Towards the QCD Phase Diagram at small Baryon Densities

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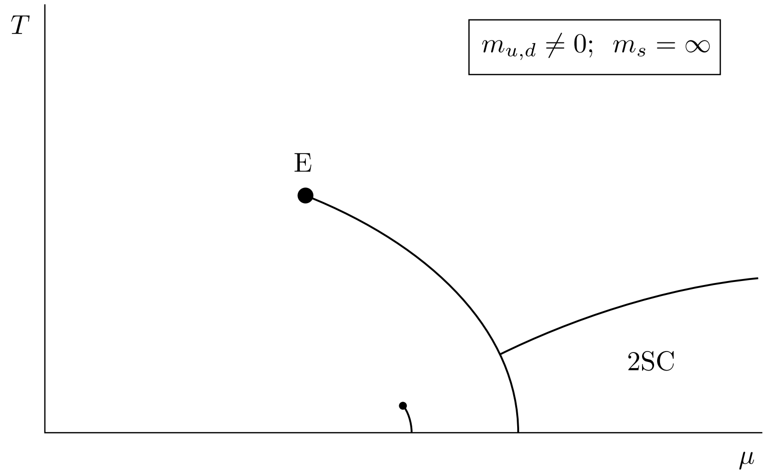
- Introduction+motivation
- The imaginary μ approach
- Numerical results for $N_f = 2, 3, 4$
- First results for $N_f = 2 + 1, \mu = 0$
- Conclusions

QCD: rich phase structure



I. Hot QCD occurs in: a) The early universe, $t \sim 10^{-6} s$

b) Heavy ion collisions

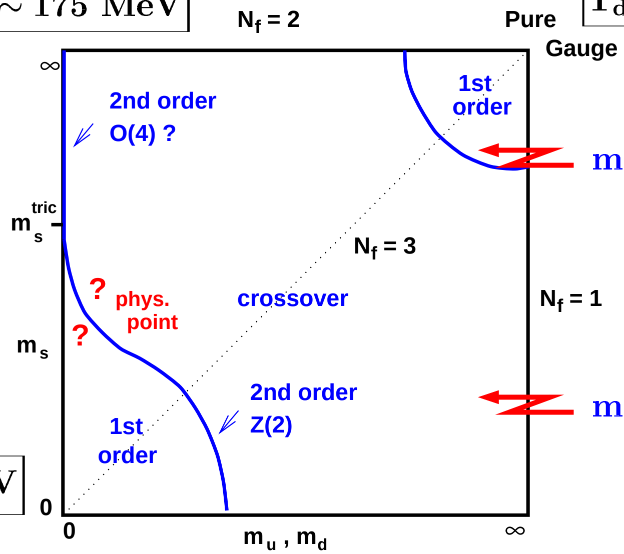
II. Dense QCD may occur in: Core of neutron stars

Non-pert. problem $\Rightarrow$Lattice 1975-2001: $\mu \neq 0$ impossible $\Rightarrow$sign problem

Here: phase diagram, $\mu_q/T \lesssim 1$ (μ-effects on EoS for heavy ion exp. very small)

The QCD phase transition at finite $T, \mu = 0$

$\boxed{\mathrm{T}_\chi^{\mathrm{n_f=2}} \sim 175\ \mathrm{MeV}}$ $\boxed{\mathrm{T_d} \sim 270\ \mathrm{MeV}}$



$\mathrm{m_{PS}^{crit}} \simeq \mathbf{2.5\ GeV}$

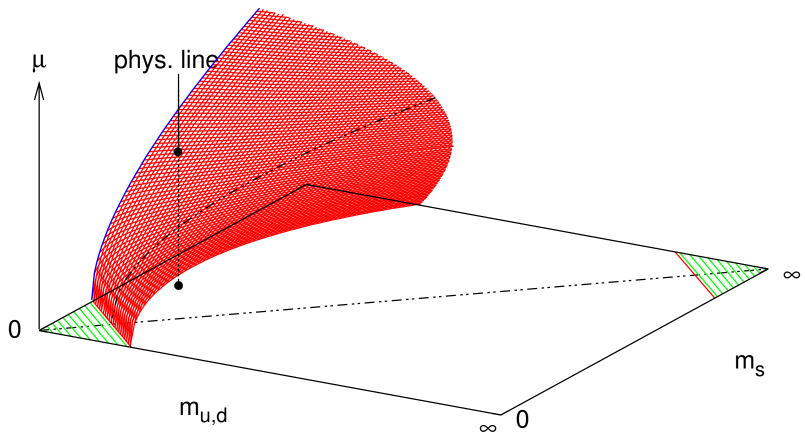
$\mathrm{m_{PS}^{crit}} \simeq \mathbf{300\ MeV} \Rightarrow m_c(\mu = 0)$ (unimproved)

