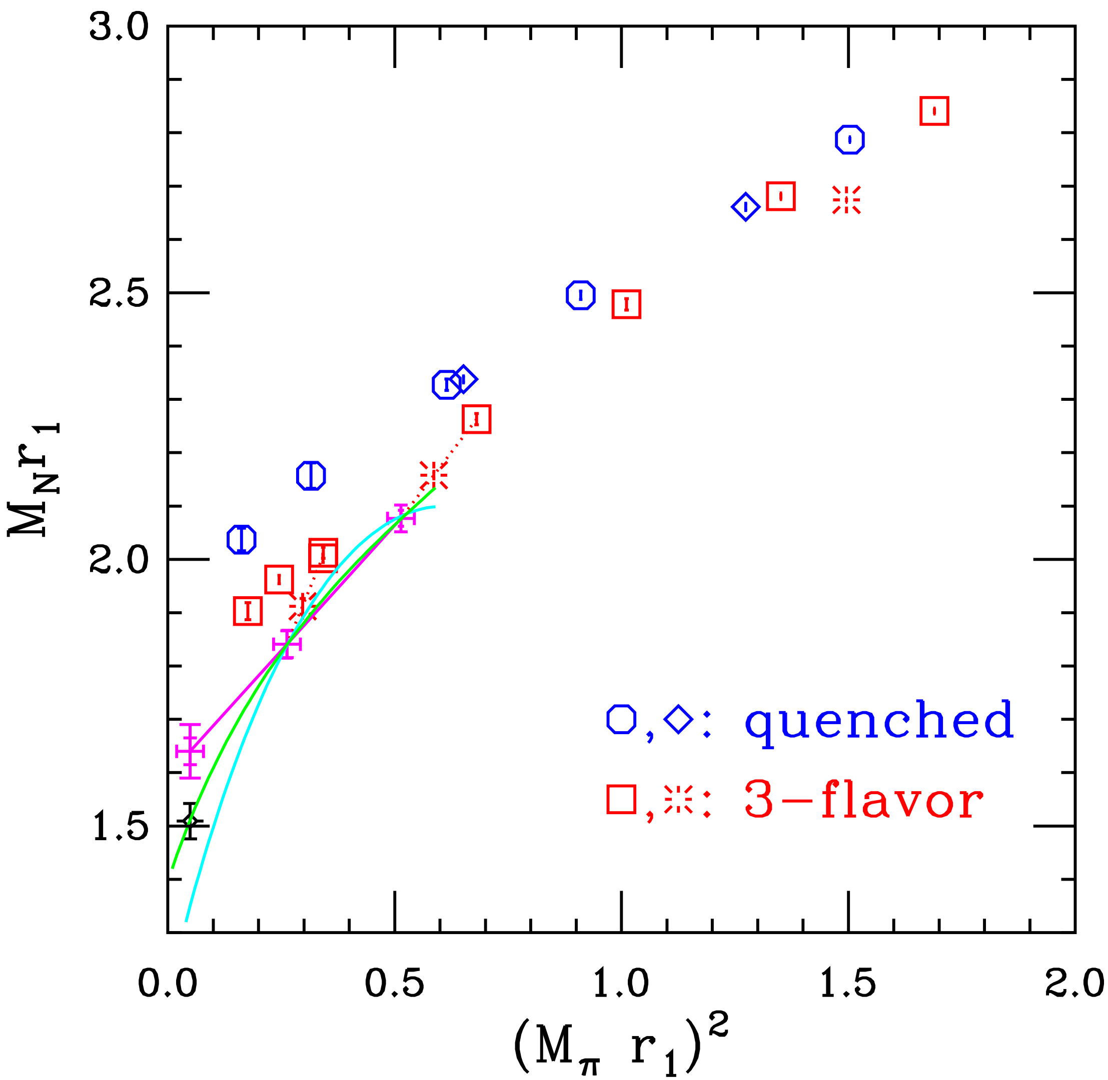
Vector meson masses



J Parameter

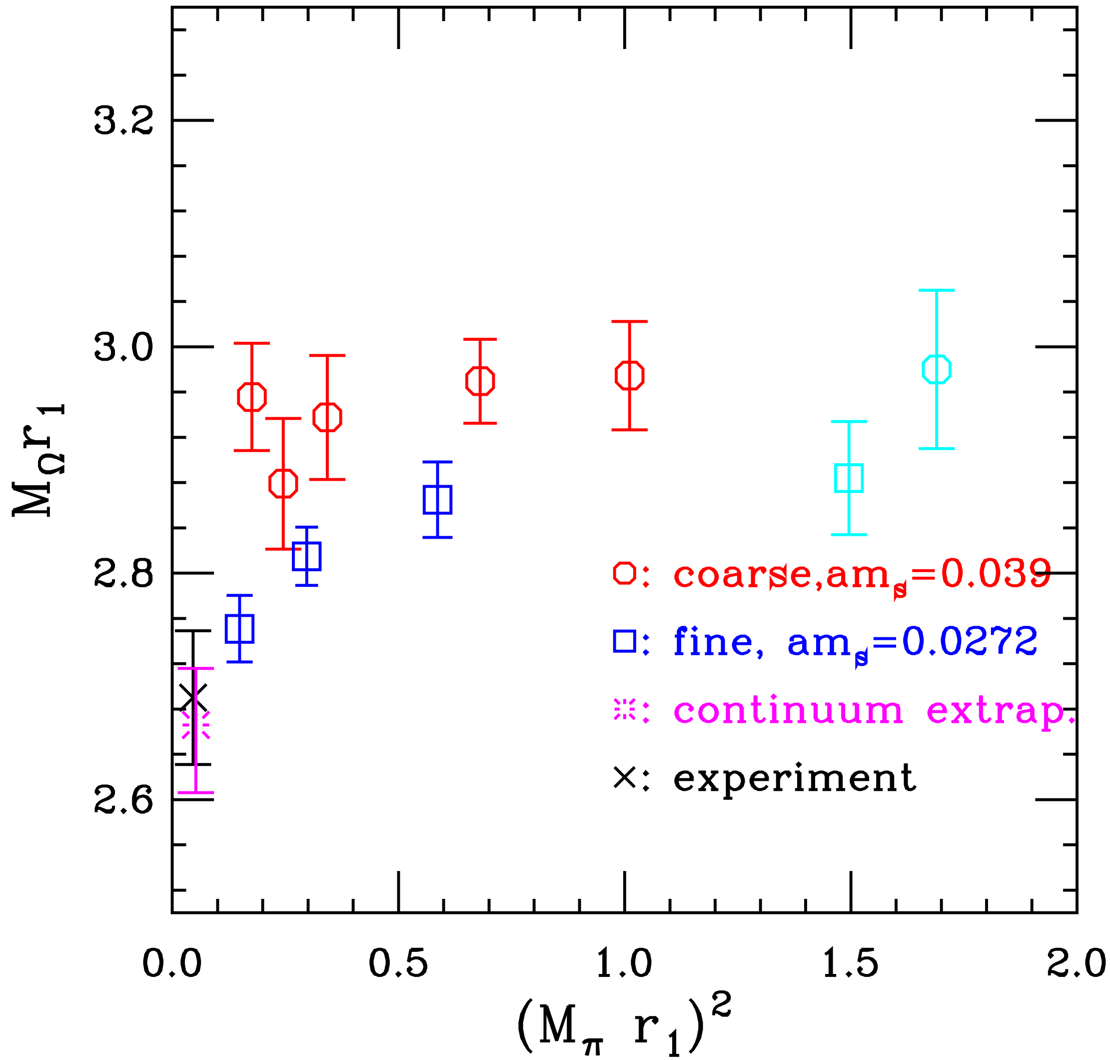


Nucleon masses



The fancy plusses are continuum extrapolations at fixed m_l'/m_s'. The two curves are two different continumm chiral extrapolations (with two parameters each).

