QCD at Small Chemical Potential

Dominique Toublan

University of Illinois at Urbana-Champaign

Modern Challenges for Lattice Field Theory

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Introduction

- QCD at nonzero T and μ_B:
  ▷ Neutron stars (high μ_B, low T)
  ▷ RHIC (low μ_B, high T)
  $\longrightarrow$ In fact, nonzero μ_B, μ_I, and μ_S

- QCD phase diagram, theory:
  ▷ Models: NJL, RMT, Ladder QCD, ...
  Bailin & Love, Casalbuoni, Shuryak, Stephanov, Verbaarschot, Wilczek & Rajagopal, DT
  ▷ Lattice QCD:
  $\longrightarrow$ Most important tool at $T \neq 0$, $\mu = 0$
  $\longrightarrow$ Usual approach impossible at $\mu_B \neq 0$
  $\longrightarrow$ Exceptional case: $\mu_B = 0$, $\mu_I \neq 0$
  Bielefeld, CP-PACS, Fodor & Katz, de Forcrand & Philipsen, Kogut, Lombardo, MILC, Swansea, DT

QCD Phase Diagram: Lattice Results

Critical temperature for small μ_q: $T_c(\mu_u, \mu_d)$

- $T_0 = T_c(0,0) = 175 \pm 6$ MeV, crossover

  MILC, CP-PACS, Bielefeld, Fodor & Katz

- $T_c(\mu,\mu)/T_0 = 1 - 0.114(46)\left(\frac{\mu}{T_0}\right)^2$

  $\longrightarrow$ Small curvature

  Bielefeld-Swansea ($m_\pi \simeq 172$ MeV), de Forcrand & Philipsen

- $T_c(\mu,-\mu)/T_0 = 1 - 0.185(16)\left(\frac{\mu}{T_0}\right)^2$

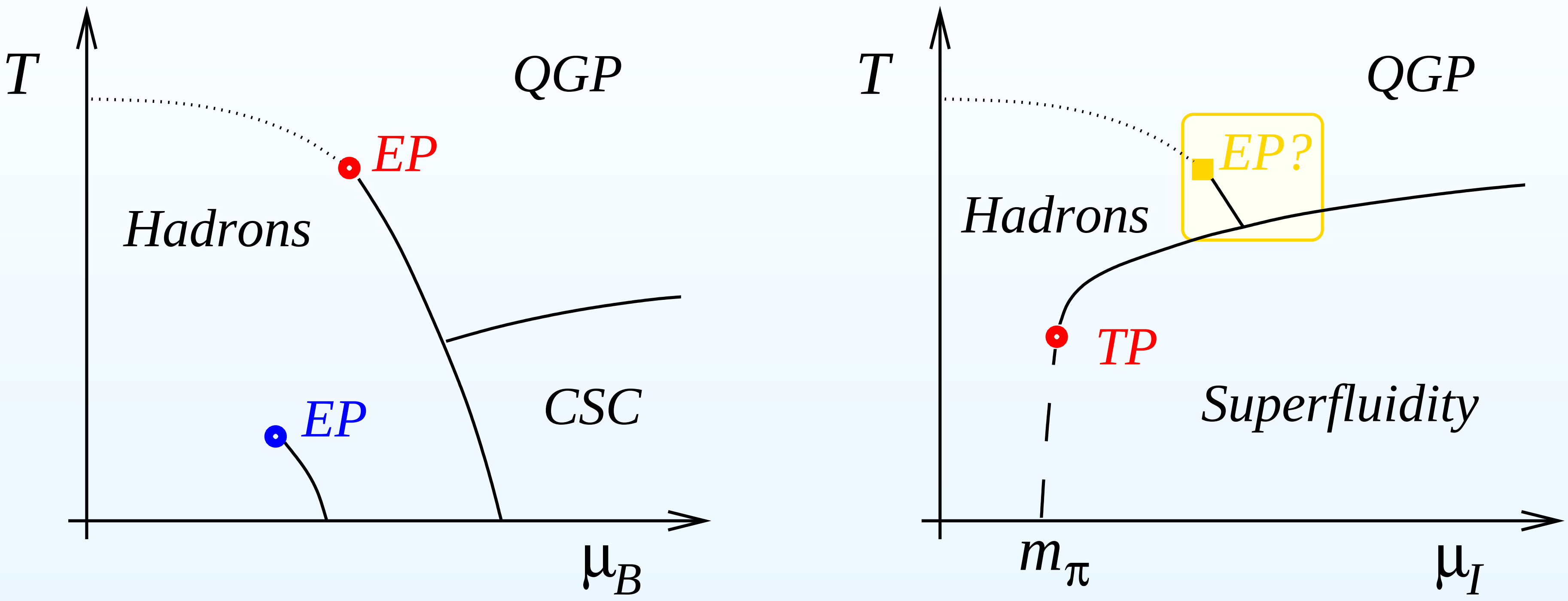
  $\longrightarrow\ T_c(\mu,\mu) \simeq T_c(\mu,-\mu)$

