Heavy Quark Physics in 2+1-Flavor Lattice QCD

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KITP Program
"Modern Challenges for Lattice Field Theory"
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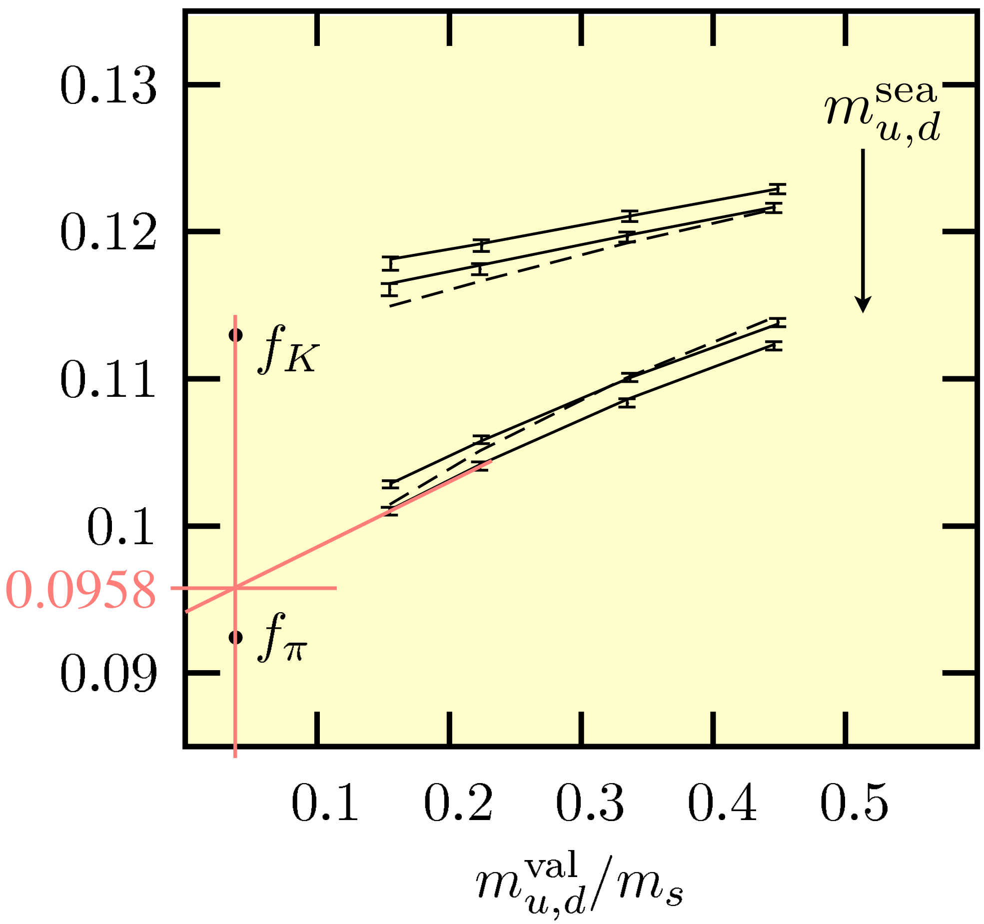
Outline

- Semileptonic *D* decays
  - Chiral extrapolation with and without BK
  - Estimation of discretization effects
- *D* Meson Decay Constant
  - Chiral extrapolation with stagPQχPT
- Mass of the B_c Meson
  - Estimation of discretization effects

Preliminaries

- 2+1 flavor calculations with improved staggered quarks have reproduced PDG values of a wide variety of masses, mass splittings, and decay constants.
- Results assume (and suggest!?)
  - $[\det_4 M]^{1/4} \doteq \det_1(\not{D} + m)$
  - staggered (partially quenched) chiral PT
  - effective field theories for heavy quarks

Chiral Extrapolation



- Dots at 0.04 are experimental.
- Error bars are lattice QCD.
- Linear extrap (by eye).
- Gasser-Leutwyler χlog gets closer (solid).
- Sharpe-Shoresh χlog even closer (dashed).

- Thus encouraged, HPQCD, MILC, and Fermilab Lattice Collaborations are using these methods to calculate matrix elements relevant to *flavor physics*.
- The stakes are high: “Are non-Standard phenomena visible in *B* decays?”

Proofs

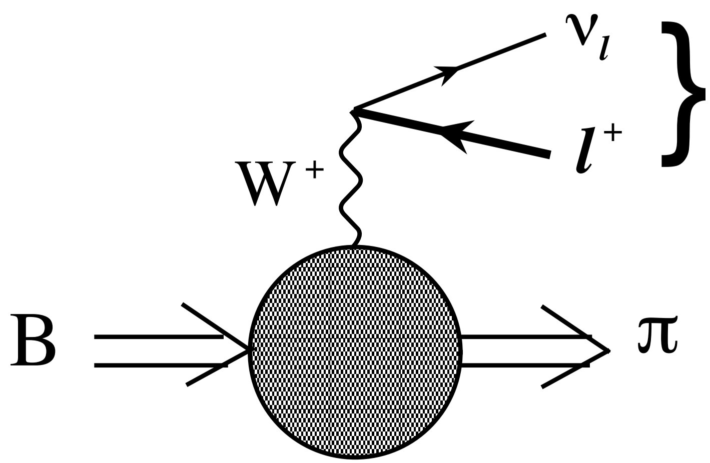
- We need physicists' proofs that the methods are sound.
  - For heavy quarks, using HQET/NRQCD as a theory of cutoff effects suffices.
  - For staggered quarks, the fourth-root trick could benefit from a better foundation, but (I think) most of the simple arguments against it are lame.

Tests

- As a complement to (quasi)-mathematical proofs, other tests are desirable.
- Experimenters suggest making *predictions*.
- *D* meson decay properties and B_c mass are being improved by ongoing experiments.

$$f_+^{D \to \pi}(q^2) \;\&\; f_+^{D \to K}(q^2)$$

Semileptonic Decay



$$q^2 = m_B^2 + m_\pi^2 - 2m_B E$$

$$\frac{d\Gamma}{dE} = \frac{G_\mu^2 m_B}{12\pi^3}|V_{ub}|^2 p^3 |f_+(E)|^2$$

$$\langle \pi(p_\pi)|\mathcal{V}^\mu|B(p_B)\rangle = f_+(E)\left[p_B + p_\pi - \frac{m_B^2 - m_\pi^2}{q^2}q\right]^\mu + f_0(E)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

Polology

- For $E < 0$, there are poles and cuts, and so on, from real states in $l\nu$ scattering.
  - vector mesons for f_+ at
  - scalar mesons for f_0
- Their effects spill into physical region $E > 0$.
- For D and B mesons, the vector is nearby.

BK *Ansatz*

- With this in mind Becirevic and Kaidalov proposed the parametrization

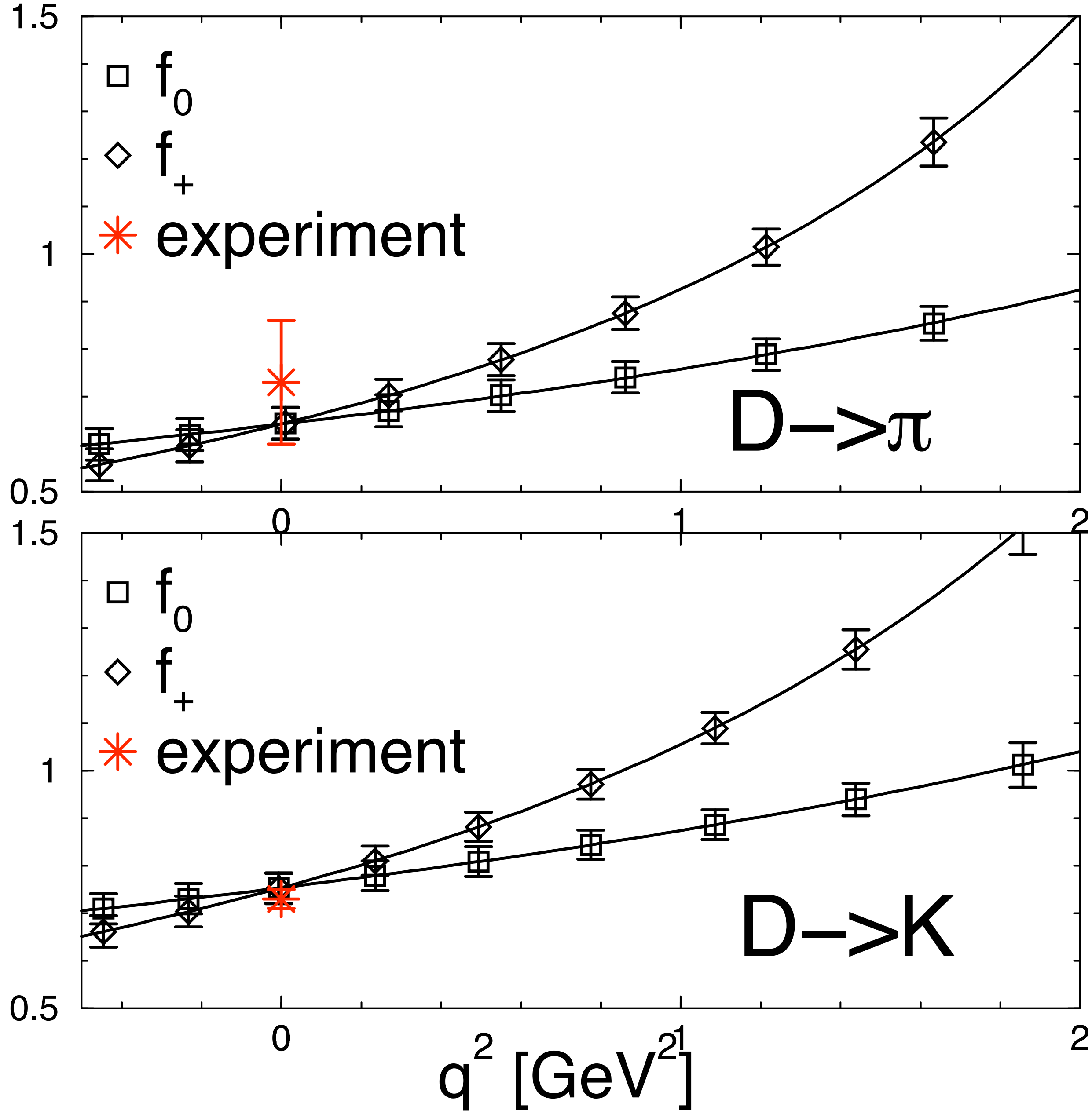
$$f_+(q^2) = \frac{f(0)}{(1-q^2/m^2_{D*})(1-\alpha q^2/m^2_{D*})}$$

$$f_0(q^2) = \frac{f(0)}{(1-q^2/m^2_{D*}/\beta)}$$

- Builds in the closest pole, and has parameters for the slop.

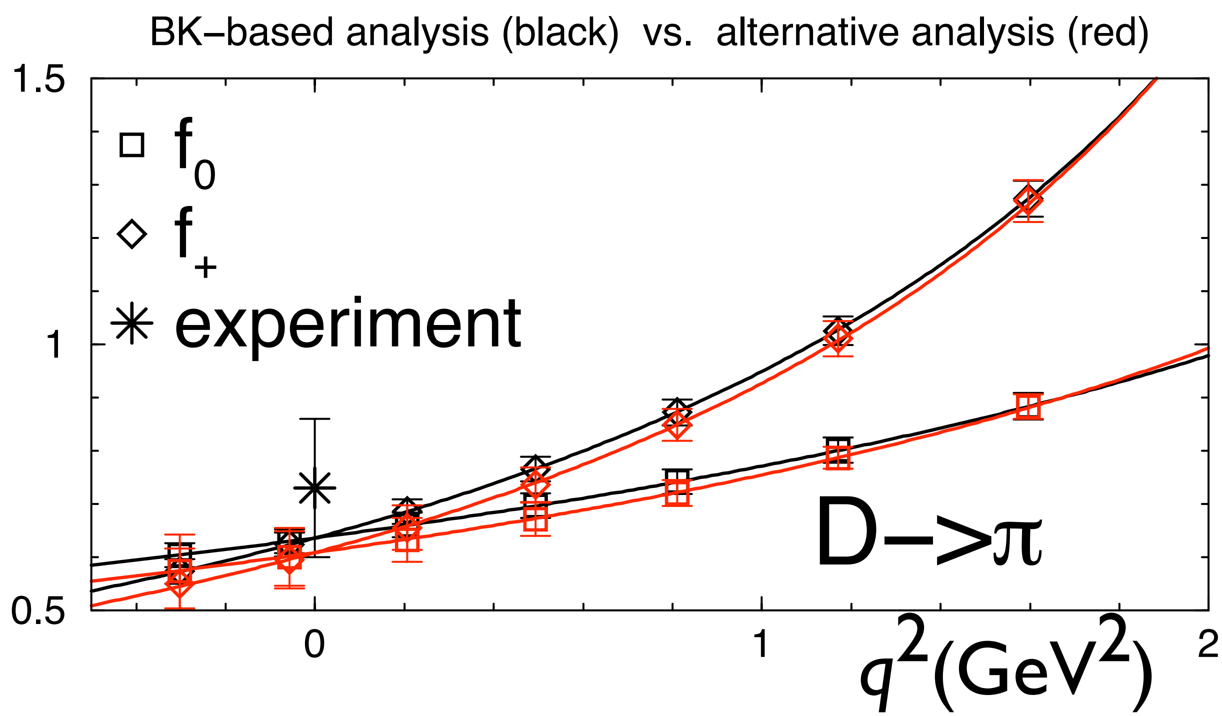
- Advantages
  - builds in pole, & also heavy-quark scaling laws
  - fit to BK is most sensitive to low energy, yet f_0 influences f_+ through $f(0)$.
- Disadvantages
  - parametrization deteriorates with E
  - fit to BK is sensitive mostly to low energy, and f_0 determines $f(0)$.