Ω^- baryon



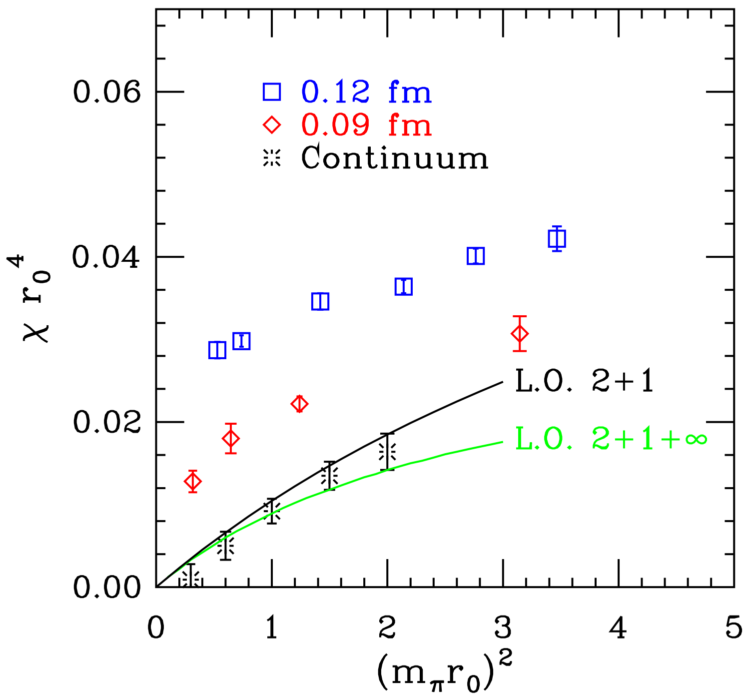
Topology

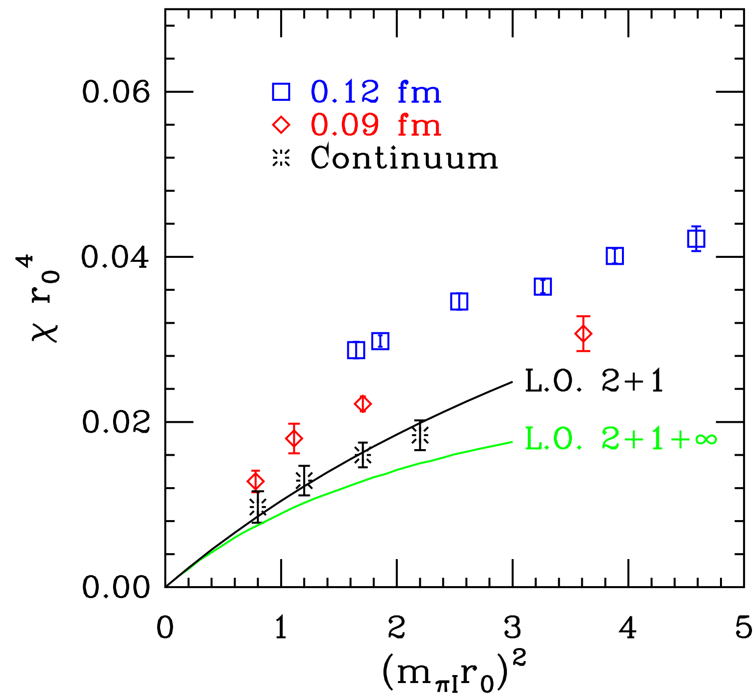
Behavior of topological susceptibility at small quark mass depends on number of flavors. Thus it provides a test of the "sqrt-trick" for staggered fermion simulations to reduce the number of tastes.
For our $N_f = 2 + 1$ simulations, following Leutwyler & Smilga, Dürr, Lee & Sharpe, Aubin & Bernard, Billeter, DeTar and Osborn (hep-lat/0406032) find:

$$\chi = \frac{f^2 m_{\pi,I}^2/8}{1 + m_{\pi,I}^2/2m_{ss,I}^2 + 3m_{\pi,I}^2/2m_0^2}$$

Identical to continuum result, except that taste singlet meson mass appears, the largest of the pseudoscalar masses from taste symmetry breaking at finite lattice spacing.

