Inclusive and Exclusive B→sll: Theory Perspective and Experimental Status

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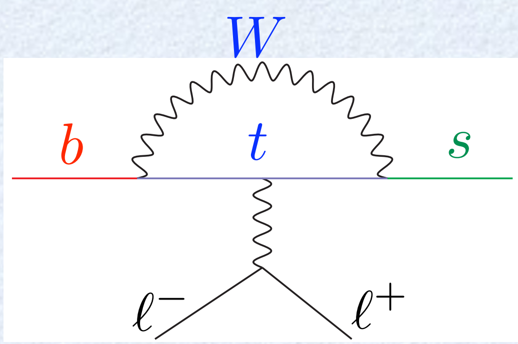
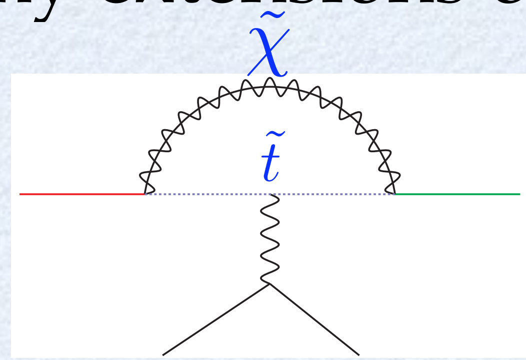
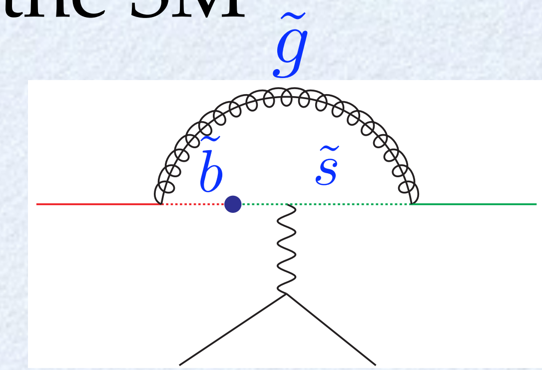
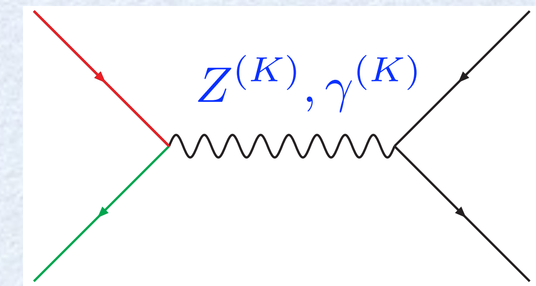
WHY $b \to s\ell^+\ell^-$

- Sensitive to many extensions of the SM









Central values of exclusive global fit require non-MFV models

- Exclusive modes are experimentally easier (LHCb) but harder to bring under theoretical control (factorization, power corrections, ...)
- Inclusive modes require a super-B machine to be fully exploited but the theoretical outlook is very impressive
- Some references (inclusive):

  Misiak; Buras, Munz, Bobeth, Urban, Asatryan, Asatrian, Greub, Walker, Ghinculov, Hurth, Isidori, Yao, Gambino, Gorbahn, Haisch, Huber, Lunghi, Wyler, Lee, Ligeti, Stewart, Tackmann, …

- Some references (exclusive):

  Beneke, Feldmann, Seidel, Grinstein, Pirjol, Bobeth, Hiller, Dyk, Wacker, Piranishvili, Altmannshofer, Ball, Bharucha, Buras, Wick, Straub, Matias, Lunghi, Virto, Descotes-Genon, Hofer, Hurth, Mahmoudi, …

WHY $b \to s\ell^+\ell^-$

SM operator basis:

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{tq}^* \left[\sum_{i=1}^{10} C_i Q_i + \frac{V_{ub}V_{uq}^*}{V_{tb}V_{tq}^*} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + \underbrace{\sum_{i=3}^{6} C_{iQ} Q_{iQ} + C_b Q_b}_{\text{for QED corrections}} \right]$$

- Magnetic & chromo-magnetic

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

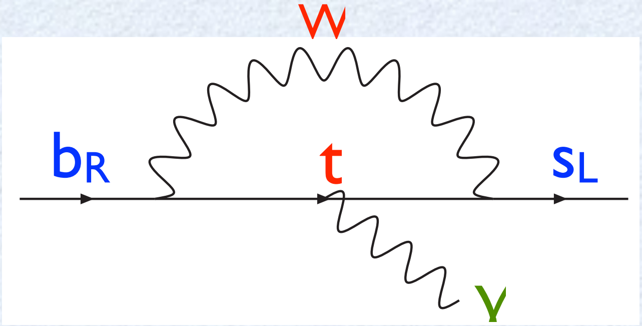
$$Q_8 = \frac{g}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

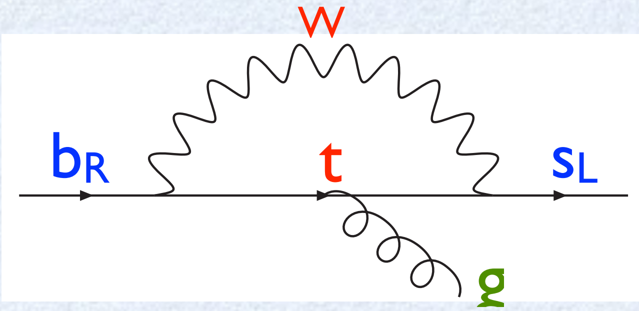
- Semileptonic

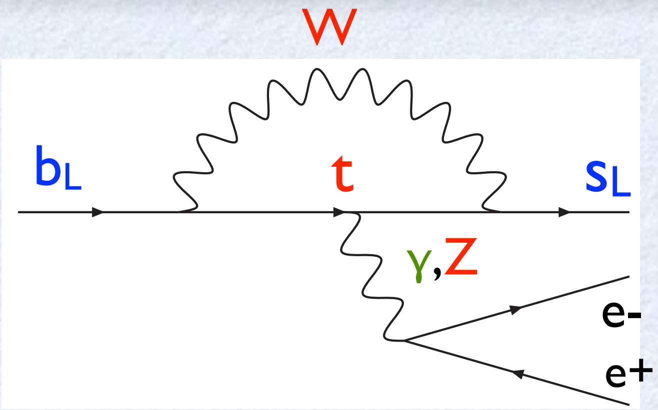
$$Q_9 = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell}\gamma^\mu \ell)$$

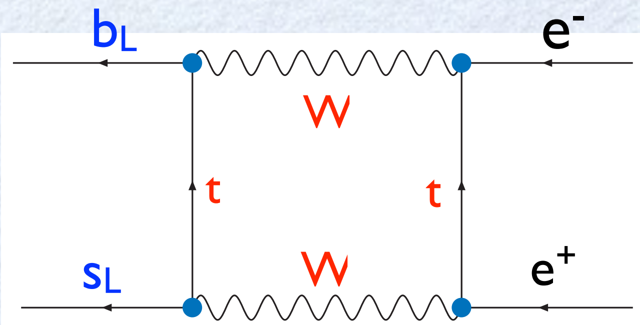
$$Q_{10} = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

Everything is known very well ($V_{ub}V_{uq}$ contribution is small for b→sll but important for b→dll)









WHY $b \to s\ell^+\ell^-$

In NP extensions we get more structures (V+A, scalar, tensor)

- Right-handed (V+A):

$$Q'_7 = \frac{e}{16\pi^2} m_b [\bar{s}_R \sigma^{\mu\nu} b_L] F_{\mu\nu}$$

$$Q'_8 = \frac{g}{16\pi^2} m_b [\bar{s}_R \sigma^{\mu\nu} T^a b_L] G^a_{\mu\nu}$$

$$Q'_9 = [\bar{s}_R \gamma_\mu b_R][\bar{\ell}\gamma^\mu \ell]$$

$$Q'_{10} = [\bar{s}_R \gamma_\mu b_R][\bar{\ell}\gamma^\mu \gamma_5 \ell]$$

- Scalar:

$$Q_S = [\bar{s}_L b_R][\bar{\ell}\bar{\ell}]$$

$$Q'_S = [\bar{s}_R b_L][\bar{\ell}\bar{\ell}]$$

$$Q_P = [\bar{s}_L b_R][\bar{\ell}\gamma_5 \bar{\ell}]$$

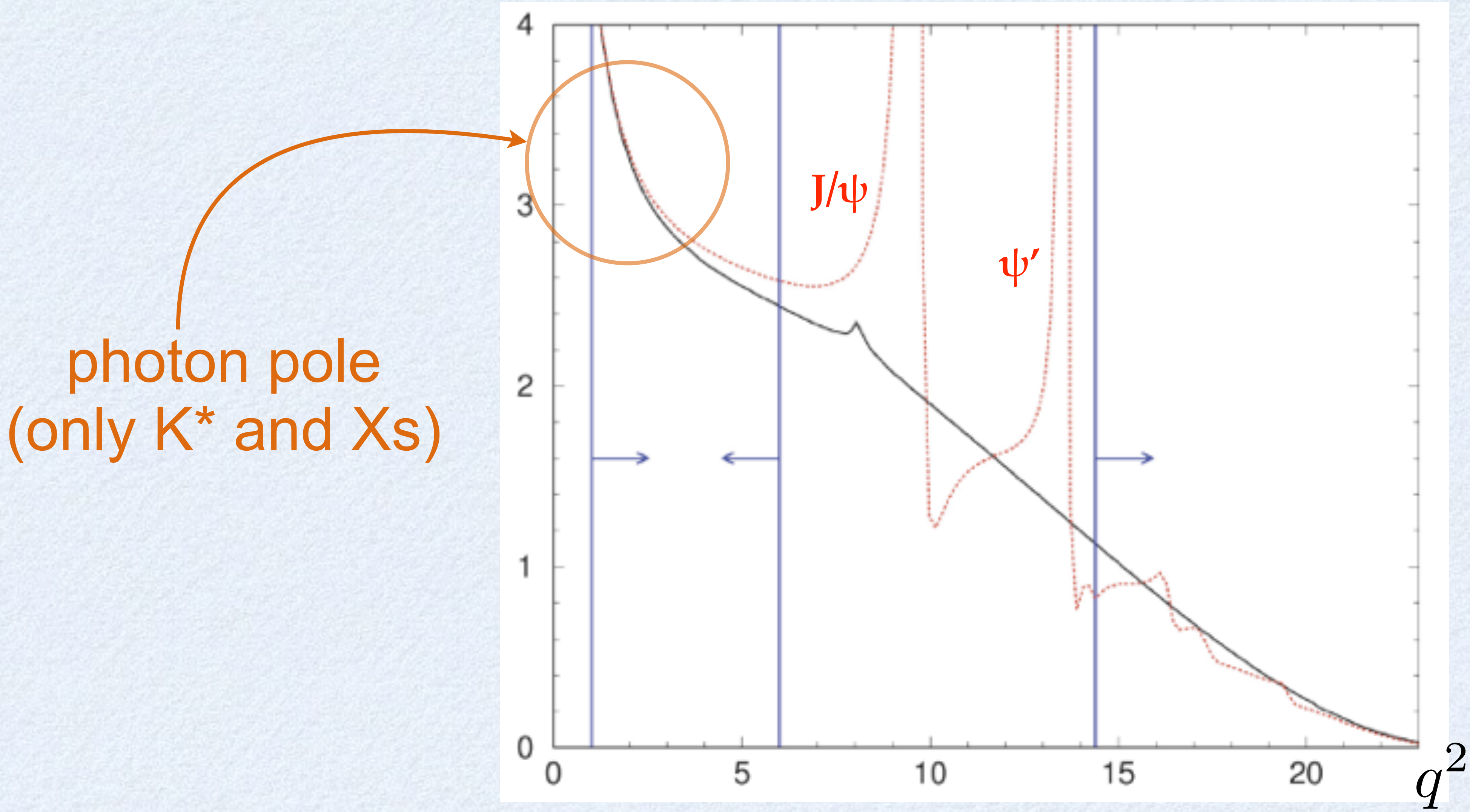
$$Q'_P = [\bar{s}_R b_L][\bar{\ell}\gamma_5 \bar{\ell}]$$

- Tensor

$$Q_T = [\bar{s}\sigma_{\mu\nu} b][\bar{e}ll\sigma^{\mu\nu}\ell]$$

$$Q_{T5} = \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}[\bar{s}\sigma_{\mu\nu} b][\bar{\ell}\sigma_{\alpha\beta}\ell]$$

Typical spectrum



- Intermediate charmonium resonances contribute via:
  $B \to (K, K^*, X_s)\ \psi_{\bar{c}c} \to (K, K^*, X_s)\ \ell^+\ell^-$
- Contributions of J/ψ and ψ' have to be dropped (for different reasons in inclusive and exclusive modes)

WHY $b \to s\ell^+\ell^-$

- Multi-objects in the final state (3 for B→K/X_s, 4 for B→K^*→Kπ) allows to isolate contributions from various operators
- $B \to X_s \ell\ell$

$$\frac{d^2\Gamma^{X_s}}{dq^2\, d\cos\theta_\ell} = \frac{3}{8}\left[(1+\cos^2\theta_\ell)\, H_T + 2(1-\cos^2\theta_\ell)\, H_L + 2\cos\theta_\ell\, H_A\right]$$

$$H_T \sim 2\hat{s}(1-\hat{s})^2\left[|C_9 + \frac{2}{\hat{s}}C_7|^2 + |C_{10}|^2\right] \qquad \hat{s} = q^2/m_b^2$$

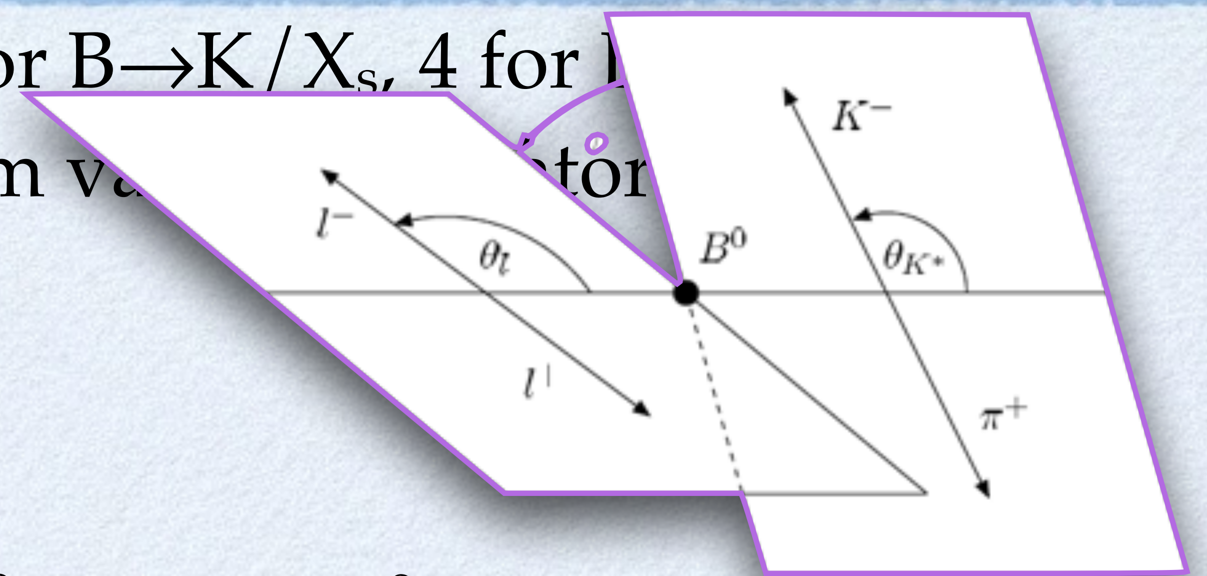
$$H_L \sim (1-\hat{s})^2\left[|C_9 + 2C_7|^2 + |C_{10}|^2\right]$$

$$H_A \sim -4\hat{s}(1-\hat{s})^2 \mathrm{Re}\left[C_{10}(C_9 + 2\frac{m_b^2}{q^2}C_7)\right]$$

- H_A is not suppressed by the lepton mass
- There are similar contributions from non-SM operators ***but there is no interference between V+A and V-A structures***
- We have three observables and those related by CP and isospin

WHY $b \to s\ell^+\ell^-$

- Multi-objects in the final state (3 for B→K/X$_s$, 4 for B
allows to isolate contributions from va tor



- $B \to K^*\ell\ell \to K\pi\ell\ell$

$$\frac{d^4\Gamma^{K^*}}{dq^2\; d\cos\theta_l\; d\cos\theta_{K^*}\; d\phi} \simeq$$

$$J_1^s \sin^2\theta_{K^*} + J_1^c \cos^2\theta_{K^*} + (J_2^s \sin^2\theta_{K^*} + J_2^c \cos^2\theta_{K^*}) \cos 2\theta_l$$
$$+ J_3 \sin^2\theta_{K^*} \sin^2\theta_l \cos 2\phi + J_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos\phi + J_5 \sin 2\theta_{K^*} \sin\theta_l \cos\phi$$
$$+ J_6 \sin^2\theta_{K^*} \cos\theta_l + J_7 \sin 2\theta_{K^*} \sin\theta_l \sin\phi$$
$$+ J_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin\phi + J_9 \sin^2\theta_{K^*} \sin^2\theta_l \sin 2\phi,$$

- ***We have 11 observables and those related by CP and isospin!***
- The J_a observables are functions of all the Wilson coefficients (V+A and V-A operators do interfere)
- In the literature one finds various combinations of these J_a

WHY $b \to s\ell^+\ell^-$

- Multi-objects in the final state (3 for B→K/X_s, 4 for B→K^*→Kπ) allows to isolate contributions from various operators
- $B \to K\ell\ell$

