

# Flavor Pheno $\leftrightarrow$ Lattice QCD

## auxiliary slides

Discussion based on 1204.4444, 1308.4379, 1506.06699

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# 1. Low recoil Region – power corrections

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In SM+SM' basis (V,A operators and flipped ones only) the effective Wilson coefficients  $C_{\pm}^{\text{eff}}(q^2) \equiv C^{\text{eff}}(q^2) \pm C^{\text{eff}'}(q^2)$  are independent of the polarization Bobeth,GH,van Dyk'12 (and as they should in agreement with endpoint relations GH,Zwicky14)

$$B \rightarrow V\ell\ell : H_{0,\parallel} = C_-^{\text{eff}}(q^2)f_{0,\parallel}(q^2), \quad H_{\perp} = C_+^{\text{eff}}(q^2)f_{\perp}(q^2),$$
$$B \rightarrow P\ell\ell : H = C_+^{\text{eff}}(q^2)f(q^2)$$

$f_i, i = 0, \perp, \parallel$  ( $f$ ): usual  $B \rightarrow V$  ( $B \rightarrow P$ ) form factors

Parameterize corrections to the lowest order OPE results as

$$f_{\lambda}(q^2) \rightarrow f_{\lambda}(q^2)(1 + \epsilon_{\lambda}(q^2)), \quad \epsilon_{\lambda}(q^2) = \mathcal{O}(\alpha_s/m_b, [\mathcal{C}_7/\mathcal{C}_9]/m_b) \quad \lambda = 0, \pm 1$$

The endpoint relations imply degeneracy at endpoint

$$\epsilon_{\lambda}(q_{\text{max}}^2) \equiv \epsilon, \quad \lambda = 0, \pm 1, \parallel, \perp \text{ with the endpoint relations already enforced by } f_{\parallel}(q_{\text{max}}^2) = \sqrt{2}f_0(q_{\text{max}}^2), f_{\perp}(q_{\text{max}}^2) = 0. \quad \rightarrow$$

# 1. Low recoil Region – power corrections

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”There are no genuine non-factorizable contributions ( $1/m_b$ , resonances,..) at zero recoil.” GH,Zwicky14

consider this in scans, uncertainty estimations.

## 2. Low recoil Region – universality

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Why is it short-distance universal?

$$B \rightarrow Vll : H_{0,\parallel} = C_-^{\text{eff}}(q^2) f_{0,\parallel}(q^2), \quad H_{\perp} = C_+^{\text{eff}}(q^2) f_{\perp}(q^2),$$

$$B \rightarrow Pl\ell : H = C_+^{\text{eff}}(q^2) f(q^2)$$

because the short-distance coefficients  $C_-^{\text{eff}}(q^2)$ ,  $C_+^{\text{eff}}(q^2)$  don't know about the endpoint.

Applications in many modes  $B \rightarrow X_J ll$ ,  $J = 0, 1, 2, \dots$

Universality in  $B \rightarrow K^* ll$  allow to extract form factor ratios (assuming no right-handed currents) Hambrock, GH '12, Hambrock, GH, Schacht, Zwicky13

# $B \rightarrow K^* \mu^+ \mu^-$ data progress 2012 to 2013

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2012:

|                           | BaBar                  | CDF                    |                         | LHCb                   |                         |
|---------------------------|------------------------|------------------------|-------------------------|------------------------|-------------------------|
| $q^2$ [GeV <sup>2</sup> ] | $F_L$                  | $F_L$                  | $A_T^{(2)}$             | $F_L$                  | $A_T^{(2)}$             |
| [14.18, 16]               | $0.43^{+0.13}_{-0.16}$ | $0.40^{+0.12}_{-0.12}$ | $0.11^{+0.65}_{-0.65}$  | $0.35^{+0.10}_{-0.06}$ | $0.06^{+0.24}_{-0.29}$  |
| [16, 19.xx]               | $0.55^{+0.15}_{-0.17}$ | $0.19^{+0.14}_{-0.13}$ | $-0.57^{+0.60}_{-0.57}$ | $0.37^{+0.07}_{-0.08}$ | $-0.75^{+0.35}_{-0.20}$ |

