HVP contribution to the muon anomalous magnetic moment from lattice QCD

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HPQCD collaboration

Santa Barbara
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Using the Darwin (9600 core) Sandybridge/infiniband cluster at Cambridge, part of STFC's DiRAC HPC facility

Muon anomalous magnetic moment

$$\vec{\mu} = g\frac{e}{2m}\vec{S} \qquad\qquad a_\mu = \frac{g-2}{2}$$

Measure using polarised muons circulating in E and B fields. At a momentum where $\beta \times E$ terms cancel, difference between spin and cyclotron frequencies:

$$\omega_a = -\frac{e}{m}a_\mu B$$

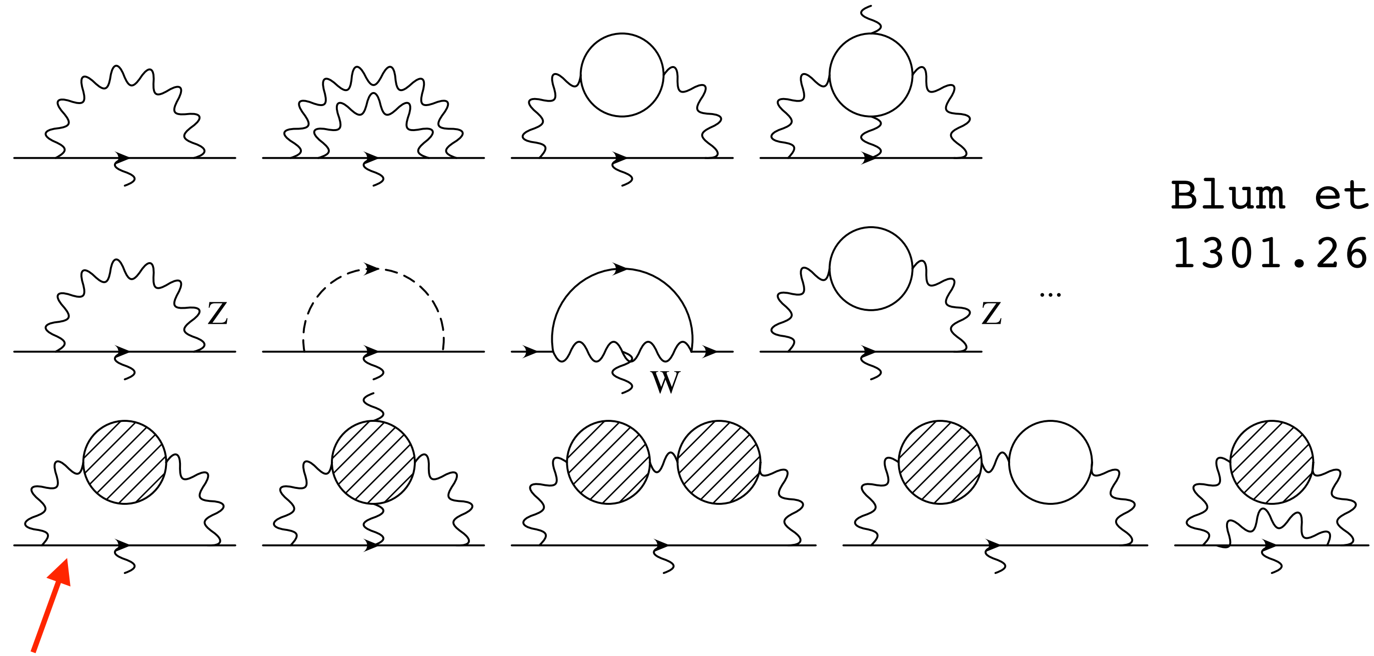
BNL result:

$$a_\mu^{expt} = 11659208.9(6.3) \times 10^{-10}$$

E989 (FNAL) will reduce exptl uncty to 1.6, starting 2017

Standard Model theory expectations

Contributions from QED, EW and QCD interactions. QED dominates.



Blum et al, 1301.2607

LO Hadronic vacuum polarisation (HVP)

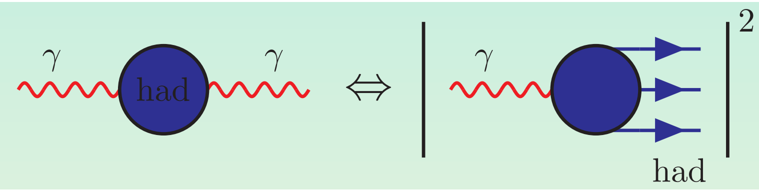
$$a_\mu^{expt} - a_\mu^{QED} - a_\mu^{EW} = 721.7(6.3) \times 10^{-10}$$

$$= a_\mu^{HVP} + a_\mu^{HOHVP} + a_\mu^{HLBL} + a_\mu^{new\, physics}$$

$$a_\mu^{HVP, no\, new\, physics} = 721.0(6.8) \times 10^{-10}$$

Best method to date for HVP uses exptl e+e- cross-section

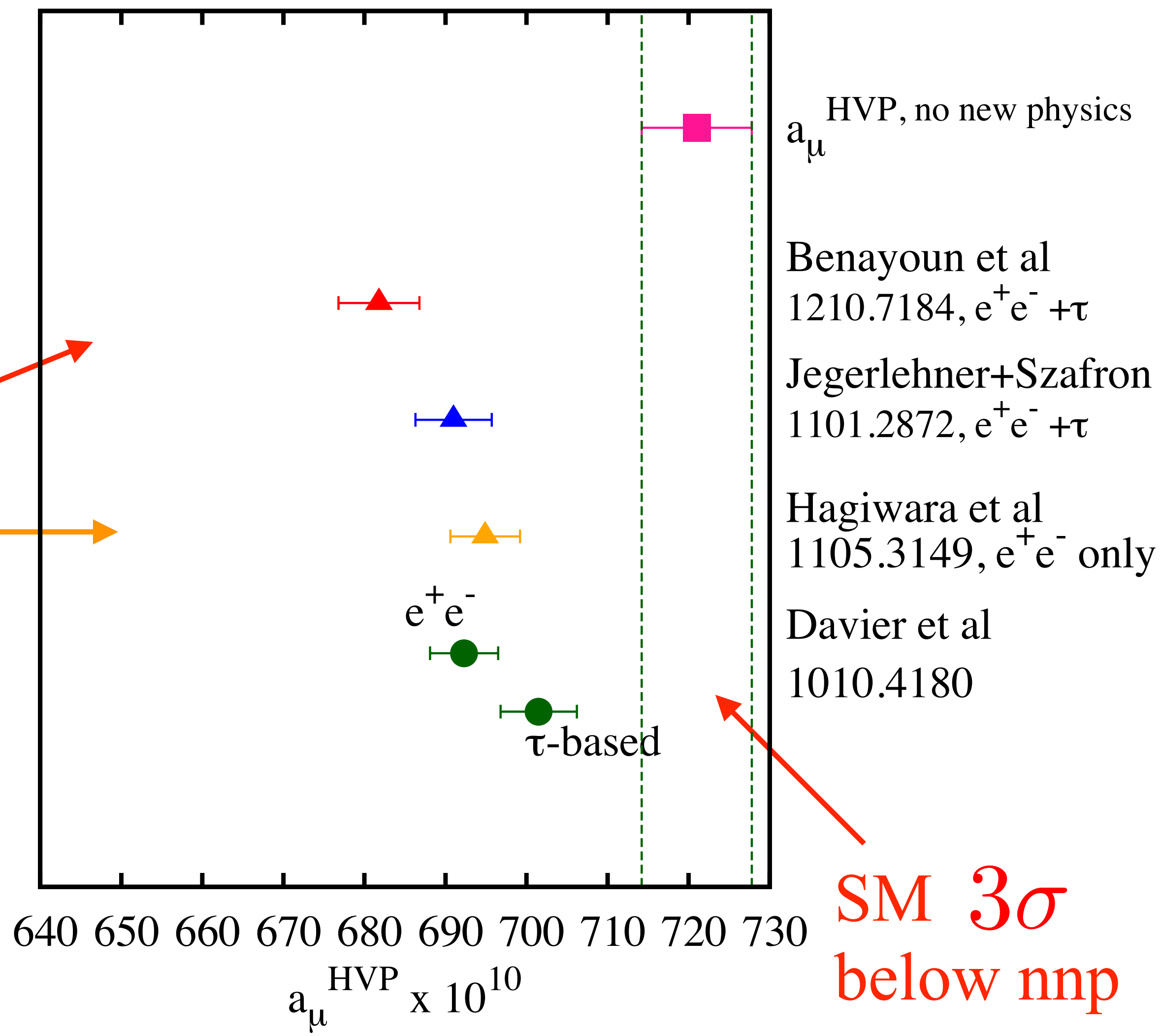
$$a_\mu^{HVP} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\,\sigma^0_{had}(s)K(s)$$



