Ph. de Forcrand, with O. Akerlund, A. Georges & P. Werner

DMFT for Lattice QFT ??

Crucial difference:

Lattice spacing a is physical in Condensed Matter

↕

Must take continuum limit $a \to 0, \quad \text{i.e.} \ \frac{\xi}{a} \to \infty$ in Quantum Field Theory

$\Longrightarrow$ Need accurate approximation near 2nd order phase transition

Impossible for a single-site approximation "by construction" ?

$\Longrightarrow$ Need accurate approximation near 2nd order phase transition

Impossible for a single-site approximation "by construction" ?

NO: DMFT - knows about free [massless] case

- provides self-consistent, approximate self-energy

$\Longrightarrow$ Potentially accurate 2-point function $\tilde{G}(\mathbf{k},\omega) = \dfrac{\mathbf{1}}{|\mathbf{k}|^{\mathbf{2}} + \omega^{\mathbf{2}} + \mathbf{m_0^2} + \tilde{\mathbf{\Sigma}}(\mathbf{k},\omega)}$

Simplest benchmark case: ϕ^4 in d dimensions, $\phi(\mathbf{x},t) \in \mathcal{R}$

arXiv: 1305.7136

Simplest benchmark case: ϕ^4 in d dimensions, $\phi(\mathbf{x},t) \in \mathcal{R}$

arXiv: 1305.7136

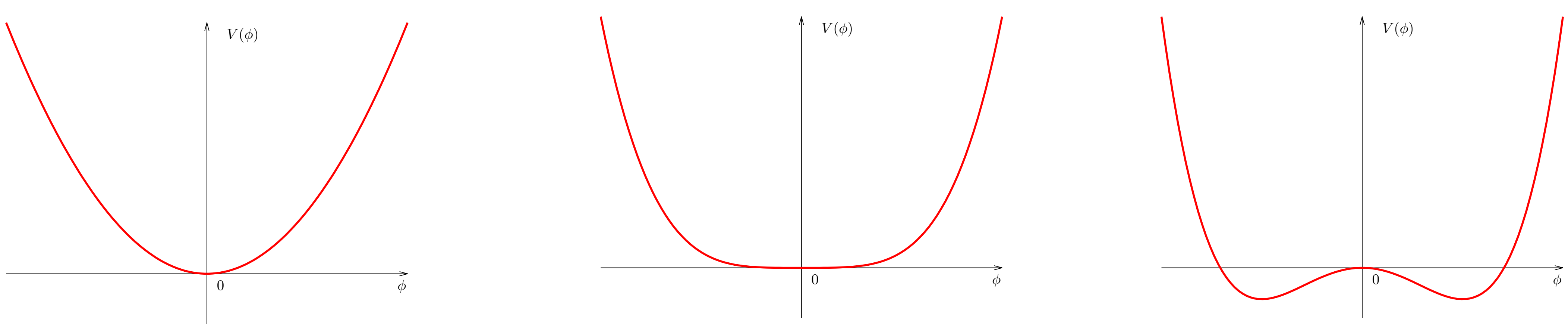
Continuum: $\mathcal{L}_E[\phi(x)] = \frac{1}{2}\partial_\mu\phi(x)\partial_\mu\phi(x) + \frac{1}{2}{m_0}^2\phi(x)^2 + \frac{g_0}{4!}\phi(x)^4$

Lattice: $a^{\frac{d-2}{2}}\phi(x) = \sqrt{2\kappa}\varphi_x;\ \ (am_0)^2 = \frac{1-2\lambda}{\kappa} - 2d;\ \ a^{4-d}g_0 = \frac{6\lambda}{\kappa^2}$

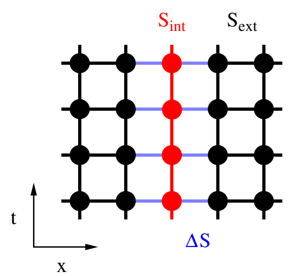
$$\Longrightarrow \quad S_L = \sum_x \left(-2\kappa \sum_\mu \varphi_{x+\widehat{\mu}}\, \varphi_x + \varphi_x^2 + \lambda(\varphi_x^2 - 1)^2 \right)$$