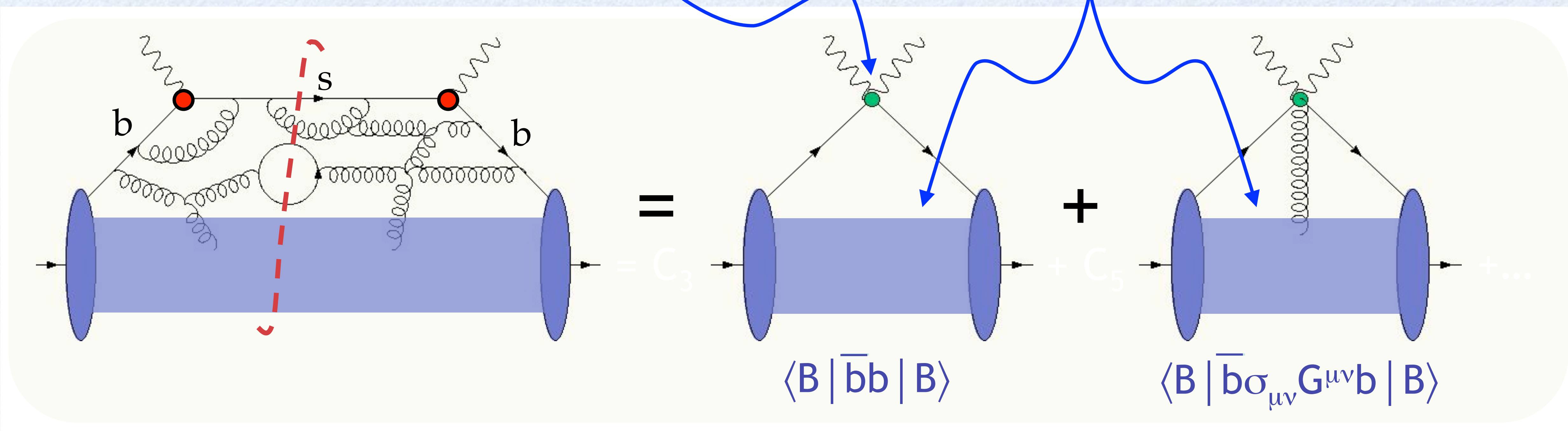
$$\frac{d^2\Gamma^K}{dq^2\ d\cos\theta_\ell} = a + b\ \cos\theta_\ell + c\ \cos\theta_\ell^2$$

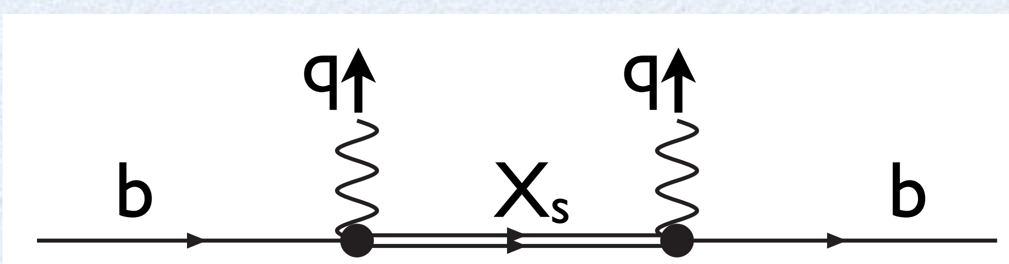
$$\begin{aligned} a &\sim C_7 + C_7',\ C_9 + C_9',\ C_{10} + C_{10}', \\ &\quad C_S + C_S',\ C_P + C_P',\ m_\ell\, C_T \\ b &\sim C_S + C_S',\ C_P + C_P',\ C_T,\ C_{T5},\ m_\ell\,(C_{10} + C_{10}') \\ c &\sim C_7 + C_7',\ C_9 + C_9',\ C_{10} + C_{10}',\ C_T,\ C_{T5} \end{aligned}$$

- In the SM ***b*** is suppressed by the lepton mass: huge sensitivity to scalar, pseudoscalar, tensor operators (e.g. forward-backward asymmetry)
- We have three observables and those related by CP and isospin
- Advantage: form factors very accessible to lattice QCD

Theory: Inclusive

$$\Gamma\left[\bar{B}\to X_s\ell^+\ell^-\right] = \Gamma\left[\bar{b}\to X_s\ell^+\ell^-\right] + O\left(\frac{\Lambda_{QCD}^2}{m_b^2}, \frac{\Lambda_{QCD}^3}{m_b^3}, \frac{\Lambda_{QCD}^2}{m_c^2}, \ldots\right)$$





$$\begin{aligned} p_{X_s}^2 &= (p_b - q)^2 = m_b^2 + q^2 - 2m_b q_0 \\ &< m_b^2 + q^2 - 2m_b\sqrt{q^2} = \left(m_b - \sqrt{q^2}\right)^2 \end{aligned}$$

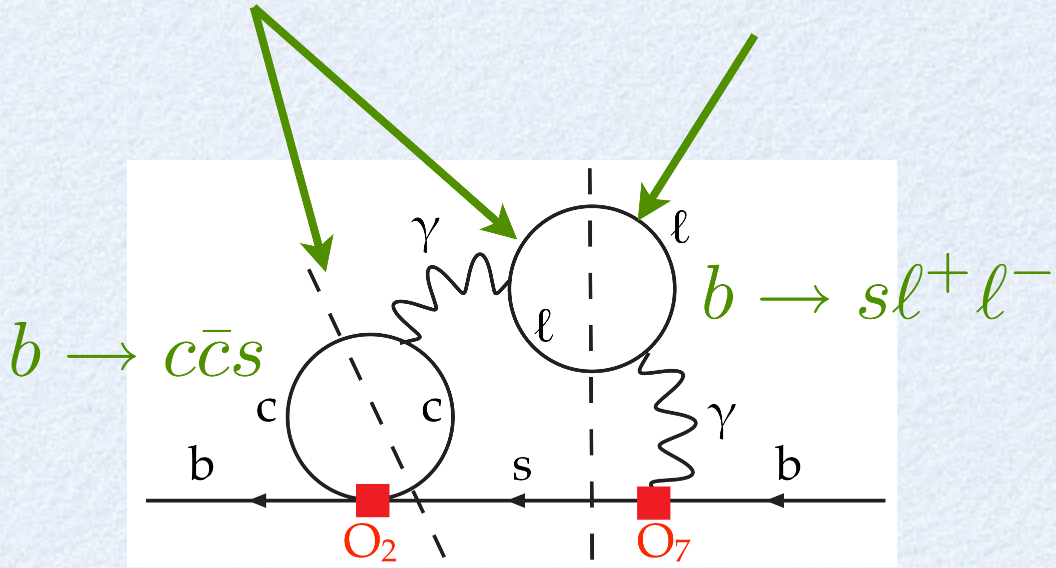
OPE is an expansion in $\Lambda_{QCD}/(m_b - \sqrt{q^2})$ and breaks down at $q^2 \sim m_b^2$

Charmonium troubles

- Optical theorem: [Beneke, Buchalla, Neubert, Sachrajda]

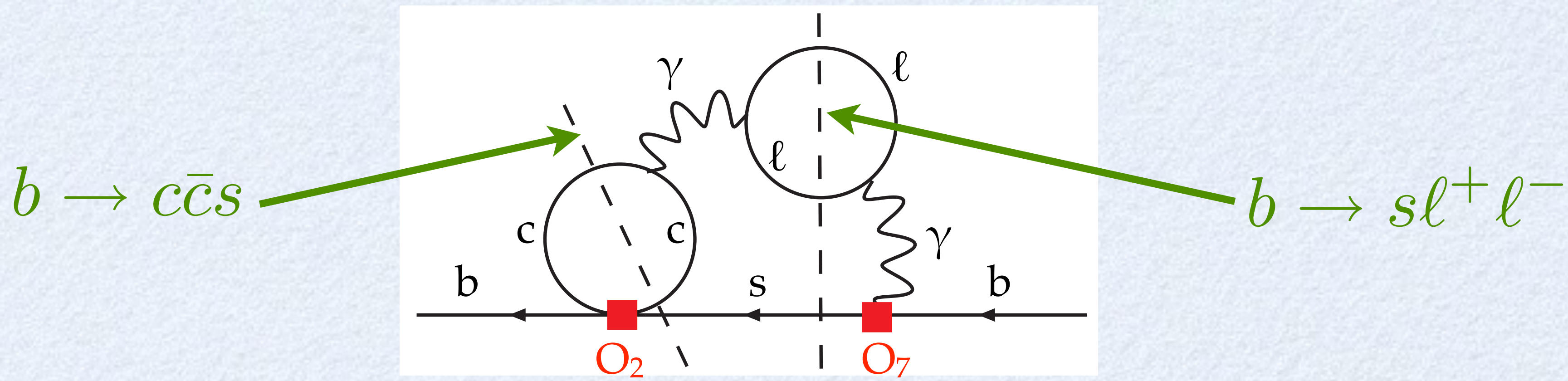
$$\mathrm{Im}\left[\sum_{ij}\langle\bar{B}|T\,Q_i(0)\,Q_j(x)|\bar{B}\rangle\right]\sim\Gamma(\bar{B}\to X_s)\neq\Gamma(\bar{B}\to X_s\ell^+\ell^-)$$

$$\Gamma(\bar{B}\to X_s)\sim 10^{-4}$$
$$\Gamma(\bar{B}\to X_s\ell^+\ell^-)\sim 10^{-6}$$



1. This is **not a violation of quark-hadron duality** (that in the inclusive is related to the integral over the real states in the Xs system)
2. The **OPE itself is perfectly fine** and it breaks down only at large q^2
3. **For $q^2 \sim m_{cc}$ the diagram is controlled by resonant long distance contributions** (think about the hadronic contribution to $(g\text{-}2)_\mu$)
4. The problem is that we are not including diagrams corresponding to open charm and hadronic decays of the charmonium resonances

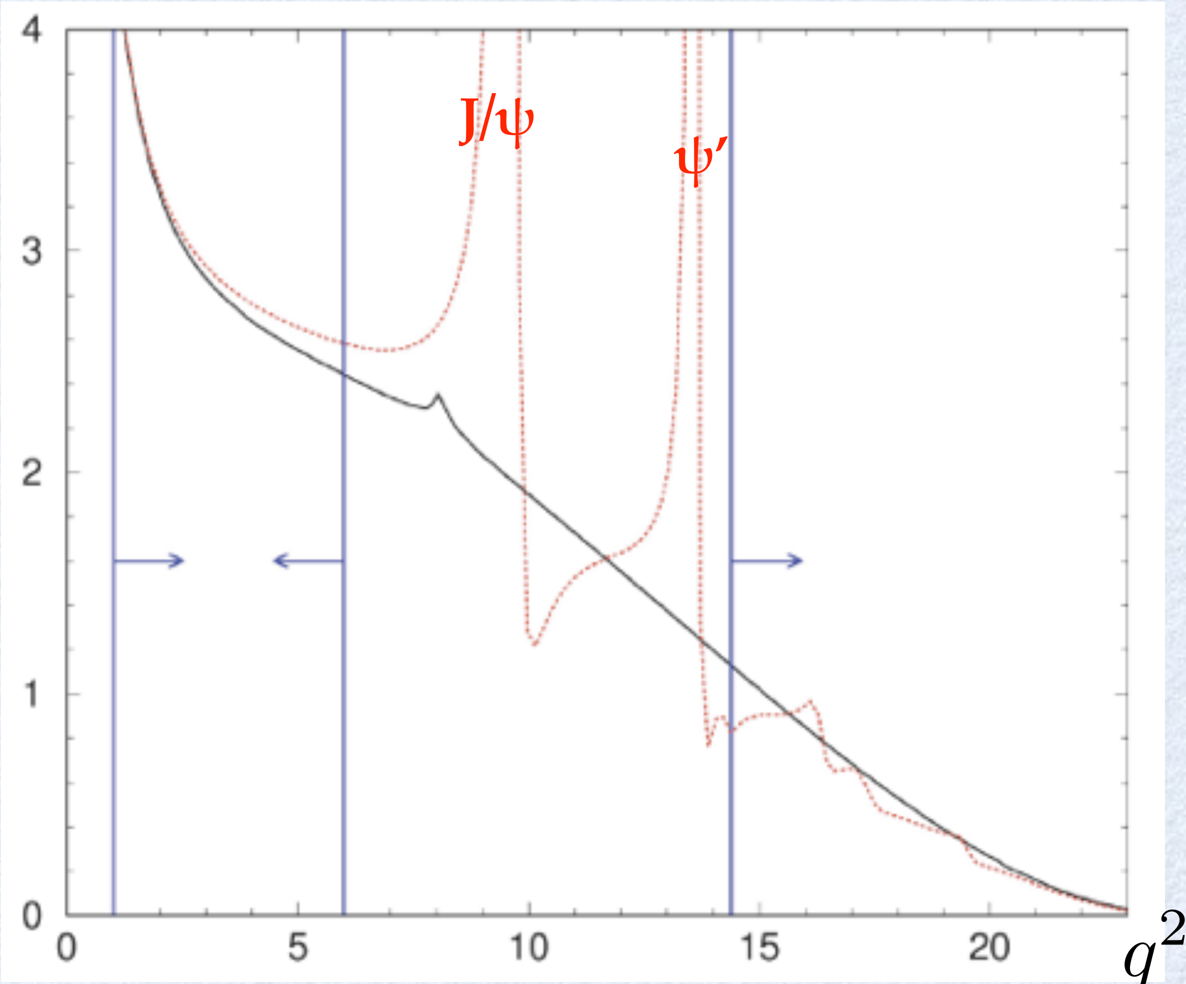
Charmonium troubles



- Three regions:
  - 0.04 GeV² < q² < 1 GeV²
  - ***1 GeV² < q² < 6 GeV²***
  - ***q² > 14.4 GeV²***

  dominated by the photon pole (b→sγ)

- Resonances model using data:
  - ★ Krüger-Sehgal (e+e- data)
  - ★ Breit-Wigner ansatz (old approach)



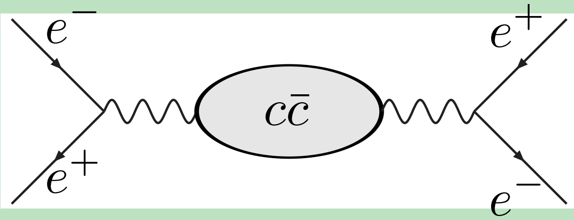
$\Gamma(\bar{B} \to X_s) \sim 10^{-4}$

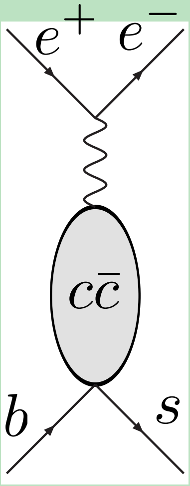
$\Gamma(\bar{B} \to X_s\ell^+\ell^-) \sim 10^{-6}$

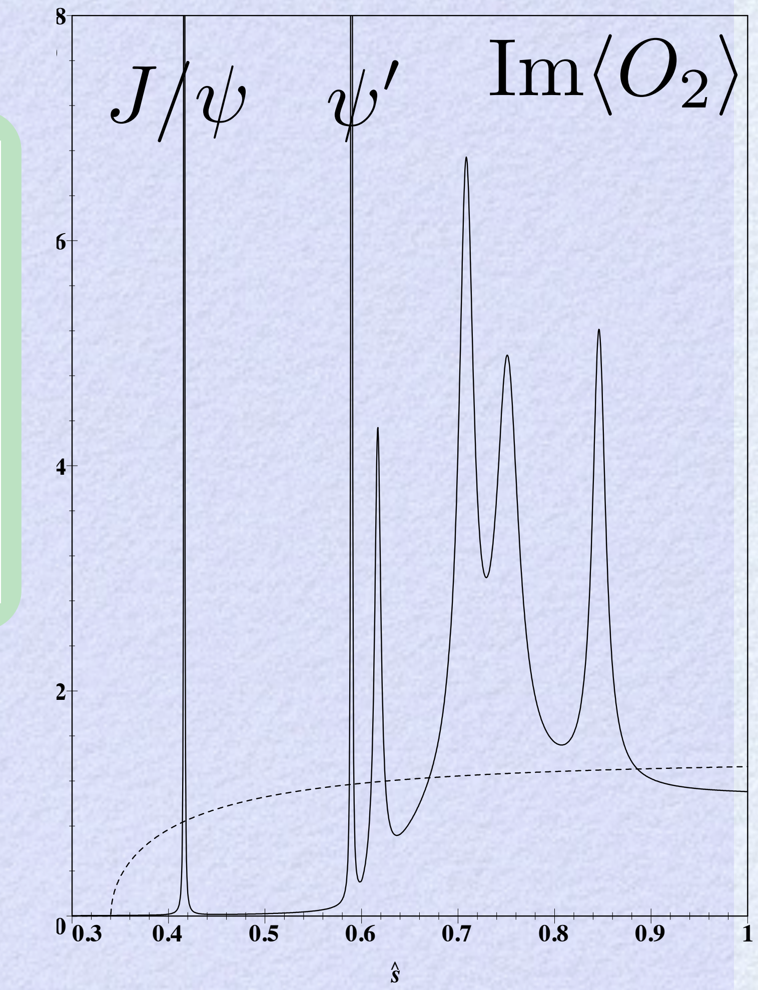
Q^2 cuts

- Kruger-Sehgal mechanism:

$$R_{\rm had}^{c\bar{c}} = \frac{\sigma(e^+e^- \to c\bar{c}\ \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$=$ 