2013:

| $q^2$ | BaBar                  | CDF                    |                         | LHCb                   |                         |                         | ATLAS                  | CMS                    |
|-------|------------------------|------------------------|-------------------------|------------------------|-------------------------|-------------------------|------------------------|------------------------|
| $F_L$ | $F_L$                  | $A_T^{(2)}$            | $F_L$                   | $A_T^{(2)}$            | ${}^a P'_4$             | $F_L$                   | $F_L$                  |                        |
| bin1  | $0.43^{+0.13}_{-0.16}$ | $0.40^{+0.12}_{-0.12}$ | $0.11^{+0.65}_{-0.65}$  | $0.33^{+0.08}_{-0.08}$ | $0.07^{+0.26}_{-0.28}$  | $-0.18^{+0.54}_{-0.70}$ | $0.28^{+0.16}_{-0.16}$ | $0.53^{+0.12}_{-0.12}$ |
| bin2  | $0.55^{+0.15}_{-0.17}$ | $0.19^{+0.14}_{-0.13}$ | $-0.57^{+0.60}_{-0.57}$ | $0.38^{+0.09}_{-0.08}$ | $-0.71^{+0.36}_{-0.26}$ | $0.70^{+0.44}_{-0.52}$  | $0.35^{+0.08}_{-0.08}$ | $0.44^{+0.08}_{-0.08}$ |

in these observables, SD-coeffs and fact. stuff drops out!

At endpoint:  $F_L = 1/3, A_T^{(2)} = -1, P'_4 = \sqrt{2}$

# Benefits of $B \rightarrow K^*$ at low recoil

At low hadr. recoil transversity amplitudes  $A_i^{L,R}$ ,  $i = \perp, \parallel, 0$  related \*:

$$A_i^{L,R} \propto C^{L,R} \cdot f_i$$

$C^{L,R}$ : universal short-dist.-physics;  $C^{L,R} = (C_9^{\text{eff}} \mp C_{10}) + \kappa \frac{2\hat{m}_b}{\hat{s}} C_7^{\text{eff}}$

$1/m_b$ - corrections parametrically suppressed  $\sim \alpha_s/m_b, C_7/(C_9 m_b)$

$f_i$ : form factors

$C^{L,R}$  drops out in ratios:

$$F_L = \frac{|A_0^L|^2 + |A_0^R|^2}{\sum_{X=L,R} (|A_0^X|^2 + |A_\perp^X|^2 + |A_\parallel^X|^2)} = \frac{f_0^2}{f_\perp^2 + f_\parallel^2 + f_0^2}$$

$$A_T^{(2)} = \frac{|A_\perp^L|^2 + |A_\perp^R|^2 - |A_\parallel^L|^2 - |A_\parallel^R|^2}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}$$

