Higgs mass and Vacuum instability (and triviality)

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Outline

- Experimental limits on Higgs mass
- More general - Standard Model good theory
- History: Landau pole and Higgs upper bound
- Full story: Triviality
- Vacuum instability and Higgs lower bound
- Full story: Triviality, again

Experimental limits

Not seen: Higgs mass > 114.4 GeV

Precision ElectroWeak measurements (W mass, ...)

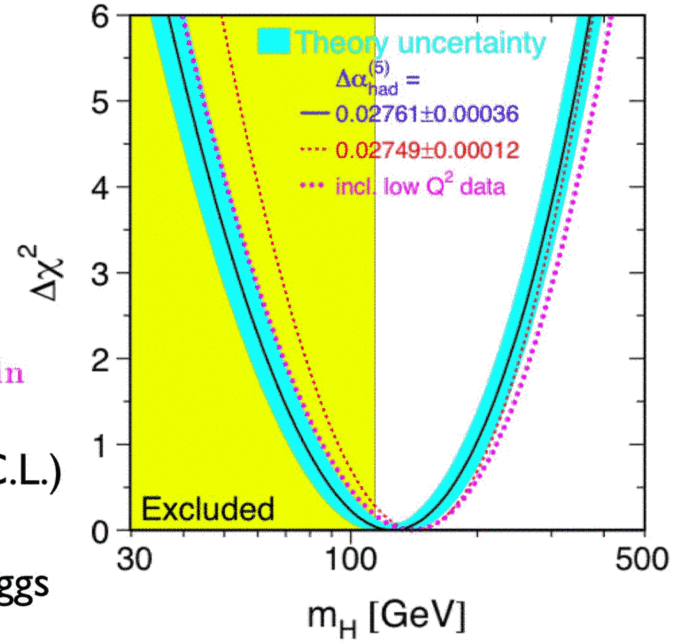
Fit SM to data with Higgs mass free parameter

Fit quality $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$

$m_{\rm H} = 126^{+73}_{-48}$ GeV (95% C.L.)

Standard Model favors light Higgs

LEP EW workgroup 2005



General Higgs bounds

Omit precision EW: how heavy or light could Higgs be?

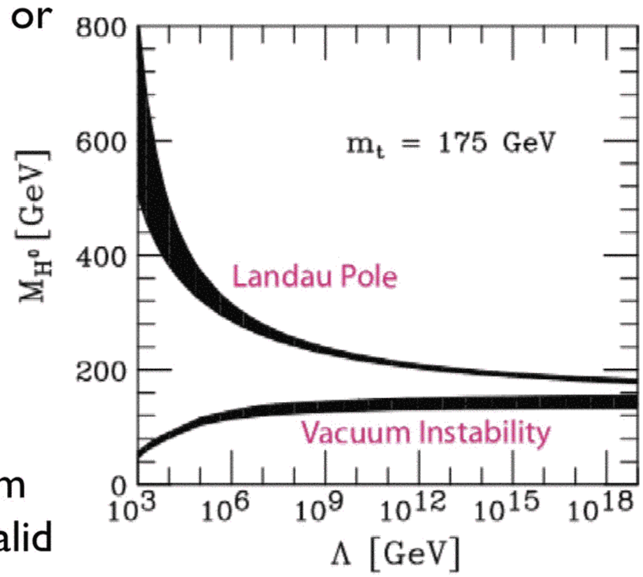
Bounds on Higgs if SM good theory up to Λ- new physics

a. Tells us where Higgs can be

- cannot have Higgs > 1 TeV

b. If Higgs seen, know maximum energy scale up to which SM valid

Uncertainty in bounds in principle reducible

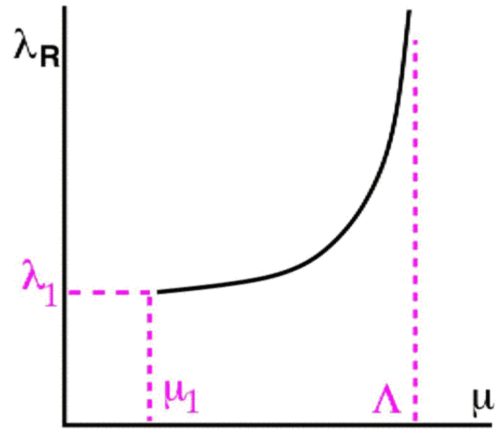


Particle Data Group 2002

Landau pole and upper bound

$4\mathrm{d}\ \lambda\phi^4 \qquad \lambda(\mu) \qquad \lambda(v_\mathrm{R}) = 3m_\mathrm{H}^2/v_\mathrm{R}^2$