  Bielefeld-Swansea ($m_\pi \simeq 172$ MeV), Kogut & Sinclair

- Critical endpoint

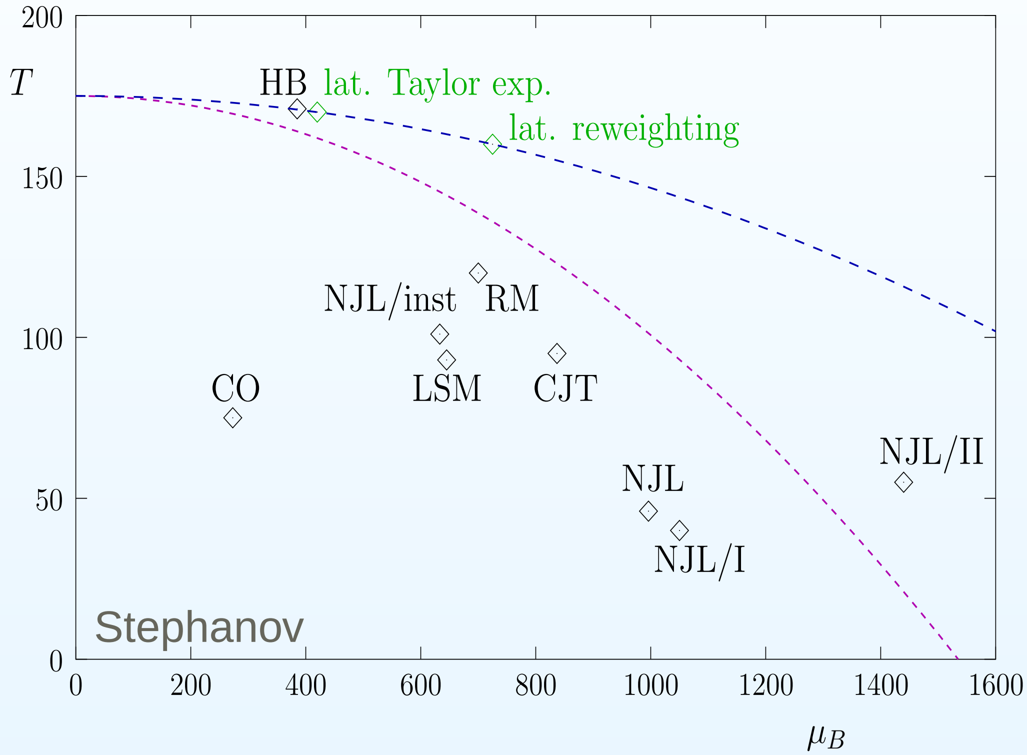
  Bielefeld-Swansea, Fodor & Katz, de Forcrand & Philipsen

QCD Phase Diagram



- Hadronic Phase to Quark Gluon Plasma:
  Crossover at $\mu = 0$, Small curvature,
  $T_c(\mu, \mu) \simeq T_c(\mu, -\mu)$, Critical endpoint
- Agreement between lattice and models

Critical Endpoint



- Lattice & Models: Existence of critical endpoint
- Problem: Location of critical endpoint

Properties of the Critical Temperature

Do we understand these 4 properties?