$\mathcal{Z}_2$ global symmetry: $\varphi_x \to -\varphi_x \ \forall x$

Spontaneous Symmetry Breaking $\longrightarrow$ 2nd-order P.T. $\longrightarrow$ Continuum limit



DMFT treatment of $S_L = \sum_x \left(-2\kappa \sum_\mu \varphi_{x+\widehat{\mu}}\, \varphi_x + \varphi_x^2 + \lambda(\varphi_x^2 - 1)^2 \right)$



Consider single site in "heatbath": $S_{\text{imp}} = \sum_{t,t'} \varphi_t K^{-1}_{\text{imp},c}(t-t')\varphi_{t'} + \lambda \sum_t (\varphi_t^2 - 1)^2 - h \sum_t \varphi_t$

Remember: $\tilde{G}(\mathbf{k},\omega) = \dfrac{\mathbf{1}}{\tilde{\mathbf{G}}_{\mathbf{0}}^{-\mathbf{1}}(\mathbf{k},\omega) + \tilde{\boldsymbol{\Sigma}}(\mathbf{k},\omega)}$ $\quad \tilde{G}_{\text{imp}}(\omega) = \dfrac{1}{\tilde{K}^{-1}_{\text{imp},c}(\omega) + \tilde{\Sigma}_{\text{imp}}(\omega)}$

1. Given $\widetilde{K}_{\text{imp},c}$, compute by Monte Carlo: $\widetilde{\Sigma}_{\text{imp}}(\omega) = \widetilde{G}^{-1}_{\text{imp}}(\omega) - \widetilde{K}^{-1}_{\text{imp},c}(\omega)$

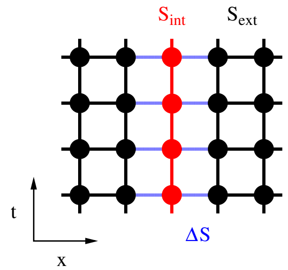
2. Take $\widetilde{\Sigma}(\mathbf{k},\omega) \approx \widetilde{\boldsymbol{\Sigma}}_{\text{imp}}(\omega)$, and obtain: $\tilde{G}_{xx}(\omega) = \sum_{\mathbf{k}} \dfrac{1}{\tilde{G}_0^{-1}(\mathbf{k},\omega) + \tilde{\boldsymbol{\Sigma}}(\mathbf{k},\omega)}$

3. Since $\tilde{G}_{xx}(\omega) = \tilde{G}_{\text{imp}}(\omega)$, obtain $\widetilde{K}^{-1}_{\text{imp},c}(\omega)|_{\text{new}} = \widetilde{G}^{-1}_{\text{imp}}(\omega) - \widetilde{\Sigma}_{\text{imp}}(\omega)$

Iterate to fixed point ! single site $\rightarrow \tilde{G}_{\text{imp}}(\omega) = \tilde{G}_{xx}(\omega) \leftarrow$ full system

(same with $h \propto \phi_{\text{ext}}$, until $\langle \varphi \rangle_{S_{\text{imp}}} = \phi_{\text{ext}}$)

DMFT treatment of $S_L = \sum_x \left(-2\kappa \sum_\mu \varphi_{x+\hat{\mu}}\, \varphi_x + \varphi_x^2 + \lambda(\varphi_x^2 - 1)^2 \right)$



Consider single site in "heatbath": $S_{\rm imp} = \sum_{t,t'} \varphi_t K^{-1}_{{\rm imp},c}(t-t')\varphi_{t'} + \lambda \sum_t (\varphi_t^2 - 1)^2 - h \sum_t \varphi_t$

Remember: $\tilde{G}(\mathbf{k},\omega) = \dfrac{\mathbf{1}}{\tilde{\mathbf{G}}_\mathbf{0}^{-\mathbf{1}}(\mathbf{k},\omega) + \tilde{\mathbf{\Sigma}}(\mathbf{k},\omega)}$ $\qquad \tilde{G}_{\rm imp}(\omega) = \dfrac{1}{\tilde{K}^{-1}_{{\rm imp},c}(\omega) + \tilde{\Sigma}_{\rm imp}(\omega)}$