$e^+e^- \to \gamma^* \to hadrons$

"bare" cross-section but inc. final-state radiation

some "tension" between results. Difference is use of BaBar radiative return data

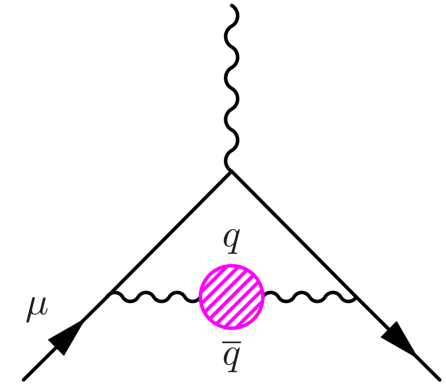


Hagiwara et al:

$$a_\mu^{HVP} = 694.9(4.3) \times 10^{-10}$$

Lattice calculation of HVP

Analytically continue to Euclidean q^2.



$$a_\mu^{HVP,i} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha e_i^2)\hat{\Pi}_i(q^2)$$

Blum, hep-lat/0212018

connected contribution for flavour i

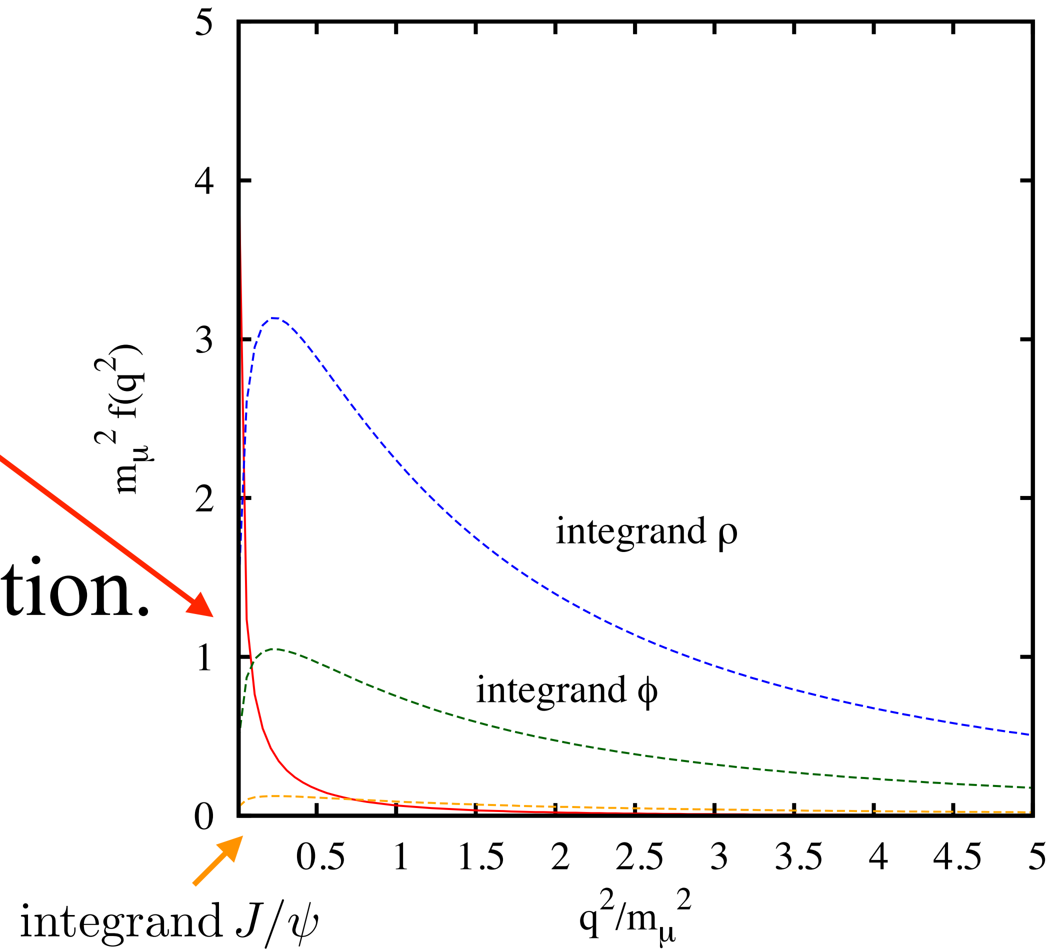
$f(q^2)$ is divergent function with scale set by m_μ

$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

is vacuum polarisation function.

Test with mesons:

$$\hat{\Pi}(q^2) \propto \frac{1}{m_V^2} - \frac{1}{q^2 + m_V^2}$$



Calculation with quarks

HPQCD 1403.1778

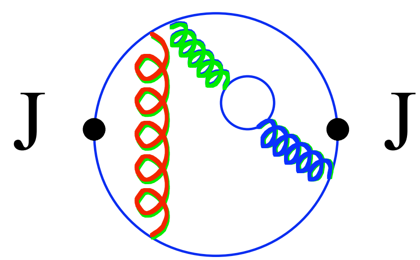
For spatial vector currents at zero spatial momentum

$$\Pi^{jj}(q^2) = q^2\Pi(q^2) = a^4 \sum_t e^{iqt} \sum_{\vec{x}} \langle j^j(\vec{x},t) j^j(0) \rangle$$

Time-moments of lattice current-current correlators

$$G_{2n} \equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^j(\vec{x},t) j^j(0) \rangle$$

$$= (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2) \Big|_{q^2=0}$$

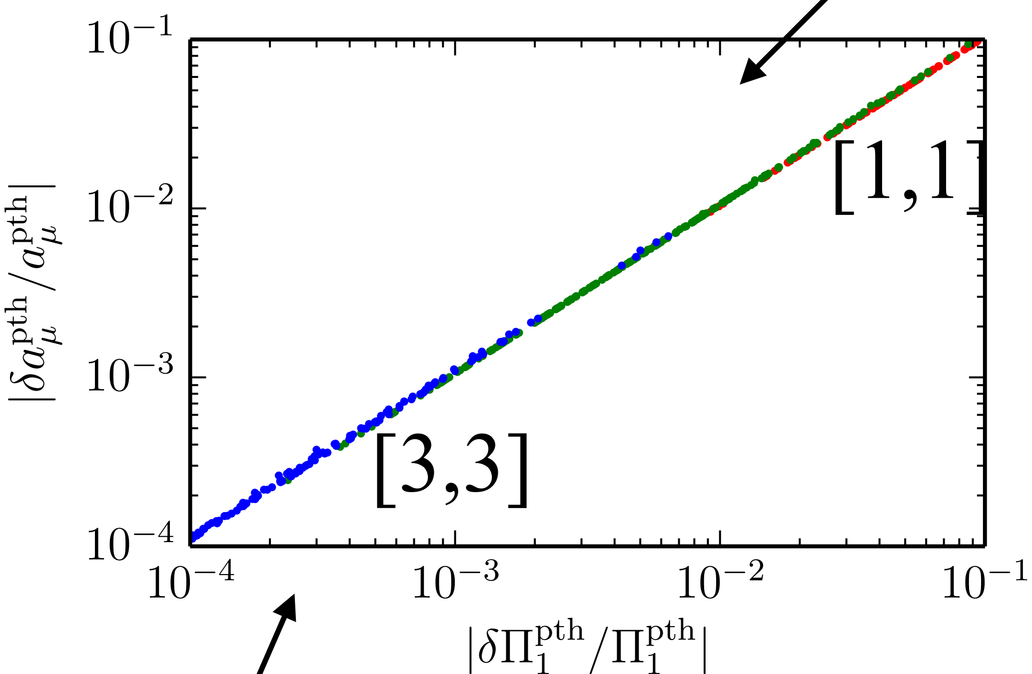


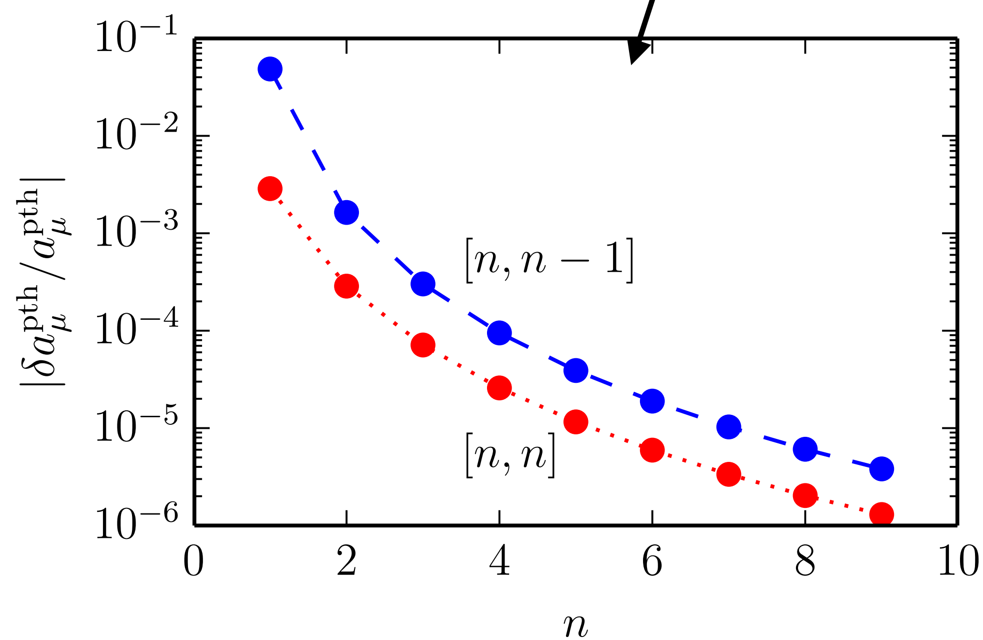
$$\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j} \Pi_j$$ with $$\Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}$$

Allows us to reconstruct $\hat{\Pi}(q^2)$ and integrate

Use Pade approximants (ratio of m/n polynomials) rather than Taylor expansion for better large q^2 behaviour.