Study $T_c(\mu_u, \mu_d)$ within two 'minimal' models

- Hadron Resonance Gas Model
  - ▷ Semi-classical model: Naive ideal gas
  - ▷ Very good description of hadronic phase

    Braun-Munzinger, Cleymans, Hagedorn, Karsch,
    Rafelski, Redlich, Stachel, Tawfik

- $1/N_c$ Expansion
  - ▷ Small parameter expansion
  - ▷ Distinguishes classes of Feynman diagrams
  - ▷ Relates different observables

    't Hooft, Witten, Cohen

Hadron Gas Model

- Free gas with all hadrons up to ~ 2 GeV
- Pressure due to particle of mass M

$$\Delta p = \sum_n g M^2 T^2 \frac{(-\eta)^{n+1}}{2n^2\pi^2} K_2\left(\frac{nM}{T}\right) e^{n(B\mu_B + I_3\mu_I)/T}$$

g degeneracy, $\eta = \pm 1$ fermion/boson

- Temperatures of interest $\lesssim 200$ MeV
- Good descript. hadronic phase (exp. & latt.)
  - ▷ Hadron contribution $\sim \exp(-M/T)$
  - ▷ Hadron interaction $\sim \exp(-(M+M')/T)$
  - ▷ Good for pions too (Chiral P. T.)

Hadron Gas Model: Criteria for T_c

Kogut, DT

- Quark-antiquark condensate (light quarks)

$$\langle \bar{q} q \rangle = \frac{\partial p}{\partial m_q} \ll 1$$

▷ Order parameter when $m_q = 0$

▷ Lattice: $\frac{\partial M}{\partial m_q} \simeq A \frac{\langle \bar{q} q \rangle_0}{F_\pi^2 M} \qquad \longrightarrow A = 1.8 - 2.4$

Cleymans, Karsch, Redlich, Tawfik

- Energy density

$$\epsilon = -p + T \frac{\partial p}{\partial T} + \mu \frac{\partial p}{\partial \mu} \simeq 0.5 - 1.0\,\mathrm{GeV/fm^3}$$

▷ Lattice

Cleymans, Karsch, Redlich, Tawfik

$\longrightarrow$ Compute $\langle \bar{q} q \rangle$ and ϵ in hadron gas model

T_0 in Hadron Gas Model

- Hadron gas model: $T_0 = 181 \pm 5$ MeV

  ▷ Determine T_0 by $\langle \bar{q} q \rangle = 0$
  $\Rightarrow \quad T_0 = 185 \pm 6$ MeV

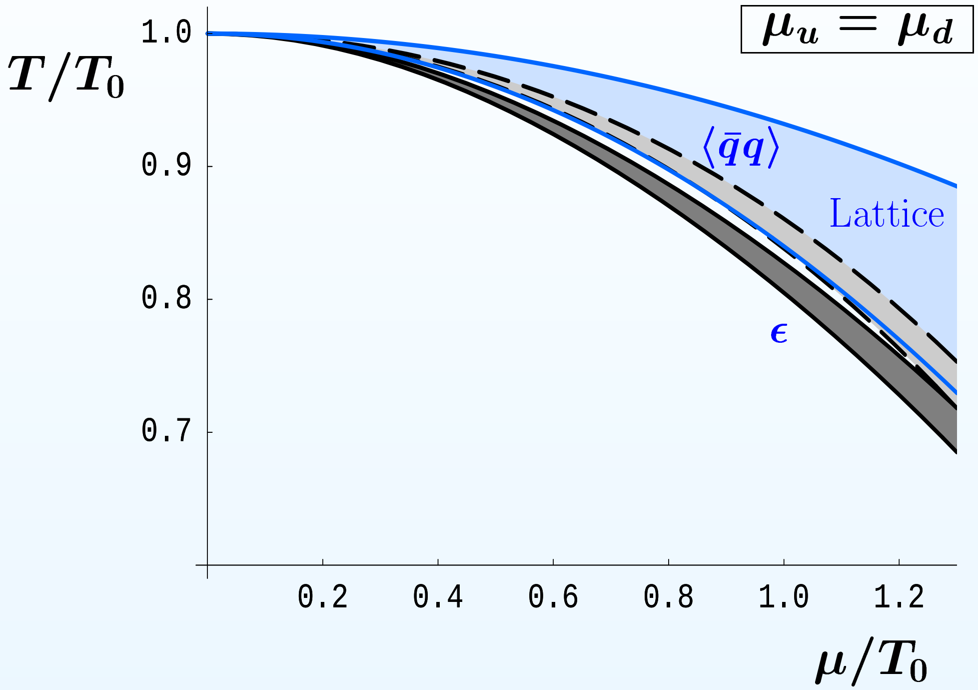
  ▷ Determine T_0 by $\epsilon \simeq 0.5 - 1.0$ GeV/fm^3
  $\Rightarrow \quad T_0 = 176 \pm 8$ MeV