1. Given $\widetilde{K}_{{\rm imp},c}$, compute by Monte Carlo: $\widetilde{\Sigma}_{\rm imp}(\omega) = \widetilde{G}^{-1}_{\rm imp}(\omega) - \widetilde{K}^{-1}_{{\rm imp},c}(\omega)$

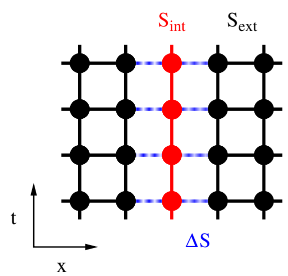
2. Take $\widetilde{\Sigma}(\mathbf{k},\omega) \approx \widetilde{\mathbf{\Sigma}}_{\rm imp}(\omega)$, and obtain: $\tilde{G}_{xx}(\omega) = \sum_\mathbf{k} \dfrac{1}{\tilde{G}_0^{-1}(\mathbf{k},\omega) + \tilde{\mathbf{\Sigma}}(\mathbf{k},\omega)}$

3. Since $\tilde{G}_{xx}(\omega) = \tilde{G}_{\rm imp}(\omega)$, obtain $\widetilde{K}^{-1}_{{\rm imp},c}(\omega)|_{\rm new} = \widetilde{G}^{-1}_{\rm imp}(\omega) - \widetilde{\Sigma}_{\rm imp}(\omega)$

Iterate to fixed point ! $\qquad$ single site $\rightarrow \tilde{G}_{\rm imp}(\omega) = \tilde{G}_{xx}(\omega) \leftarrow$ full system

(same with $h \propto \phi_{\rm ext}$, until $\langle\varphi\rangle_{S_{\rm imp}} = \phi_{\rm ext}$)

DMFT treatment of $S_L = \sum_x \left(-2\kappa \sum_\mu \varphi_{x+\hat{\mu}}\, \varphi_x + \varphi_x^2 + \lambda(\varphi_x^2 - 1)^2 \right)$



Consider single site in "heatbath": $S_{\rm imp} = \sum_{t,t'} \varphi_t K^{-1}_{{\rm imp},c}(t-t')\varphi_{t'} + \lambda \sum_t (\varphi_t^2 - 1)^2 - h \sum_t \varphi_t$

Remember: $\tilde{G}(\mathbf{k},\omega) = \frac{1}{\tilde{\mathbf{G}}_{\mathbf{0}}^{-1}(\mathbf{k},\omega) + \tilde{\mathbf{\Sigma}}(\mathbf{k},\omega)}$ $\quad \tilde{G}_{\rm imp}(\omega) = \frac{1}{\tilde{K}^{-1}_{{\rm imp},c}(\omega) + \tilde{\Sigma}_{\rm imp}(\omega)}$

1. Given $\widetilde{K}_{{\rm imp},c}$, compute by Monte Carlo: $\widetilde{\Sigma}_{\rm imp}(\omega) = \widetilde{G}^{-1}_{\rm imp}(\omega) - \widetilde{K}^{-1}_{{\rm imp},c}(\omega)$

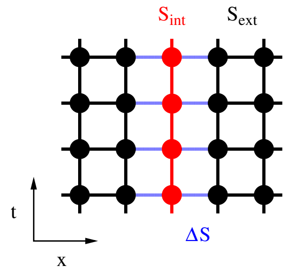
2. Take $\widetilde{\Sigma}(\mathbf{k},\omega) \approx \widetilde{\mathbf{\Sigma}}_{\rm imp}(\omega)$, and obtain: $\tilde{G}_{xx}(\omega) = \sum_{\mathbf{k}} \frac{1}{\tilde{G}_0^{-1}(\mathbf{k},\omega) + \tilde{\mathbf{\Sigma}}(\mathbf{k},\omega)}$

3. Since $\tilde{G}_{xx}(\omega) = \tilde{G}_{\rm imp}(\omega)$, obtain $\widetilde{K}^{-1}_{{\rm imp},c}(\omega)|_{\rm new} = \widetilde{G}^{-1}_{\rm imp}(\omega) - \widetilde{\Sigma}_{\rm imp}(\omega)$

