Heavy-to-light form factors in SCET

Modern Challenges for Lattice Field Theory
KITP, UCSB
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Based on:

hep-ph/0404217, hep-ph/0408344 (ff's)
hep-ph/0411073 (B vs. D)
hep-ph/0502xxx ($K^*\gamma$, ..)

Thanks especially to my collaborators
Thomas Becher, Matthias Neubert,
Bjorn Lange, Seung Lee,

Outline

- modifications to HQET at large recoil
- SCET in a nutshell (in a Feynman diagram)
- form factors and symmetry relations
- the BK parameterization
- heavy-quark symmetry in SCET and B vs. D

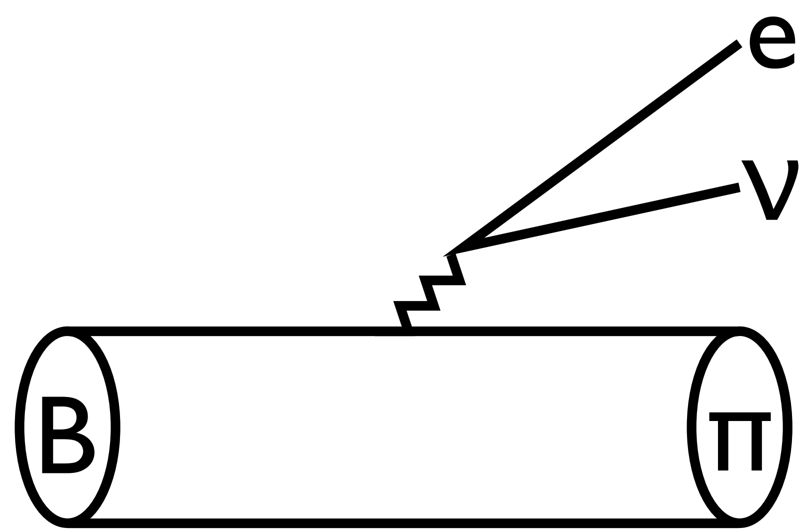
Why this should be of interest to lattice field theory

- better understanding of the underlying symmetries and relevant degrees of freedom
- constraints and relations for form factor parameters, extrapolations
- can apply existing lattice data (e.g. form factors) to NECGP quantities (e.g. $B \to \pi\pi$)

Not Even Close to Gold-Plated

Limits of HQET

Consider semileptonic B→π decay (the "hydrogen atom problem" for B decays to light hadrons)



The most important case is with pion energy $E \sim m_b >> \Lambda$, but consider first the easier case, $E \sim \Lambda$

Hadronic physics described by form factors:

$$\langle \pi(p')|\bar{q}\gamma^\mu b|\bar{B}(p)\rangle \equiv$$
$$F_+(q^2)\left(p^\mu + p'^\mu - \frac{m_B^2 - m_\pi^2}{q^2}q^\mu\right) + F_0(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

At E~ Λ ,

$$\begin{aligned} p^\mu &\equiv m_B v^\mu \\ p'^\mu &\equiv m_\pi v'^\mu = \mathcal{O}(\Lambda) \\ q^2 &= m_B^2 - 2Em_B + m_\pi^2 = m_B^2 + \mathcal{O}(m_B\Lambda) \end{aligned}$$

With this power counting, find e.g. heavy-quark scaling laws:

$$\frac{F_+^{B\to\pi}(E)}{F_+^{D\to\pi}(E)} = \sqrt{\frac{m_B}{m_D}}, \quad \frac{F_0^{B\to\pi}(E)}{F_0^{D\to\pi}(E)} = \sqrt{\frac{m_D}{m_B}}$$

When are these scaling laws valid?

In general, at $q^2=0$ for heavy meson H, light meson M:

$$(v\cdot v')_{\max} = \frac{E_{\max}}{m_M} = \frac{\frac{m_H^2+m_M^2}{2}}{\sqrt{m_H^2 m_M^2}}$$

For example:

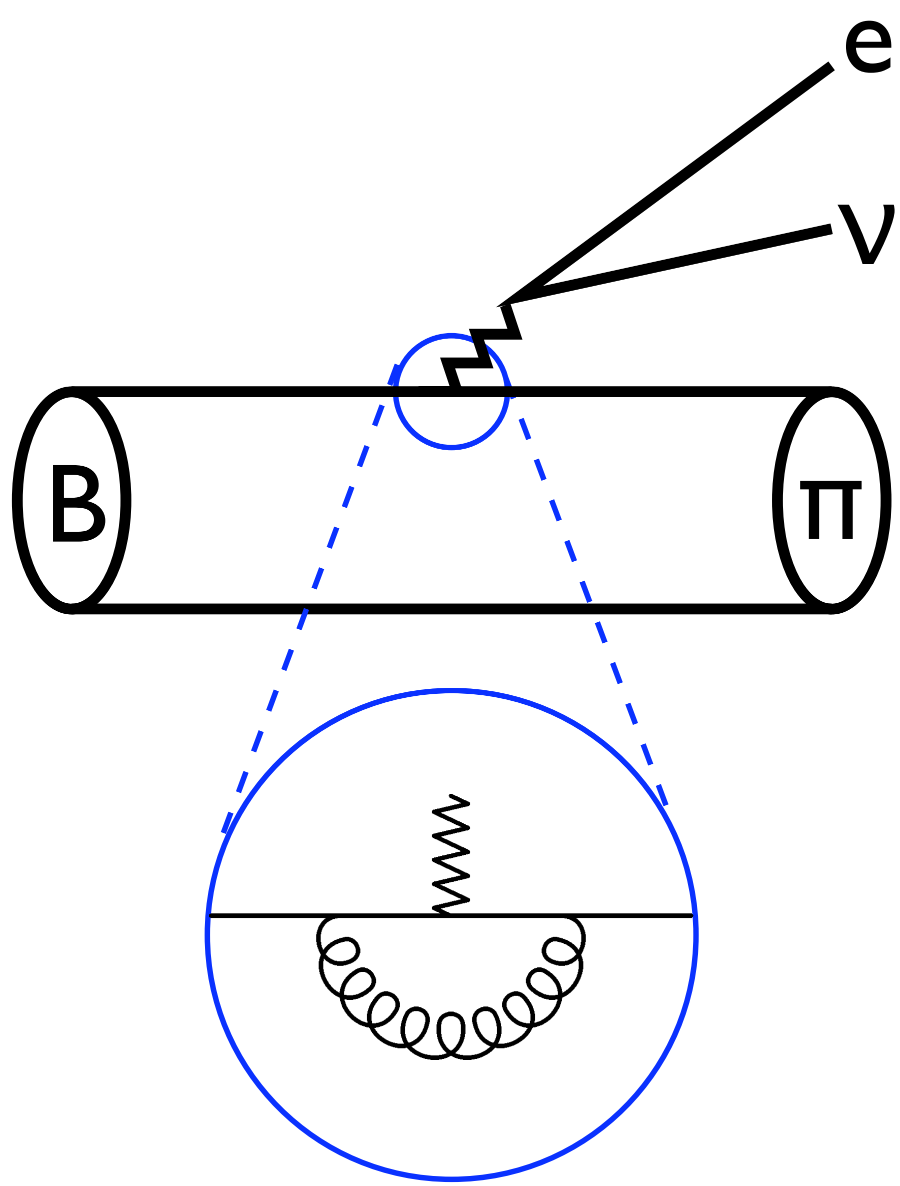
$$B\to D: \quad (v\cdot v')_{\max}\sim 1.6$$
$$D\to \pi: \quad (v\cdot v')_{\max}\sim 6.5$$
$$B\to \pi: \quad (v\cdot v')_{\max}\sim 19$$

Compare the heavy-quark expansion parameter:

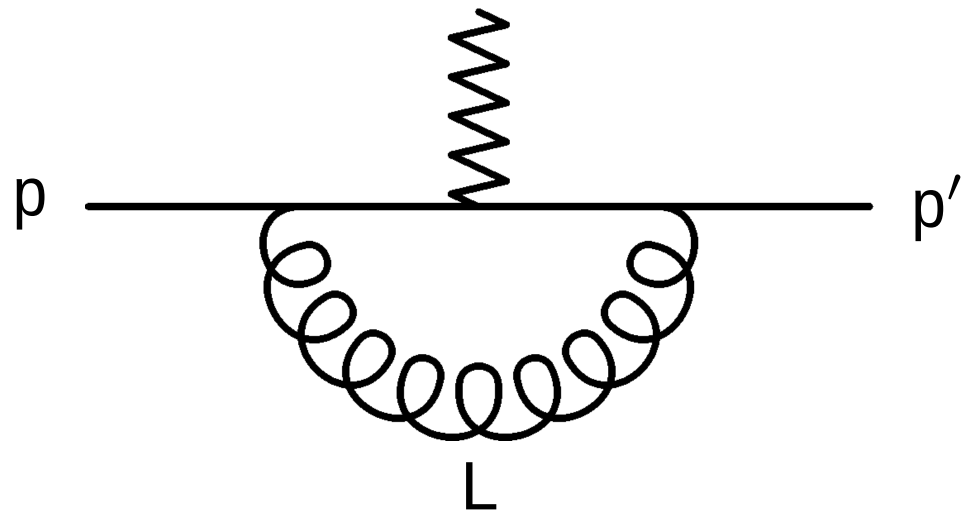
$$\frac{m_c}{\Lambda}\sim 3-4\,, \quad \frac{m_b}{\Lambda}\sim 10$$

⇒ Usual HQET power counting OK (but not obvious) for B→D, but for D→π and B→π, expect that the new scale $E\Lambda=\Lambda^2 v\cdot v'$ will give rise to new effects

SCET (≈ moving HQET)



Analyze momentum regions at small and large recoil



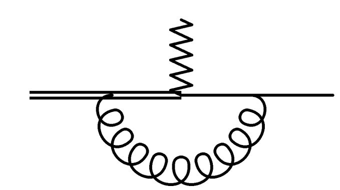
$$p = m_b v + k\,, \quad k \sim \Lambda v$$

$$\begin{aligned} p^2 &\sim m_b^2 \\ p'^2 &\sim \Lambda^2 \\ p\cdot p' &\sim m_b E \sim ? \end{aligned}$$

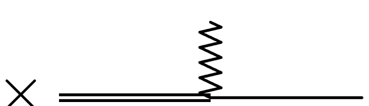
$$\gamma^\alpha \frac{\not L + \not p'}{(L+p')^2} \gamma^\mu \frac{\not L + \not p + m_b}{(L+p)^2 - m_b^2} \gamma_\alpha \frac{1}{L^2}$$

Case 1: small recoil (E~ Λ)

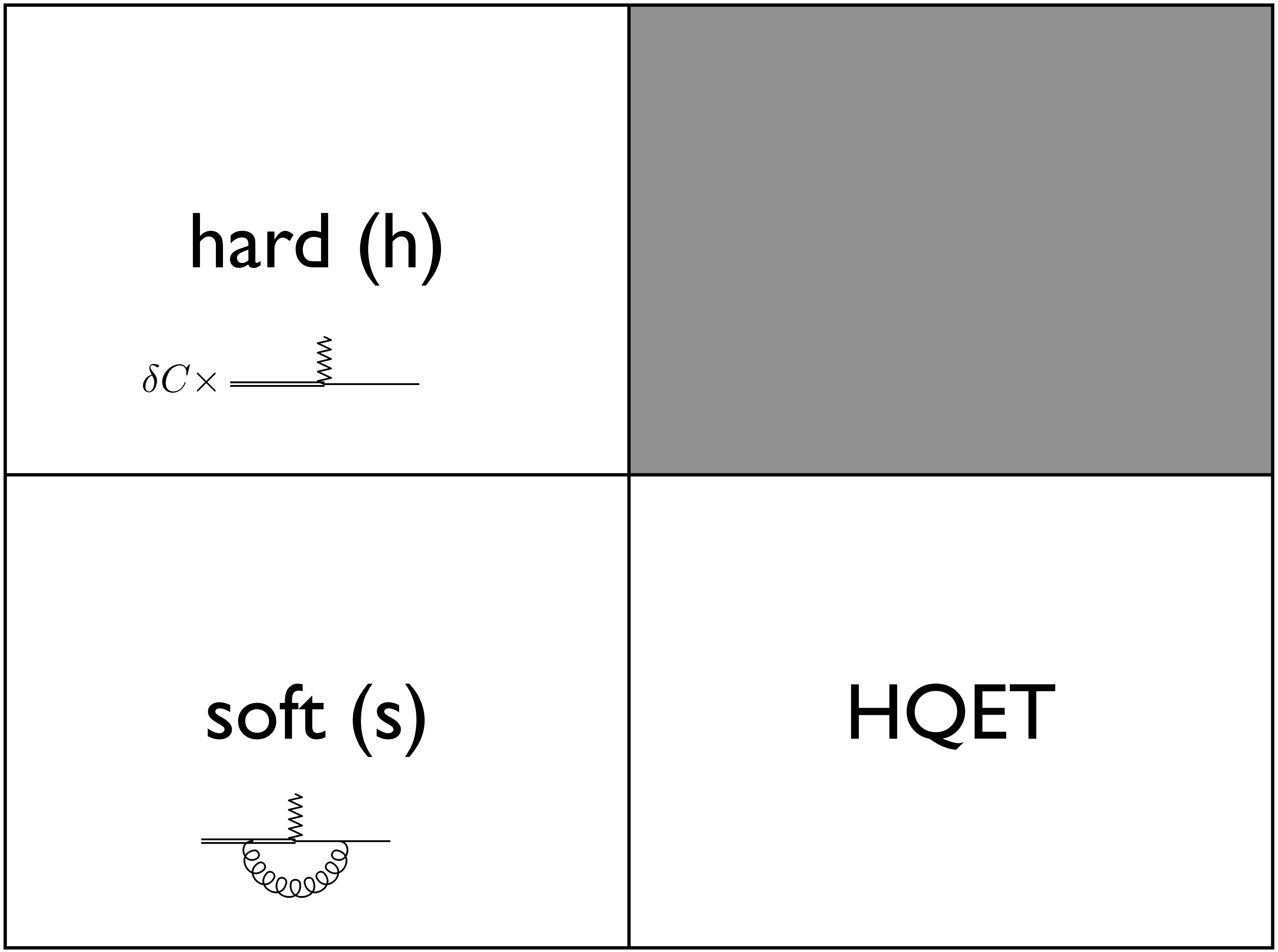
L~ Λ : soft (s) $\quad \gamma^\alpha \frac{\not L + \not p'}{(L+p')^2} \gamma^\mu \frac{1}{v\cdot(L+k)} \frac{1+\not v}{2} \gamma_\alpha \frac{1}{L^2}$