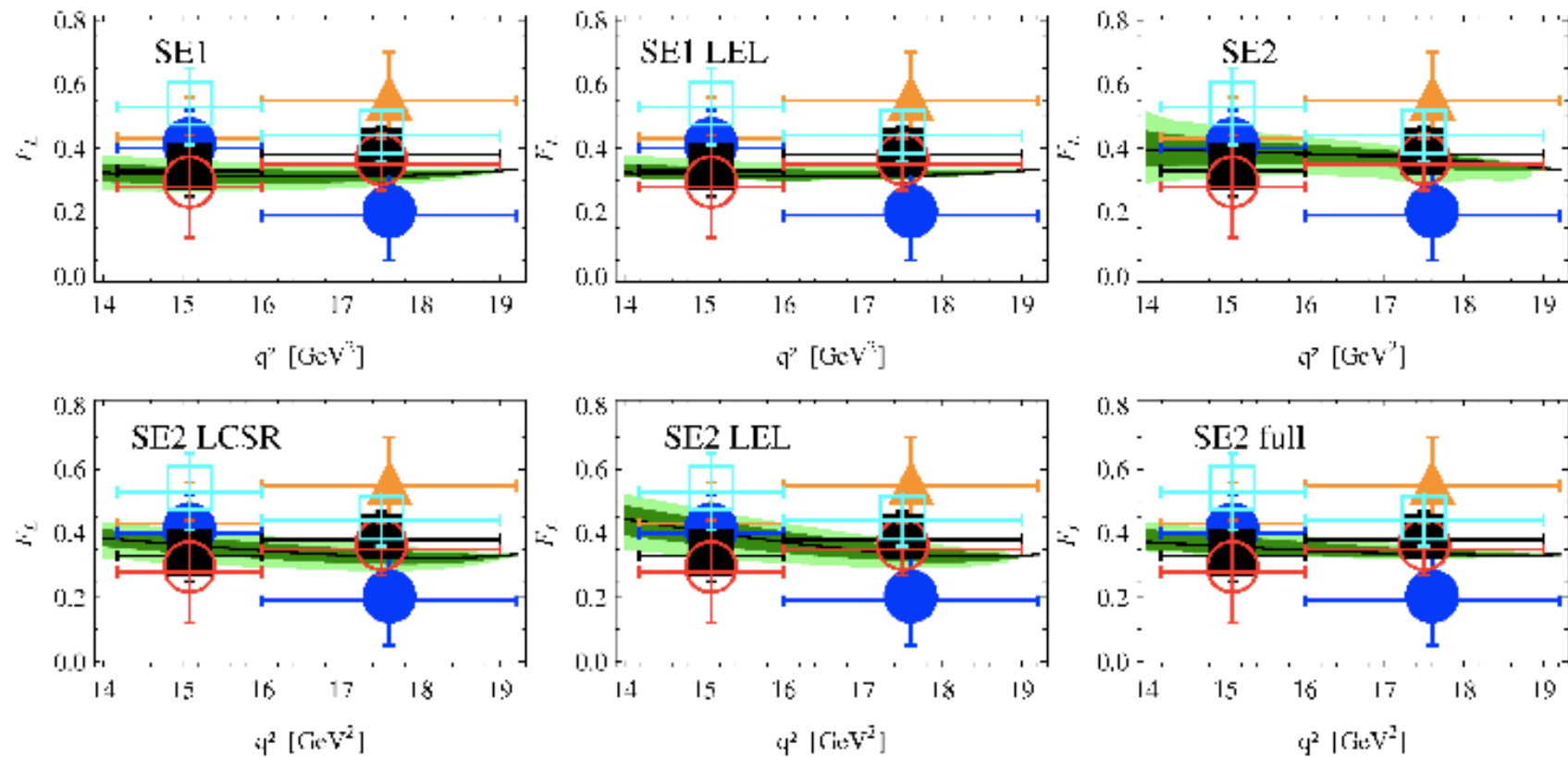
$$P_4'(q^2) = \frac{\sqrt{2} f_\parallel(q^2)}{\sqrt{f_\parallel^2(q^2) + f_\perp^2(q^2)}}$$

\* assuming only V-A operators

# Advances in ... Extracting $B \rightarrow K^*$ form factors

Higher order Series Expansion; use theory input from low  $q^2$ : LCSR (sum rules) or  $V(0)/A_1(0) = (m_B + m_{K^*})^2 / (2m_B E_{K^*}) + \mathcal{O}(1/m_b) = 1.33 \pm 0.4$  (LEL)

$F_L$ :