3d Ising

$\boxed{\mathrm{T}_\chi^{\mathrm{n_f=3}} \sim 155\ \mathrm{MeV}}$

N.B: m_{PS}^{crit} has huge cut-off effects! (factor 1/4?) Bielefeld, MILC

Finite density, $\mu \neq 0$:

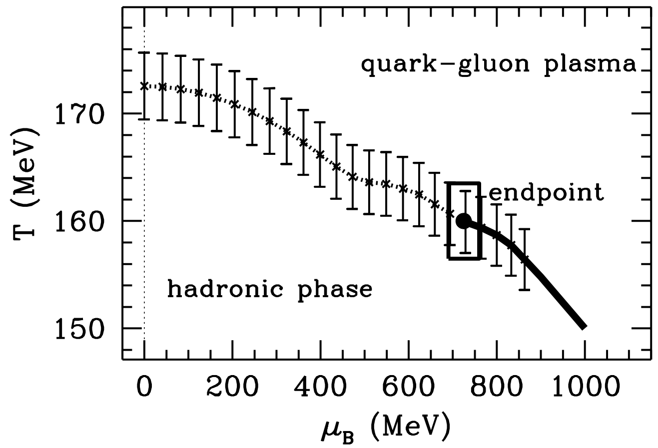


Evading the sign problem:

I. Multi-dimensional reweighting in (μ, β)

Fodor, Katz

$$Z = \left\langle \frac{\mathrm{e}^{-S_g(\beta)} \det(M(\mu))}{\mathrm{e}^{-S_g(\beta_0)} \det(M(\mu = 0))} \right\rangle_{\mu=0,\beta_0}$$



idea: simulate at $\beta_0 = \beta_c(0)$, better overlap by sampling both phases; errors? ovlp.?

II. Reweighting and/or Taylor expansion

Bielefeld/Swansea

Gavai, Gupta

idea: for small μ/T, compute coeffs. of Taylor series $\Rightarrow$local ops. $\Rightarrow$gain V

convergence?

III. Imaginary μ + analytic continuation

de Forcrand, O.P.

D'Elia, Lombardo; Azcoiti et al.

fermion determinant positive $\Rightarrow$no sign problem

idea: for small μ/T, fit **full** simulation results of imag. μ by Taylor series $\Rightarrow$continuation

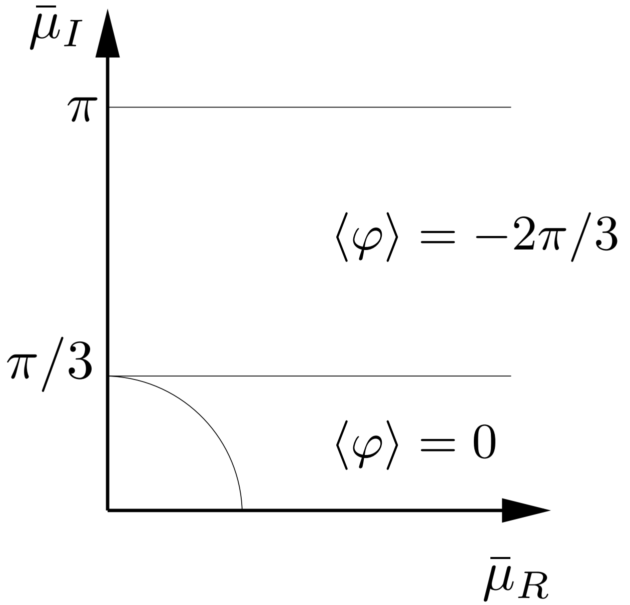
- vary **two** parameters $(\mu, T)$$\Rightarrow$uncorrelated results
- control over systematics