Test Pade approximants in similar scenarios (1-loop quark vacuum polarisation, with noise added)





Improved precision allows higher order Pade - we use [2,2]

CHARM contribution

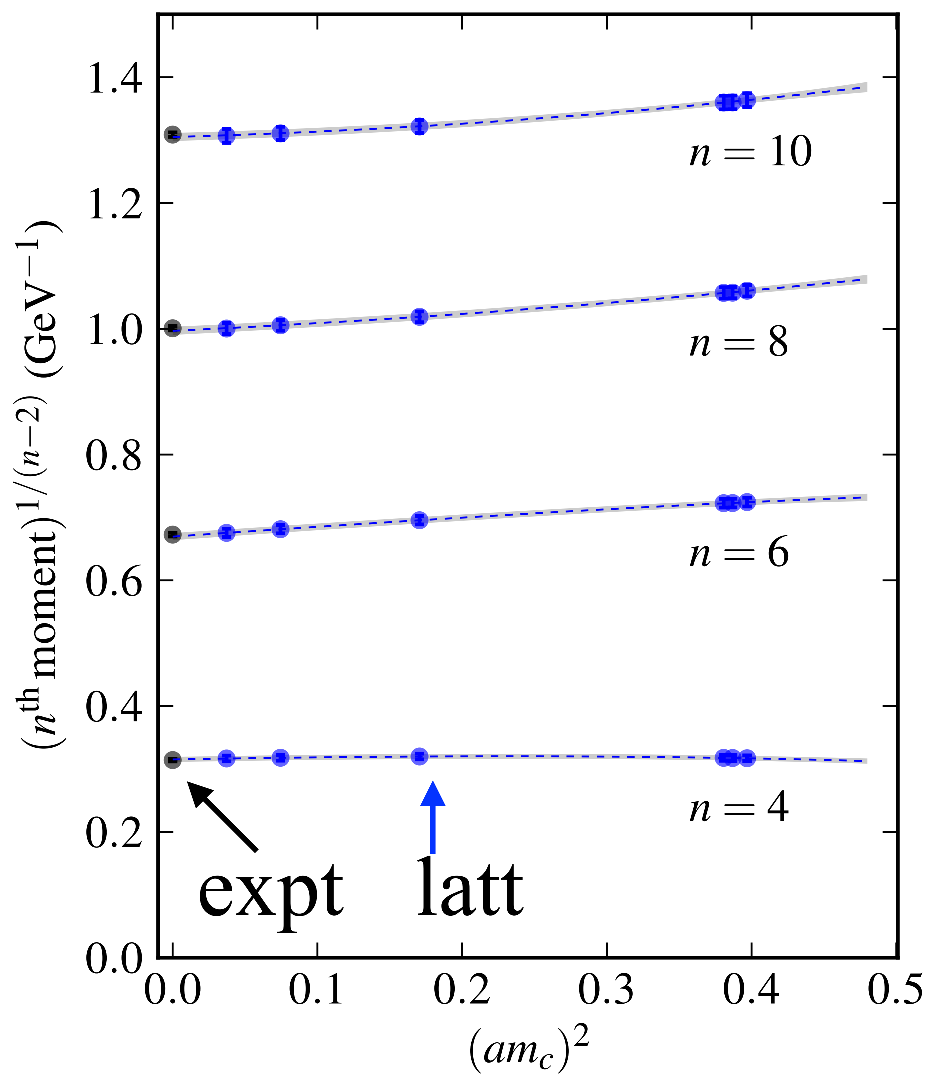
HPQCD 1004.4285, 1208.2855

Part of the set of calculations that gave

$m_c, M(J/\psi) - M(\eta_c), \Gamma(J/\Psi \to e^+e^-), \Gamma(J/\psi \to \eta_c\gamma)$

Used HISQ valence quarks on MILC 2+1 asqtad configs. Z_V from contnm QCD pert. th.

Extrapolation to physical point allows us to compare directly to moments from e+e- expt. in charm region



$$a_\mu^{HVP,c} = 14.4(4) \times 10^{-10}$$

HPQCD 1403.1778

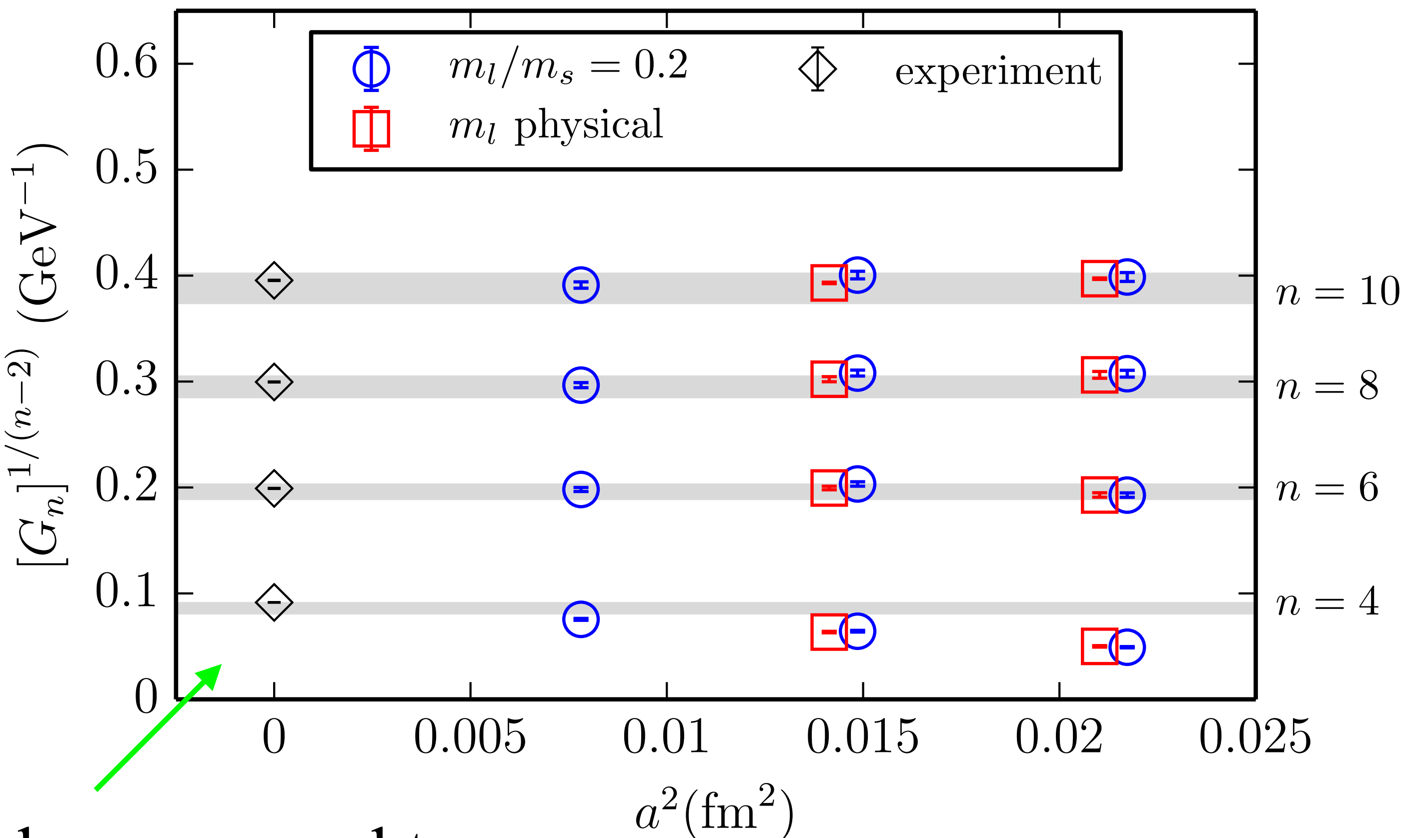
BOTTOM contribution

HPQCD 1110.6887,
1309.5797,
1408.5768

Part of the set of calculations that gave

$m_b, M(\Upsilon) - M(\eta_b), M(\Upsilon') - M(\eta_b'), \Gamma(\Upsilon \to e^+e^-), \Gamma(\Upsilon' \to e^+e^-)$