$\beta(\lambda) = 3\lambda^2/16\pi^2 + \mathcal{O}(\lambda^3) > 0$



Run RG from $\lambda(\mu_1 = v_\mathrm{R}) = \lambda_1$

Divergence at some scale Λ

If SM valid up to $\Lambda \Rightarrow \lambda(\mu_1) \leq \lambda_1$

To increase Λ must decrease λ_1

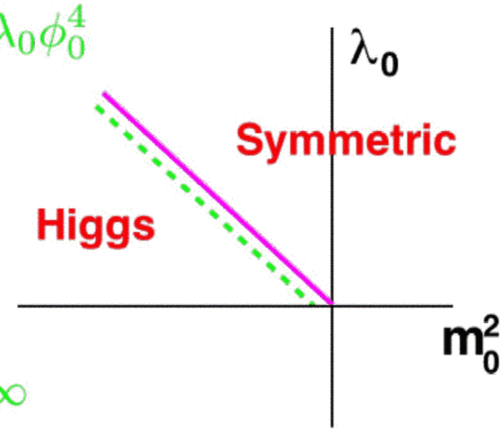
- Higgs mass bound decreases

Λ interpreted as scale of new physics

Really breakdown of renormalized perturbation theory

Full story: triviality

$4\mathrm{d}\ \lambda\phi^4 \qquad \mathcal{L} = \frac{1}{2}m_0^2\phi_0^2 + \frac{1}{24}\lambda_0\phi_0^4$



Regulate theory e.g. lattice

Cut-off $\Lambda = \pi/a$

Phase diagram critical surface

$v_\mathrm{R}a \to 0 \qquad \xi/a = 1/(m_\mathrm{H}a) \to \infty$

Close to critical surface, cut-off large $\Lambda/v_\mathrm{R} = \pi/v_\mathrm{R}a$

Theory physically acceptable if cut-off sufficiently large

For fixed cut-off (dashed line), how heavy can Higgs be?

Neuberger & Dashen (1983) Numerical experiment

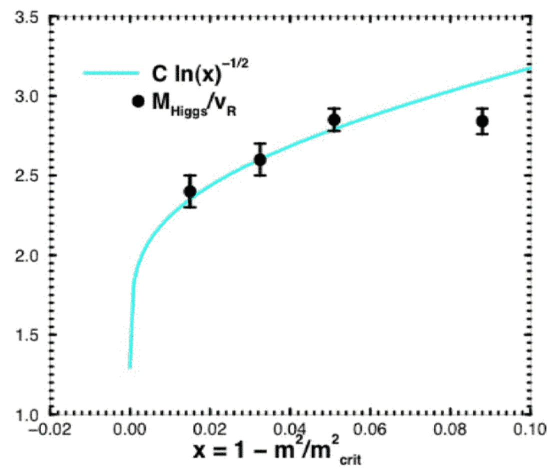
Trivial

$\lambda = 3m_{\rm H}^2/v_{\rm R}^2 \propto 1/\ln(\Lambda/v_{\rm R})$

$\lambda \to 0$ as $\Lambda/v_{\rm R} \to \infty$

Continuum theory
noninteracting - trivial

$x = 0.1 \Rightarrow \Lambda/v_{\rm R} \approx 8,\ m_{\rm H}/v_{\rm R} \approx 2.8$



a. For fixed cut-off, $m_{\rm H}$ largest for $\lambda_0 \to \infty$ non-perturbative

b. For $\xi/a = 1/(m_{\rm H}a) \geq 2$ Euclidean invariance broken at % level

Upper bound
$\Lambda \sim$ few TeV $\Rightarrow m_{\rm H} <$ 600-700 GeV
$\Lambda \sim 10^{19}$ GeV $\Rightarrow m_{\rm H} <$ 140-150 GeV

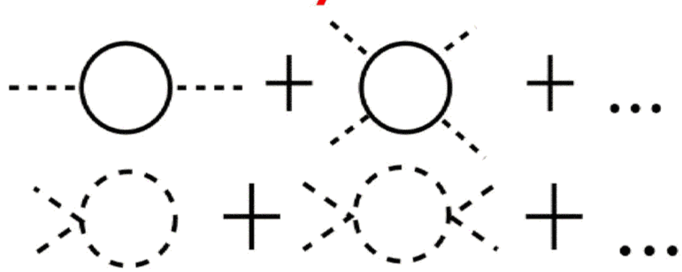
Neuberger, Bhanot, Heller, Vranas, Luscher, Weisz, Montvay, Jansen, Kuti, Hasenfratz,...

