What lattice can do for quantum gravity

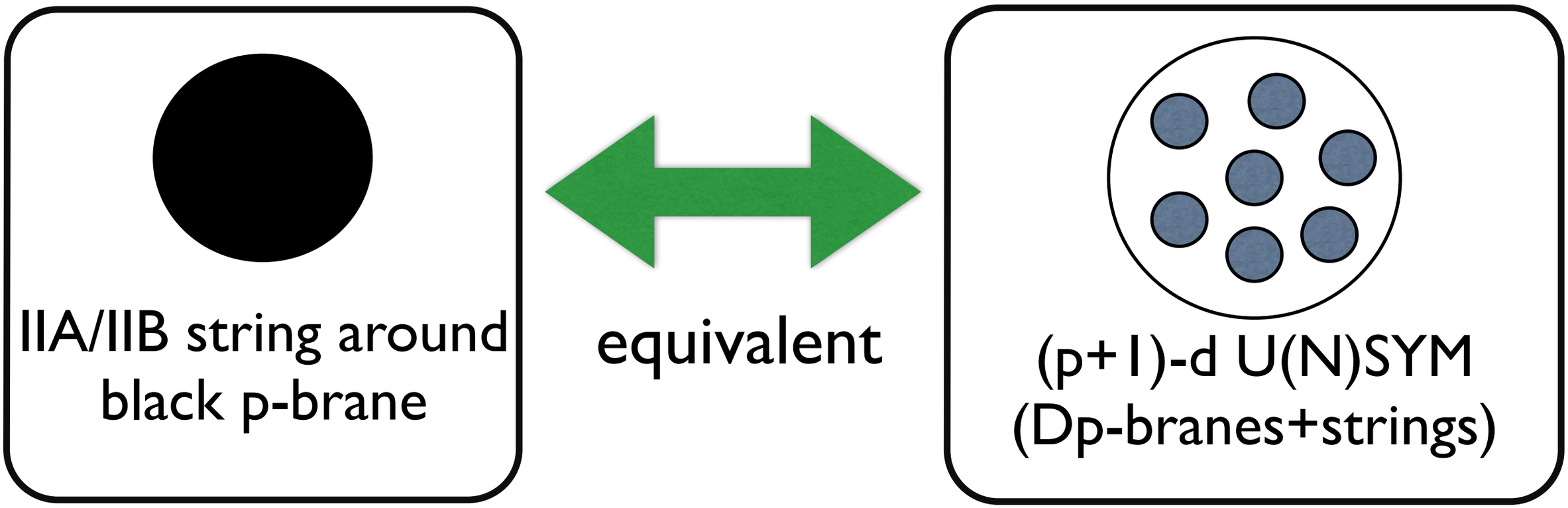
Masanori Hanada
花田 政範
Hana Da Masa Nori

Kyoto/Stanford

Gur-Ari, M.H., Shenker, to appear
+Aoki-M.H.-Iizuka, 1503.05562[hep-th]
+work in progress
+ a short review of Euclidean theory

15 Sept 2015 @ KITP, Santa Barbara

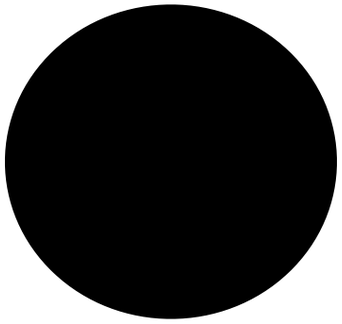
Gauge/Gravity Duality



We can learn about quantum gravity and BH
by solving gauge theory.

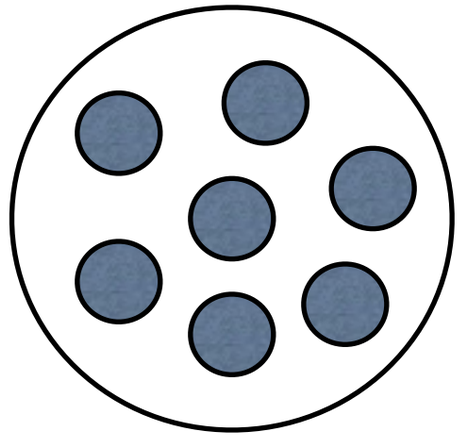
But SYM is hard! → numerical calculation.

Numerically easiest example



IIA string around
black 0-brane
(near horizon)

equivalent



(0+1)-d maximal SYM
(D0-branes+strings)

(Maldacena 1997, Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

Matrix model of super-membrane (de Wit-Hoppe-Nicolai, 1988)
Matrix model of M-theory (Banks-Fischler-Shenker-Susskind, 1996)

D0-brane quantum mechanics

$$S = \frac{N}{\lambda}\int_0^{\beta=1/T} dt\ Tr\Big\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i [X_i, \psi]\Big\}$$

(dimensional reduction of 4d N=4 SYM)

It should reproduce thermodynamics of black 0-brane.

effective dimensionless temperature $T_{eff} = \lambda^{-1/3} T$

high-T = weak coupling = stringy (large α' correction)

By deriving various field theories from string theory and considering their large N limit we have shown that they contain in their Hilbert space excitations describing supergravity on various spacetimes. We further conjectured that the field theories are dual to the full quantum M/string theory on various spacetimes. In principle, we can use this duality to give a definition of M/string theory on flat spacetime as (a region of) the large N limit of the field theories. Notice that this is a non-perturbative proposal for defining such theories, since the corresponding field theories can, *in principle*, be defined non-perturbatively. We

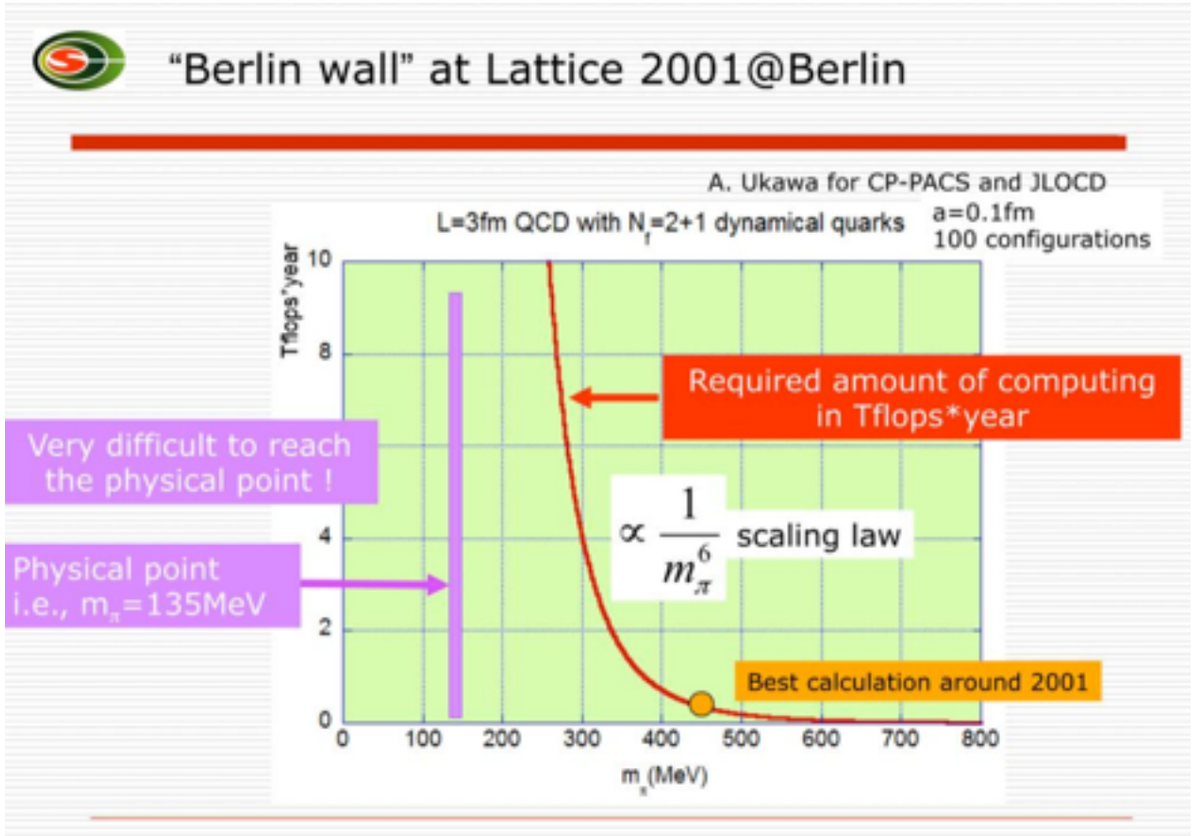
Maldacena,
"The Large N Limit of Superconformal Field Theories and Supergravity"
(1997)

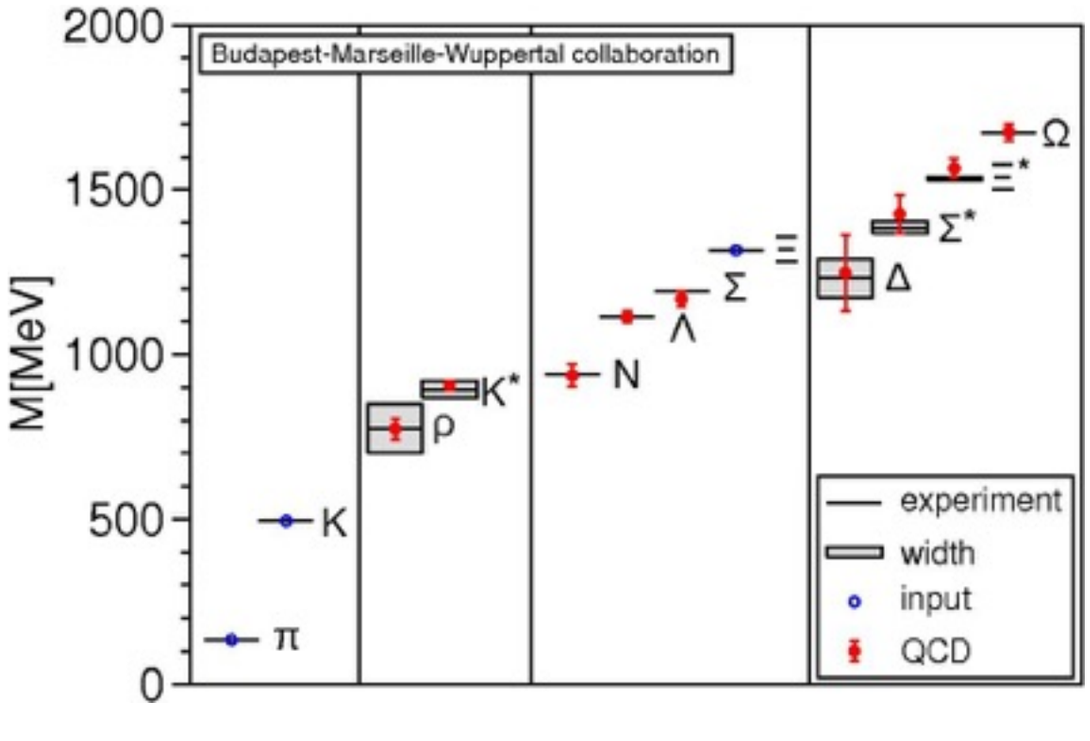
Now we know how to regularize such theories!

Kaplan, Katz, Unsal, Cohen, Sugino, Catterall, Kawamoto,
Kanamori, Suzuki, Maldacena, Seikh-Jabbari, Van Raamsdonk,
M.H., Matsuura, Ishii, Ishiki,Shimasaki, Tsuchiya, Nishimura, ….

If the conjecture is correct, it would provide us with the first well defined nonperturbative formulation of a quantum theory which includes gravitation. In principle, with a sufficiently big and fast computer any scattering amplitude could be computed in the finite N matrix model with arbitrary precision. Numerical extrapolation to infinite N is in principle, if not in practice, possible. The situation is much like that in QCD where the only known definition of the theory is in terms of a conjectured limit of lattice gauge theory. Although the practical utility of the lattice theory may be questioned, it is almost certain that an extrapolation to the continuum limit exists. The existence of the lattice gauge Hamiltonian formulation insures that the the theory is unitary and gauge invariant.

Banks-Fischler-Shenker-Susskind,
"M Theory As A Matrix Model: A Conjecture"
(1996)





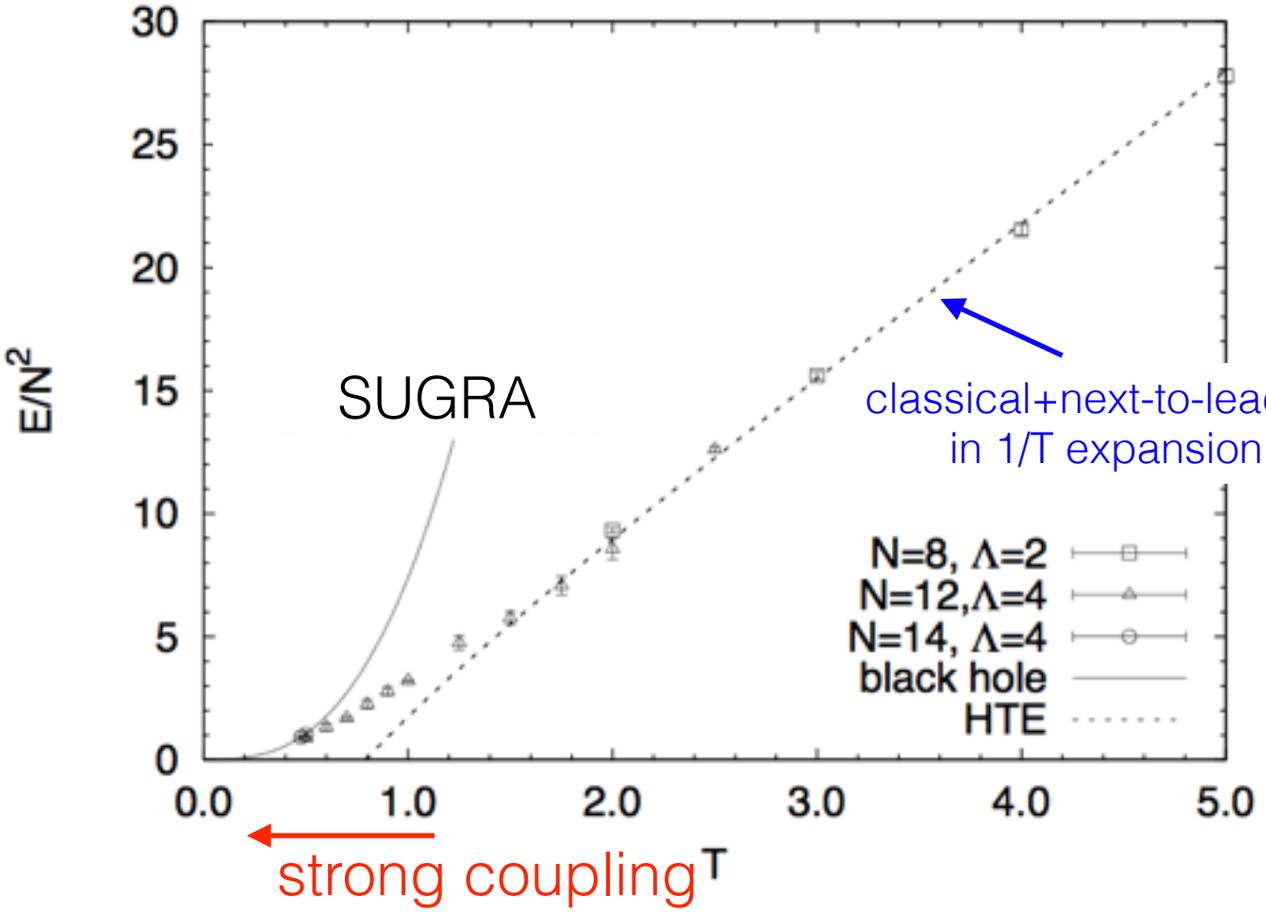
BMW-collaboration, Science, 2008

Practical utility is unquestionable by now.

