Standard-model prediction for direct CP violation in $K \to \pi\pi$ decay

KITP: Lattice Gauge Theory for the LHC and Beyond

August 19, 2015

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Outline

- Physics of CP violation and $K \rightarrow \pi\pi$ decay
- Calculating $K \rightarrow \pi\pi$ using lattice QCD: $\Delta I = 3/2$ & $1/2$
- Calculation of ε'
- Outlook

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CP violation and

$K \rightarrow \pi\,\pi$ decay

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$K \rightarrow \pi\,\pi$ and CP violation

- Final $\pi\pi$ states can have $I = 0$ or 2.

$$\langle \pi\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$

$$\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i e^{\delta_2 - \delta_0}}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| \left(\frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} - \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \right)$$

Direct CP violation

$K^0 - \overline{K^0}$ mixing

- ΔS=1 weak decays allow K^0 and $\overline{K^0}$ to decay to the same $\pi - \pi$ state.
- Resulting mixing described by Wigner-Weisskopf:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

- Decaying states are mixtures of K^0 and K^0

$$|K_S\rangle = \frac{K_+ + \overline{\epsilon} K_-}{\sqrt{1+|\overline{\epsilon}|^2}}$$
$$|K_L\rangle = \frac{K_- + \overline{\epsilon} K_+}{\sqrt{1+|\overline{\epsilon}|^2}}$$

$$\overline{\epsilon} = \frac{i}{2}\left\{ \frac{\mathrm{Im}\, M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\, \Gamma_{0\bar{0}}}{\mathrm{Re}\, M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\, \Gamma_{0\bar{0}}} \right\}$$

Indirect CP violation

CP violation

- CP violating, experimental amplitudes:

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- | H_w | K_L \rangle}{\langle \pi^+\pi^- | H_w | K_S \rangle} = \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0 | H_w | K_L \rangle}{\langle \pi^0\pi^0 | H_w | K_S \rangle} = \epsilon - 2\epsilon'$$

- Where: $\epsilon = \overline{\epsilon} + i\frac{\mathrm{Im}\boldsymbol{A}_0}{\mathrm{Re}\boldsymbol{A}_0}$

Indirect: $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$

Direct: $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.66 \pm 0.23) \times 10^{-3}$

$K \to \pi\,\pi$ decay from lattice QCD

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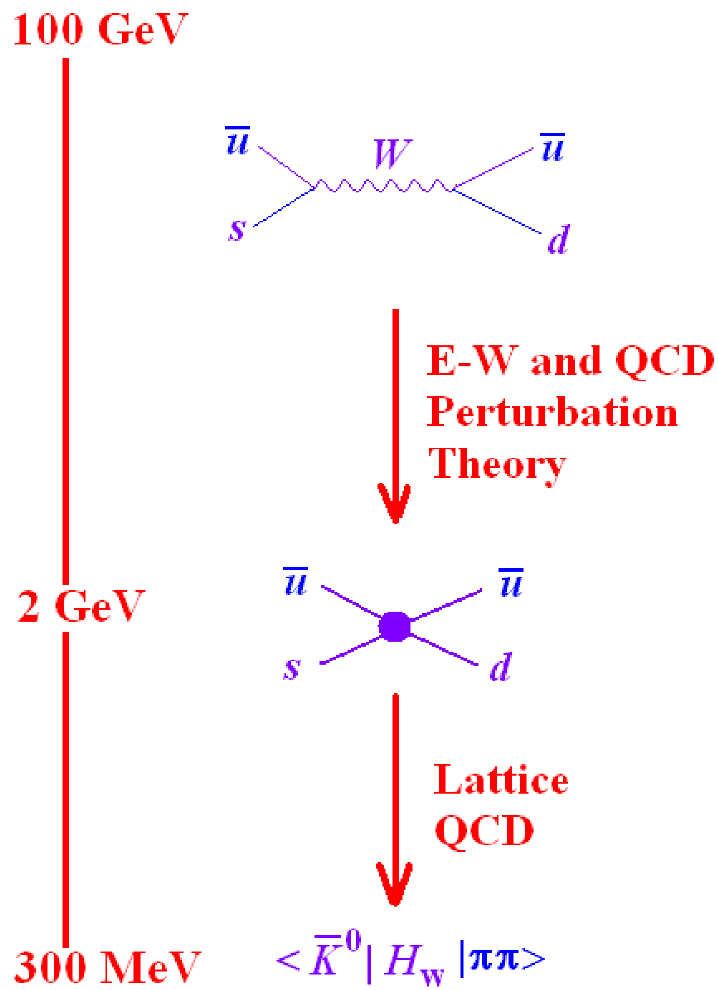
Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

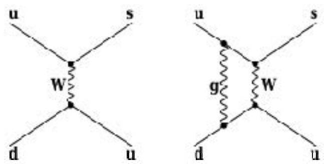
$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i \right\}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = (1.543 + 0.635i) \times 10^{-3}$
- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



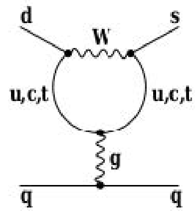
Four quark operators

- **Current-current operators**



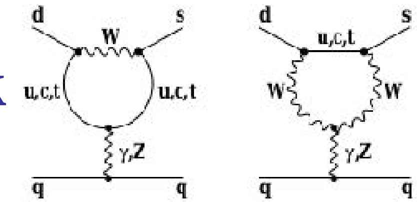
$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$
$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**



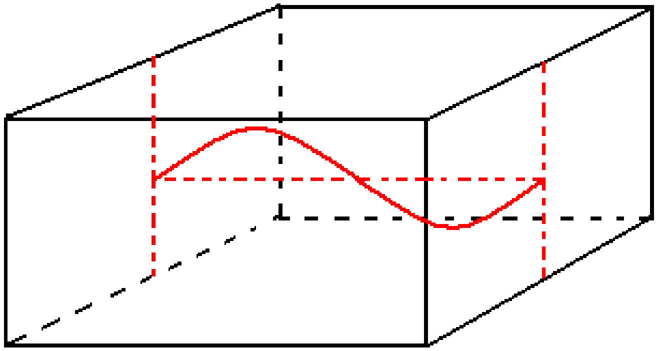
$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$
$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

- **Electro-Weak Penguins**



$$Q_7 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_8 \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V+A}$$
$$Q_9 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_{10} \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V-A}$$

Physical $\pi\pi$ states – Lellouch-Luscher



- Euclidean e^{-Ht} projects onto
  $|\pi\pi(\vec{p}=0)>$
- Use finite-volume quantization.
- Adjust volume so 1st or 2nd excited state has correct p.
- Requires extracting signal from non-leading large ***t*** behavior:

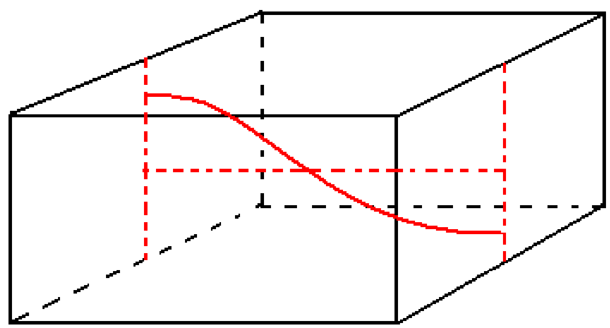
$$G(t) \sim c_0 e^{-E_0 t} + c_1 e^{-E_1 t}$$