$\langle O_2 \rangle =$ 



$$\mathrm{Im}\langle O_2 \rangle \to \langle O_9 \rangle_{\rm tree} \left(\frac{\pi}{3} R_{\rm had}^{c\bar{c}}(\hat{s}) \right)$$

$$\mathrm{Re}\langle O_2 \rangle \to \langle O_9 \rangle_{\rm tree} \left(-\frac{8}{9}\log m_c/m_b - \frac{4}{9} + \frac{\hat{s}}{3} P \int_{4\hat{m}_D^2}^{\infty} \frac{R_{\rm had}^{c\bar{c}}(\hat{s}')}{\hat{s}'(\hat{s}' - \hat{s})} d\hat{s}' \right)$$

- Alternatively use a Breit-Wigner ansatz to parametrize $\langle O_2 \rangle$

$$Y_{\rm amm}(\hat{s}) = Y_{\rm pert}(\hat{s}) + \frac{3\pi}{\alpha^2} C^{(0)} \sum_{V_i = \psi(1s),\ldots,\psi(6s)} \kappa_i \frac{\Gamma(V_i \to \ell^+\ell^-)\, m_{V_i}}{m_{V_i}{}^2 - \hat{s}\, m_B{}^2 - i m_{V_i} \Gamma_{V_i}}$$

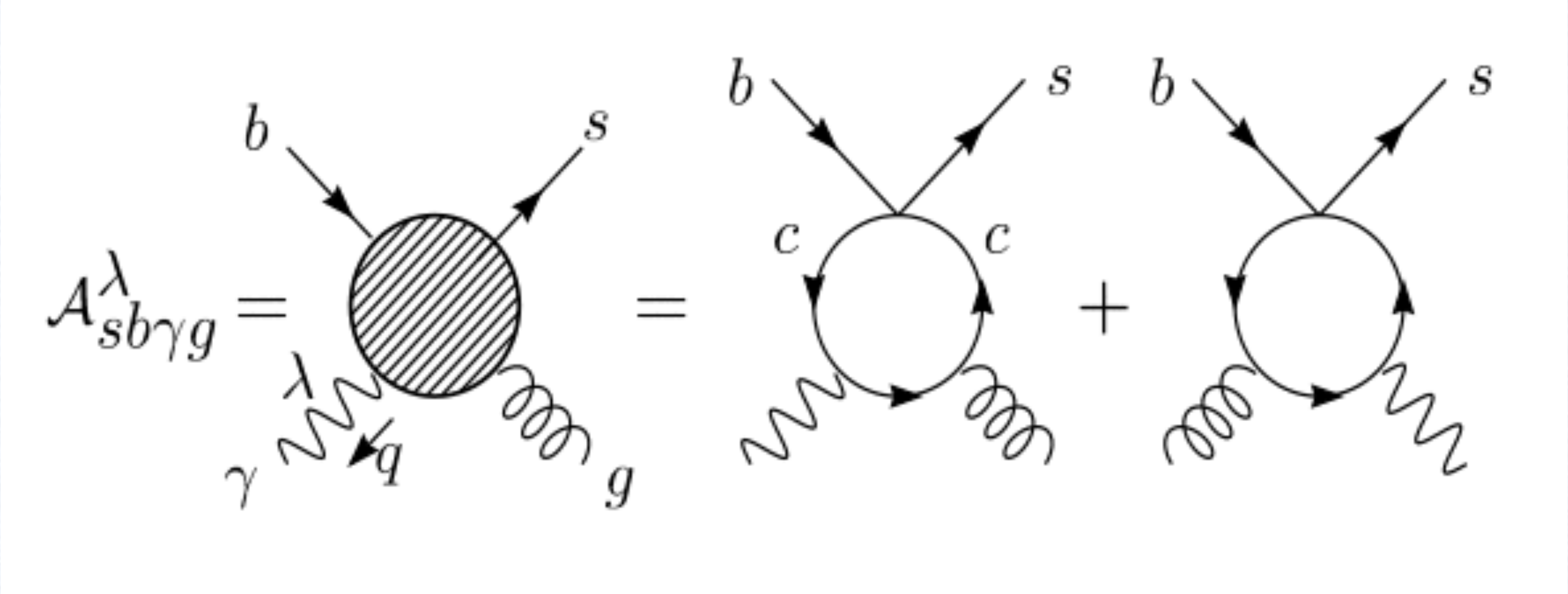
Fudge factors

- The impact in the low q^2 region is +1.8%, in the high q^2 region is -10%
- Historically $\kappa_i \approx 2$. Using NNLO Wilson coefficients one finds $\kappa_i \approx 1$

[Bobeth, Hiller, van Dyk]

Theory: inclusive

- The KS mechanism captures the long distance contribution that corresponds to cc pair in color singlet state (J/ψ)
- The color octet contribution is non-resonant, is captured by Λ^2/m_c^2 power corrections



and yields a local contribution proportional to $\langle \bar{B}|\bar{b}\sigma_{\mu\nu}G^{\mu\nu}b|\bar{B}\rangle \sim \lambda_2$

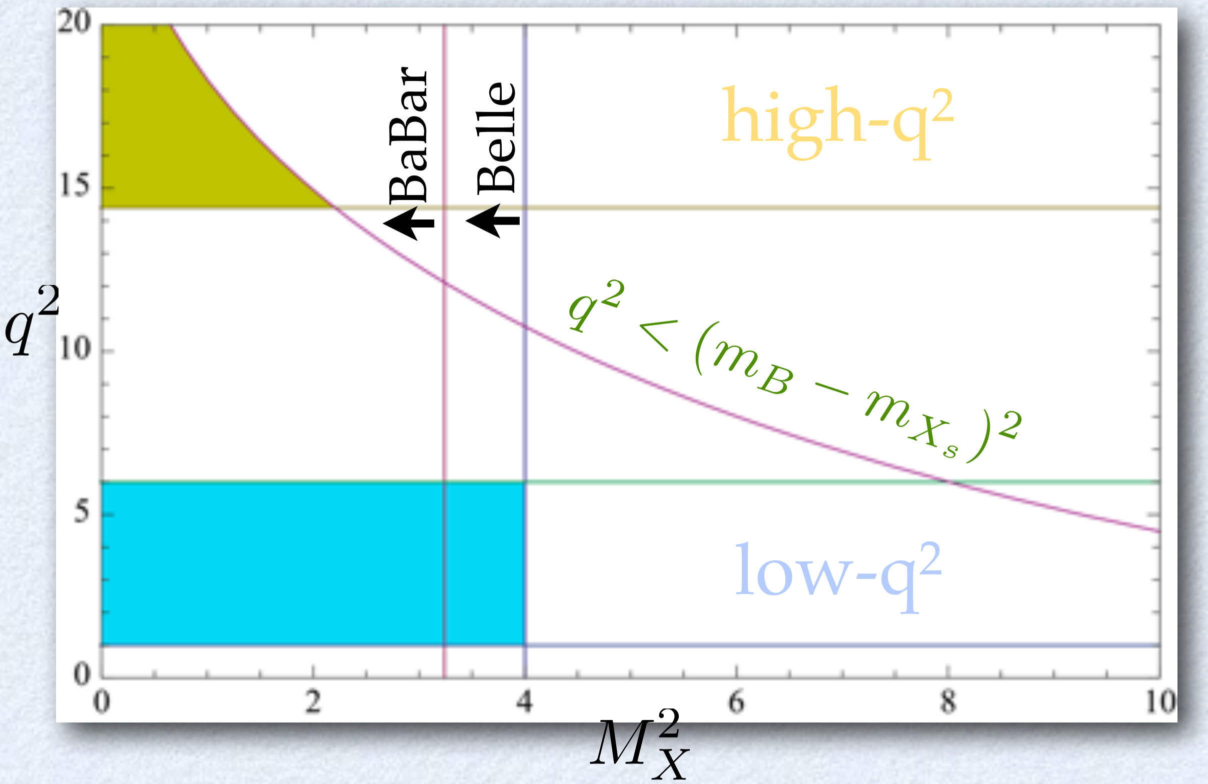
Theory: Inclusive

$$\Gamma\left[\bar{B} \to X_s \ell^+ \ell^-\right] = \Gamma\left[\bar{b} \to X_s \ell^+ \ell^-\right] + O\left(\frac{\Lambda_{QCD}^2}{m_b^2}, \frac{\Lambda_{QCD}^3}{m_b^3}, \frac{\Lambda_{QCD}^2}{m_c^2}, \ldots\right)$$

local OPE, optical theorem
quark-hadron duality

HQET

Phase space cuts introduce sensitivity to new scales, the rate becomes less inclusive and new non-perturbative effects appear



$M_{X_s} < [1.8, 2]$ GeV **cut to remove double semileptonic decay background**

- High-q^2 region unaffected
- **Experiments correct using Fermi motion model**
- $SCET_I$ suggests cuts are universal (same for b→sll and b→ulν)

Effect of cc resonances can be included using data from ee→hadrons

Theory: Inclusive

$$\Gamma\left[\bar{B}\to X_s\ell^+\ell^-\right] = \Gamma\left[\bar{b}\to X_s\ell^+\ell^-\right] + O\left(\frac{\Lambda^2_{QCD}}{m_b^2}, \frac{\Lambda^3_{QCD}}{m_b^3}, \frac{\Lambda^2_{QCD}}{m_c^2}, \ldots\right)$$

local OPE, optical theorem
quark-hadron duality

HQET

- Low-q^2: theory in excellent shape
- High-q^2: the OPE starts to break down and only integrated quantities are reliable
  - mismatch between partonic and hadronic phase space
  - power corrections are larger
  - higher charmonium resonances must be integrated over
  - things improve dramatically by normalizing the rate to the semileptonic rate with the same q^2 cut [Ligeti et al.]

$$\mathcal{R}(s_0) = \int_{s_0}^1 \mathrm{d}\hat{s}\,\frac{\mathrm{d}\Gamma(\bar{B}\to X_s\ell^+\ell^-)}{\mathrm{d}\hat{s}} \Big/ \int_{s_0}^1 \mathrm{d}\hat{s}\,\frac{\mathrm{d}\Gamma(\bar{B}^0\to X_u\ell\nu)}{\mathrm{d}\hat{s}}$$

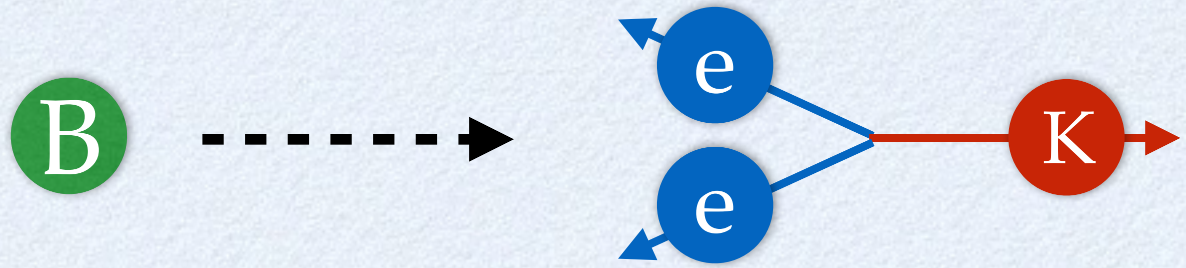
Theory: Exclusive (low Q^2)

- The central problem is the calculation of matrix elements:

$$\langle K^{(*)}\ell\ell|O(y)|B\rangle \approx \langle K^{(*)}|T\, J_\mu^{\rm em}(x)\, O(y)|B\rangle$$

if O contains a leptonic current (i.e. $O_{7,9,10}$) the matrix elements reduces to a form factor

- At low-q^2 the $K^{(*)}$ recoils strongly:



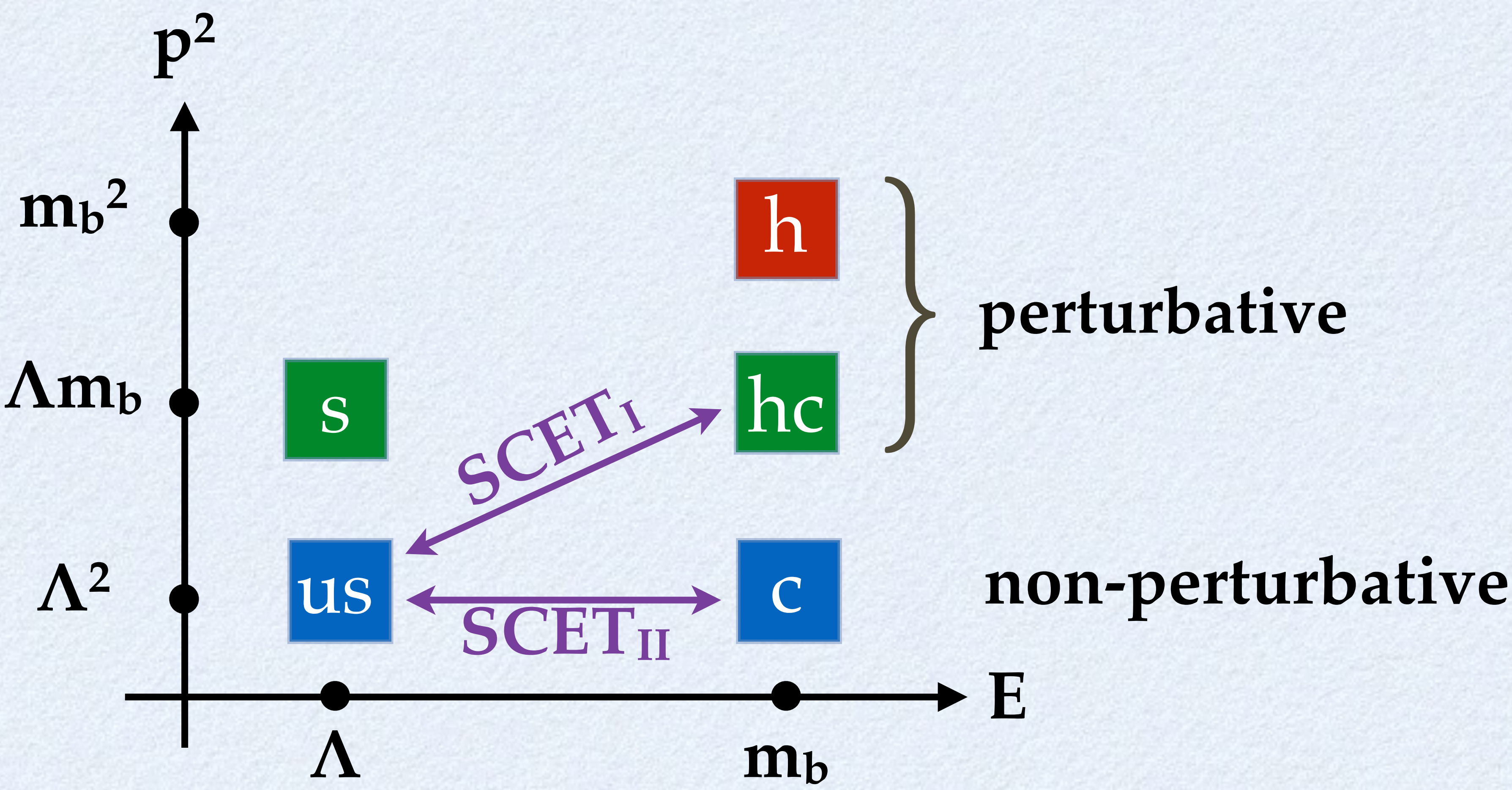
- The large energy of the $K^{(*)}$ introduces three scales: m_b^2, Λm_b and Λ^2:

$$\langle K^{(*)}|T\, J_\mu^{\rm em}(x)\, O(y)|B\rangle \sim \underbrace{C}_{m_b^2} \times [\underbrace{\text{Form Factor}}_{\Lambda^2} + \underbrace{\phi_B}_{\Lambda^2} \star \underbrace{J}_{\Lambda m_b} \star \underbrace{\phi_K}_{\Lambda^2}] + O\left(\frac{\Lambda}{m_b}\right)$$

$\mathrm{SCET_{II}}$

Theory: Exclusive (low Q^2)

- **Soft Collinear Effective Theory**



- us-hc factorization is rock solid (inclusive modes, collider physics)
- us-c factorization is more problematic (exclusive modes) because both collinear and ultrasoft modes have $p^2 \sim \Lambda^2$ and sometimes they don't factorize (zero-bin, messenger modes …)

Theory: Exclusive (low q^2)

- For example, the B→Kll rate is given by:

$$\frac{d\Gamma}{dq^2} \sim \left| f_+(q^2)\, C_9^{\rm eff}(q^2) + \frac{2m_b}{m_B+m_K} f_T(q^2)\, C_7^{\rm eff}(q^2) \right.$$

$$\left. + \frac{2m_b}{m_B}\frac{\pi^2}{N_c}\frac{f_B f_K}{m_B}\sum_{\pm}\int \frac{d\omega}{\omega}\Phi_{B,\pm}(\omega)\int_0^1 du\,\Phi_K(u)\left[T_{P,\pm}^{(0)} + \tilde\alpha_s C_F\, T_{P,\pm}^{(\rm nf)}\right]\right|^2$$

$$+ \left| f_+(q^2)\, C_{10}\right|^2$$

- The form factor f_T can be expressed in terms of f_+ (it is now preferable to use directly the lattice determination of f_T):

$$\frac{m_B}{m_B+m_K} f_T = f_+\left[1 + \tilde\alpha_s C_F\left(\log\frac{m_b^2}{\mu^2} + 2L\right)\right]$$

$$- \frac{\pi}{N_c}\frac{f_B f_K}{E}\alpha_s C_F \overbrace{\int \frac{d\omega}{\omega}\Phi_{B,+}(\omega)}^{\lambda_{B,+}^{-1}}\int_0^1 \frac{du}{\bar u}\,\Phi_K(u)$$