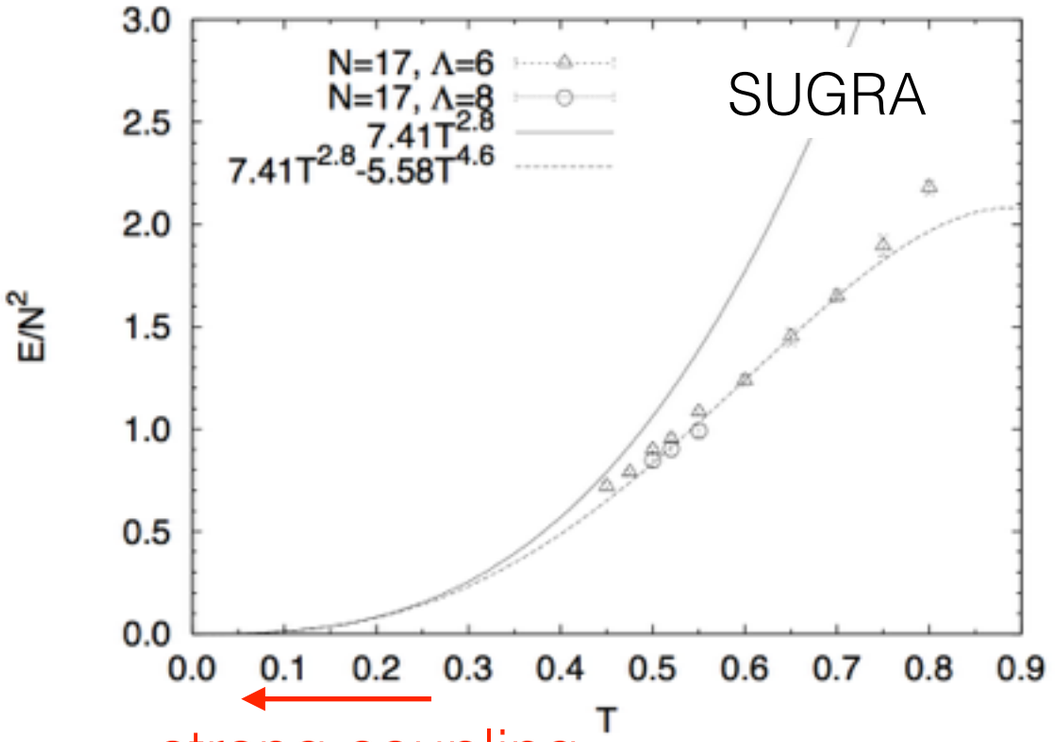
Thermodynamics

→ Successful so far

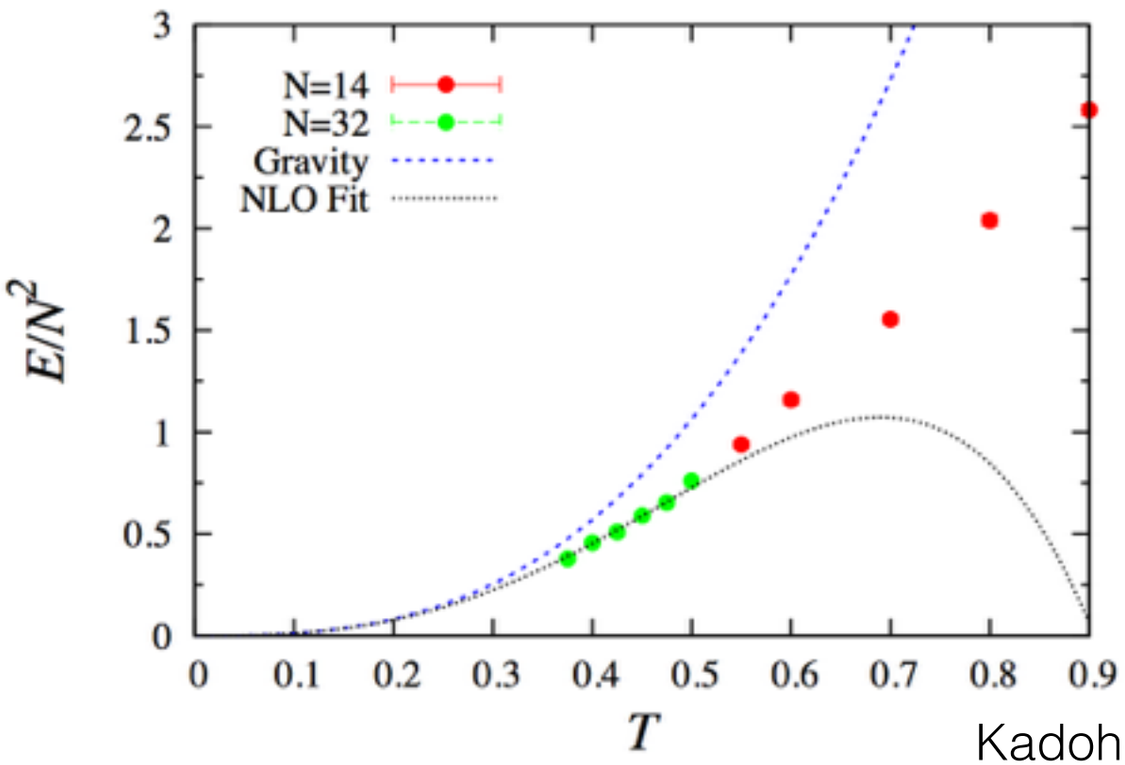
Energy of BH & MQM



Anagnostopoulos-M.H.-Nishimura-Takeuchi, PRL 2007



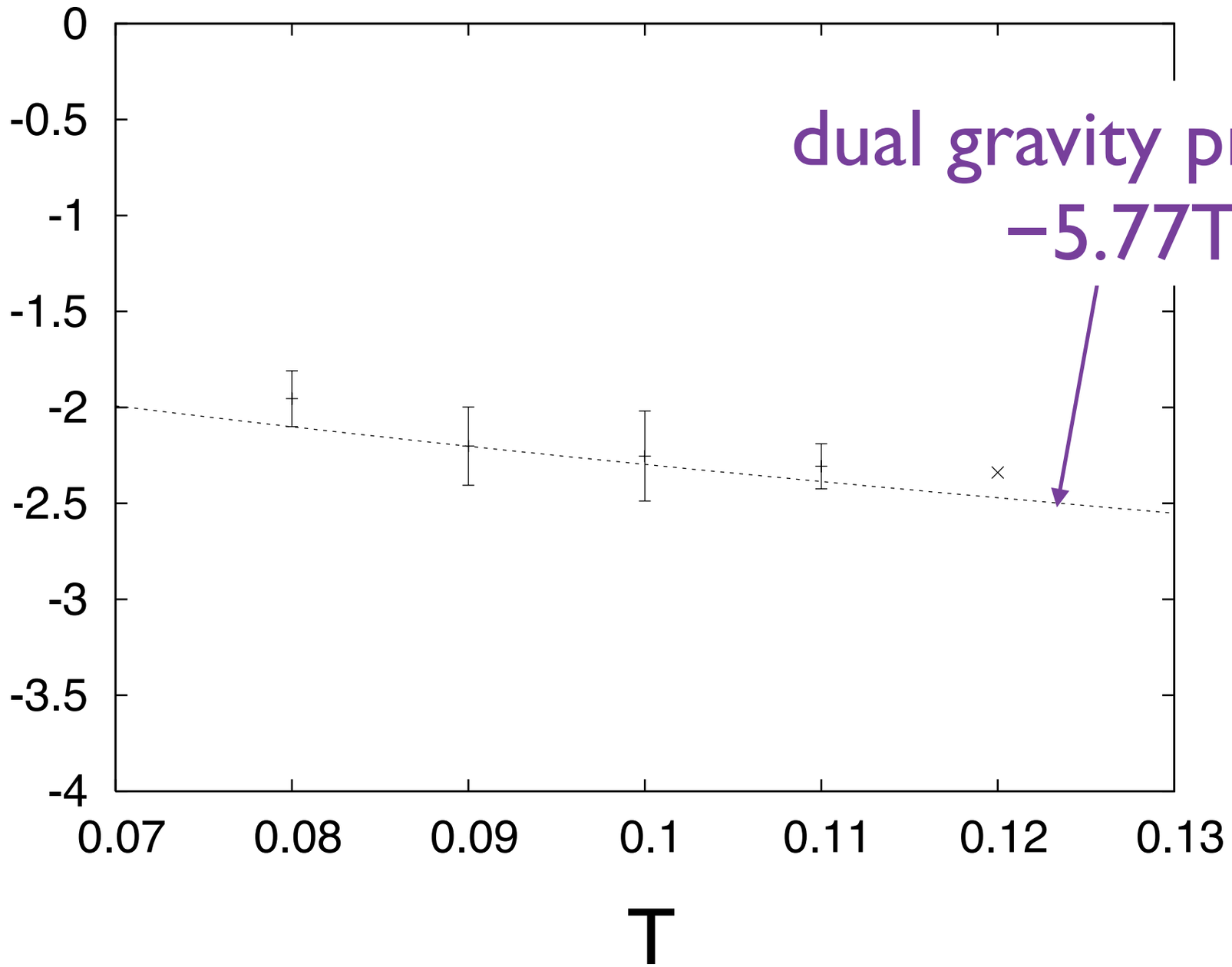
M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2008



Kadoh-Kamata, 2015

Disagreement?
We will go closer to continuum.
(stay tuned.)

$$E/N^2 = 7.41T^{2.8} - 5.77T^{0.4}/N^2 + \ldots$$

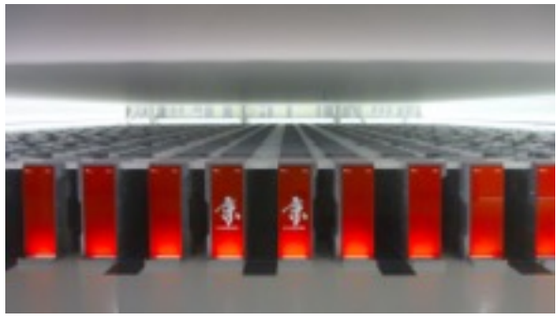


M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

Monte Carlo String/M-theory Collaboration (MCSMC)

Evan Berkowitz (LLNL)
Masanori Hanada (Stanford U./Kyoto U.)
Goro Ishiki (U. of Tsukuba)
So Matsuura (Keio U.)
Enrico Rinaldi (LLNL)
Shinji Shimasaki (KEK)
Pavlos Vranas (LLNL)

Vulcan
(LLNL, Livermore, USA)



K-supercomputer
(RIKEN, Kobe, Japan)

Both string and lattice theorists will be welcomed.

Lattice Holograhic Cosomology Collaboration (LHC Collaboration)

Southampton, UK

A. Jüttner
A. Portelli
F. Sanfilippo
K. Skenderis

Livermore & Stanford, USA

E. Berkowitz
P. Powel
E. Rinaldi
P. Vranas

Kyoto, Japan

M. Hanada

What lattice Monte Carlo can do

- Black hole thermodynamics
- 4d SYM → AdS_5/CFT_4
- Matrix model of M-theory
- 6d superconformal theory from matrix model
- Application to cosmology
- de Sitter/CFT correspondence?

and more.

You should ask string theorists at UCSB

Real-time study

Aoki, M.H., Iizuka, 1503.05562[hep-th]
Gur-Ari, M.H., Shenker, in preparation
+ work in progress with Berkowitz, Maltz

- **Full quantum study is impossible with current technology.**

  stochastic quantization (complex Langevin)?
  brute-force diagonalization?
  quantum simulator? → experimental quantum gravity?

- **Strong coupling lattice gauge theory** (+improvement)

M.H.,Maltz,Susskind 2014

stringy d.o.f. is manifest; still numerically demanding, but should be possible in near future.

- **Classical real time evolution**

i.e. just solve classical EOM

high temperature = weak coupling = highly stringy

highly nonlinear & nonperturbative

"BH" = soliton (or resonance) of matrix model

We will see the formation & evaporation of "BH" in this limit.

Gauge/gravity duality

Quantum Field Theory

$N=\infty$, $\lambda=\infty$ (T=0) ⟺ Classical Gravity

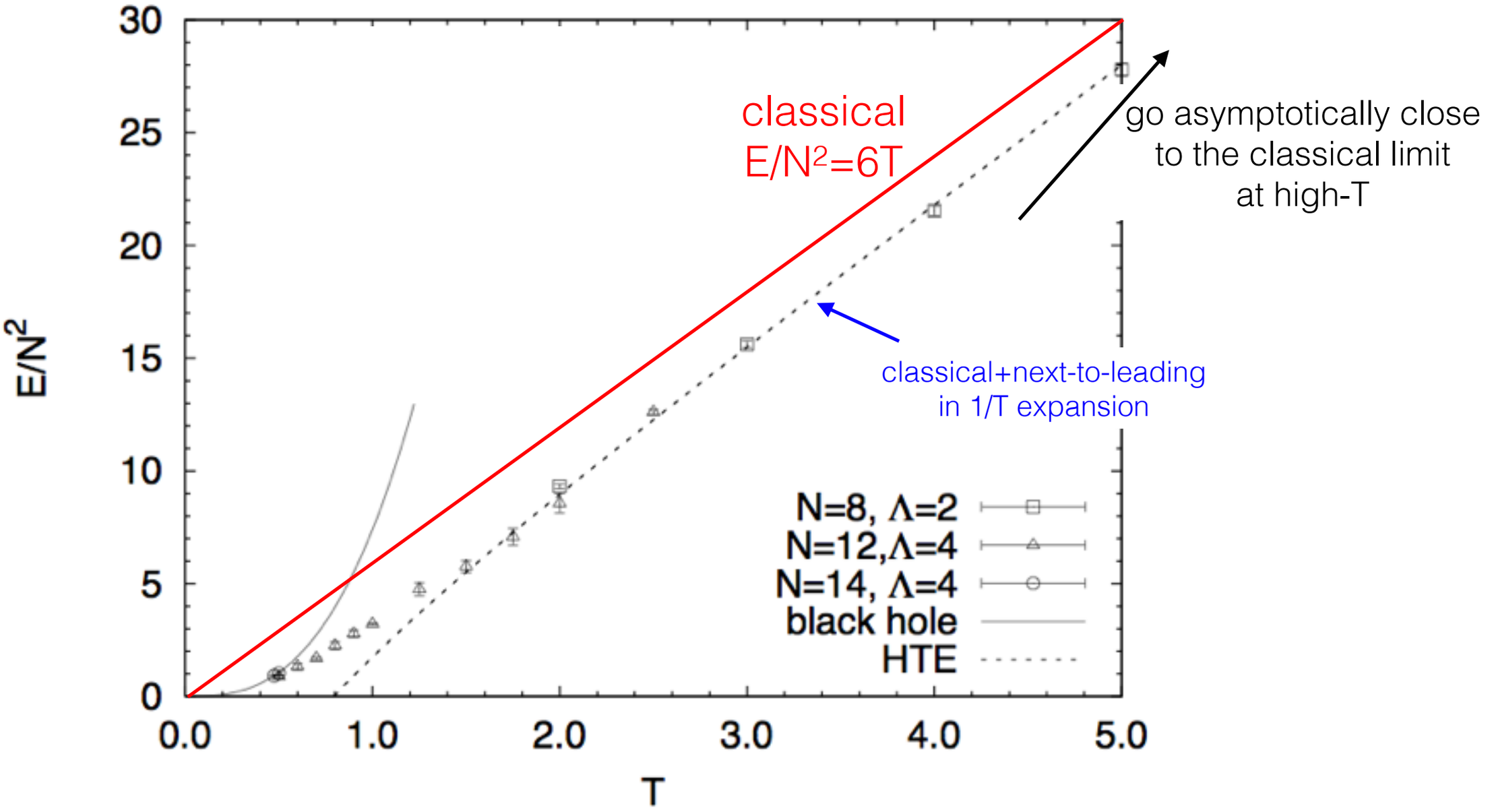
$N=\infty$, $\lambda<\infty$ (T>0) ⟺ Classical String Theory ($\alpha' > 0$)

$N<\infty$, $\lambda<\infty$ (T>0) ⟺ Quantum String Theory ($\alpha' > 0$, $g_s>0$)

Classical simulation might tell us something about this part

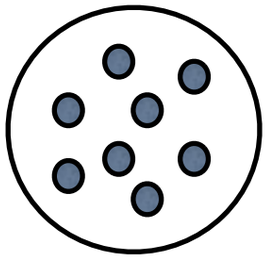
Remark

There is no phase transition between low- and high-T.



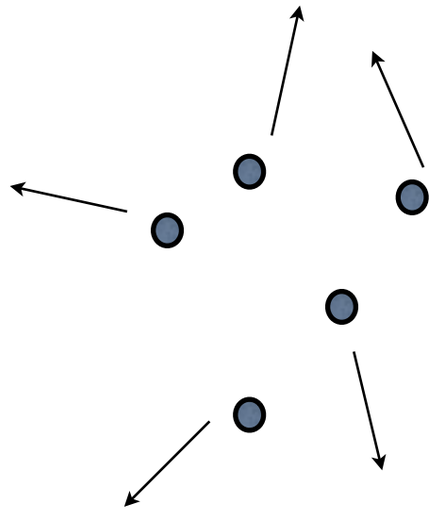
E/N^2 in BFSS vs 0-brane mass (0707.4454[hep-th])

‘eigenvalues’ = D0-branes



bound state of eigenvalues

= black hole

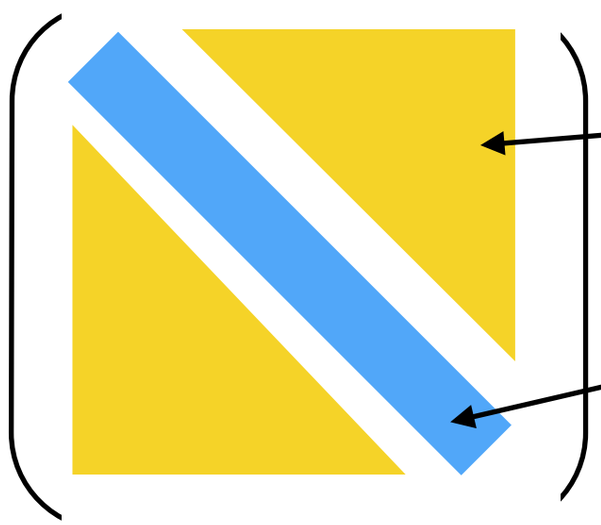


flat direction

= gas of D0-branes

This phase reproduced the dual BH thermodynamics.

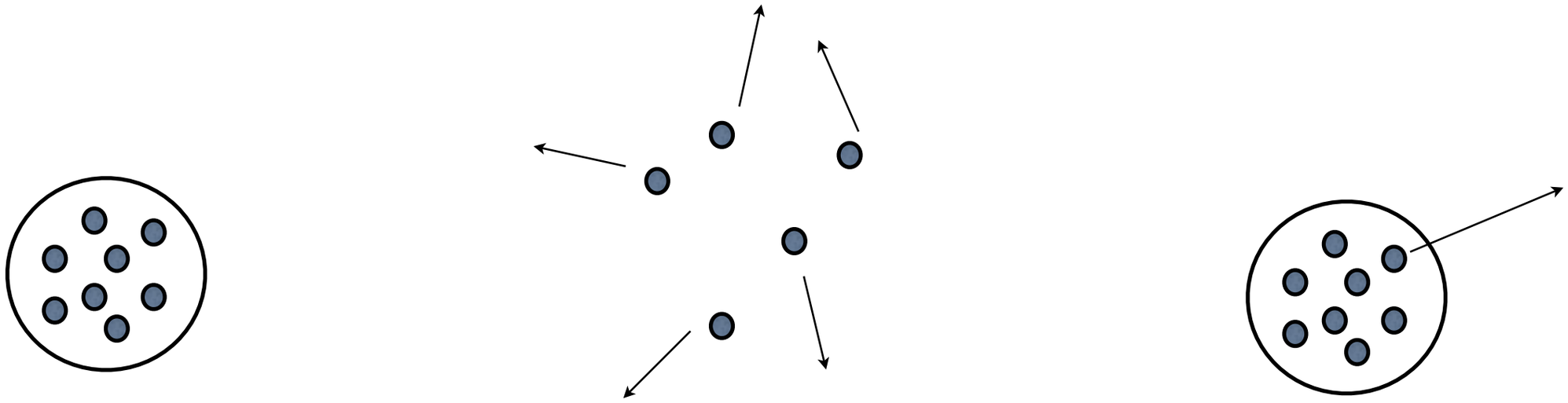
$X_M =$



open strings

D0-branes

‘eigenvalues’ = D0-branes



bound state of eigenvalues
= black hole

flat direction
= gas of D0-branes

emission of eigenvalue
= evaporation of BH
(emission of D0)

This model can describe BH evaporation!