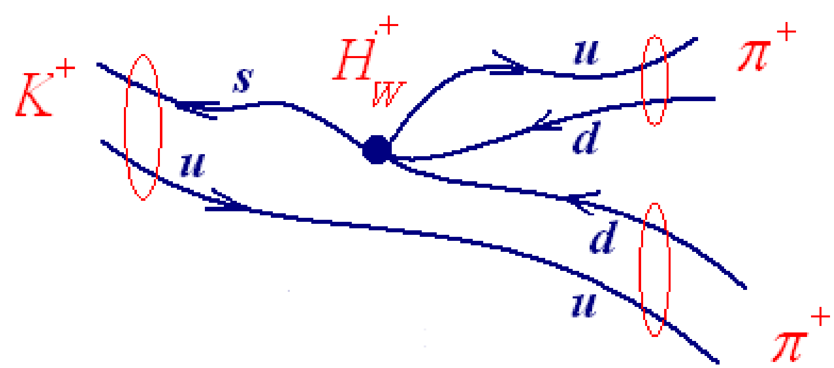
- Correctly include π - π interactions, including normalization.

$\Delta I = 3/2$

Δ I = 3/2 $K \rightarrow \pi\pi$

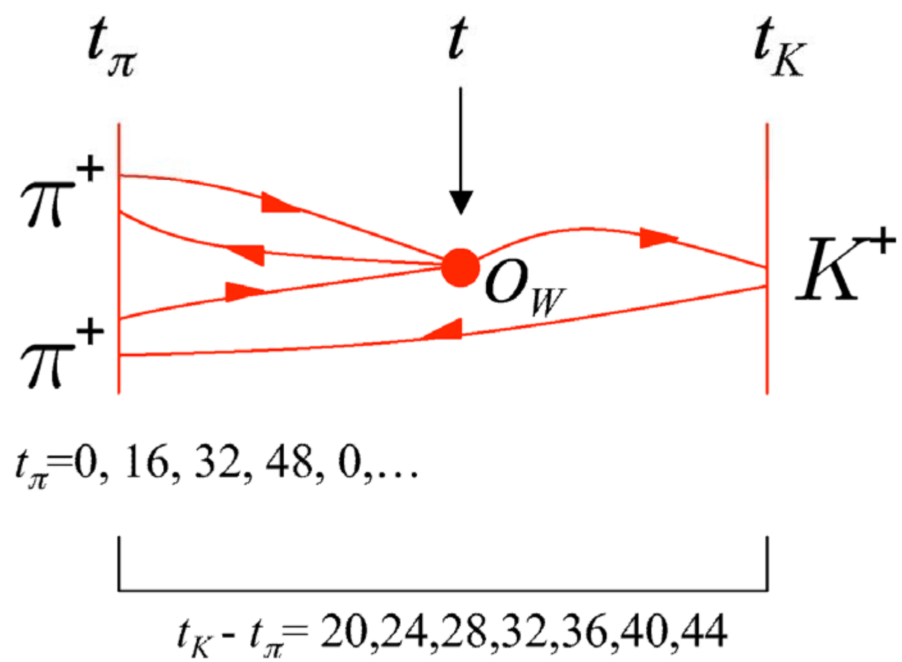
- Three operators contribute $O^{(27,1)}$, $O^{(8,8)}$ and $O^{(8,8)m}$.
- Use isospin to relate to $K^+ \rightarrow \pi^+\pi^+$.
- Use anti-periodic boundary conditions for *d* quark. (Changhoan Kim, hep-lat/0210003).
- Achieve essentially physical kinematics for 32^3 x 64 DSDR ensemble (146 configurations)
  - m_π = 142.9(1.1) MeV
  - m_K = 511.3(3.9) MeV
  - $E_{\pi\pi}$ = 492(5.5) MeV



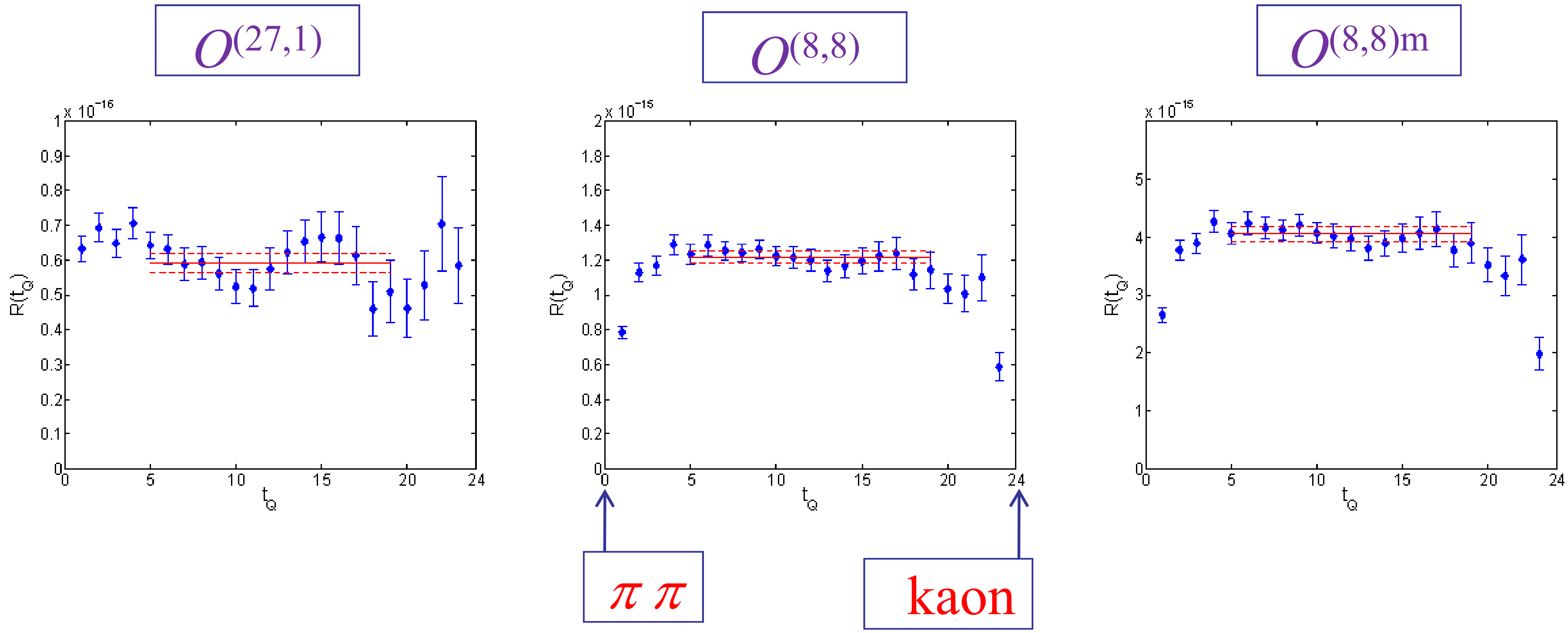


Computational Set-up
(Lightman and Goode)

- Use anti-periodic boundary conditions for d quark in two directions (average over three choices).
- Fix $\pi - \pi$ source at $t = 0$, vary location of O_W and kaon source.