Topology

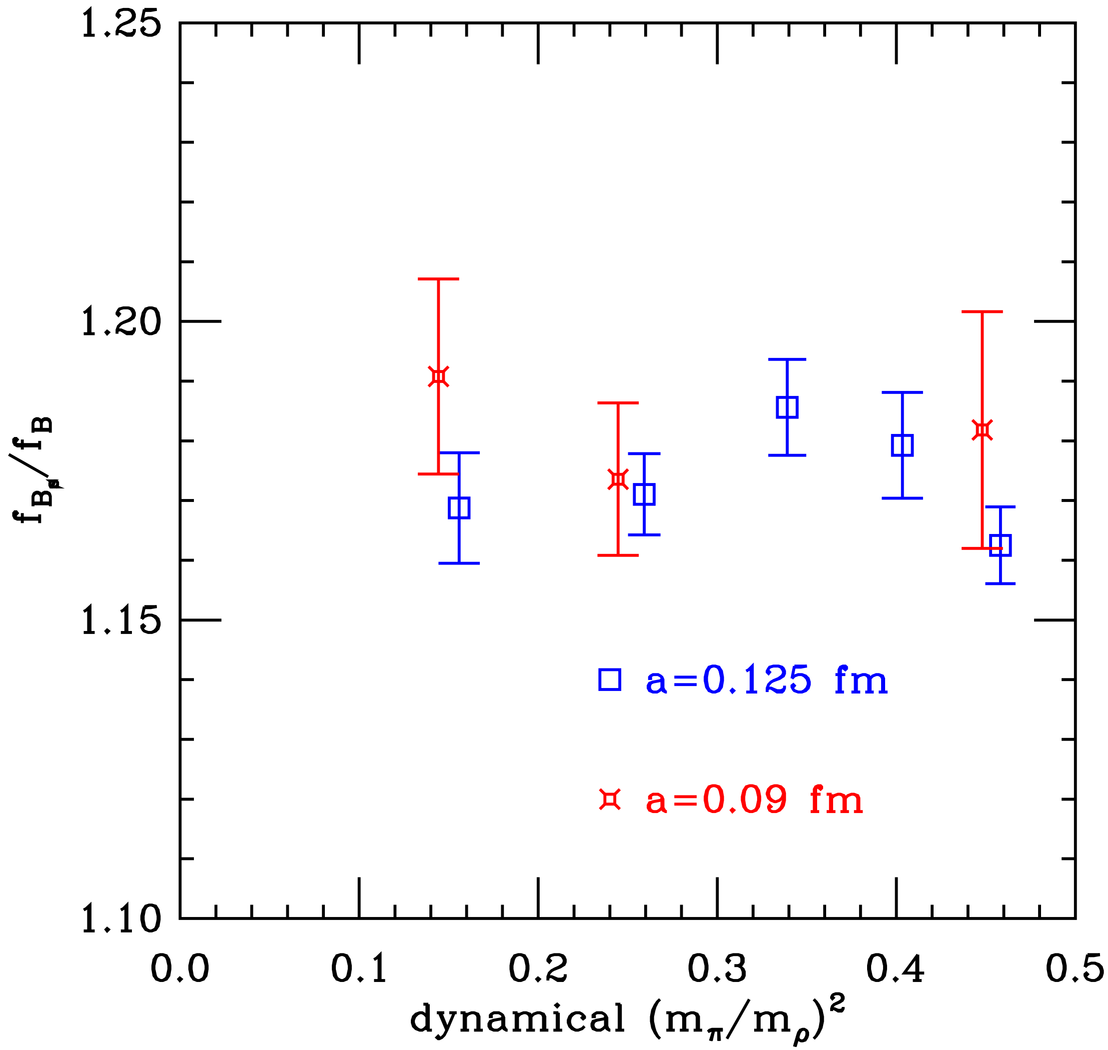




Plotted with taste symmetry breaking, at finite lattice spacing, taken into account, the agreement with theoretical expectations improves.

Heavy-light decay constants

MILC has computed f_B, f_{B_s}, f_D and f_{D_s} with clover valence quarks. Z-factors are not available yet. As an example, we show ratio the f_{B_s}/f_B.



