NUCLEON STRUCTURE ON THE LATTICE: CONNECTION TO NEW PHYSICS?

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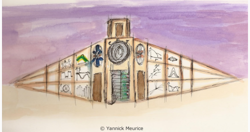


- I am very grateful for the opportunity to be here, thanks to the KITP program organizers and coordinators.

- The program is **Lattice Gauge Theory for the LHC and Beyond**.
- Looking at the Wikispace pages, I saw very few talks from the *Nuclear Physics* perspective.
- I will try to provide that perspective.

- An overview will be given instead of specific calculations.

THE REAL TALK

INTRODUCTION

- The internal structure of the nucleon, such as the spin, charge and current distributions of the nucleon, gives us a (more) detailed view of the internal landscape of the nucleon.

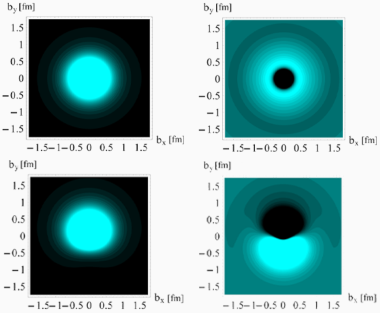


Figure 1: Quark transverse charge densities in the proton (left) and neutron (right). Top: Unpolarized. Bottom: Polarized. Images from: `http://wwwth.kph.uni-mainz.de/556.php`

- Lattice calculations of the nucleon structure are usually done through the nucleon matrix elements $\langle N(P')|O(x)|N(P)\rangle$.
- which can be extracted from the ratios between nucleon three-point and two-point functions.

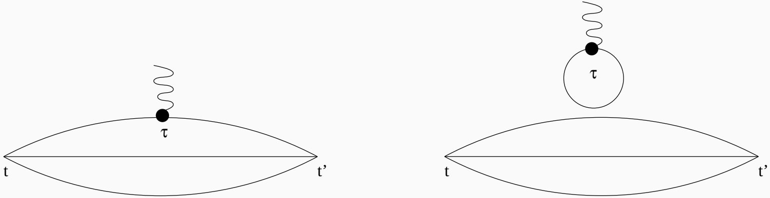


Figure 2: Nucleon three-point functions. Connected (left) vs. disconnected (right).

- Disconnected diagrams need to include

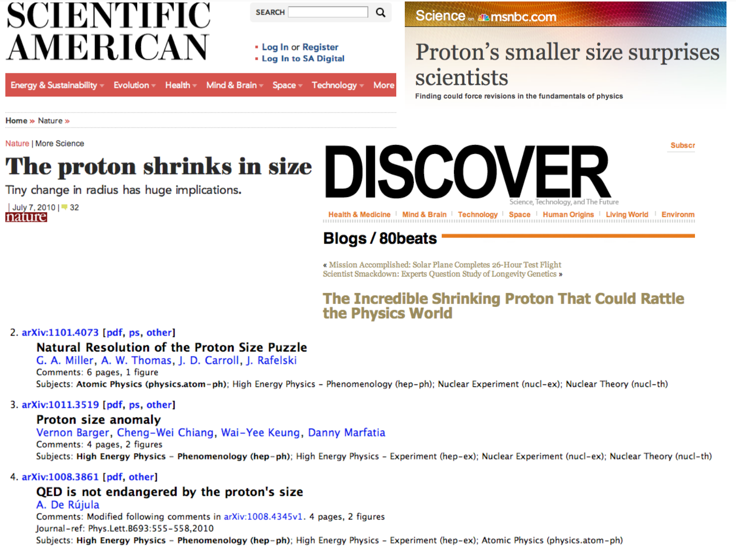
$$C_1(\tau) = \sum_{\vec{x}} Tr[\Gamma\, D^{-1}(\vec{x},\tau)] \tag{1}$$

  Number of matrix inversions $\propto L^3 \times T \Rightarrow$ Computation-intensive!
- We often calculate the *isovector* $(p-n)$ quantities in which the disconnected diagrams cancel out.