This evaporation is suppressed at N=∞.

(The instability has been observed in imaginary time simulation.)

$$L = \frac{1}{2g_{YM}^2}\mathrm{Tr}\left(\sum_i (D_t X^i)^2 + \frac{1}{2}\sum_{i\neq j}[X_i, X_j]^2\right)$$

$$\begin{cases} \dfrac{d^2X^i}{dt^2} - \displaystyle\sum_j [X^j, [X^i, X^j]] = 0 \\ \displaystyle\sum_i \left[X^i, \frac{dX^i}{dt}\right] = 0 \end{cases}$$ (A=0 gauge)

discretize & solve it numerically.

It takes only 15 - 30 minutes for average graduate students to write C or Fortran codes.
[cf) Monte Carlo code for thermodynamics → a few months ～1 year for smart students]

Discretization of EOM

continuum

$$\dot{X}^i(t) = V^i(t)\,,$$

$$\dot{V}^i(t) = F^i(t) \equiv \sum [X^j(t), [X^i(t), X^j(t)]]\,.$$

discretized

$$X^i(t+\delta t) = X^i(t) + V^i(t)\cdot\delta t + F^i(t)\cdot\frac{\delta t^2}{2}\,,$$

$$V^i(t+\delta t) = V^i(t) + \big(F^i(t) + F^i(t+\delta t)\big)\cdot\frac{\delta t}{2}\,.$$

Gauss's law is exact at regularized level.

Remark

$$\frac{d^2X^i}{dt^2} - \sum_j [X^j, [X^i, X^j]] = 0$$

Invariant under the scaling $t \to t/\alpha,\ X_M \to \alpha X_M$

All values of the energy (or 'temperature') are equivalent.

$E, T \to \alpha^4 E, \alpha^4 T$

We take T=1 unless otherwise stated.

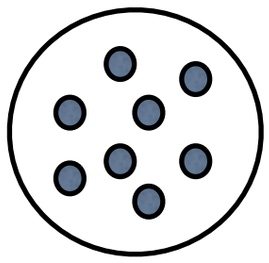
$E = 6(N^2-1)T = 6(N^2-1)$

(traceless condition)

Formation & thermalization of BH

- This system is chaotic. (Savvidy, 1984; Berenstein et al, 2012)
- Almost all initial conditions end up with ‘typical’ matrix configurations — BH.

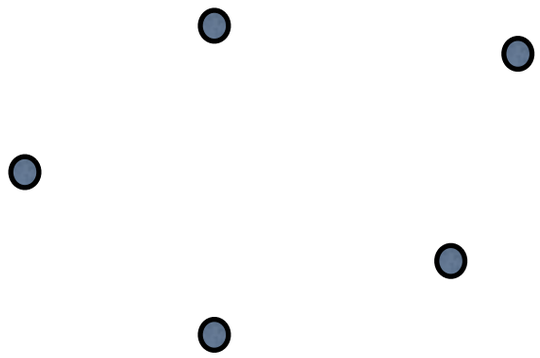
BH



open strings
(off-diagonal components)
are excited

entropy $\sim N^2$

gas



open strings are suppressed

entropy $\sim N$

Example:
Collision of 2 BHs

Formation & Thermalization of “BH” can be seen.

After thermalization,

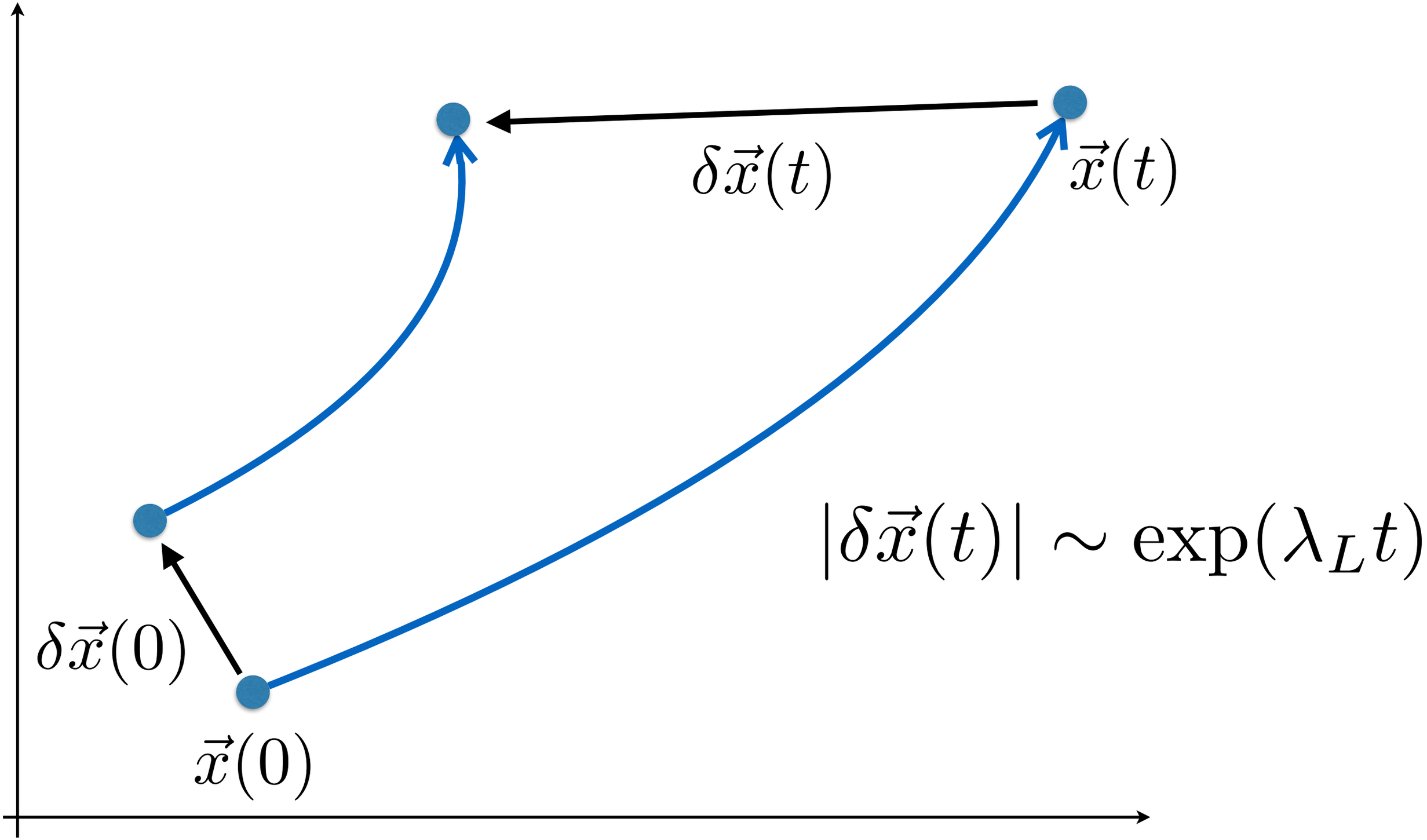
- $(\mathrm{Tr}\, X^2)/N$, $(\mathrm{Tr}\, V^2)/N$ etc are t-independent at large-N.
- SO(9) rotational symmetry emerges.
- (X(t), V(t)) (→‘micro-state’) changes rapidly.

(I don’t have a time to explain the detail, sorry)

Fast scrambling

- Take a 'micro-state' (X,V) from a thermalized "BH."
- Then add a small perturbation:
  $X \to X+\delta X$, $V \to V+\delta V$.

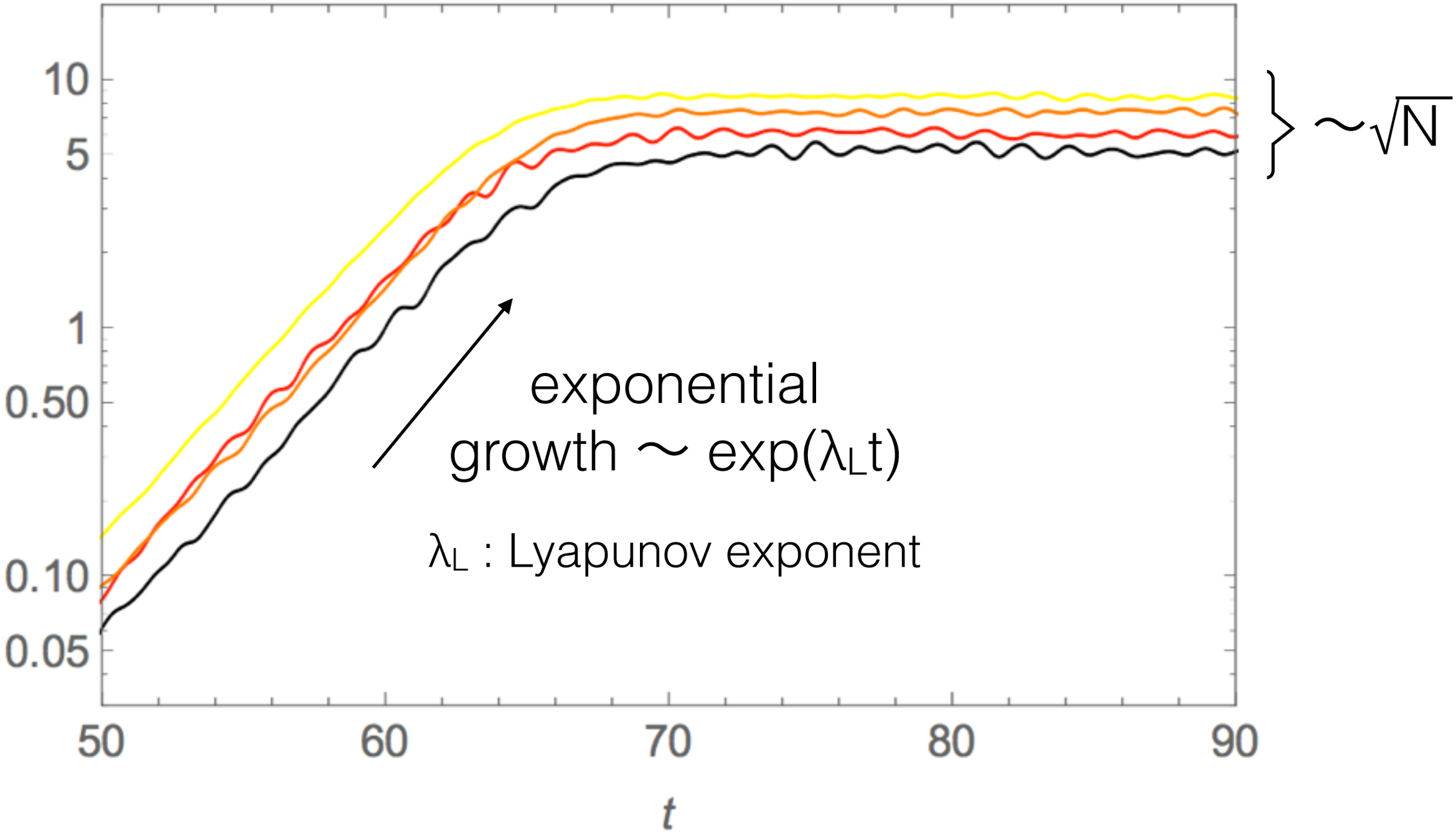
Lyapunov Exponent



$$|\delta\vec{x}(t)| \sim \exp(\lambda_L t)$$

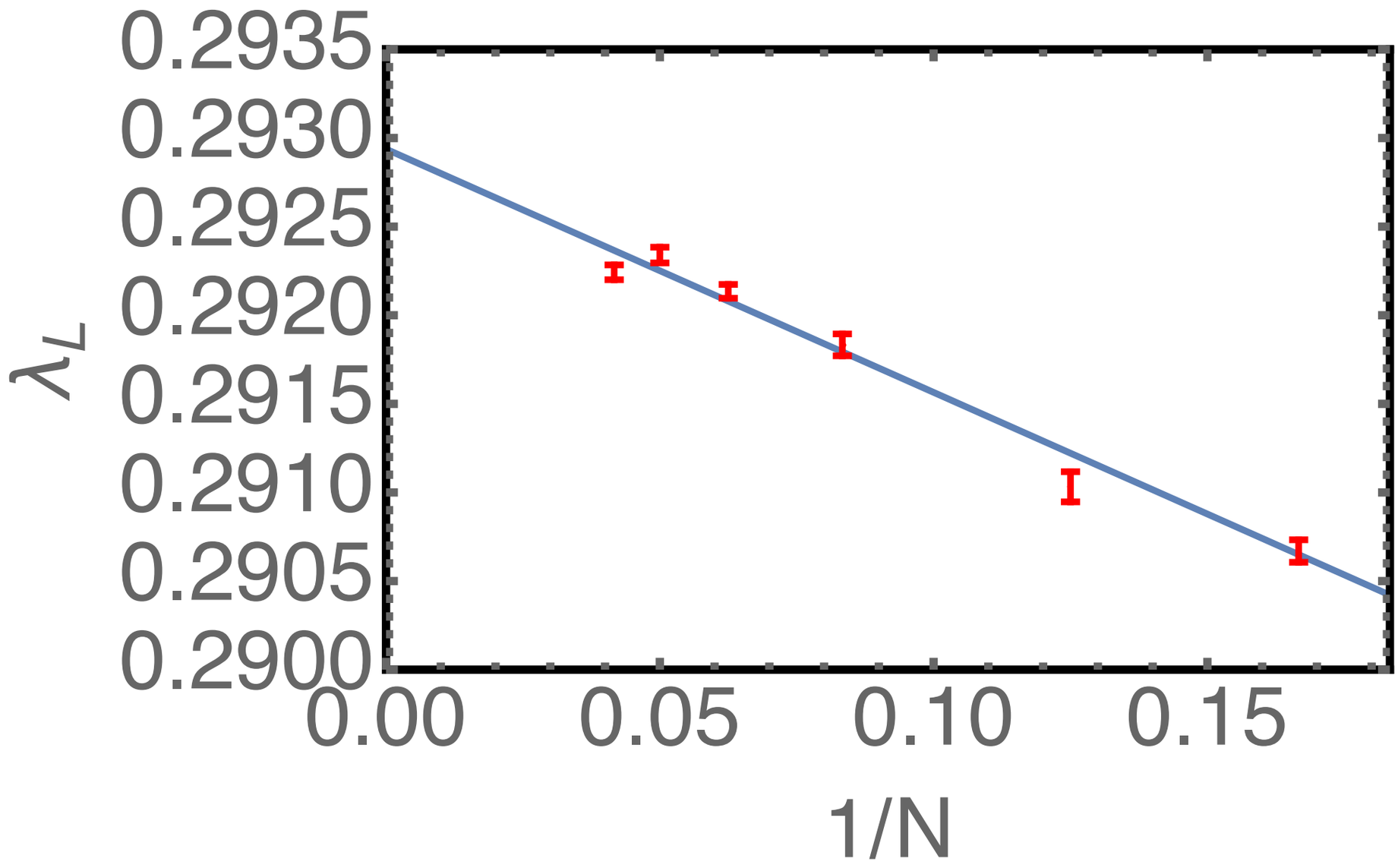
Fast scrambling

- Take a 'micro-state' (X,V) from a thermalized "BH."
- Then add a small perturbation:
  $X \to X+\delta X$, $V \to V+\delta V$.
- δX and δV grows quickly, i.e. information of the initial state is scrambled.
- 'scrambling time' $t_s \sim \log N$. (Sekino-Susskind, 2008; Shenker-Stanford 2013, 2014; Maldacena-Shenker-Stanford 2015)
- Let's test this conjecture.