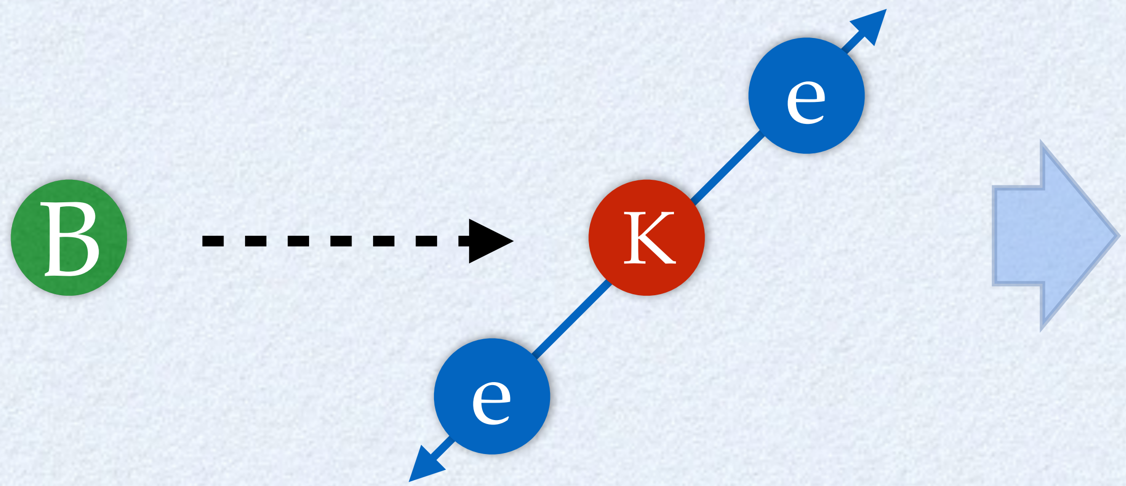
Theory: exclusive (high Q^2)

- The central problem is the calculation of matrix elements:

$$\langle K^{(*)}\ell\ell|O(y)|B\rangle \approx \langle K^{(*)}|T\, J_\mu^{\rm em}(x)\, O(y)|B\rangle$$

if O contains a leptonic current (i.e. $O_{7,9,10}$) the matrix elements reduces to a form factor (lattice, QCD sum rules)

- At high-q^2 the $K^{(*)}$ doesn't recoil:

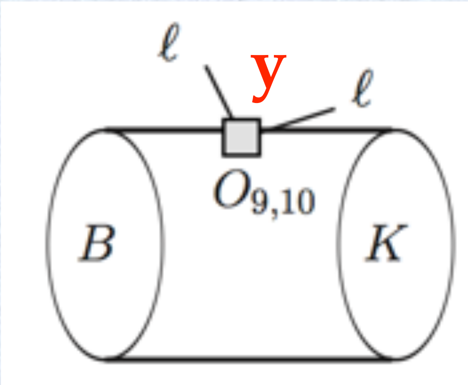


$$(x-y)^2 \sim \frac{1}{q^2} \sim \frac{1}{m_b^2}$$

Grinstein & Pirjol showed how to write a simple OPE in which **all matrix elements** are given in terms of calculable hard coefficients and **form factors** (up to power corrections)

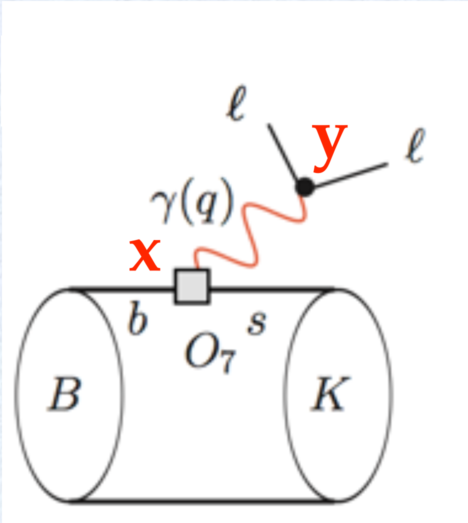
Theory: exclusive (high q^2)

- b→sll matrix elements are controlled by the large q^2



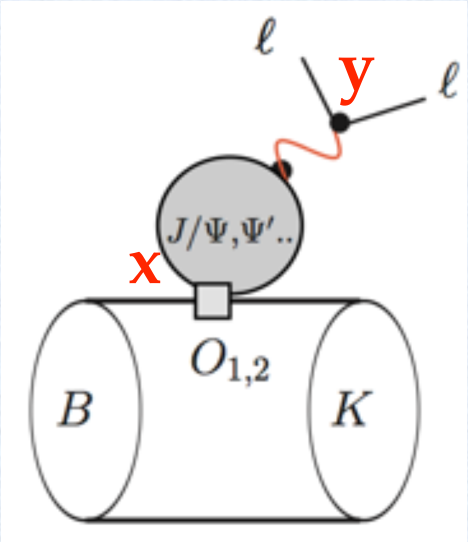
$$\langle K^{(*)}|O_{9,10}(y)|B\rangle \sim f_+(q^2)$$

local



$$\langle K^{(*)}|TJ^\mu(x)O_7(y)|B\rangle \sim \frac{1}{q^2} f_T(q^2)$$

local



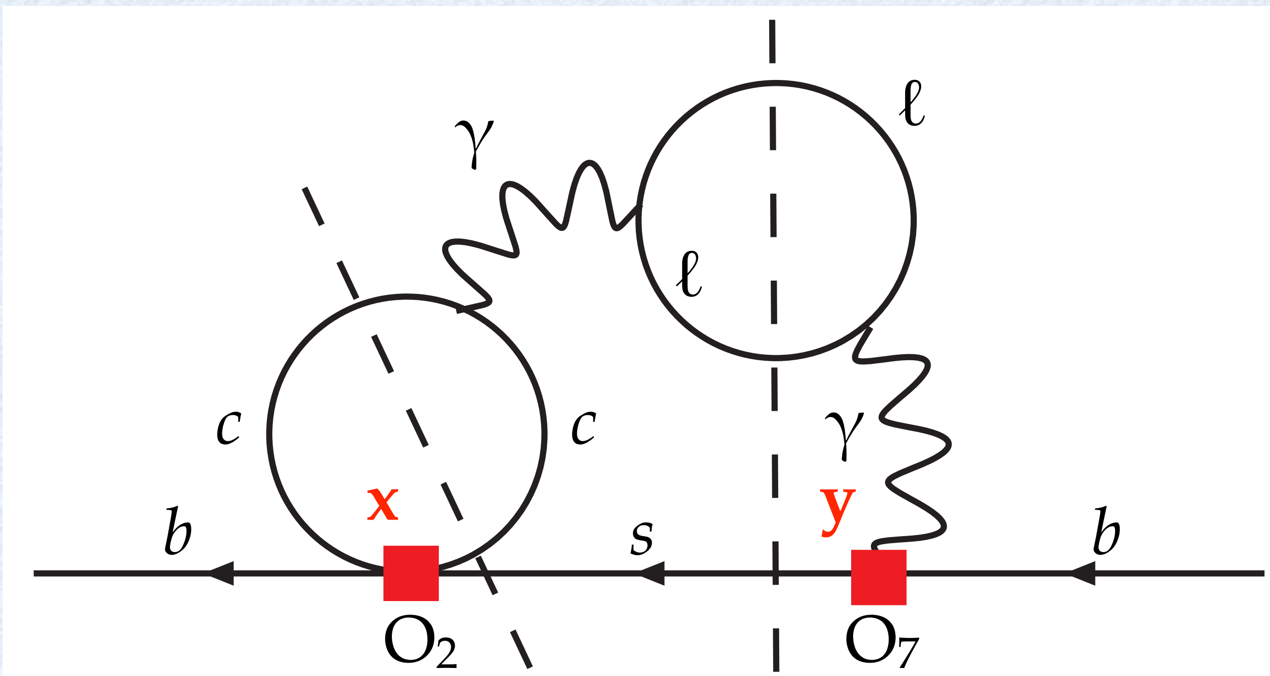
$$\langle K^{(*)}|TJ^\mu(x)O_{1,2}(y)|B\rangle \sim h(q^2)\ f_+(q^2)$$

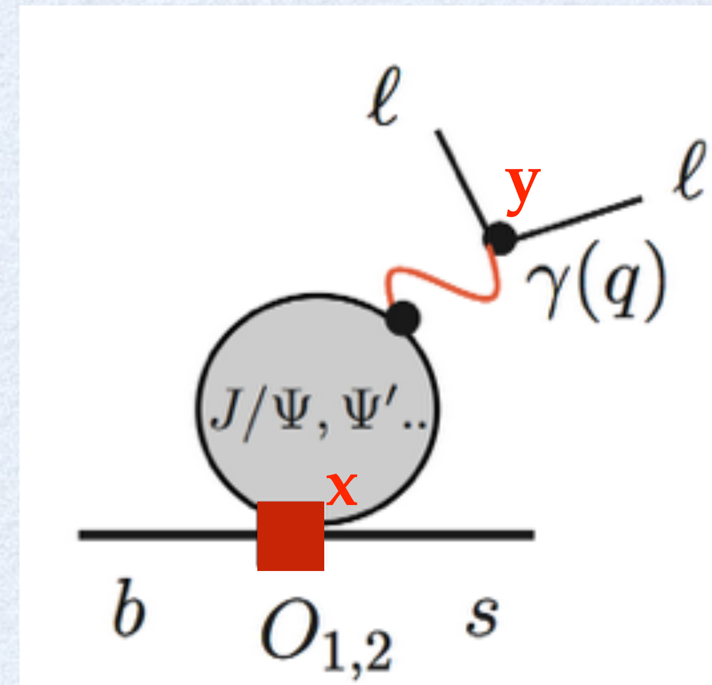
highly non-local

Does this signal a breakdown of the OPE?

Theory: Exclusive (high Q^2)

- Note the difference between inclusive and exclusive (high-q^2) OPE:





$$(x-y)^2 \sim \frac{1}{\left(m_b - \sqrt{q^2}\right)^2}$$

The breakdown of the OPE at very large q^2 is independent of the presence of resonant charm loops

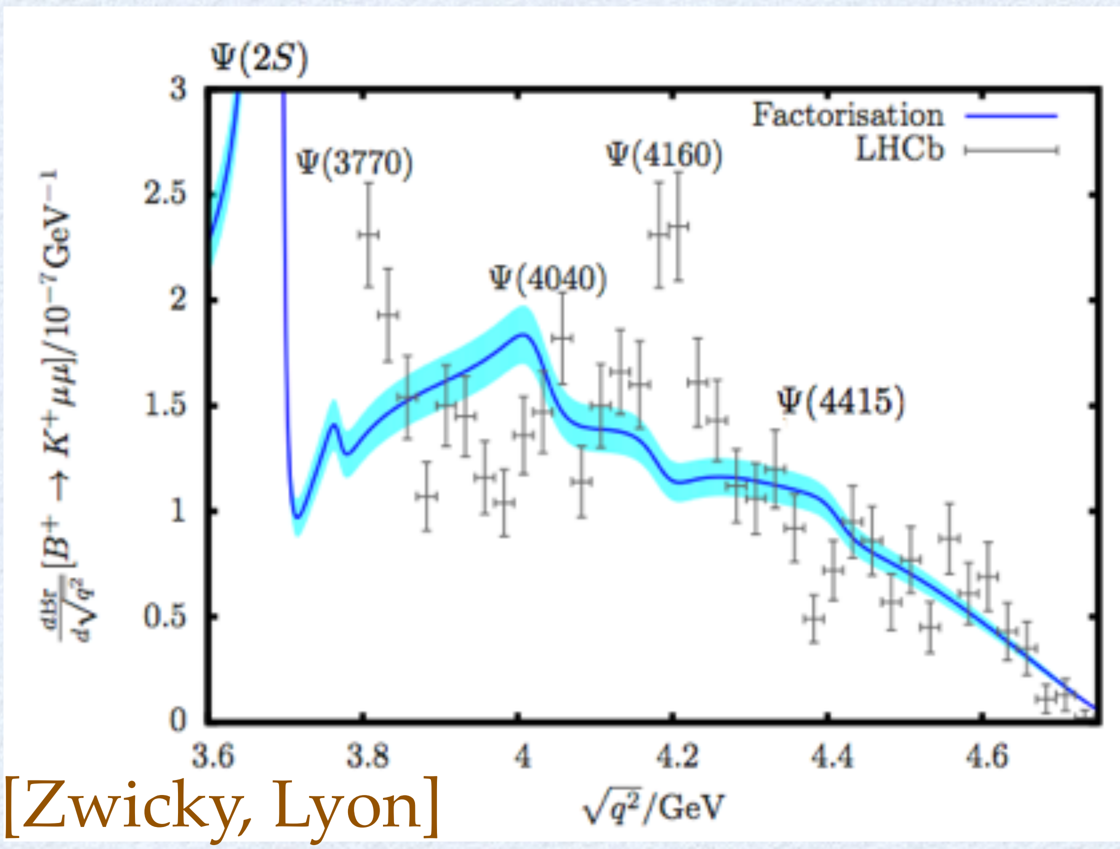
$$(x-y)^2 \gg \frac{1}{q^2}$$

The presence of resonant charm loops jeopardize the OPE itself and one has to rely on quark-hadron duality

[Beylich, Buchalla, Feldmann]

Theory: Exclusive (high q^2)

- Does the KS mechanism to include resonant effects work?
- For B→Kll these attempts seem to fail:



[Zwicky, Lyon]

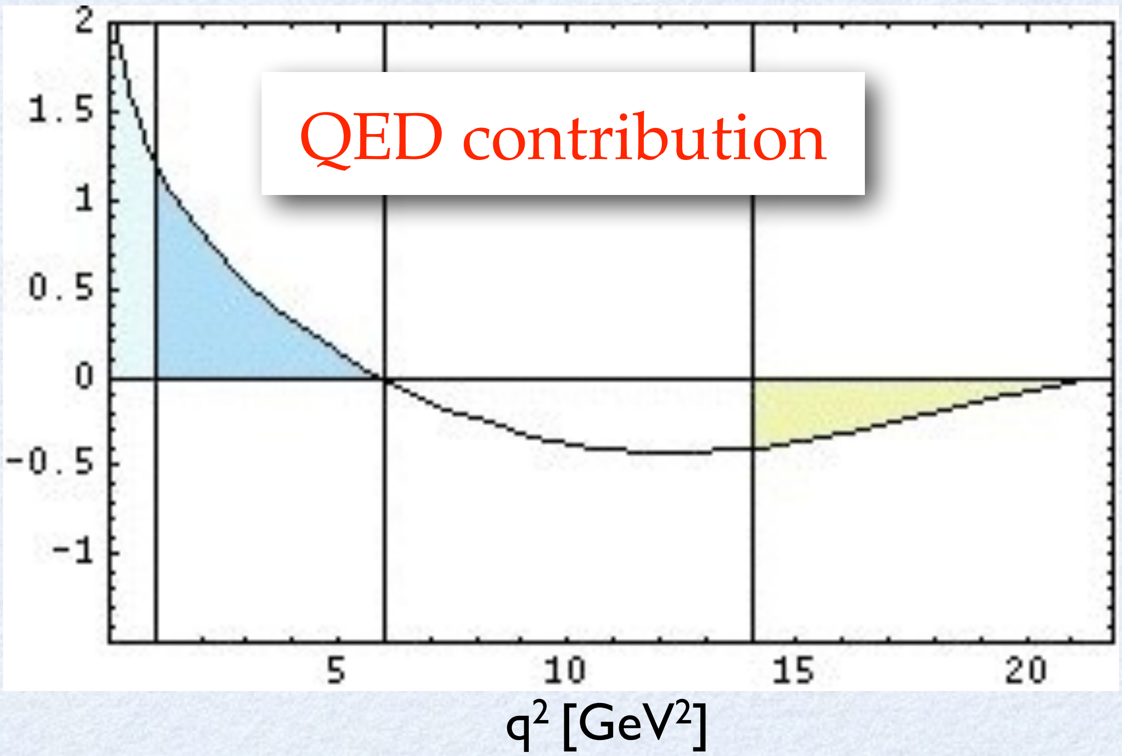
Experimental and theoretical valley and peaks do not match

Beylich, Buchalla and Feldmann argue that integrating over the high-q^2 region and invoking quark-hadron duality yields accurate predictions

- What is going on? Apparently this seems to be a failure of QCD factorization in describing the hadronic B→ψ_{cc} K process (i.e. color octet contributions might be important)
- Will this persists for the K* and Xs modes?
  Apparently not [Bobeth, Hiller, van Dyk]

Inclusive: QED Logs

- The *rate is proportional to* $\alpha_{\rm em}^2(\mu^2)$. Without QED corrections the scale μ is undetermined $\rightarrow \pm 4\%$ uncertainty
- Focus on corrections enhanced by large logarithms:
  - $\alpha_{\rm em} \log(m_W/m_b) \sim \alpha_{\rm em}/\alpha_s$ [WC, RG running] [Bobeth,Gambino,Gorbahn,Haisch]
  - $\alpha_{\rm em} \log(m_\ell/m_b)$ [Matrix Elements]
- The differential rate is not IR safe with respect to photon emission the results in the presence of a physical collinear logarithm, $\log(m_\ell/m_b)$



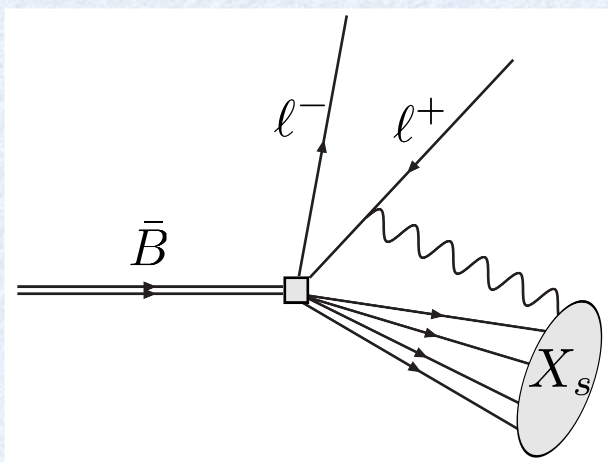
$$\text{virtual} = \frac{A_{\rm soft+collinear}}{\epsilon^2} + \frac{B_{\rm collinear} + B_{\rm soft}}{\epsilon} + C$$

$$\text{real} = -\frac{A_{\rm soft+collinear}}{\epsilon^2} - \frac{B'_{\rm collinear} + B_{\rm soft}}{\epsilon} + C'$$

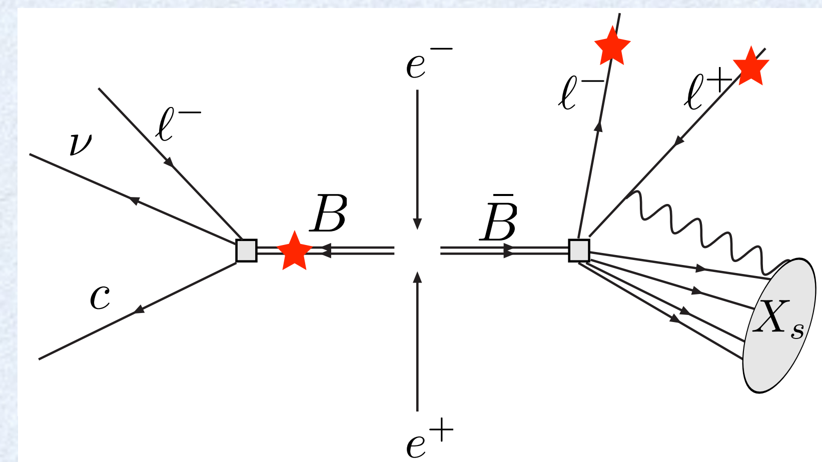
$$\int dq^2 \left(B_{\rm collinear} - B'_{\rm collinear}\right) = 0$$

QED Logs: Thery vs Experiment

- *Theory*
  include all bremsstrahlung photons into the X_s system:



- *Experiment (fully inclusive, Super-B only)*
  One B is identified; on the other side only the two leptons are reconstructed:



- *Experiment (Xs system reconstructed as a sum over exclusive states):*
  At BaBar (Belle) photons with energies smaller than 30 (20) MeV are not resolved. There is an attempt to identify photons emitted inside a small cone (35x50 mrad) around the electrons.