Lattice calculations of nucleon structure can have connections to new physics in three ways:

- Decipher theory-experiment or experiment-experiment discrepancies, e.g. proton size puzzle.
- Provide inputs to BSM phenomenology, e.g. nucleon strangeness content, scalar and tensor charges, ...
- Direct calculations of BSM nucleon matrix elements, e.g. proton decay.

THE PROTON SIZE PUZZLE



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The proton shrinks in size

Tiny change in radius has huge implications.

July 7, 2010 | 32

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Science on msnbc.com

Proton's smaller size surprises scientists

Finding could force revisions in the fundamentals of physics

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« Mission Accomplished: Solar Plane Completes 26-Hour Test Flight
Scientist Smackdown: Experts Question Study of Longevity Genetics »

The Incredible Shrinking Proton That Could Rattle the Physics World

2. arXiv:1101.4073 [pdf, ps, other]
Natural Resolution of the Proton Size Puzzle
G. A. Miller, A. W. Thomas, J. D. Carroll, J. Rafelski
Comments: 6 pages, 1 figure
Subjects: **Atomic Physics (physics.atom-ph)**; High Energy Physics - Phenomenology (hep-ph); Nuclear Experiment (nucl-ex); Nuclear Theory (nucl-th)

3. arXiv:1011.3519 [pdf, ps, other]
Proton size anomaly
Vernon Barger, Cheng-Wei Chiang, Wai-Yee Keung, Danny Marfatia
Comments: 4 pages, 2 figures
Subjects: **High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Experiment (hep-ex); Nuclear Experiment (nucl-ex); Nuclear Theory (nucl-th)

4. arXiv:1008.3861 [pdf, other]
QED is not endangered by the proton's size
A. De Rújula
Comments: Modified following comments in arXiv:1008.4345v1. 4 pages, 2 figures
Journal-ref: Phys.Lett.B693:555-558,2010
Subjects: **High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Experiment (hep-ex); Atomic Physics (physics.atom-ph)

Figure 3: Snapshots in 2011.

Possible explanations:

- New Physics:
  New particles that interact with only muons but not electrons.
  [Karshenboim, McKeen & Pospelov, 2014]
  [Carlson & Rislow, 2012] [Batell, McKeen & Pospelov, 2011]
- New interpretation of the e-p data: [Lorenz, Hammer & Meisner 2012]
  They reanalyzed the Mainz e-p scattering data using a dispersive framework, and found a much smaller proton charge radius.
  0.84(1) fm [reanalysis of e-p] vs 0.84087(39) fm [muonic hydrogen]
- Errors are underestimated? [Paz, 2012]

What's the QCD contribution? It's a non-perturbative question.
$\Rightarrow$ Enters lattice QCD.

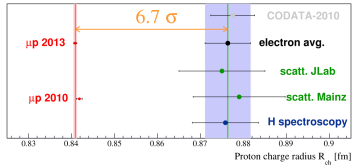
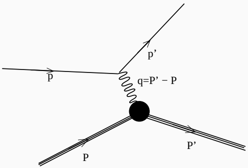


Figure 4: The proton charge radius discrepancies in experiments. [Arlington 2015]

The proton charge radius can be determined from the proton electromagentic form factors, defined from the matrix element:

$$\langle \mathcal{N}(P')|J^{\mu}_{EM}(x)|\mathcal{N}(P)\rangle = e^{i(P'-P)\cdot x}\bar{u}(P')\left[\gamma^{\mu}F_1(Q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2M_N}F_2(Q^2)\right]u(P)$$

- $q = P' - P$, $Q^2 = -q^2$.
- $F_1(Q^2), F_2(Q^2)$: Dirac and Pauli form factors.