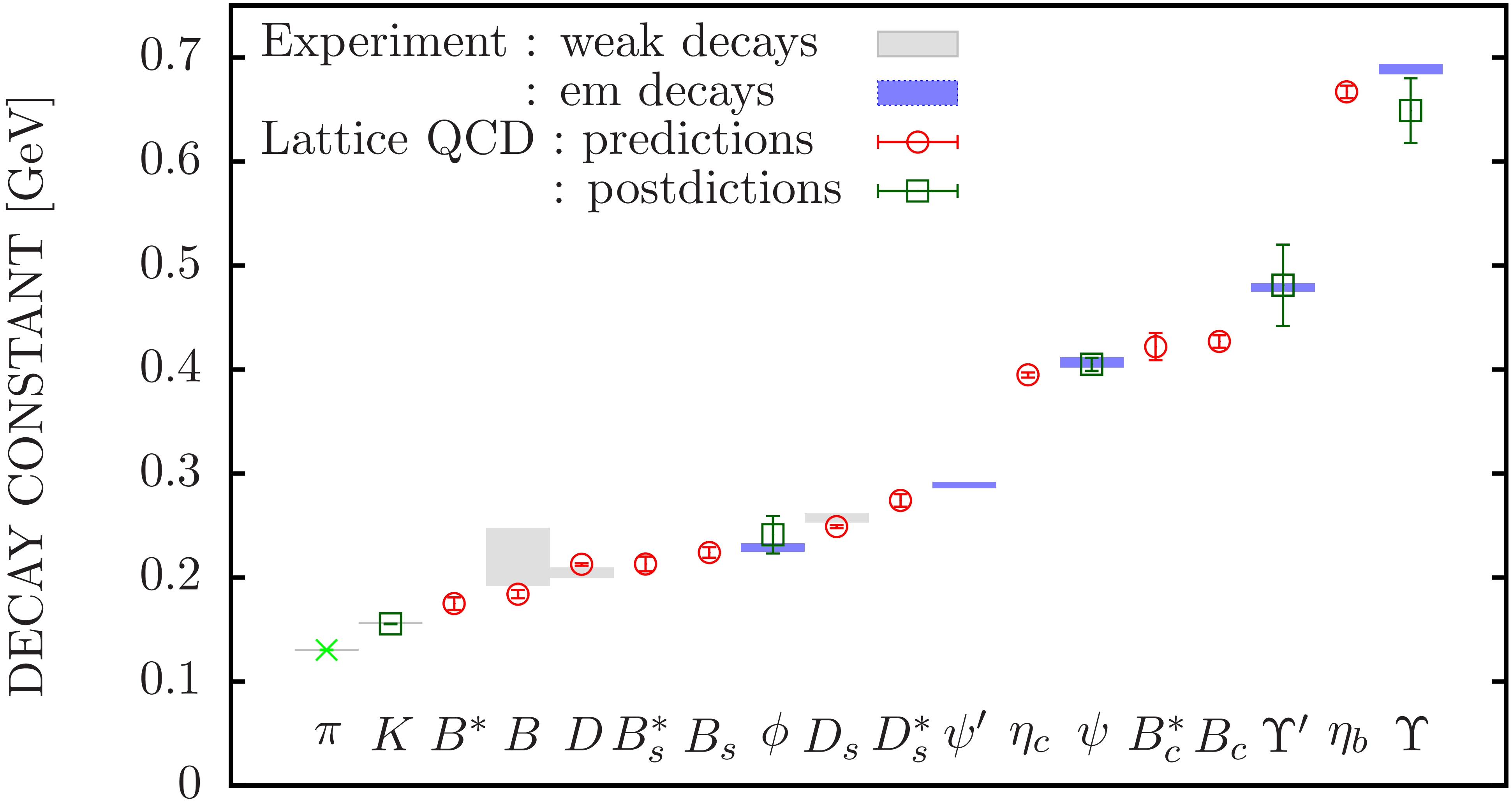
Used NRQCD valence quarks on MILC 2+1+1 HISQ configs. Z_V from contnm QCD pert. th.



Again, moments can be compared to those extracted from expt.

$a_\mu^{HVP,b} = 0.27(4) \times 10^{-10}$

Keep an eye on the 'big'picture whilst doing this .....



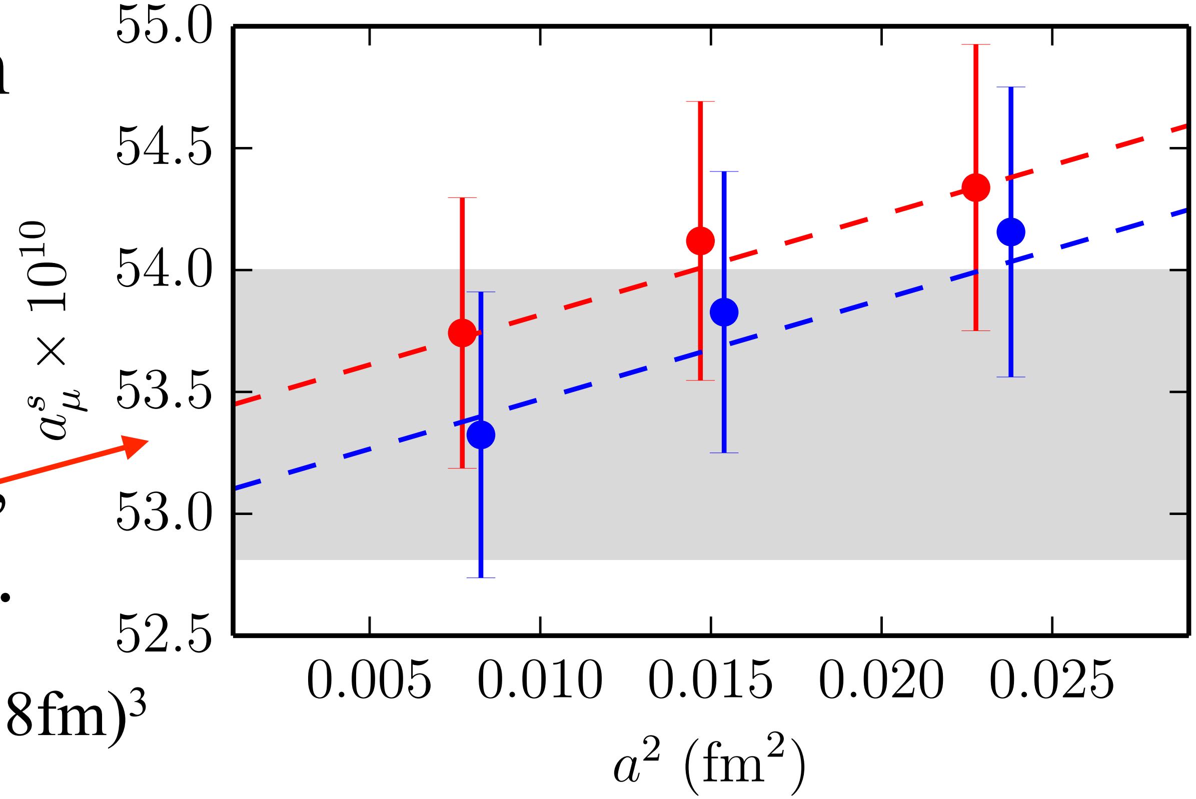
HPQCD 1503.05762

STRANGE contribution

HPQCD 1403.1778

HISQ valence quarks on MILC 2+1+1 HISQ configs. Local Jv - nonpert. Z_V.
multiple a (fixed by w_0), m_l (inc. phys.), volumes.
Tune s from η_s