  ***Photons* inside the cone are included in the definition of the q^2.**

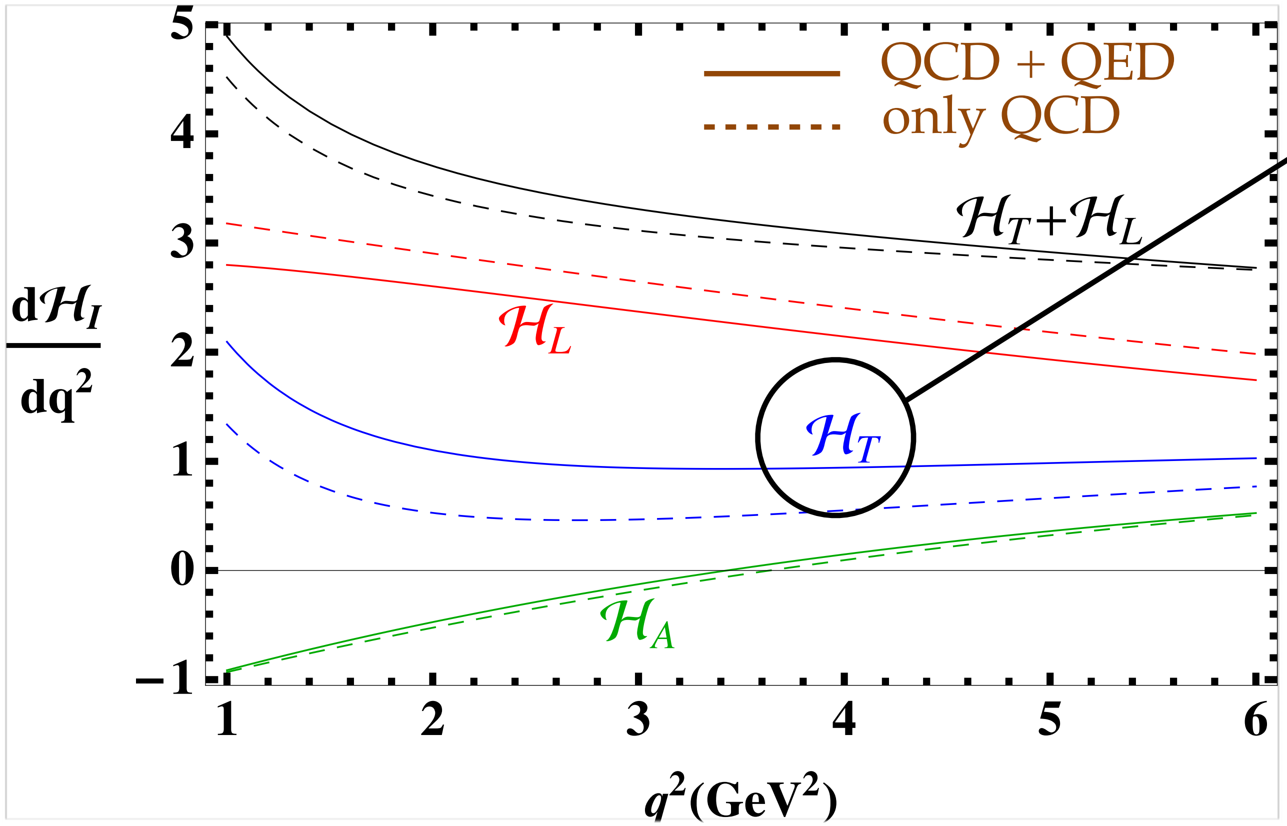
- *Measured rates are sensitive to the soft photon cutoff and to the size of the cone*

$$\frac{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\%$$

$$\frac{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$

QED LOGS: SIZE OF THE EFFECT

- We calculated the effect of collinear photon radiation and found large effects on some observables



Shift on H_T is ~70%!

H_T is smaller than H_L ($\hat{s} \lesssim 0.3$):

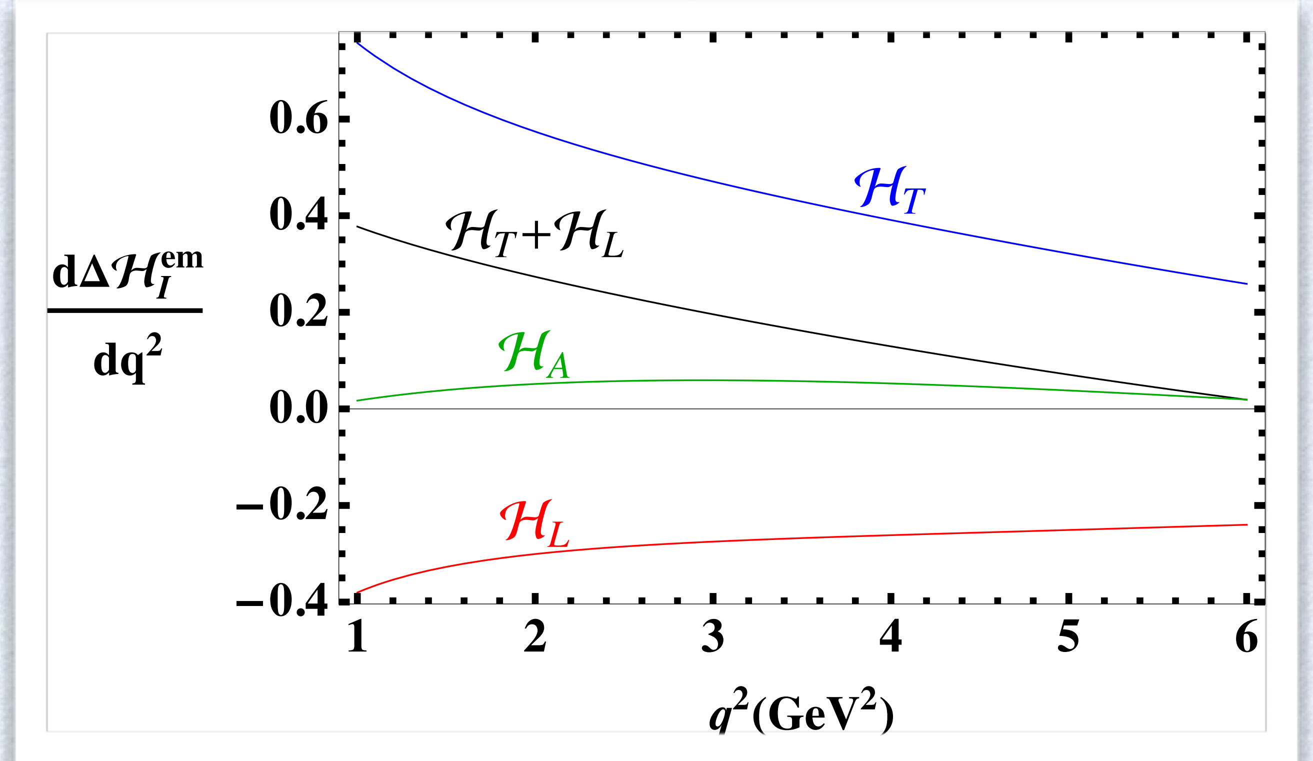
$$H_T \sim 2\hat{s}(1-\hat{s})^2 \left[|C_9 + \frac{2}{\hat{s}}C_7|^2 + |C_{10}|^2\right]$$

$$H_L \sim (1-\hat{s})^2 \left[|C_9 + 2C_7|^2 + |C_{10}|^2\right]$$

	$q^2 \in [1,6]$ GeV^2			$q^2 \in [1,3.5]$ GeV^2			$q^2 \in [3.5,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$	$\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$	$\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$
$\mathcal{B}$	100	5.1	5.1	54.6	3.7	6.8	45.4	1.4	3.1
$\mathcal{H}_T$	19.5	14.1	72.5	9.5	8.8	92.1	10.0	5.4	53.6
$\mathcal{H}_L$	80.0	-8.7	-10.9	44.7	-4.7	-10.6	35.3	-4.0	-11.3
$\mathcal{H}_A$	-3.3	1.4	-43.6	-7.2	0.8	-10.7	4.0	0.6	16.2

QED logs: size of the effect

- We calculated the effect of collinear photon radiation and found large effects on some observables

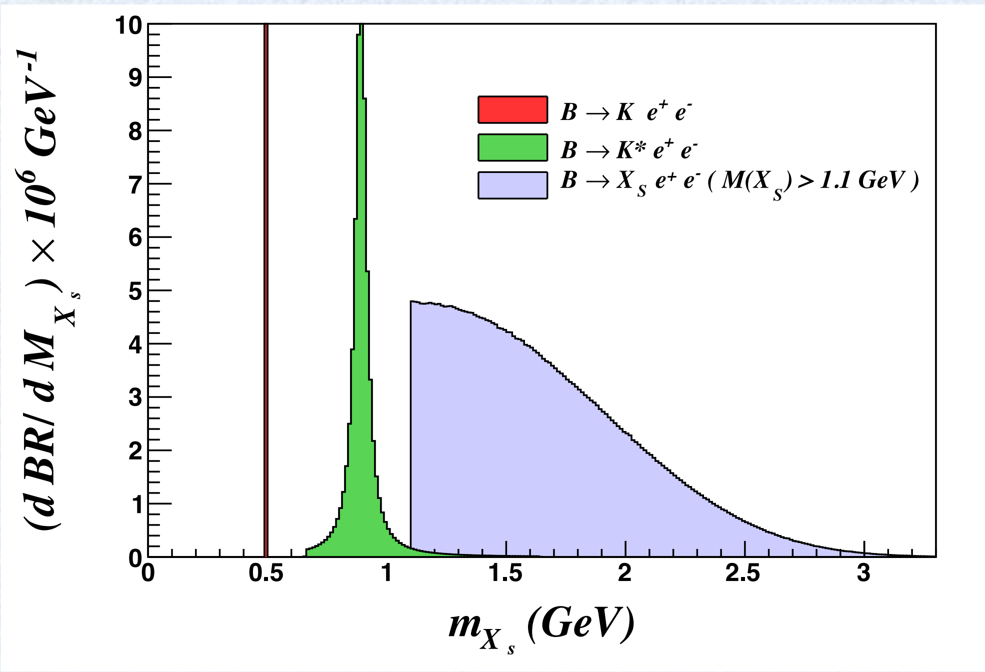


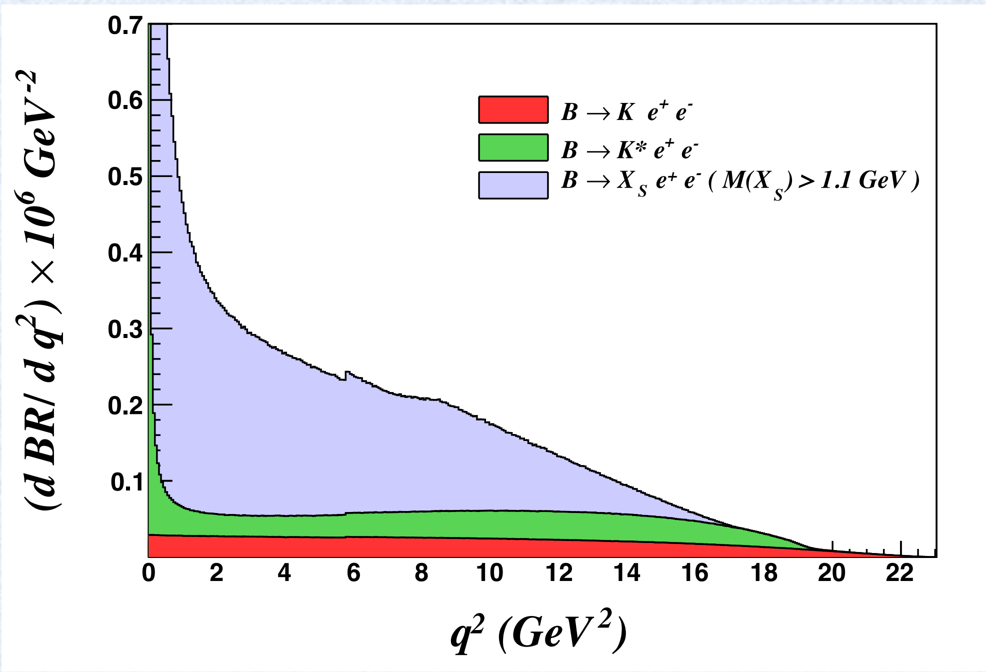
Size of QED contributions to the H_T and H_L is similar

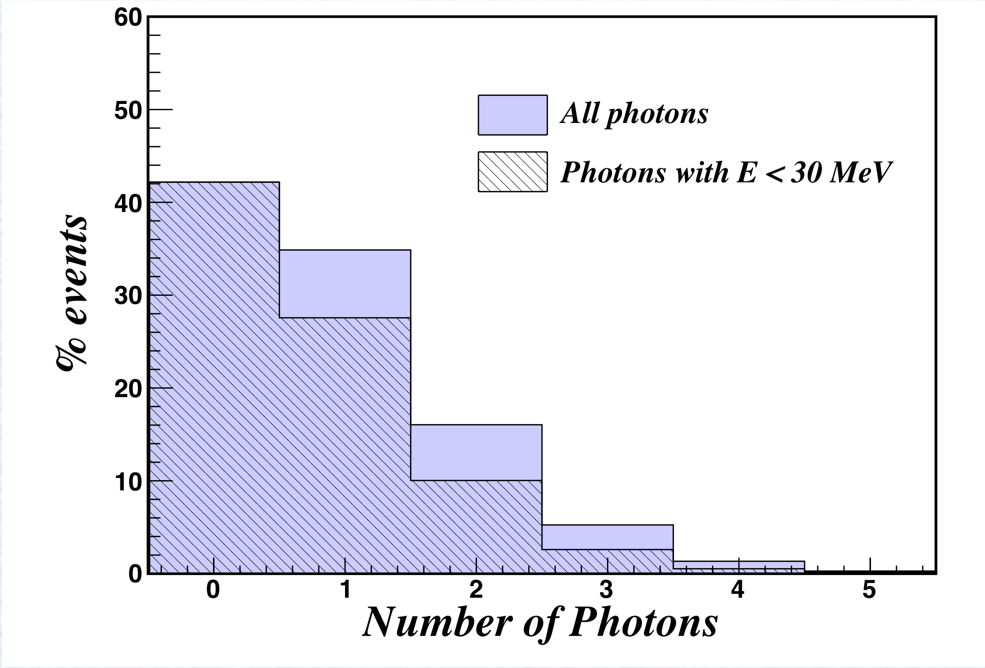
	$q^2 \in [1,6]$ GeV^2			$q^2 \in [1,3.5]$ GeV^2			$q^2 \in [3.5,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$	$\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$	$\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$
$\mathcal{B}$	100	5.1	5.1	54.6	3.7	6.8	45.4	1.4	3.1
$\mathcal{H}_T$	19.5	14.1	72.5	9.5	8.8	92.1	10.0	5.4	53.6
$\mathcal{H}_L$	80.0	-8.7	-10.9	44.7	-4.7	-10.6	35.3	-4.0	-11.3
$\mathcal{H}_A$	-3.3	1.4	-43.6	-7.2	0.8	-10.7	4.0	0.6	16.2