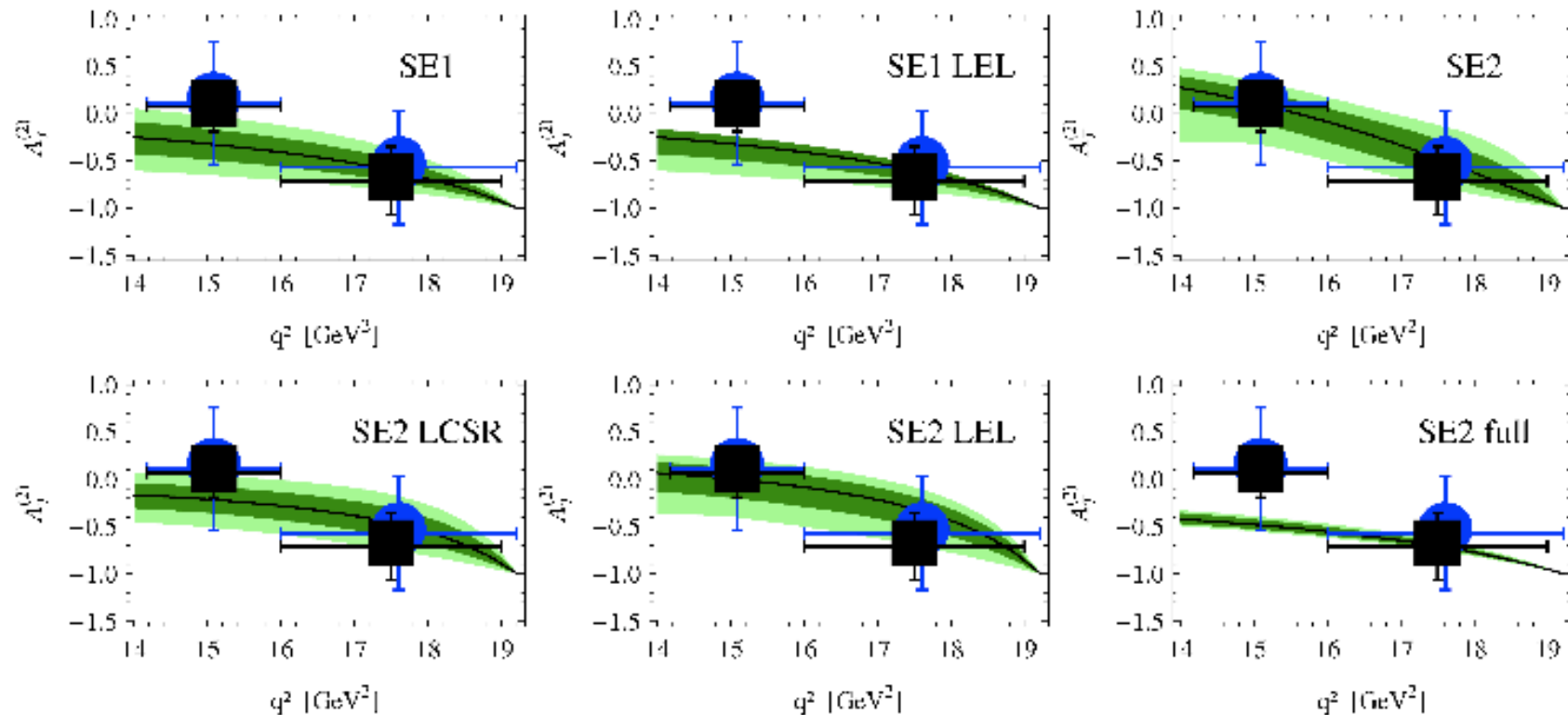


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$A_T^{(2)}$ :