- Under one-photon exchange approximation, the elastic $e-p$ scattering cross section

$$\frac{d\sigma}{d\Omega} \propto F_1^2(Q^2) + \frac{Q^2}{4M^2}\left[F_2^2(Q^2) + 2(F_1(Q^2) + F_2(Q^2))\tan^2\frac{\theta}{2}\right]$$

- Experiments use Rosenbluth separation method:
  at fixed Q^2, from $\frac{d\sigma}{d\Omega}$ vs. $\tan^2\frac{\theta}{2}$ obtain $F_1(Q^2)$ and $F_2(Q^2)$.

- Sachs electric and magnetic form factors are commonly used:

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

- The proton charge radius is defined as $r_p = \sqrt{\langle r_E^2 \rangle}$, where

$$\langle r_E^2 \rangle = -6 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2 \to 0}$$

- Determination of $\langle r_E^2 \rangle$ may be model dependent (derivative of an unknown function).
- Data of $G_E(Q^2)$ typically fit well with a dipole ansatz.

$$G_E(Q^2) = \frac{1}{(1 + Q^2/M^2)^2},$$

and

$$\langle r_E^2 \rangle = 12/M^2.$$

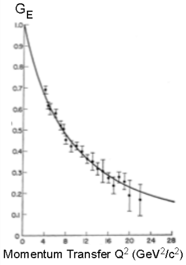


Figure 5: [Janssens et. al. 1966]

- On the lattice we compute the appropriate two- and three-point correlation functions in order to extract $\langle N, \vec{p}', s' | O | N, \vec{p}, s\rangle$.
- Nucleon two-point functions:

$$C^{2\text{pt}}_{\beta\alpha}[(t', \vec{p}'); (t, \vec{x})] = \sum_{\vec{x}'} e^{-i\vec{p}'\cdot\vec{x}'} \langle N_\beta(t', \vec{x}')\overline{N}_\alpha(t, \vec{x})\rangle$$

$$(t' \gg t) \quad \to \quad \frac{1}{2E_{\vec{p}'}} e^{-i\vec{p}'\cdot\vec{x}} e^{-E_{\vec{p}'}(t'-t)} \langle 0|N_\beta|N, \vec{p}', s\rangle\langle N, \vec{p}', s|\overline{N}_\alpha|0\rangle$$

- Nucleon three-point functions:

$$C^{3\text{pt}}_{\beta\alpha}[(t', \vec{p}'); (\tau, \vec{q}); (t, \vec{x})] = \sum_{\vec{x}'} e^{-i\vec{p}'\cdot\vec{x}'} \sum_{\vec{y}} e^{i\vec{q}\cdot\vec{y}} \langle N_\beta(t', \vec{x}') O(\tau, \vec{y}) \overline{N}_\alpha(t, \vec{x})\rangle$$

$$(t' \gg \tau \gg t) \quad \to \quad \frac{1}{2E_{\vec{p}'} 2E_{\vec{p}}} e^{-E_{\vec{p}'}(t'-\tau)} e^{-E_{\vec{p}}(\tau-t)} e^{-i\vec{p}\cdot\vec{x}}$$

$$\times \quad \langle 0|N_\beta|N, \vec{p}', s'\rangle\langle N, \vec{p}, s|\overline{N}_\alpha|0\rangle \langle N, \vec{p}', s'|O|N, \vec{p}, s\rangle$$

LATTICE METHODS (II)

- Proper 3pt-to-2pt ratios give the nucleon MEs.

$$R_{\mathcal{O}}(\tau, p', p) = \frac{C^{3\text{pt}}_{\mathcal{O}}(\tau, p', p)}{C^{2\text{pt}}(t', p')} \left[\frac{C^{2\text{pt}}(t' - \tau + t, p)\ C^{2\text{pt}}(\tau, p')\ C^{2\text{pt}}(t', p')}{C^{2\text{pt}}(t' - \tau + t, p')\ C^{2\text{pt}}(\tau, p)\ C^{2\text{pt}}(t', p)} \right]^{1/2}$$

$$\to R_0 \left[1 + a e^{-(t'-\tau)\epsilon(p')} + b e^{-(\tau - t)\epsilon(p)} + \ldots \right]$$

- If excited-state contributions are small, this ratio should show a plateau.