- Lattice: $T_0 = 175 \pm 6$ MeV

  MILC, CP-PACS, Bielefeld, Fodor & Katz

$\longrightarrow$ Very good agreement

Curvature in Hadron Gas Model



- Hadron Gas Model:

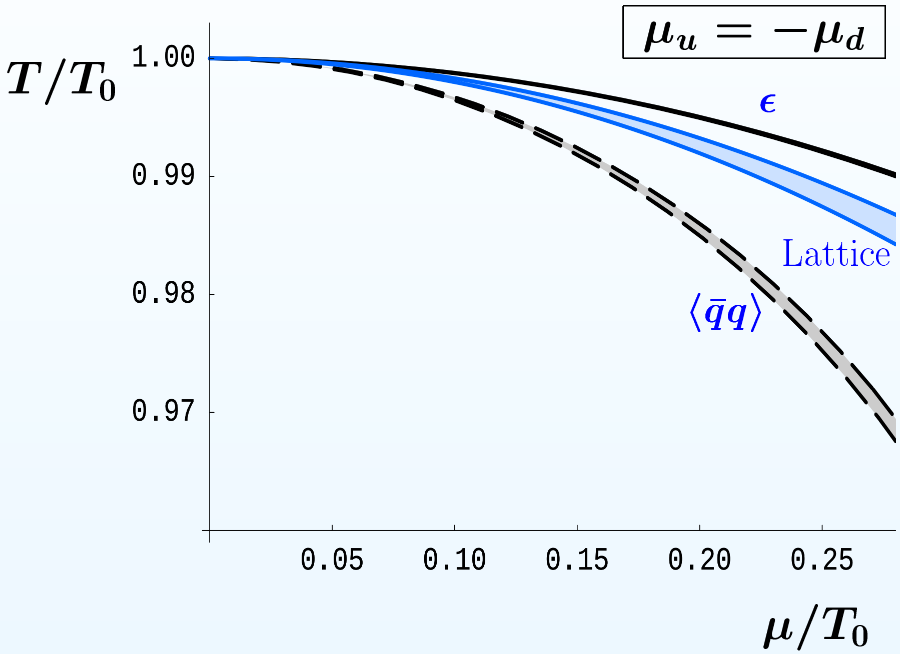
$$T_c(\mu,\mu)/T_0 = 1 - 0.17(1)\left(\frac{\mu}{T_0}\right)^2$$

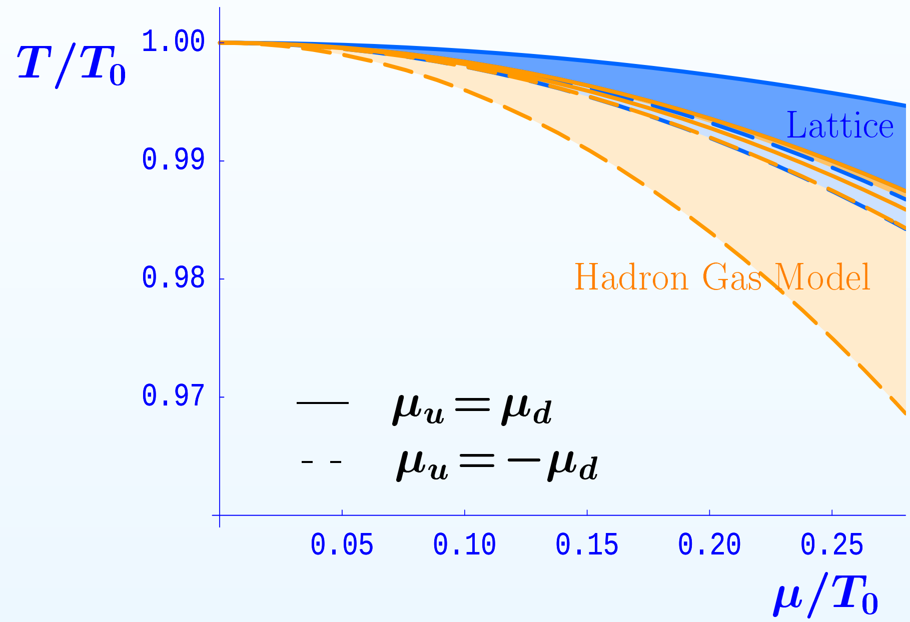
- Lattice: $T_c(\mu,\mu)/T_0 = 1 - 0.114(46)\left(\frac{\mu}{T_0}\right)^2$