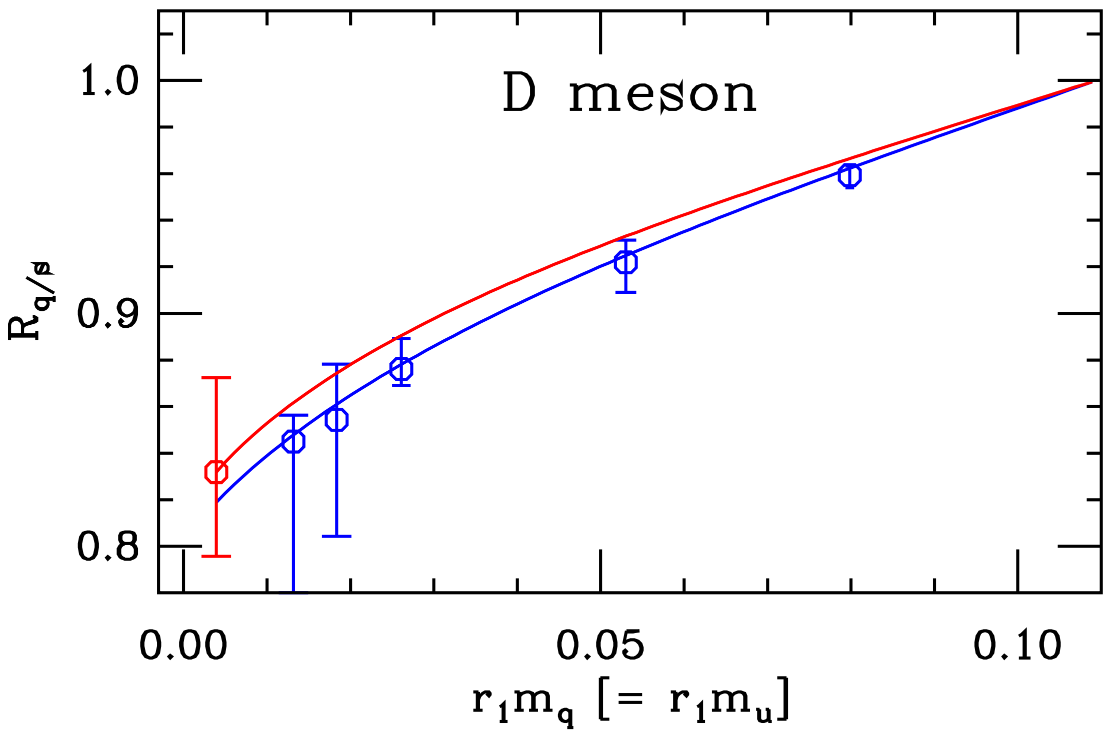
Heavy-light decay constants

With the Fermilab and HPQCD collaborations, we are computing the decay constant also with improved staggered light and heavy clover (Fermilab) quarks. Advantages are:

- Can go to lower light valence quarks
- Use SχPT (Aubin & Bernard) for chiral extrapolation to m_d
- Have Z-factors, written as $Z_V^{Qq} = \rho_V (Z_V^{QQ} Z_V^{qq})^{1/2}$, with Z_V^{QQ} and Z_V^{qq} from charge normalization, non-perturbatively, and $\rho_V \approx 1$ to one-loop.

Heavy-light decay constants

Use of SχPT is illustrated in the fit of $R_{q/s} = f_D\sqrt{m_D}/f_{D_s}\sqrt{m_{D_s}}$.
The red line and extrapolated point are obtained after removing the $\mathcal{O}(a^2)$ effects from the fit.



Heavy-light decay constants

We find:

$$
\begin{aligned}
f_{D_s} &= 263^{+5}_{-9} \pm 24 \text{ MeV} \, , \\
f_D &= 224^{+10}_{-14} \pm 21 \text{ MeV} \, , \\
\frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} &= 1.20 \pm .06 \pm .06 \, .
\end{aligned}
$$

The computations for B-mesons are in progress.

Experimentally measured is only f_{D_s} from leptonic decays — as of Oct 2004, there was only one event for $D^+ \to \mu^+\nu_\mu$ —

$$f_{D_s^+} = 266 \pm 32 \text{ MeV} \, .$$

Heavy-light decay constants

We find:

$$
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\frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} &= 1.20 \pm .06 \pm .06 \ .
\end{aligned}
$$

The computations for B-mesons are in progress.

Experimentally measured, from leptonic decays
— Cleo-c now has 8 events for $D^+ \to \mu^+\nu_\mu$
(hep-ex/0411050, PRD70 (2004) 112004) —

$$
\begin{aligned}
f_{D_s} &= 266 \pm 32 \text{ MeV} \ , \\
f_D &= 202 \pm 41 \pm 17 \text{ MeV} \ .
\end{aligned}
$$

Semileptonic B/D decays

With the Fermilab and HPQCD collaborations, we are computing also form factors for semileptonic $D- > \pi/K$ and $B- > \pi/D$ decays. The heavy-to-light decay amplitudes are parametrized as

$$\langle P|V^\mu|H\rangle = f_+(q^2)(p_H + p_P - \Delta)^\mu + f_0(q^2)\Delta^\mu \ ,$$