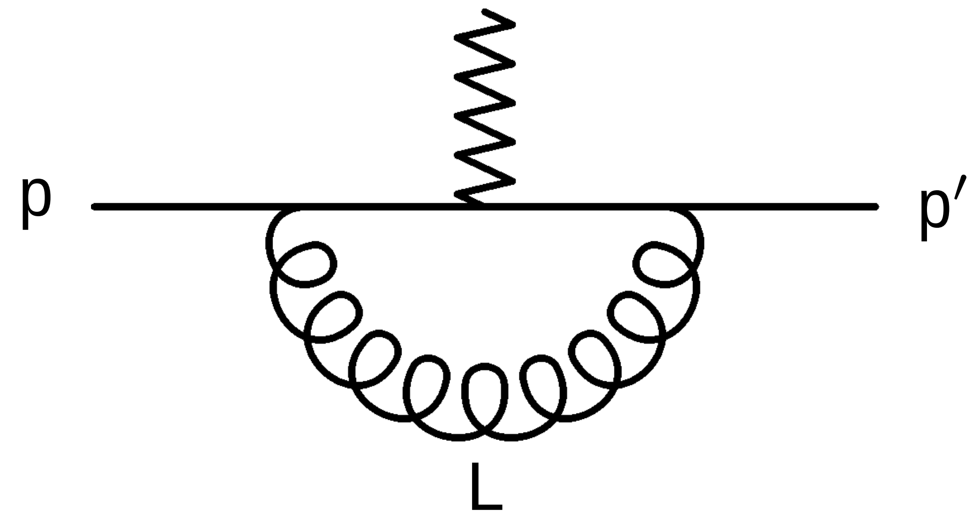


L~m$_b$: hard (h) $\quad \gamma^\alpha \frac{\not L}{L^2} \gamma^\mu \frac{\not L + m_b(1+\not v)}{L^2 + 2 m_b v\cdot L} \gamma_\alpha \frac{1}{L^2}$ $\quad \delta C \times$



$$\Rightarrow \quad \bar q \gamma^\mu b \to \sum_i C_i(m_b)\, \bar q_s \Gamma_i^\mu h \qquad \left(\frac{1+\not v}{2} h = h \right)$$


hard (h)
$\delta C \times$
soft (s)
HQET



$$n^\mu\,(\bar{n}^\mu) = (1,0,0,\pm 1) \quad \lambda = \frac{\Lambda}{m_b} \ll 1$$

$$P^\mu = n\cdot P\,\frac{\bar{n}^\mu}{2} + \bar{n}\cdot P\,\frac{n^\mu}{2} + P_\perp^\mu$$
$$\leftrightarrow\ (P_+, P_-, P_\perp)$$

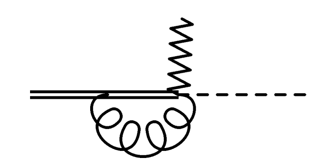
$$\gamma^\alpha \frac{\not{L}+\not{p}'}{(L+p')^2}\gamma^\mu \frac{\not{L}+\not{p}+m_b}{(L+p)^2-m_b^2}\gamma_\alpha \frac{1}{L^2}$$

$$p - m_b v = k \quad \sim \quad m_b(\lambda,\lambda,\lambda)$$
$$p' \quad \sim \quad m_b(\lambda^2,1,\lambda)$$

Case 2: large recoil (E~m_b)

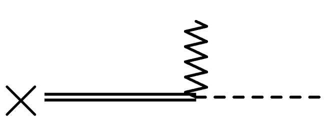
L~(λ,λ,λ): soft (s)

$$\gamma^\alpha \frac{1}{n\cdot L}\frac{\not{n}}{2}\gamma^\mu \frac{1}{v\cdot(L+k)}\frac{1+\not{v}}{2}\gamma_\alpha \frac{1}{L^2}$$



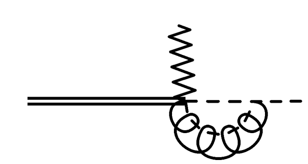
L~(1,1,1): hard (h)

$$\gamma^\alpha \frac{\not{L}+\bar{n}\cdot p'\,\frac{\not{n}}{2}}{L^2+\bar{n}\cdot p'\,n\cdot L}\gamma^\mu \frac{\not{L}+m_b(1+\not{v})}{L^2+2m_b v\cdot L}\gamma_\alpha \frac{1}{L^2}$$

$\delta C\times$ 

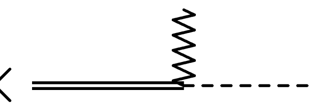
L~(λ^2,1,λ): collinear (c)

$$\gamma^\alpha \frac{\bar{n}\cdot(L+p')\,\frac{\not{n}}{2}}{(L+p')^2}\gamma^\mu \left[\frac{1+\not{v}}{n\cdot v\,\bar{n}\cdot L} + \frac{\not{n}}{2n\cdot v\, m_b}\right]\gamma_\alpha \frac{1}{L^2}$$

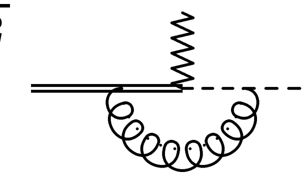


L~$(\lambda, 1, \lambda^{1/2})$): $$\gamma^\alpha \frac{\bar{n}\cdot(L+p')\frac{\not{n}}{2}}{L^2+\bar{n}\cdot p'\, n\cdot L}\gamma^\mu\left[\frac{1+\not{v}}{n\cdot v\,\bar{n}\cdot L}+\frac{\not{n}}{2n\cdot v\, m_b}\right]\gamma_\alpha\frac{1}{L^2}$$

hard-collinear (hc)

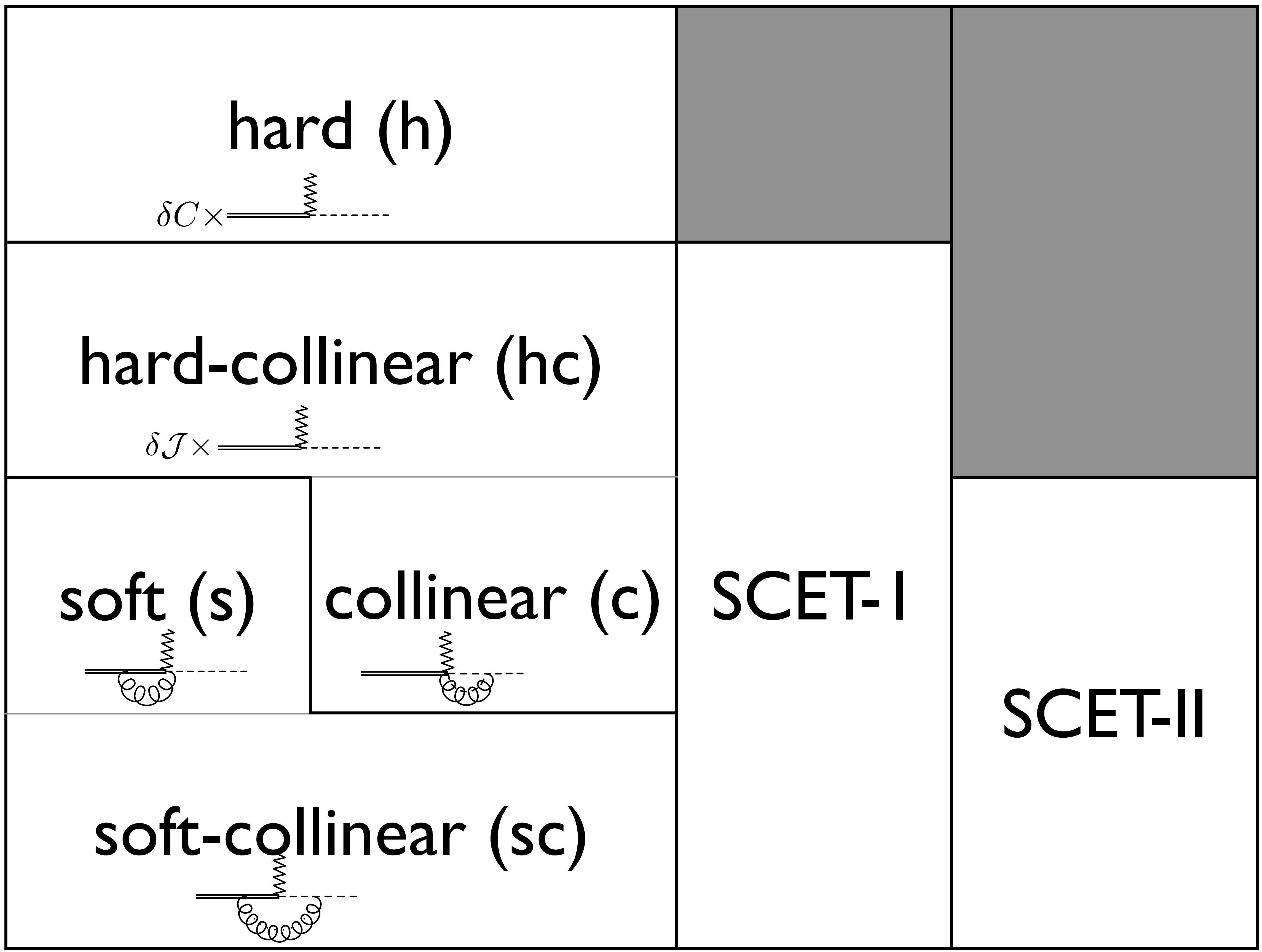
$\delta\mathcal{J}\times$ 

L~$(\lambda^2, 1, \lambda^{3/2})$: $$\gamma^\alpha \frac{\bar{n}\cdot p'\frac{\not{n}}{2}}{p'^2+\bar{n}\cdot p'\, n\cdot L}\gamma^\mu\frac{1}{v\cdot k+\frac{1}{2}n\cdot v\,\bar{n}\cdot L}\frac{1+\not{v}}{2}\gamma_\alpha\frac{1}{L^2}$$