Iterate to fixed point ! single site $\rightarrow \tilde{G}_{\rm imp}(\omega) = \tilde{G}_{xx}(\omega) \leftarrow$ full system

(same with $h \propto \phi_{\rm ext}$, until $\langle \varphi \rangle_{S_{\rm imp}} = \phi_{\rm ext}$)

DMFT treatment of $S_L = \sum_x \left(-2\kappa \sum_\mu \varphi_{x+\hat\mu}\, \varphi_x + \varphi_x^2 + \lambda(\varphi_x^2 - 1)^2 \right)$



Consider single site in "heatbath": $S_{\rm imp} = \sum_{t,t'} \varphi_t K^{-1}_{{\rm imp},c}(t-t')\varphi_{t'} + \lambda \sum_t (\varphi_t^2 - 1)^2 - h \sum_t \varphi_t$

Remember: $\tilde G(\mathbf{k},\omega) = \dfrac{\mathbf{1}}{\tilde{\mathbf{G}}_{\mathbf{0}}^{-1}(\mathbf{k},\omega) + \tilde{\mathbf{\Sigma}}(\mathbf{k},\omega)}$ $\qquad \tilde G_{\rm imp}(\omega) = \dfrac{1}{\tilde K^{-1}_{{\rm imp},c}(\omega) + \tilde\Sigma_{\rm imp}(\omega)}$

1. Given $\widetilde{K}_{{\rm imp},c}$, compute by Monte Carlo: $\widetilde\Sigma_{\rm imp}(\omega) = \widetilde G^{-1}_{\rm imp}(\omega) - \widetilde K^{-1}_{{\rm imp},c}(\omega)$

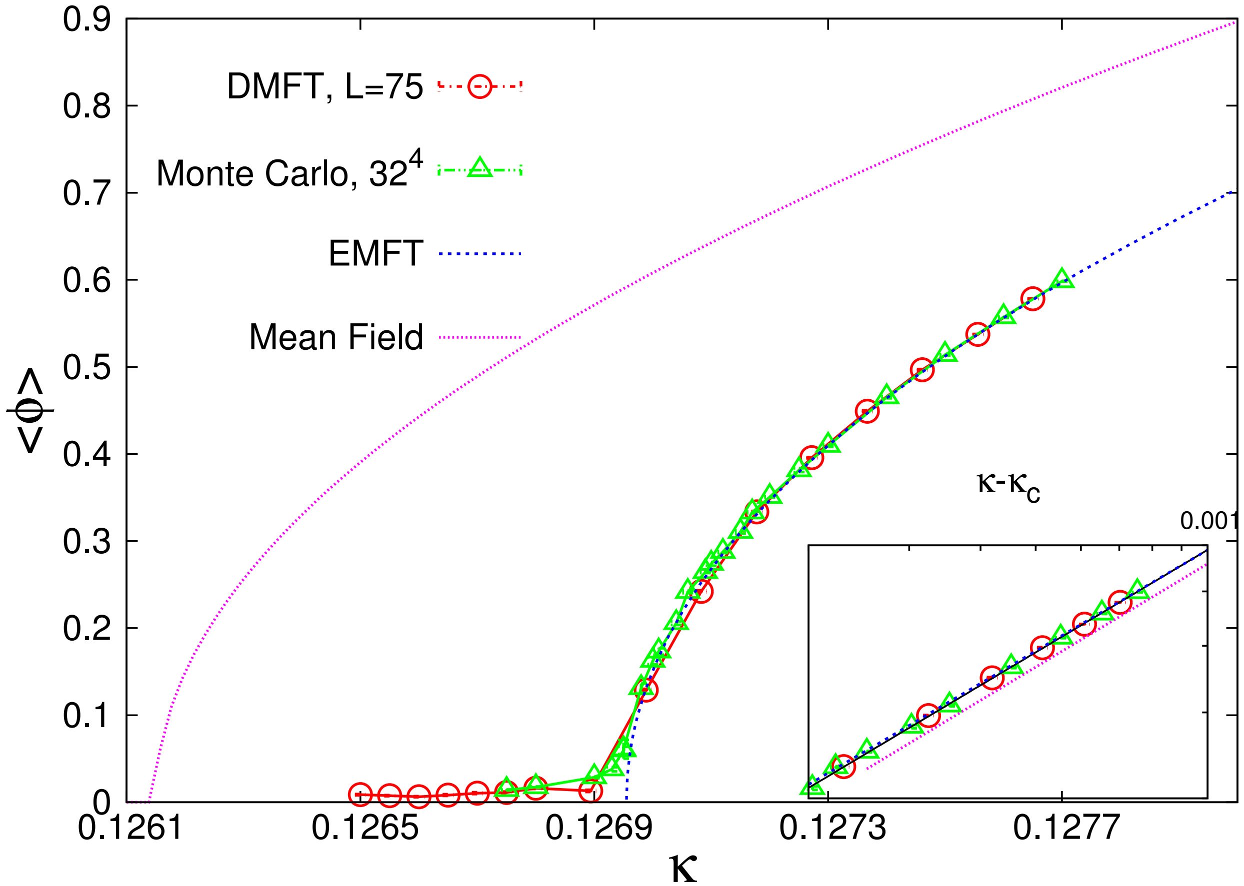
2. Take $\widetilde\Sigma(\mathbf{k},\omega) \approx \widetilde{\mathbf{\Sigma}}_{\rm imp}(\omega)$, and obtain: $\tilde G_{xx}(\omega) = \sum_{\mathbf{k}} \dfrac{1}{\tilde G_0^{-1}(\mathbf{k},\omega) + \tilde{\mathbf{\Sigma}}(\mathbf{k},\omega)}$