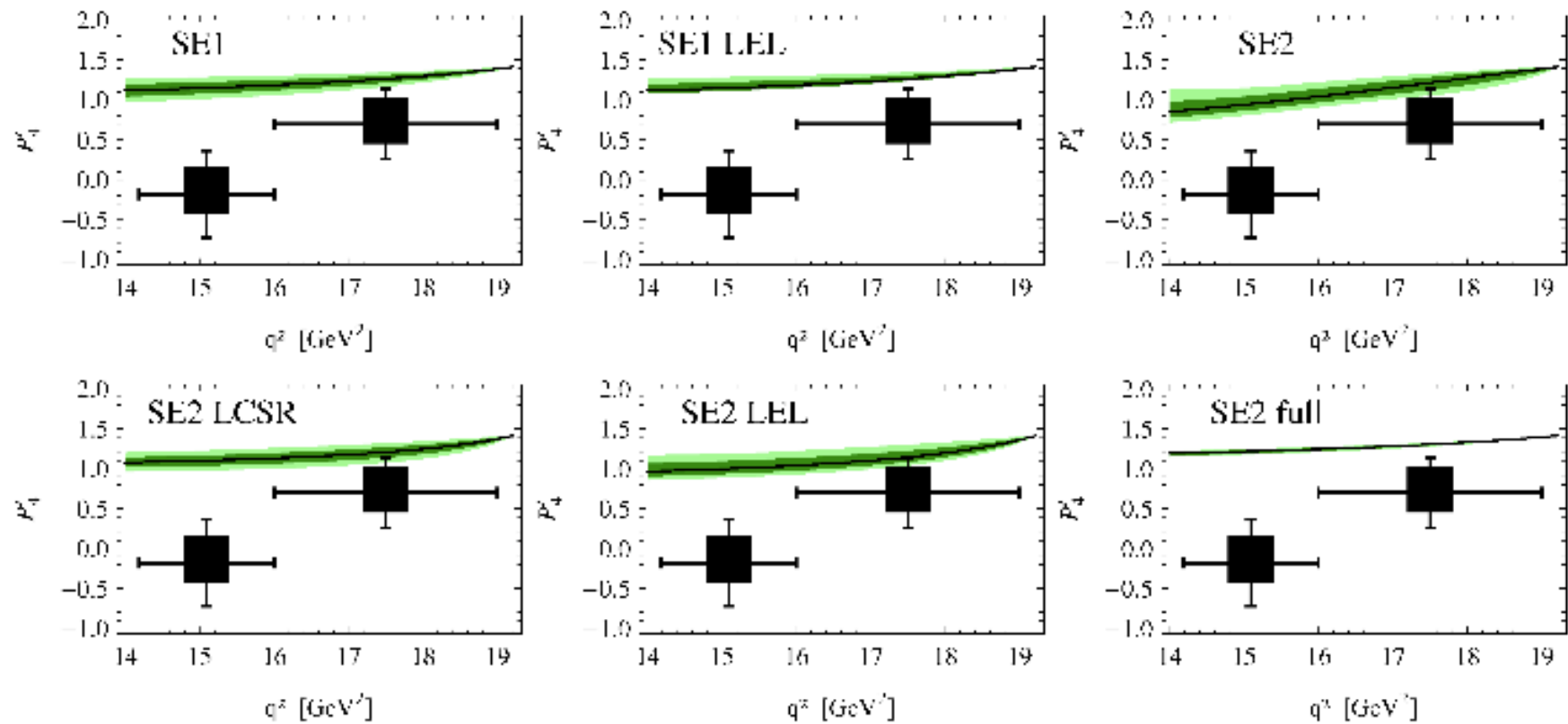




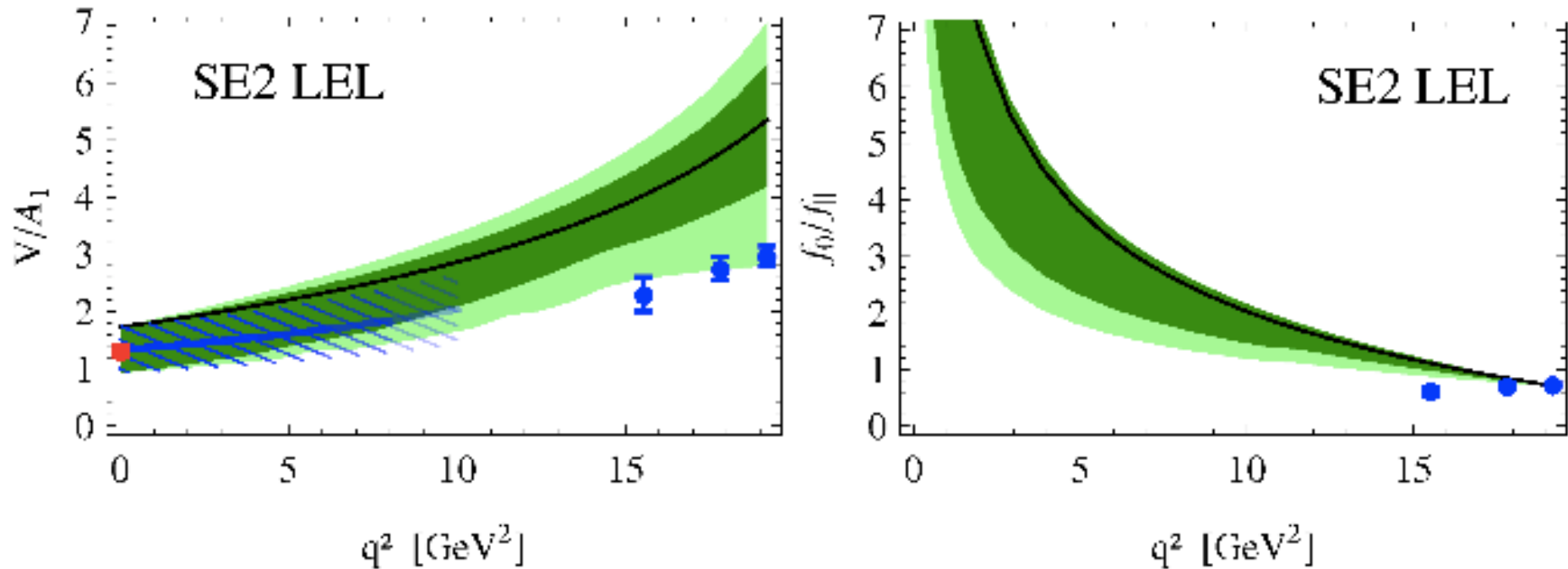
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$P'_4$ :