QCD at complex μ: general properties

$$Z(V,\mu,T) = \mathrm{Tr}\left(\mathrm{e}^{-(\hat{H}-\mu\hat{Q})/T}\right); \quad \mu = \mu_R + i\mu_I; \quad \bar{\mu} = \mu/T$$

exact symmetries: μ-reflection and μ_I-periodicity — Roberge, Weiss

$$Z(\bar{\mu}) = Z(-\bar{\mu}), \qquad Z(\bar{\mu}_R, \bar{\mu}_I) = Z(\bar{\mu}_R, \bar{\mu}_I + 2\pi/N_c)$$



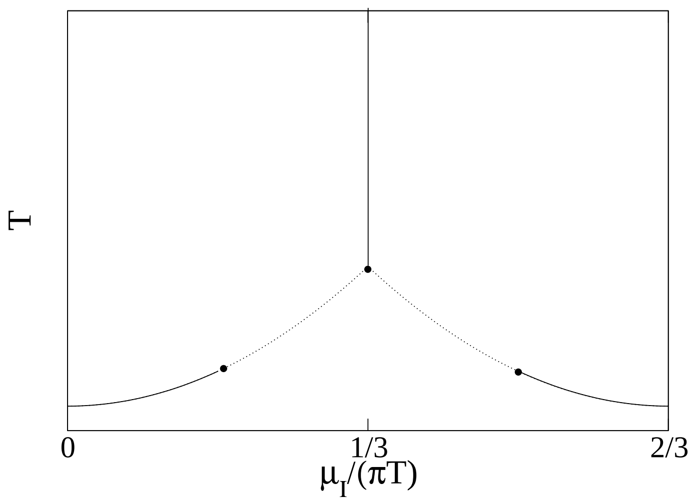
Z(3)-transitions:

$$\bar{\mu}_I^c = \frac{2\pi}{3}\left(n + \frac{1}{2}\right)$$

pert./strong coupling:

1rst order for high T

crossover for low T



analytic continuation within arc, $\mu_B \lesssim 600\mathrm{MeV}$: $\langle O \rangle = \sum_n^N c_n \bar{\mu}_I^{2n} \Rightarrow \mu_I \longrightarrow i\mu_I$

Analyticity of the (pseudo-) critical line on finite V

de Forcrand, O.P.

location of phase transition from maximum of susceptibilites:

$$\chi(\beta, a\mu, V) = V N_t \left\langle (\mathcal{O} - \langle \mathcal{O} \rangle)^2 \right\rangle , \quad \mathcal{O} \in \{\text{plaq}, \bar{\psi}\psi, |P(x)|\}$$

- finite volume: suscept. always finite and analytic

Critical line $\beta_c(a\mu)$ defined by peak $\chi_{max} \equiv \chi(a\mu_c, \beta_c)$

- implicit function theorem: $\chi(\beta, a\mu)$ analytic $\Rightarrow \beta_c(a\mu)$ analytic!

symmetries: $\Rightarrow$ $\boxed{\beta_c(a\mu) = \sum_n c_n (a\mu)^{2n}}$

What to expect in physical units?

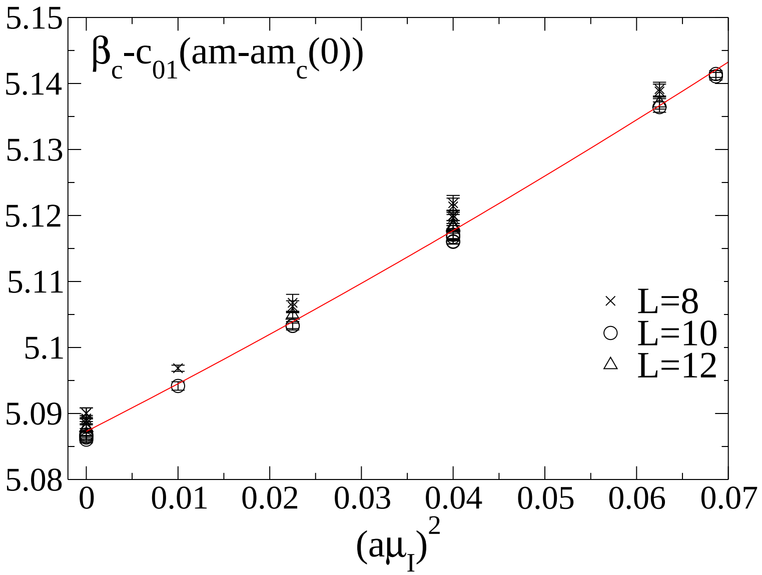
Natural expansion parameter is $\frac{\mu}{\pi T}$:

- thermal perturbation theory
- $\mu_I \longrightarrow$ shift in Matsubara frequencies $(2n+1)\pi T$

$$\Rightarrow \frac{T_c(\mu)}{T_c(\mu=0)} = 1 - O(1) \left(\frac{\mu}{\pi T_c(0,m)} \right)^2 + O(1) \left(\frac{\mu}{\pi T_c(0,m)} \right)^4 + \ldots$$

$N_f = 3$ results, quark mass dependence

$$\beta_c(a\mu, am) = \sum_{k,l=0} c_{kl} \, (a\mu)^{2k} \, (am - am_c(0))^l$$

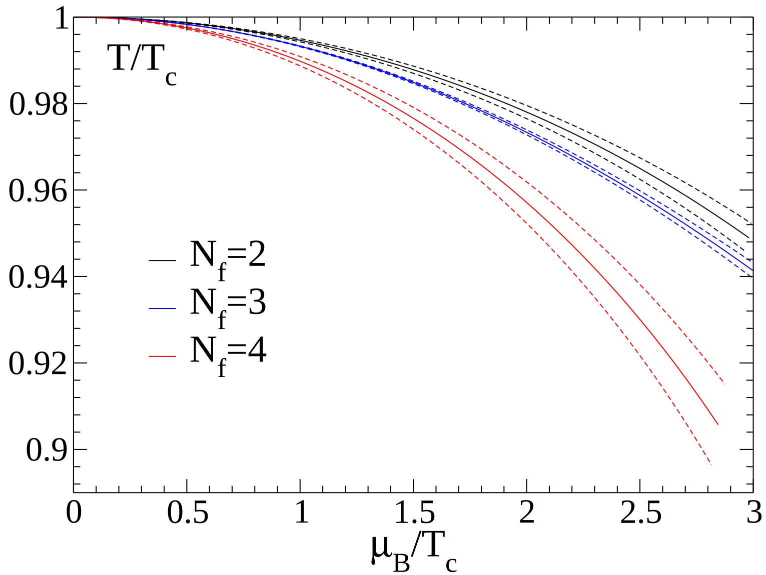


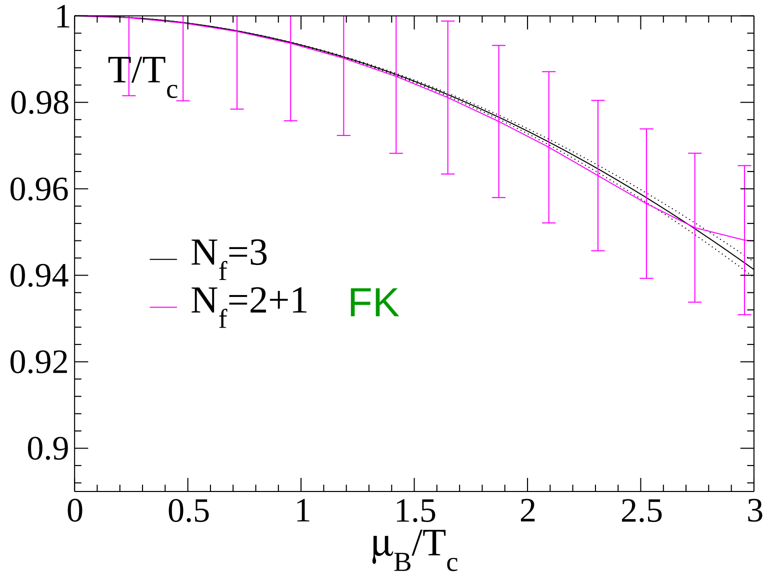
⇒sensitive to μ^4, but data very well described by μ^2 fit !