- Analysis method
  - calculate matrix elements for various (m_q, $\boldsymbol{p}$).
  - use BK to interpolate to fiducial values of *E*, same for each ensemble.
  - use staggered χPT for chiral extrapolation
  - use BK to extrapolate to full kinematic range



f_0
f_+
experiment
D->π
D->K
1.5
1
0.5
0
1
2
q^2 [GeV^2]

- An alternative is to avoid BK altogether, and use χPT to extrapolate jointly in (m_q, E):



- Consistent, but no-BK has larger error in low q^2 (high E) region.

hep-ph/0408306

dominant error: heavy quark discretization

- $D \to Kl\nu$

$$f_+^{D\to K}(0) = 0.73(3)(7)$$

$$f_+^{D\to K}(0) = 0.78(5)$$ [BES, hep-ex/0406028]

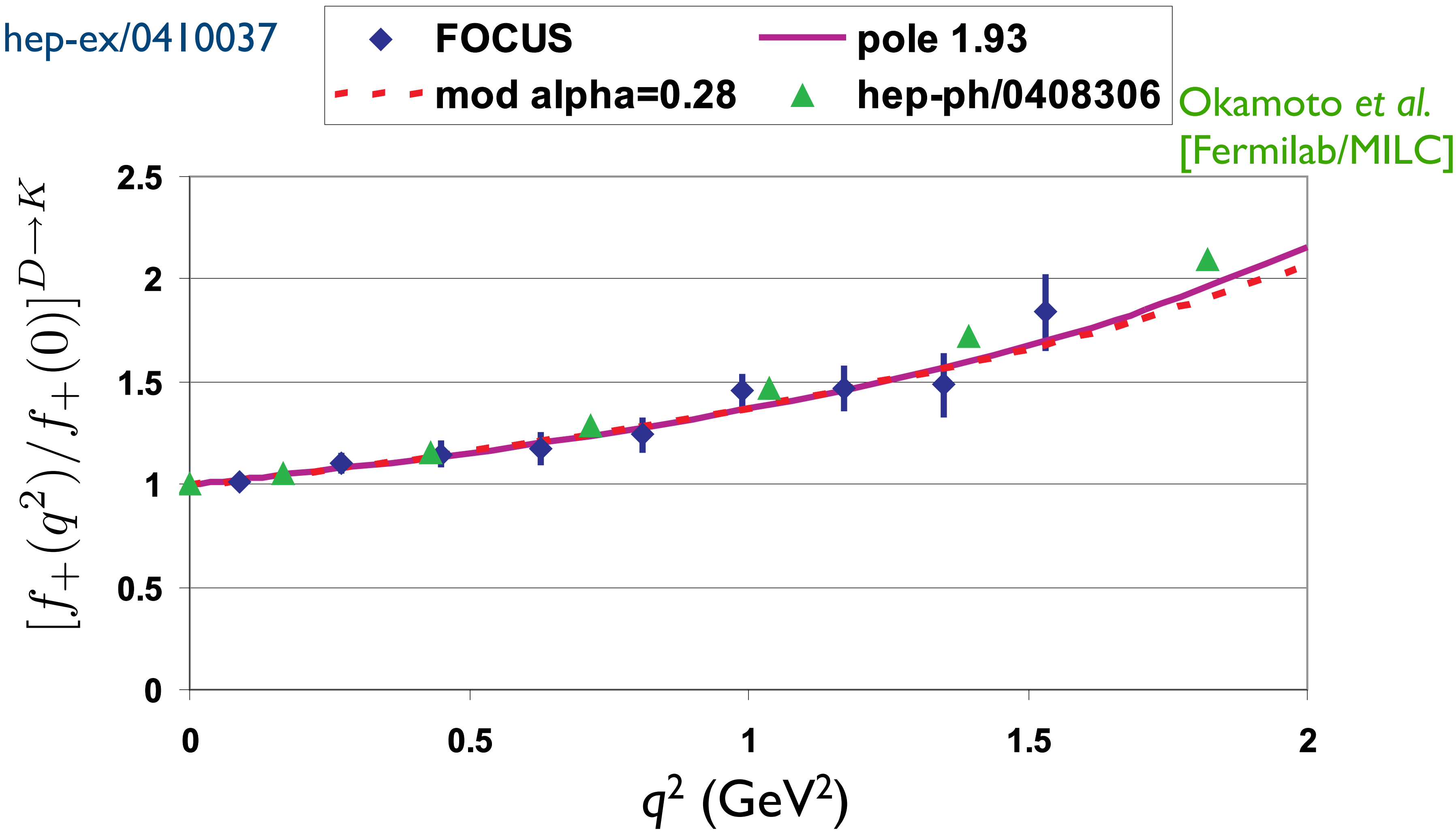
- $D \to \pi l\nu$:

$$f_+^{D\to \pi}(0) = 0.64(3)(6)$$

$$f_+^{D\to \pi}(0) = 0.87(3)(9) f_+^{D\to K}$$

$$f_+^{D\to \pi}(0) = 0.86(9) f_+^{D\to K}$$ [CLEO, hep-ex/0407035]

$D \to K l \nu$ vs. q^2



Discretization Effects

- Dominant error, but only one sentence!
- Both QCD and LGT can be described by

$$\mathcal{L}_{\text{QCD}} \doteq \mathcal{L}_{\text{HQET}} = \sum_i \mathcal{C}_i^{\text{cont}}(m_Q)\mathcal{O}_i$$

$$\mathcal{L}_{\text{LGT}} \doteq \mathcal{L}_{\text{HQET}(m_0 a)} = \sum_i \mathcal{C}_i^{\text{lat}}(m_Q, m_0 a)\mathcal{O}_i$$

- Discretization error is in mismatch of coefficients.

- In general,

$$\texttt{error}_i = \left| \left[\mathcal{C}_i^{\rm lat}(m_Q, m_0 a) - \mathcal{C}_i^{\rm cont}(m_Q) \right] \mathcal{O}_i \right|$$

- For Wilson(-like) quarks write

$$\mathcal{C}_i^{\rm lat}(m_Q, m_0 a) - \mathcal{C}_i^{\rm cont}(m_Q) = a^{\dim \mathcal{O}_i - 4} f_i(m_0 a)$$

- For heavy-light use HQET to order and estimate

$$\texttt{error}_i = f_i(m_0 a)(a\Lambda_{\rm QCD})^{\dim \mathcal{O}_i - 4}$$

- What would you use for Λ_{QCD}?
- Based on estimates of the Λ that appears in the heavy-quark expansion—from lattice, sum rules, and *experiment*—the sensible range is
  - Λ_{QCD} = 500–700 MeV

Λ (MeV):	400	500	600	700	800	900	1000
$\texttt{error}_B$ [$O(\alpha_s a)$ Lagrangian]	1.8	2.2	2.6	3.1	3.5	3.9	4.4
$\texttt{error}_3$ [$O(\alpha_s a)$ current]	1.1	1.4	1.7	2.0	2.2	2.5	2.8
$\texttt{error}_E$ [$O(a^2)$ Lagrangian]	0.4	0.6	0.9	1.2	1.5	1.9	2.4
($c_E = 0$)	1.1	1.6	2.4	3.2	4.2	5.3	6.6
$\texttt{error}_X$ [$O(a^2)$ current]	0.9	1.3	1.9	2.6	3.4	4.3	5.4
(d_1 off)	1.3	2.0	2.8	3.9	5.0	6.4	7.9
$\texttt{error}_Y$ [$O(a^2)$ current]	0.4	0.6	0.8	1.1	1.5	1.8	2.3
temporal total	2.8	3.6	4.7	5.9	7.2	8.7	10.5
spatial total	3.2	4.1	5.3	6.6	7.8	9.4	11.2

Pending studies on finer lattices, we quoted sum in quadrature of both currents, at Λ_{QCD} = 700 MeV

All of CKM

- Okamoto has combined our (*i.e.*, his) calculations of $D \to \pi$ and $D \to K$ with preliminary calculations of $B \to D$ to obtain the middle row of the CKM matrix.
  - add $B \to \pi$, $K \to \pi$ and unitarity to get the top row, the right column and $|V_{ts}|$.
  - add $\sin(2\beta)$ to get the last element

hep-lat/0412044

f_{D_s} & f_D

f_{D_s} & f_D

- *D* meson decay constants either
  - determine $|V_{cs}|$ and $|V_{cd}|$
  - check QCD (with $|V_{cs}|$ and $|V_{cd}|$ from CKM unitarity).
- CLEO-*c* is measuring them.
- A test of light quarks and (staggered) PQχPT.

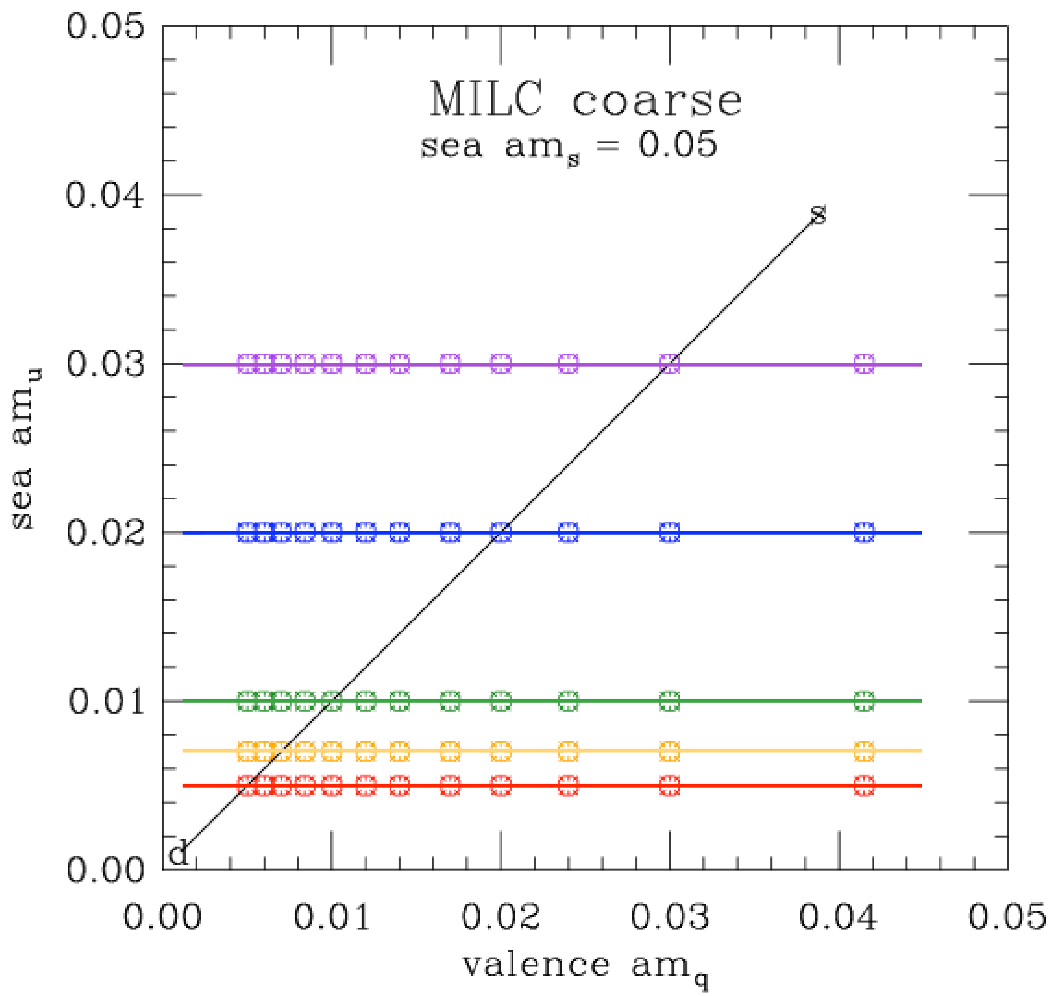
Staggered PQχPT

- In the case of decay constants, chiral logs are important.
- In staggered PQχPT, Aubin & Bernard find