Vacuum instability

Higgs-Yukawa model

$N_{\rm F}$ degenerate fermions

Effective potential



$$U_{\rm eff}(\phi) = V + 1/2\int_k \ln[k^2 + V''] - 2N_{\rm F}\int_k \ln[k^2 + y^2\phi^2] \qquad V = m^2\phi^2/2 + \lambda\phi^4/24$$

Regulate, add counterterms to absorb divergences, remove cut-off

$$U_{\rm eff} = V + \{(V'')^2/64\pi^2\}\{\ln[V''/\mu^2] - 3/2\} - \{N_{\rm F}y^4\phi^4/16\pi^2\}\{\ln[y^2\phi^2/\mu^2] - 3/2\}$$

Negative fermion term dominates if $\lambda^2 < 16N_{\rm F}y^4$

Potential has no ground state - vacuum is unstable

Couplings can still be small - perturbative

Linde, Politzer, Wolfram, Cabibbo, Maiani, Parisi, Altarelli, Sher, Casas, Espinosa, Quiros, ...

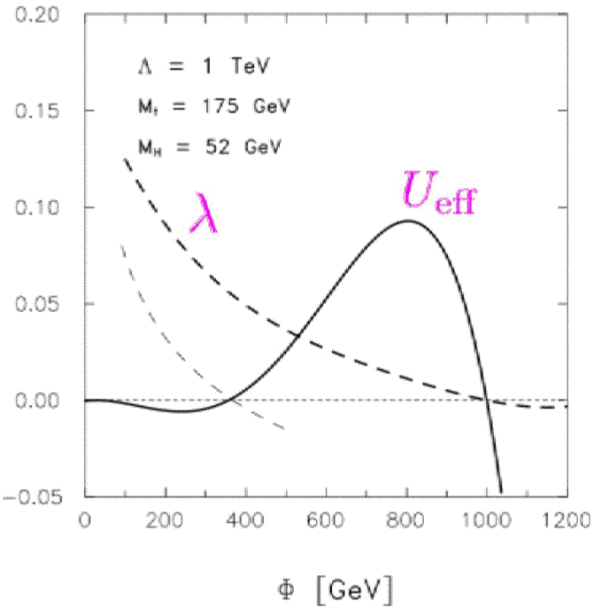
Standard Model

2-loop SM calculation
with RG improvement

If $m_{\rm H} = 52$ GeV EW vacuum at $v_{\rm R} = 246$ GeV stable only if new physics enters at $\phi \sim 1$ TeV

To push instability to higher energies, must increase $m_{\rm H}$

Higgs lower bound increases



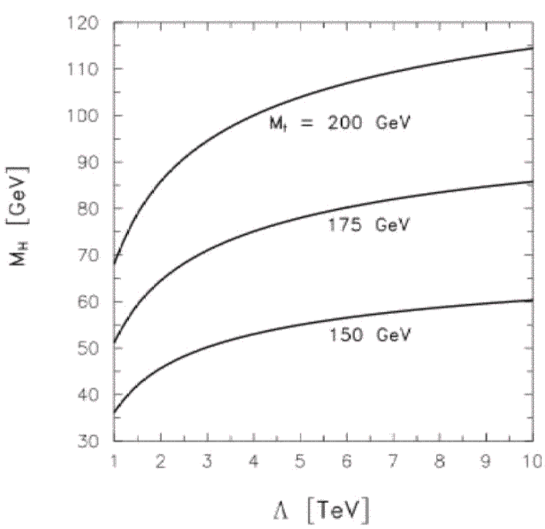
Casas, Espinosa & Quiros (1996)