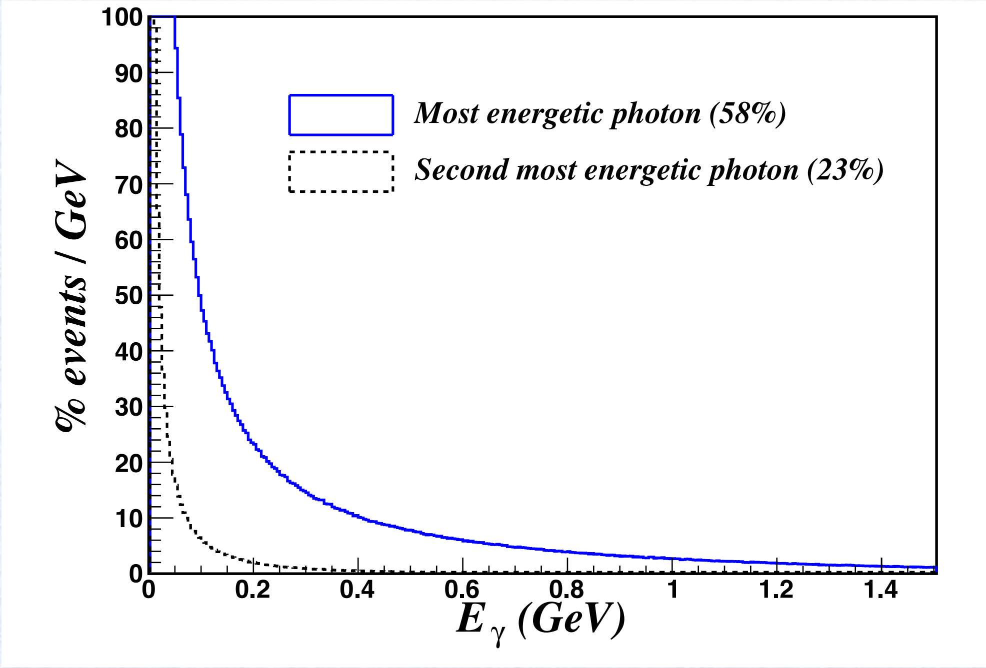
QED Logs: Monte Carlo

- EM effects have been calculated analytically and cross checked against Monte Carlo generated events (EVTGEN + PHOTOS)



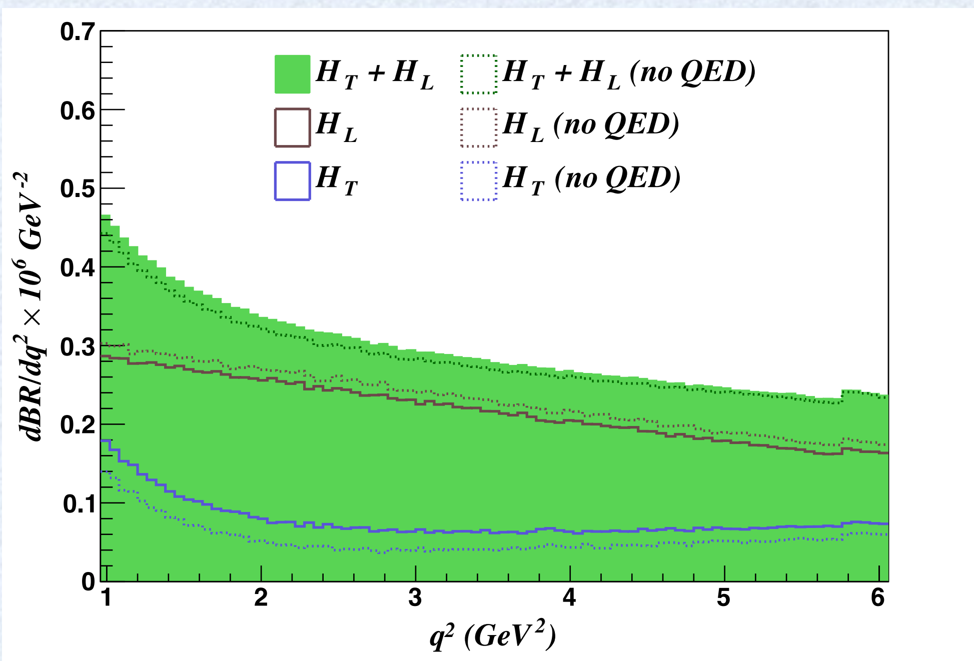


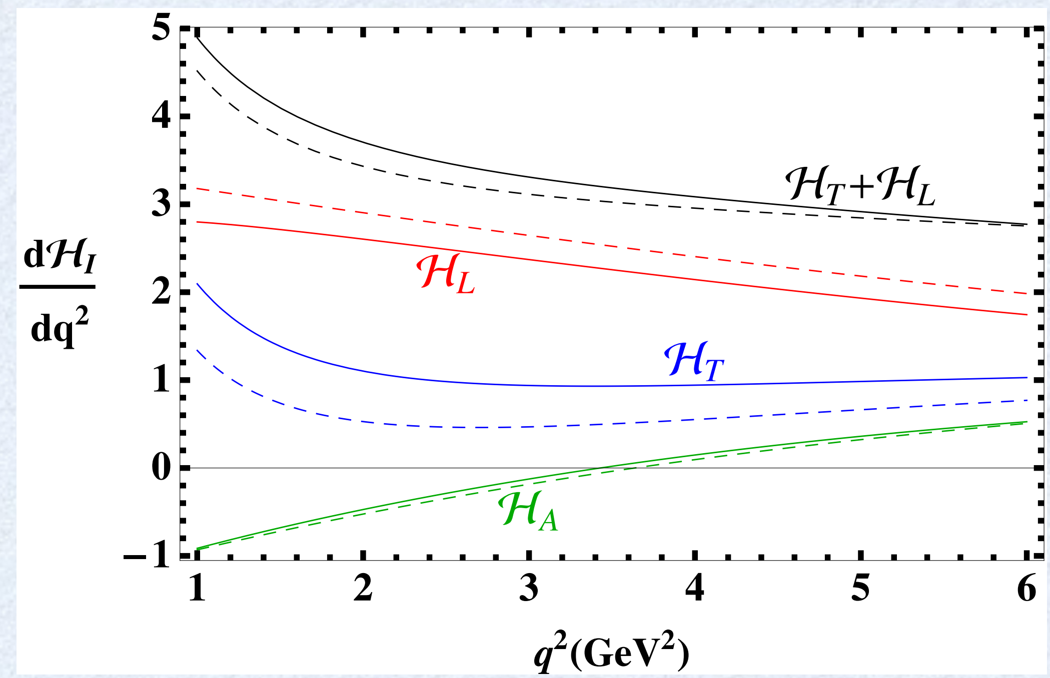


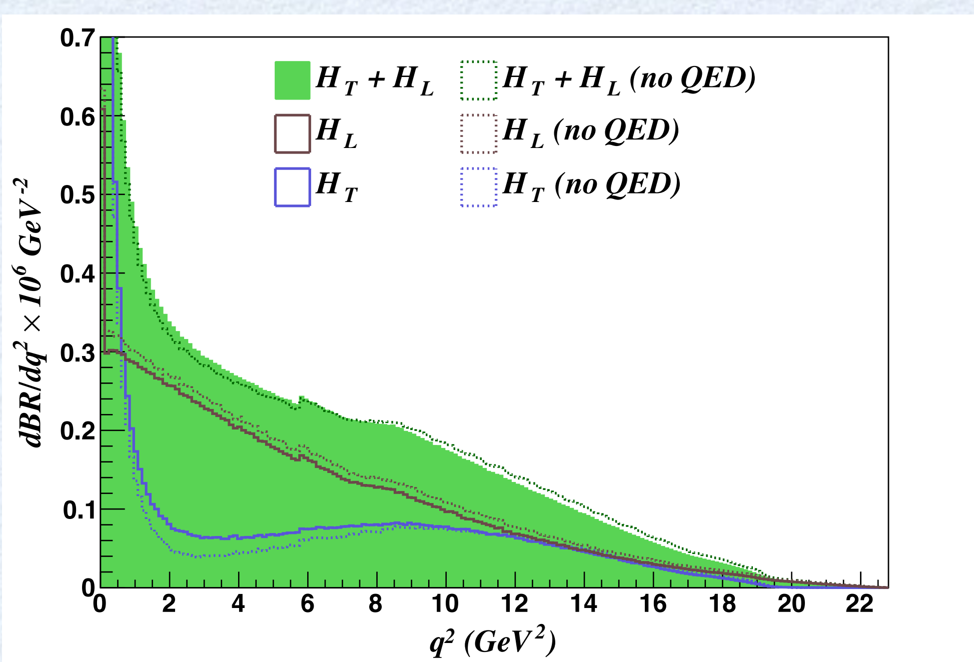


QED Logs: Monte Carlo

- The Monte Carlo study reproduces the main features of the analytical results







Monte Carlo:

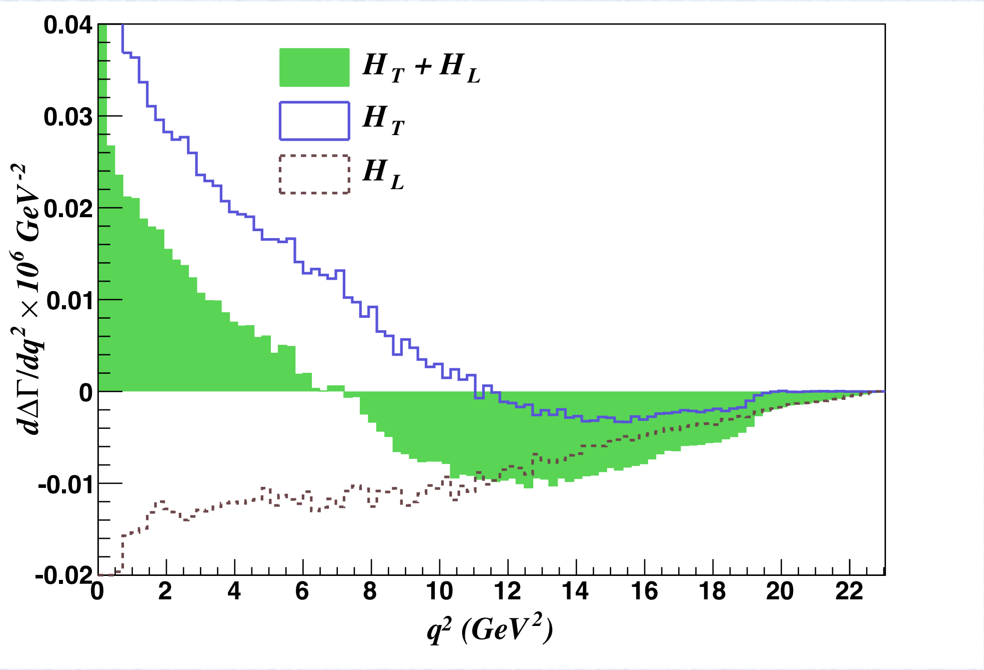
	$q^2 \in [1,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$
$\mathcal{B}$	100	3.5	3.5
$\mathcal{H}_T$	19.0	8.0	43.0
$\mathcal{H}_L$	81.0	-4.5	-5.5

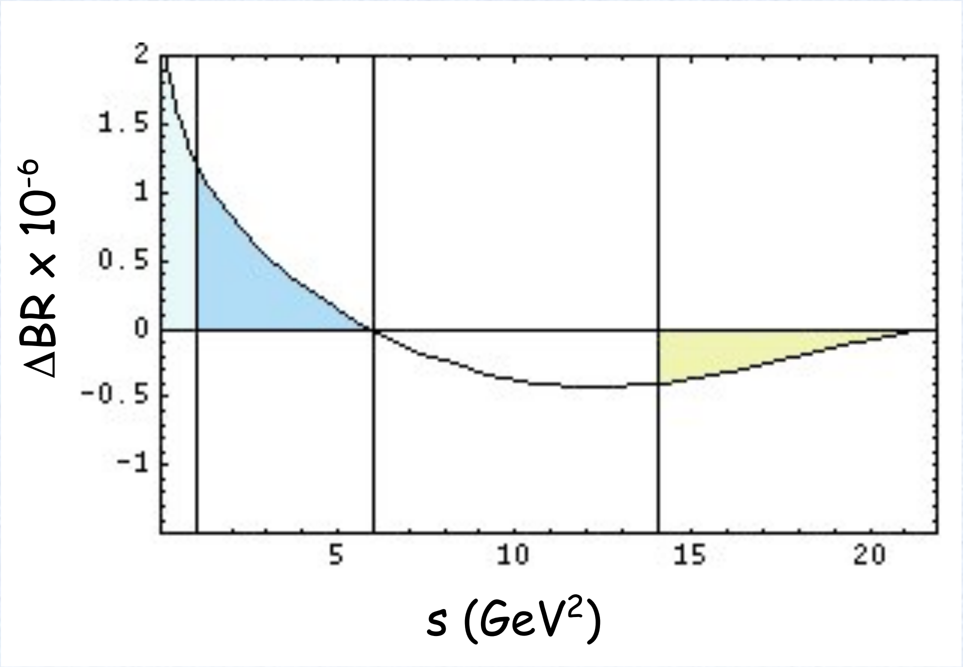
Analytical:

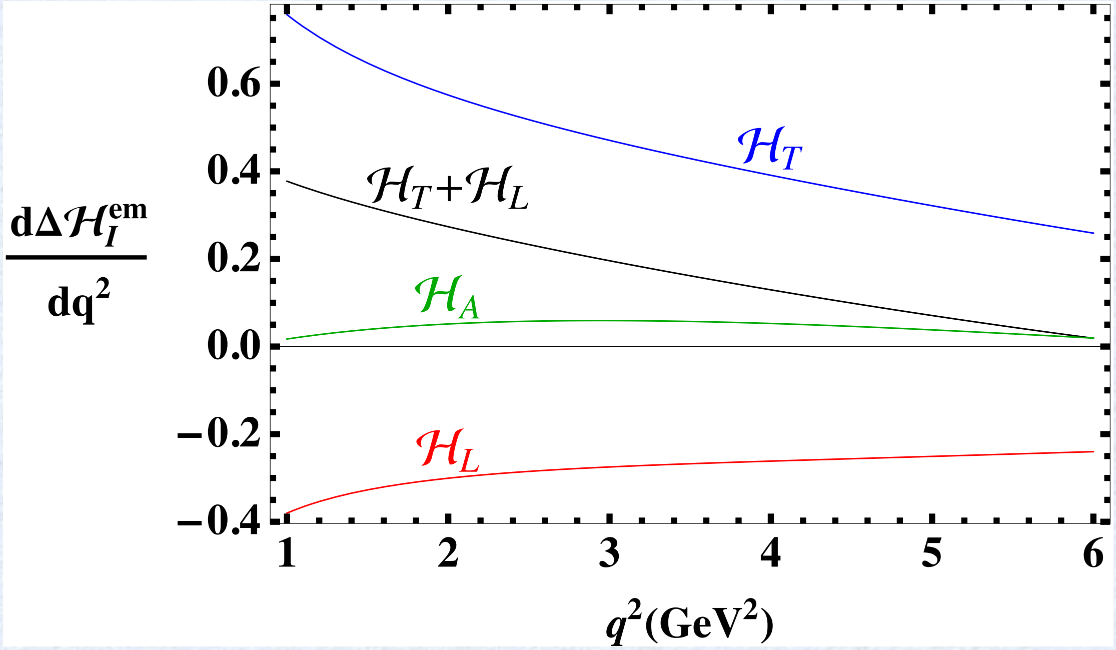
	$q^2 \in [1,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$
$\mathcal{B}$	100	5.1	5.1
$\mathcal{H}_T$	19.5	14.1	72.5
$\mathcal{H}_L$	80.0	-8.7	-10.9

QED Logs: Monte Carlo

- The Monte Carlo study reproduces the main features of the analytical results







Definition of observables

- At leading order in QED and at all orders in QCD, the double differential width is a quadratic polynomial: $\Gamma \sim a \cos^2\theta + b \cos\theta + c$.
- Γ receives non polynomial log-enhanced QED corrections
- Best strategy: **measure individual observables (BR, A_{FB}) and use Legendre polynomial as projectors**

$$H_I(q^2) = \int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} W_I(z) dz$$

$$W_T = \frac{2}{3} P_0(z) + \frac{10}{3} P_2(z), \qquad W_3 = P_3(z)$$

$$W_L = \frac{1}{3} P_0(z) - \frac{10}{3} P_2(z), \qquad W_4 = P_4(z)$$

$$W_A = \frac{4}{3} \mathrm{sign}(z).$$

new observables

$$\frac{d\Gamma}{dq^2} = \int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} dz = H_T + H_L$$

$$\frac{dA_{\mathrm{FB}}}{dq^2} = \int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} \mathrm{sign}(z) dz = \frac{3}{4} H_A$$

$$\frac{d\overline{A}_{\mathrm{FB}}}{dq^2} = \frac{\int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} \mathrm{sign} dz}{\int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} dz} = \frac{3}{4} \frac{H_A}{H_T + H_L}$$

Inclusive: present status

	δ_{th}		R(μ/e)
$\mathcal{H}_T[1,6]_{\mu\mu} = (4.03 \pm 0.28) \cdot 10^{-7}$	±7%	$\mathcal{H}_T[1,6]_{ee} = (5.34 \pm 0.38) \cdot 10^{-7}$	0.75
$\mathcal{H}_L[1,6]_{\mu\mu} = (1.21 \pm 0.07) \cdot 10^{-6}$	±6%	$\mathcal{H}_L[1,6]_{ee} = (1.13 \pm 0.06) \cdot 10^{-6}$	1.07
$\mathcal{H}_A[1,3.5]_{\mu\mu} = (-1.10 \pm 0.05) \cdot 10^{-7}$	±5%	$\mathcal{H}_A[1,3.5]_{ee} = (-1.03 \pm 0.05) \cdot 10^{-7}$	1.07
$\mathcal{H}_A[3.5,6]_{\mu\mu} = (+0.67 \pm 0.12) \cdot 10^{-7}$	±18%	$\mathcal{H}_A[3.5,6]_{ee} = (+0.73 \pm 0.12) \cdot 10^{-7}$	0.92
$\mathcal{H}_3[1,6]_{\mu\mu} = (3.71 \pm 0.50) \cdot 10^{-9}$	±13%	$\mathcal{H}_3[1,6]_{ee} = (8.92 \pm 1.20) \cdot 10^{-9}$	0.42
$\mathcal{H}_4[1,6]_{\mu\mu} = (3.50 \pm 0.32) \cdot 10^{-9}$	±9%	$\mathcal{H}_4[1,6]_{ee} = (8.41 \pm 0.78) \cdot 10^{-9}$	0.42
$\mathcal{B}[1,6]_{\mu\mu} = (1.62 \pm 0.09) \cdot 10^{-7}$	±5%	$\mathcal{B}[1,6]_{ee} = (1.67 \pm 0.10) \cdot 10^{-7}$	0.97
$\mathcal{B}[> 14.4]_{\mu\mu} = (2.53 \pm 0.70) \cdot 10^{-7}$	±28%	$\mathcal{B}[> 14.4]_{ee} = (2.20 \pm 0.70) \cdot 10^{-7}$	1.15

- Scale uncertainties dominate at low-q^2
- Power corrections and scale uncertainties dominate at high-q^2
- Log-enhanced QED corrections at low and high q^2 are correlated

QED logs in R_K?