$$\frac{T_c(\mu, m)}{T_c(\mu = 0, m_c(0))} = 1 + 1.937(17)\frac{m - m_c(0)}{\pi T_c}$$

$$+0.602(9)\left(\frac{\mu}{\pi T_c(0,m)}\right)^2 + 0.23(9)\left(\frac{\mu}{\pi T_c(0,m)}\right)^4$$

N_f-dependence



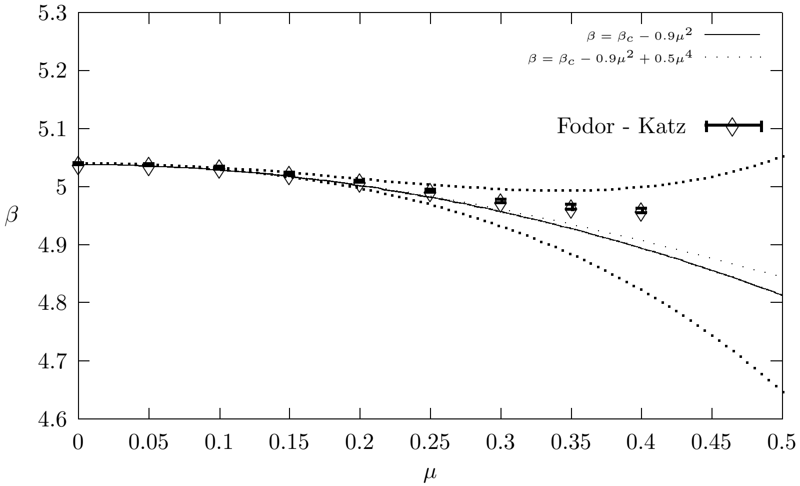


Comparison:

$N_f = 4$

imag. μ vs. reweighting

D'Elia, Lombardo

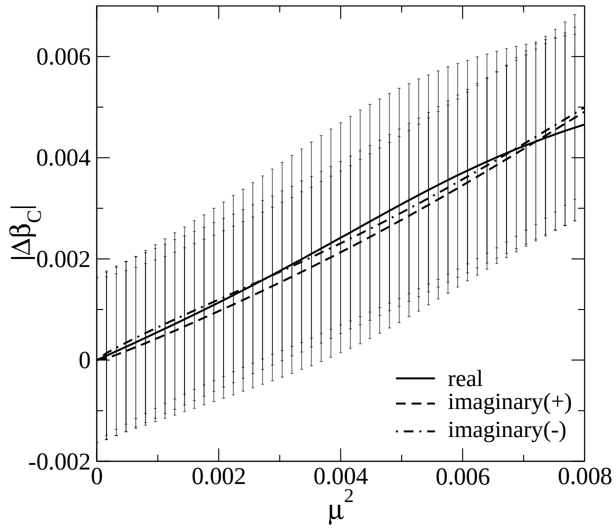


Comparison:

real μ vs. imag. μ

to leading order

Bielefeld/Swansea



Comparison: analyt. continuation ok in SU(2) Giudice, Papa

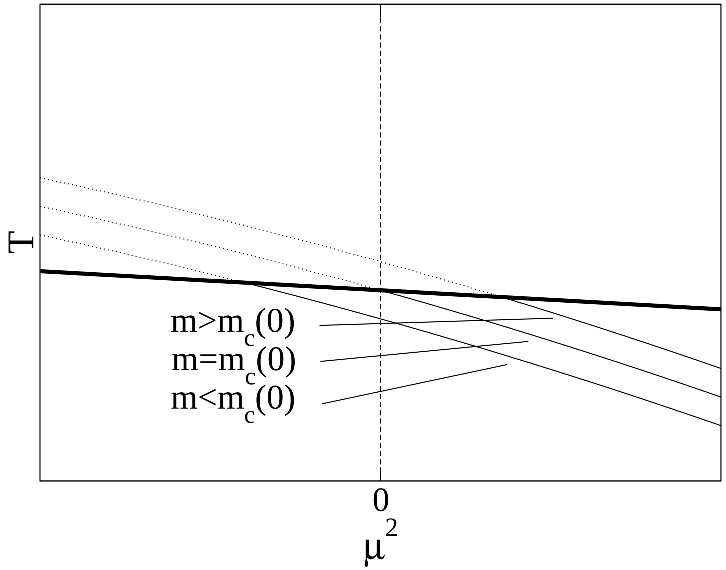
The critical endpoint and its quark mass dependence