3. Since $\tilde G_{xx}(\omega) = \tilde G_{\rm imp}(\omega)$, obtain $\widetilde K^{-1}_{{\rm imp},c}(\omega)|_{\rm new} = \widetilde G^{-1}_{\rm imp}(\omega) - \widetilde\Sigma_{\rm imp}(\omega)$

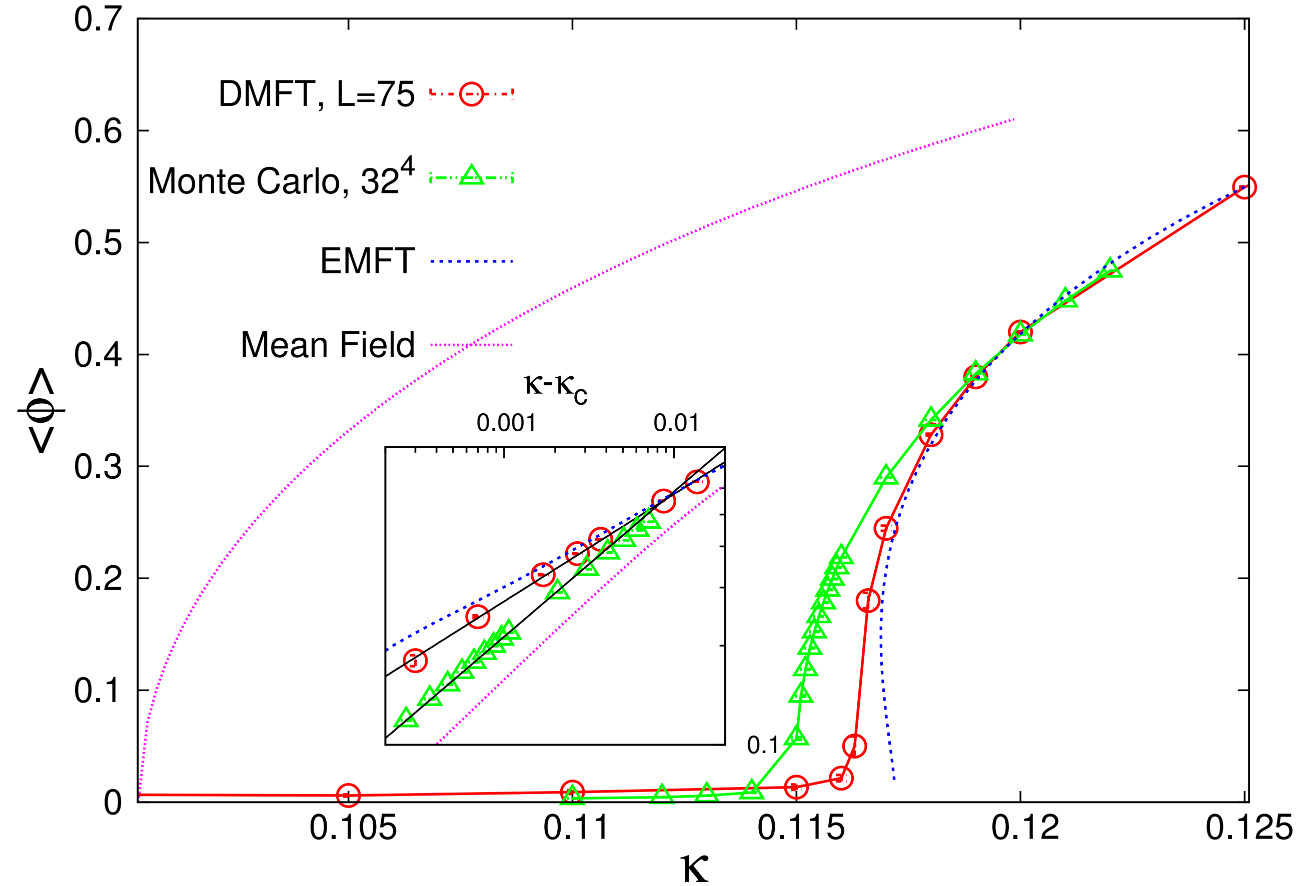
Iterate to fixed point ! single site $\rightarrow \tilde G_{\rm imp}(\omega) = \tilde G_{xx}(\omega) \leftarrow$ full system