$\searrow m_\pi \Rightarrow$ steeper

Bielefeld-Swansea ($m_\pi \simeq 172$ MeV), de Forcrand & Philipsen

μ_b vs μ_i in Hadron Gas Model





- Hadron Gas Model:

$$T_c(\mu, -\mu)/T_0 = 1 - 0.3(1) \left(\frac{\mu}{T_0}\right)^2$$

- Lattice: $T_c(\mu, -\mu)/T_0 = 1 - 0.185(16) \left(\frac{\mu}{T_0}\right)^2$

Bielefeld-Swansea ($m_\pi \simeq 172$ MeV), Kogut & Sinclair

$\Rightarrow T_c(\mu, \mu) \sim T_c(\mu, -\mu)$

Hadron Gas Model: Results

- $T_0 = 181 \pm 5$ MeV
- $T_c(\mu, \mu)/T_0 = 1 - 0.17(1) \left(\frac{\mu}{T_0}\right)^2$
- $T_c(\mu, -\mu)/T_0 = 1 - 0.3(1) \left(\frac{\mu}{T_0}\right)^2$

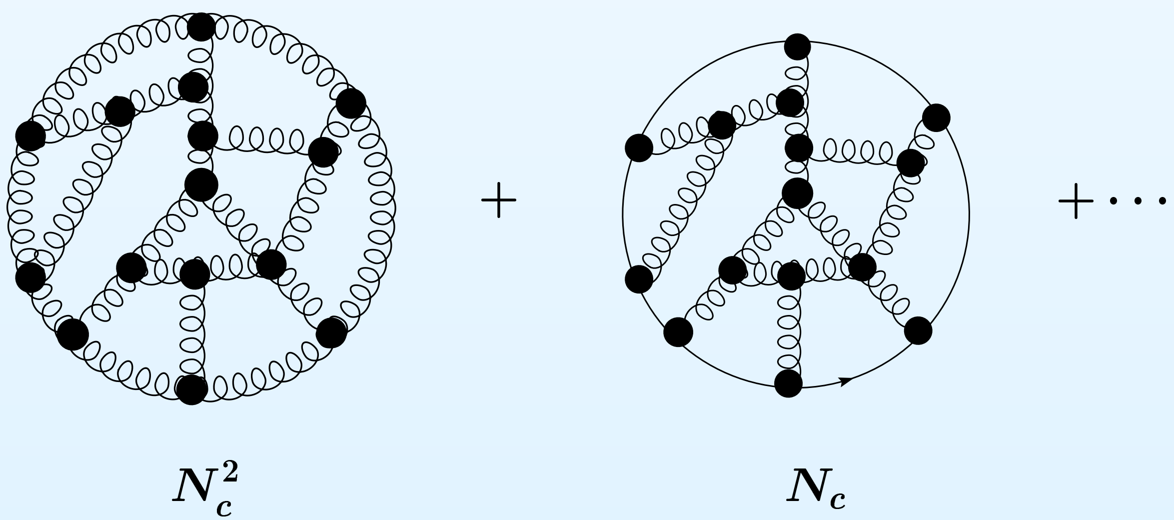
  $\rightarrow T_c(\mu, \mu) \sim T_c(\mu, -\mu)$
- Critical Endpoint: No access to order of trans. in Hadron Gas Model