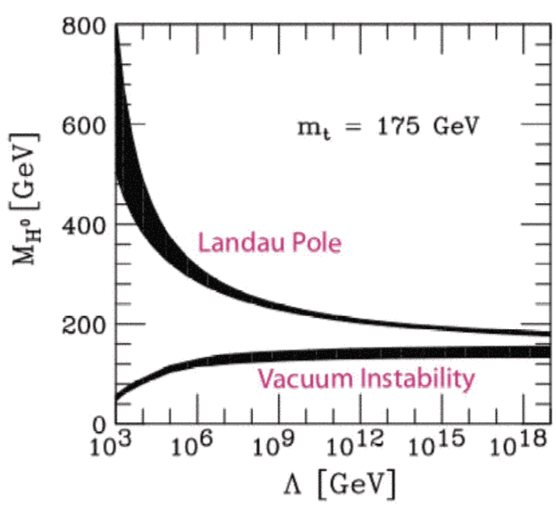
If $\lambda \ll y^4 \Rightarrow \mu(d\lambda/d\mu) < 0$

Thick dashed line $\lambda(\mu = c\phi)$

Vacuum instability coincides with $\lambda = 0$

Bounds





Lower bound theoretical uncertainty estimated < 5 GeV

In principle can be made more accurate

Lower bound relevant for current experimental situation

Is potential really unstable?

Can measure effective potential non-perturbatively Kuti & Shen (1988)

Constraint effective potential $U^c_{\rm eff}$ in finite volume Ω

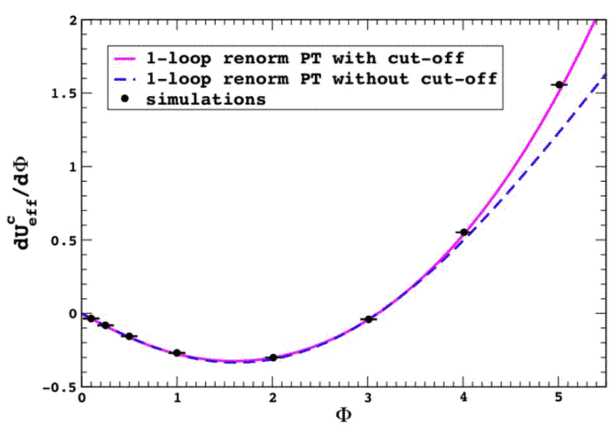
$$\exp(-\Omega U^c_{\rm eff}(\Phi)) = \int [D\phi]\delta(\Phi - 1/\Omega \sum_x \phi(x)) \exp(-S[\phi])$$

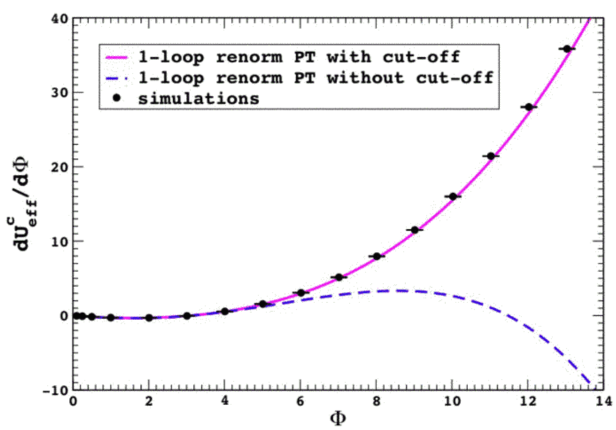
Constraint potential has absolute minimum in finite volume

O'Raifeartaigh, Wipf & Yoneyama (1986)

Distinguish Higgs and Symmetric phases in finite volume

Measure potential





Higgs-Yukawa model single scalar $N_{\rm F} = 8$ degenerate fermions

$$dU^c_{\rm eff}/d\Phi = m_0^2\Phi + 1/6\lambda_0\langle\phi^3\rangle_\Phi - N_{\rm F} y_0 \langle {\rm Tr}(D^{-1}[\phi])\rangle_\Phi$$

$\langle \ldots \rangle_\Phi$ expectation value with fixed Φ

Simulations: absolute minimum at $\Phi \approx 3.1$ Higgs phase

No sign of instability at large Φ

Perturbation theory

Regulate, add counter-terms to absorb divergences

$$U^c_{\rm eff} = V + 1/2 \int_{k\neq 0}^{\Lambda} \ln[1 + V''/k^2] - 2N_{\rm F} \int_{k\neq 0}^{\Lambda} \ln[1 + y^2\Phi^2/k^2]$$

$$-1/2 \int_{k\neq 0}^{\Lambda} \{V''/k^2 - (V'')^2/2[k^2+\mu^2]^2\}$$

$$+2N_{\rm F} \int_{k\neq 0}^{\Lambda} \{y^2\Phi^2/k^2 - y^4\Phi^4/2[k^2+\mu^2]^2\}$$

Can keep cut-off in integrals finite, or naively remove

Compare renormalized perturbation theory to simulations

Renorm. PT with cut-off in perfect agreement with simulations

Why does continuum PT incorrectly predict instability?