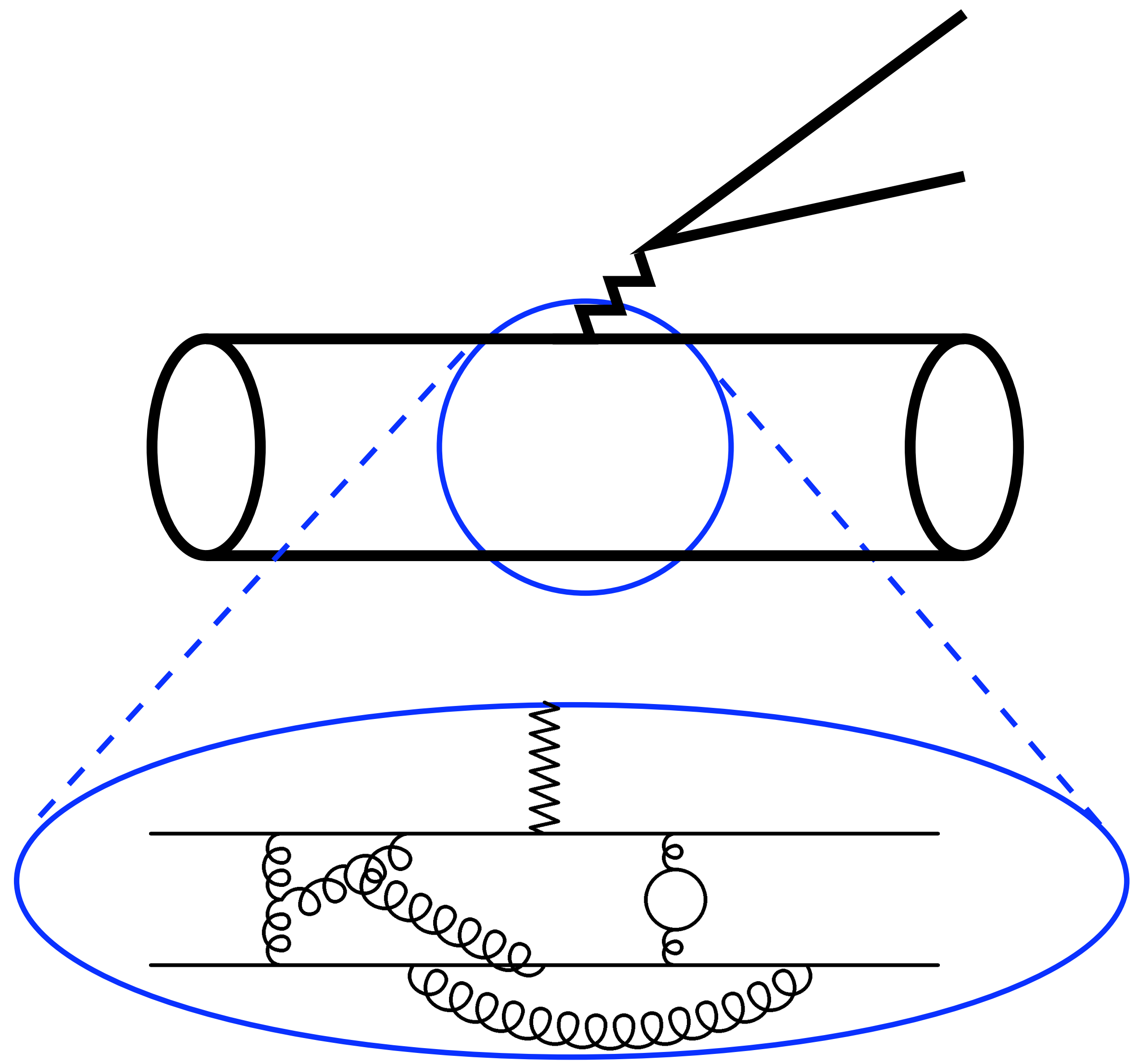
soft-collinear (sc)

$$\Rightarrow \quad \bar{q}\gamma^\mu b \to \sum_i C_i(m_b,E)\bar{\mathcal{X}}_c\Gamma_i^\mu\mathcal{H}_s+\ldots \left(\begin{array}{rcl} \frac{\not{n}\bar{\not{n}}}{4}\mathcal{X}_c &=& \mathcal{X}_c \\ \frac{1+\not{v}}{2}\mathcal{H}_s &=& \mathcal{H}_s \end{array}\right)$$



hard (h)
$\delta C \times$
hard-collinear (hc)
$\delta \mathcal{J} \times$
soft (s)
collinear (c)
soft-collinear (sc)
SCET-I
SCET-II

Can do the same for:



(left as homework)

- At arbitrary loop order, the same regions appear

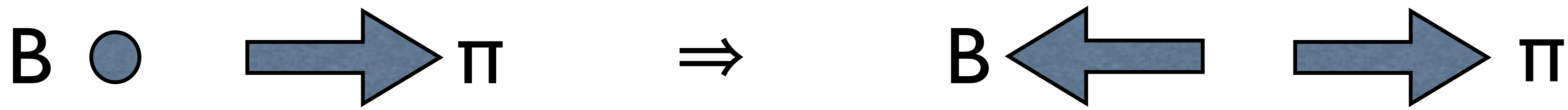
- Can organize the analysis by introducing a field (=particle) for each momentum region

- Integrate out the heavy particles, find the Lagrangian for the light particles

- Especially elegant when implemented in dimensional regularization ⇒ "strategy of regions"

Not easy, but a well-defined problem

Can make the description more symmetric:



Boost to a frame where the light degrees of freedom carry the same energy:

$$k \sim (\Lambda, \Lambda, \Lambda) \quad \rightarrow \quad (e^{\eta}\Lambda, e^{-\eta}\Lambda, \Lambda)$$
$$\sim \quad \left(E_{cm}, \frac{\Lambda^2}{E_{cm}}, \Lambda\right)$$
$$\sim \quad E_{cm}\left(1, \lambda_{cm}^2, \lambda_{cm}\right)$$

$$p' \sim (\frac{\Lambda^2}{m_b}, m_b, \Lambda) \quad \rightarrow \quad (e^{\eta}\frac{\Lambda^2}{m_b}, e^{-\eta}m_b, \Lambda)$$
$$\sim \quad \left(\frac{\Lambda^2}{E_{cm}}, E_{cm}, \Lambda\right)$$
$$\sim \quad E_{cm}(\lambda_{cm}^2, 1, \lambda_{cm})$$

Lorentz invariance: $\quad p' \cdot k \sim m_b\Lambda \sim E_{cm}^2$

$$\boldsymbol{e^{\eta} \sim \frac{m_b}{E_{cm}} \sim \sqrt{\frac{m_b}{\Lambda}} = \frac{1}{\sqrt{\lambda}}, \quad \lambda_{cm} \equiv \frac{\Lambda}{E_{cm}} \sim \sqrt{\frac{\Lambda}{m_b}} \sim \sqrt{\lambda}}$$

Soft mode in the boosted frame:

$$s: \; m_b(\lambda,\lambda,\lambda) \to m_b(\lambda^{1/2},\lambda^{3/2},\lambda)$$

$$= (m_b\Lambda)^{1/2}(1,\lambda,\lambda^{1/2}) = E_{cm}(1,\lambda^2{}_{cm},\lambda_{cm})$$

Similarly for other modes:

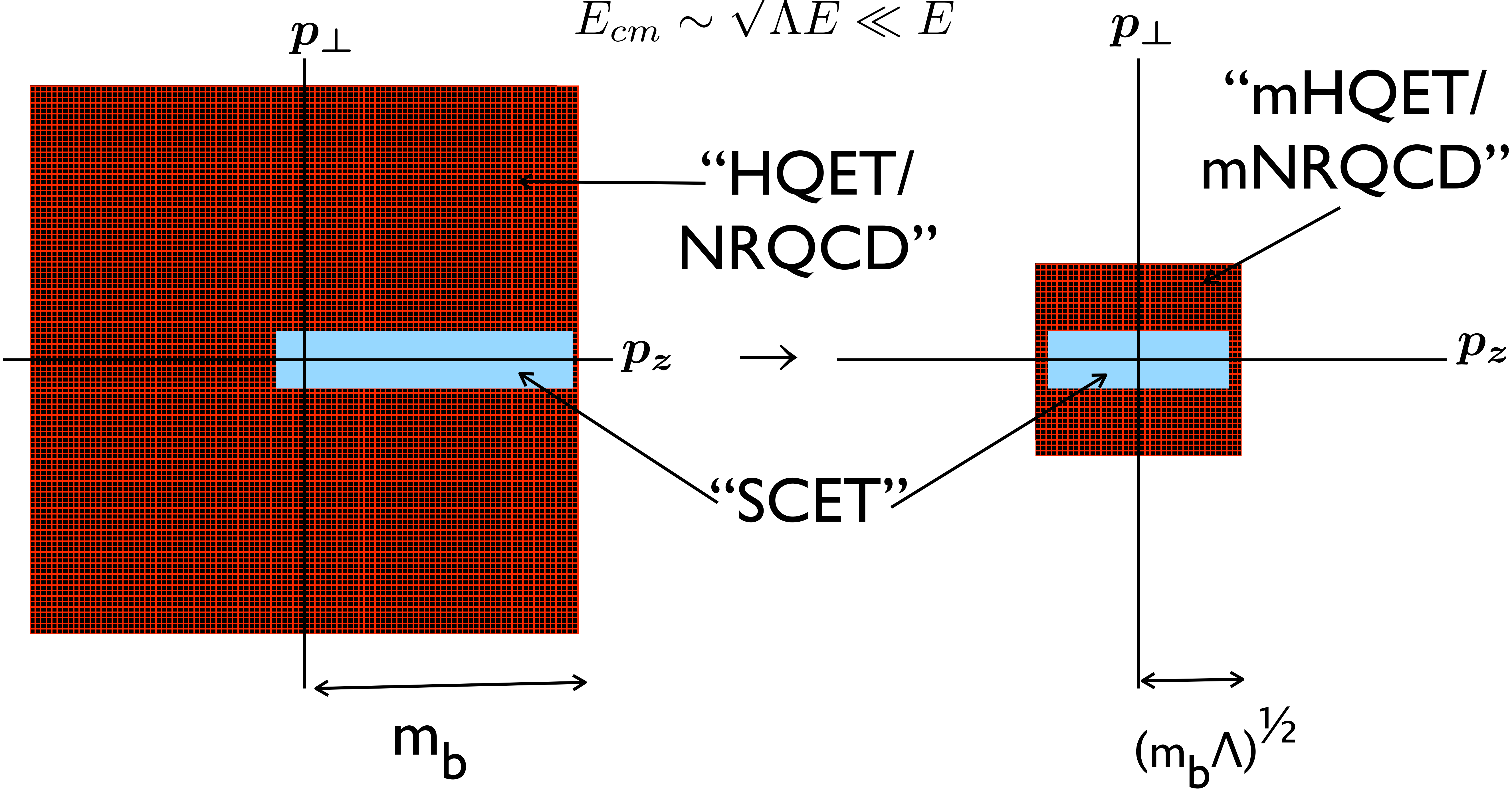
hc	$m_b(\lambda,1,\lambda^{1/2})$	$\to$	$E_{cm}(1,1,1)$	h
c	$m_b(\lambda^2,1,\lambda)$	$\to$	$E_{cm}(\lambda^2{}_{cm},1,\lambda_{cm})$	c
s	$m_b(\lambda,\lambda,\lambda)$	$\to$	$E_{cm}(1,\lambda^2{}_{cm},\lambda_{cm})$	$\bar{c}$
sc	$m_b(\lambda^2,\lambda,\lambda^{3/2})$	$\to$	$E_{cm}(\lambda^2{}_{cm},\lambda^2{}_{cm},\lambda^2{}_{cm})$	us

$$\mathrm{SC(hc,sc)ET} \to \bar{\mathrm{C}}\mathrm{C(h,us)ET}$$

- SCET (or $\bar{C}$CET) is the effective field theory for hard exclusive processes (e.g form factors: BD, $B\pi$, $D\pi$, $\pi\pi$, $\rho\pi$, …)

Descriptions are of course equivalent in a Lorentz-invariant field theory, but note that

$$E_{cm} \sim \sqrt{\Lambda E} \ll E$$



⇒ Much more efficient lattice simulation

Form factors in SCET

Gauge-invariant, boost-invariant building blocks:

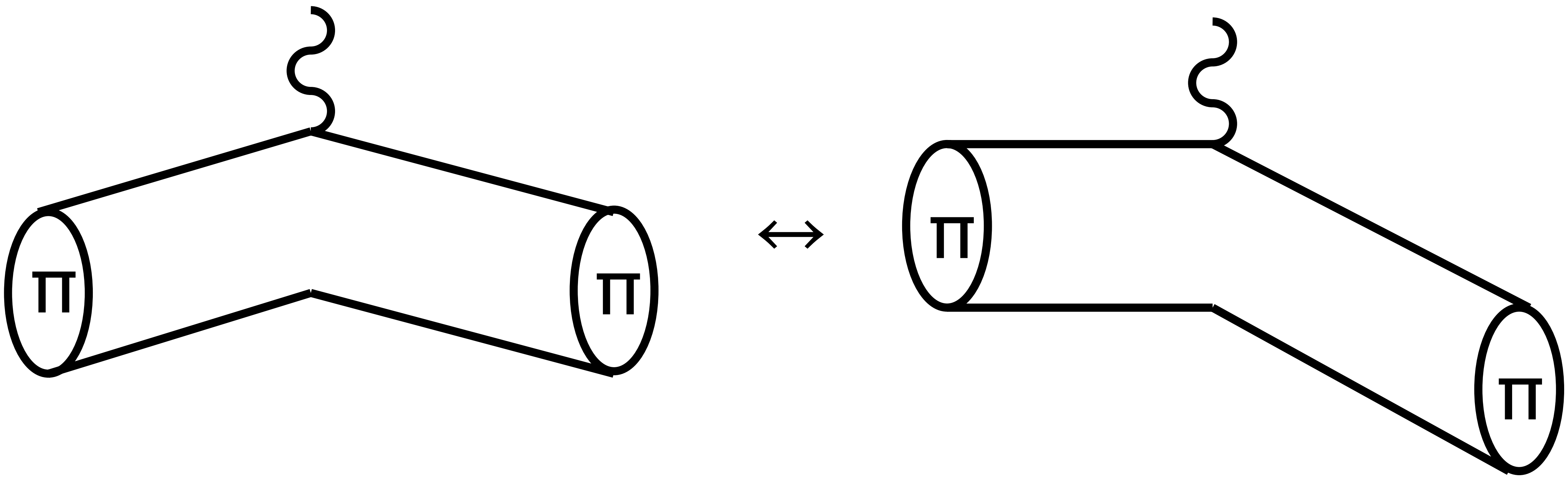
	d	$[\lambda]$
$\frac{1}{\bar{n}\cdot\partial_c}\,\bar{\mathcal{X}}_c\frac{\bar{\not{n}}}{2}\Gamma'\mathcal{X}_c$	2	2
$\frac{1}{n\cdot\partial_s}\,\bar{\mathcal{Q}}_s\frac{\not{n}}{2}\Gamma'\mathcal{Q}_s$	2	2
$\bar{\mathcal{Q}}_s\Gamma''\mathcal{Q}_s$	3	3
$n\cdot\partial_s\,\bar{\mathcal{Q}}_s\frac{\bar{\not{n}}}{2}\Gamma'\mathcal{Q}_s$	4	4

	d	$[\lambda]$
$g_\perp^{\mu\nu},\,\epsilon_\perp^{\mu\nu}$	0	0
$\partial^\mu_{c\perp},\,\mathcal{A}^\mu_{c\perp},\,\partial^\mu_{s\perp},\,\mathcal{A}^\mu_{s\perp}$	1	1
$n\cdot\partial_s\bar{n}\cdot\partial_s,\,n\cdot\partial_s\bar{n}\cdot\mathcal{A}_s$	2	2
$\bar{n}\cdot\partial_c n\cdot\partial_c,\,\bar{n}\cdot\partial_c\,n\cdot\mathcal{A}_c$	2	2
$\frac{1}{\bar{n}\cdot\partial_c\,n\cdot\partial_s}$	-2	-1