$\rightarrow$ Quantitative agreement with lattice

$1/N_c$ Expansion

- Distinguishes classes of Feynman diagrams
- Relates different observables
  ▷ Hadron phenomenology
- $1/N_c$ expansion of the pressure:

$$p = N_c^2\, p_0(T) + N_c\, p_1(T, m_q) + \cdots$$

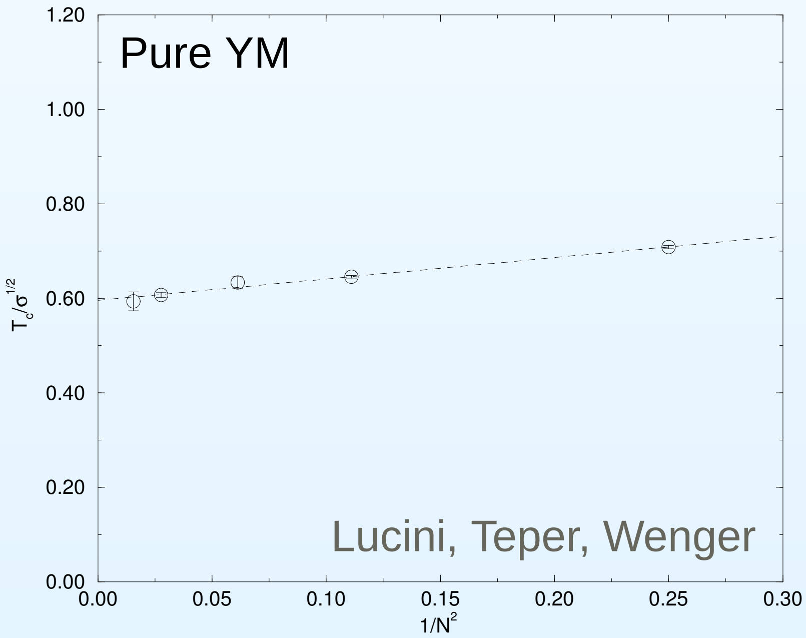


T_0 in $1/N_c$ Expansion

- Critical temperature: $\partial C_V/\partial T|_{T_c} = 0$

$$C_V = \frac{\partial \epsilon}{\partial T}\bigg|_V \Rightarrow C_V = N_c^2\left(c_0(T) + \frac{1}{N_c}c_1(T, m_q) + \cdots\right)$$

$$\Rightarrow T_0(N_c, N_f)/T_0^\infty = 1 + \frac{1}{N_c}t_1(N_f, m_q) + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$



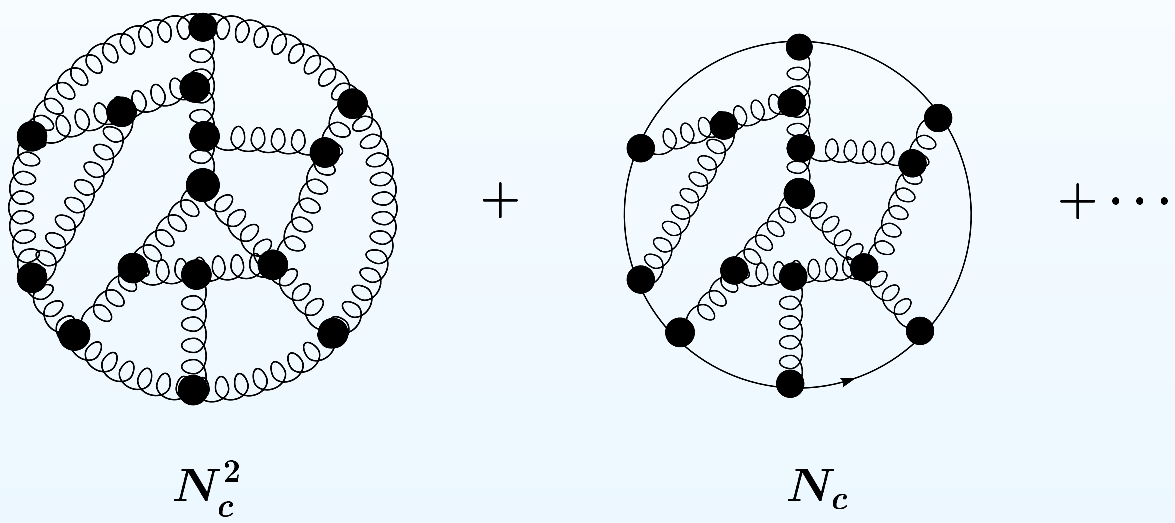
$$\frac{T_0^{\rm QCD} - T_0^{\rm YM}}{T_0^{\rm YM}} \sim 1/N_c \sim 30\%$$