- Inclusive: at BaBar and Belle the Xs system is reconstructed as sum over exclusive final states. **Most of the photons are not recovered nor searched for**. The analysis is performed by letting them be part of the hadronic system: **$\log(m_{e,\mu}/m_b)$ is physical**.

- Exclusive: At LHCb the B meson are massively boosted and collinear photons can be extremely energetic. LHCb uses PHOTOS to put back into the leptons all soft/collinear emissions. This procedure is cross checked on $J/\psi \to (ee,\mu\mu)$.
  There are no $\log(m_{e,\mu}/m_b)$ enhanced corrections.

- Given the not-so-great agreement between the analytic calculation and the MC simulation, LHCb is pursuing a data-driven approach to the reconstruction of missing photons

High-Q^2: Reducing the errors

- Normalize the decay width to the semileptonic $B \to X_u l \nu$ rate with the ***same dilepton invariant mass cut***:

$$\mathcal{R}(s_0) = \frac{\int_{\hat{s}_0}^{1} \mathrm{d}\hat{s} \, \frac{\mathrm{d}\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{\mathrm{d}\hat{s}}}{\int_{\hat{s}_0}^{1} \mathrm{d}\hat{s} \, \frac{\mathrm{d}\Gamma(\bar{B}^0 \to X_u \ell \nu)}{\mathrm{d}\hat{s}}}$$

[Ligeti,Tackmann]

- ***Impact of*** $1/m_b^2$ ***and*** $1/m_b^3$ ***power corrections drastically reduced:***

$$\begin{aligned} \mathcal{R}(14.4)_{\mu\mu} =& (2.62 \pm 0.09_{\text{scale}} \pm 0.03_{m_t} \pm 0.01_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.23_{\text{CKM}} \\ & \pm 0.0002_{\lambda_2} \pm 0.09_{\rho_1} \pm 0.04_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s}) \cdot 10^{-3} \\ =& (2.62 \pm 0.30) \cdot 10^{-3} \end{aligned}$$

$$\begin{aligned} \mathcal{R}(14.4)_{ee} =& (2.25 \pm 0.12_{\text{scale}} \pm 0.03_{m_t} \pm 0.02_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.20_{\text{CKM}} \\ & \pm 0.02_{\lambda_2} \pm 0.14_{\rho_1} \pm 0.08_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s}) \cdot 10^{-3} \\ =& (2.25 \pm 0.31) \cdot 10^{-3} \end{aligned}$$

- ***The largest source of uncertainty is*** V_{ub}

Present Status

BaBar: 471x10[6] BB pairs (424 fb^{-1})
Belle: 152x10[6] BB pairs (140 fb^{-1})
711 fb^{-1} on tape!!

- World averages (Babar, Belle):

$$\mathrm{BR}^{\mathrm{exp}} = (1.58 \pm 0.37) \times 10^{-6} \qquad q^2 \in [1,6]$$

$$\mathrm{BR}^{\mathrm{exp}} = (0.48 \pm 0.10) \times 10^{-6} \qquad q^2 > 14.4$$

$$\overline{A}_{\mathrm{FB}}^{\mathrm{exp}} = \begin{cases} 0.34 \pm 0.24 & q^2 \in [0.2, 4.3] \\ 0.04 \pm 0.31 & q^2 \in [4.3, 7.3(8.1)] \end{cases}$$

$\delta_{\mathrm{exp}} \approx 23\%$
$\delta_{\mathrm{exp}} \approx 21\%$

non-optimal binning

- Theory:

$$\mathrm{BR}^{\mathrm{th}} = (1.65 \pm 0.10) \times 10^{-6} \qquad q^2 \in [1,6]$$

$$\mathrm{BR}^{\mathrm{th}} = (0.237 \pm 0.070) \times 10^{-6} \qquad q^2 > 14.4$$

$$\overline{A}_{\mathrm{FB}}^{\mathrm{th}} = \begin{cases} -0.077 \pm 0.006 & q^2 \in [0.2, 4.3] \\ 0.05 \pm 0.02 & q^2 \in [4.3, 7.3(8.1)] \end{cases}$$

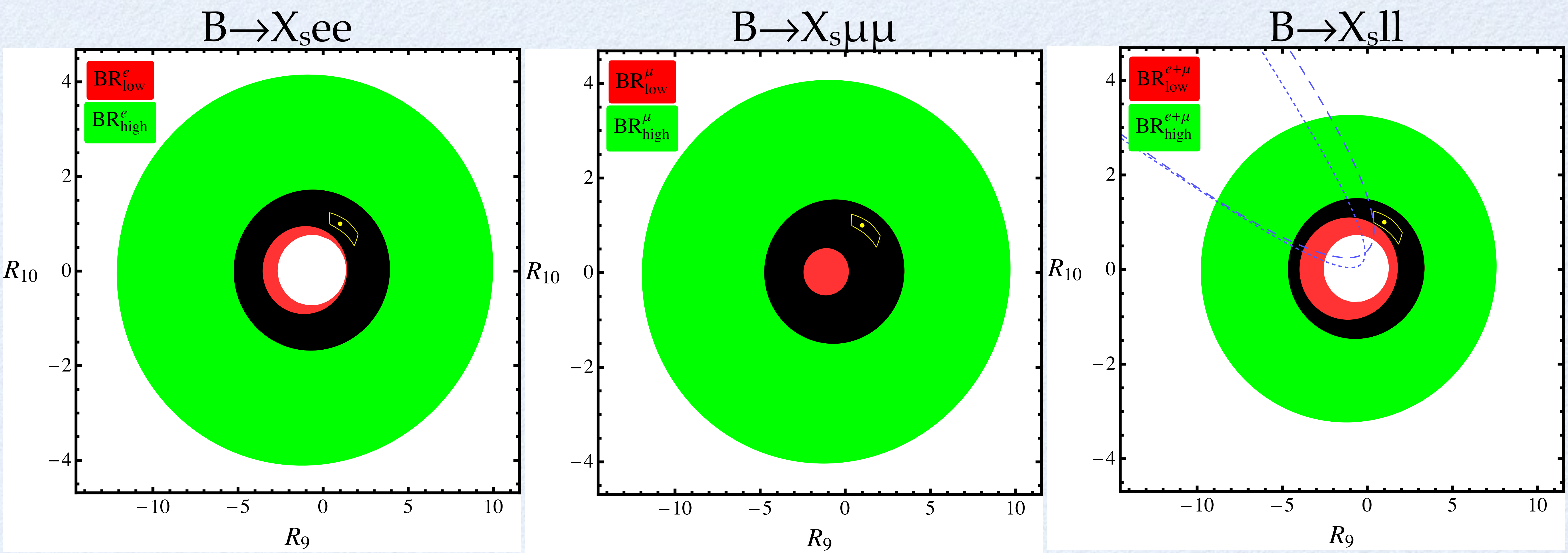
$\delta_{\mathrm{th}} \approx 6\%$
$\delta_{\mathrm{th}} \approx 30\%$

non-optimal binning

- $$\mathrm{BR} = H_T + H_L \qquad \overline{A}_{\mathrm{FB}} = \frac{3}{4}\frac{H_A}{H_T + H_L}$$

PRESENT STATUS

- Constraints in the $[R_9,R_{10}]$ plane ($R_i = C_i(\mu_0)/C_i^{\rm SM}(\mu_0)$):



- Note that $C_9^{\rm SM}(\mu_0) = 1.61$ and $C_{10}^{\rm SM}(\mu_0) = -4.26$
- Best fits from the exclusive anomaly translate in $R_9 \sim 0.3$ (for the single WC fit) or $R_9 \sim 0.65$ and $R_{10} \sim 0.9$ (for the $C_9^{\rm NP} = -C_{10}^{\rm NP}$ scenario)

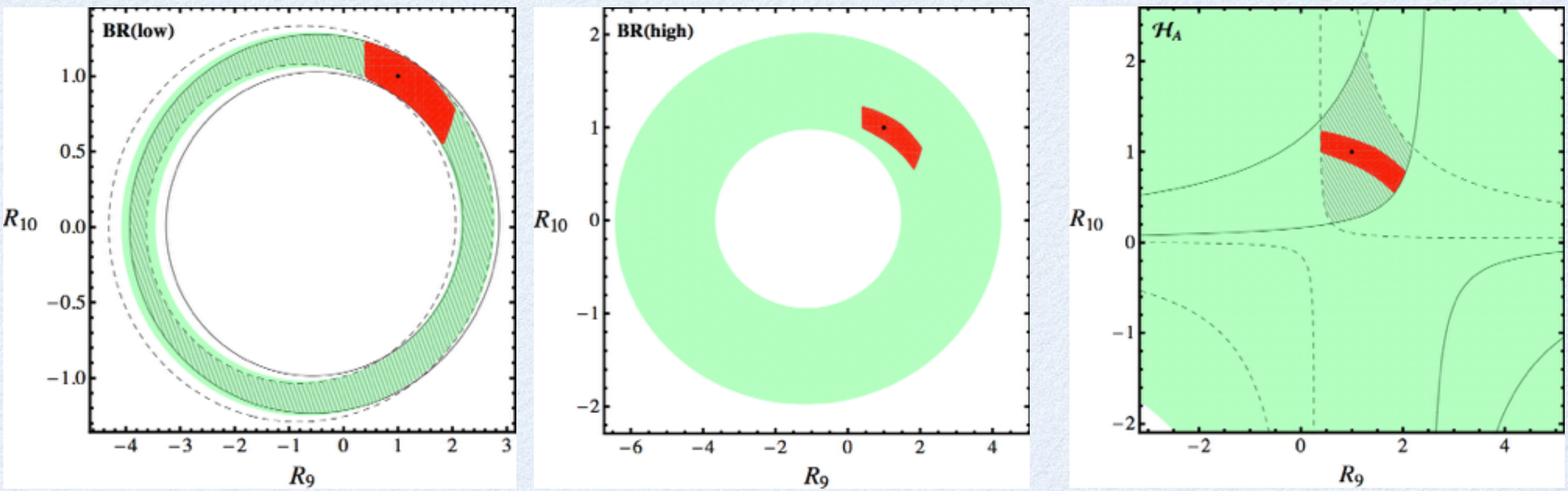
Projections

- Projected reach with 50 ab^{-1} of integrated luminosity

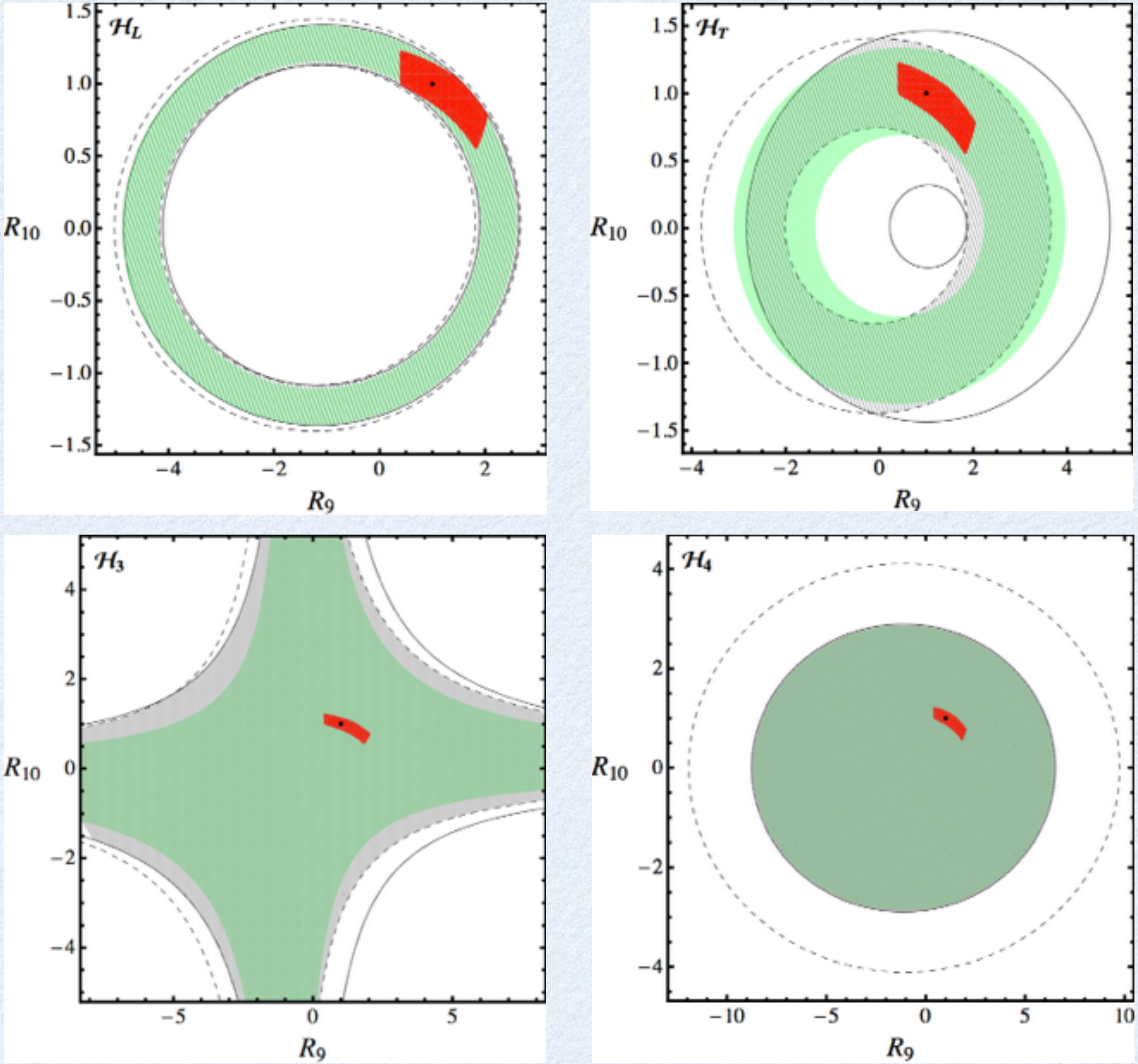
$$\mathcal{O}_{\rm exp} = \int \frac{d^2\mathcal{N}}{d\hat{s}dz} W[\hat{s}, z]\, d\hat{s}\, dz \;,$$

$$\delta\mathcal{O}_{\rm exp} = \left[\int \frac{d^2\mathcal{N}}{d\hat{s}dz} W[\hat{s}, z]^2\, d\hat{s}\, dz\right]^{\frac{1}{2}}$$

	[1, 3.5]	[3.5, 6]	[1, 6]	> 14.4
$\mathcal{B}$	3.7 %	4.0 %	3.0 %	4.1%
$\mathcal{H}_T$	24 %	21 %	16 %	-
$\mathcal{H}_L$	5.8 %	6.8 %	4.6 %	-
$\mathcal{H}_A$	37 %	44 %	200 %	-
$\mathcal{H}_3$	240 %	180 %	150 %	-
$\mathcal{H}_4$	140 %	360 %	140 %	-