Strategy:

Starting from a (gauge-invariant, boost-invariant) QCD operator of a given dimension, construct the most general SCET operator order by order in λ

The Factorizable case:
The ππ (electromagnetic) form factor
(Like a good form factor should be)



$$\langle \pi(p')|\bar{q}\gamma^\mu q|\pi(k)\rangle = (k+p')^\mu F(Q^2)$$

$$k^\mu \sim (\Lambda, \Lambda, 0)\,, \quad p'^\mu \sim \left(\frac{\Lambda^2}{E}, E, 0\right)\,, \qquad Q^2 = -(k-p')^2 = n\cdot k\,\bar{n}\cdot p'$$

$$\begin{aligned} \langle \bar{q}\gamma_\perp^\mu q\rangle &= 0 \\ \bar{n}\cdot p'\langle \bar{q}\,\not{n}\,q\rangle &= Q^2 F(Q^2) \\ n\cdot k\langle \bar{q}\,\not{\bar{n}}\,q\rangle &= Q^2 F(Q^2) \end{aligned}$$

	d	$[\lambda]$
$\frac{1}{\bar{n}\cdot\partial_c}\,\bar{\mathfrak{X}}_c\frac{\bar{\not{n}}}{2}\Gamma'\mathfrak{X}_c$	2	2
$\frac{1}{n\cdot\partial_s}\,\bar{\mathfrak{Q}}_s\frac{\not{n}}{2}\Gamma'\mathfrak{Q}_s$	2	2
$\bar{\mathfrak{Q}}_s\Gamma''\mathfrak{Q}_s$	3	3
$n\cdot\partial_s\,\bar{\mathfrak{Q}}_s\frac{\bar{\not{n}}}{2}\Gamma'\mathfrak{Q}_s$	4	4

	d	$[\lambda]$
$g_\perp^{\mu\nu},\ \epsilon_\perp^{\mu\nu}$	0	0
$\partial^\mu_{c\perp},\ \mathcal{A}^\mu_{c\perp},\ \partial^\mu_{s\perp},\ \mathcal{A}^\mu_{s\perp}$	1	1
$n\cdot\partial_s\bar{n}\cdot\partial_s,\ n\cdot\partial_s\bar{n}\cdot\mathcal{A}_s$	2	2
$\bar{n}\cdot\partial_c n\cdot\partial_c,\ \bar{n}\cdot\partial_c\,n\cdot\mathcal{A}_c$	2	2
$\frac{1}{\bar{n}\cdot\partial_c\,n\cdot\partial_s}$	-2	-1

Find the representation of:

$$\boldsymbol{\bar{n}\cdot p'\,\bar{q}\,\not{n}\,q \quad (d=4)}$$

See immediately from the table that $[\lambda]_{\text{min}}=4$:

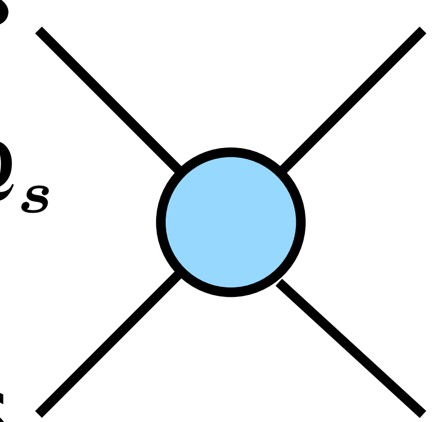
$$\Rightarrow \qquad Q^2F(Q^2)\sim\Lambda^4\frac{1}{\Lambda^2}\sim\Lambda^2\,, \quad \boldsymbol{F(Q^2)\sim\frac{\Lambda^2}{Q^2}}$$

- Dimensional/helicity counting rules are built into the formalism

	d	$[\lambda]$
$\frac{1}{\bar{n}\cdot\partial_c}\,\bar{\mathcal{X}}_c\frac{\not{\bar{n}}}{2}\Gamma'\mathcal{X}_c$	2	2
$\frac{1}{n\cdot\partial_s}\,\bar{\mathcal{Q}}_s\frac{\not{n}}{2}\Gamma'\mathcal{Q}_s$	2	2
$\bar{\mathcal{Q}}_s\Gamma''\mathcal{Q}_s$	3	3
$n\cdot\partial_s\,\bar{\mathcal{Q}}_s\frac{\not{\bar{n}}}{2}\Gamma'\mathcal{Q}_s$	4	4

	d	$[\lambda]$
$g_\perp^{\mu\nu},\ \epsilon_\perp^{\mu\nu}$	0	0
$\partial^\mu_{c\perp},\ \mathcal{A}^\mu_{c\perp},\ \partial^\mu_{s\perp},\ \mathcal{A}^\mu_{s\perp}$	1	1
$n\cdot\partial_s\bar{n}\cdot\partial_s,\ n\cdot\partial_s\bar{n}\cdot\mathcal{A}_s$	2	2
$\bar{n}\cdot\partial_c n\cdot\partial_c,\ \bar{n}\cdot\partial_c\, n\cdot\mathcal{A}_c$	2	2
$\frac{1}{\bar{n}\cdot\partial_c\, n\cdot\partial_s}$	-2	-1

Require mimimal field content of
valence partons: $\bar{\mathcal{X}}_c(\ldots)\mathcal{X}_c\,\bar{\mathcal{Q}}_s(\ldots)\mathcal{Q}_s$

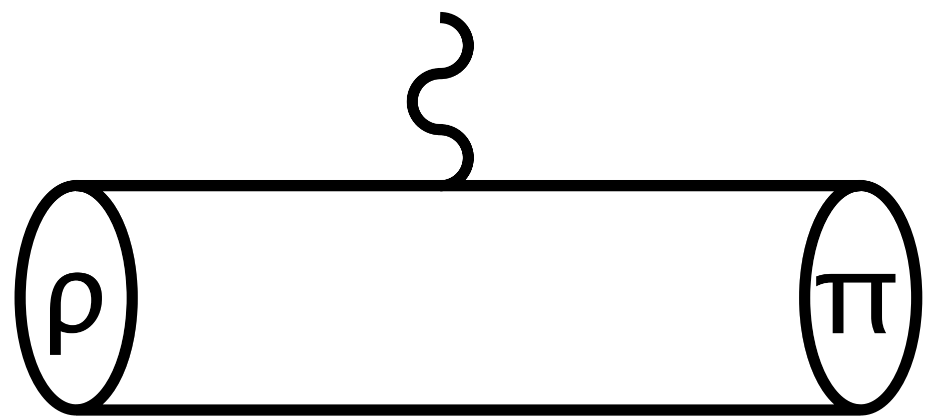


Only possibility at leading power:

$$F(Q^2) = \frac{1}{Q^2}\langle\pi(p')|\ \frac{1}{\bar{n}\cdot\partial_c}\,\bar{\mathcal{X}}_c\frac{\not{\bar{n}}}{2}\Gamma'\mathcal{X}_c\frac{1}{n\cdot\partial_s}\,\bar{\mathcal{Q}}_s\frac{\not{n}}{2}\Gamma'\mathcal{Q}_s\ |\pi(k)\rangle$$

$$= \frac{f_\pi^2}{Q^2}\int_0^1 dx\int_0^1 dy\,\phi_\pi(x)T_H(x,y,\alpha_s)\phi_\pi(y)$$

The Nonfactorizable Case:
The ρπ form factor
(Unsafe at any speed)



$$\langle \pi(p')|\bar{q}\gamma^\mu q|\rho(k,\eta)\rangle = 2i\epsilon^{\mu\nu\rho\sigma}\eta_\nu k_\rho p'_\sigma \frac{F(Q^2)}{m_\pi + m_\rho}$$

$$k^\mu \sim (\Lambda, \Lambda, 0)\,, \quad p'^\mu \sim \left(\frac{\Lambda^2}{E}, E, 0\right)\,, \qquad Q^2 = -(k-p')^2 = n\cdot k\,\bar{n}\cdot p'$$

$$\begin{aligned} \langle \bar{q}\gamma_\perp^\mu q\rangle &= \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\eta_\nu \bar{n}_\rho n_\sigma \frac{Q^2 F(Q^2)}{m_\pi + m_\rho} \\ \langle \bar{q}\,\not{n}\,q\rangle &= 0 \\ \langle \bar{q}\,\not{\bar{n}}\,q\rangle &= 0 \end{aligned}$$

	d	$[\lambda]$
$\frac{1}{\bar{n}\cdot\partial_c}\,\bar{\mathcal{X}}_c\frac{\bar{\not{n}}}{2}\Gamma'\mathcal{X}_c$	2	2
$\frac{1}{n\cdot\partial_s}\,\bar{\mathcal{Q}}_s\frac{\not{n}}{2}\Gamma'\mathcal{Q}_s$	2	2
$\bar{\mathcal{Q}}_s\Gamma''\mathcal{Q}_s$	3	3
$n\cdot\partial_s\,\bar{\mathcal{Q}}_s\frac{\bar{\not{n}}}{2}\Gamma'\mathcal{Q}_s$	4	4

	d	$[\lambda]$
$g_\perp^{\mu\nu},\ \epsilon_\perp^{\mu\nu}$	0	0
$\partial^\mu_{c\perp},\ \mathcal{A}^\mu_{c\perp},\ \partial^\mu_{s\perp},\ \mathcal{A}^\mu_{s\perp}$	1	1
$n\cdot\partial_s\bar{n}\cdot\partial_s,\ n\cdot\partial_s\bar{n}\cdot\mathcal{A}_s$	2	2
$\bar{n}\cdot\partial_c n\cdot\partial_c,\ \bar{n}\cdot\partial_c\,n\cdot\mathcal{A}_c$	2	2
$\frac{1}{\bar{n}\cdot\partial_c\,n\cdot\partial_s}$	-2	-1

Find the representation of:

$$\bar{q}\,\gamma_\perp^\mu\,q \quad (d=3)$$

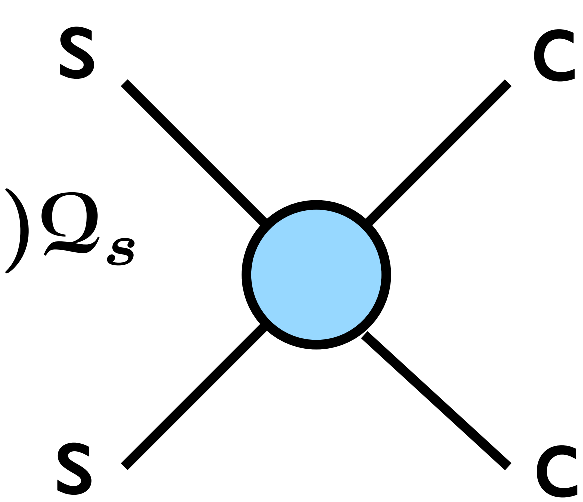
See immediately from the table that $[\lambda]_{\text{min}} \geq 4$:

$$\Rightarrow \qquad Q^2\frac{F(Q^2)}{\Lambda} \sim \frac{\Lambda^5}{Q^2}\frac{1}{\Lambda^2} \sim \frac{\Lambda^3}{Q^2}\,, \quad \boldsymbol{F(Q^2) \sim \frac{\Lambda^4}{Q^4}}$$

	d	$[\lambda]$
$\frac{1}{\bar{n}\cdot\partial_c}\,\bar{\mathcal{X}}_c\frac{\bar{\not{n}}}{2}\Gamma'\mathcal{X}_c$	2	2
$\frac{1}{n\cdot\partial_s}\,\bar{\mathcal{Q}}_s\frac{\not{n}}{2}\Gamma'\mathcal{Q}_s$	2	2
$\bar{\mathcal{Q}}_s\Gamma''\mathcal{Q}_s$	3	3
$n\cdot\partial_s\,\bar{\mathcal{Q}}_s\frac{\bar{\not{n}}}{2}\Gamma'\mathcal{Q}_s$	4	4

	d	$[\lambda]$
$g_\perp^{\mu\nu},\ \epsilon_\perp^{\mu\nu}$	0	0
$\partial_{c\perp}^\mu,\ \mathcal{A}_{c\perp}^\mu,\ \partial_{s\perp}^\mu,\ \mathcal{A}_{s\perp}^\mu$	1	1
$n\cdot\partial_s\bar{n}\cdot\partial_s,\ n\cdot\partial_s\bar{n}\cdot\mathcal{A}_s$	2	2
$\bar{n}\cdot\partial_c n\cdot\partial_c,\ \bar{n}\cdot\partial_c\, n\cdot\mathcal{A}_c$	2	2
$\frac{1}{\bar{n}\cdot\partial_c\, n\cdot\partial_s}$	-2	-1