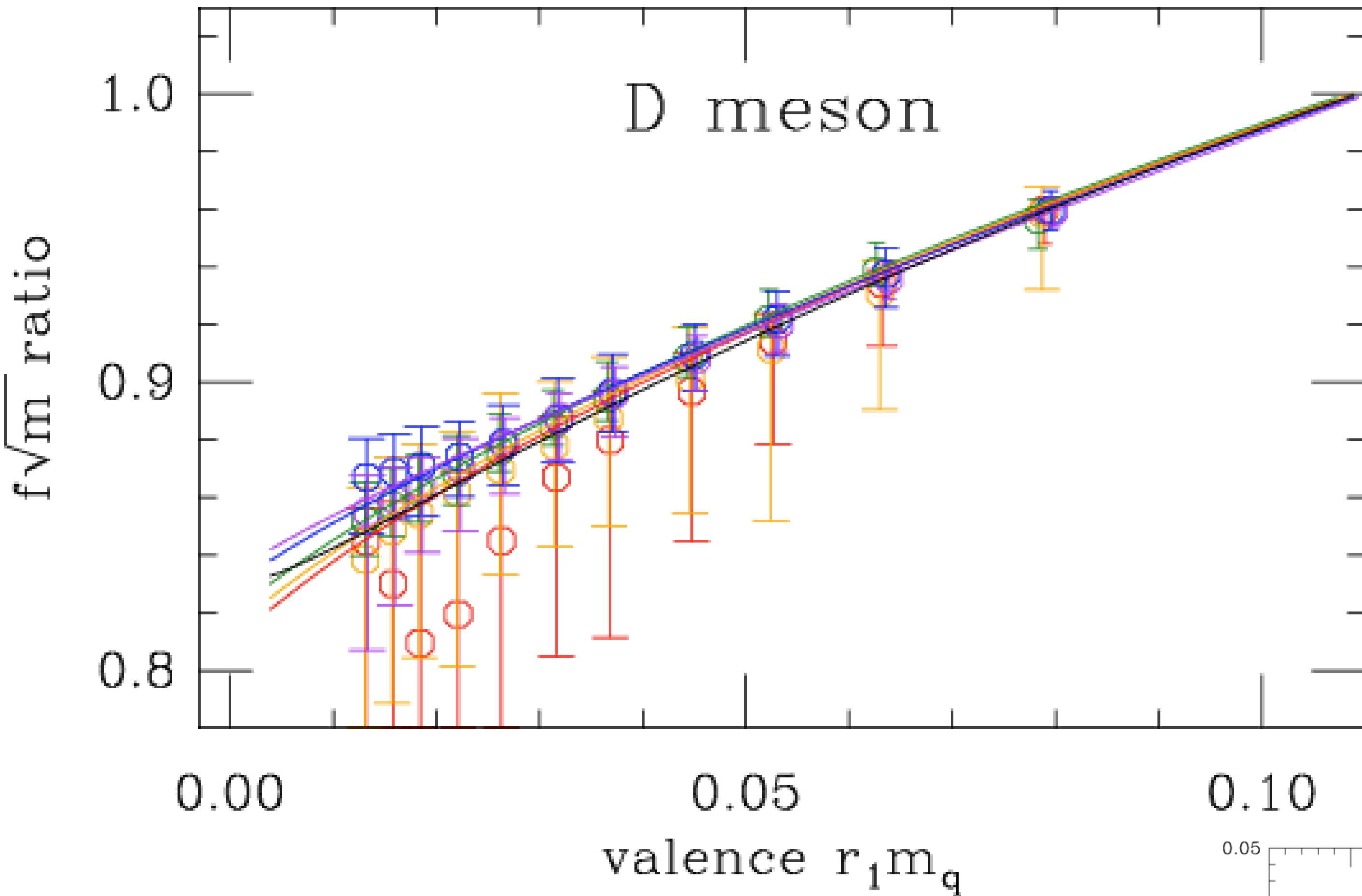
$$m_{uu}^2 \ln m_{qq}^2 \to \begin{cases} m_{uu}^2 \ln m_{\text{average}}^2 \\ m_{uu}^2 \ln m_{\text{taste singlet}}^2 \end{cases}$$

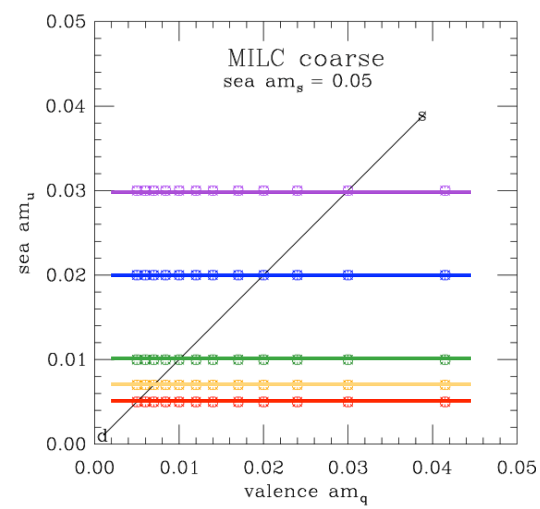
so singularity of PQχPT softened.

Chiral Extrapolation f_D



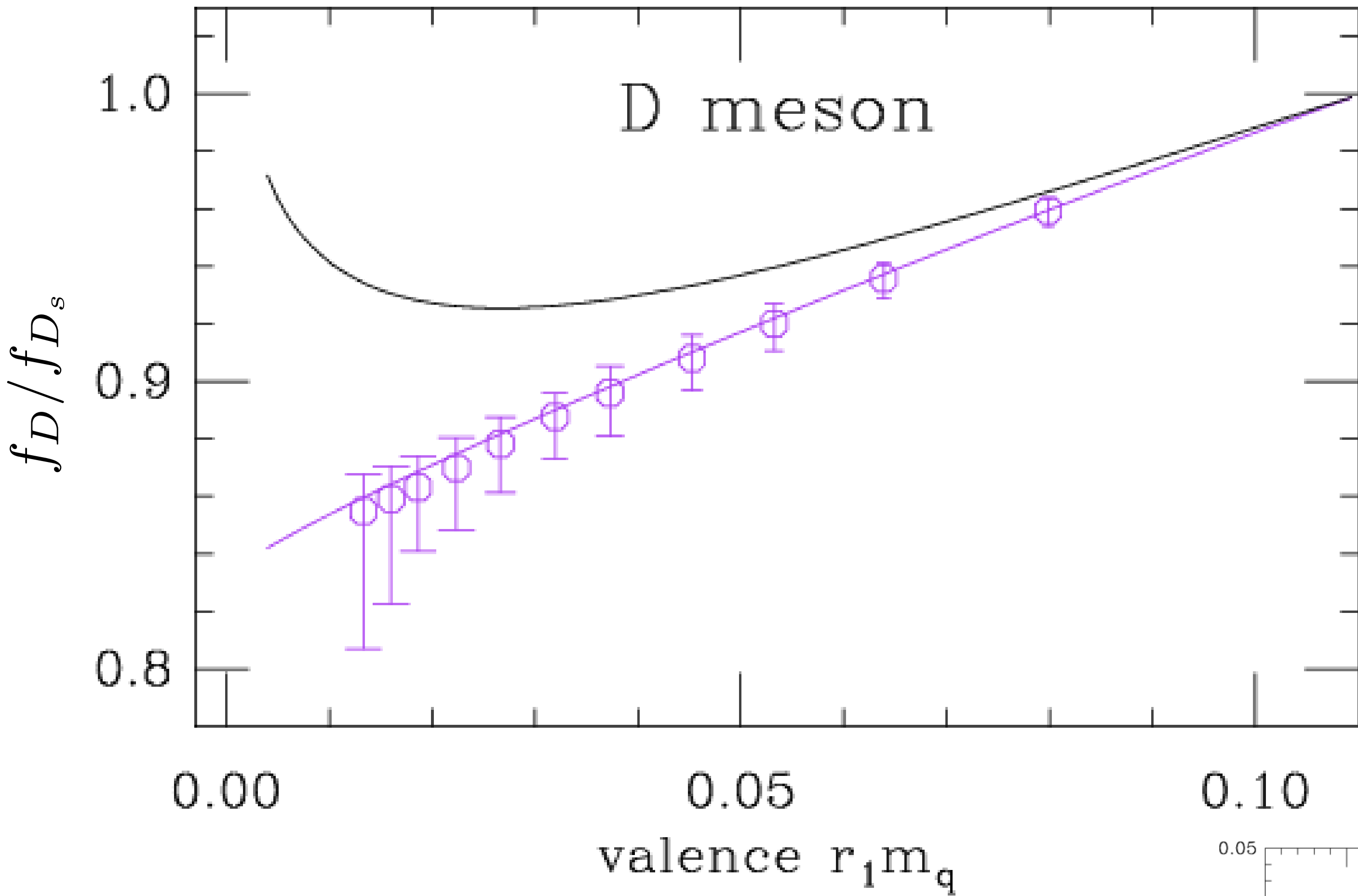
- Extrapolate in sea m_u and valence m_q to get down to real m_l.
- Single fit to all data constrains χPT better.
- Staggered PQχPT treats all a in same fit.

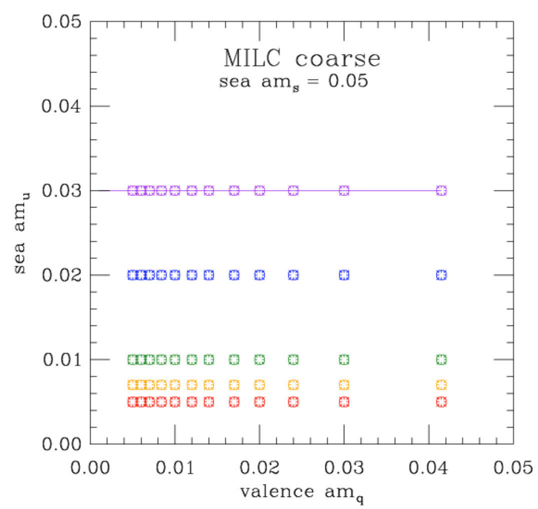




Fit all 60 combinations of (am_u, am_q).

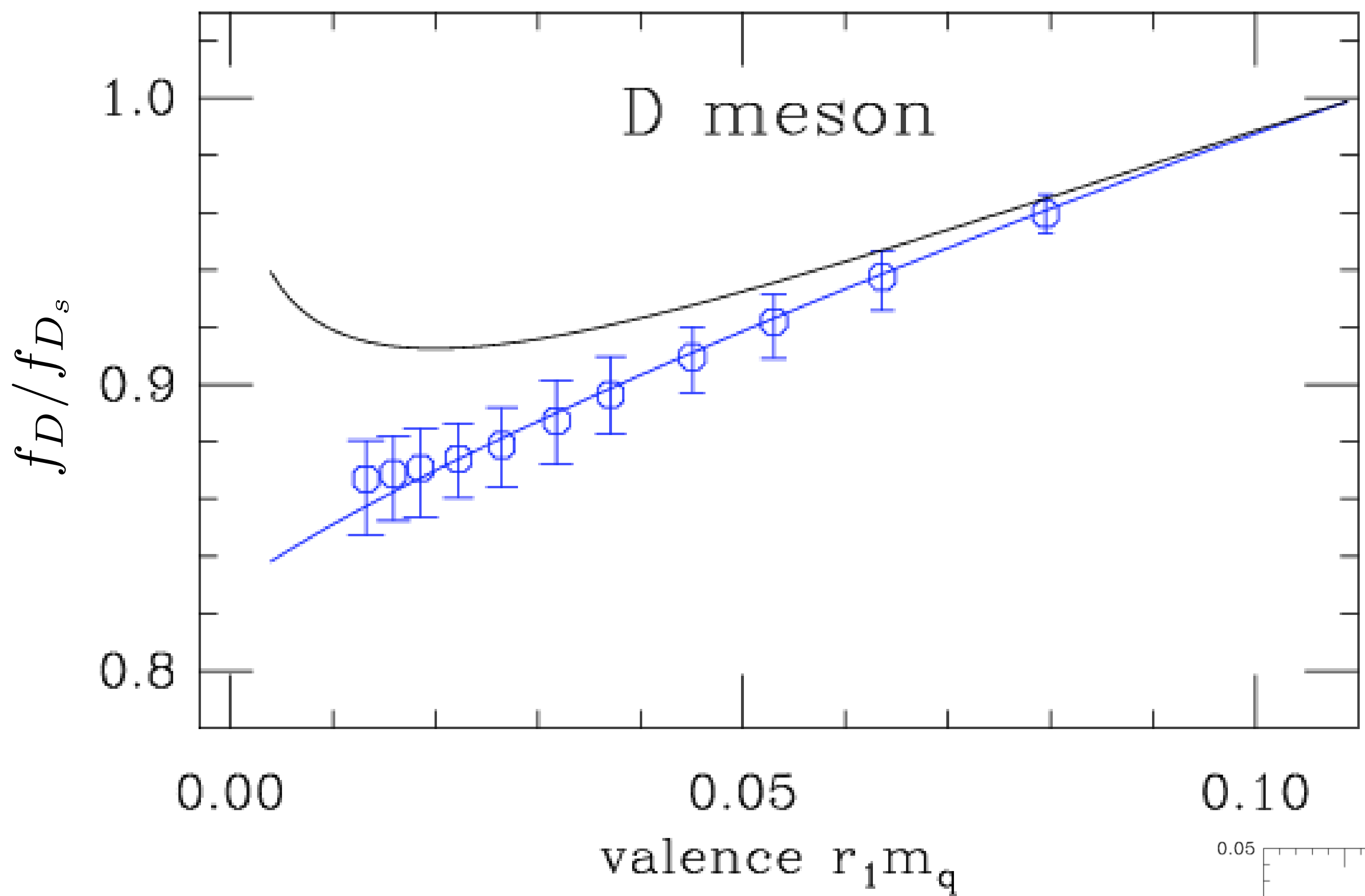
Quality of the fit is obvious, right?