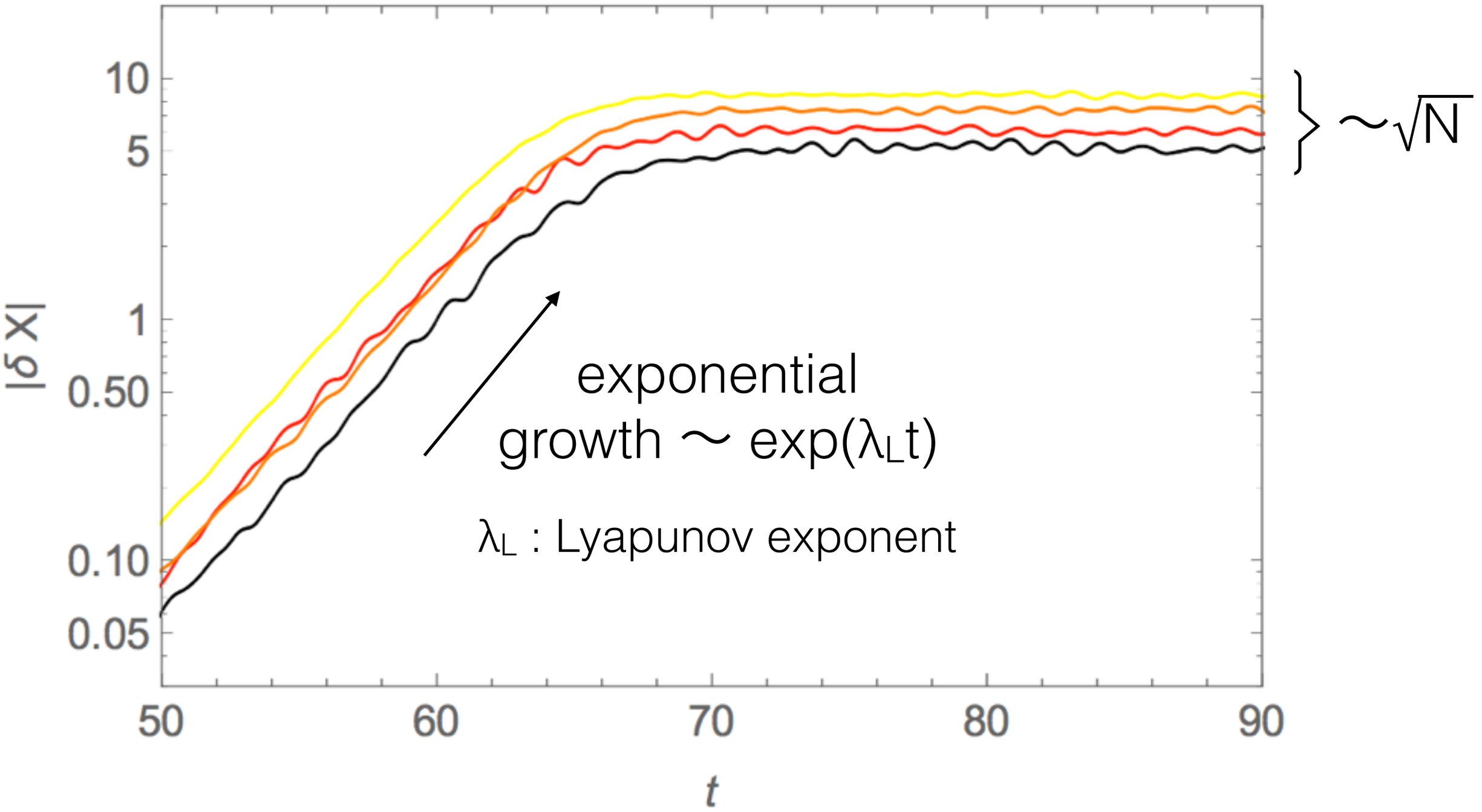

10
5
1
0.50
0.10
0.05
|δ X|
50
60
70
80
90
t
exponential
growth ∼ exp(λLt)
λL : Lyapunov exponent
∼√N

N=6,8,12,16

1/N Behavior



$$\lambda_L = 0.293 - \frac{0.014}{N} + O(1/N^2) \qquad (N = 8, \ldots, 24)$$

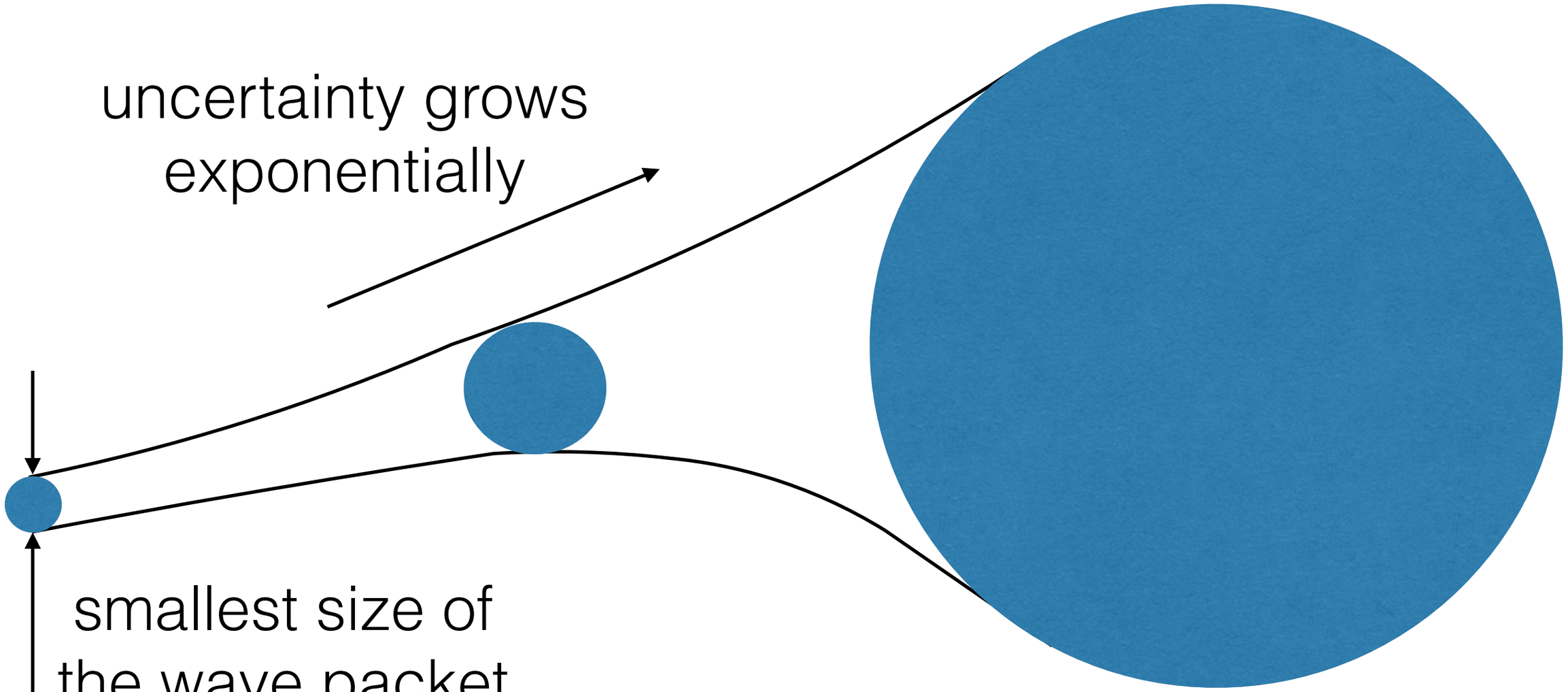


$\exp(\lambda_L t_s) \sim \sqrt{N}$

→ “scrambling time” $t_s = (\log N)/\lambda_L \sim \log N$

Fast scrambling!

uncertainty grows
exponentially



smallest size of
the wave packet
in phase space

$$\sim \hbar \sim N^0$$

maximum uncertainty
$\sim$ size of the system

$$\sim \sqrt{N}$$

$$\boxed{\lambda_L = 0.293\ (\lambda\text{'t Hooft}\, T)^{1/4}}$$

$$\frac{d^2X^i}{dt^2} - \sum_j [X^j, [X^i, X^j]] = 0$$

Invariant under the scaling

$$t \to t/\alpha,\ X_M \to \alpha X_M$$

E, T, λ → α^4E, α^4T, αλ

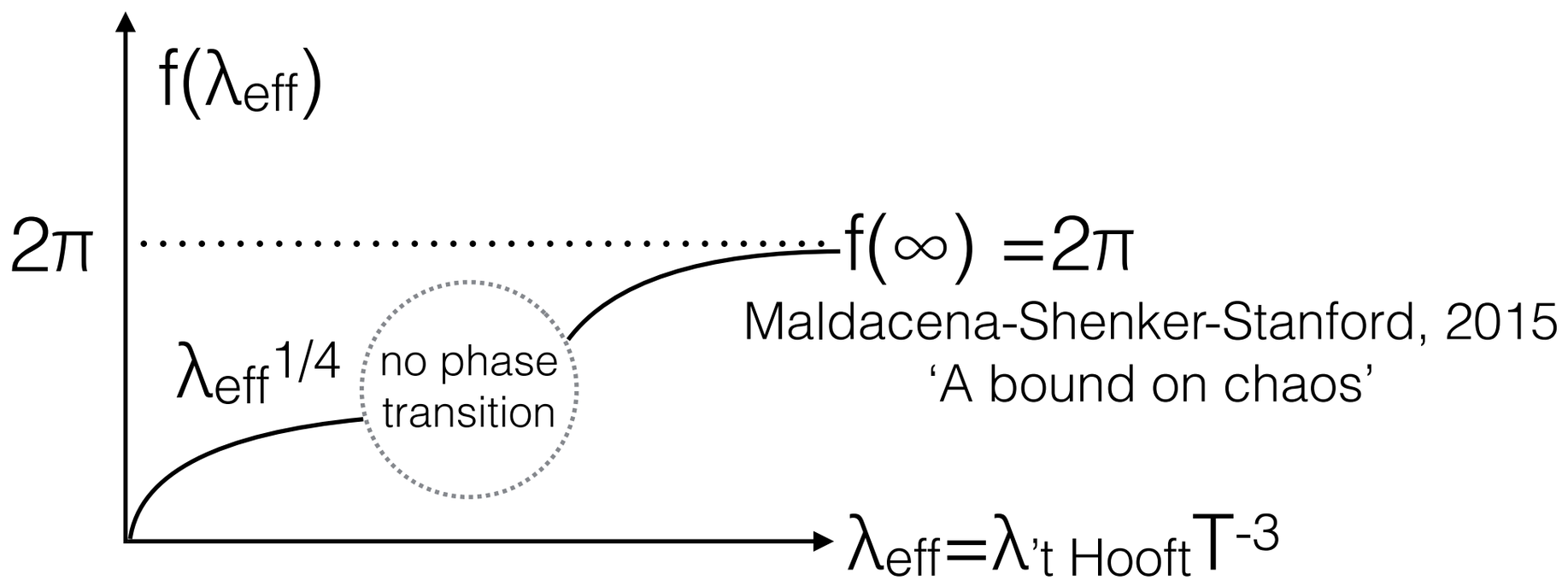
strong coupling vs. weak coupling

effective dimensionless temperature $T_{eff} = (\lambda_{'t\ Hooft})^{-1/3} T$

effective dimensionless 't Hooft coupling $\lambda_{eff} = \lambda_{'t\ Hooft} T^{-3}$

$\lambda_L = 0.293\ (\lambda_{'t\ Hooft} T)^{1/4} = (0.293\ \lambda_{eff}^{1/4}) T$

$\lambda_L = f(\lambda_{eff}) T$



Maldacena-Shenker-Stanford, 2015
'A bound on chaos'

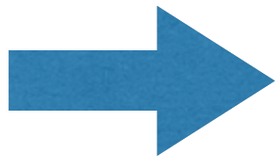
Lyapunov spectrum

$$\begin{pmatrix} \delta\dot{V}_M \\ \delta\dot{X}_M \end{pmatrix} = \begin{pmatrix} 0 & \mathcal{M}_{MN} \\ \delta_{MN} & 0 \end{pmatrix} \begin{pmatrix} \delta V_N \\ \delta X_N \end{pmatrix}$$

$$\mathcal{M}_{MN}\delta X_N = \delta[X_N, [X_M, X_N]]$$

$$\begin{pmatrix} \delta X(t') \\ \delta V(t') \end{pmatrix} = U(t', t) \begin{pmatrix} \delta X(t) \\ \delta V(t) \end{pmatrix}$$

$$U(t + \delta t, t) = \begin{pmatrix} 1 & \delta t \\ M(t)\delta t & 1 \end{pmatrix}$$



Singular values of U
= growth rate $\exp(\lambda_L(t'-t))$
(Lyapunov spectrum)

$16(N^2-1)$ physical modes
$8(N^2-1)$ positive modes

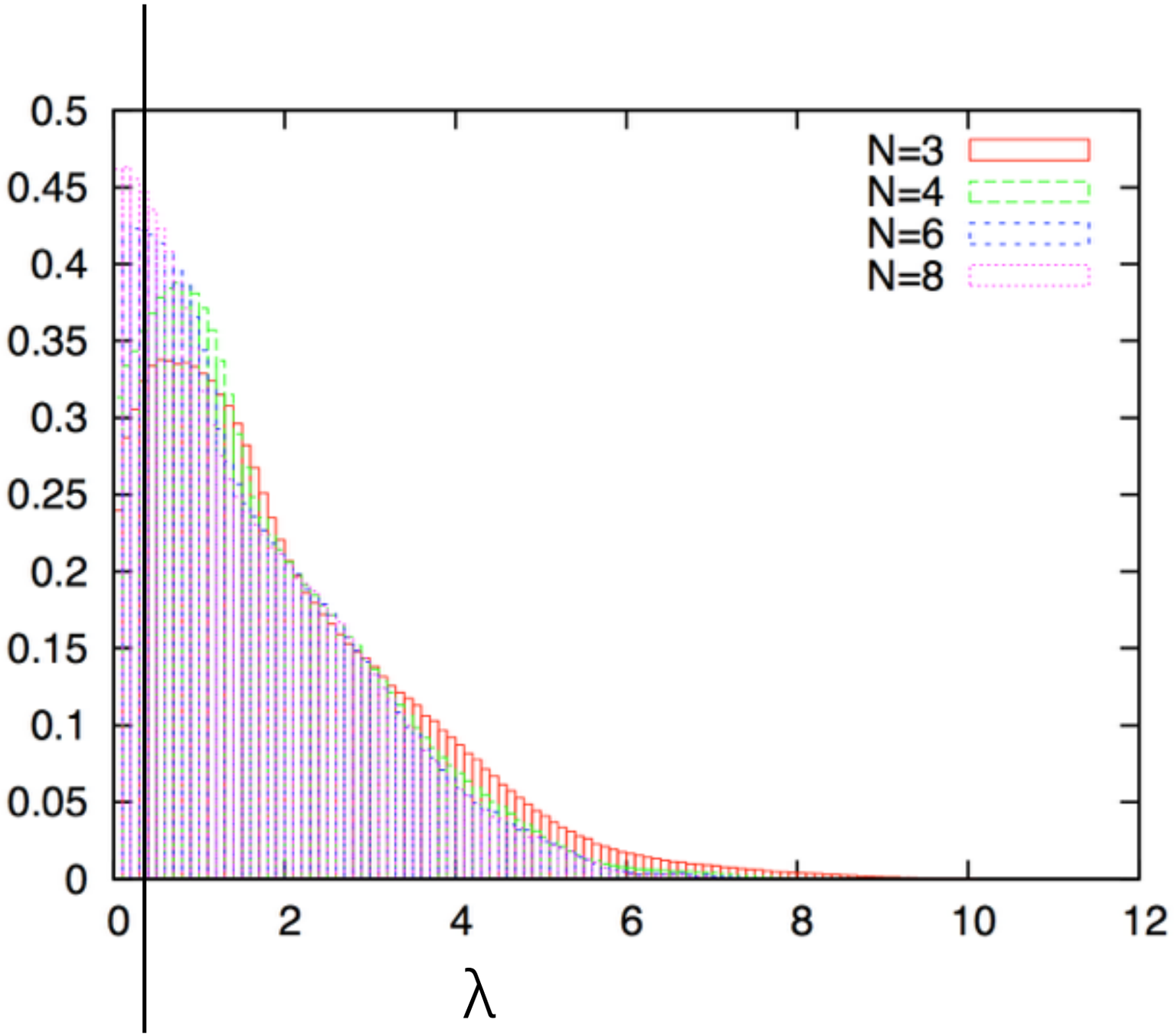
‘local Lyapunov exponent’

- Lyapunov spectrum is defined at t’–t $\to\infty$
- At finite t’–t, λ_L can depend on t’–t.
- t’–t $\to$ 0 gives ‘local’ Lyapunov exponent.

Our guess: at large-N, exponents are t-independent.

surprise.

λ_L=0.293 (the largest Lyapunov exponent)



why?

$$\mathcal{M}_{MN}\delta X_N = \delta[X_N,[X_M,X_N]]$$

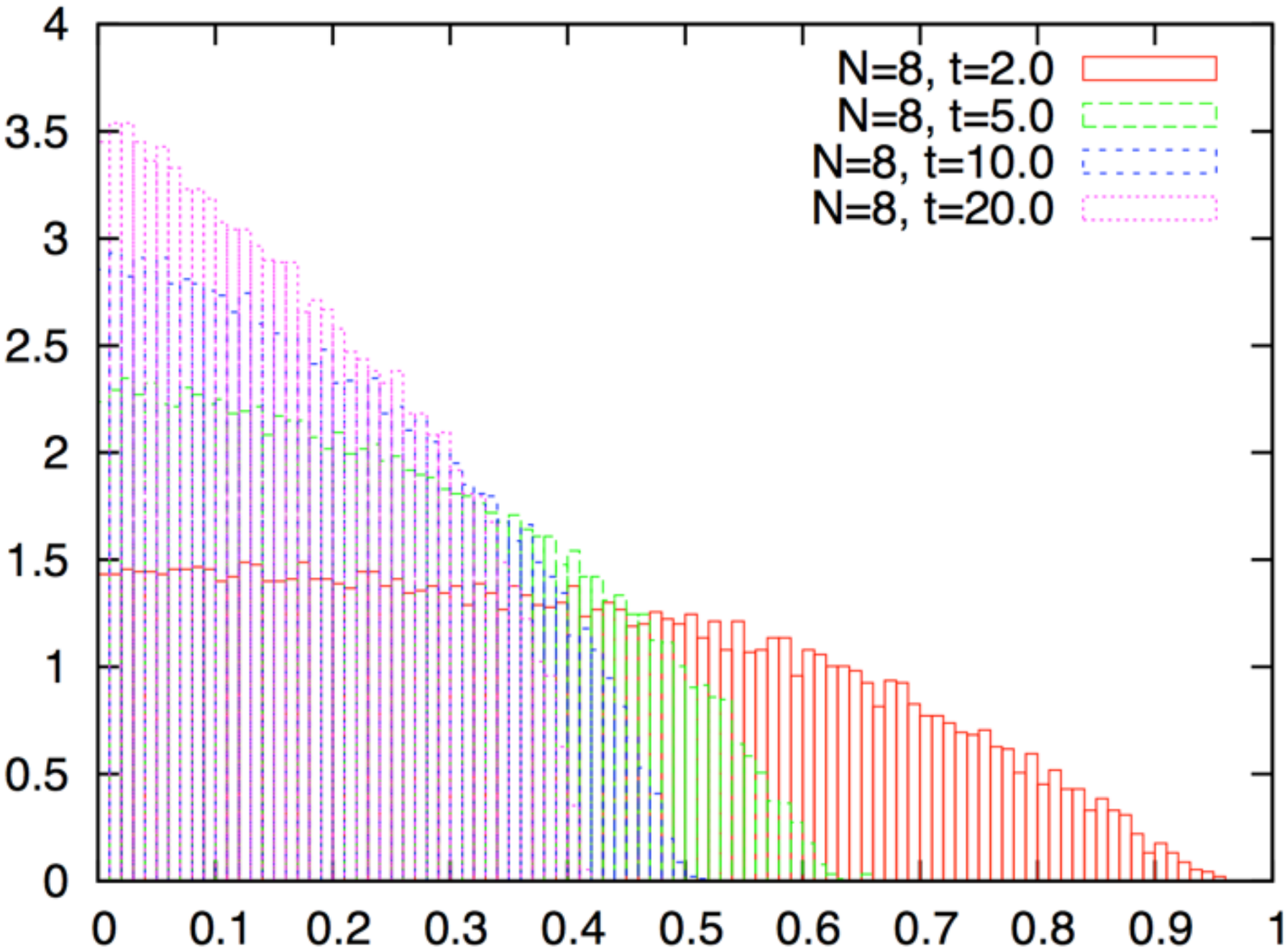
it depends on $X_M(t)$, and hence on t.