Require mimimal field content of valence partons: $\bar{\mathcal{X}}_c(\ldots)\mathcal{X}_c\,\bar{\mathcal{Q}}_s(\ldots)\mathcal{Q}_s$



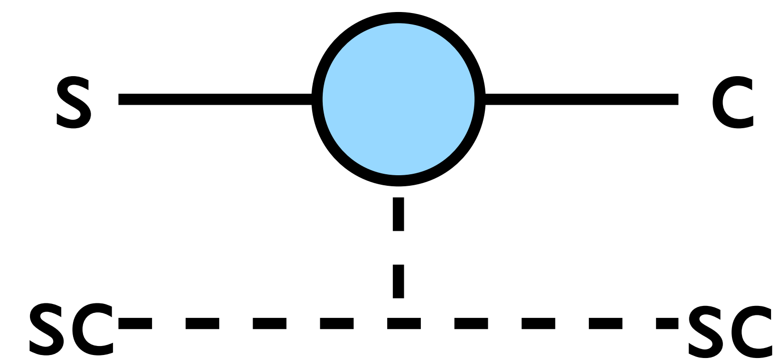
Many possibilities at leading power:

$$\frac{1}{\bar{n}\cdot\partial_c\,n\cdot\partial_s}\left[\frac{1}{\bar{n}\cdot\partial_c}\bar{\mathcal{X}}_c\frac{\bar{\not{n}}}{2}\Gamma'\mathcal{X}_c\right]\left[\frac{1}{n\cdot\partial_s}\,\bar{\mathcal{Q}}_s\frac{\not{n}}{2}\Gamma'\mathcal{Q}_s\right]\{\partial_{c\perp}^\mu,\ \mathcal{A}_{c\perp}^\mu,\ \partial_{s\perp}^\mu,\ \mathcal{A}_{s\perp}^\mu\}$$

$$\frac{1}{\bar{n}\cdot\partial_c\,n\cdot\partial_s}\left[\frac{1}{\bar{n}\cdot\partial_c}\bar{\mathcal{X}}_c\frac{\bar{\not{n}}}{2}\Gamma'\mathcal{X}_c\right]\left[\bar{\mathcal{Q}}_s\Gamma''\mathcal{Q}_s\right]$$

	d	$[\lambda]$
$\mathcal{X}_c$	$\frac{3}{2}$	1
$\frac{1}{\bar{n}\cdot\partial_c}\frac{\bar{\not{n}}}{2}$	-1	0
$\mathcal{Q}_s$	$\frac{3}{2}$	$\frac{3}{2}$
$\frac{1}{n\cdot\partial_s}\frac{\not{n}}{2}$	-1	-1
Γ''	0	0

Consider also endpoint configurations: $\bar{\mathcal{X}}_c(\ldots)\mathcal{Q}_s$



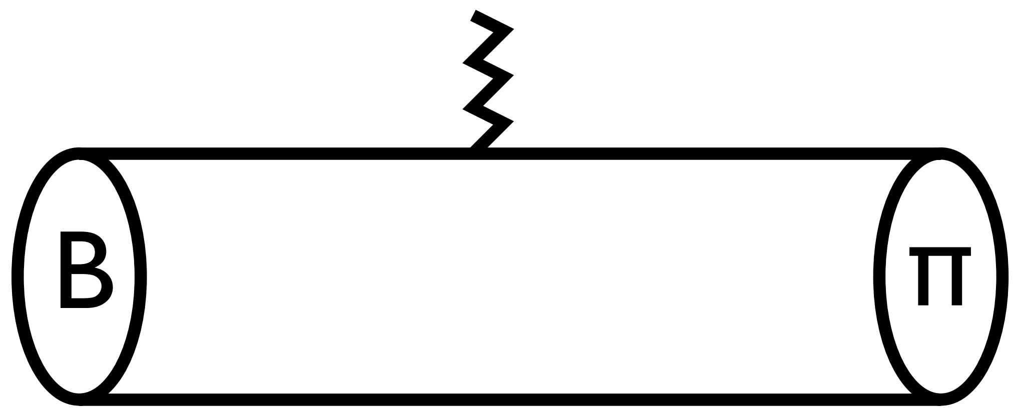
Can show: endpoint suppression of $\lambda^{3/2}$

$\Rightarrow$ Need d=3, $[\lambda]=5/2$

At leading power:

$$\bar{\mathcal{X}}_c\Gamma''\mathcal{Q}_s$$

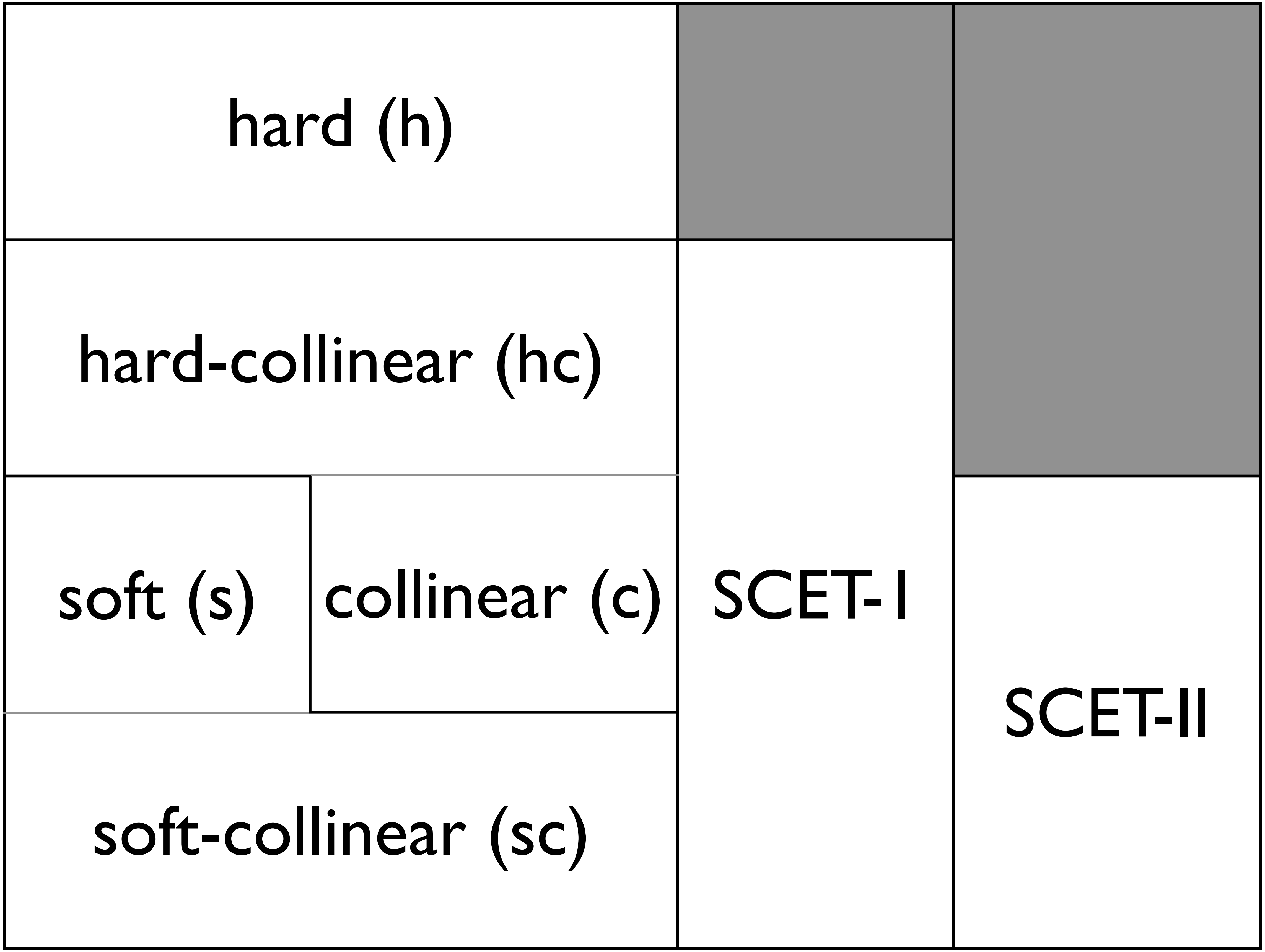
The Factorizable plus Nonfactorizable Case: The Bπ form factor