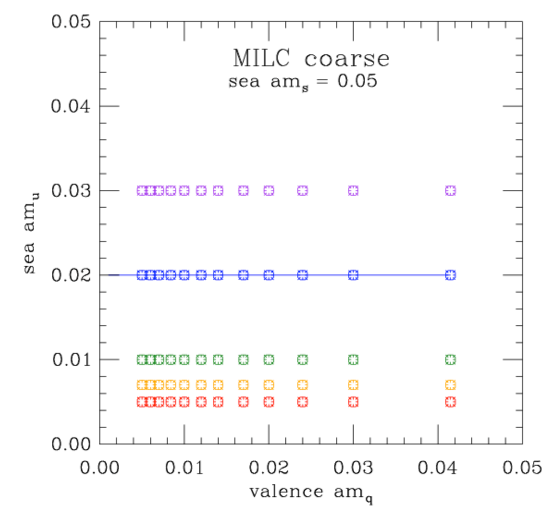




am$_u$ = 0.03 part of stagPQχPT fit.

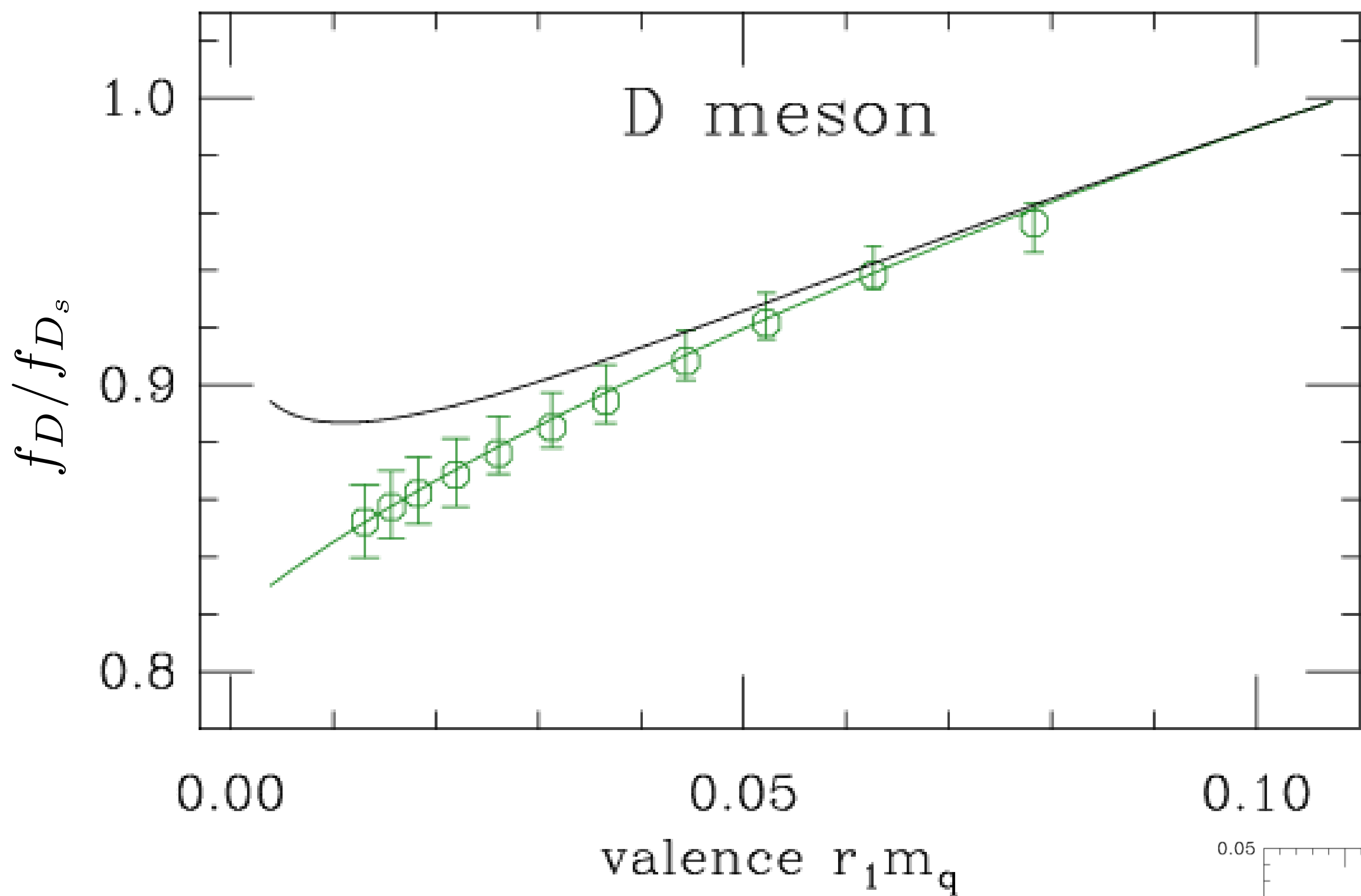
With $O(a^2)$ bits turned off.

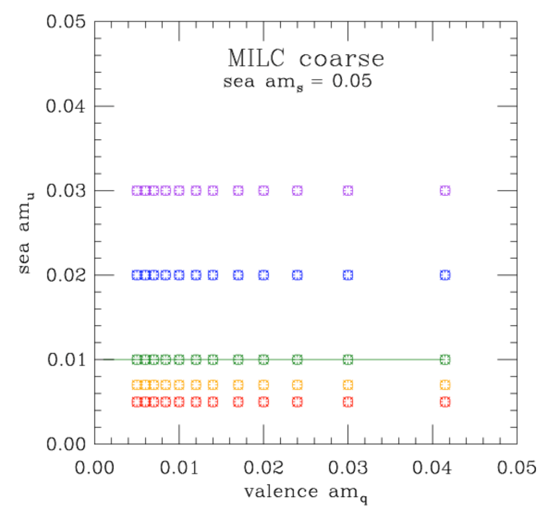




am_u = 0.02 part of stagPQχPT fit.

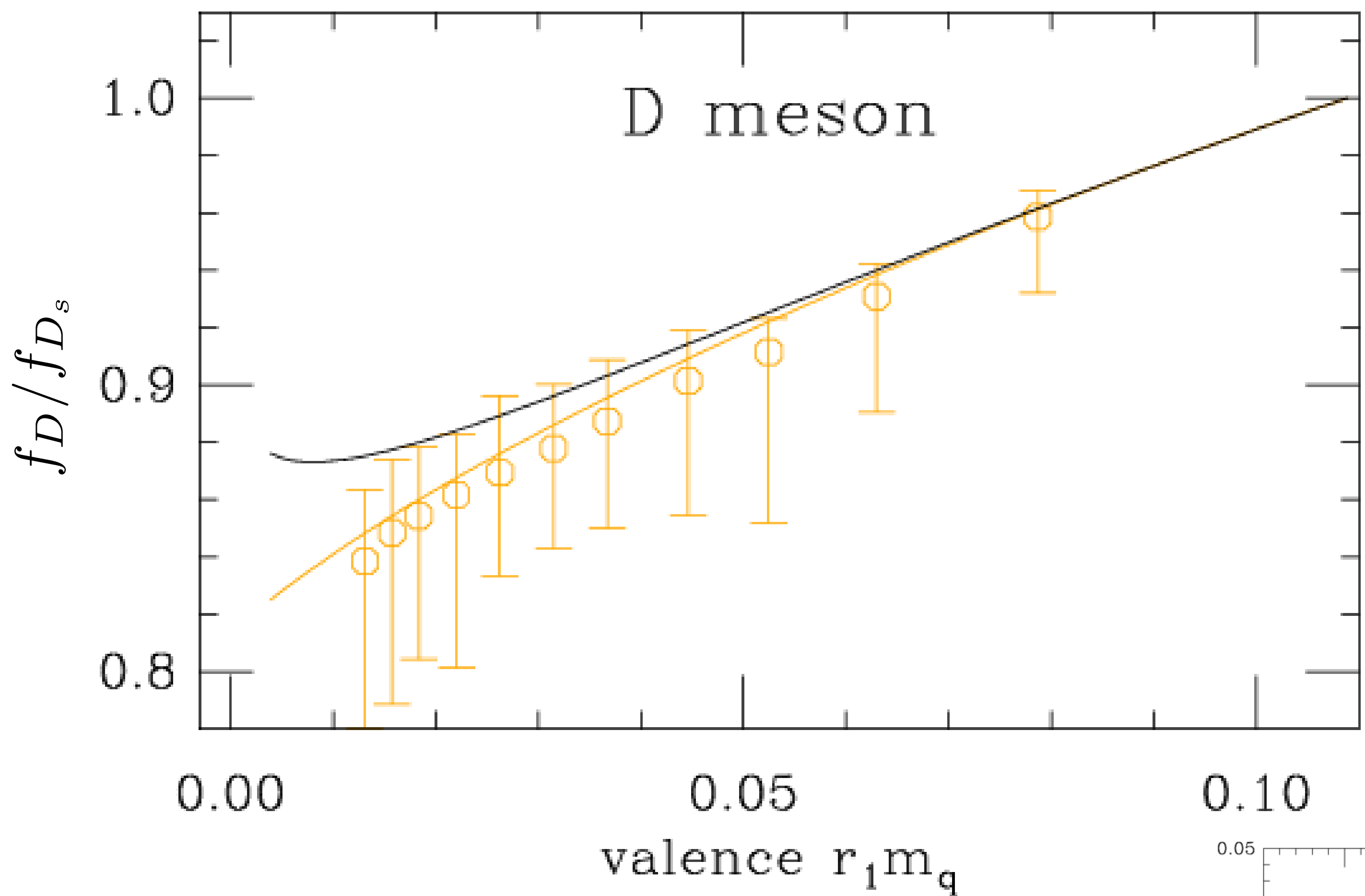
With $O(a^2)$ bits turned off.

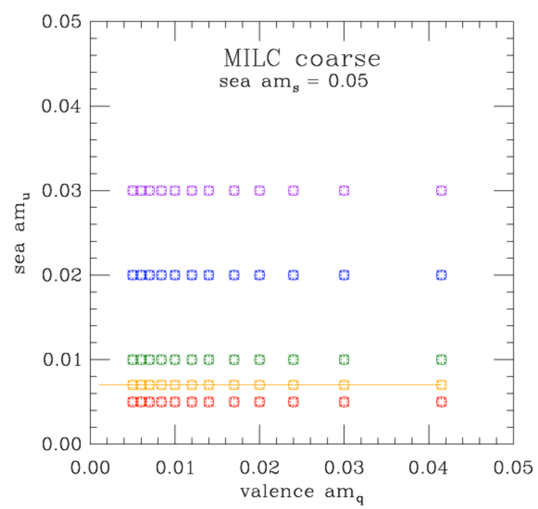




am_u = 0.01 part of stagPQχPT fit.

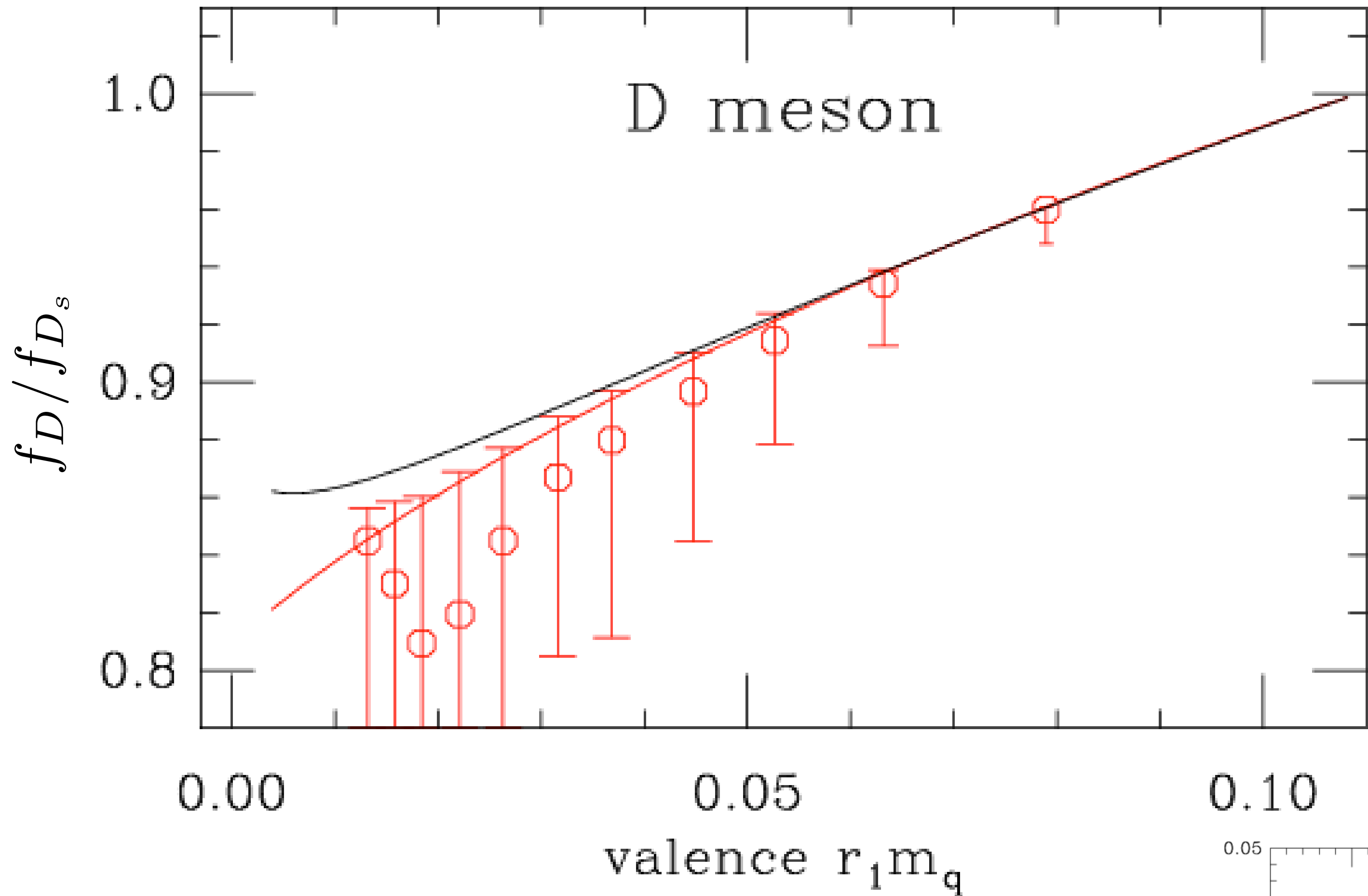
With $O(a^2)$ bits turned off.





am_u = 0.007 part of stagPQχPT fit.