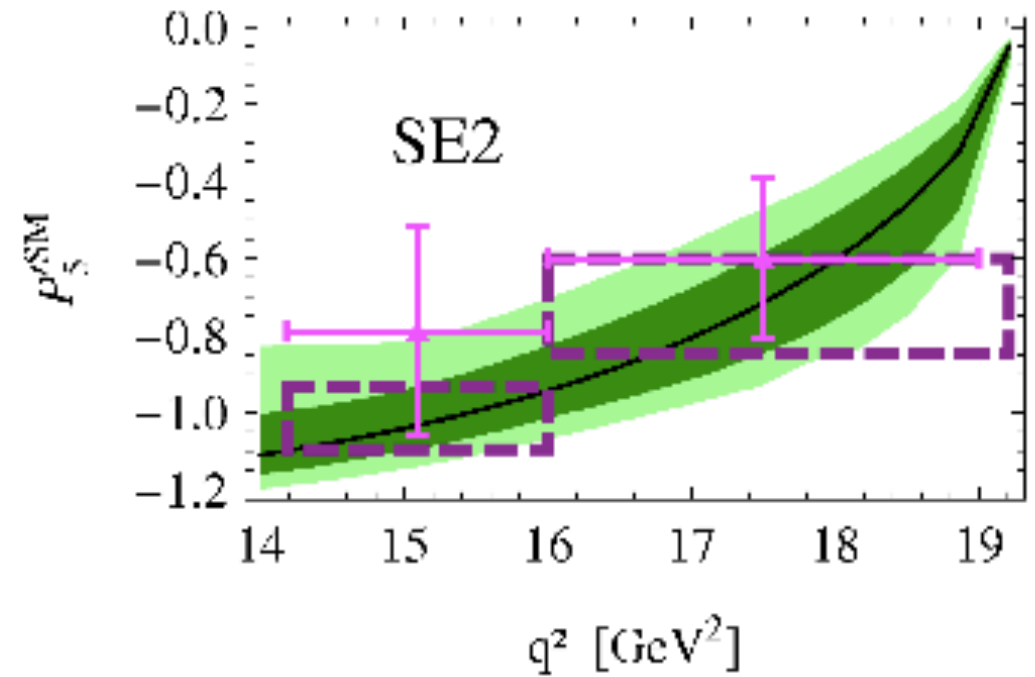
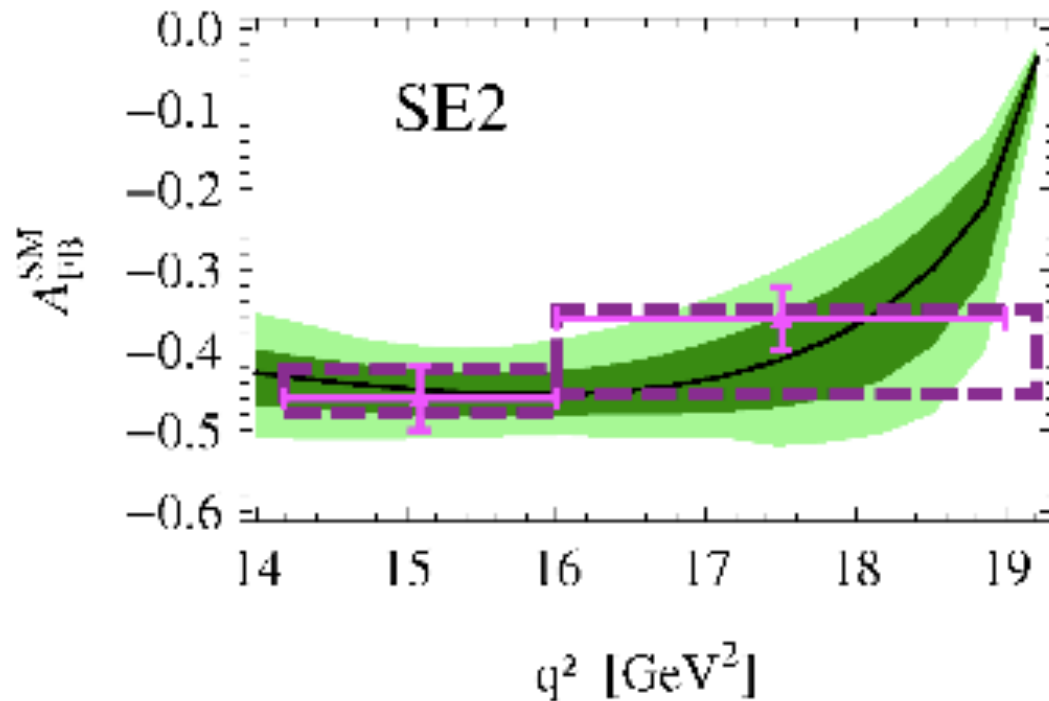
# Advances in ... Extracting $B \rightarrow K^*$ form factors