$N_c = 3,\ N_f = 3$: $\sim\ 35\%$

$1/N_c \Rightarrow$ relations for critical temperature in different cases

Curvature in $1/N_c$ Expansion

$$p = N_c^2\, p_0(T) + N_c\, p_1(T, \mu^2) + \cdots$$



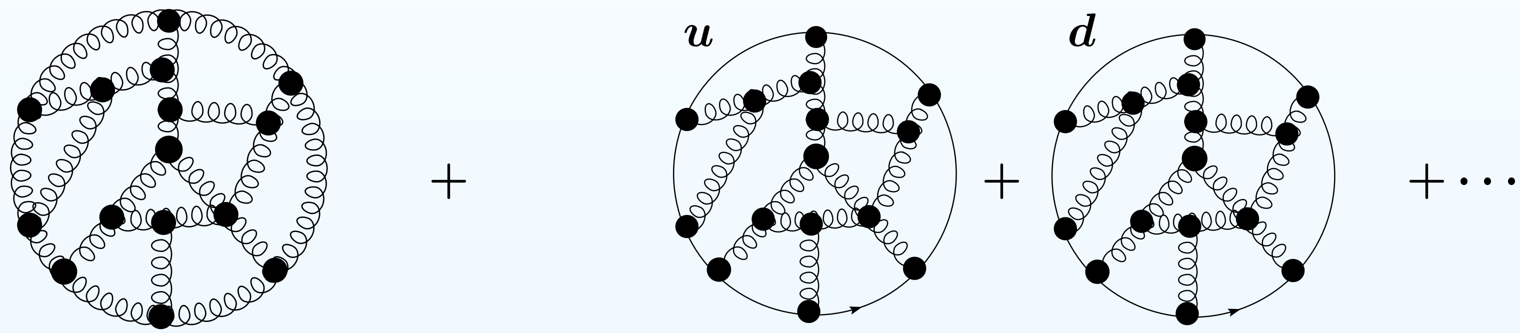
▷ Critical temperature: $\partial C_V / \partial T|_{T_c} = 0$

$$\Rightarrow T_c(\mu,\mu)/T_0 = 1 + \frac{t_1}{N_c}\left(\frac{\mu}{T_0}\right)^2 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

$\longrightarrow$ Curvature $\sim 1/N_c$: small!

μ_b vs μ_i in $1/N_c$ Expansion

$$p = N_c^2\, p_0(T) + N_c \left(p_1(T,\mu_u^2) + p_1(T,\mu_d^2) \right) + \cdots$$



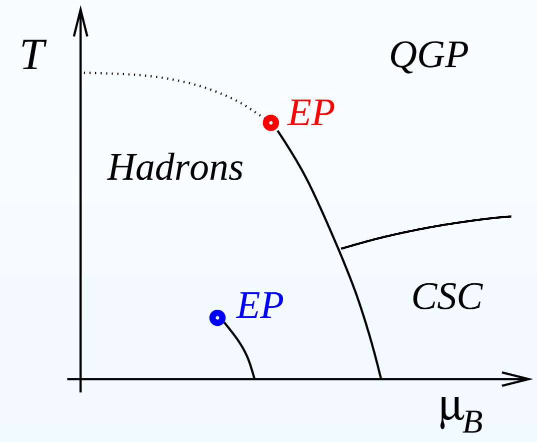
▷ Critical temperature: $\partial C_V / \partial T|_{T_c} = 0$

$$\Rightarrow T_c(\mu_u,\mu_d)/T_0 = 1 + \frac{1}{N_c}\left(t_1(\mu_u^2) + t_1(\mu_d^2) \right) + \mathcal{O}(\tfrac{1}{N_c^2})$$