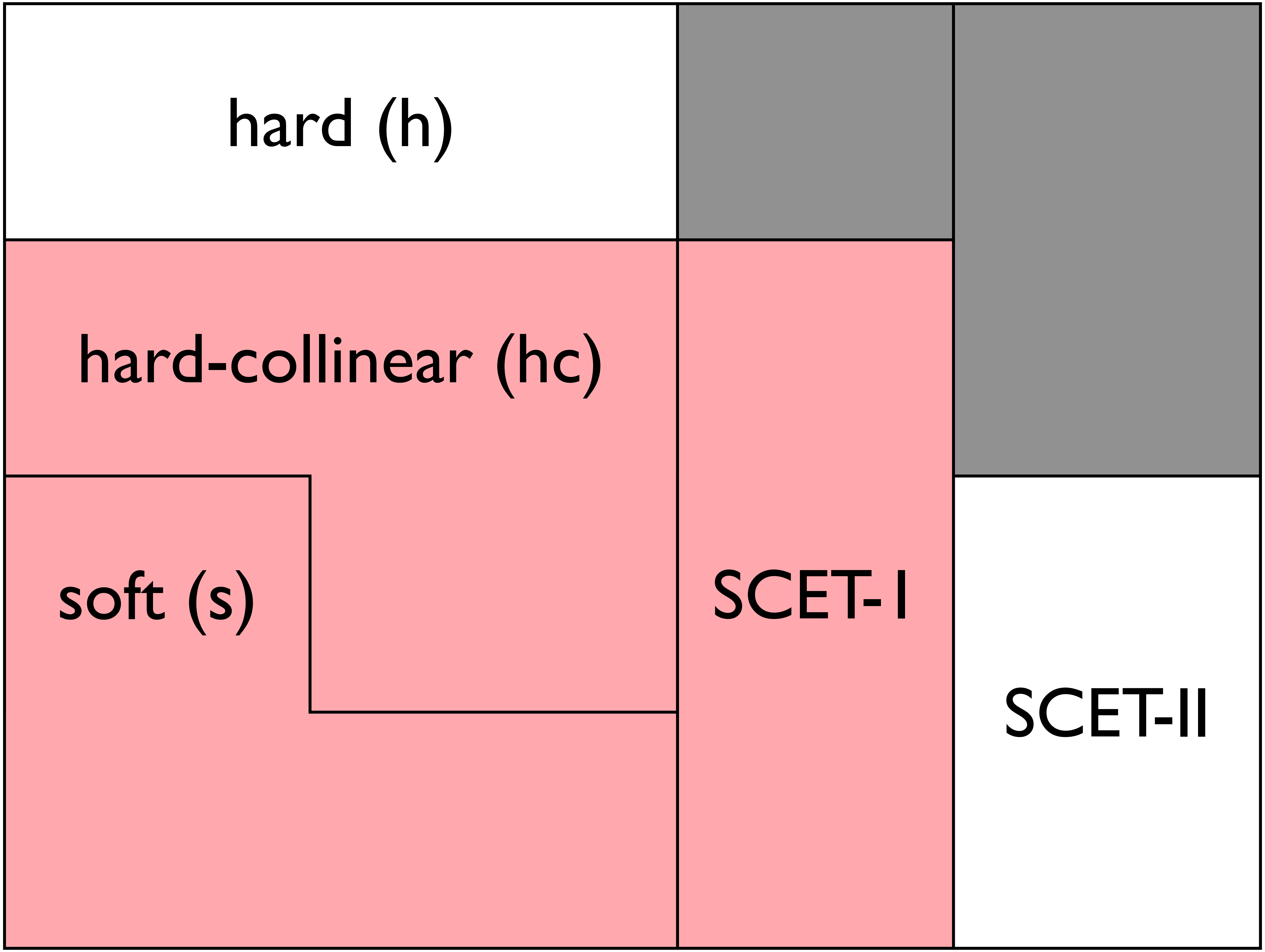


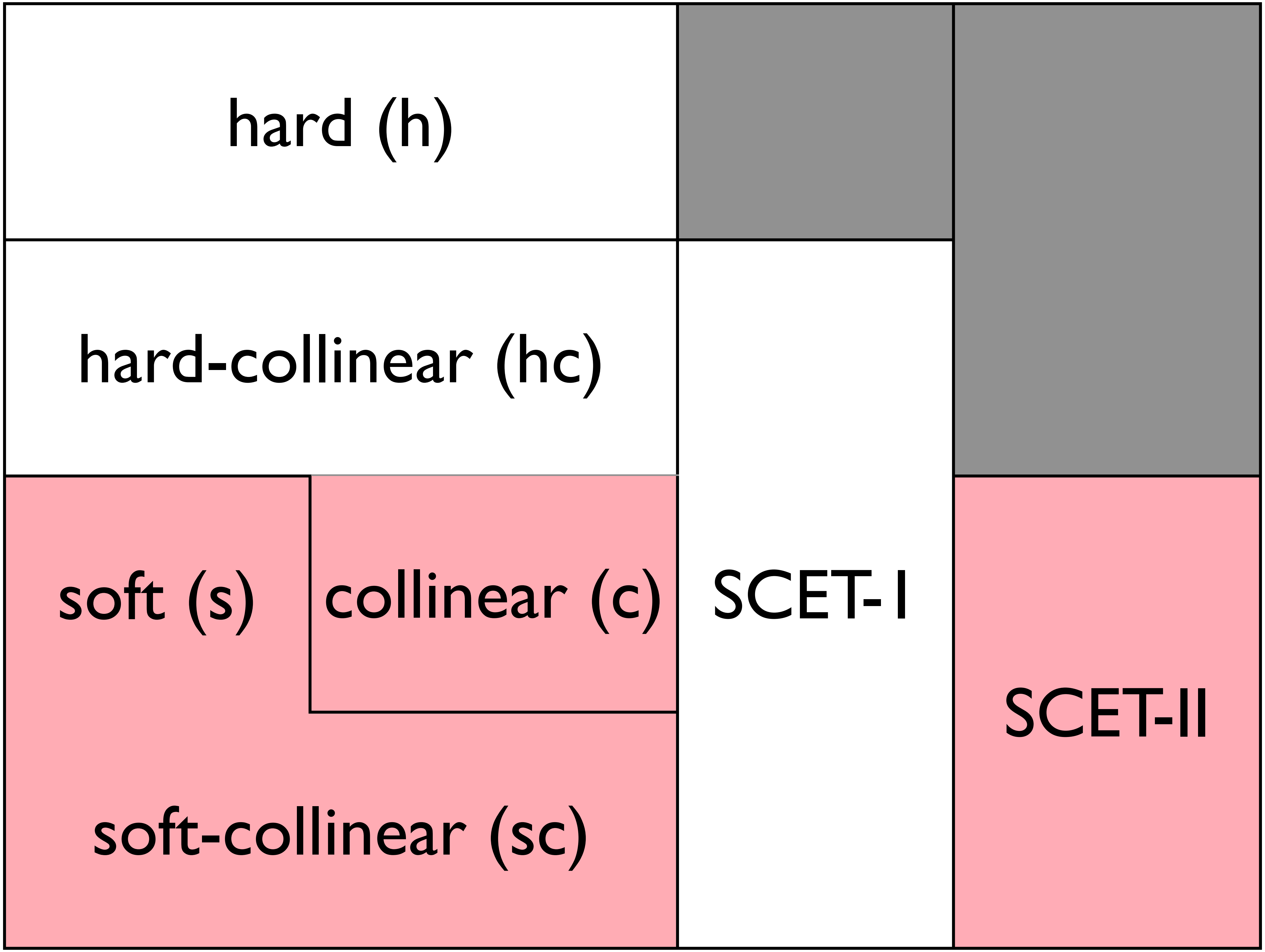
$$\langle \pi(p')|\bar{q}\gamma^{\mu}b|\bar{B}(p)\rangle =$$
$$F_{+}(q^2)\left(p^{\mu}+p'^{\mu}-\frac{m_B^2-m_\pi^2}{q^2}q^{\mu}\right)+F_0(q^2)\frac{m_B^2-m_\pi^2}{q^2}q^{\mu}$$

To isolate different contributions, convenient to perform the analysis in two steps:

QCD →SCET-I→SCET-II


hard (h)
hard-collinear (hc)
soft (s)
collinear (c)
soft-collinear (sc)
SCET-I
SCET-II


hard (h)
hard-collinear (hc)
soft (s)
SCET-I
SCET-II


hard (h)
hard-collinear (hc)
soft (s)
collinear (c)
soft-collinear (sc)
SCET-I
SCET-II

A general field Φ decomposes as:

$$\begin{array}{rcl} \mathrm{SCET_I} & \rightarrow & \mathrm{SCET_{II}} \\ \phi_{hc} & \rightarrow & \phi_{hc} + \phi_c \\ \phi_s & \rightarrow & \phi_s + \phi_{sc} \end{array}$$

As in SCET-II, consider general building blocks for gauge-invariant, boost-invariant operators in SCET-I (easier, since no inverse derivatives)

$$\bar{q}\gamma^\mu b \rightarrow C_i^A(E, m_b) J_i^A + \int_0^1 du\, C_j^B(E, m_b, u) J_j^B(u) + \ldots$$

$$\begin{array}{rcl} J_i^A & = & \bar{\mathfrak{X}}_{hc}\Gamma_i^A h \quad (d=3) \\ J_j^B & = & \bar{\mathfrak{X}}_{hc}\mathcal{A}_{hc\perp\mu}\Gamma_j^{B\mu} h \quad (d=4) \end{array}$$

	d	$[\lambda]$
$\frac{1}{\bar{n}\cdot\partial_c}\,\bar{\mathcal{X}}_c\frac{\bar{n}\!\!\!/}{2}\Gamma'\mathcal{X}_c$	2	2
$\frac{1}{n\cdot\partial_s}\,\bar{\mathcal{Q}}_s\frac{n\!\!\!/}{2}\Gamma'\mathcal{Q}_s$	2	2
$\bar{\mathcal{Q}}_s\Gamma''\mathcal{Q}_s$	3	3
$n\cdot\partial_s\,\bar{\mathcal{Q}}_s\frac{\bar{n}\!\!\!/}{2}\Gamma'\mathcal{Q}_s$	4	4

	d	$[\lambda]$
$g_\perp^{\mu\nu},\,\epsilon_\perp^{\mu\nu}$	0	0
$\partial_{c\perp}^\mu,\,\mathcal{A}_{c\perp}^\mu,\,\partial_{s\perp}^\mu,\,\mathcal{A}_{s\perp}^\mu$	1	1
$n\cdot\partial_s\bar{n}\cdot\partial_s,\,n\cdot\partial_s\bar{n}\cdot\mathcal{A}_s$	2	2
$\bar{n}\cdot\partial_c n\cdot\partial_c,\,\bar{n}\cdot\partial_c\,n\cdot\mathcal{A}_c$	2	2
$\frac{1}{\bar{n}\cdot\partial_c\,n\cdot\partial_s}$	-2	-1

$$F_i^{B\to\pi} = C_i^A\,\zeta_\pi + \Delta F_i^{B\to\pi}$$

A-type (d=3): nonfactorizable:

$$\langle\pi(p')|\bar{\mathcal{X}}_{hc}\Gamma h|\bar{B}_v\rangle = -2E\,\zeta_\pi(E)\,\mathrm{tr}\left\{\gamma_5\frac{\bar{n}\!\!\!/ n\!\!\!/}{4}\Gamma\frac{1+v\!\!\!/}{2}(-\gamma_5)\right\}$$

B-type (d=4): factorizable:

$$\bar{\mathcal{X}}_{hc}\mathcal{A}_{hc\perp\mu}\Gamma^\mu h \to \frac{1}{\bar{n}\cdot\partial_c}\,\bar{\mathcal{X}}_c\frac{\bar{n}\!\!\!/}{2}\Gamma'\mathcal{X}_c\frac{1}{n\cdot\partial_s}\,\bar{\mathcal{Q}}_s\frac{n\!\!\!/}{2}\Gamma'\mathcal{H}_s$$

$$\Delta F_i^{B\to\pi} = \frac{1}{8E}\sqrt{m_B}F\phi_B(\omega)\otimes f_\pi\phi_\pi(u)\otimes\mathcal{J}_\parallel\otimes C_i^B$$

Perturbative power counting:

$$E\,\zeta \sim \langle \pi(p')|\mathcal{O}^{(d=3\,,[\lambda]=4)}|\bar{B}_v\rangle \sim \frac{1}{\Lambda}\,\frac{\Lambda^4}{E}\,\frac{\sqrt{m_B}}{\Lambda^{3/2}}$$

$$\Rightarrow \quad \boldsymbol{\zeta_\pi \equiv \sqrt{m_B}\hat{\zeta}_\pi \sim \frac{\sqrt{m_B}\Lambda^{3/2}}{E^2}}$$

Symmetry Relations

$$F_i^{B\to\pi} = C_i^A\,\zeta_\pi + \Delta F_i^{B\to\pi}$$

- All B→π form factors are in principle calculable in terms of any one of them
- Given the uncertainties in the remaining hadronic parameters (especially Φ_B), important to find exact or approximate relations that are independent of these parameters

$$F_i^{B\to\pi} = C_i^A\,\zeta_\pi + \Delta F_i^{B\to\pi}$$

$$\Delta F_i^{B\to\pi} = \frac{1}{8E}\sqrt{m_B} F\phi_B \otimes f_\pi\phi_\pi \otimes \mathcal{J}_\| \otimes C_i^B$$

Three classes of relations:

- 1st class: exact up to power corrections
- 2nd class: C^B at tree level - corrections of order $\alpha_s(m_b)$
- 3rd class: $C^B=0$

Look at 2nd class in more detail:

$$\begin{aligned}
\Delta F_i^{B\to\pi} &= \frac{1}{8E}\int_0^\infty \frac{d\omega}{\omega}\sqrt{m_B}F\phi_B(\omega)\int_0^1 du f_\pi\phi_\pi(u)\int_0^1 du'\mathcal{J}_\parallel(u,u')C_i^B(u') \\
&\equiv \frac{1}{8E}\sqrt{m_B}F\phi_B\otimes f_\pi\phi_\pi\otimes\mathcal{J}_\parallel\otimes C_i^B \\
&\approx C^{B(\mathrm{tree})}\left[\frac{1}{8E}\sqrt{m_B}F\phi_B\otimes f_\pi\phi_\pi\otimes\mathcal{J}_\parallel\otimes 1\right] \\
&\equiv -C^{B(\mathrm{tree})}\left[\left(\frac{m_B}{2E}\right)^2 H_\pi\right]
\end{aligned}$$

$$\Rightarrow \quad \left(\frac{2E}{m_B}\right)^2 H_\pi \equiv \sqrt{m_B}\hat{H}_\pi \sim \frac{\sqrt{m_B}\Lambda^{3/2}}{E^2}$$

Three form factors, two unknowns:

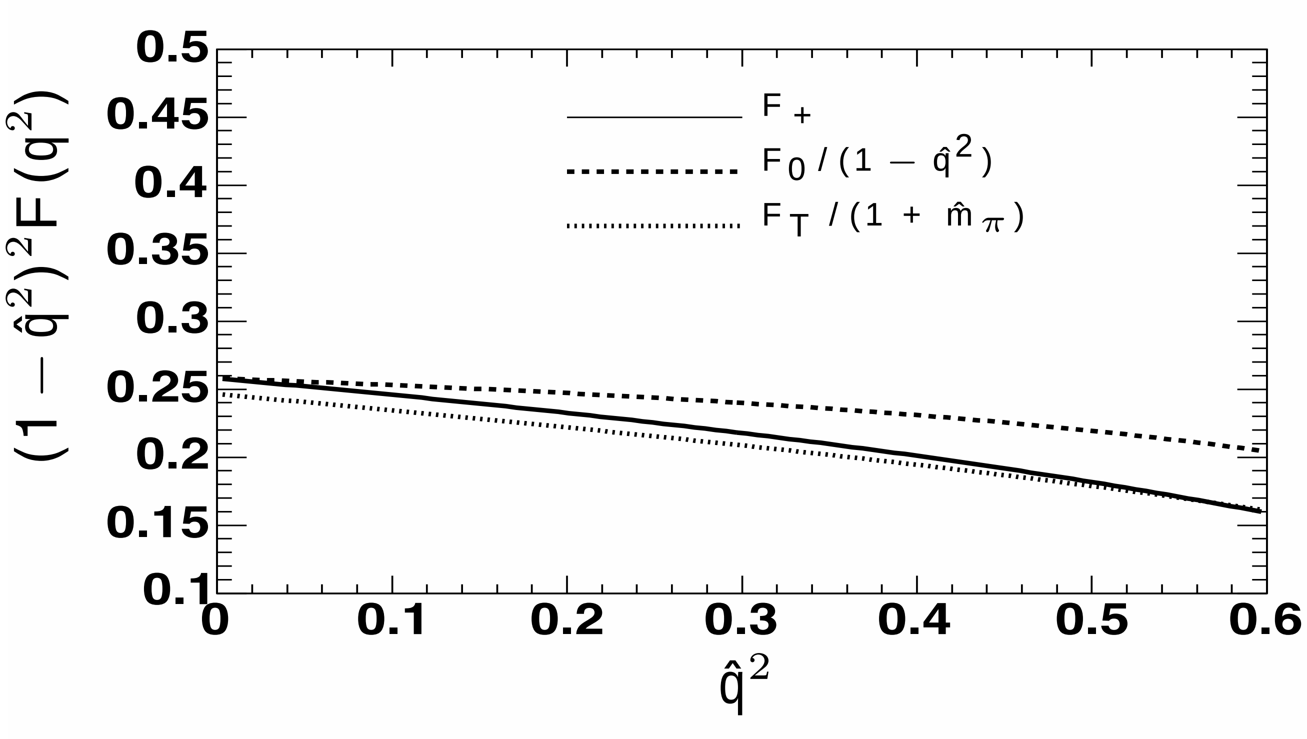
$$\begin{aligned}
F_+ &= \sqrt{m_B}\left[\hat{\zeta}_\pi + \left(\frac{4E}{m_B}-1\right)\hat{H}_\pi\right] \\
F_0 &= \sqrt{m_B}\left[\hat{\zeta}_\pi + \hat{H}_\pi\right] \\
\frac{m_B}{m_B+m_\pi}F_T &= \sqrt{m_B}\left[\hat{\zeta}_\pi - \hat{H}_\pi\right]
\end{aligned}$$