‘growing direction’ changes very rapidly.
Perturbation grows to some directions
and shrinks along other directions.

A nontrivial cancellation makes
the Lyapunov exponent rather small.

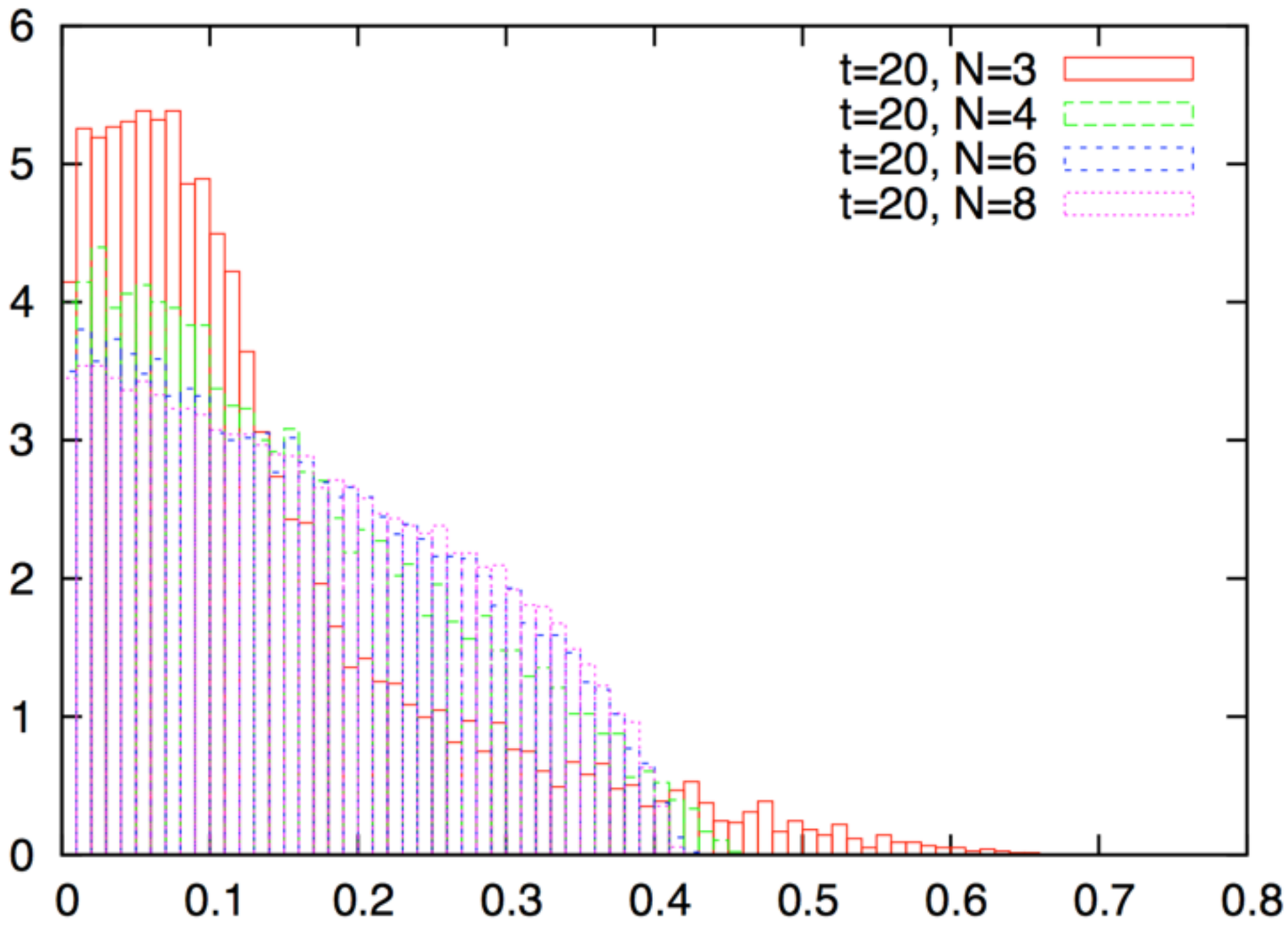
If X_M moved even faster, the exponent could be zero.

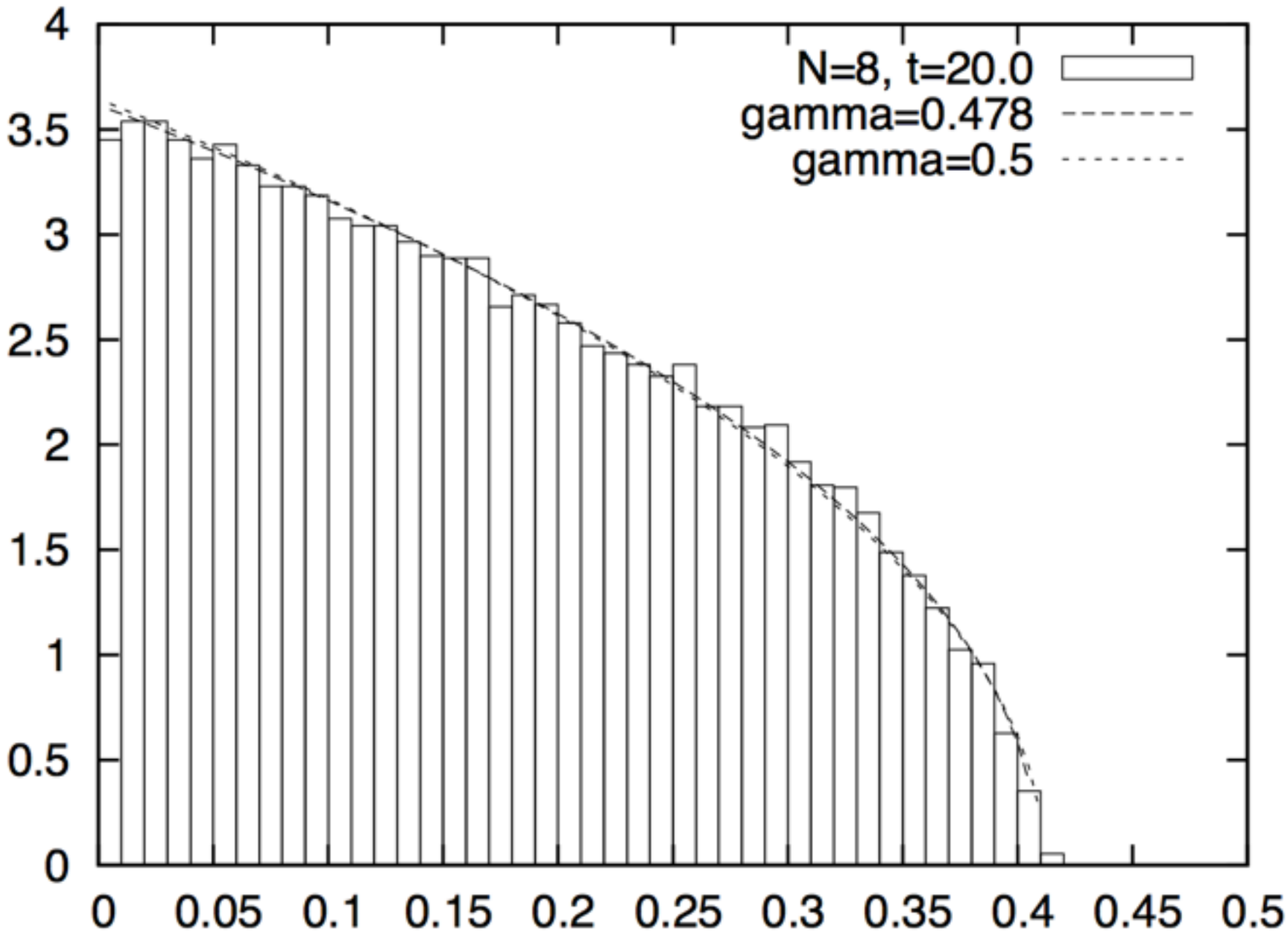
Fixed N, different t



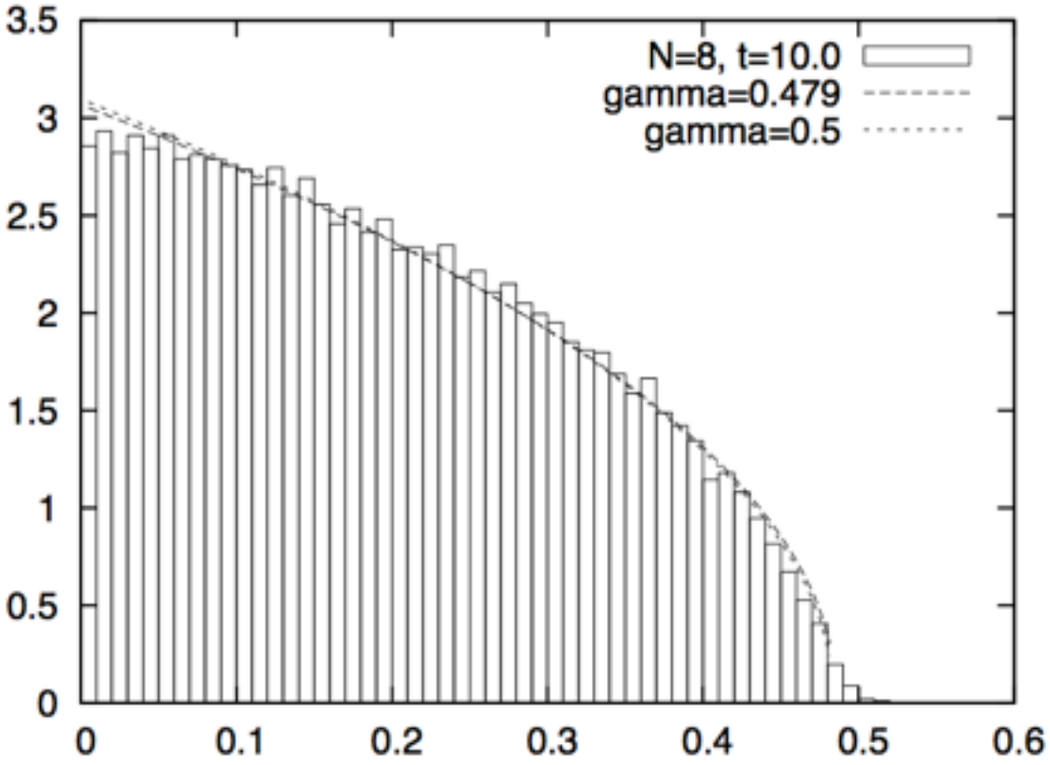
exponents become smaller

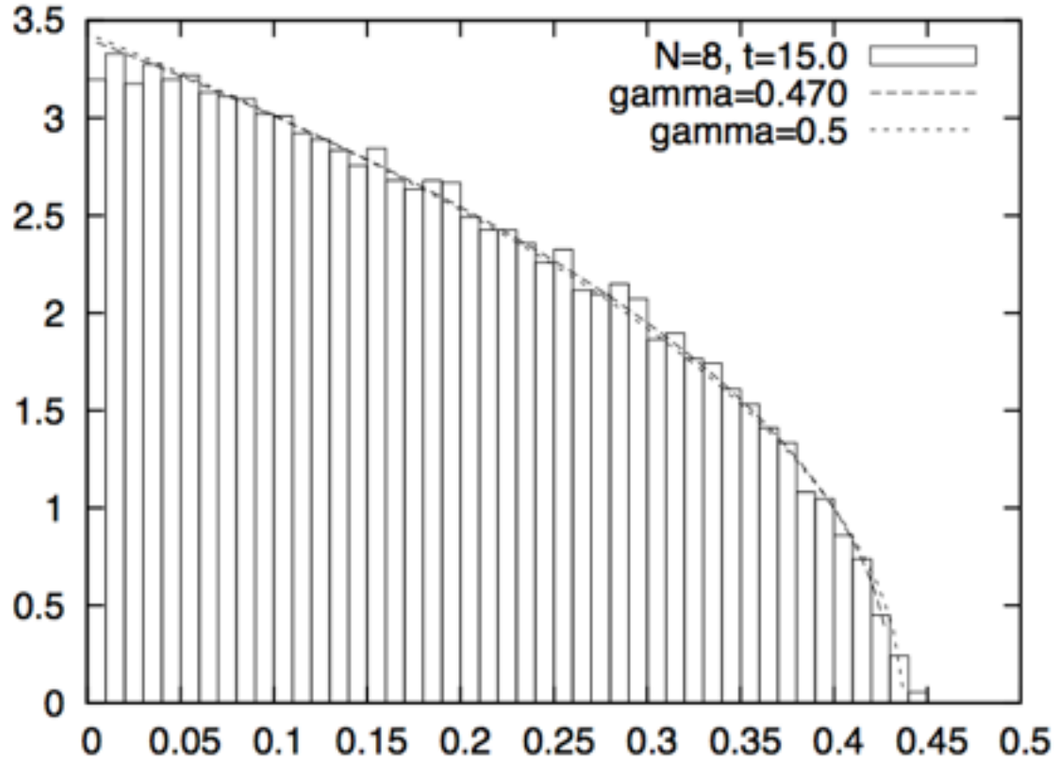
Fixed t, different N

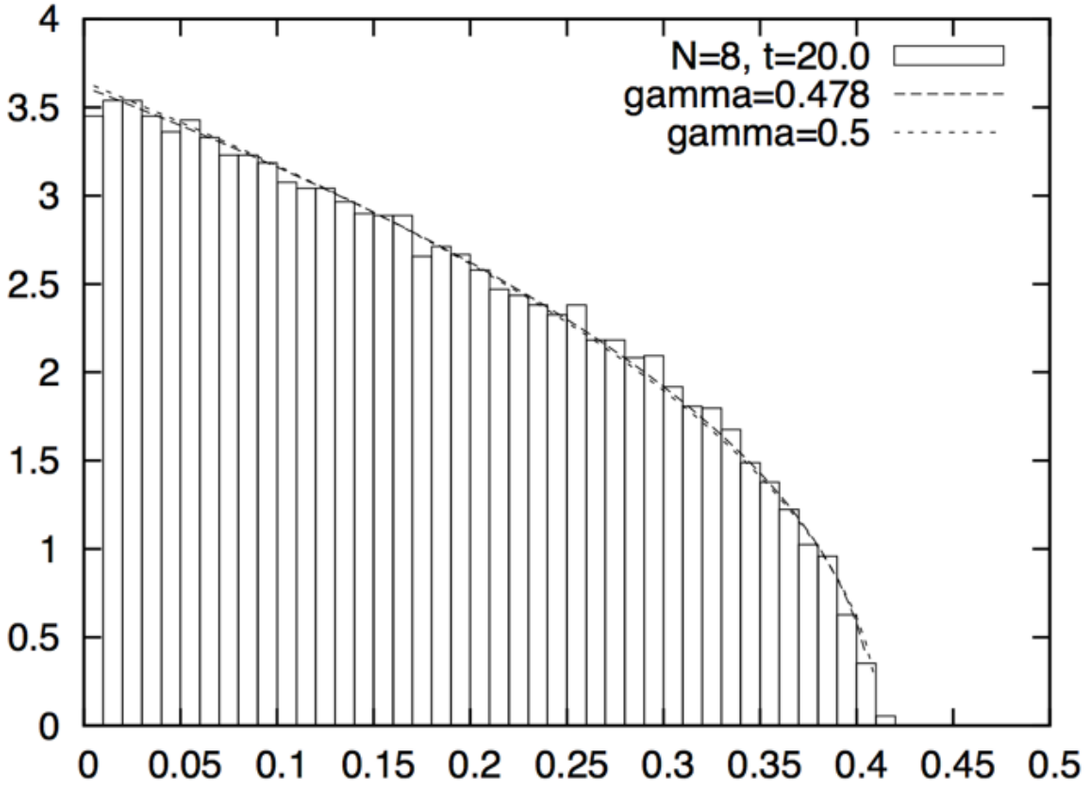




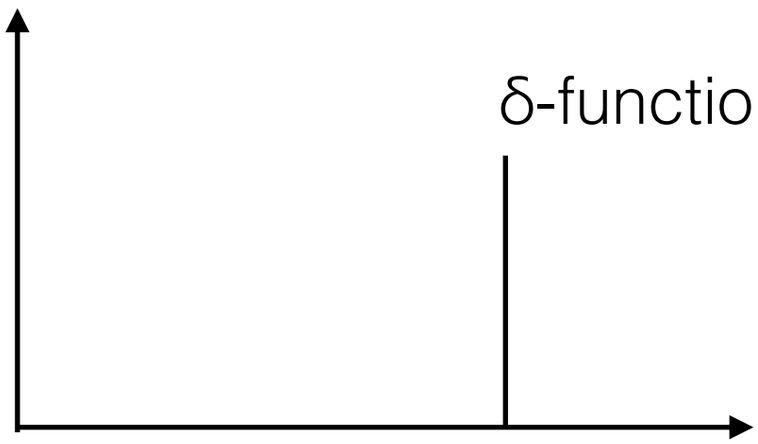
Fitting ansatz $\rho(\lambda) = \frac{(\gamma+1)(\lambda_{max}-\lambda)^{\gamma}}{\lambda_{max}^{\gamma+1}}$