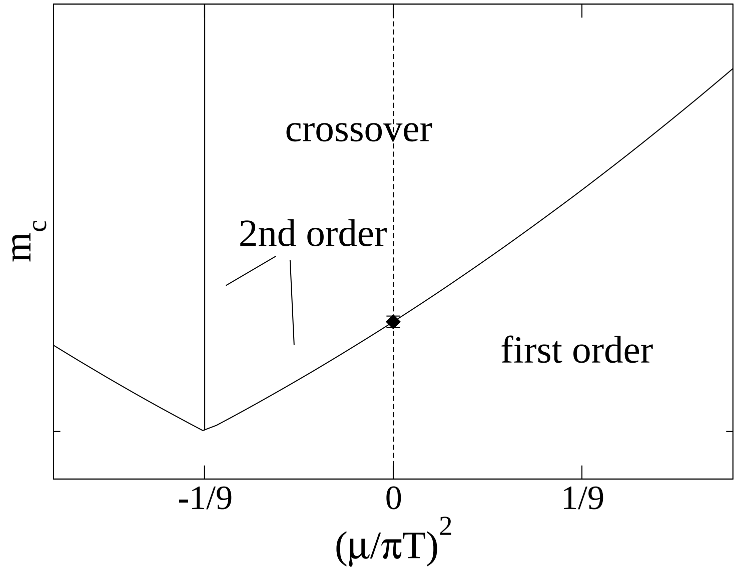
Phase diag. 3d: (T, μ, m)

●confined/deconfined $\Rightarrow$pseudo-crit. surface $T_c(\mu, m)$

On this surface,

●1.O./crossover $\Rightarrow$line of crit.points $T^*(\mu) = T_c(\mu, m_c(\mu))$





Expect: $$\frac{m_c(\mu)}{m_c(\mu{=}0)} = 1 + c_1 \left(\frac{\mu}{\pi T}\right)^2 + \ldots$$

$m = 0$ $\Rightarrow$true chiral phase transition $\Rightarrow$ $\boxed{c_1 \leq 9}$

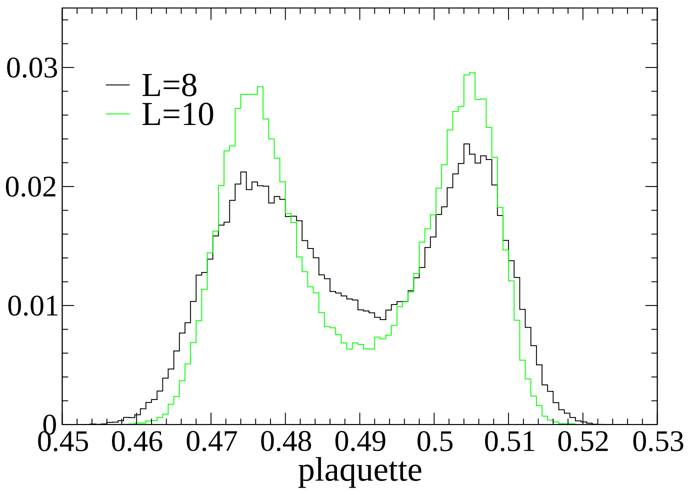
Criticality: cumulant ratios

3d Ising universality :
$$B_4(m_c,\mu_c) = \frac{\langle (\delta\bar{\psi}\psi)^4 \rangle}{\langle (\delta\bar{\psi}\psi)^2 \rangle^2} \rightarrow 1.604, \quad V \rightarrow \infty$$

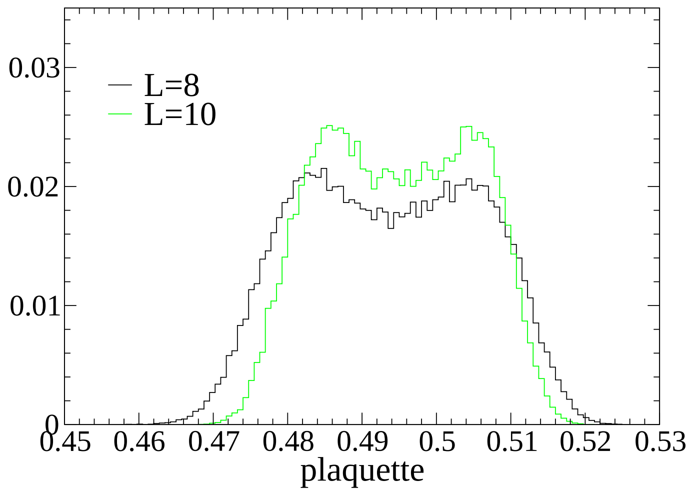
($<, > \Rightarrow$ first-order, crossover)