(same with $h \propto \phi_{\rm ext}$, until $\langle\varphi\rangle_{S_{\rm imp}} = \phi_{\rm ext}$)

Some results



$4d, \ \lambda = 0.01$



$4d, \ \lambda = 2$

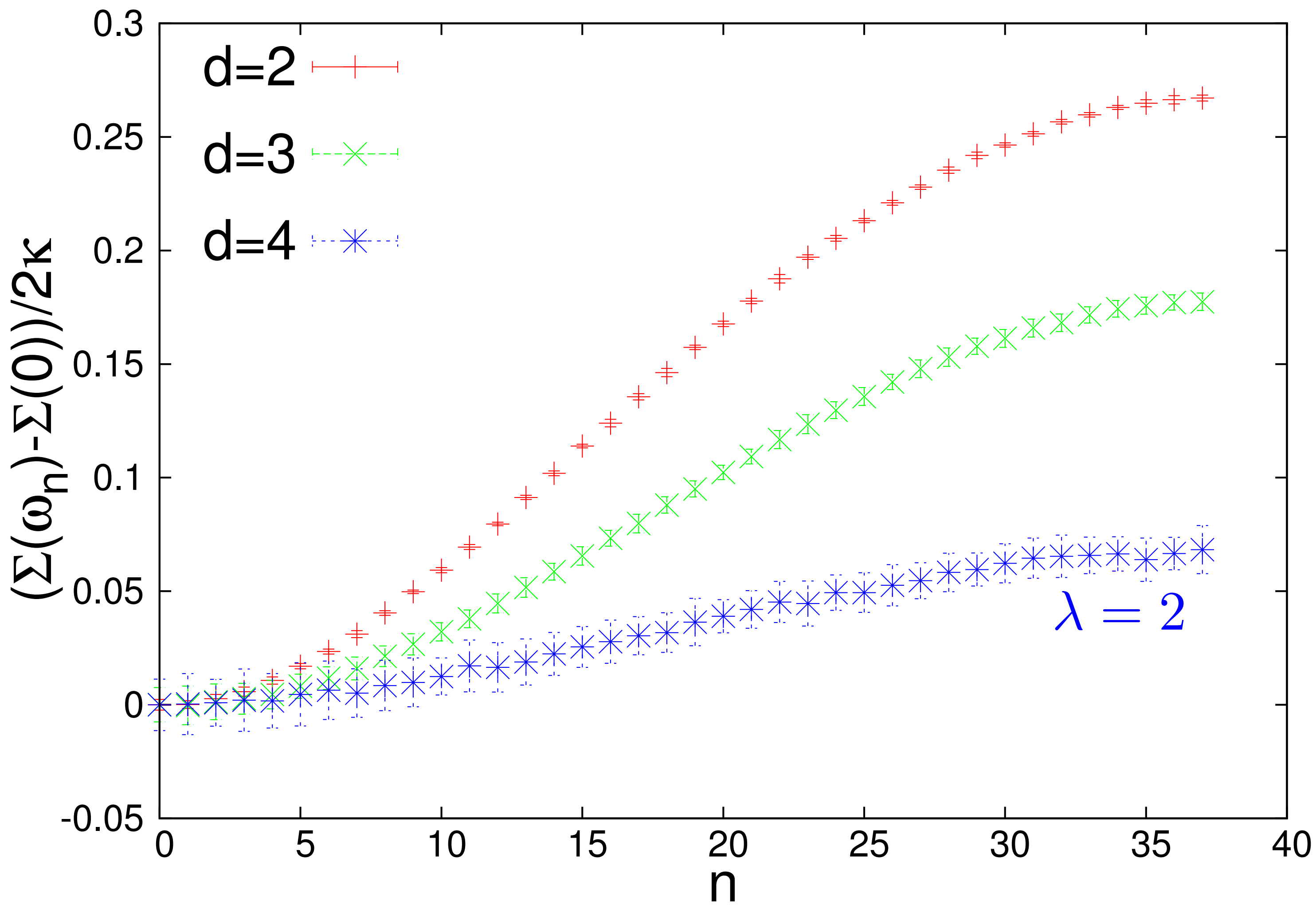
DMFT prefers weak interactions (self-energy), high dimension (mean field):