$$F_+(q^2) = F_0(q^2) + \frac{m_B}{m_B+m_\pi}\frac{q^2}{m_B^2}F_T(q^2)$$

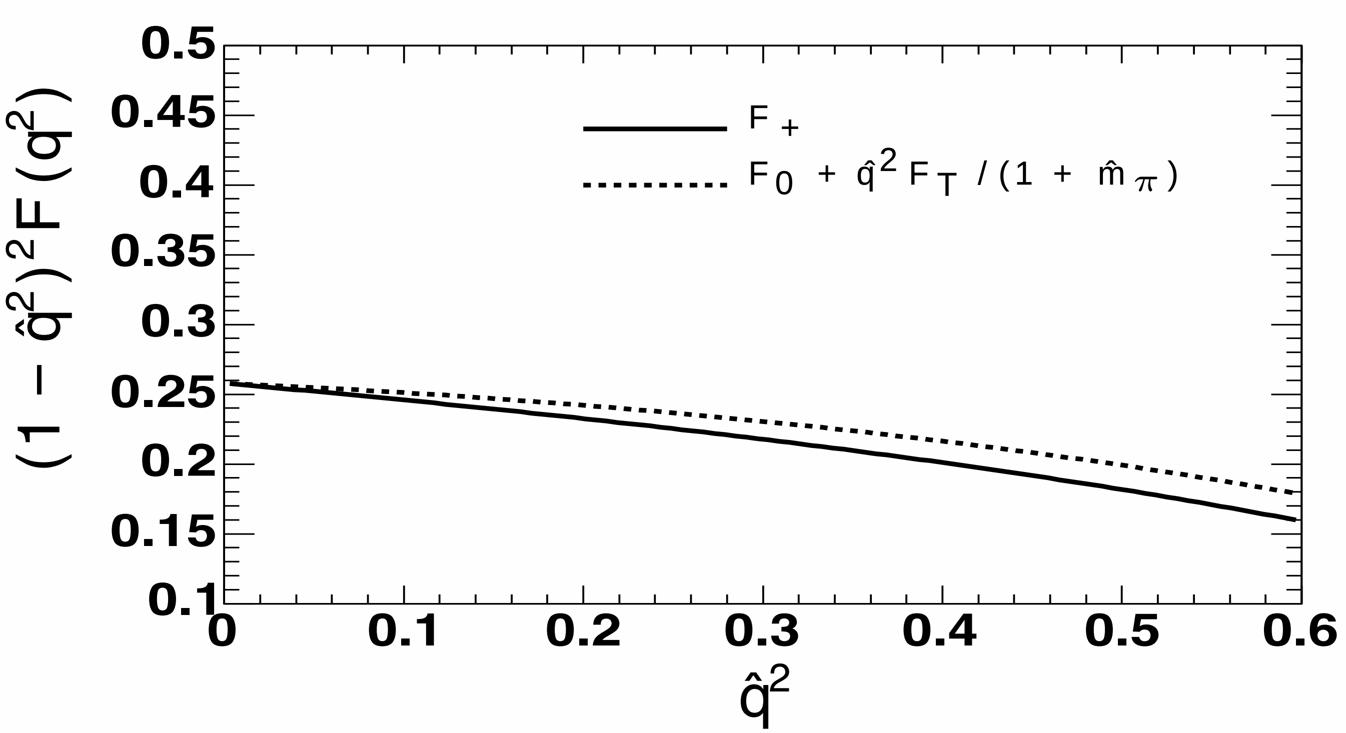
- Test of lattice form factor determinations

Comparison to Light-Cone QCD Sum Rules

B→π 3rd class



B→π 2nd class

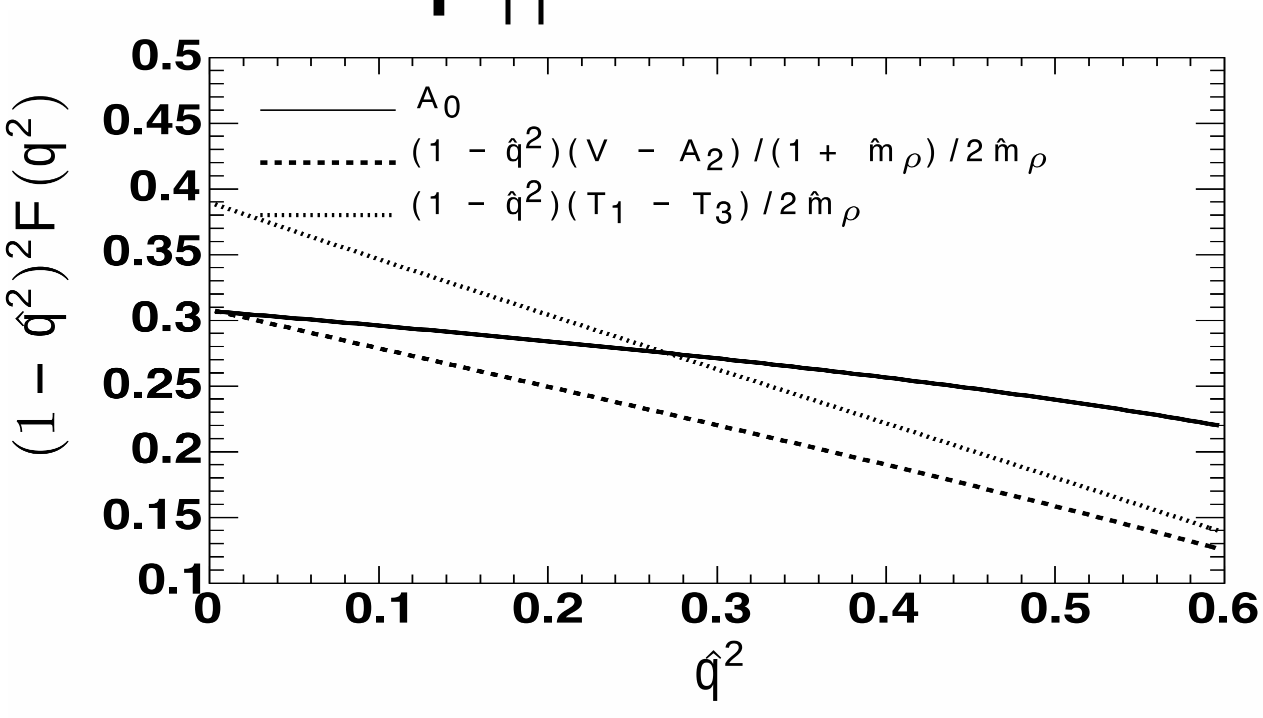


(in units of m_B)