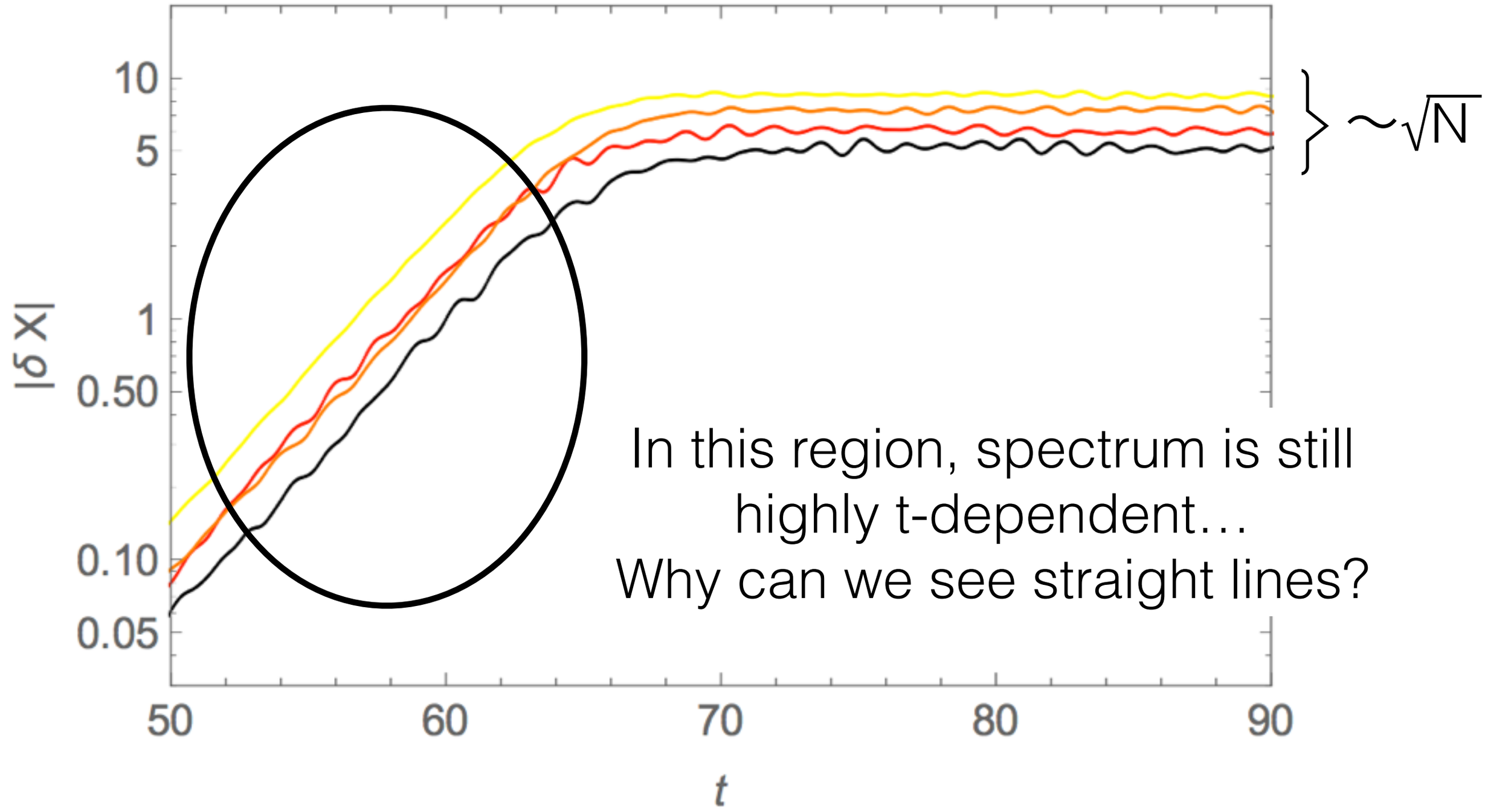




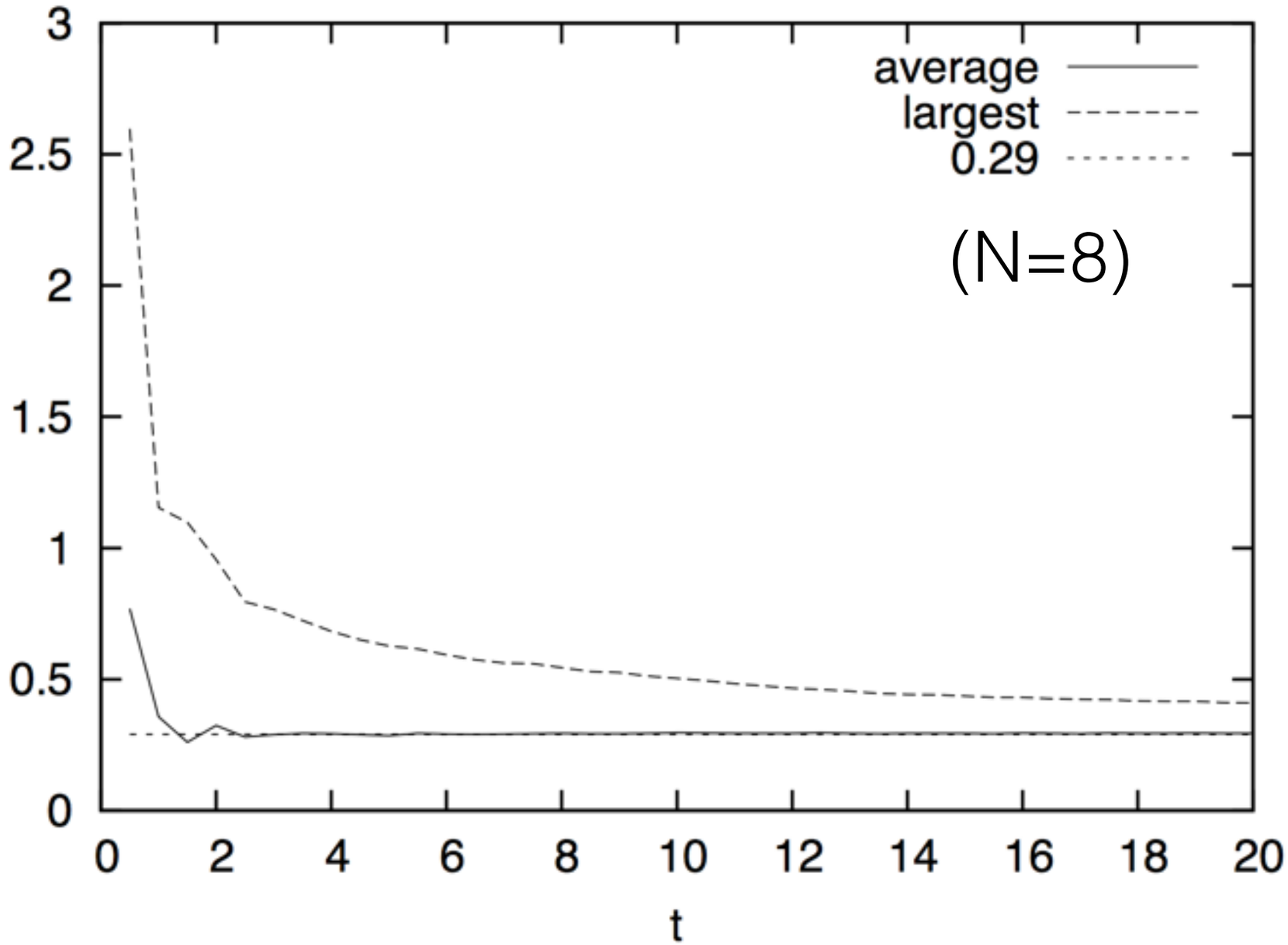
cf) strong couplong by SUGRA



The spectrum spreads due to stringy effect.

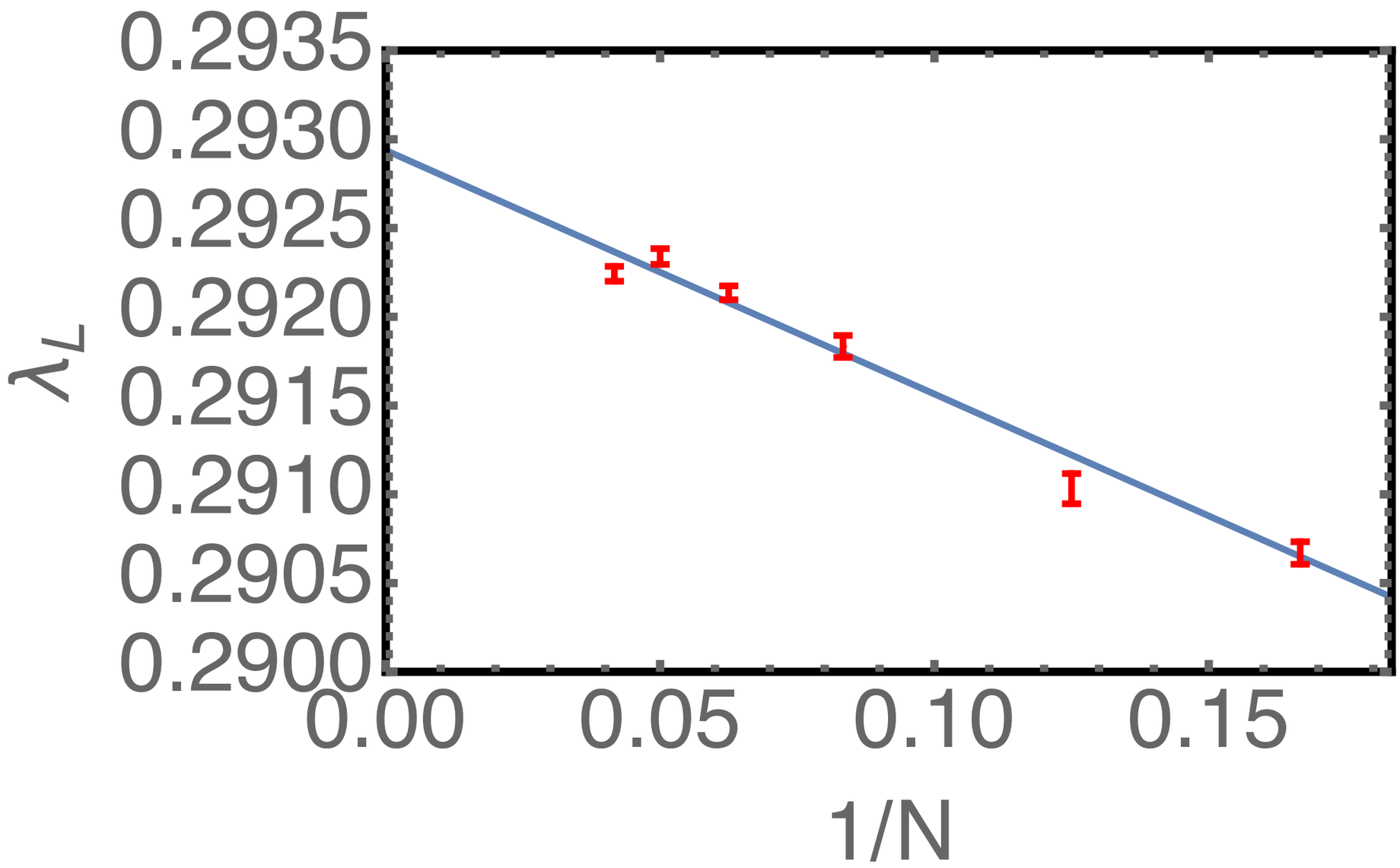

10
5
1
0.50
0.10
0.05
|δ X|
50
60
70
80
90
t
~√N
In this region, spectrum is still highly t-dependent…
Why can we see straight lines?

N=6,8,12,16



$$\lambda_{av} \equiv \frac{1}{t} \log \sqrt{(\sum_{i,\pm} e^{\pm 2\lambda_i t})/16(N^2-1)}$$

Don’t worry:
These values are not obtained by fits at t < 60.
We used Sprott’s algorithm, t > 1000.



$$\lambda_L = 0.293 - \frac{0.014}{N} + O(1/N^2) \quad (N = 8, \ldots, 24)$$

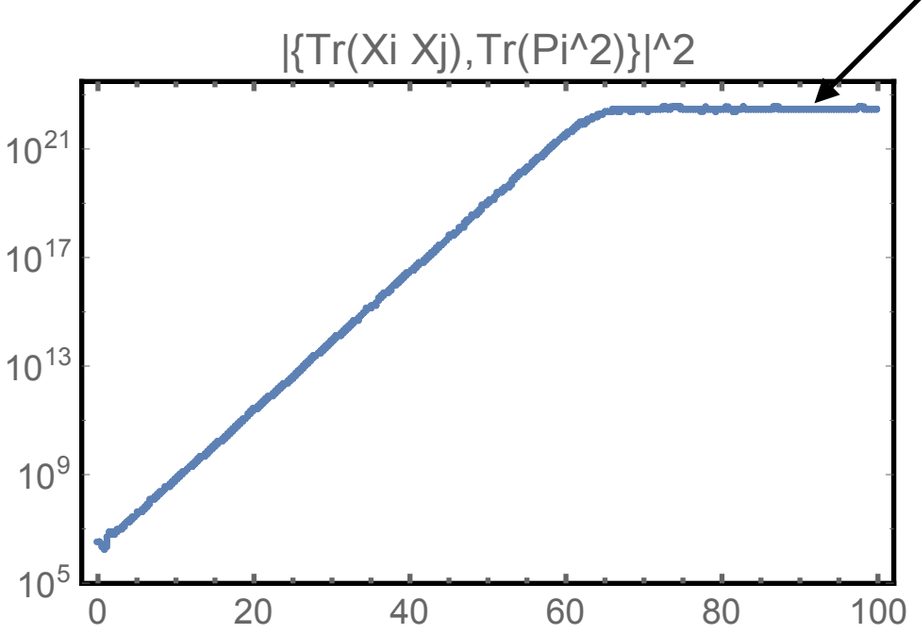
Lyapunov exponent from correlators

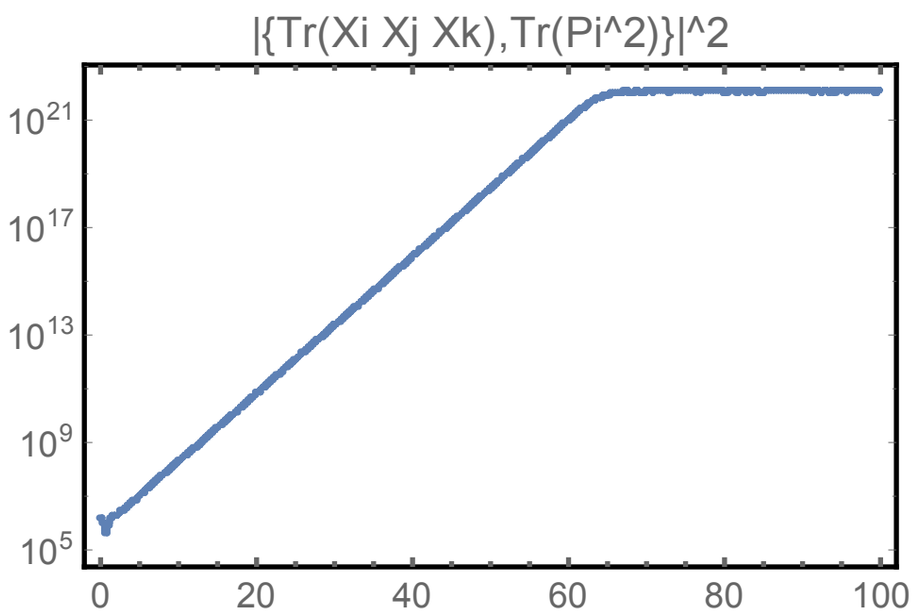
- Maldacena, Shenker and Stanford calculated Lyapunov exponent by using

$$-\langle [W(t), V(0)]^2 \rangle \sim \hbar^2 e^{2\lambda_L t}$$

- commutator in Q.M. → Poisson bracket in classical mechanics

This saturation is a numerical artifact.
(We used finite difference, rather than derivative.)