In $d = 2, 3$, DMFT finds a weak first-order transition

In $d = 4$, DMFT finds critical coupling with 2-3 digits, crit. exponents with 1-2 digits

Compare with ordinary mean-field, and with **EMFT**

EMFT: "extended" mean-field theory



$\omega_n = 2\pi n/75$

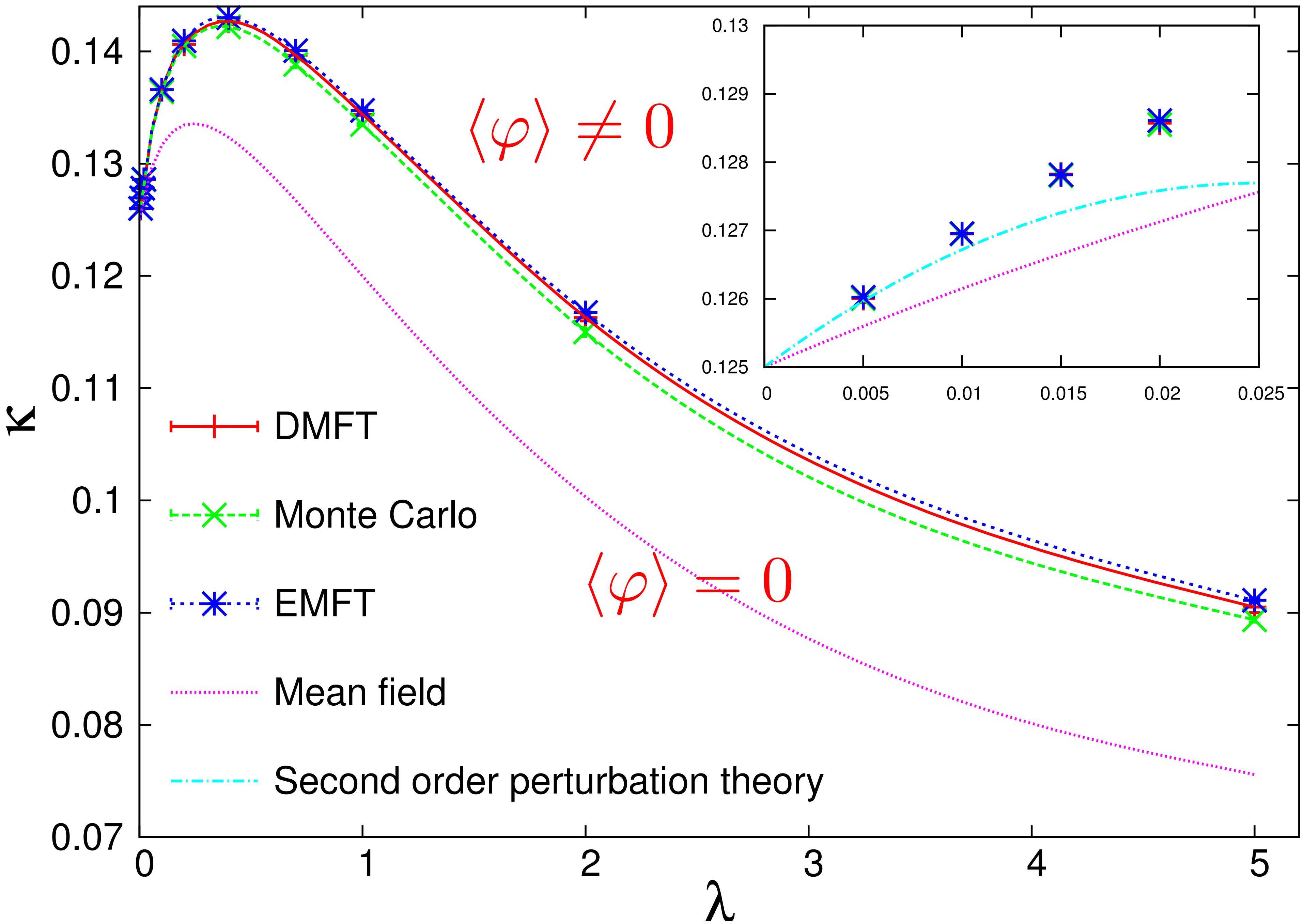
Self-energy depends weakly on frequency, esp. in high dimension: $\tilde{\Sigma}(\omega) \approx \tilde{\Sigma}(0)$

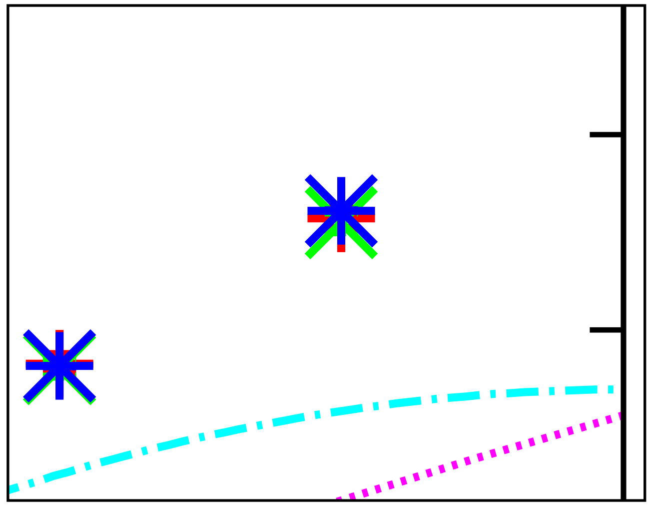
Drop ω - dependence in DMFT: single-site "EMFT" approximation

Solve self-consistently for $G_{xx} = \langle\varphi^2\rangle - \langle\varphi\rangle^2$ and for $\langle\varphi\rangle$

Non-trivial 2-pt function: $\tilde{G}(\mathbf{k},\omega) = \dfrac{\mathbf{1}}{\tilde{\mathbf{G}}_{\mathbf{0}}^{-\mathbf{1}}(\mathbf{k},\omega) + \tilde{\mathbf{\Sigma}}}$

φ_4^4 phase diagram in (λ, κ) plane





DMFT extremely good, EMFT almost as good

Summary

DMFT: - cheap, very good approximation

- info on 2pt-function $\longrightarrow$ masses, finite temperature, finite size

EMFT: - even cheaper

- no Monte Carlo: no jitter at phase transition, no sign problem!

→ Oscar