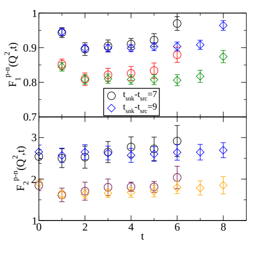
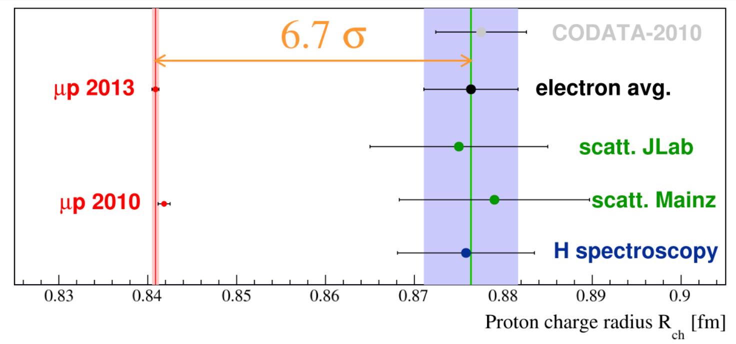


Figure 6: Example plateaus for the isovector Dirac and Pauli form factors. [ML 2014]

- A constant fit to the plateau region can then be performed to determine the relevant matrix element.



- The discrepancy between ep and μp results is 4%.
- To see a 2σ effect, we need 1% to 2% combined error (statistical + systematic) for r_p, or 2-4% combined error for r_p^2.
- $\rightarrow$ This is not trivial for lattice.

- Different lattice calculations have good agreement. (Good)
- As m_π in simulations gets smaller, the values go up. (Good)
- Uncertainties are generally still fairly large. (Bad)
- The results are systematically below the experimental values, even at or near the physical pion mass. (Bad)

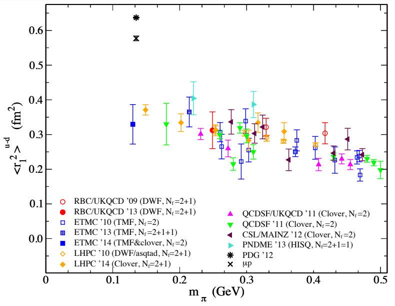


Figure 7: (Incomplete) Lattice results for isovector Dirac radii. [Constantinou, Lattice 2014]

Possible systematic errors:

- Excited-state contaminations.
- Dipole fits problematic.
- Chiral extrapolations.
- Finite lattice spacing and finite volume?

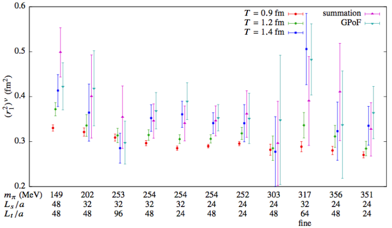


Figure 8: [Green et al. 2014]

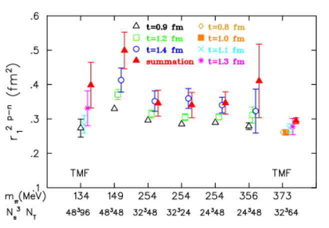


Figure 9: [Abdel-Rehim et al. 2014]

- Several groups have extensively studied the source-sink separation dependence of the charge radii, to investigate possible excited-state contaminations.
- Having small short-sink separations tend to give smaller values for the radii.
- The effects are bigger at smaller pion masses.