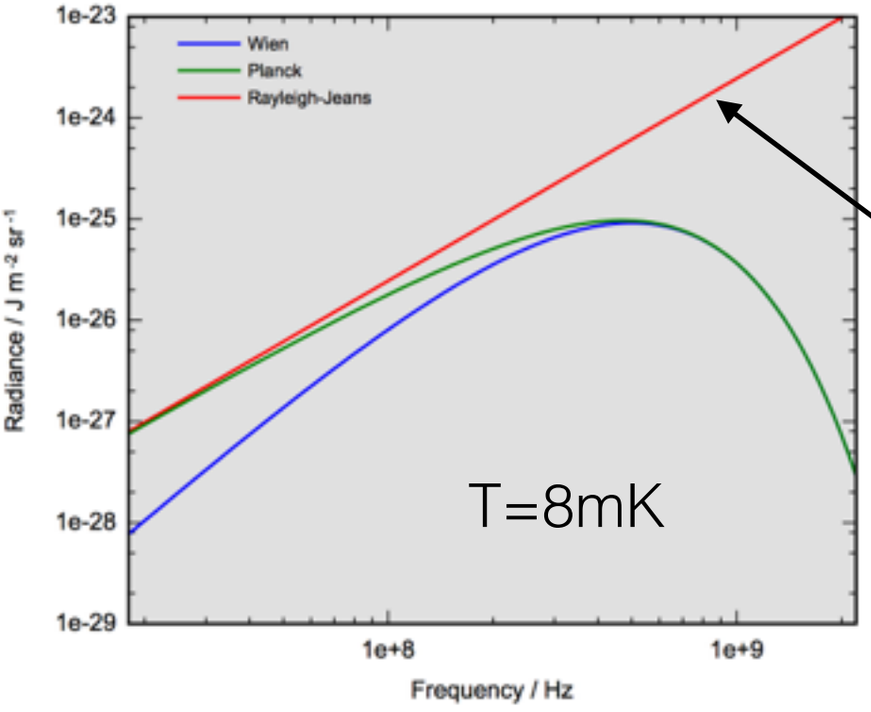




The same λ_L is obtained.

Classical Yang-Mills theory?

(nonzero spatial dimensions)



(wikipedia)

oops!

equipartition of energy
+ infinite d.o.f. in UV

→ *UV catastrophe*

In classical YM, energy flows to UV;
thermal equilibrium is never reached.

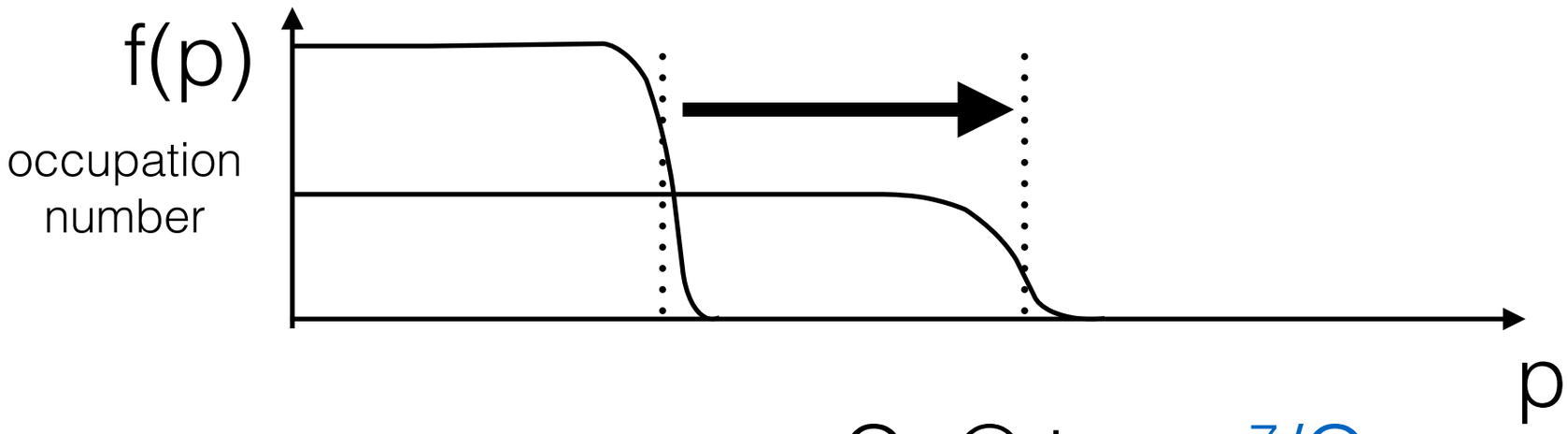
Lord Rayleigh
1842-1919

James Jeans
1877-1946

Max Planck
1858-1947

But UV catastrophe might not be so catastrophic

- Energy flow to UV is slow. (Kurkela-Moore, 2012)



$p_{max} \sim aQ \ @\ t \sim a^7/Q$

where $\varepsilon = Q^4N^2/\lambda$: 't Hooft energy density

scrambling time $\sim$ (log N)/Q

no problem when (log N) << a^7

$\exp((3/2)^7) \sim 2.6\times10^7$

'thermalization' at IR is achieved, then very slow flow to UV follows.

What can we do?

- Thermalization of black brane.
- Correlation functions.
- Scrambling in 2d, 3d and 4d theories; how perturbations grow in color space and in spatial dimensions.
- Black hole / black string topology change.
- What is the ‘stringy effect’ ?

Similarity to & difference from
strong coupling limit (supergravity) ?

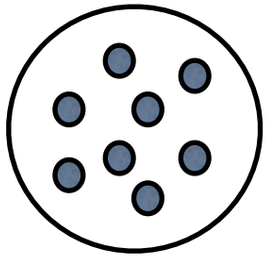
Evaporation

(in progress)

chaos (or ergodicity) + flat direction

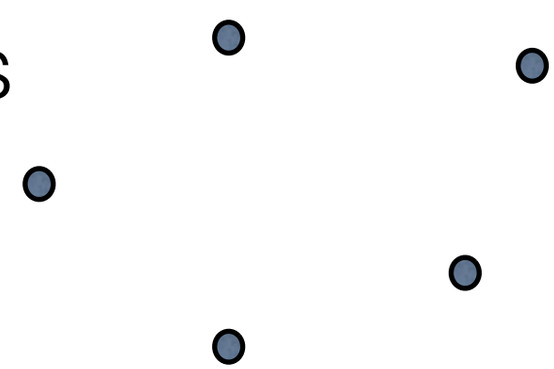
→ evaporation

BH



entropy ∼ N^2

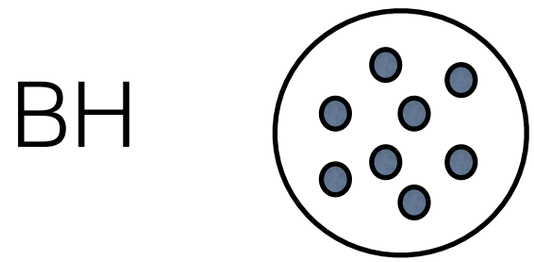
gas



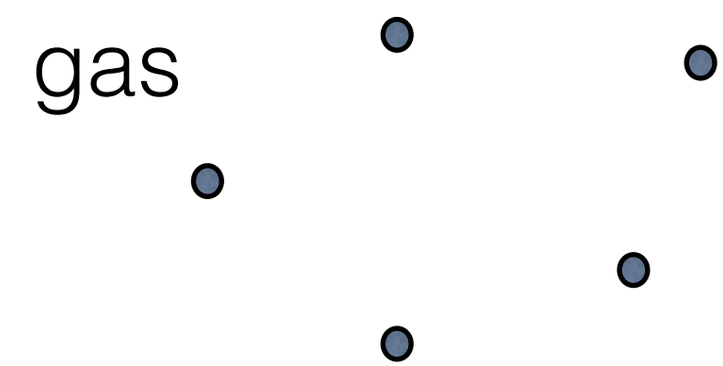
entropy ∼ N

Exponentially suppressed,
but still can appear after long time.

chaos (or ergodicity) + flat direction
→ evaporation



entropy ~ N^2



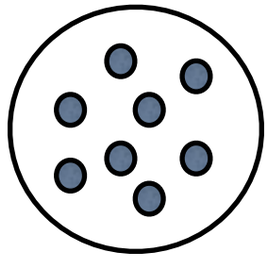
entropy ~ N × ∞

(space volume)

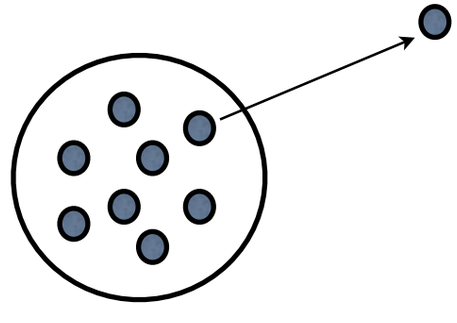
※ Flat direction must be sufficiently flat. Will be explained shortly. →

Once brane is emitted,
it does not come back.

‘eigenvalues’ = position of D0-branes



entropy ~ N^2



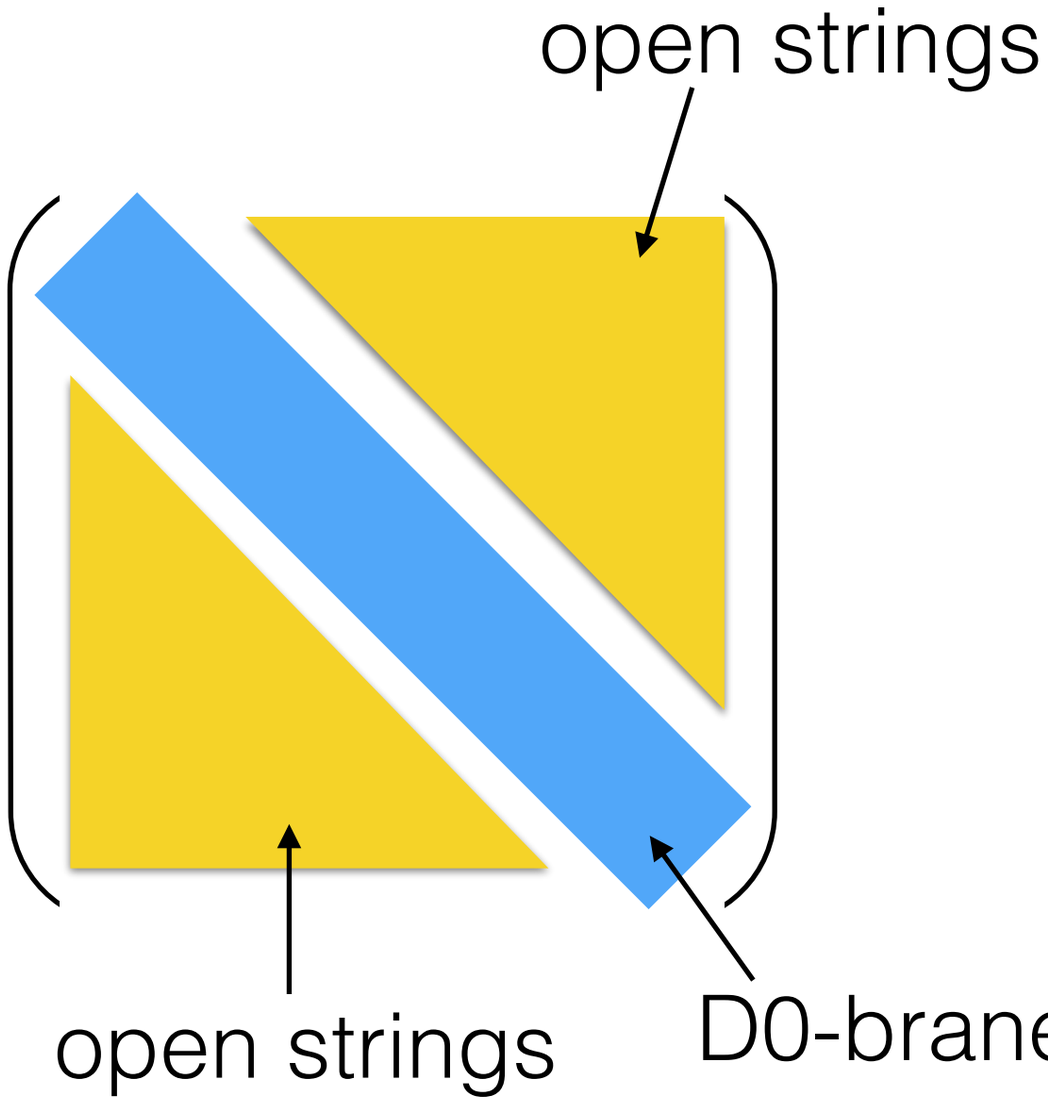
entropy ~ $(N-1)^2$

Emission rate ~ exp(–N)

※ This is different from the emission of massless particles, in the sense $m_{D0} \sim N$.

※ However the same mechanism would work for light particles at low-T, M-theory region, due to ‘quantum chaos’ + flat direction