up to $(5.8\text{fm})^3$

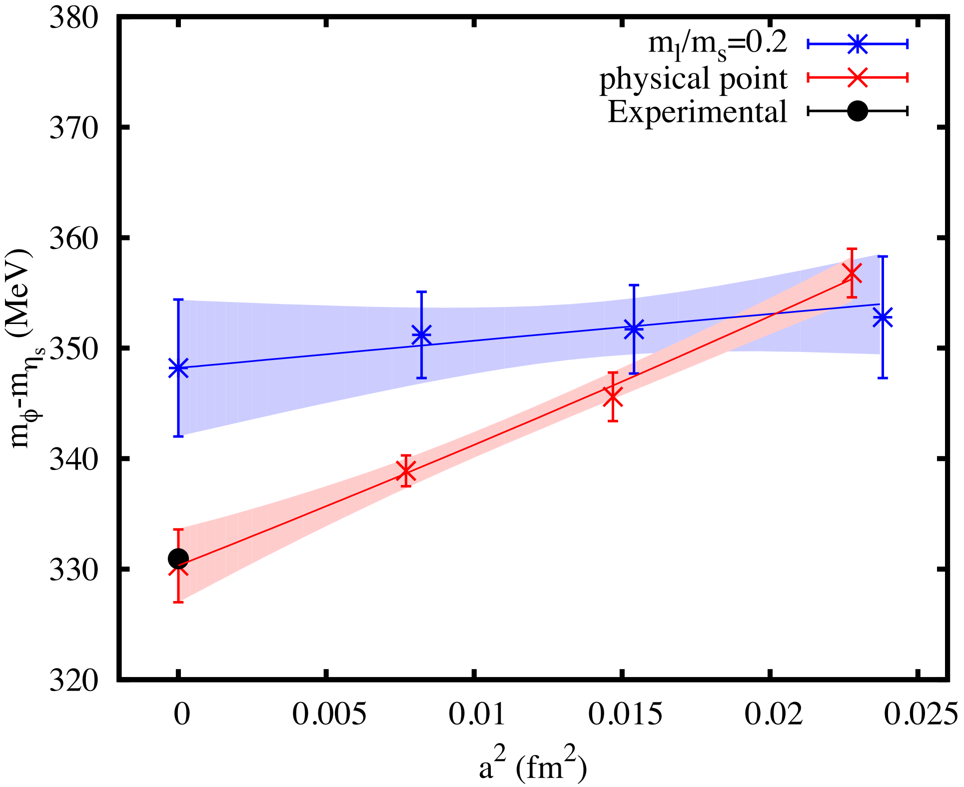


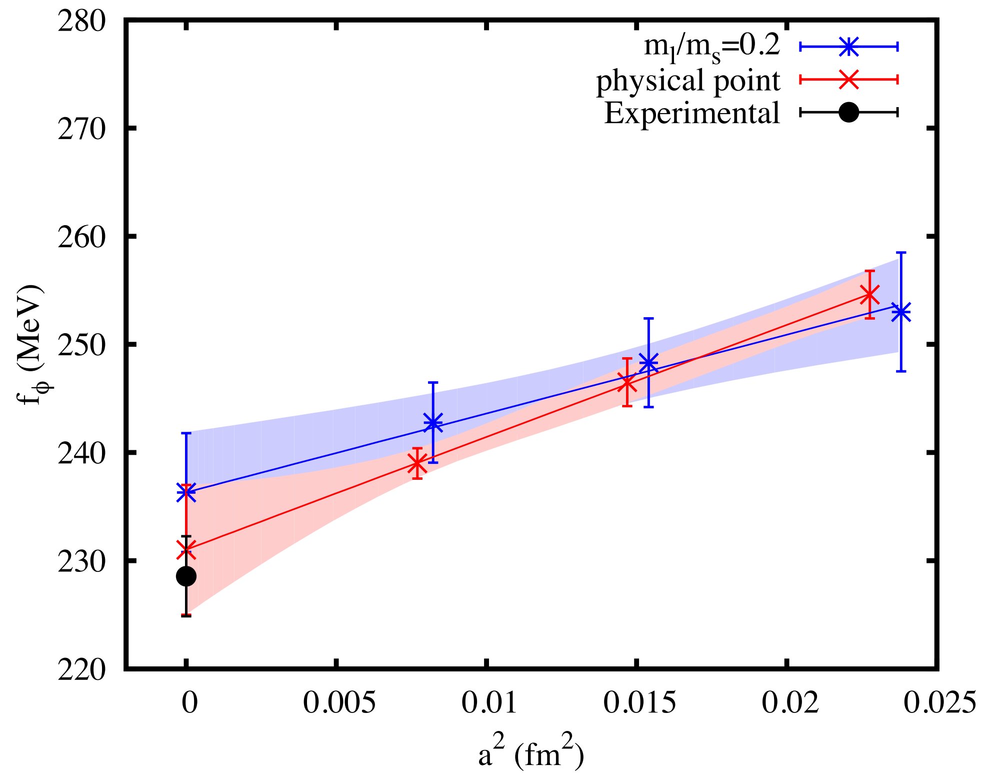
	a_μ^s
Uncertainty in lattice spacing (w_0, r_1):	1.0%
Uncertainty in Z_V:	0.4%
Monte Carlo statistics:	0.1%
$a^2 \to 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.0%
Finite lattice volume:	$< 0.1\%$
Padé approximants:	$< 0.1\%$
Total:	1.1%

$$a^s_{\mu,\text{lat}} = a^s_\mu \times (1 + c_{a^2}(a\Lambda_{\text{QCD}}/\pi)^2 + c_{\text{sea}}\delta x_{\text{sea}} + c_{\text{val}}\delta x_{\text{val}})$$

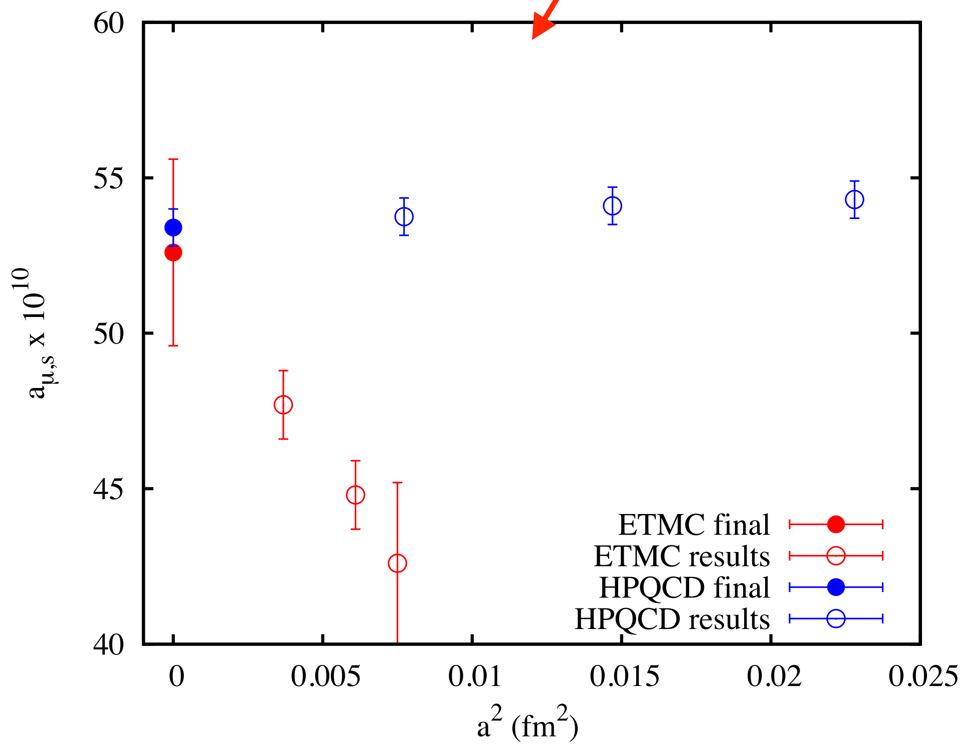
$$a_\mu^{HVP,s} = 53.41(59) \times 10^{-10}$$

Check mass and decay constant of ϕ from these correlators against expt

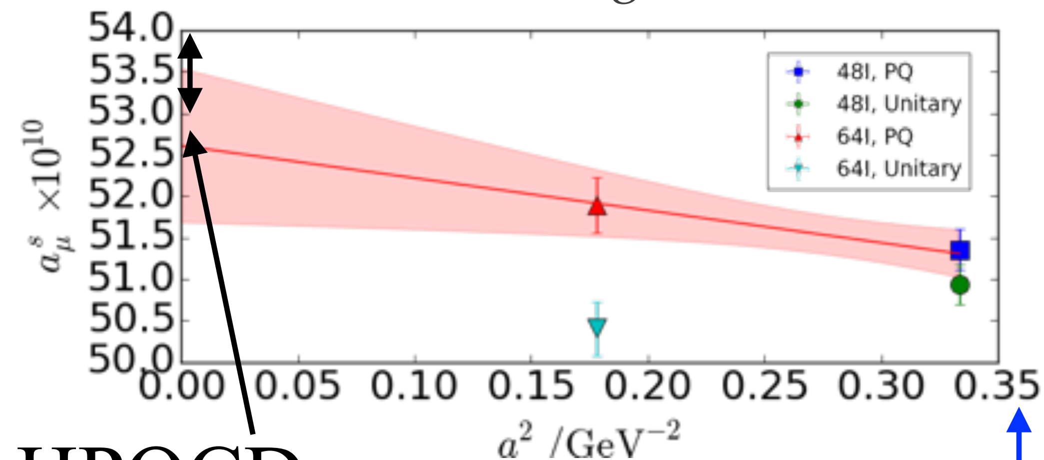




Results from ETMC, RBC/UKQCD

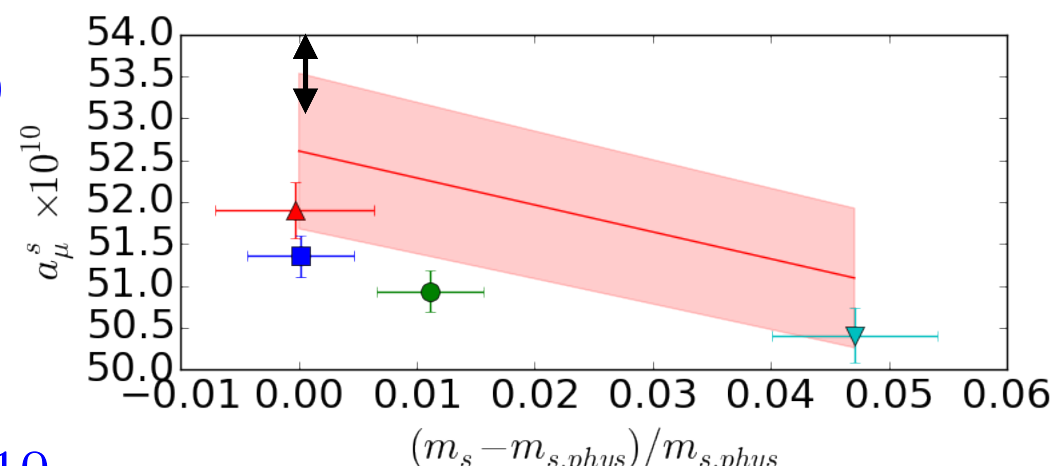


continuum limit for strange contribution to a_μ



HPQCD

$0.014\mathrm{fm}^2$



$$a_\mu^{HVP,s,ETMC} = 53(3) \times 10^{-10}$$

Continuum estimate/
upper limit:

$$a_\mu^{HVP,s,cont} = 55.3(8) \times 10^{-10}$$

1403.1778

LIGHT contribution $m_u = m_d$

HISQ valence quarks on MILC 2+1+1 HISQ configs. Use Z_V from s calc.

Multiple a (use w_0), m_l (inc. phys.), volumes (at ml/ms=0.1).

New ingredient since correlators much noisier. Use:

$$G(t) = \begin{cases} G_{\rm data}(t) & \text{for } t \leq t^* \\ G_{\rm fit}(t) & \text{for } t > t^* \end{cases}$$

← from Monte Carlo

← from multi-exponential fit

$t^* = 1.5\text{fm} = 6/m_\rho$ so 70% of result from $G_{\rm data}$

- 80% of result comes from ρ meson pole, so need to understand ρ on lattice, inc. finite-volume from $\pi\pi$.
- 10% from $\pi\pi$, sensitive to finite-volume and m_π (so taste-issues for staggered quarks).

One approach is to correct Taylor coefficients

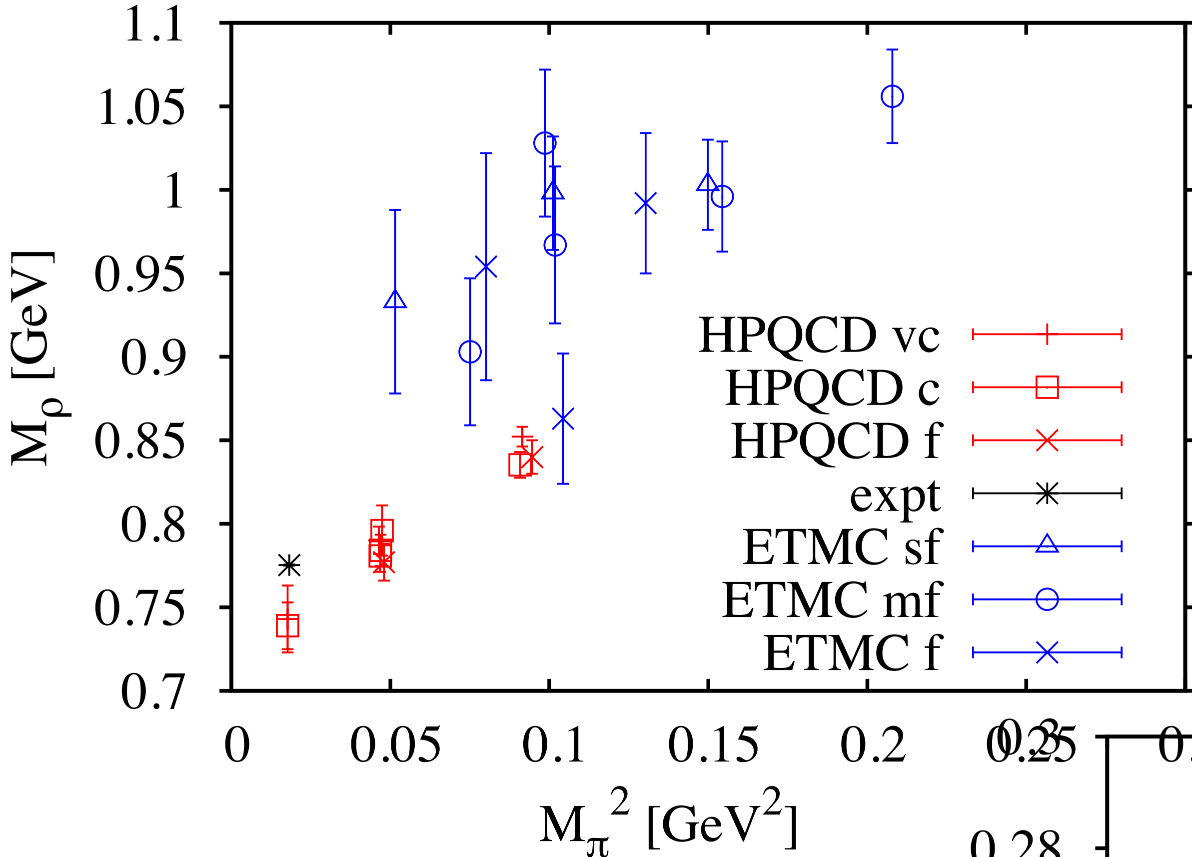
$$\hat{\Pi}_j^{latt} \to (\hat{\Pi}_j^{latt} - \hat{\Pi}_j^{latt}(\pi\pi)) \left[\frac{m_\rho^{2j+2}}{f_\rho^2}\right]_{latt} \left[\frac{f_\rho^2}{m_\rho^{2j+2}}\right]_{expt} + \hat{\Pi}_j^{cont}(\pi\pi)$$

Remove lattice $\pi\pi$ using one-loop, staggered quark chiral pert. theory

Rescale using exptl f_ρ and m_ρ elaborating on ETMC : 1308.4327. Reduces lattice systematics, but intro. uncties from f_ρ

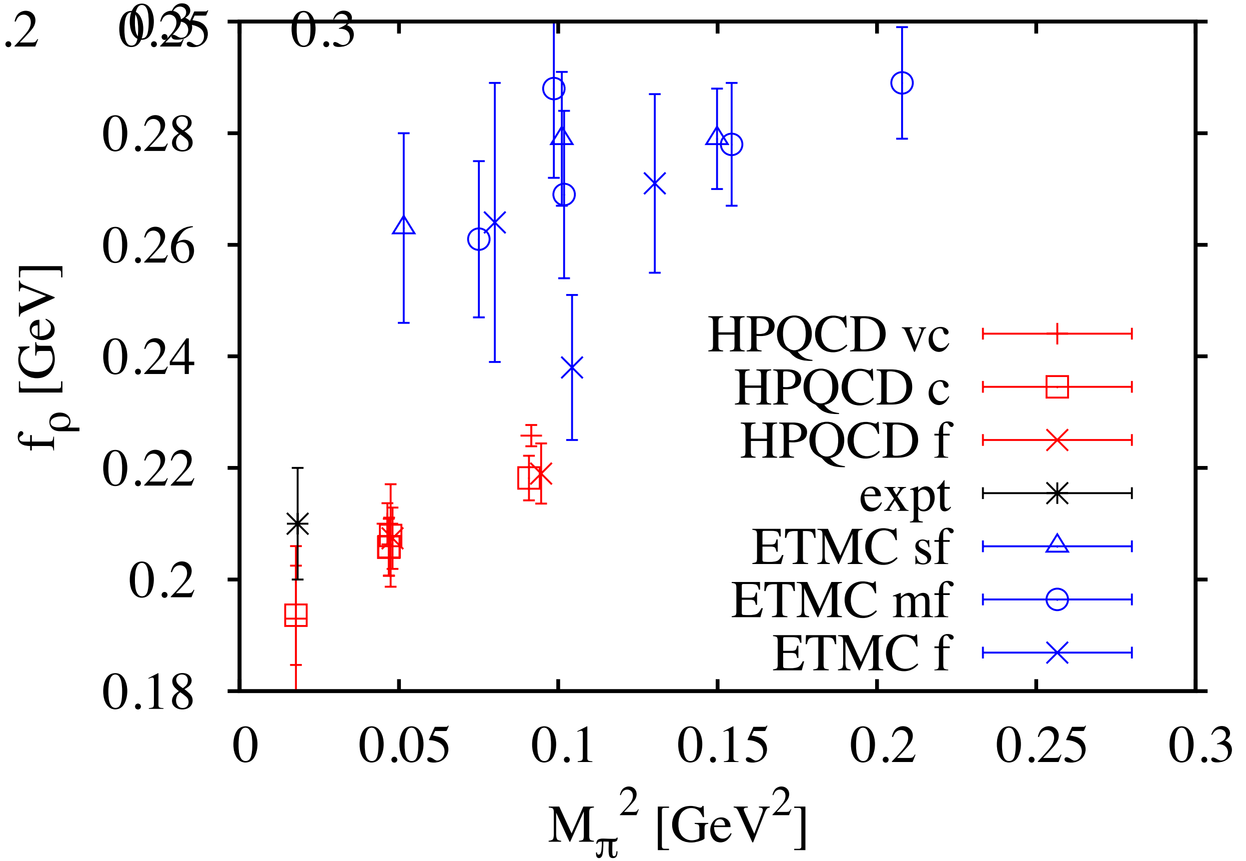
Restore $\pi\pi$ from continuum chiral pert. th.