need VERY LONG MC runs for sufficient tunneling statistics

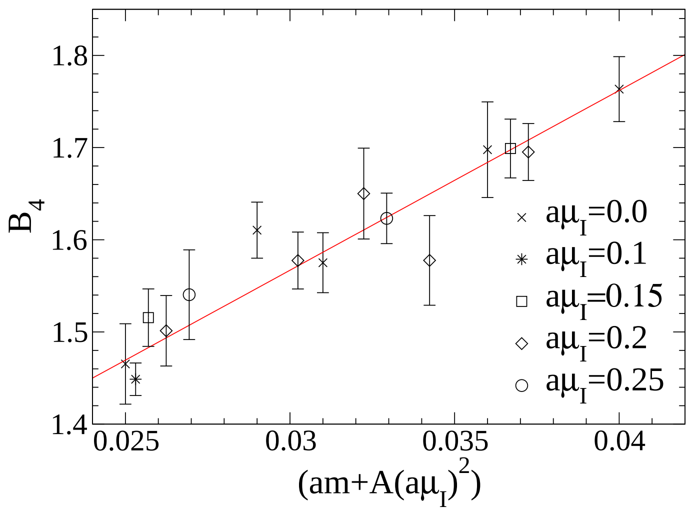
first-order



crossover



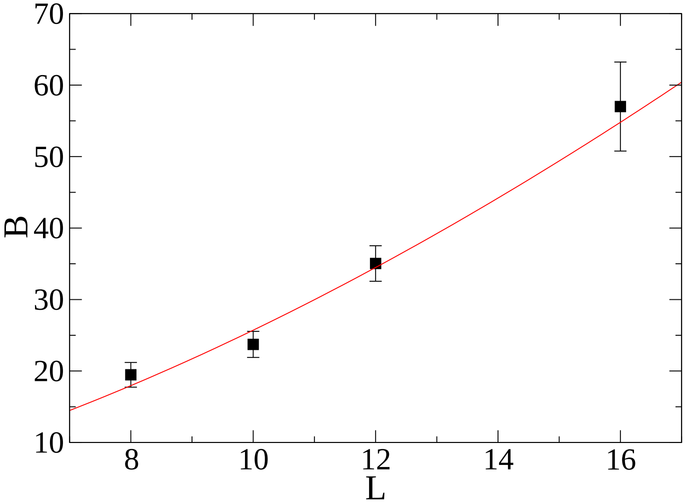
Taylor series $B_4(am, a\mu) = 1.604 + B\left(am - am_c(0) + A(a\mu)^2\right) + \ldots$



$$\Rightarrow \frac{m_c(\mu)}{m_c(\mu=0)} = 1{+}0.84(36)\left(\frac{\mu}{\pi T}\right)^2{+}\ldots$$ **high quark mass sensitivity of μ_c!**

$m_c(0)$ in agreement with Bielefeld, Columbia; c_1(Bielefeld,impr.) $\sim 600(300)$ **?**

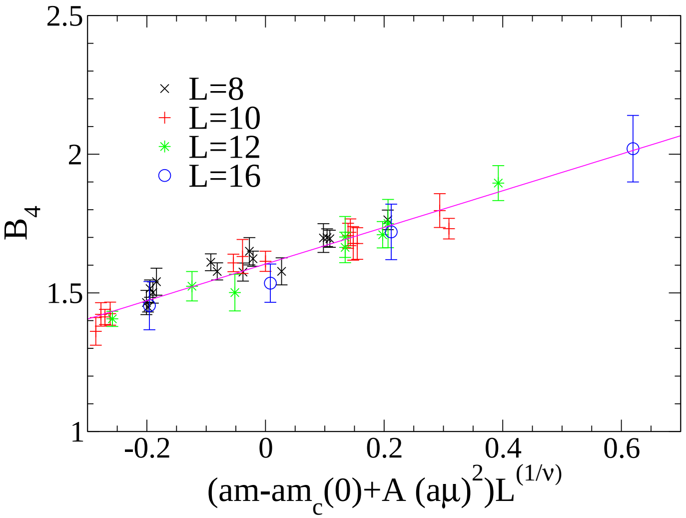
Finite size scaling



FSS:

$\nu = 0.62(3)$

$\nu(Ising) = 0.63$



Can one expect the critical point to be at "small" μ?