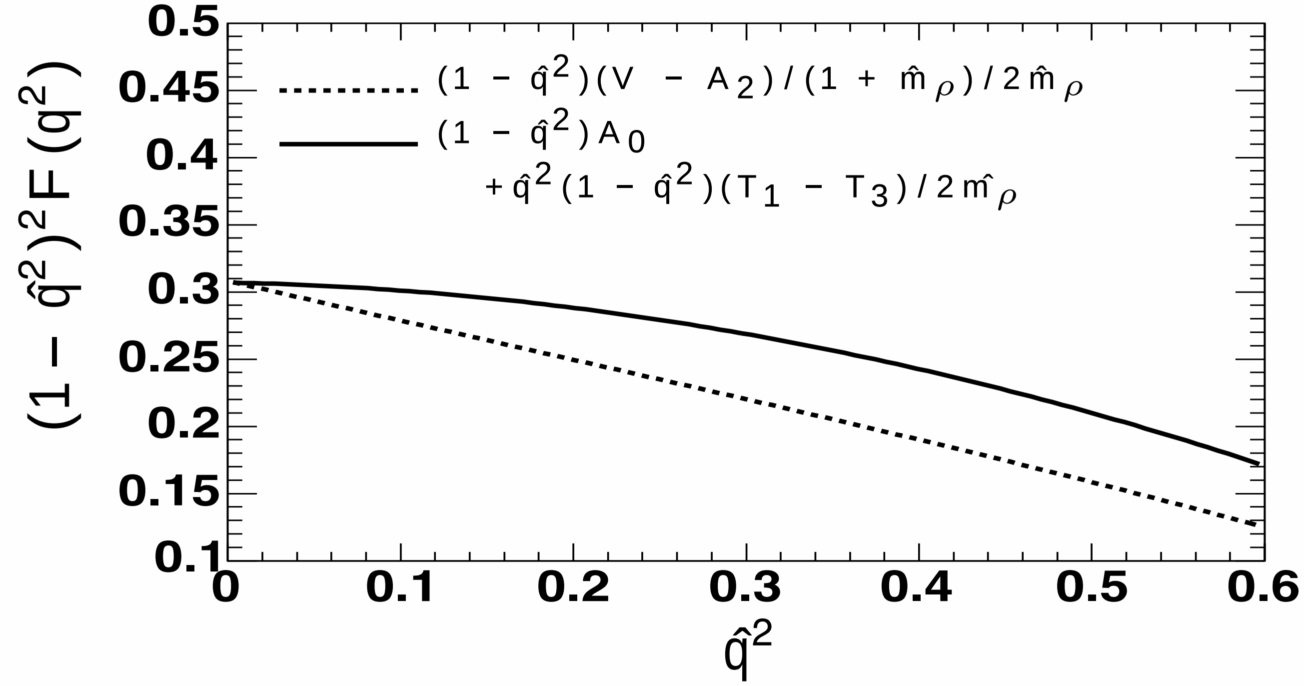
Ball and Zwicky, hep-ph/0406232

$$1-\frac{q^2}{m_B^2}=\frac{2E}{m_B}+\mathcal{O}(m_\pi^2)$$

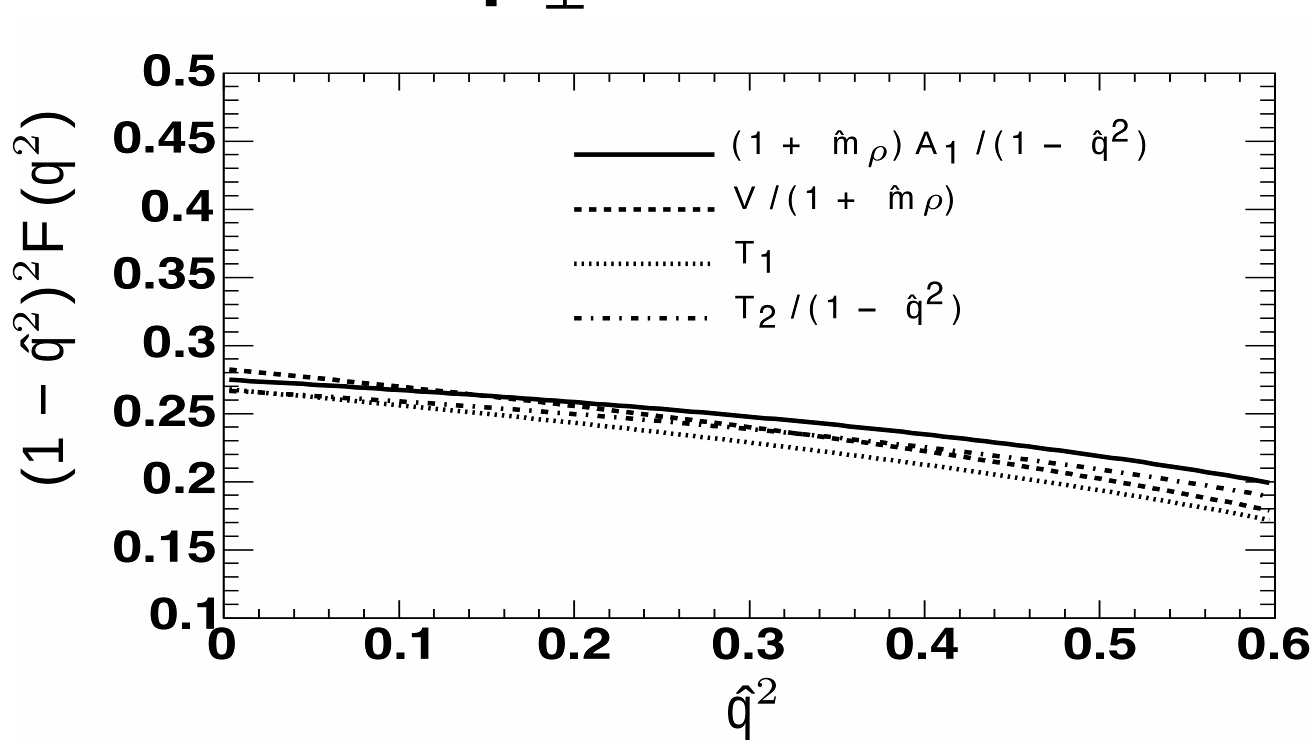
$B \to \rho_{||}$ 3rd class



$B \to \rho_{||}$ 2nd class



$B \to \rho_{\perp}$ 1st class

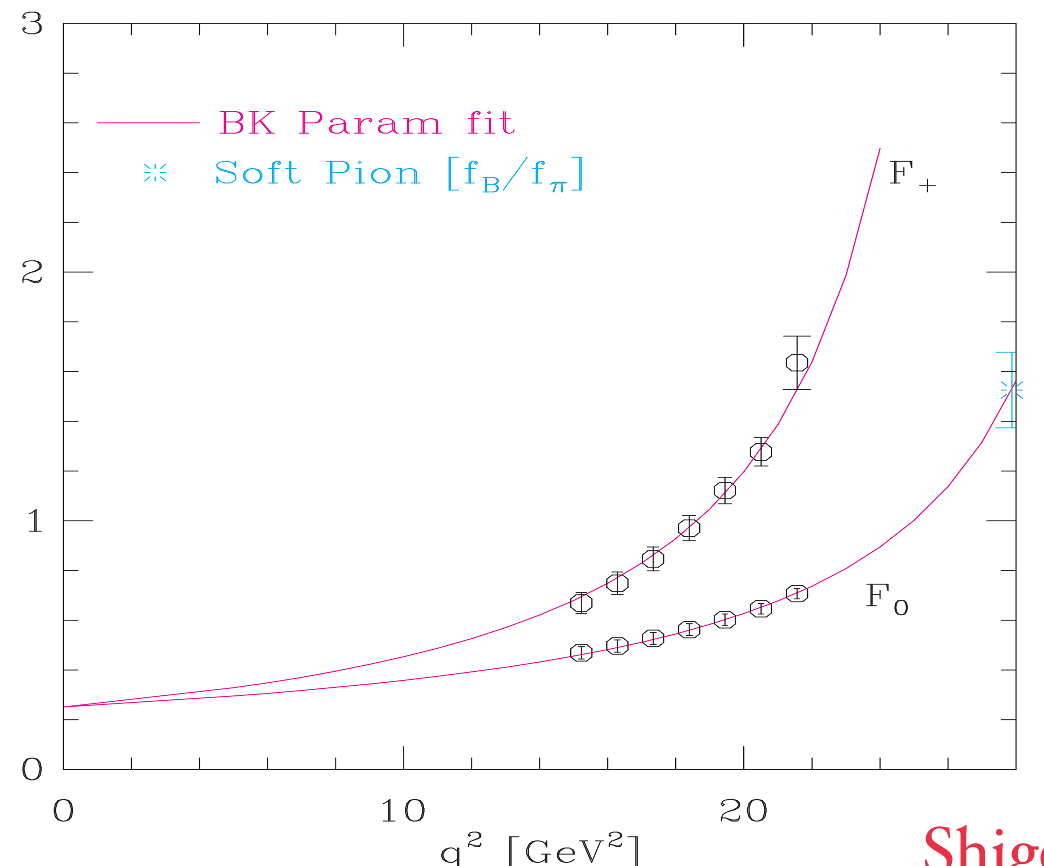


Ball and Zwicky, hep-ph/0412079

Form factor extrapolations

(More interesting than it sounds)

- Need to extrapolate present-day lattice data from large q^2 (small recoil) to small q^2 (large recoil)



Shigemitsu et.al. hep-lat/0408019

Becirevic-Kaidalov (BK) parameterization (and generalizations)

We know:

1) $$F_+(q^2) = \frac{\mathrm{Res}_{q^2=m_B^{*2}}(F_+)}{m_{B^*}^2 - q^2} + \frac{1}{\pi}\int_{(m_B+m_\pi)^2}^{\infty} dt\,\frac{\mathrm{Im}F_+(t)}{t-q^2-i\epsilon}$$
$$F_0(q^2) = \frac{1}{\pi}\int_{(m_B+m_\pi)^2}^{\infty} dt\,\frac{\mathrm{Im}F_0(t)}{t-q^2-i\epsilon}$$

2) $F_+(0) = F_0(0)$

3) $F_+(E\sim\Lambda) \sim \sqrt{\frac{m_B}{\Lambda}}, \quad F_0(E\sim\Lambda)\sim\sqrt{\frac{\Lambda}{m_B}}$ (HQET)

4) $F_+(E\sim m_B) \sim \left(\frac{\Lambda}{m_B}\right)^{3/2}, \quad F_0(E\sim m_B)\sim\left(\frac{\Lambda}{m_B}\right)^{3/2}$ (SCET)

Ansatz: represent dispersive integral by an effective pole (with mass ~ $m_B + \Lambda$)

$$\Rightarrow \quad \begin{aligned} F_+ &= c_B \left(\frac{1}{1 - \frac{q^2}{m_{B*}^2}} - \frac{\alpha}{1 - \frac{1}{\gamma}\frac{q^2}{m_{B*}^2}} \right) \\ F_0 &= \frac{c_B(1-\alpha)}{1 - \frac{1}{\beta}\frac{q^2}{m_{B*}^2}} \end{aligned}$$

Becerivic and Kaidalov, hep-ph/9904490

Using the heavy quark scaling laws, easy to show:

$$\begin{aligned} \alpha &= 1 - \frac{\alpha_0}{m_B} + \ldots, \quad \beta = 1 + \frac{\beta_0}{m_B} + \ldots \\ \gamma &= 1 + \frac{\gamma_0}{m_B} + \ldots, \quad c_B = \frac{c_{B0}}{\sqrt{m_B}} + \ldots \end{aligned}$$

At large recoil:

$$F_+ = \frac{\sqrt{m_B} c_{B0}}{(2E)^2}\left[\frac{2E}{m_B}\alpha_0 + \left(1 - \frac{2E}{m_B}\right)\gamma_0\right]$$

$$\frac{m_B}{2E}F_0 = \frac{\sqrt{m_B} c_{B0}}{(2E)^2}\,\alpha_0$$

But recall:

$$F_+ = \sqrt{m_B}\left[\hat{\zeta}_\pi + \left(\frac{4E}{m_B} - 1\right)\hat{H}_\pi\right]$$

$$\frac{m_B}{2E}F_0 = \sqrt{m_B}\left[\hat{\zeta}_\pi + \hat{H}_\pi\right]$$

In this ansatz, α and ϒ are related to the universal SCET functions:

$$\hat{\zeta}_\pi = \frac{c_B}{(2E)^2}\left(\frac{\alpha_0 + \gamma_0}{2}\right), \quad \hat{H}_\pi = \frac{c_B}{(2E)^2}\left(\frac{\alpha_0 - \gamma_0}{2}\right)$$

Solve for α, γ:

$$\alpha = 1 - \frac{\alpha_0}{m_B} + \ldots$$
$$\frac{1}{\gamma} = 1 - \frac{\gamma_0}{m_B} + \ldots \approx \alpha + \delta(1-\alpha)$$

$$\delta = \frac{2\hat{H}_\pi}{\hat{\zeta}_\pi + \hat{H}_\pi}, \quad 1 - \delta = \frac{\hat{\zeta}_\pi - \hat{H}_\pi}{\hat{\zeta}_\pi + \hat{H}_\pi}$$

$$\Rightarrow \quad F_+(q^2) = \frac{f(0)\left(1-\delta\frac{q^2}{m_{B*}^2}\right)}{\left(1-\frac{q^2}{m_{B*}^2}\right)\left(1-[\alpha+\delta(1-\alpha)]\frac{q^2}{m_{B*}^2}\right)}$$

$$F_0(q^2) = \frac{f(0)}{1-\frac{1}{\beta}\frac{q^2}{m_{B*}^2}}$$

δ=0: original 3-parameter BK parameterization

What if $\delta \neq 0$?

$$\delta = \frac{2\hat{H}_\pi}{\hat{\zeta}_\pi + \hat{H}_\pi}$$

$\delta = 0$?   1 ?   2 ?

$0 < \delta < 1$?

What if $\delta \neq 0$?

δ	$f(0)$	α	β	$2f(0)/(1-\alpha)$	"$d\ln\hat{\zeta}_\pi/d\ln E$"	χ^2
0	0.251(15)	0.407(70)	1.18(5)	0.85	-1.73	2.3
0.4	0.253	0.482	1.19	0.98	-1.81	1.7
0.722	0.259	0.675	1.21	1.63	-1.84	0.7
0.8	0.262	0.810	1.21	2.75	-1.84	0.75
0.95	0.269	2.238	1.24	-0.44	-1.83	1.32
$\to 1$	0.273	∞	1.25	0	-1.82	1.7
1.05	0.277	-1.57	1.27	0.22	-1.81	2.3
1.2	0.297	-0.15	1.35	0.52	-1.75	5.8
1.4	0.335	0.002	1.53	0.67	-1.66	21

(preliminary analysis of preliminary numbers)

At $\delta = 1$, simple pole parameterization:

$$F_+(q^2) = \frac{f(0)}{1 - \frac{q^2}{m_{B*}^2}}, \quad F_0(q^2) = \frac{f(0)}{1 - \frac{1}{\beta}\frac{q^2}{m_{B*}^2}}$$

Physical interpretation

δ	$f(0)$	α	β	$2f(0)/(1-\alpha)$	"$d\ln\hat{\zeta}_\pi/d\ln E$"	χ^2
0	0.251(15)	0.407(70)	1.18(5)	0.85	-1.73	2.3
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1.2	0.297	-0.15	1.35	0.52	-1.75	5.8
1.4	0.335	0.002	1.53	0.67	-1.66	21

$$f(0) \quad \leftrightarrow \quad F_+(0)$$

$$\frac{2f(0)}{1-\alpha} \quad \leftrightarrow \quad \frac{f_{B^*}g_{B^*B\pi}}{m_{B^*}} = 1.13 \left[\frac{f_{B^*}}{200\,\mathrm{MeV}}\right]\left[\frac{g_{B^*B\pi}}{30}\right]$$

$$\delta \quad \leftrightarrow \quad \frac{2\hat{H}_\pi}{\hat{\zeta}_\pi + \hat{H}_\pi}$$

$$-\frac{2+\alpha+\alpha^2+\frac{1}{\beta}-\frac{1}{\beta^2}-2\alpha^2\delta-\alpha(1-\alpha)\delta^2}{2+\alpha-\frac{1}{\beta}-\alpha\delta} \quad \leftrightarrow \quad \frac{d\ln\hat{\zeta}_\pi}{d\ln E}$$

- The 3-parameter BK parameterization is motivated by a 3rd-class symmetry relation ($F_0=(2E/m_B)F_+$ at large recoil)

- Equally good parameterizations allow $\delta \neq 0$, and have a nice physical interpretation

Heavy quark symmetry in SCET

Recall the HQET relation:

$$\frac{F_+^{B\to\pi}(E)}{F_+^{D\to\pi}(E)} = \sqrt{\frac{m_B}{m_D}}$$

In SCET this becomes:

$$\frac{F_+^{B\to\pi}}{F_+^{D\to\pi}} = \sqrt{\frac{m_B}{m_D}}\frac{\hat{\zeta}_\pi(E) + \left(\frac{4E}{m_B} - 1\right)\hat{H}_\pi(E)}{\hat{\zeta}_\pi(E) + \left(\frac{4E}{m_D} - 1\right)\hat{H}_\pi(E)}$$

At maximum recoil in semileptonic D→π (E≈m$_D$/2):

$$\sqrt{\frac{m_D}{m_B}}\,\frac{F_+^{B\to\pi}}{F_+^{D\to\pi}} = \frac{\hat\zeta_\pi(E)+\left(\frac{4E}{m_B}-1\right)\hat H_\pi(E)}{\hat\zeta_\pi(E)+\left(\frac{4E}{m_D}-1\right)\hat H_\pi(E)} \to \frac{\hat\zeta_\pi+\left(\frac{2m_D}{m_B}-1\right)\hat H_\pi}{\hat\zeta_\pi+\hat H_\pi}$$

$$= 1-\left(1-\frac{m_D}{m_B}\right)\delta$$

$$\Rightarrow \quad \delta = \frac{1-\sqrt{\frac{m_D}{m_B}}\frac{F_+^{B\to\pi}(q_*^2)}{F_+^{D\to\pi}(0)}}{1-\frac{m_D}{m_B}} = 0.38\pm 0.1\pm 0.3$$

$$q_*^2 = m_B^2 - m_B m_D \approx 18\,\mathrm{GeV}^2$$

$$F_+^{B\to\pi}(18\,\mathrm{GeV}^2) = 0.93(10)\,, \qquad F_+^{D\to\pi}(0) = 0.73(15)$$

MILC hep-lat/0408019 (preliminary) BES hep-ex/0406028

Questions

- Can more form factors be calculated, and tested by the symmetry relations? (F_+,F_0,F_T)
- What is δ ?
- Can SCET be put on the lattice? SCET-II? SCET-I? (cf. pNRQCD)

Conclusions (Why this should be of interest to lattice field theory)

- better understanding of the underlying symmetries and relevant degrees of freedom
- constraints and relations for form factor parameters, extrapolations
- can apply existing lattice data (e.g. form factors) to NECGP quantities (e.g. $B \to \pi\pi$)

Not Even Close to Gold-Plated