$<\pi\,\pi\,|\,O\,|\,K>$ from 146 configurations



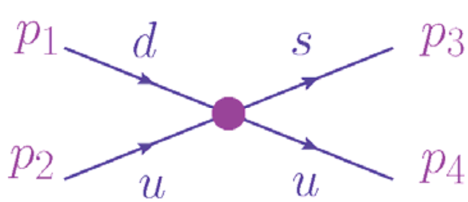
Plot ratio of correlators:

$$\frac{C^{i}_{K\pi\pi}(t)}{C_K(t_K - t)\,C_{\pi\pi}(t)} = \frac{\mathcal{M}_i}{Z_K Z_{\pi\pi}}$$

Operator Normalization
(Rome-Southampton)

- Effective weak Hamiltonian H_W contains four-quark operators normalized in the $\overline{\text{MS}}$ scheme.

- Impose non-perturbative RI scheme on lattice operators:
  - Evaluate Landau-gauge, off-shell Green's functions:



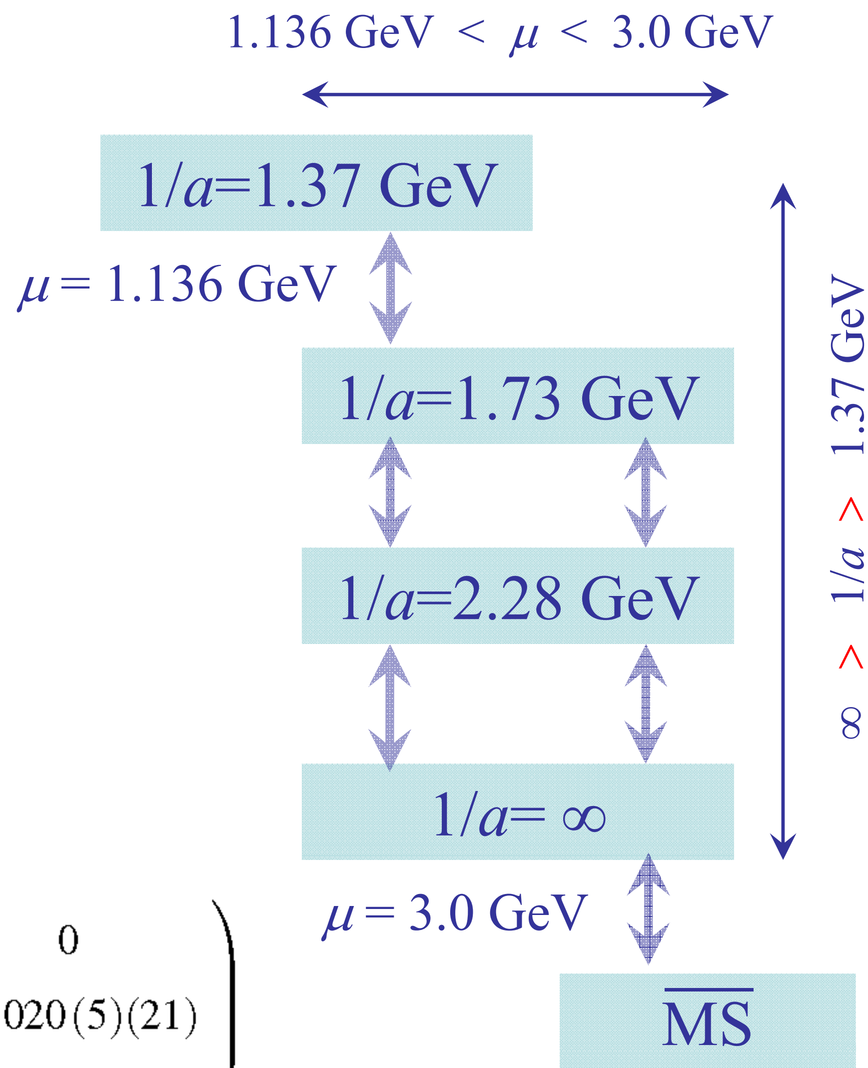
$$\left(\Gamma(p_1, p_2, p_3, p_4)_j\right)^{\alpha\beta\gamma\delta}_{abcd} = \prod_{i=1}^{4}\left(\int d^4x_i e^{ip_i\cdot x_i}\right)\left\langle \overline{q}^{\alpha}_a(x_1)\overline{q}^{\beta}_b(x_2) O_j q^{\delta}_c(x_3) q^{\gamma}_d(x_4)\right\rangle$$

  - Impose normalization conditions: $\mathrm{tr}\{P_i \Gamma_j\} = F_{ij}$

- Use continuum perturbation theory to convert RI to $\overline{\text{MS}}$

Relate lattice and continuum operators

- Normalize off-shell, gauge-fixed 4-quark Greens functions.
- Calculation is performed on $1/a$=1.37 GeV lattice.
- Converting to perturbative $\overline{\mathrm{MS}}$ scheme is unreliable at scale $\mu \sim 1/a$!
- Carry out sequence of NP RI matching steps:

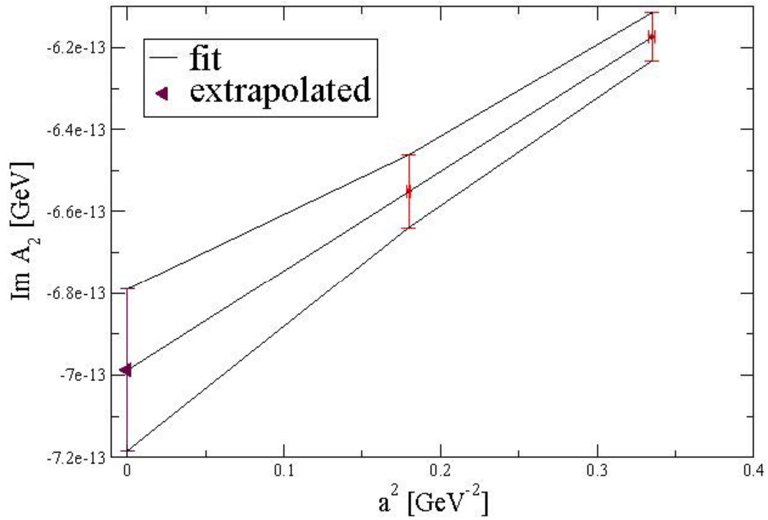


$$Z^{\overline{\mathrm{MS}},(\mathrm{latt})}_{(\not{a},\not{a})}(\mu) = \begin{pmatrix} 0.424(4)(4) & 0 & 0 \\ 0 & 0.472(6)(8) & -0.020(5)(21) \\ 0 & -0.067(23)(30) & 0.572(28)(20) \end{pmatrix}$$

Δ I = 3/2 – Continuum Results

(Tadeusz Janowski)

- Use two new large ensembles to remove a^2 error (m_π=135 MeV, L=5.4 fm)
  - 48^3 x 96, 1/a=1.73 GeV
  - 64^3 x 128, 1/a=2.28 GeV