If $\mu_c \sim 120$ MeV (FK), then

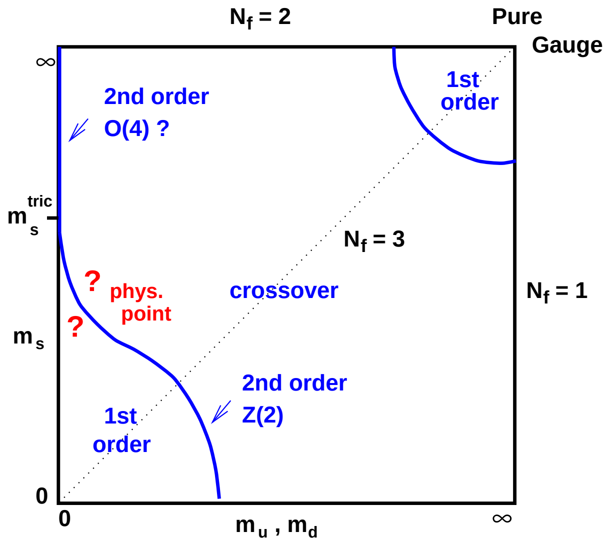
$$1 < \frac{m}{m_c(\mu=0)} \lesssim 1.05$$

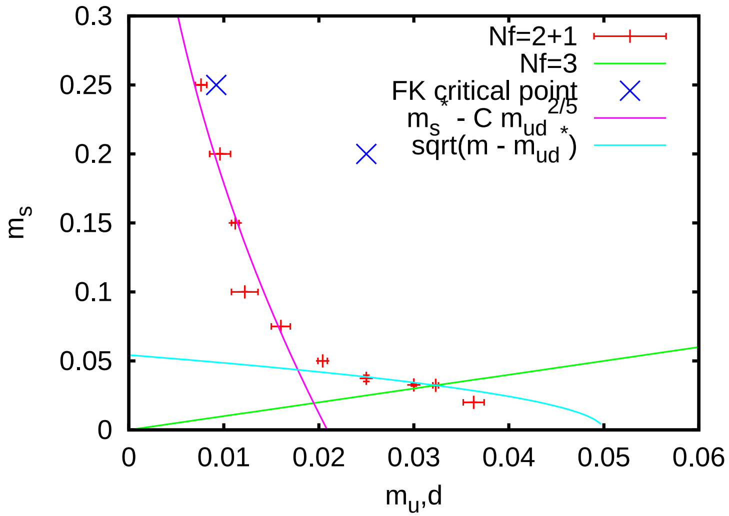
fine tuning of quark masses!

Outlook: $N_f = 2+1$ Phase diag. 4d: $(T, \mu, m_{u,d}, m_s)$

Two-step procedure:

I. $(m_s, m_{u,d})$ phase-diagram at $\mu = 0$ $\Rightarrow m_s^c(m_{u,d})$





$\Rightarrow$ strong non-linearities, no linear extrapolations from $N_f = 3$!

$\Rightarrow$ new Fodor, Katz qualitatively consistent with our results ($m_{u,d}/m_{u,d}^c \sim 1.1$)

$\Rightarrow$ consistent with $O(4)$, tri-critical point, $m_s^{\rm tric}/T \sim 2.8$

II. repeat calculation for $\mu \neq 0$

Conclusions

Simulations of small baryon densities possible for $\mu_q/T \sim 1$

Location of transition line consistent in all approaches**!**

- Critical line very flat, small quark mass dependence

- Critical endpoint extremely quark mass sensitive

$\Rightarrow \mu_c \lesssim 400$ MeV requires nature to fine tune m_q's

Hard work (cut-off effects), but high T phase diagram in reach !