$X_M =$

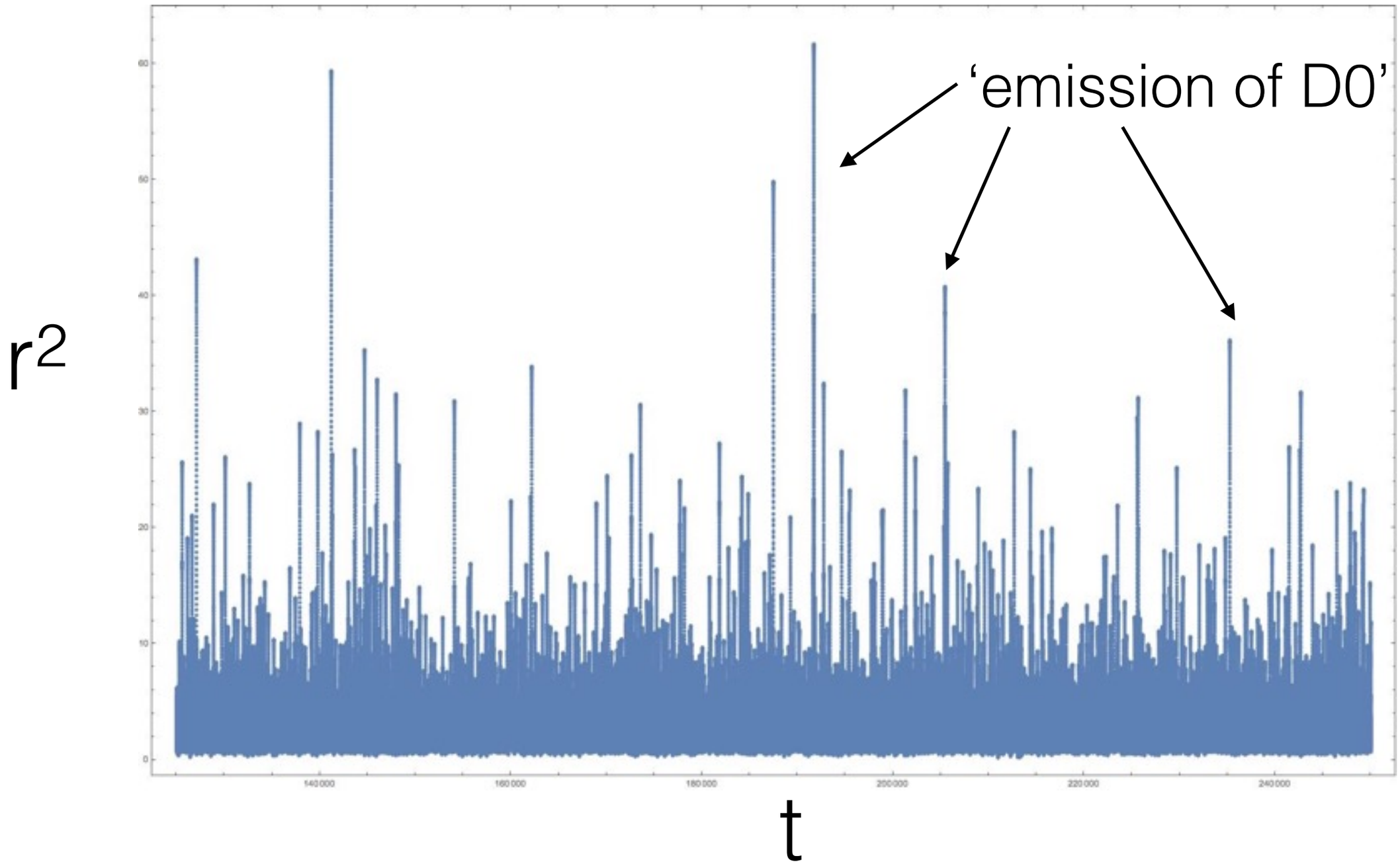


eigenvalue of $(X_M^2)_{ij}$
≒ radial coordinate of D0

the largest eigenvalue: r^2

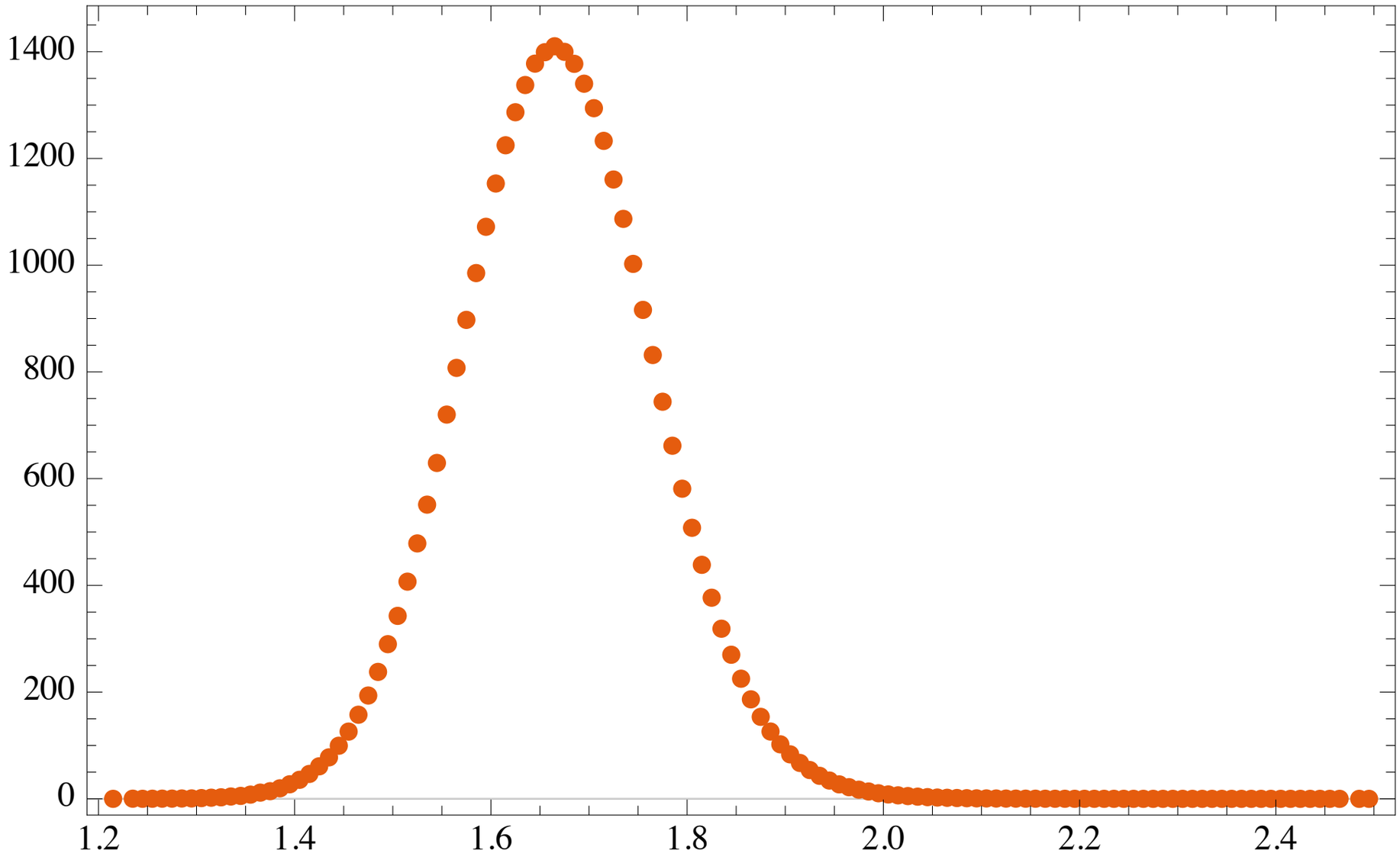
Distribution of r^2

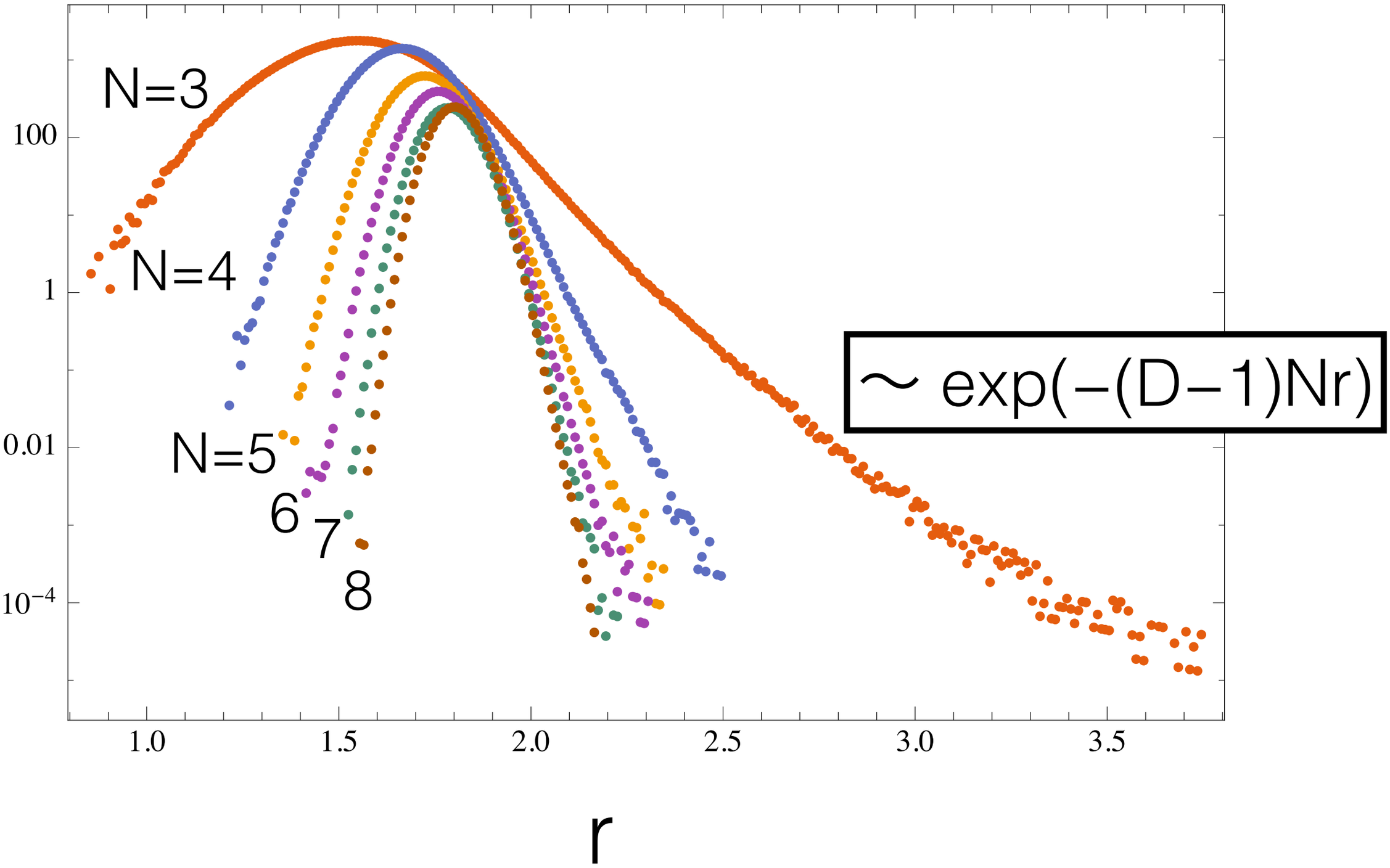
D=3 (3-matrix model), N=3



Rather unstable. BFSS (D=9) has the same instability.

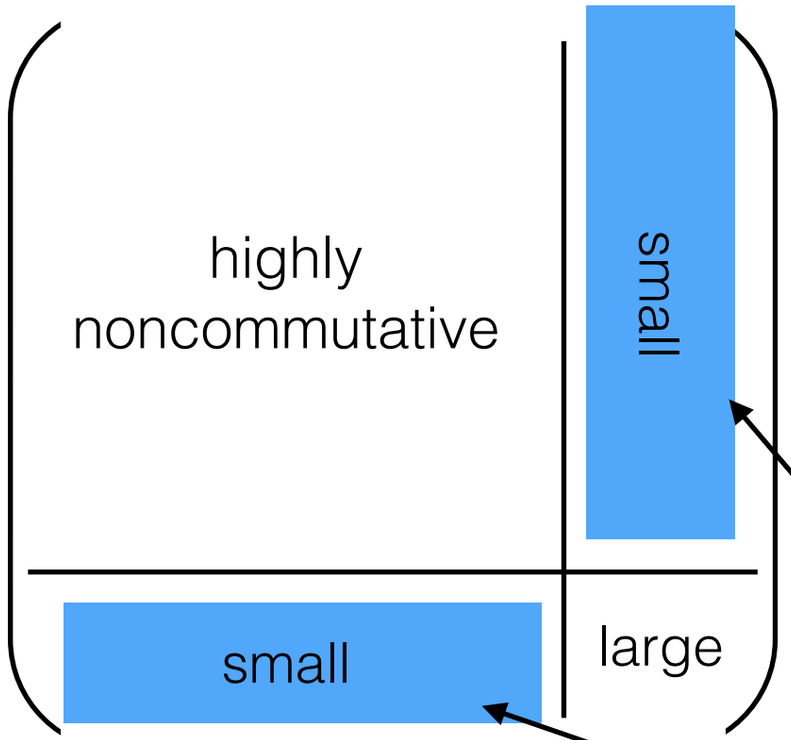
Histogram of r
D=9, N=4

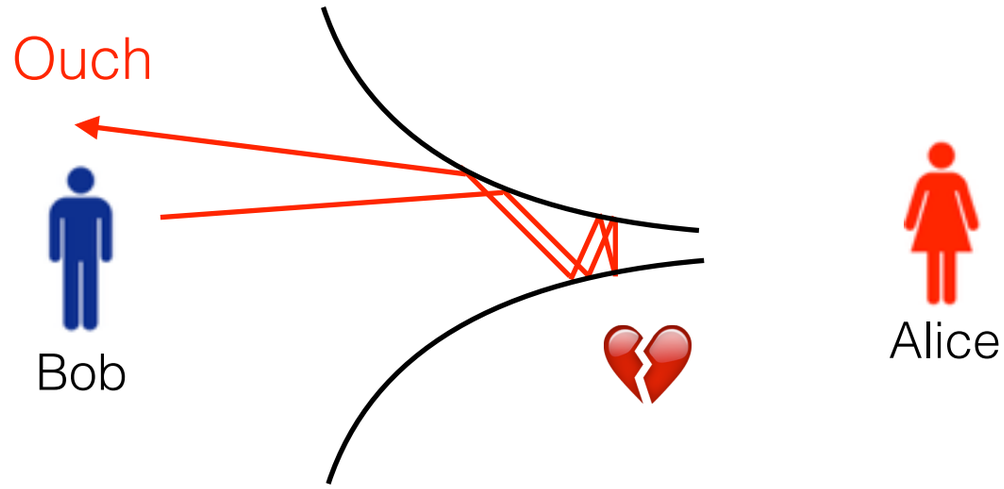



N=3
N=4
N=5
6
7
8
100
1
0.01
10^{-4}
1.0
1.5
2.0
2.5
3.0
3.5
r
∼ exp(−(D−1)Nr)

but flat direction is too narrow…

$X_M =$



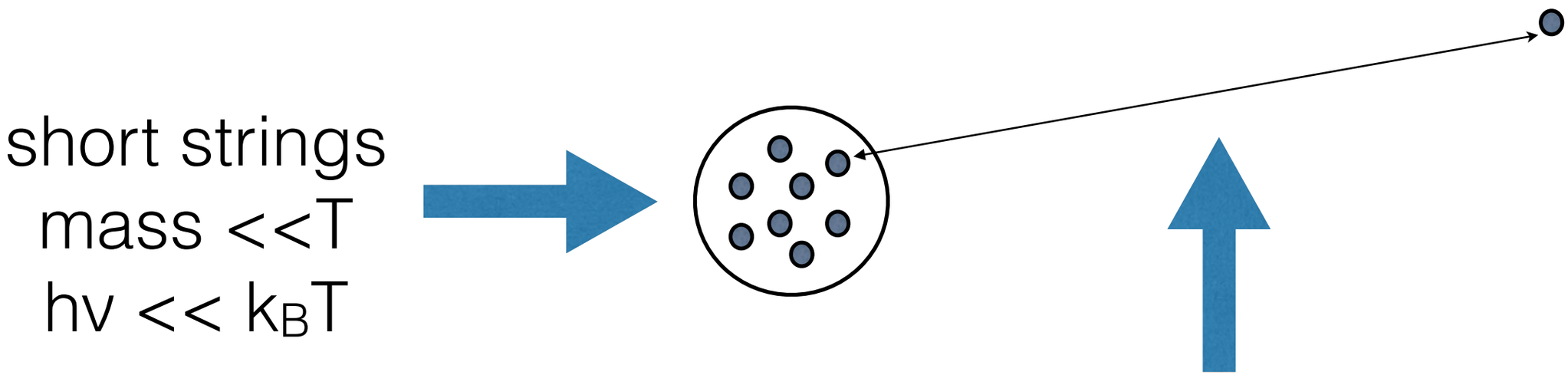


must be exactly zero
in order for D0 to
roll to infinity

This is not Bob's fault.

This is just an artifact of the classical treatment.

What was wrong?

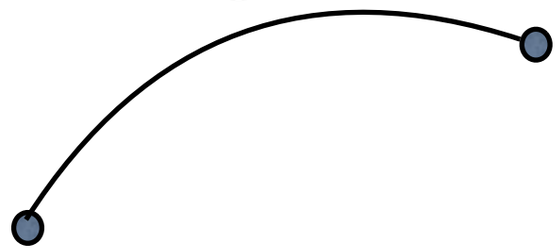


classical approximation is valid when hν << k_BT

classical approximation is valid when $h\nu \ll k_B T$

open string mass = hv of harmonic oscillator

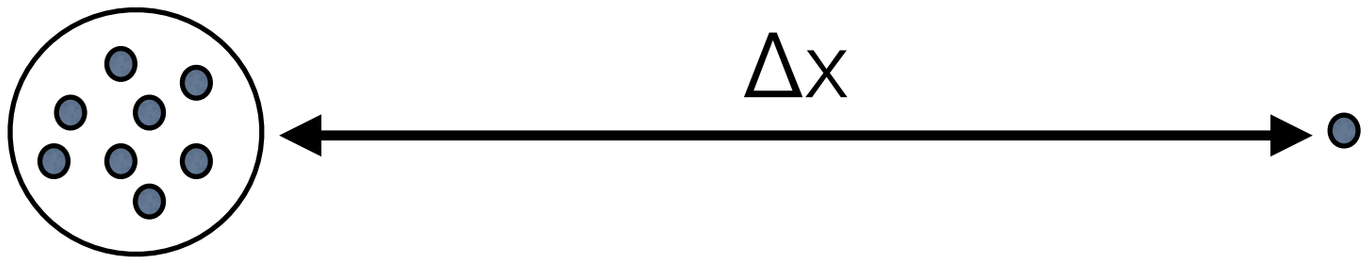
off diagonal element
= open string



diagonal element
= D0-brane

$$L = \frac{1}{2g_{YM}^2}\mathrm{Tr}\left(\sum_i (D_t X^i)^2 + \frac{1}{2}\sum_{i\neq j}[X_i, X_j]^2\right)$$

SUSY makes flat direction flatter



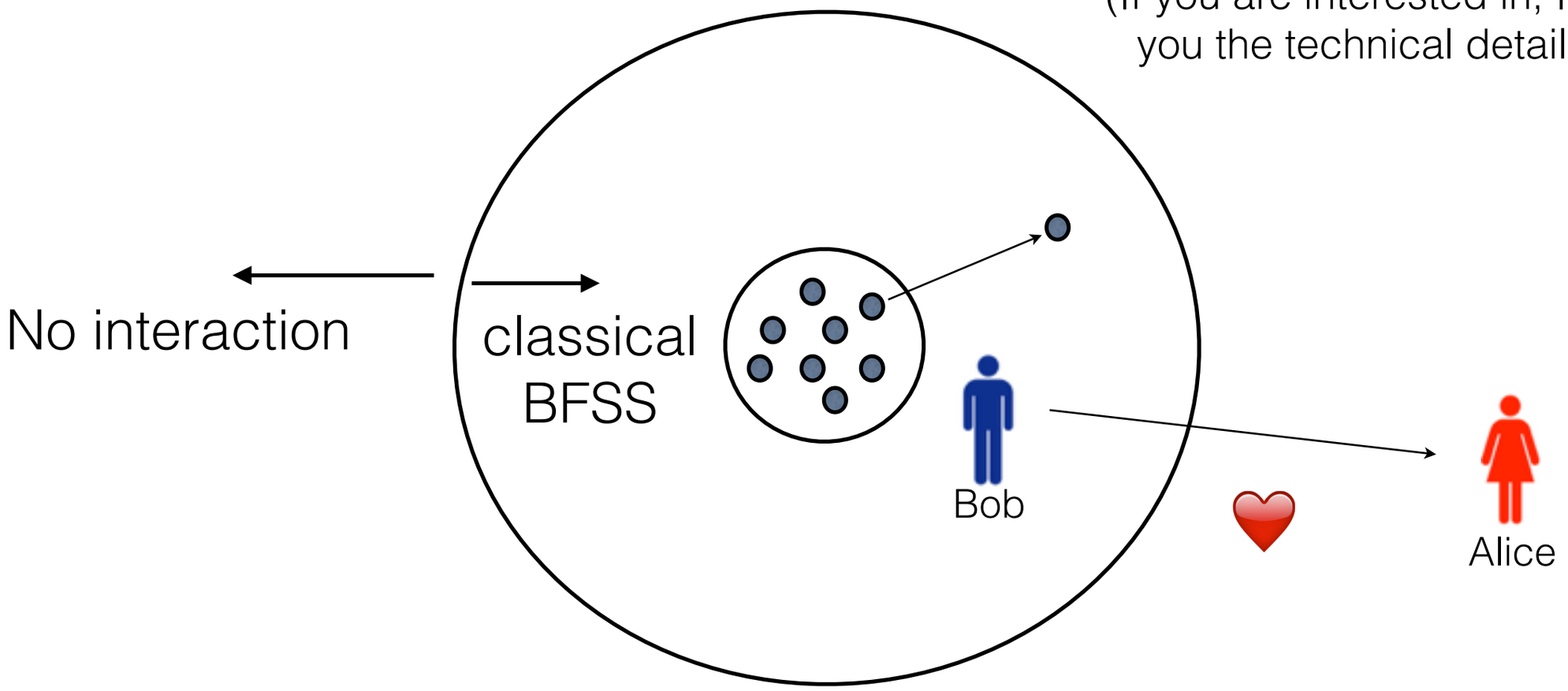
One-loop approximation
should be valid when Δx is large.

There, fermions are not negligible,
they cancel the attraction coming from bosons.

An effective model

- Turn-off the interaction (off-diagonal elements) once D0 goes beyond a threshold value.
- It can be done by keeping full SU(N) symmetry.

(If you are interested in, I can tell you the technical detail later.)



Classical time evolution mimics formation and evaporation of BH.

Future directions

- More on the thermalization & scrambling processes.
- What can we learn from 1/N corrections?
- Can we make a better effective theory? Learn from QGP industry? Determine the potential by Euclidean simulation?
- Can we somehow mimic emission of massless particles? (→ Information Puzzle)
- (1+1)-, (1+2)- and (1+3)-d YM; BH/BS topology change. (→ How stringy effects resolve the singularity)
- Full quantum simulation?
- Firewall? No Firewall?