$\pi\pi$ contribution distorted at physical point using staggered quarks on these coarse lattices. Important to inc. other masses. But note: need 7fm lattice to get this piece below 1% for contnm $\pi\pi$



Analysis of ρ parameters

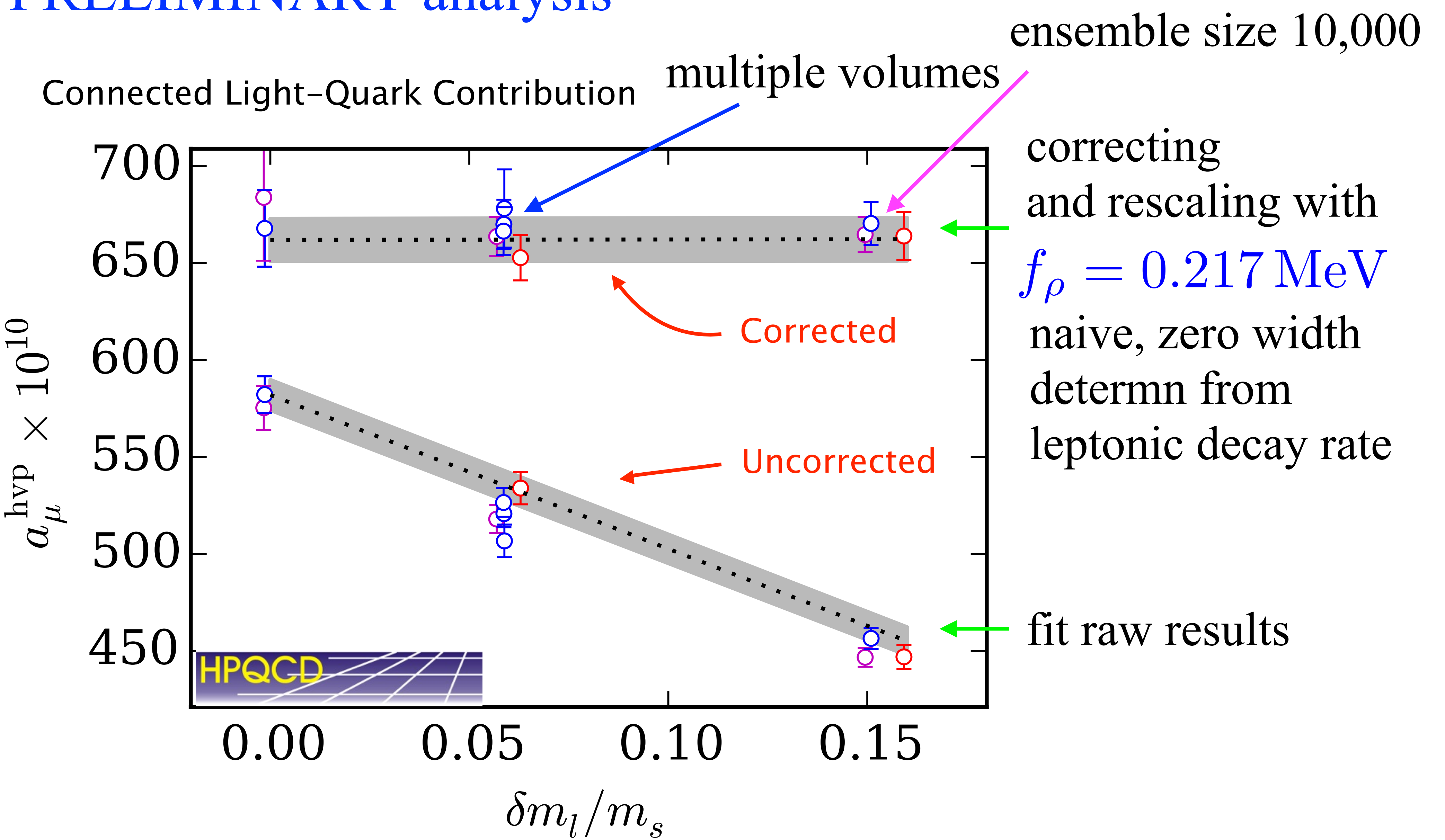
Direct comparison with ETMC possible



ETMC a 0.06-0.08fm
L 2.5- 2.9fm

HPQCD a 0.09-0.15fm
L 2.5-5.8fm

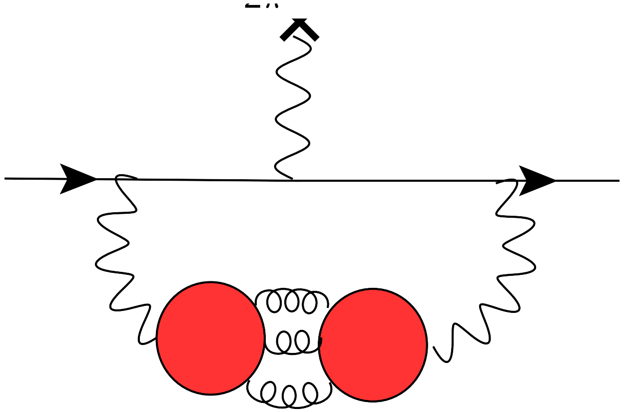
PRELIMINARY analysis



Improve by estimating systematics with an effective theory that includes $\rho, \gamma, \pi\pi$ e.g. Jegerlehner +Szafron, 1101.2872

Future: improve statistics, finer lattices

DISCONNECTED contribution to HVP

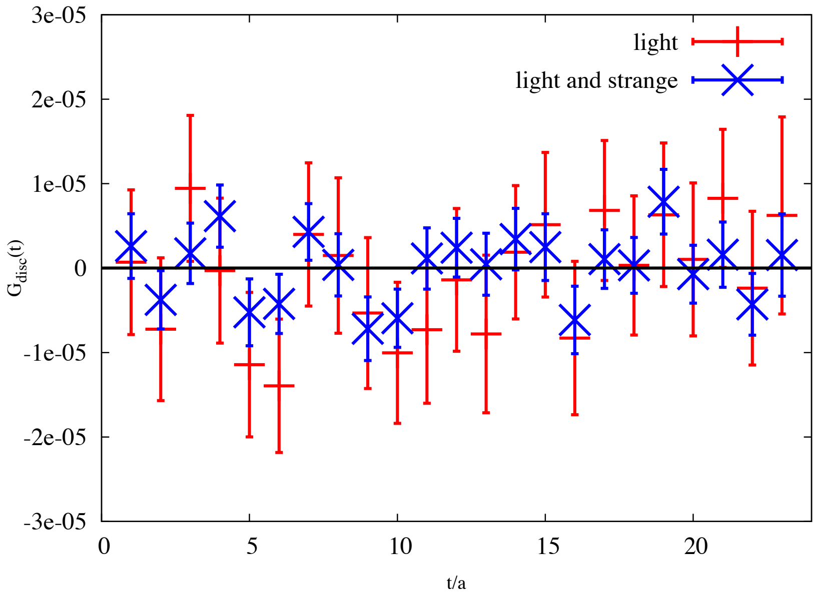


Vanishes if $m_u = m_d = m_s$

since $\sum_i e_i = 0$ Blum, hep-lat/0212018

For real masses, result is disconn. correlator for (l-s) current with charge 1/3 (so e^2 factor is 1/5 of connected)

Focus has been on stochastic methods. Using same source for l and s helps Guelpers, Mainz,LAT14