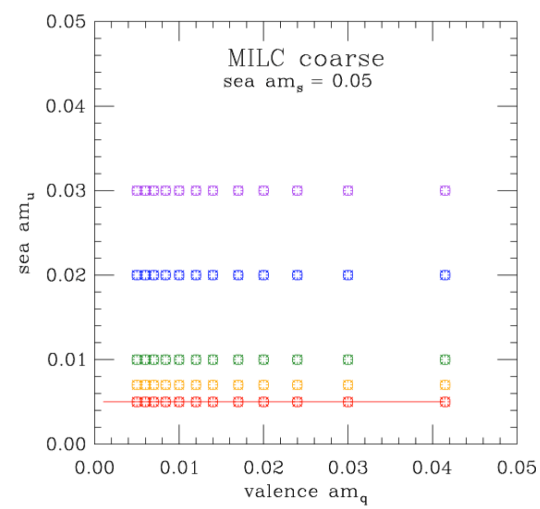
With $O(a^2)$ bits turned off.

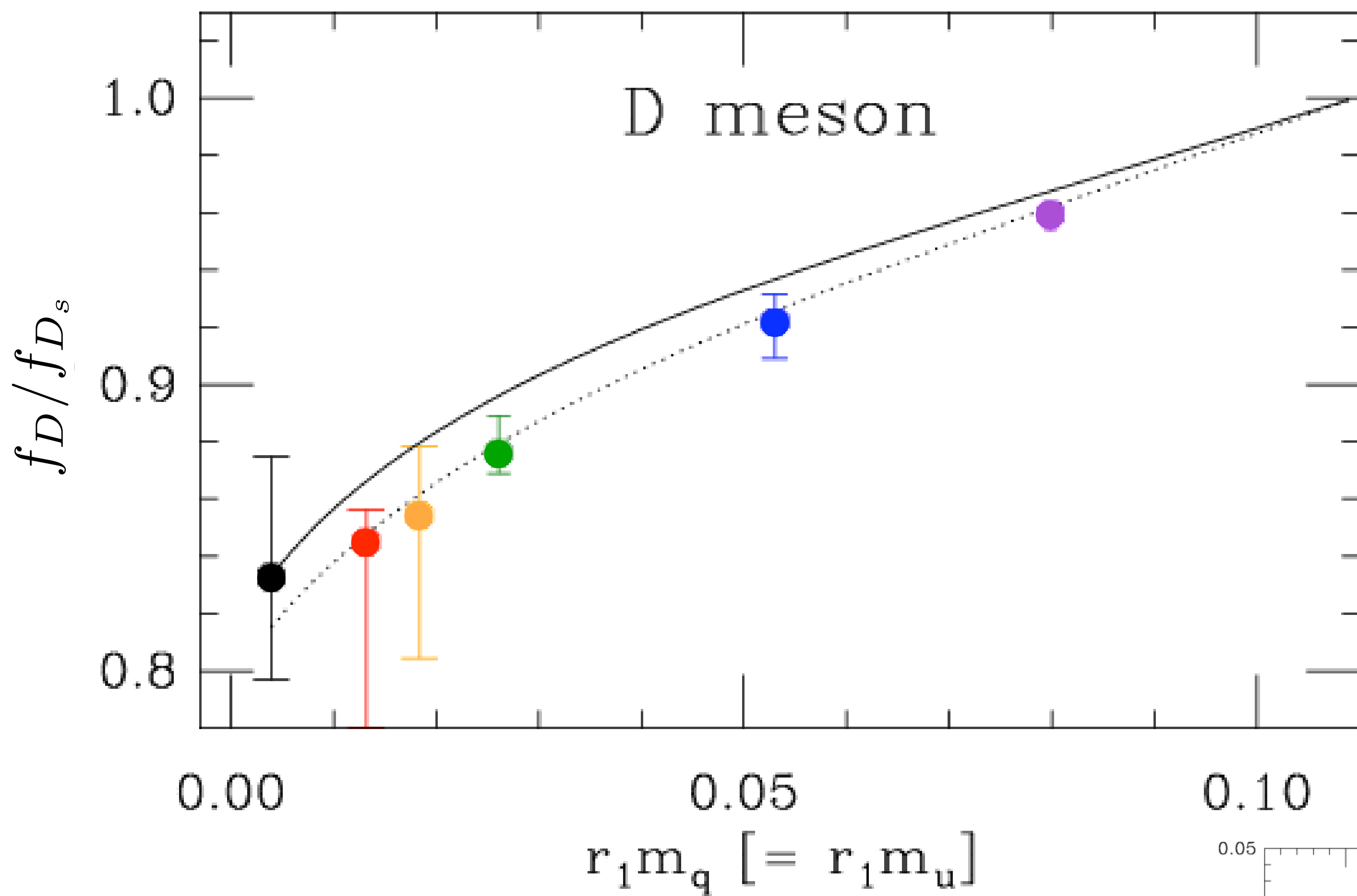


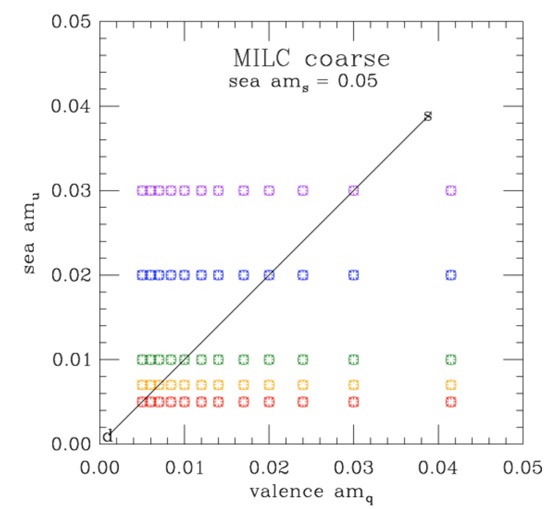
only 200 configs so far

am_u = 0.005 part of stagPQχPT fit.

With $O(a^2)$ bits turned off.





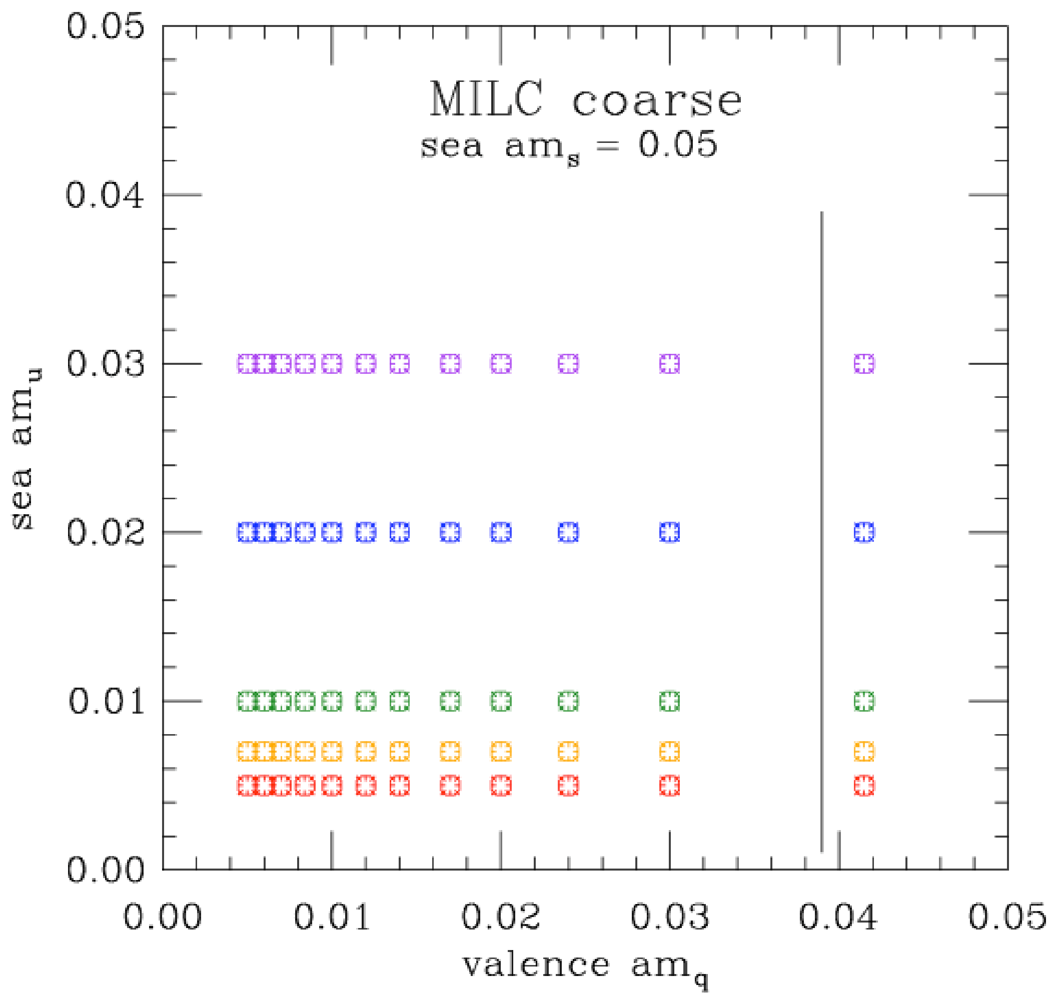


$am_u = am_q$ part

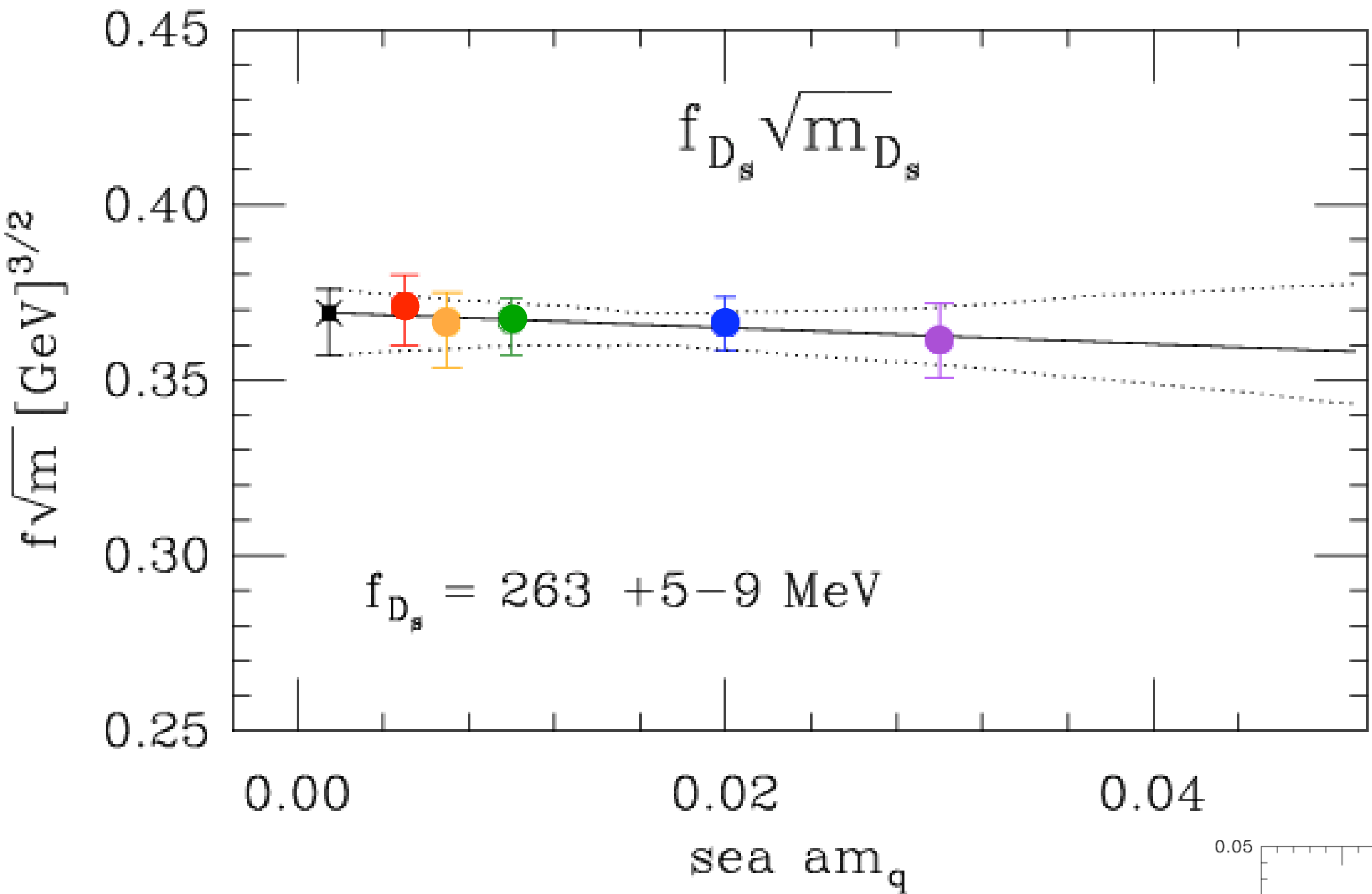
of stagPQχPT fit (dotted)