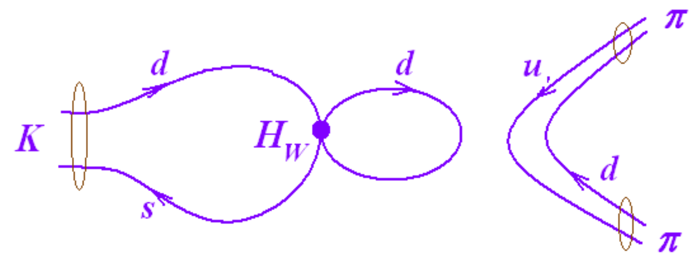


- Continuum results:
  - $\mathrm{Re}(A_2) = 1.50(0.04_{stat})\ (0.14_{syst})\times 10^{-8}$ GeV
  - $\mathrm{Im}(A_2) = -6.99(0.20)_{stat}\ (0.84)_{syst}\times 10^{-13}$ GeV
- Experiment: $\mathrm{Re}(A_2) = 1.479(4)\ 10^{-8}$ GeV
- $E_{\pi\pi} \rightarrow \delta_2 = -11.6(2.5)(1.2)^o$
- Phys.Rev. **D91**, 074502 (2015)

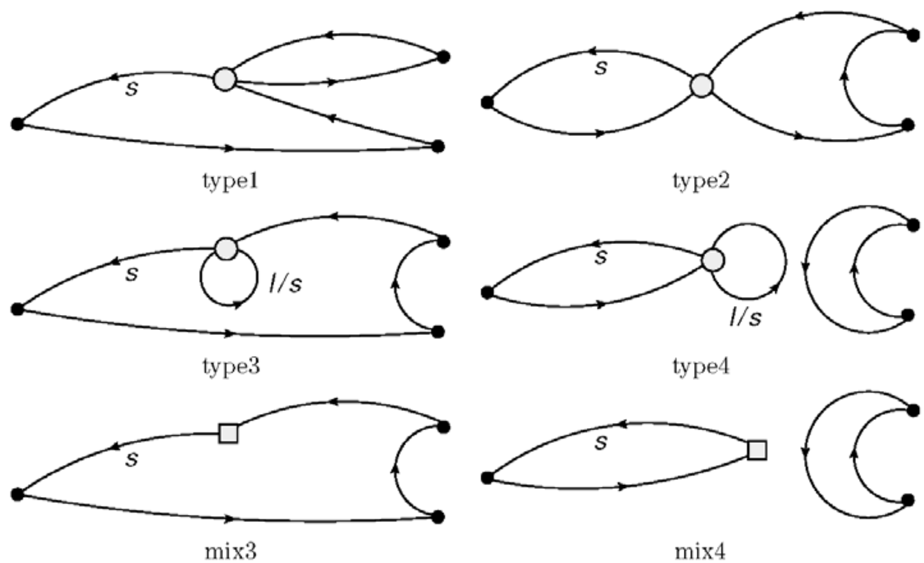
Δ I = 1/2

$\Delta I = 1/2 \;\; K \rightarrow \pi\pi$

- Made much more difficult by disconnected diagrams:

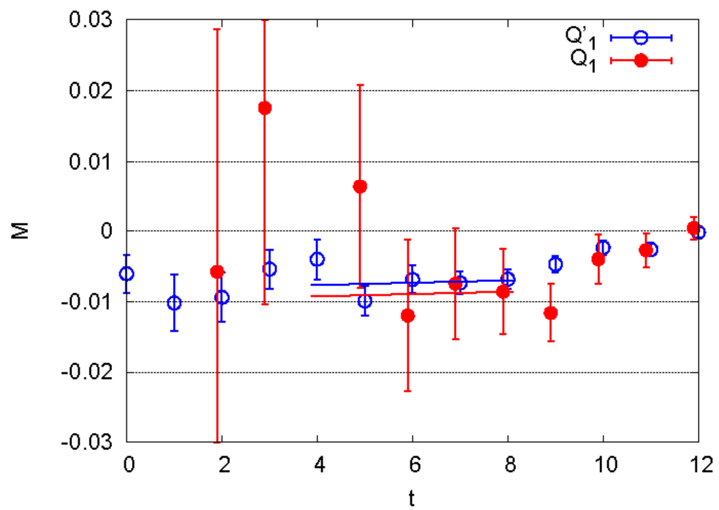


- Many more diagrams (48) than $\Delta I = 3/2$:

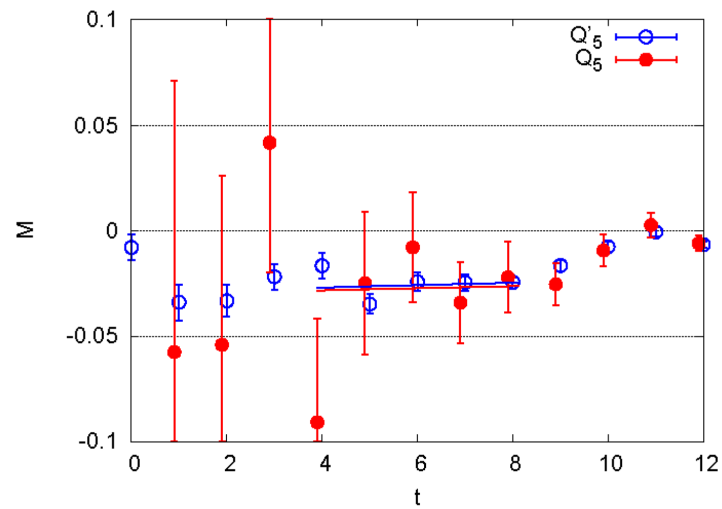


$\Delta I = 1/2$ $K \rightarrow \pi\pi$ at threshold
(Qi Liu)

- Initial threshold decay calculation successful
  - Re (A_0): 25% statistical errors
  - Im (A_0): 50% statistical errors



Q2 - largest part of Re(A_0)

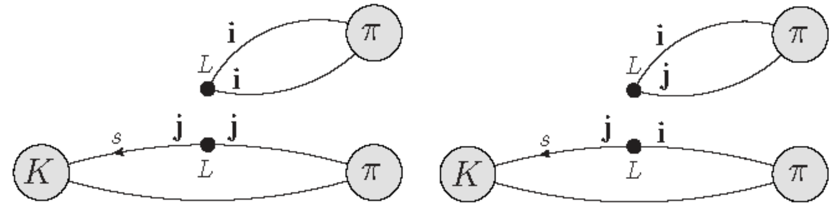


Q6 - largest part of Im(A_0)

Explain ΔI = 1/2 rule

(Q Liu)

- Two current-current diagrams dominate:



①   ②

- Where

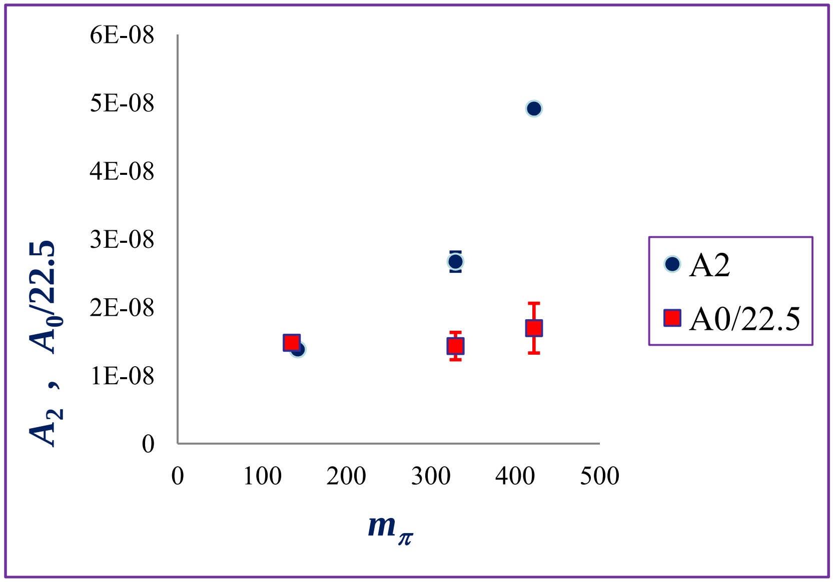
$$A_{0,2}(t_\pi, t_{\mathrm{op}}, t_K) \approx i\frac{1}{\sqrt{3}}\{2 \cdot ① - ②\}$$

$$A_{2,2}(t_\pi, t_{\mathrm{op}}, t_K) = i\sqrt{\frac{2}{3}}\{① + ②\}$$

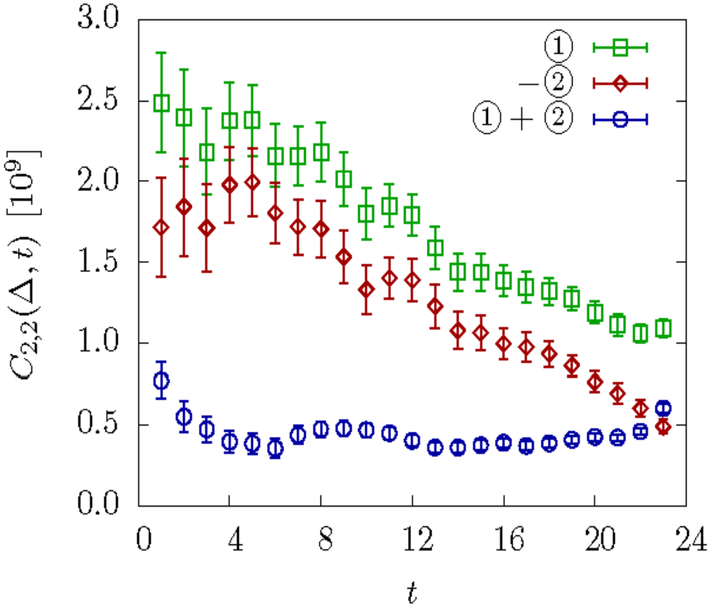
- Factorization: ② = 1/3 ①
- Actual calculation: ② = –0.7 ①

ΔI = 1/2 rule – Emerging explanation

Compare A_2 and A_0 /22.5



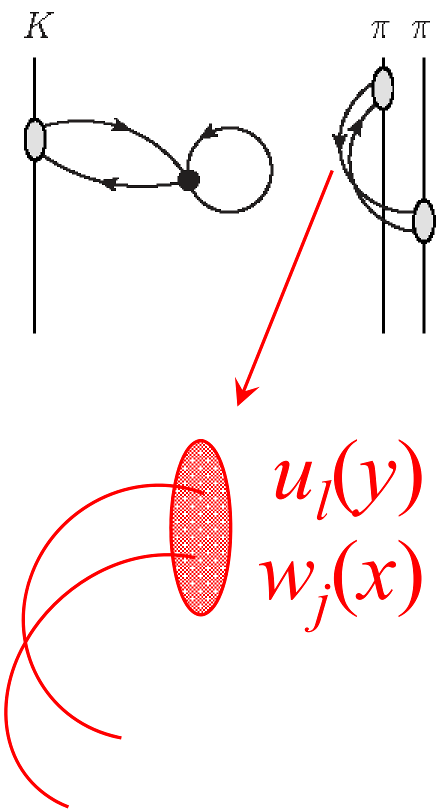
Cancellation in A_2



- 50 year puzzle resolved!
- Is this an explanation or the absence of one?

$\Delta I = ½$ $K \rightarrow \pi\pi$ – suppress vacuum
(Qi Liu & Daiqian Zhang)

- Separate two pion operators.
- Use **all-2-all** propagators (Trinity/KEK)
  - Sum over localized sources – further suppress vacuum coupling
  - See 5x improvement in statistics for $I = 0$, π–π scattering

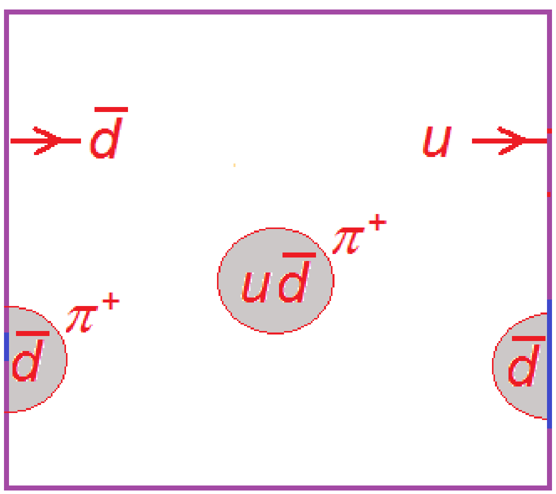


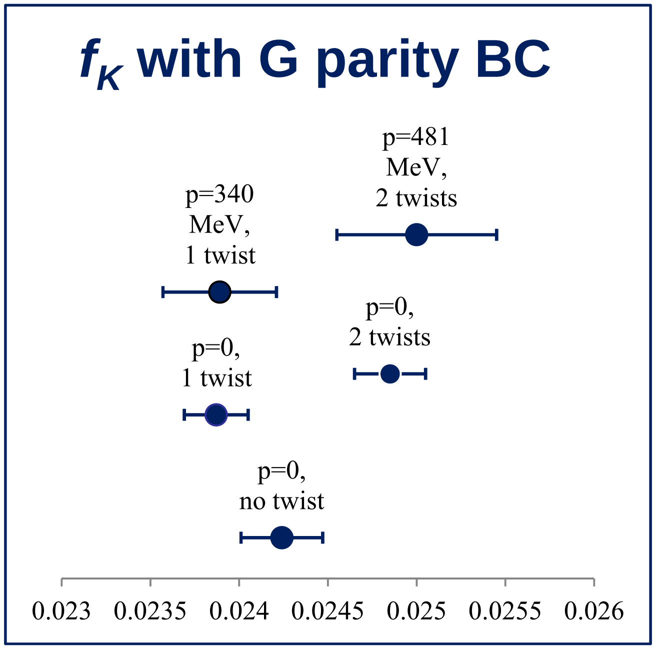
$$\begin{aligned}\langle q(x)\overline{q}(y)\rangle &= \langle x|\frac{1}{D_{\rm DWF}}|y\rangle \\ &= \sum_{n=1}^{N_{\rm modes}} \phi_n(x)\frac{1}{\lambda_n}\phi_n(y)^\dagger \\ &\quad + \sum_{k=1}^{N_{\rm noise}} \langle x|\frac{1}{D}\left(I - P_{n\le N_{\rm modes}}\right)|\eta_k\rangle\eta_k(y)^\dagger \\ &= \sum_{l=1}^{N_{\rm modes}+N_{\rm noise}} w_l(x)u_l(y)^\dagger\end{aligned}$$

$$\int d^3x d^3y \; u_l(\vec{y},t)^\dagger \psi_\pi(\vec{x},\vec{y}) w_j(\vec{x},t)$$