Renormalization

$$\mathcal{L} = m_0^2\phi_0^2/2 + \lambda_0\phi_0^4/24 + y_0\phi_0\bar{\Psi}_0^a\Psi_0^a + \text{K.E.}$$

$$= m_0^2 Z_\phi \phi^2/2 + \lambda_0 Z_\phi^2\phi^4/24 + y_0 Z_\phi^{1/2} Z_\Psi \phi\bar{\Psi}^a\Psi^a + \text{K.E.}$$

$$= (m^2 + \delta m^2)\phi^2/2 + (\lambda + \delta\lambda)\phi^4/24 + (y + \delta y)\phi\bar{\Psi}^a\Psi^a + \text{K.E.}$$

Large $N_{\rm F}$ limit   fermion loops only $Z_\Psi = 1$

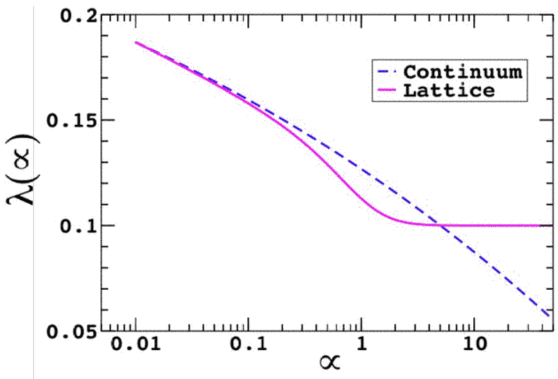
$m_0^2 Z_\phi = m^2 + \delta m^2$ $\lambda_0 Z_\phi^2 = \lambda + \delta\lambda$ $y_0 Z_\phi^{1/2} = y + \delta y$

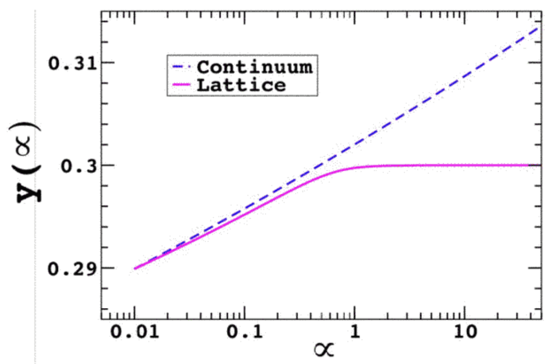
$\delta m^2 = 4N_{\rm F}y^2 \int_k^{\Lambda} 1/k^2$ $\delta\lambda = -24N_{\rm F}y^4 \int_k^{\Lambda} 1/[k^2+\mu^2]^2$ $\delta y = 0$

Fix bare couplings   physical properties (e.g. masses) fixed

$m^2(\mu), \lambda(\mu), y(\mu), Z_\phi(\mu)$ run with μso theory unchanged

Triviality, again





Arbitrary choice $\lambda_0 = 0.1, y_0 = 0.3$ μ lattice spacing units

Continuum RG $y^2(\mu) = y_1^2/[1 - (N_F y_1^2/4\pi^2)\ln\mu]$

$\lambda(\mu) = 12y^2(\mu) + c_1 y^4(\mu)$

$\mu \ll 1 \Rightarrow y(\mu), \lambda(\mu) \to 0$ Trivial

$\mu \gg 1 \Rightarrow y(\mu) \to y_0, \lambda(\mu) \to \lambda_0$ continuum RG breaks down