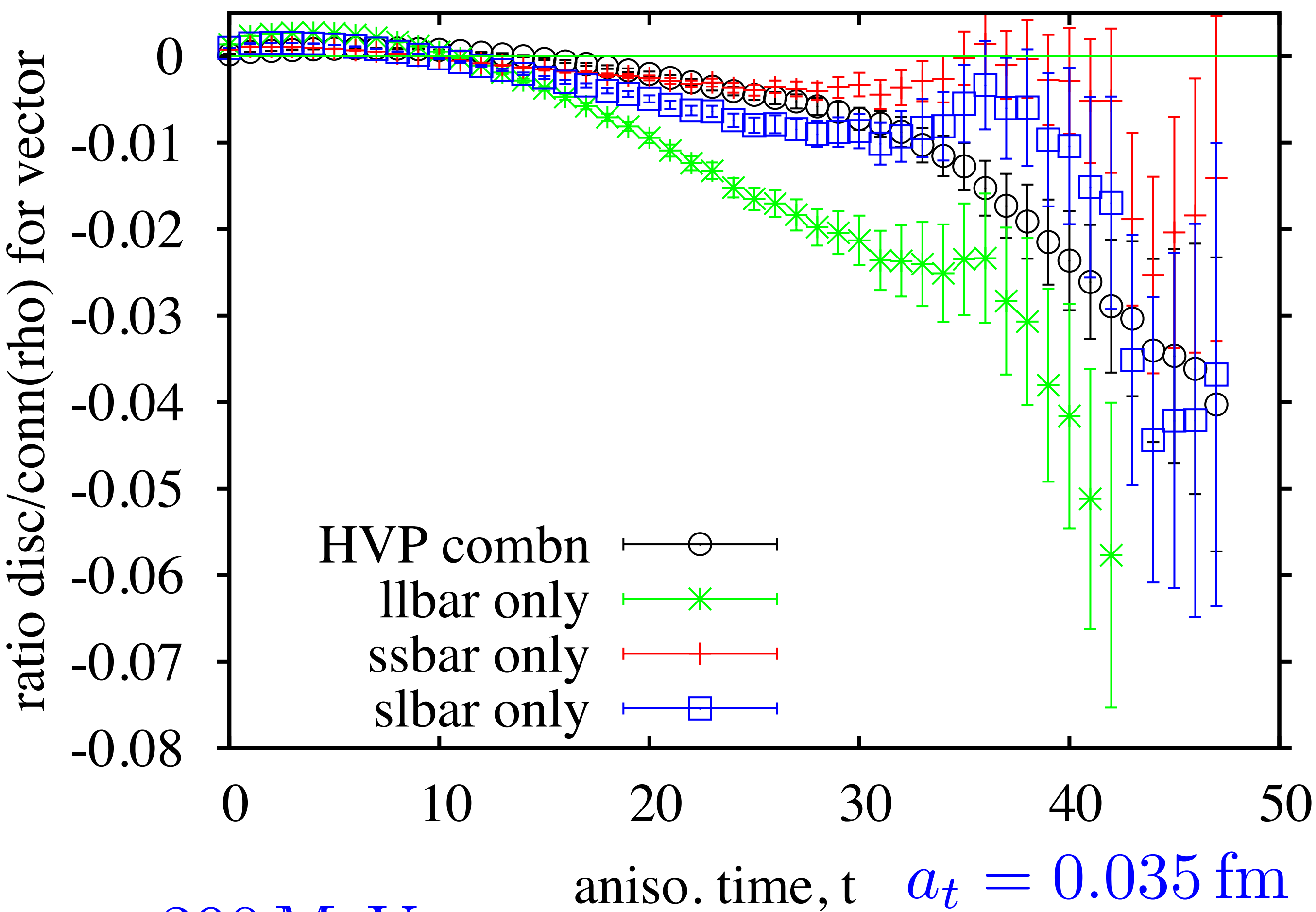
Chakraborty,HPQCD,LAT15

no signal from 50+50 sources (all-to-all) per config. One-link J

HadSpec results

e.g. Hadspec,1309.2608

Use instead many (~150) source vectors (eigenvectors of gauge-covariant Laplacian) for both conn. and disc. correlators to obtain good signal.



$a_t = 0.035$ fm

$m_\pi = 390\,\mathrm{MeV}$

PRELIMINARY

Fitting and normalising to connected light, gives HVP disc. contribn of ~ -1%

Hadspec+HPQCD,in prep.

anisotropic clover action

Simple (but conservative) argument on size of disc. pieces

l-l disc.pieces provide key difference between ω and ρ

$$2D_{ll} = -\frac{f_\rho^2 m_\rho}{2} e^{-m_\rho t} + \frac{f_\omega^2 m_\omega}{2} e^{-m_\omega t}$$

$$\frac{\hat{\Pi}_{j,disc}}{\hat{\Pi}_{j,conn}} = \frac{1}{2}\left[\frac{m_\rho^{2j+2} f_\omega^2}{m_\omega^{2j+2} f_\rho^2} - 1\right]$$

We do not have accurate information on decay constants because of width of ρ , mixing of ω etc

Taking $f_\rho = 0.21(1)\ \mathrm{GeV}, f_\omega = 0.20(1)\ \mathrm{GeV}$

HVP : disc-ll/conn-ll = -1.5(1.5) %

Adding contributions to (g-2)/2

Largest value we can get? (since uses rescaling with largest f_ρ

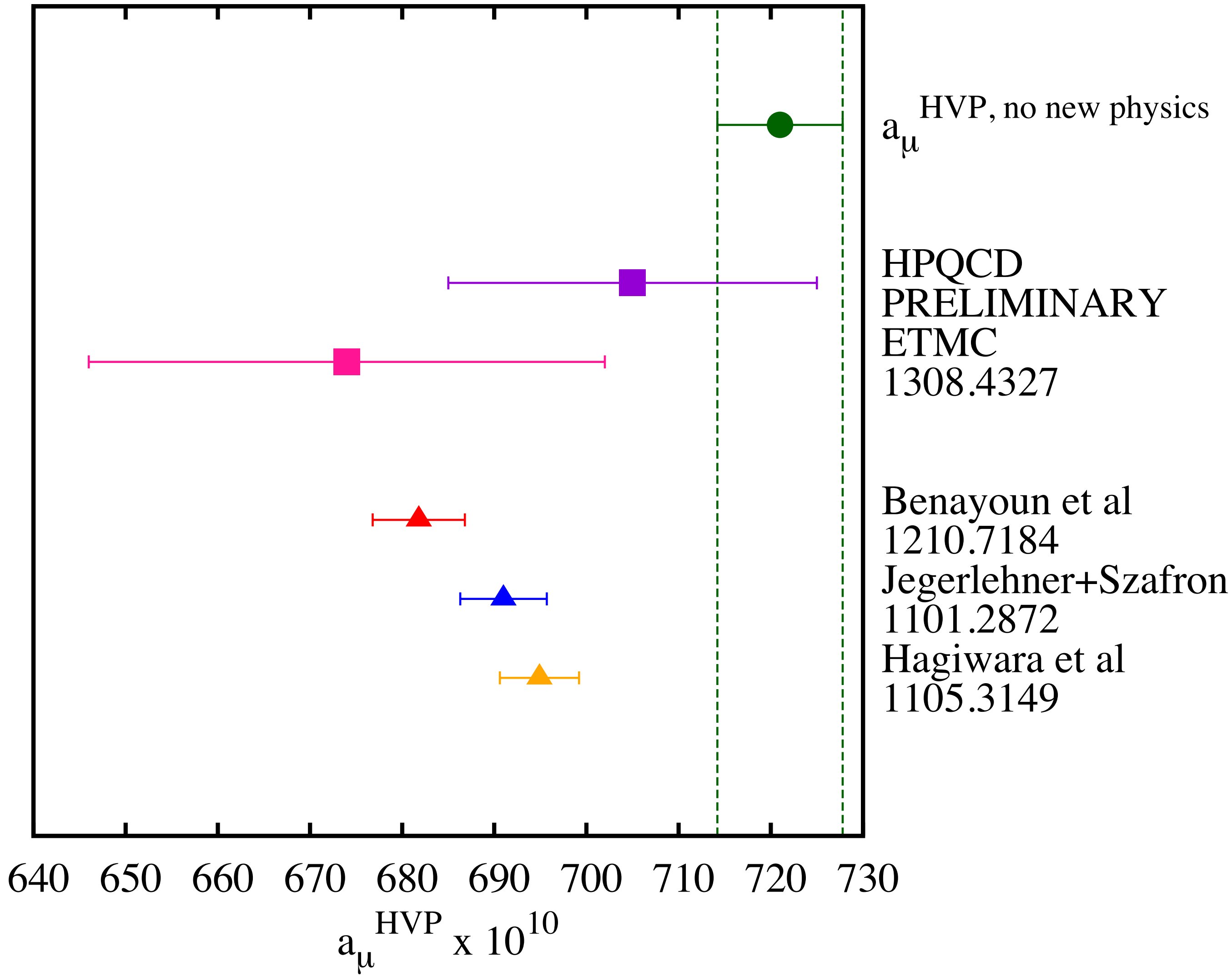
ETMC 1308.4327

HPQCD:

x 10^{-10}

ETMC		HPQCD	
567(11)stat	light, connected	662(11)	preliminary
	strange connected	53.4(6)	1403.1778
	charm connected	14.4(4)	1403.1778, 1208.2855
	bottom connected	0.27(4)	1408.5768
	disconn. (estimate)	-25(15)	add −7 from $\pi\pi$ to maximum simple estimate
674(28)	TOTAL	705(20)	

Lattice - continuum comparison



Lattice calcs inc. u,d,s,c

Good agreement but lattice uncty (all from u/d) still too big.
Need to add in QED, m_u/m_d effects (~1% and positive?)