$\Delta I = ½\ K \rightarrow \pi\pi$ – above threshold
(Chris Kelly & Daiqian Zhang)

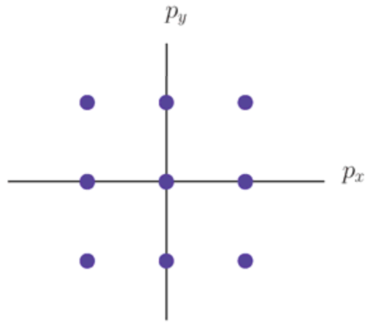
- Use **G-parity** BC to obtain $p_\pi = 205$ MeV (Changhoan Kim, hep-lat/0210003)
  - $G = C\, e^{i\pi I_y}$
  - Non-trivial: $\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$
  - Extra $I = 1/2$, s' quark adds $e^{-m_K L}$ error.
  - Tests: f_K and B_K correct within errors.





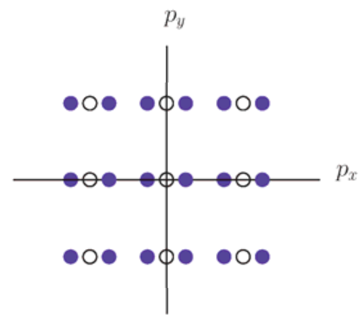
$\Delta I = 1/2\ \ K \to \pi\pi$ – G-parity

$(2\pi n_x/L\,,\ 2\pi n_x/L)$



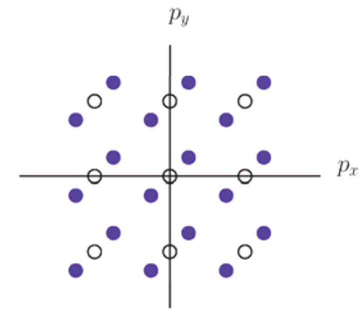
quark: 0 twists

$(2\pi(n_x \pm 1/4)/L\,,\ 2\pi n_x/L)$



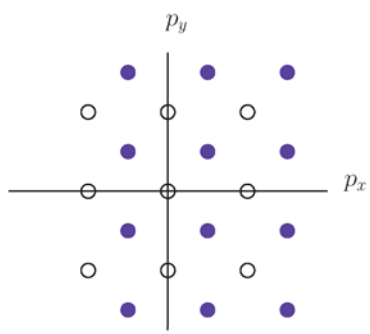
quark: 1 twists

$(2\pi(n_x \pm 1/4)/L\,,\ 2\pi(n_y \pm 1/4)/L)$



quark: 2 twists

$(2\pi(n_x \pm 1/2)/L\,,\ 2\pi(n_y \pm 1/2)/L)$



meson: 2 twists

- Allowed momenta with G-parity links *x* and *y*
- Diagonal structure results
- Breaks cubic symmetry

Calculation of A_0 and ε'

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Overview of calculation

- Use 32^3 x 64 ensemble
  - $1/a = 1.3784(68)$ GeV, $L = 4.53$ fm.
  - G-parity boundary condition in 3 directions
    - Usual $u - d$ iso-doublet
    - Unusual $s - s'$ with rooted determinant.
  - 216 configurations separated by 4 time units
  - 300 time units discarded for equilibration
  - 900 low modes for all-to-all propagators
  - Solve for $\pi\pi$ and kaon sources on each of 64 time slices
- Computer resources
  - 6 hours/trajectory – BG/Q ½ rack
  - 20 hours/trajectory – BG/Q ½ rack
  - One year to generate configurations, one year for measurements.

Overview of calculation

- Achieve essentially physical kinematics:
  - M_π = 143.1(2.0)
  - M_K = 490.6(2.2) MeV
  - $E_{\pi\pi}$ = 498(11) MeV
  - m_{res} = 0.001842(7) (90% of physical light quark mass)

Overview of results

- Determine the complex ΔI=1/2 amplitude A_0
  - $\mathrm{Re}(A_0) = (4.66 \pm 1.00_{\mathrm{stat}} \pm 1.26_{\mathrm{sys}}) \times 10^{-7}$ GeV
  - Expt: $(3.3201 \pm 0.0018) \times 10^{-7}$ GeV
  - $\mathrm{Im}(A_0) = (-1.90 \pm 1.23_{\mathrm{stat}} \pm 1.08_{\mathrm{sys}}) \times 10^{-11}$ GeV

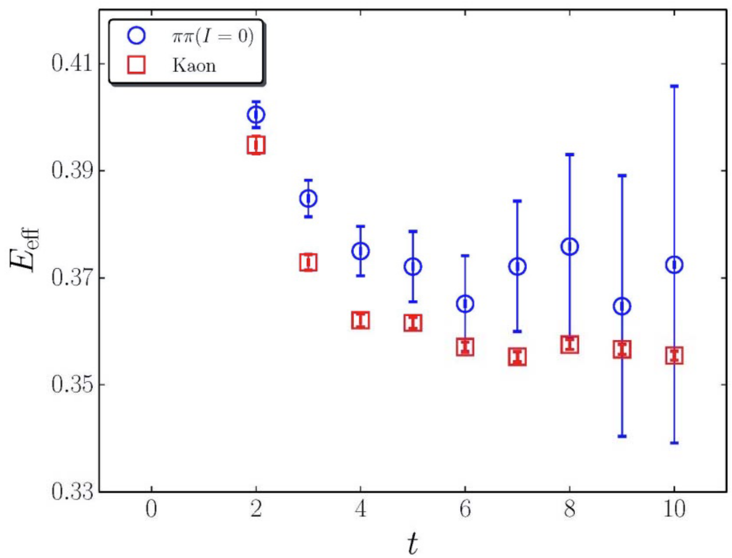
- Calculate $\mathrm{Re}(\varepsilon'/\varepsilon)$:
- $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.38 \pm 5.15_{\mathrm{stat}} \pm 4.59_{\mathrm{sys}}) \times 10^{-4}$
  - Expt.: $(16.6 \pm 2.3) \times 10^{-4}$
  - [2.1 σ difference]

Overview of systematic errors

Description	Error
Operator renormalization	15%
Wilson coefficients	12%
Finite lattice spacing	12%
Lellouch-Luscher factor	11%
Finite volume	7%
Parametric errors	5%
Excited states	5%
Unphysical kinematics	3%
Total	27%