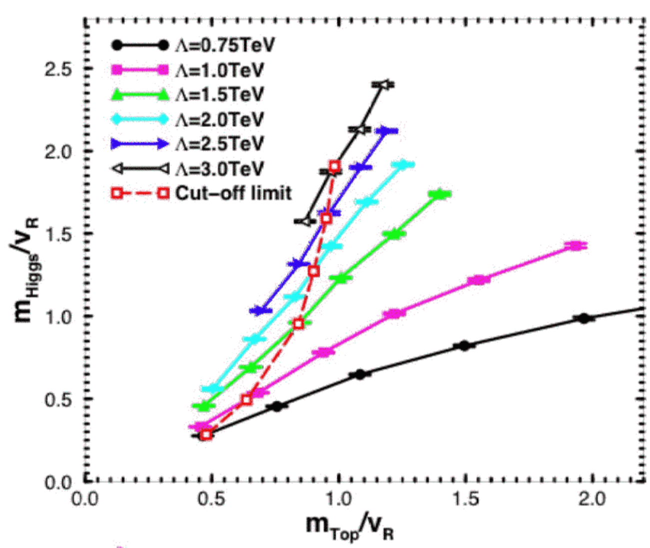
Vacuum instability $\lambda(\mu) < 0$ does not actually occur

Is there a Higgs lower bound?

Very similar to upper bound

For fixed cutoff, explore all possible bare couplings

Summary of Higgs-Yukawa simulations $N_F = 8$



Fixed m_{Top}, Λ

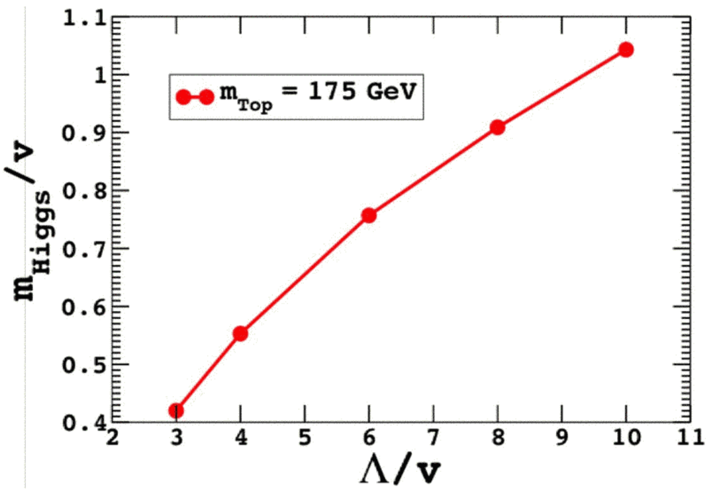
m_H smallest for $\lambda_0 \to 0$

recall upper bound: m_H largest for $\lambda_0 \to \infty$

Cut-off limit: $\xi/a = 1/(m_{ph}a) \geq 2$

Left of dashed line, cut-off effects acceptably small

lower bound



Fixed $m_{\rm Top}$, extract Higgs lower bound from summary plot

Convert to physical units $v_{\rm R} = 246$ GeV

Cannot have cut-off too small otherwise cut-off effects dominate

Ambiguity in bound

Large $N_{\rm F}$ limit

Extract lower bound from perturbation theory

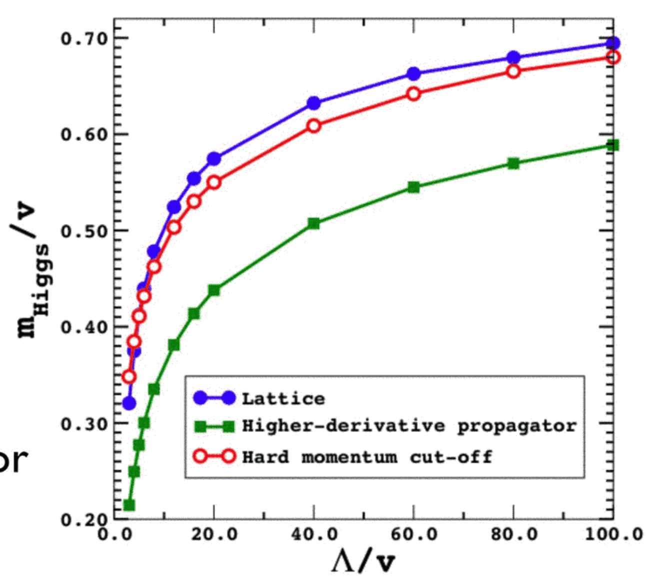
Compare different regulators

a. lattice

b. hard momentum cut-off

c. higher-derivative propagator

$$k^2 \to k^2(1 + k^2/\Lambda^2)^2$$



Bound not universal quantity

Cannot make arbitrarily precise regulator-independent prediction

Summary

- Vacuum instability is triviality in disguise
- can determine Higgs lower bound, but beware
- realistic model: O(4) Higgs, single Top, gluons
- crossing the gap between lattice and continuum