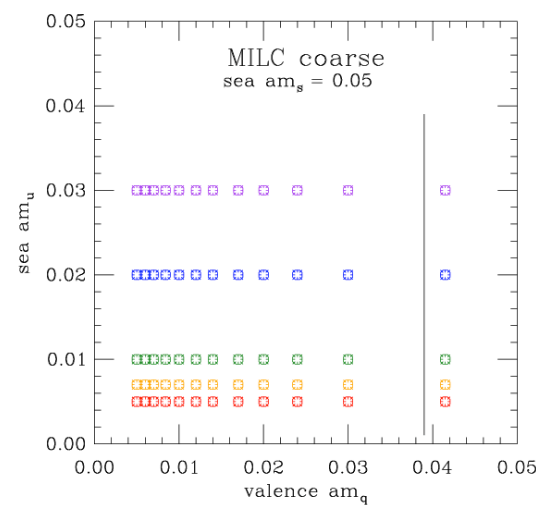
with $O(a^2)$ bits turned off (solid)

Chiral Extrapolation f_{Ds}



- Interpolate in valence m_q to get down to real m_s.
- Extrapolate in sea m_u to get down to real m_l.





Currently obtained in a separate linear fit.

Preliminary Results

- J. Simone *et al.*, hep-lat/0410030 (Lattice '04)

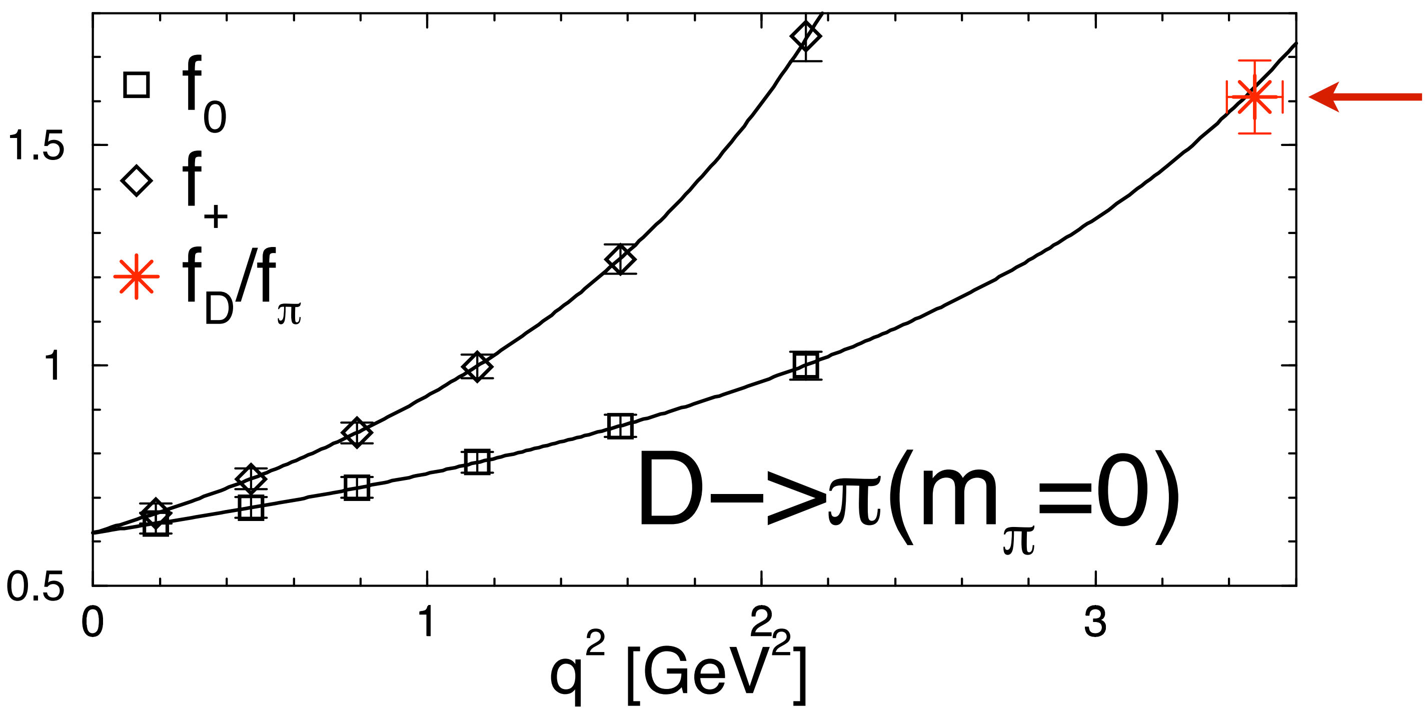
$$
\begin{aligned}
\frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} &= 1.20 \pm 0.06 \pm 0.06 \;, \\
f_{D_s} &= 263^{+5}_{-9} \pm 24 \quad \text{MeV} \;, \\
f_D &= 225^{+11}_{-13} \pm 21 \quad \text{MeV} \;.
\end{aligned}
$$

$f_D = 202 \pm 41 \pm 17$ MeV
CLEO-c, `hep-ex/0411050`

discretization uncertainty as in form factors.

Soft pion theorem

$$f_0^{D\to\pi}(q^2_{\rm max}) = \frac{f_D}{f_\pi}$$



$$\frac{f_D^{\rm LQCD}}{f_\pi^{\rm PDG}}$$

Outlook

- We will combine form factors and decay constants to obtain combinations that can be compared directly to experiment, with no CKM input:

$$\frac{1}{\Gamma_{D\to l\nu}} \frac{d\Gamma_{D\to\pi l\nu}}{dq^2} \propto \left| \frac{f_+^{D\to\pi}(q^2)}{f_D} \right|^2$$

$$\frac{1}{\Gamma_{D_s\to l\nu}} \frac{d\Gamma_{D\to K l\nu}}{dq^2} \propto \left| \frac{f_+^{D\to K}(q^2)}{f_{D_s}} \right|^2$$

B_c

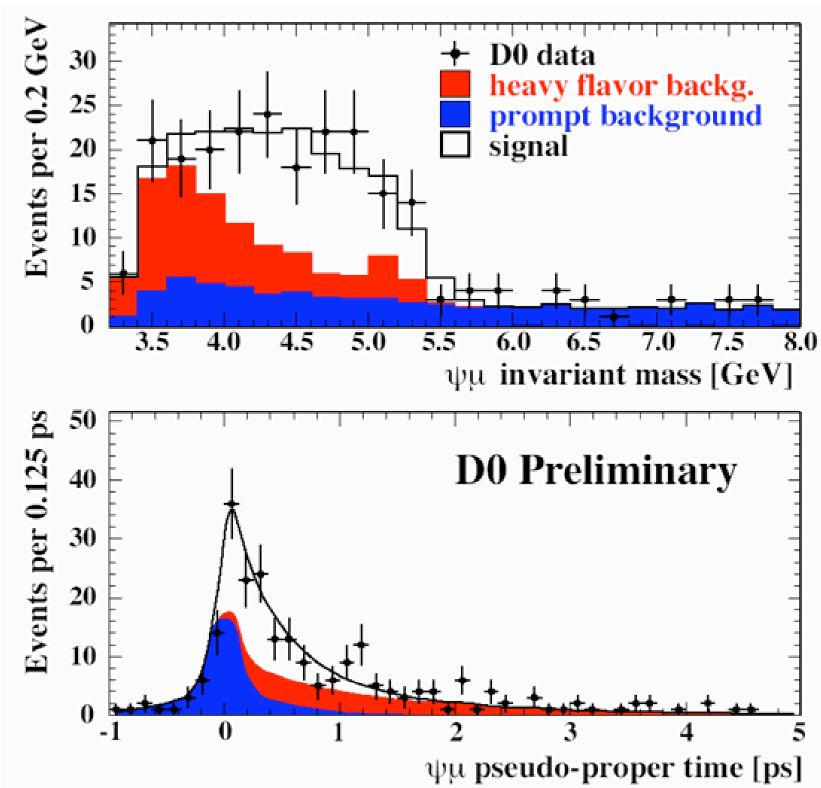
B_c

- Meson composed of a beautiful anti-quark and a charmed quark.
- Unusual beast
  - contrast with B_s & D_s, ψ & Υ: $v_c = 0.7$.
  - no annihilation to gluons

Fermilab Today — Thursday & Friday, December 2 & 3, 2004

Fermilab Result of the Week

DØ



Fermilab Result of the Week

CDF

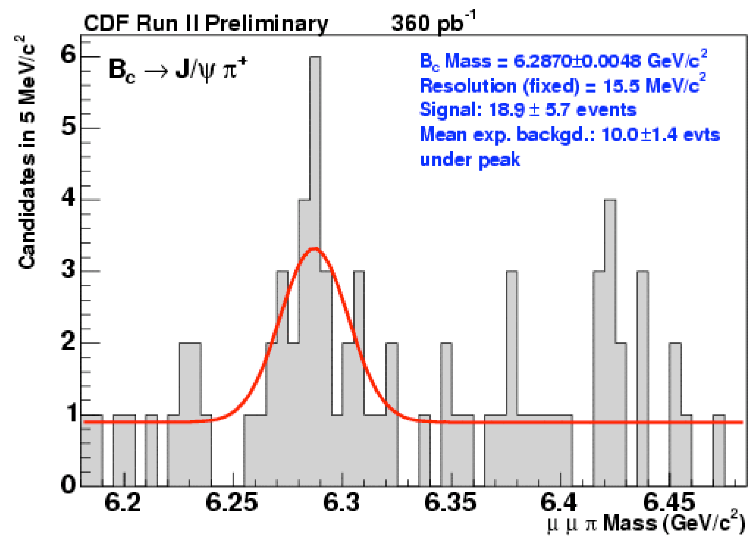
4:00 p.m. One West

Joint Experimental Theoretical Physics Seminar

Saverio D'Auria, University of Glasgow

B_c: Fully Reconstructed Decays and

Mass Measurement at CDF



QCD Theory & B_c

- Three main tools
  - potential models
  - potential NRQCD
  - lattice QCD
- All treat both quarks as non-relativistic
  - charmed quark is pushing it, $v_c^2 = 0.5$.