$I = 0$, $\pi\pi - \pi\pi$ correlator

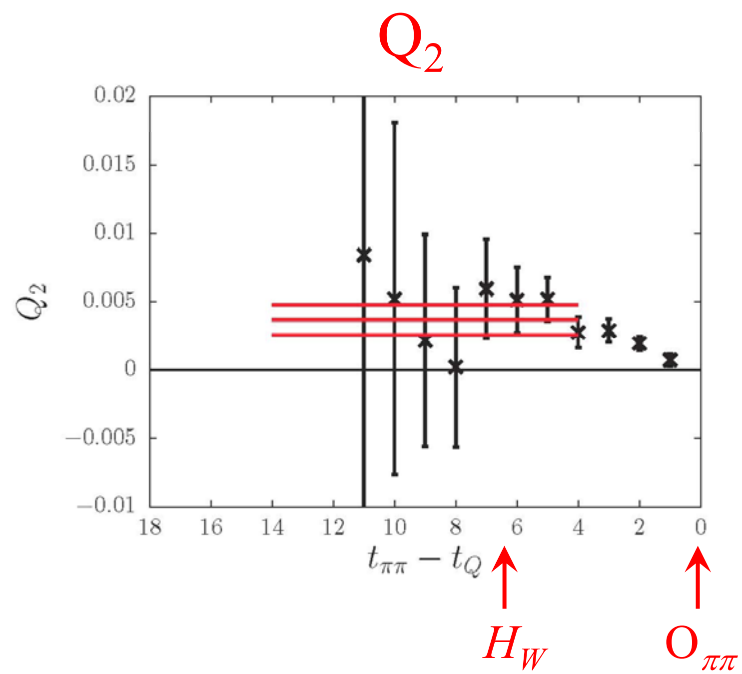
- Determine normalization of $\pi\pi$ interpolating operator.
- Determine energy of finite volume, I=0, $\pi\pi$ state: $E_{\pi\pi} =$ 498(11) MeV.
- Determine $I = 0$ $\pi\pi$ phase shift: $\delta_0 = 23.8(4.9)(2.2)^o$.

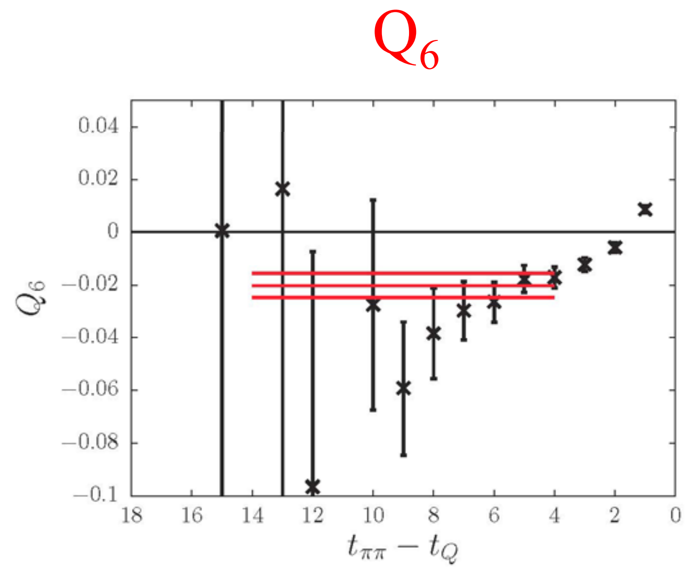


- $E_{\pi\pi}$ from a correlated 1-state fit, $6 \leq t \leq 25$, χ^2/dof=1.56(68)
- Consistent result obtained from 2-state fit, $3 \leq t \leq 25$.
- Leading-term amplitude changes by 5% between these two fits.

$\Delta I = ½$ $K \rightarrow \pi\pi$ matrix elements

- Vary time separation between H_W and $\pi\pi$ operator.
- Show data for all $K - H_W$ separations t_Q - $t_K \geq 6$ and $t_{\pi\pi}$ - t_K = 10, 12, 14, 16 and 18.
- Fit correlators with $t_{\pi\pi}$ - $t_Q \geq 4$
- Obtain consistent results for $t_{\pi\pi}$ - $t_Q \geq 3$ or 5





Test of rotational symmetry

- Normalization of $O_{\pi\pi}$ requires cubic symmetry.
- Extracting matrix elements for the ratio assumes the same A_1 state enters numerator and denominator.

$$\langle \pi\pi | Q_i | K \rangle = \frac{\langle O_{\pi\pi}(t_{\pi\pi}) Q_i(t_Q) K(t_K) \rangle}{\langle O_{\pi\pi}(t_{\pi\pi}) O_{\pi\pi}(t_Q) \rangle \langle K(t_Q) K(t_K) \rangle^{1/2}} e^{E_{\pi\pi}(t_{\pi\pi}-t_Q)/2} e^{m_K(t_Q-t_K)/2}$$

- Choose as symmetrical a pion wave function as possible:

$$\begin{aligned}(-\frac{\pi}{L},\frac{\pi}{L},\frac{\pi}{L}) &= (\frac{\pi}{2L},\frac{\pi}{2L},\frac{\pi}{2L}) + (\frac{-3\pi}{2L},\frac{\pi}{2L},\frac{\pi}{2L}) \\ &= (\frac{-\pi}{2L},\frac{-\pi}{2L},\frac{-\pi}{2L}) + (\frac{-\pi}{2L},\frac{3\pi}{2L},\frac{3\pi}{2L})\end{aligned}$$

	p=(+,+,+)	p=(-,+,+)	p=(+,-,+)	p=(+,+,-)
E_π	0.19852(85)	0.19823(82)	0.19839(72)	0.19866(88)
Z_π	6.167(69)e+06	6.081(63)e+06	6.183(50)e+06	6.170(61)e+06

Lattice matrix elements

Chiral basis

$Q'_1 = 3Q_1 + 2Q_2 - Q_3$

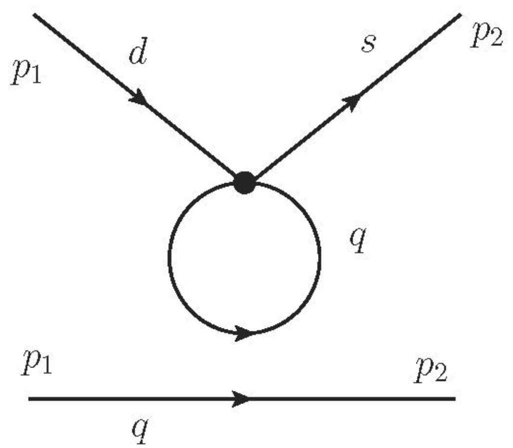
$Q'_2 = (2Q_1 - 2Q_2 + Q_3)/5$

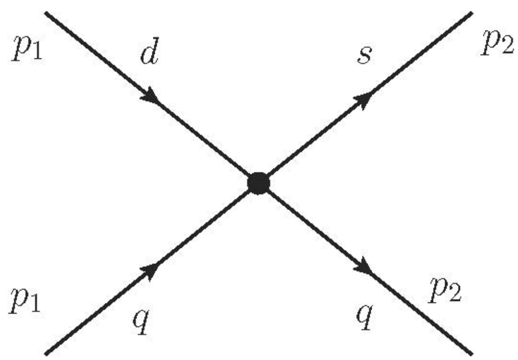
$Q'_3 = (3Q_1 - 3Q_2 + Q_3)/5$

	Conventional 10 operators	Chiral basis	
i	$\mathcal{M}^{(i)}_{\text{lat}}$ $(\text{GeV})^3$	$\mathcal{M}'^{(i)}_{\text{lat}}$ $(\text{GeV})^3$	
1	-0.247(62)	-0.147(242)	(27,1)
2	0.266(72)	-0.218(54)	(8,1)
3	-0.064(183)	0.295(59)	(8,1)
4	0.444(189)	—	(8,1)
5	-0.601(146)	-0.601(146)	(8,1)
6	-1.188(287)	-1.188(287)	(8,1)
7	1.33(8)	1.33(8)	(8,8)
8	4.65(14)	4.65(15)	(8,8)
9	-0.345(97)	—	
10	0.176(100)	—	

RI/SMOM normalization of chiral operators

- For (8,1) operators must include disconnected diagrams.
- Use $p_1 = 2\pi(4,4,0,0)/L$ and $p_2 = 2\pi(0,4,4,0)/L$
- $p_1^2 = p_2^2 = (p_1 - p_2)^2 = 1.531 \text{ GeV}^2$
- Use 100 configurations





RI/SMOM normalization of chiral operators

- For (8,1) operators must include disconnected diagrams.