Predictivity at low  $q^2$  is obtained from low  $q^2$  input. (Required at higher order)

Data-extracted form factor ratios constitute benchmark for lattice form factor estimations at low recoil. Blue points: Wingate '13 et al, red: LCSR, band:LEL

# Advances in ... Extracting $B \rightarrow K^*$ form factors



SM predictions for  $A_{FB}$  and  $P_5'$  at low recoil (assuming  $V - A$  currents). Good agreement with data in fits in both low recoil bins.

$P_4'$  escapes explanation within factorization [Altmannshofer, Straub '13, Hambrock, GH,](#)

[Schacht, Zwicky '13, Beaujean, Bobeth, vanDyk '13, Descotes-Genon, Matias, Virto '13](#)

Yes, we would like to have correlations between them.

At least, please provide ratios, we use them.

LCSR example:

$\delta V(0)/A_1(0) = 15\%$  (gaussian error prop. of Ball,Zwicky)

$\delta V(0)/A_1(0) = 8\%$  including error correlations a la Hambrock,GH,  
Schacht Zwicky '13 (parametric, continuum threshold and EOM)

# Status Low recoil $B \rightarrow K^*(\rightarrow K\pi)\mu\mu$ –largest bin

| observable      | LHCb[15,19] <sup>a,b</sup> | SM[15,19] <sup>d</sup>               |
|-----------------|----------------------------|--------------------------------------|
| $F_L$           | $0.344 \pm 0.031$          | $0.351(0.342) \pm 0.010 \pm 0.003$   |
| $A_{\text{FB}}$ | $-0.355 \pm 0.029$         | $-0.391(-0.396) \pm 0.016 \pm 0.005$ |
| $S_3$           | $-0.122 \pm 0.026$         | $-0.129(-0.131) \pm 0.009 \pm 0.007$ |
| $S_4$           | $0.214 \pm 0.029$          | $0.215(0.218) \pm 0.005 \pm 0.002$   |
| $S_5$           | $-0.244 \pm 0.029$         | $-0.230(-0.233) \pm 0.009 \pm 0.006$ |

<sup>a</sup>Uncertainties added in quadrature and symmetrized. <sup>b</sup>Values adopted to common theory definitions.

LHCb ( $3 \text{ fb}^{-1}$ ): LHCb-CONF-2015-002, CERN-LHCb-CONF-2015-002

<sup>d</sup>OPE with  $K\pi$  background; central values in parenthesis S-wave subtracted; second uncertainty due to interference (unknown strong phase) 1506.06699

# Status Low recoil $B \rightarrow K^*(\rightarrow K\pi)\mu\mu$ –endpoint bin

| observable      | LHCb[17, 19] <sup>a,b</sup> | SM[17, 19] <sup>d</sup>              | endpoint |
|-----------------|-----------------------------|--------------------------------------|----------|
| $F_L$           | $0.354 \pm 0.054$           | $0.338(0.333) \pm 0.006 \pm 0.002$   | $1/3$    |
| $A_{\text{FB}}$ | $-0.306 \pm 0.049$          | $-0.349(-0.351) \pm 0.015 \pm 0.007$ | $0^c$    |
| $S_3$           | $-0.145 \pm 0.062$          | $-0.167(-0.169) \pm 0.007 \pm 0.005$ | $-1/4$   |
| $S_4$           | $0.202 \pm 0.052$           | $0.226(0.227) \pm 0.003 \pm 0.002$   | $+1/4$   |
| $S_5$           | $-0.245 \pm 0.050$          | $-0.191(-0.193) \pm 0.008 \pm 0.006$ | $0^c$    |

<sup>a</sup>Uncertainties added in quadrature and symmetrized. <sup>b</sup>Values adopted to common theory definitions.

LHCb ( $3 \text{ fb}^{-1}$ ): LHCb-CONF-2015-002, CERN-LHCb-CONF-2015-002

<sup>d</sup>OPE with  $K\pi$  background; central values in parenthesis S-wave subtracted; second uncertainty due to interference (unknown strong phase)  $1506.06699$  <sup>c</sup>goes to zero with non-negligible slope