Energy Scales

- Several energy scales in (this) quarkonium
  - $2m_b, 2m_c > 2$ GeV
  - $m_b v_b = m_c v_c \approx 1000$ MeV
  - $\frac{1}{2} m_c v_c^2 \approx 350$ MeV, $\quad \frac{1}{2} m_b v_b^2 \approx 50$ MeV
  - $\Lambda_{QCD} \sim 500$ MeV

NRQCD

integrate out scale m_Q

$$\mathcal{L}_{\rm QCD} \doteq \mathcal{L}_{\rm HQ}$$

$$\begin{aligned}
\mathcal{L}_{\rm HQ} &= \mathcal{L}_{\rm light} - \bar{h}_v(m_1 + iv\cdot D)h_v + \frac{\bar{h}_v D_\perp^2 h_v}{2m_2} \\
&+ z_B(\mu)\frac{\bar{h}_v s_{\mu\nu}B^{\mu\nu}h_v}{2m_2} - z_R(\mu)\frac{\bar{h}_v (D_\perp^2)^2 h_v}{8m_2^3} \\
&+ z_{\rm D}(\mu)\frac{\bar{h}_v D_\perp\cdot E h_v}{4m_2^2} + z_{\rm s.o.}(\mu)\frac{\bar{h}_v s_{\mu\nu} D_\perp^\mu E^\nu h_v}{4m_2^2} \\
&+ \cdots \\
&\doteq \sum_i \quad \mathcal{C}_i(m_Q, m_Q/\mu) \quad \mathcal{O}_i(\mu/m_Q v^n)
\end{aligned}$$

Lattice errors (m_1, $z_B(\mu)$, $z_R(\mu)$, $z_{\rm D}(\mu)$)

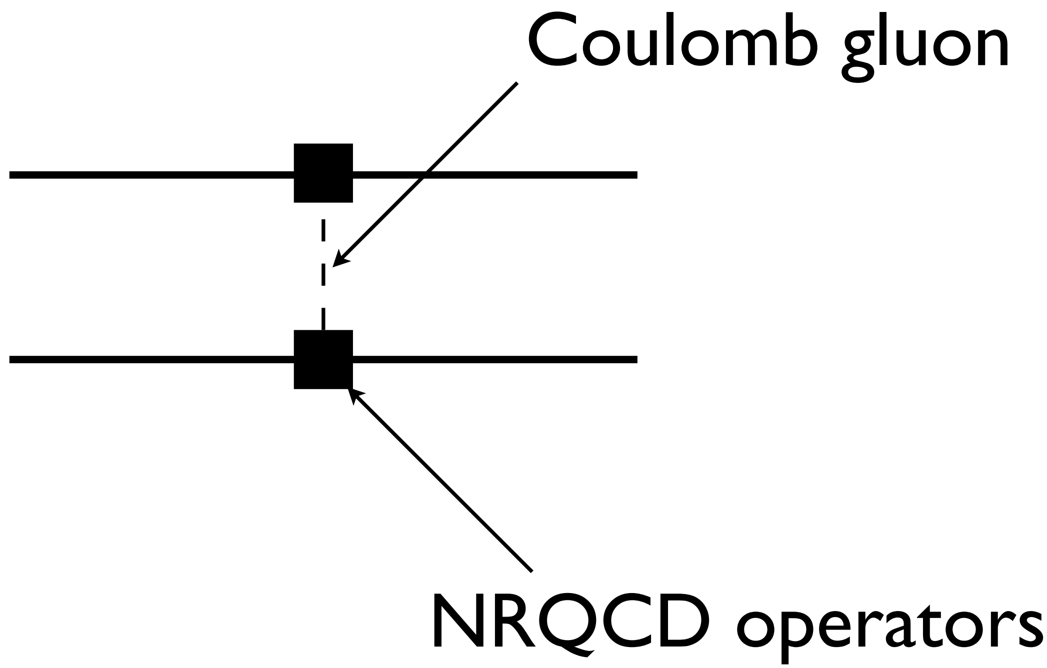
short distances: $(m_Q)^{-1}$, a: lumped into coefficients

long distances: $(m_Q v^n)^{-1}$, L: described by operators

(Same Lagrangian as HQET, but different power counting.)

Potential NRQCD

- Integrate out scale $m_Q v_Q$
- Hamiltonian contains kinetic terms, potentials, and their radiative corrections
- radiative corrections from $m_Q v_Q$ in pQCD
- bound-state solved a la positronium: assumes small shifts from scales $\Lambda, m_c v_c^2$



$$H = \frac{\boldsymbol{p}_c^2}{2m_c} + \frac{\boldsymbol{p}_b^2}{2m_b} - \frac{(\boldsymbol{p}_c^2)^2}{8m_c^3} - \frac{(\boldsymbol{p}_b^2)^2}{8m_b^3} + \cdots + V(r)$$

$$V(r) = -\frac{C_F\alpha_s}{r} + C_F\alpha_s(1+\alpha_s+\cdots)\left(\frac{1}{4m_c^2} + \frac{1}{4m_b^2}\right)4\pi\delta(\boldsymbol{r}) + \ldots$$

Potential Models

- Truncate at leading order (in α_s, v^2).
- Linear confining potential added by hand.
- Potential model α_s, m_Q not connected to QCD Lagrangian α_s, m_Q.
- Provide excellent empirical understanding.

Lattice Calculation

- Ian Allison, Christine Davies, Alan Gray, ASK, Paul Mackenzie, & James Simone
  - conference: hep-lat/0409090
  - publication: hep-lat/0411027
- Prediction: α_s, m_b, m_c taken from bottomonium and charmonium
- Use latNRQCD for *b* and Fermilab for *c*.

Essentials

- We calculate two mass splittings

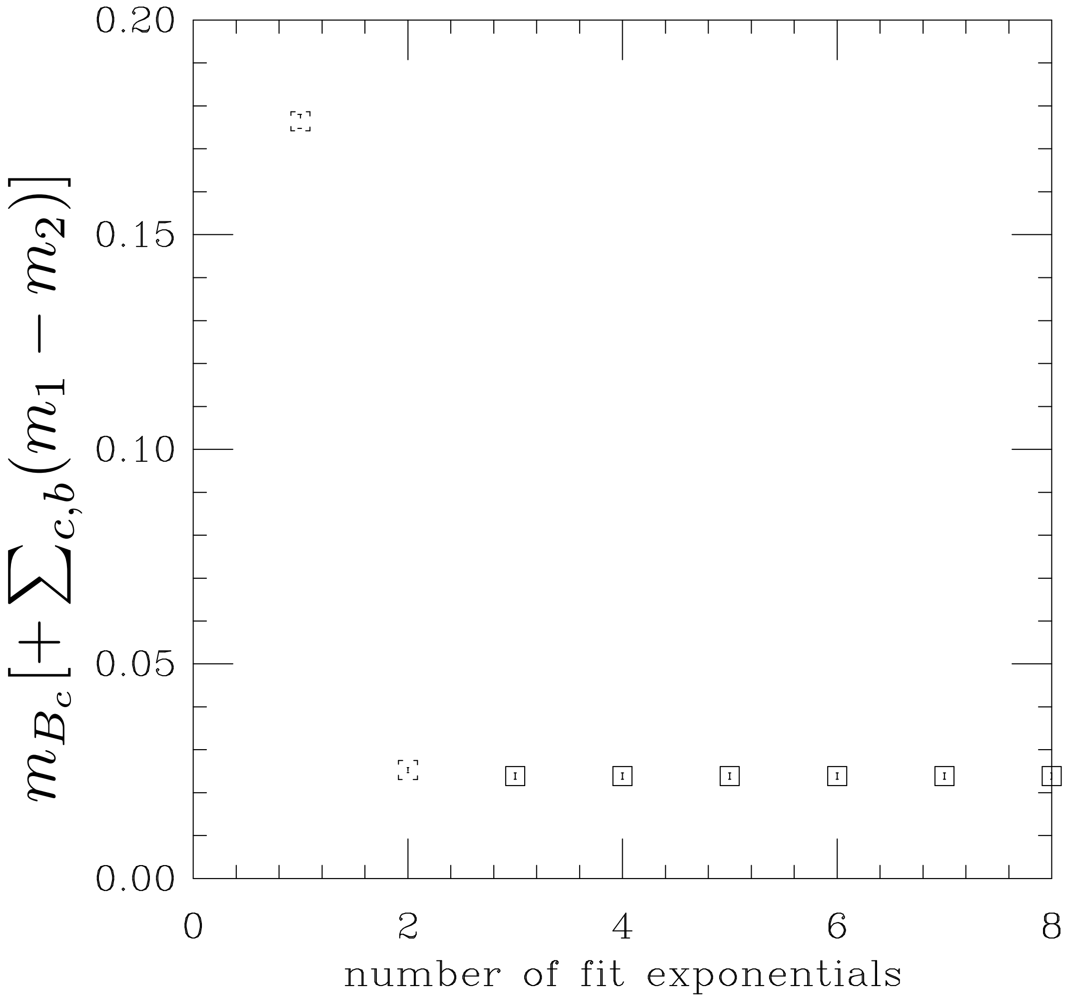
$$\Delta_{\psi\Upsilon} = m_{B_c} - \tfrac{1}{2}(\bar{m}_\psi + m_\Upsilon) \quad \text{quarkonium baseline}$$

$$\Delta_{D_s B_s} = m_{B_c} - (m_{D_s} + m_{B_s}) \quad \text{heavy-light baseline}$$

- Everything is gold-plated, in the sense that the mesons are all stable, and far from threshold.

- Chiral extrapolations mild.

Isolating Lowest State

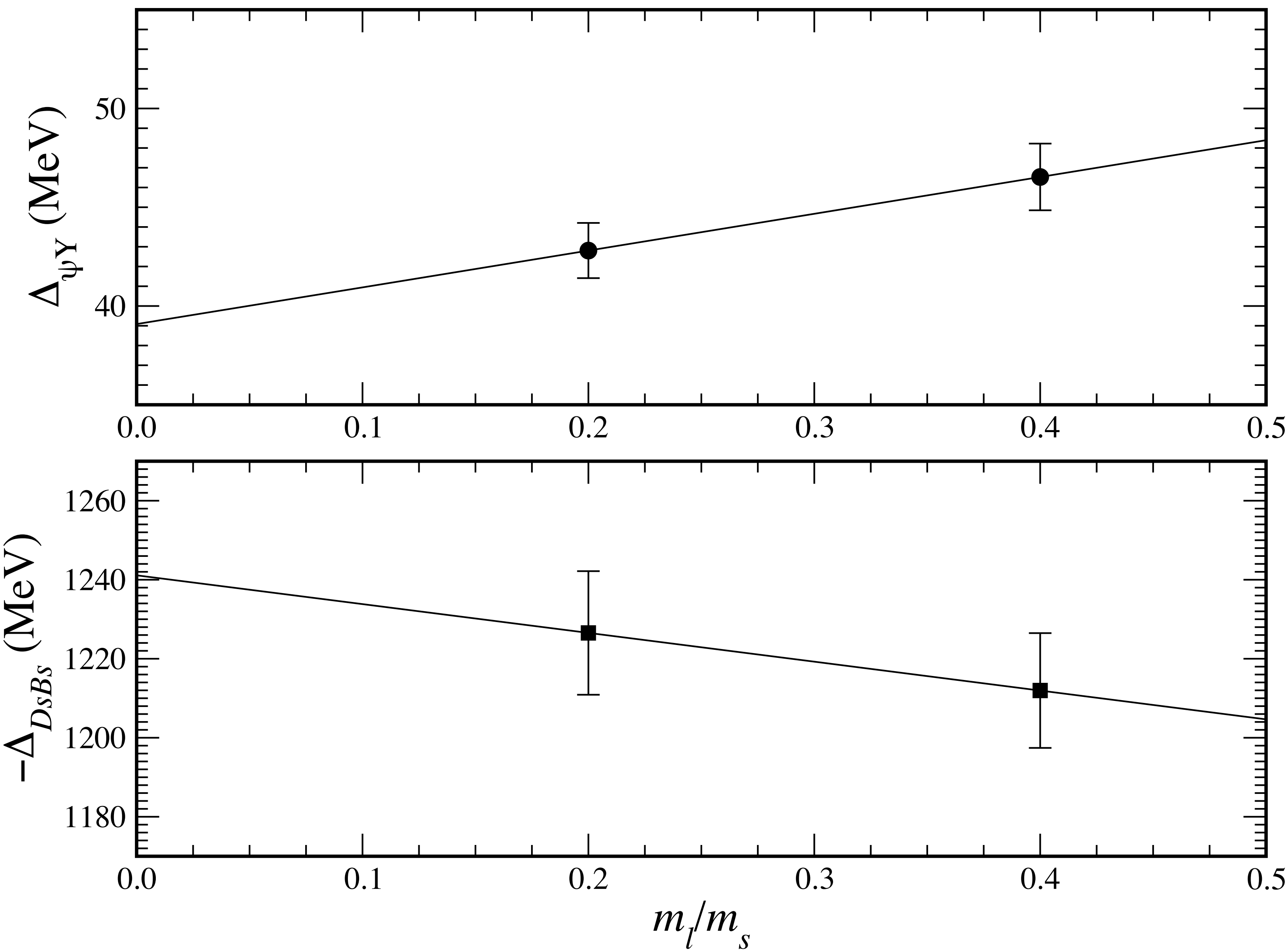


Correlator is sum of exponentials, lowest exponent is m_{B_c}

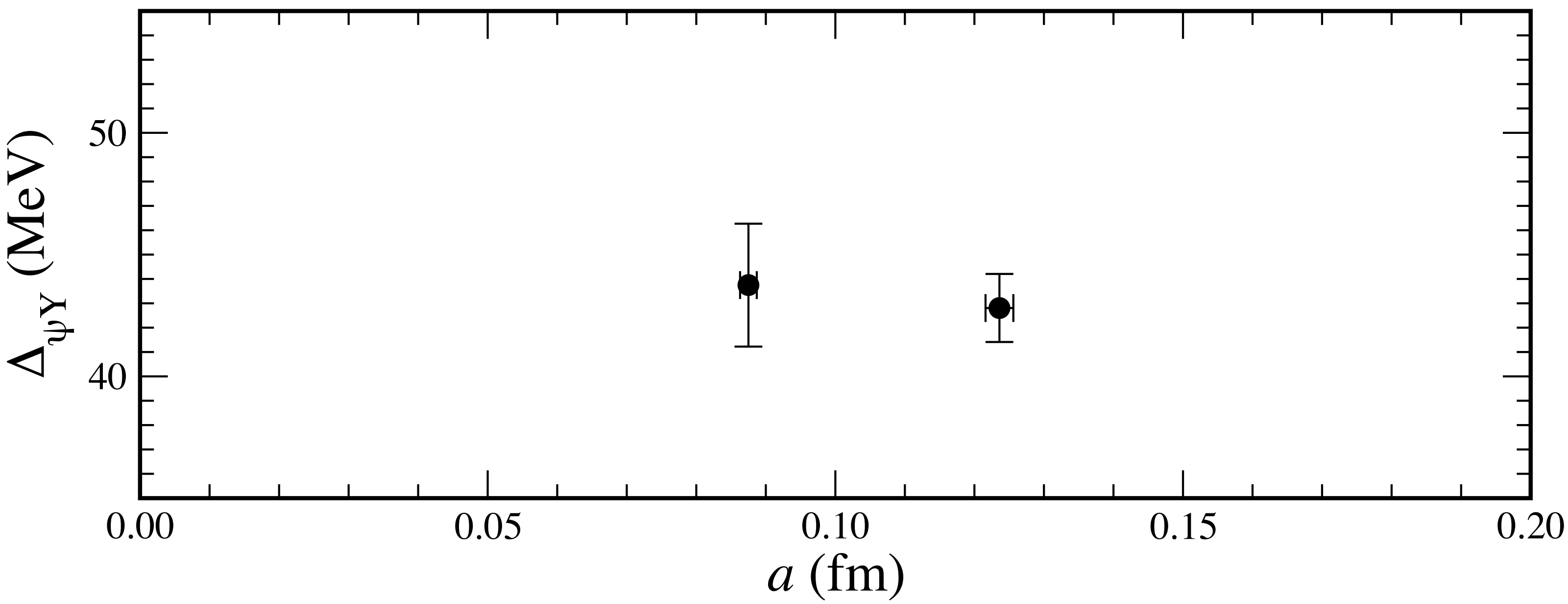
Error Cancellation

- Correlated statistics
- Unphysical shift in rest mass m_1
- Contributions from higher-in-v^2 operators, at least from quarkonium baseline.