Note:

- Summation method:

$$\sum_{\tau=t}^{t'} R_O(\tau, p', p) \to const. + R_0(t' - t) + \mathcal{O}(e^{-(t'-t)\epsilon(p')}) + \mathcal{O}(e^{-(t'-t)\epsilon(p)})$$

$\to$ use multiple source-sink separations $(t' - t)$ to determine the slope of the summed ratio.

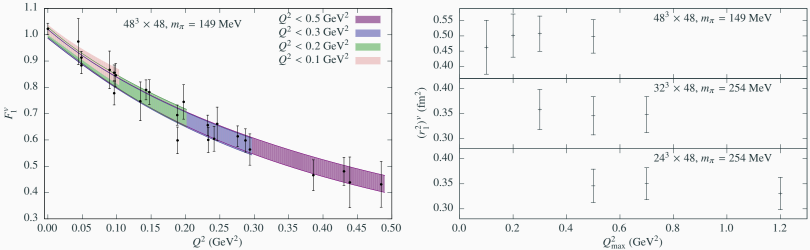


Figure 10: The dependence of the isovector Dirac radii on Q^2_{max} in the dipole fits. [Green et al. 2014]

- While the available lattice data do not indicate much dependence on the range of the dipole fits, it is preferable to use small Q^2 data to reduce the model dependence.
- When Q^2 is sufficiently small, simple linear fits will work just as well.
- Ways to calculate r_p directly?

- Usually we extract the radii from the form factors first, and then perform chiral extrapolations for r^2.
- You can also perform chiral extrapolations w.r.t. both Q^2 and m_π dependence.
- Caveats:
    - several variants of HBChPT.
    - Convergence not well understood.
    - Many LECs. Need to fix some of them using phenomenological inputs.

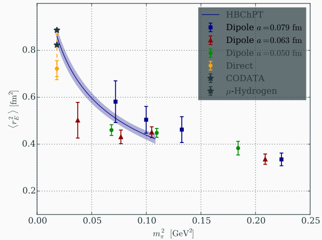


Figure 11: [Capitani et al., 2015]

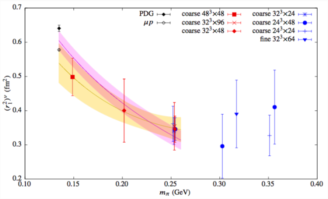
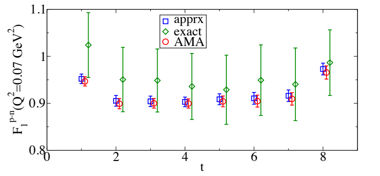


Figure 12: [Green et al. 2014]

- Nucleon observables are notoriously hard to calculate as the signal-to-noise deceases exponentially with the pion mass.

$$S/N(t) \propto \sqrt{N_{\text{meas}}} \exp\left[-(M_N - \frac{3}{2}M_\pi)t\right],$$

- The error-reduction technique, All-Mode-Averaging [Blum, Izubuchi, Shintani 2013], makes it cheaper to increase N_{meas}.
- The idea is to construct an "improved" operator $O_{\text{imp}} = O_{\text{approx}} + O_{\text{rest}}$ with O_{approx} cheaper to calculate but less precise.
- O_{rest} can be estimated with fewer, but precise, calculations of the exact (original) operator O_{exact}.
- O_{approx}: deflation + sloppy CG.



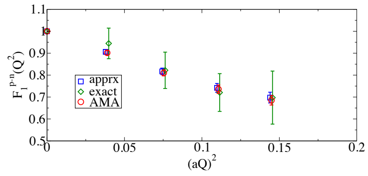


Figure 13: Error reduction for the form factors with AMA. [ML 2014]

- Yes we can calculate the proton charge radius on the lattice.
- However, challenges still remain:
  - Need to reduce statistical errors.
  - Need to thoroughly investigate systematic errors.
  - Disconnected diagrams! More studies are coming in. All suggest contributions are small for the nucleon electric form factors.