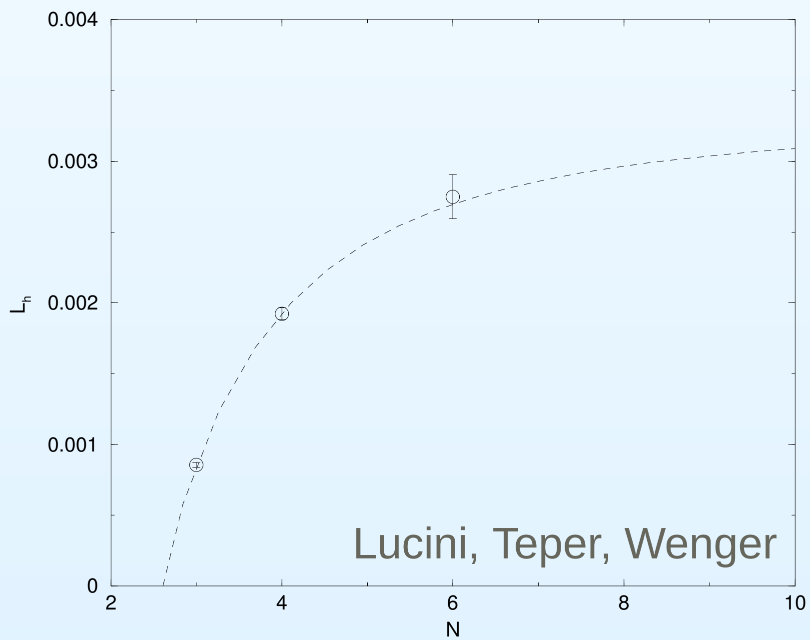
$$\Rightarrow \frac{T_c(\mu,\mu) - T_c(\mu,-\mu)}{T_c(\mu,\mu)} \sim 1/N_c^2 \sim 10\%$$

Critical Endpoint in $1/N_c$ Expansion

- If first order phase transition
  $\Rightarrow$ Latent heat: $L = T_c \, \mathrm{disc}\, s$
- $s = \partial p / \partial T|_V \Rightarrow L \sim N_c^2$



▷ Pure Yang-Mills: $L_h = L/N_c^2 = l_0 + \frac{1}{N_c^2} l_2 + \cdots$



Critical Endpoint in $1/N_c$ Expansion

- Clausius-Clapeyron: Leutwyler, Barducci, Casalbuoni, Gatto, Pettini

$$L = \frac{T_c}{\partial T_c/\partial m_q|_{\{\mu_f\}}} \mathrm{disc}\ \langle \bar{q}q \rangle$$

$$\longrightarrow \partial T_c/\partial m_q|_{\{\mu_f\}} \sim 1/N_c$$

$$\longrightarrow \langle \bar{q}q \rangle|_{T=0} \sim N_c$$

- If $T \lesssim 140$ MeV: $\langle \bar{q}q \rangle \sim$ constant

$$\Rightarrow L(\mu_u, \mu_d) = N_c^2 \left(l_0 + \frac{1}{N_c} \left(l_1(\mu_u^2) + l_1(\mu_d^2) \right) + \mathcal{O}(\frac{1}{N_c^2}) \right)$$

$$\Rightarrow \frac{L(\mu,\mu) - L(\mu,-\mu)}{L(\mu,\mu)} \sim 1/N_c^2 \sim 10\%$$

1st order at $(\mu_b^c, T_c) \Leftrightarrow$ 1st order at $(\mu_i^c, T_c) + \mathcal{O}(1/N_c^2)$

$1/N_c$ Expansion: Results

- $T_0 = T_\infty \left(1 + \mathcal{O}(\frac{1}{N_c})\right)$
- $T_c(\mu,\mu) = T_0 \left(1 + \frac{t_1}{N_c} \left(\frac{\mu}{T_0}\right)^2 \right)$
- $T_c(\mu,-\mu) = T_c(\mu,\mu) \left(1 + \mathcal{O}(\frac{1}{N_c^2})\right)$

  $\longrightarrow T_c(\mu,\mu) \sim T_c(\mu,-\mu)$
- $L(\mu,-\mu) = L(\mu,\mu) \left(1 + \mathcal{O}(\frac{1}{N_c^2})\right)$

  $\longrightarrow$ Constraints on location of critical endpt.

$\longrightarrow$ Qualitative agreement with lattice

Conclusion and Outlook

- Properties of the critical temperature
  - ▷ Crossover
  - ▷ Small curvature of $T_c(\mu,\mu)$
  - ▷ $T_c(\mu,\mu) \sim T_c(\mu,-\mu)$
  - ▷ Critical endpoint

  can be understood in Hadron Gas Model and $1/N_c$ Expansion

- $1/N_c$ Expansion $\Rightarrow$ new perspective:
  - ▷ Relation between (μ_b, T) and (μ_i, T)
  - ▷ Lattice at $\mu_i \neq 0$ useful for $\mu_b \neq 0$