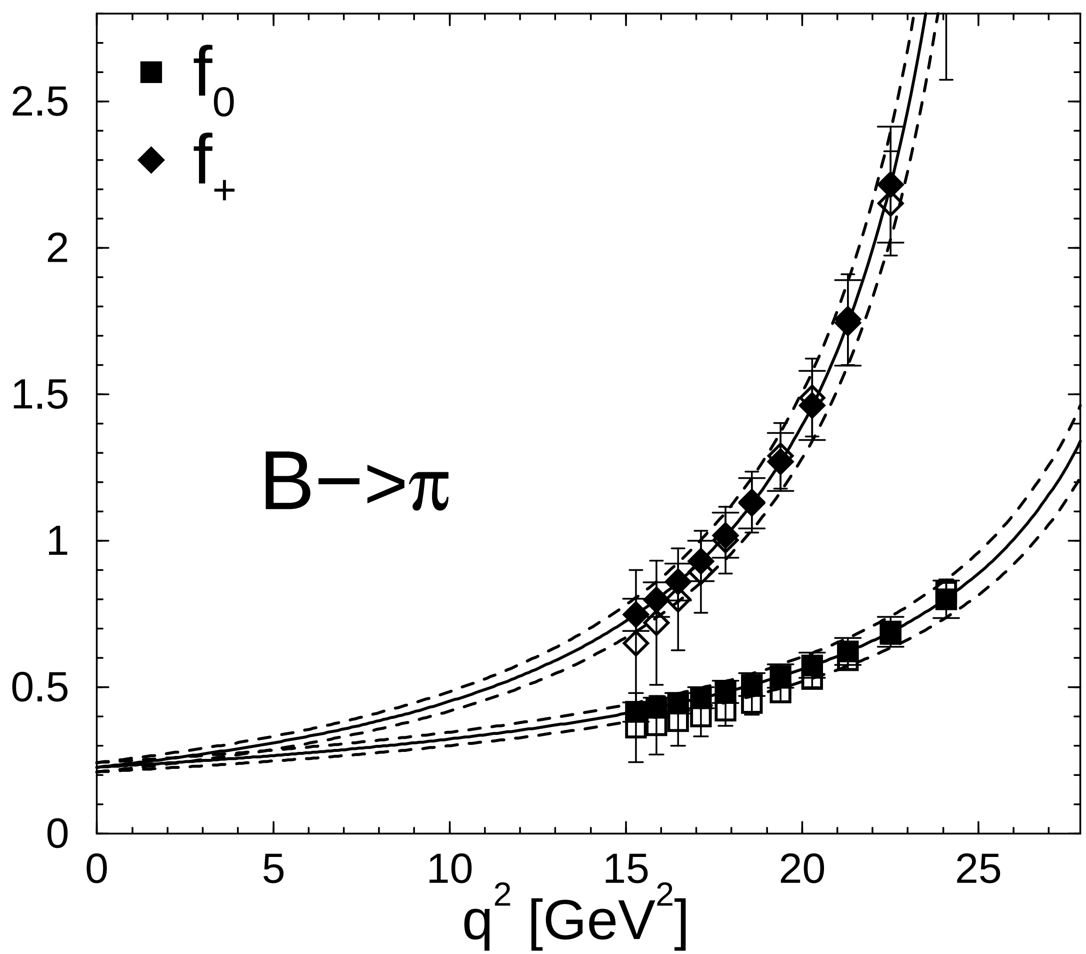
where $\Delta^\mu = (m_H^2 - m_P^2)q^\mu/q^2$.
The differential decay rate $d\Gamma/dq^2$ is proportional to $|V_{\rm CKM}|^2|f_+(q^2)|^2$. Knowing $f_+(q^2)$ allows us to extract CKM matrix elements from experiment. We find:

$$\begin{aligned} |V_{ub}| &= 3.0(4)(6)\times 10^{-3} \ , \ \ |V_{cd}| = 0.24(3)(2) \ , \\ |V_{cs}| &= 0.97(10)(2) \ , \ \ |V_{cb}| = 3.8(1)(6)\times 10^{-2} \ , \end{aligned}$$

with $\quad (|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(10)(2) \ .$

Semileptonic B/D decays

As an example we show the $B \to \pi$ form factors f_0 and f_+.



Summary and Outlook

Simulations at two lattice spacings and several sea quark masses in full 2+1 flavor QCD lead to precision results in the pseudoscalar sector, including decay constants, V_{us} and quark masses.

Many other "gold-plated" observables also show good agreement with experiment.

The configurations are used for predictions for heavy-light meson decay constants and semileptonic form factors.

Improvements will include:

- Simulations with a smaller strange sea quark mass (in progress)
- More statistics for the lightest sea quark mass (in progress)
- A 3rd, smaller lattice spacing: $a \sim 0.06$ fm (planned with SciDAC resources and collaboration with UKQCD)

Spectrum summary