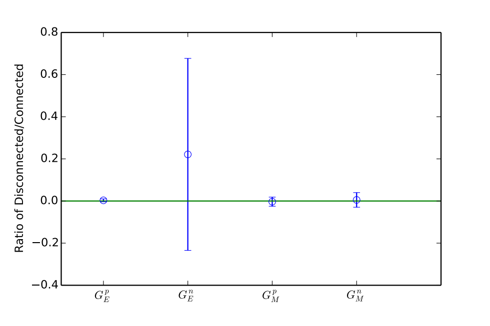


Figure 14: [A. Abdel-Rehim et al. 2014]

NUCLEON CHARGES

- The nucleon light and strange quark σ terms

$$\sigma_l = m_l \langle N|\bar{u}u + \bar{d}d|N\rangle$$
$$\sigma_s = m_s \langle N|\bar{s}s|N\rangle$$

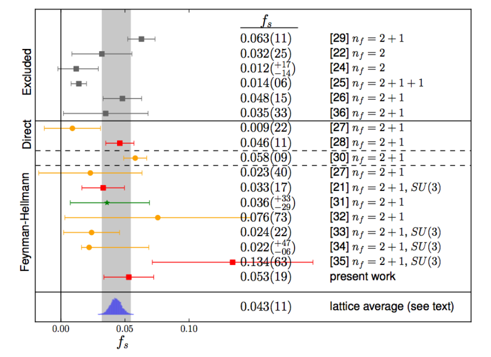
- Enters spin-independent WIMP-nucleon scattering cross section in the form of of $f_q = \sigma_q / M_N$. [Ellis, Olive & Savage 2008].
- Can be determined in two approaches:
- *Direct approach*: uses the same method as the form factor calculations. You calculate the three-point functions of the nucleon scalar density and determine the scalar matrix element.
  - Expensive, as disconnected diagrams cannot be ignored.
- *Spectrum approach*: uses Hellmann-Feynman theorem

$$m_q \langle N|\bar{q}q|N\rangle = m_q \frac{\partial M_N}{\partial m_q}$$

  - Need to take numerical derivative. Multiple quark masses are needed.

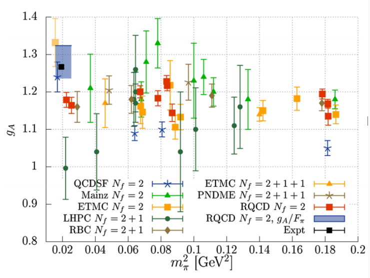
The strangeness of the nucleon is particularly hard to calculate on the lattice.

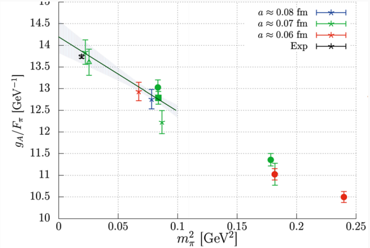
- The direct approach has only the disconnected contributions. $\rightarrow$ Very noisy!
- The spectrum approach requires multiple strange quark masses. Partial quenching doesn't work (no valence strange quark in nucleon) $\rightarrow$ Either need new dynamical ensembles or reweighting in m_s.



[Junnarkar & Walker-Loud 2013]

- g_A is known very precisely experimentally. $g_{A,\mathrm{exp}} = 1.2701(25)$
- In principle a clean quantity to calculate on the lattice (zero recoil, no disconnected diagrams, relatively good signal).
- The ability to reproduce the experimental value serves as a precision test.
- However, lattice values have been systematically 10-20% lower.
- It may be particularly susceptible to finite volume effects.





[Bali et al. 2014]

CONCLUSIONS

CONCLUSIONS

- Lattice nucleon structure calculations are very relevant to the search for new physics.
- However, many challenges remain for the lattice to get to the precision level required.
- A lot of progress has been made in the past few years.
- With improved algorithms and more powerful computers, next few years will be an exciting time to see some questions answered.