Chiral Extrapolation



Lattice Spacing Dependence



at lighter of the two sea quark masses

Error Analysis

- Statistical error is straightforward & small.
- Uncertainty from a^{-1}, m_b, m_c easy to propagate: latter two are ±10, ±5 MeV.
- Main problem is to estimate the discretization effect for the heavy quarks

Discretization Effects

(short distance mismatch) • (matrix element)

- Use calculations of tree-level mismatches
- Wave hands for one-loop mismatches
- Estimate matrix elements in potential models
- *Check* framework with other calculations

Hyperfine *i***Σ•*B***

- The mismatch of the hyperfine interaction is

$$\alpha_s a b_B(m_0 a) \times \bar{h} i \boldsymbol{\Sigma} \cdot \boldsymbol{B} h$$

  in both NRQCD and Fermilab Lagrangians.

- Estimate coefficient by comparing the simulation hyperfine splitting with experiment, where latter is known.
- Propagate to m_{B_c} and m_{Υ}.

Darwin ***D•E***

- The mismatch of the Darwin interaction is

$$\{\alpha_s, 1\} a^2 b_{\mathrm{Darwin}}(m_0 a) \times \bar{h} \boldsymbol{D} \cdot \boldsymbol{E} h$$

  for {NRQCD, Fermilab}.

- Latter dominates; use known form of the coefficient and (Richardson) potential model estimate of matrix element.
- Matrix element is small.

Relativistic $(\boldsymbol{p}^2)^2$ & p_i^4

- The mismatch of the Darwin interaction is

  $\{\alpha_s, 1\}a^3 b_4(m_0 a) \times \{\bar{h}(\boldsymbol{p}^2)^2 h, \bar{h}\sum_i p_i^4 h\}$

  for {NRQCD, Fermilab}.

- Latter dominates; use known form of the coefficient and (Richardson) potential model estimate of matrix element.

- Matrix element is not small, but check total estimate with charmonium 1P-1S.

TABLE I: Estimated shifts in masses and the splittings $\Delta_{\psi\Upsilon}$ and $\Delta_{D_sB_s}$. Entries in MeV. Dashes (—) imply the entry is negligible.

operator	m_{B_c}	$\frac{1}{2}\bar{m}_\psi$	$\frac{1}{2}m_\Upsilon$	$\Delta_{\psi\Upsilon}$	$\bar{m}_{D_s}$	$\bar{m}_{B_s}$	$\Delta_{D_sB_s}$
				$a = \frac{1}{8}$ fm			
$\boldsymbol{\Sigma}\cdot\boldsymbol{B}$	-14	0	$+3$	-17	0	0	-14
Darwin	-3	-3	∓1	±1	-4	—	$+1$
$(\boldsymbol{D}^2)^2$	$+34$	$+10$	±3	$+24$	—	—	$+34$
D_i^4	$+16$	$+5$	±2	$+11$	—	—	$+16$
total				$+18$			$+37$
				$a = \frac{1}{11}$ fm			
$\boldsymbol{\Sigma}\cdot\boldsymbol{B}$	-12	0	$+3$	-15	0	0	-12
Darwin	-2	-2	∓1	±1	-2	—	—
$(\boldsymbol{D}^2)^2$	$+17$	$+5$	±3	$+12$	—	—	$+17$
D_i^4	$+7$	$+2$	±2	$+5$	—	—	$+7$
total				$+2$			$+12$

Results

- Splittings:

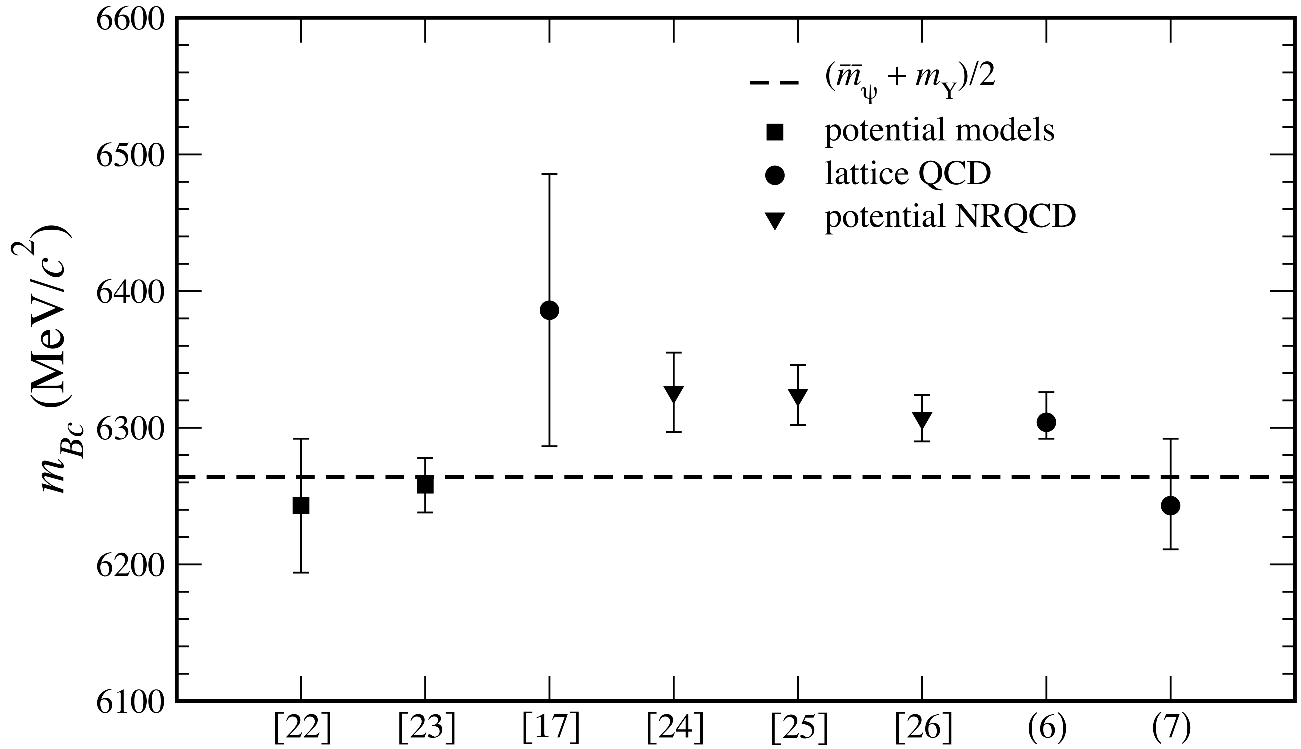
$$\begin{aligned}\Delta_{\psi\Upsilon} &= 39.8 \pm 3.8 \pm 11.2^{+18}_{-\ 0}\ \text{MeV}, \\ \Delta_{D_s B_s} &= -\left[1238 \pm 30 \pm 11^{+\ 0}_{-37}\right]\ \text{MeV},\end{aligned}$$

- Meson mass:

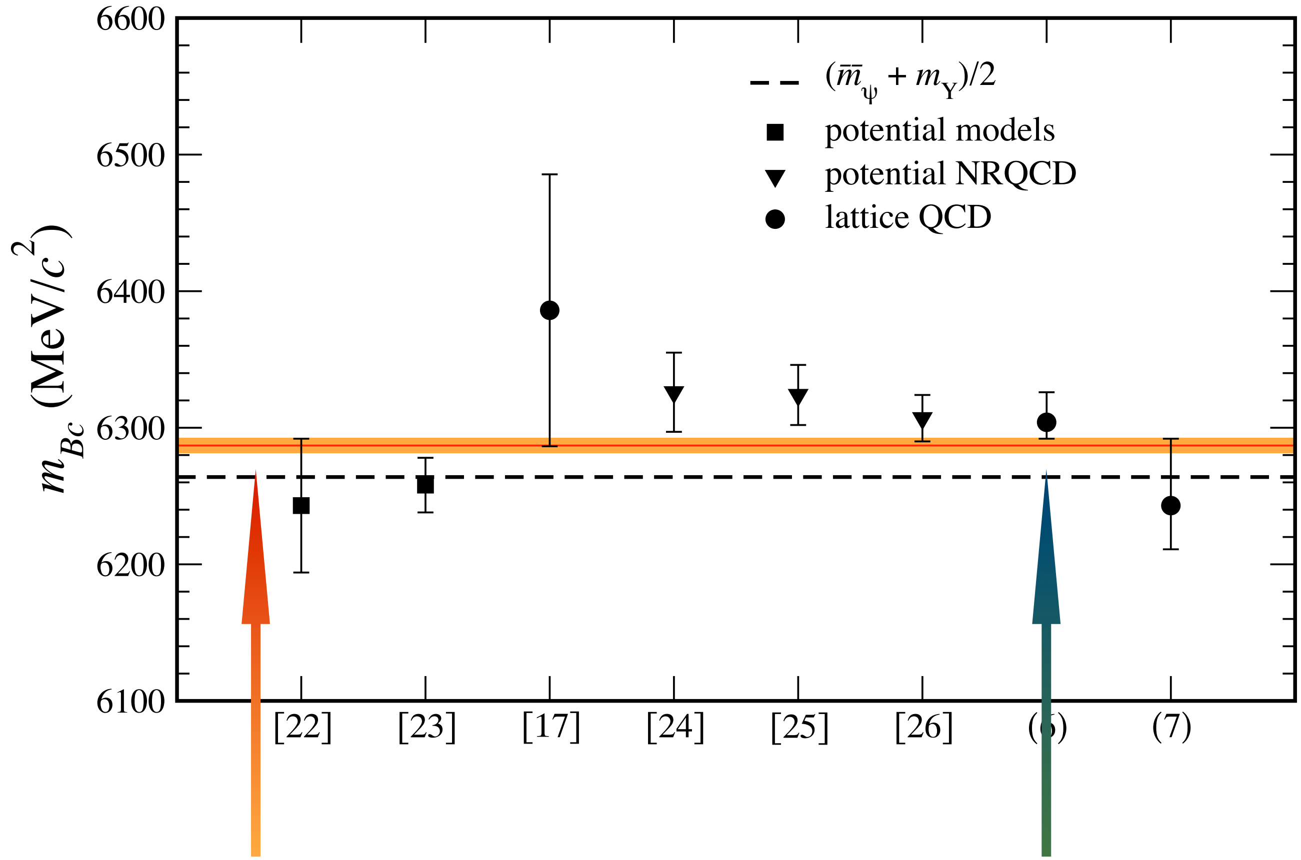
$$\begin{aligned}m_{B_c} &= 6304 \pm\ \ 4 \pm 11^{+18}_{-\ 0}\ \text{MeV}, \\ m_{B_c} &= 6243 \pm 30 \pm 11^{+37}_{-\ 0}\ \text{MeV},\end{aligned}$$

- More checks on quarkonium baseline, so it is our main result.

Compare with Models



Compare with CDF



$m_{B_c} = 6287 \pm 5$ MeV
CDF, W&C seminar, 12/03/04

$m_{B_c} = 6304 \pm 12^{+18}_{-0}$ MeV
[hep-lat/0411027]

Summary

- Results for leptonic and semi-leptonic *D* decays and the mass of the B_c meson.
- Estimates of uncertainties.
- Agreement with BES, CLEO, FOCUS, and CDF with similar time-scale and error, including *pre*dictions.

pre- *pref.*
1. a. Earlier; before; prior to: *prehistoric*.
   b. Preparatory; preliminary: *premedical.*
   c. In advance: *prepay.*
2. Anterior; in front of: *preaxial.*

[Middle English, from Old French, from Latin `prae–`, from `prae`, *before, in front*. See `per1` in Indo-European Roots.]