7 x 7

$$
Z_{\mathrm{lat}\to\mathrm{RI}} = \begin{pmatrix}
\mathbf{0.45828(3)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbf{0.293(30)} & -0.313(35) & -0.012(11) & 0.006(7) & 0 & 0 \\
0 & 0.137(48) & \mathbf{0.835(57)} & -0.005(21) & 0.006(14) & 0 & 0 \\
0 & -0.160(134) & -0.160(143) & \mathbf{0.439(49)} & -0.071(296) & 0 & 0 \\
0 & 0.034(79) & 0.106(100) & -0.036(37) & \mathbf{0.346(24)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{0.49235(3)} & -0.09216(3) \\
0 & 0 & 0 & 0 & 0 & -0.05931(6) & \mathbf{0.3724(1)}
\end{pmatrix}
$$

10 x 7

$$
Z_{RI \to MS} = \begin{pmatrix}
0.19744 & 1.07389 & 0.13830 & 0 & 0 & 0 & 0 \\
0.19744 & -0.12667 & 0.75559 & 0.00208 & -0.00625 & 0 & 0 \\
0 & 2.96832 & 1.92607 & 0.00417 & -0.01250 & 0 & 0 \\
0 & 1.74719 & 2.49297 & 0.01554 & -0.04636 & 0 & 0 \\
0 & 0 & 0 & 1.00121 & -0.00364 & 0 & 0 \\
0 & -0.04687 & -0.10936 & -0.00164 & 0.99519 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.00121 & -0.00364 \\
0 & 0 & 0 & 0 & 0 & -0.01726 & 1.04206 \\
0.29616 & 0.12667 & -0.75559 & -0.00208 & 0.00625 & 0 & 0 \\
0.29616 & -1.07389 & -0.13830 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Physical matrix elements

i	$\mathcal{M}'^{(i)}_{\mathrm{SMOM}}$ (GeV)^3	$\mathcal{M}^{(i)}_{\overline{\mathrm{MS}}}$ (GeV)^3
1	-0.0675(1109)(128)	-0.151(29)(36)
2	-0.156(27)(30)	0.169(42)(41)
3	0.212(52)(40)	-0.0492(652)(118)
4	—	0.271(93)(65)
5	-0.193(62)(37)	-0.191(48)(46)
6	-0.366(103)(70)	-0.379(97)(91)
7	0.225(37)(43)	0.219(37)(53)
8	1.65(5)(31)	1.72(6)(41)
9	—	-0.202(54)(49)
10	—	0.118(42)(28)

Contributions to A_0

i	$\mathrm{Re}(A_0)$(GeV)	$\mathrm{Im}(A_0)$(GeV)
1	$1.02(0.20)(0.07) \times 10^{-7}$	0
2	$3.63(0.91)(0.28) \times 10^{-7}$	0
3	$-1.19(1.58)(1.12) \times 10^{-10}$	$1.54(2.04)(1.45) \times 10^{-12}$
4	$-1.86(0.63)(0.33) \times 10^{-9}$	$1.82(0.62)(0.32) \times 10^{-11}$
5	$-8.72(2.17)(1.80) \times 10^{-10}$	$1.57(0.39)(0.32) \times 10^{-12}$
6	$3.33(0.85)(0.22) \times 10^{-9}$	$-3.57(0.91)(0.24) \times 10^{-11}$
7	$2.40(0.41)(0.00) \times 10^{-11}$	$8.55(1.45)(0.00) \times 10^{-14}$
8	$-1.33(0.04)(0.00) \times 10^{-10}$	$-1.71(0.05)(0.00) \times 10^{-12}$
9	$-7.12(1.90)(0.46) \times 10^{-12}$	$-2.43(0.65)(0.16) \times 10^{-12}$
10	$7.57(2.72)(0.71) \times 10^{-12}$	$-4.74(1.70)(0.44) \times 10^{-13}$
Tot	$4.66(0.96)(0.27) \times 10^{-7}$	$-1.90(1.19)(0.32) \times 10^{-11}$

Stat. error NPR

Stat. error ME

$\mathrm{Re}(A_0) = 4.66(1.00)_{stat}(1.26)_{sys} \times 10^{-7}$ GeV

Expt: $3.3201(0.0018) \times 10^{-7}$ GeV

$\mathrm{Im}(A_0) = -1.90(1.23)_{stat}(1.08)_{sys} \times 10^{-11}$ GeV

Systematic errors

Description	Error
Operator renormalization	15%
Wilson coefficients	12%
Finite lattice spacing	12%
Lellouch-Luscher factor	11%
Finite volume	7%
Parametric errors	5%
Excited states	5%
Unphysical kinematics	3%
Total	27%

Calculate Re(ε'/ε)

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= (1.38 \pm 5.15_{\mathrm{stat}} \pm 4.59_{\mathrm{sys}}) \times 10^{-4}$$

Expt: $= (16.6 \pm 2.3) \times 10^{-4}$ [2.1 σ difference]

- Im(A_0), Im(A_2), δ_0 and δ_2 from lattice QCD
- Re(A_2) and Re(A_0) from measured decay rates
- $|\varepsilon| = 2.228(0.011) \times 10^{-3}$ from experiment
- arg(ε) = arctan($2\Delta M_K/\Gamma_S$) = 42.52° (Bell-Steinberger relation)

Testing Correctness

- RHMC: G-parity and "doubled lattice" evolutions agree
- Results for f_K and B_K agree with earlier DSDR values
- Calculation of matrix elements done by two people with independent code.
- G-parity code applied to $\Delta I = 3/2$ amplitudes and results agreed with earlier method.
- G-parity and standard RBC/UKQCD code agreed for a free field calculation with large mass and large volume to remove effects of boundary (with anti-periodic time boundary to ensure that loop graphs are non-zero).

Outlook

- Present calculation of Im(A_0) and ε' can be improved with added statistics:
  - Reduce statistical error 2 x
  - Use step-scaling to reduce NPR error
  - $(1.38 \pm 5.15_{stat} \pm 4.59_{sys})$ x 10^{-4} becomes $(1.38 \pm 2.58_{stat} \pm 3.93_{sys})$ x 10^{-4} [2.9 σ difference]
- Accurate NPR and theoretical control of rescattering effects allow many critical kaon quantities to be computed:
  - $K \to \pi\pi$, $\Delta I = 3/2$ and 1/2, ε'
  - $m_{K_L} - m_{K_S}$
  - Long-distance parts of ε and $K^0 \to \pi^0 l \bar{l}$, $K^+ \to \pi^+ \nu\bar{\nu}$