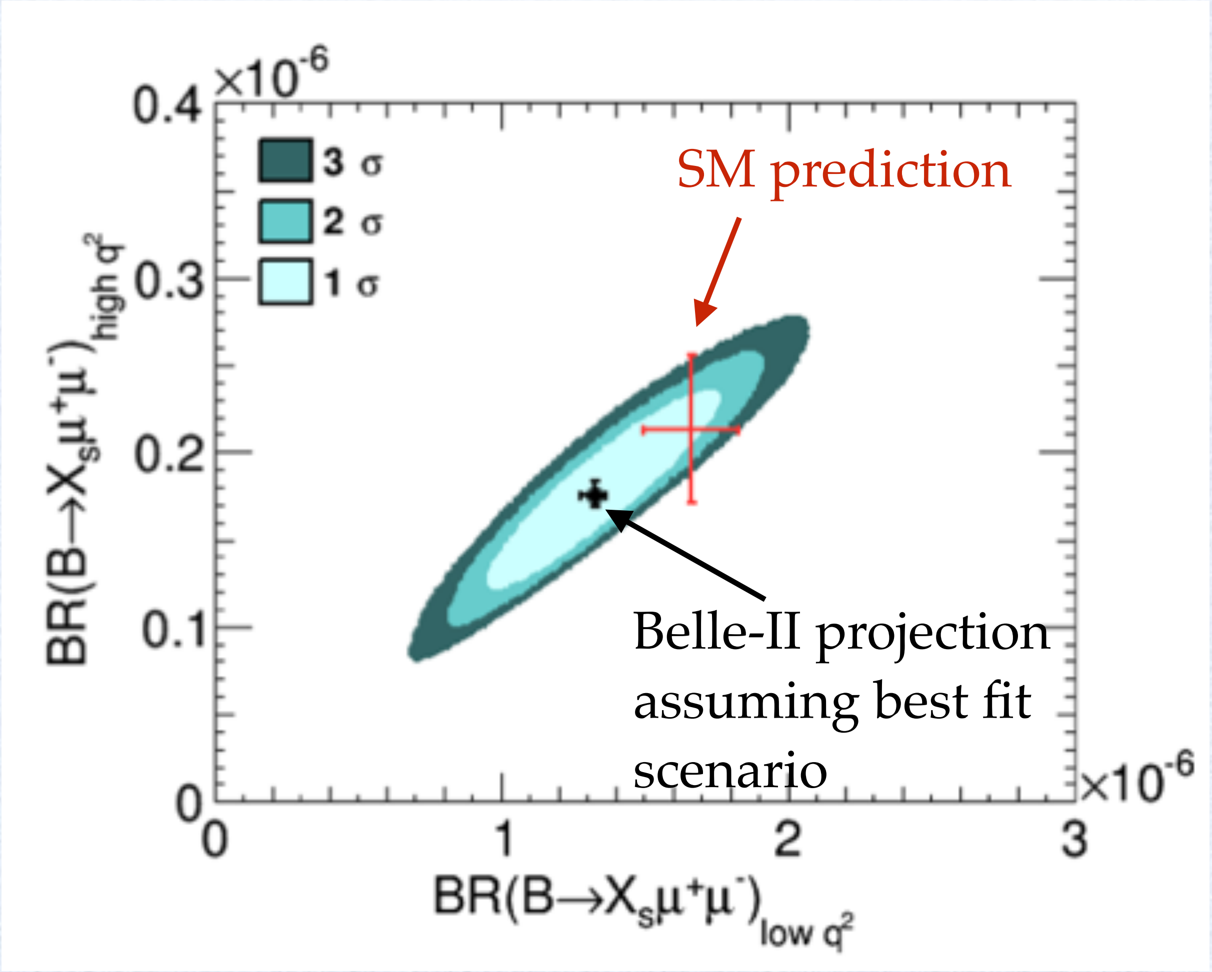


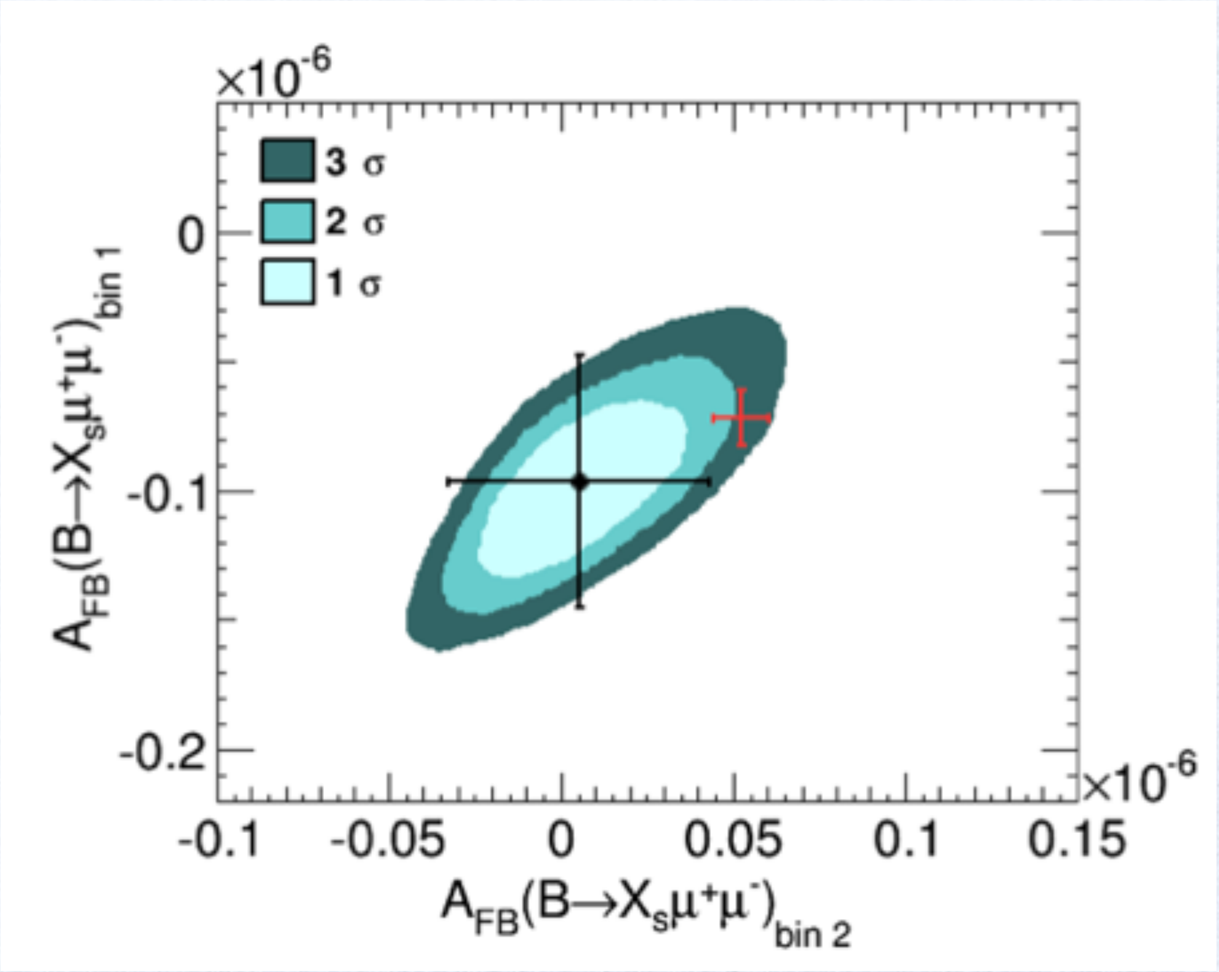
PROJECTIONS



Inclusive/Exclusive Interplay

- The effects on C_9 and C_9' are large enough to be easily checked at Belle II with inclusive decays (free of most uncertainties that plague the exclusive modes)





[Hurth, Mahmoudi 1411.2786]

Conclusions

- Inclusive calculations are almost at the "end-of-the-road", are clean but require Belle II
- Inclusive modes are sensitive to the treatment of QED radiation. The effect can be very large (depending on the observable) and can be exploited to test further combinations of Wilson coefficients
- Exclusive modes have a rich phenomenology but are plagued by form factor uncertainties (progress from lattice QCD expected), parametric uncertainties (light-cone wave functions, …) and power corrections
- LHCb data are in general agreement with the SM predictions with the exception of an angular distribution (P_5'), the BR at high-q^2 and a lepton flavor universality breaking ratio (R_K)

Backup Slides

Exclusive: Observables (K*)

- LHCb measured the complete angular distribution for the K* channel:

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\vec{\Omega}}\bigg|_{\mathrm{P}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_{\mathrm{L}})\sin^2\theta_K + F_{\mathrm{L}}\cos^2\theta_K$$

$$+\tfrac{1}{4}(1-F_{\mathrm{L}})\sin^2\theta_K\cos 2\theta_l$$

$$-F_{\mathrm{L}}\cos^2\theta_K\cos 2\theta_l + S_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi$$

$$+S_4\sin 2\theta_K\sin 2\theta_l\cos\phi + S_5\sin 2\theta_K\sin\theta_l\cos\phi$$

$$+\tfrac{4}{3}A_{\mathrm{FB}}\sin^2\theta_K\cos\theta_l + S_7\sin 2\theta_K\sin\theta_l\sin\phi$$

$$+S_8\sin 2\theta_K\sin 2\theta_l\sin\phi + S_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$

$$P'_{i=4,5,6,8} = \frac{S_{j=4,5,7,8}}{\sqrt{F_{\mathrm{L}}(1-F_{\mathrm{L}})}}.$$

Exclusive: Observables (K*)

- All these observables are given by simple formulas in terms of helicity amplitudes:

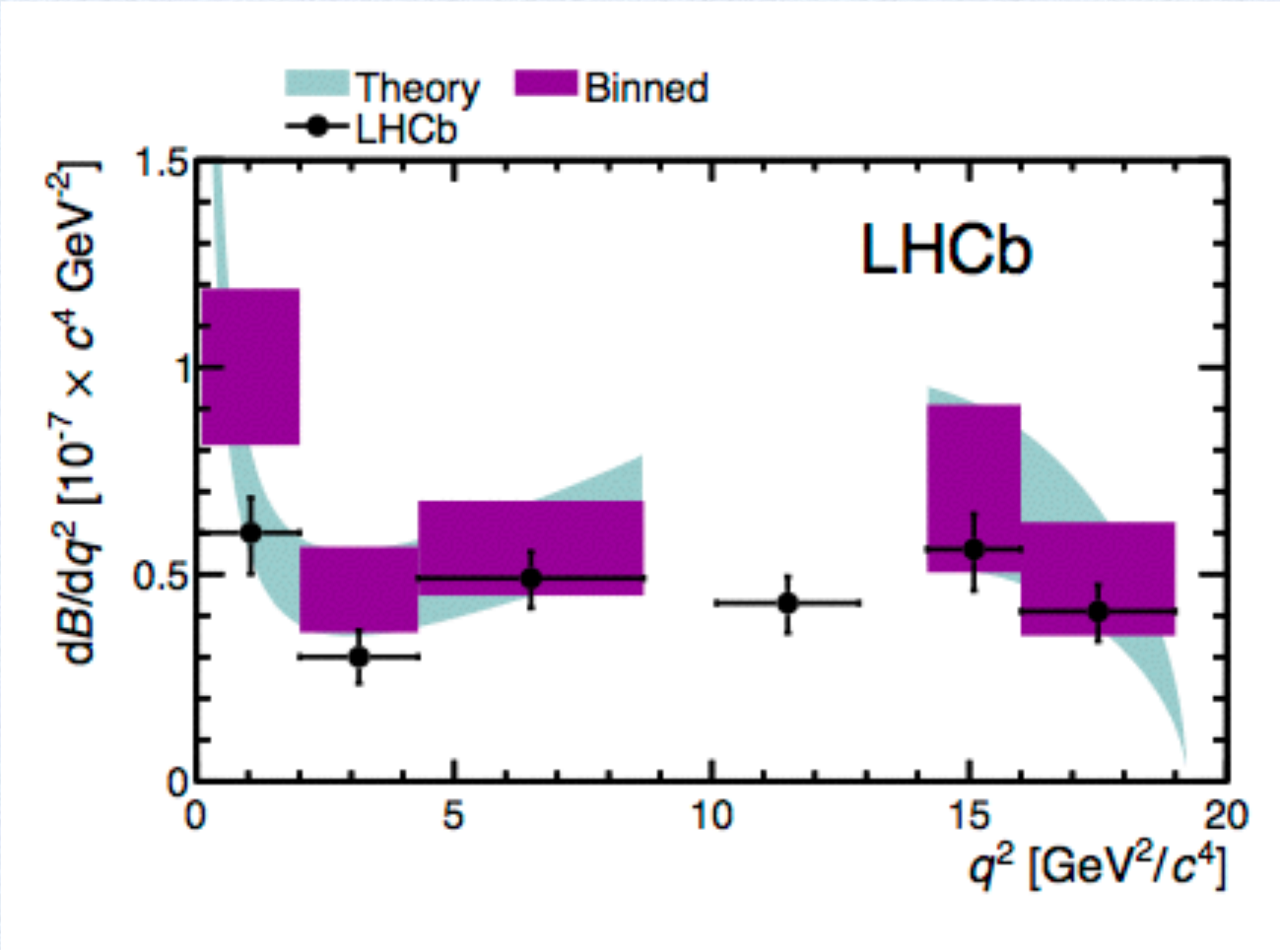
$$A_\perp^{L,R} = \sqrt{2}Nm_B(1-\hat{s})\left[(\mathcal{C}_9^{\rm eff}+\mathcal{C}_9^{\rm eff\prime}) \mp (\mathcal{C}_{10}+\mathcal{C}_{10}^\prime) + \frac{2\hat{m}_b}{\hat{s}}(\mathcal{C}_7^{\rm eff}+\mathcal{C}_7^{\rm eff\prime})\right]\xi_\perp(E_{K^*}),$$

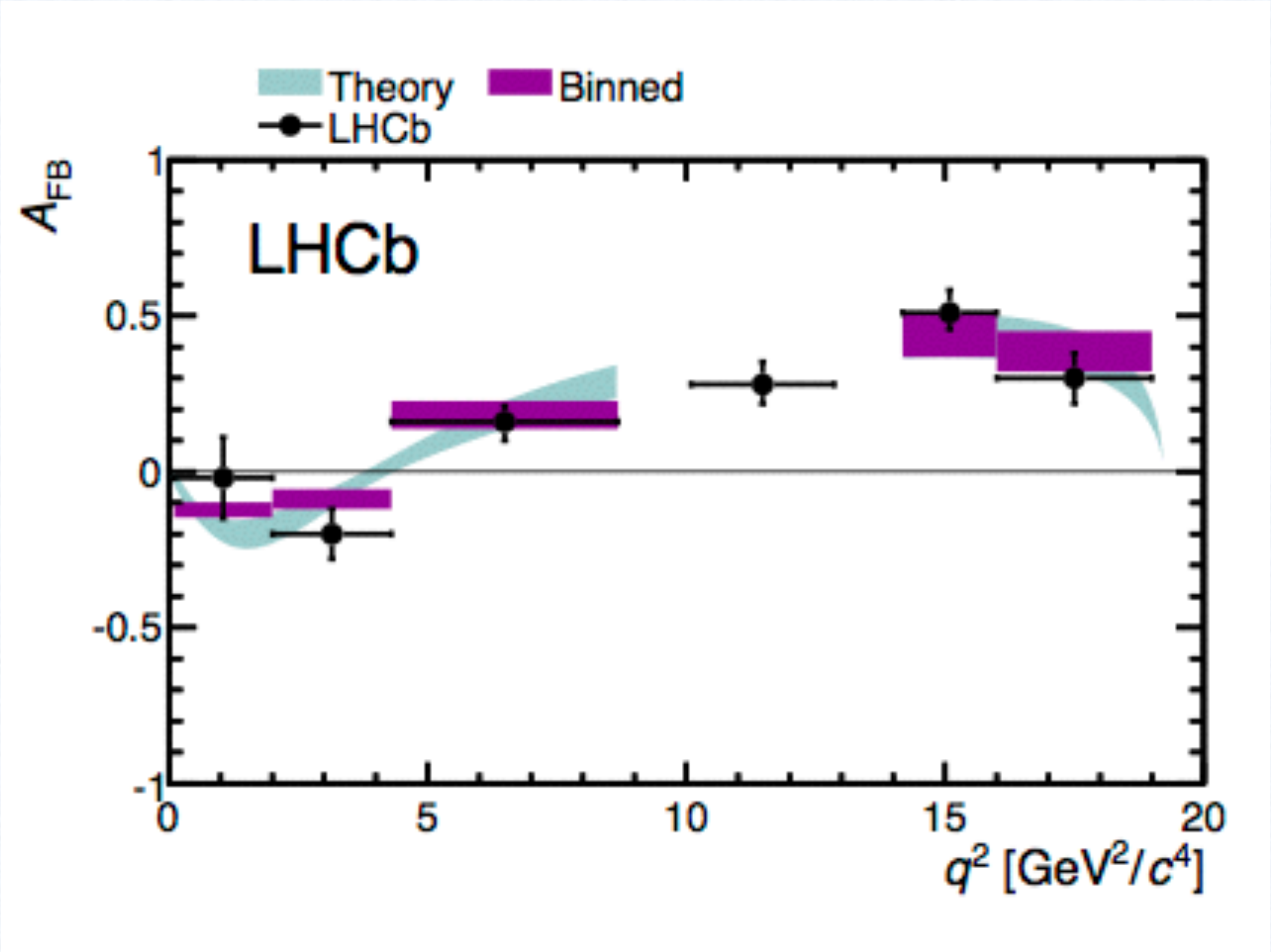
$$A_\parallel^{L,R} = -\sqrt{2}Nm_B(1-\hat{s})\left[(\mathcal{C}_9^{\rm eff}-\mathcal{C}_9^{\rm eff\prime}) \mp (\mathcal{C}_{10}-\mathcal{C}_{10}^\prime) + \frac{2\hat{m}_b}{\hat{s}}(\mathcal{C}_7^{\rm eff}-\mathcal{C}_7^{\rm eff\prime})\right]\xi_\perp(E_{K^*})$$

$$A_0^{L,R} = -\frac{Nm_B(1-\hat{s})^2}{2\hat{m}_{K^*}\sqrt{\hat{s}}}\left[(\mathcal{C}_9^{\rm eff}-\mathcal{C}_9^{\rm eff\prime}) \mp (\mathcal{C}_{10}-\mathcal{C}_{10}^\prime) + 2\hat{m}_b(\mathcal{C}_7^{\rm eff}-\mathcal{C}_7^{\rm eff\prime})\right]\xi_\parallel(E_{K^*}).$$

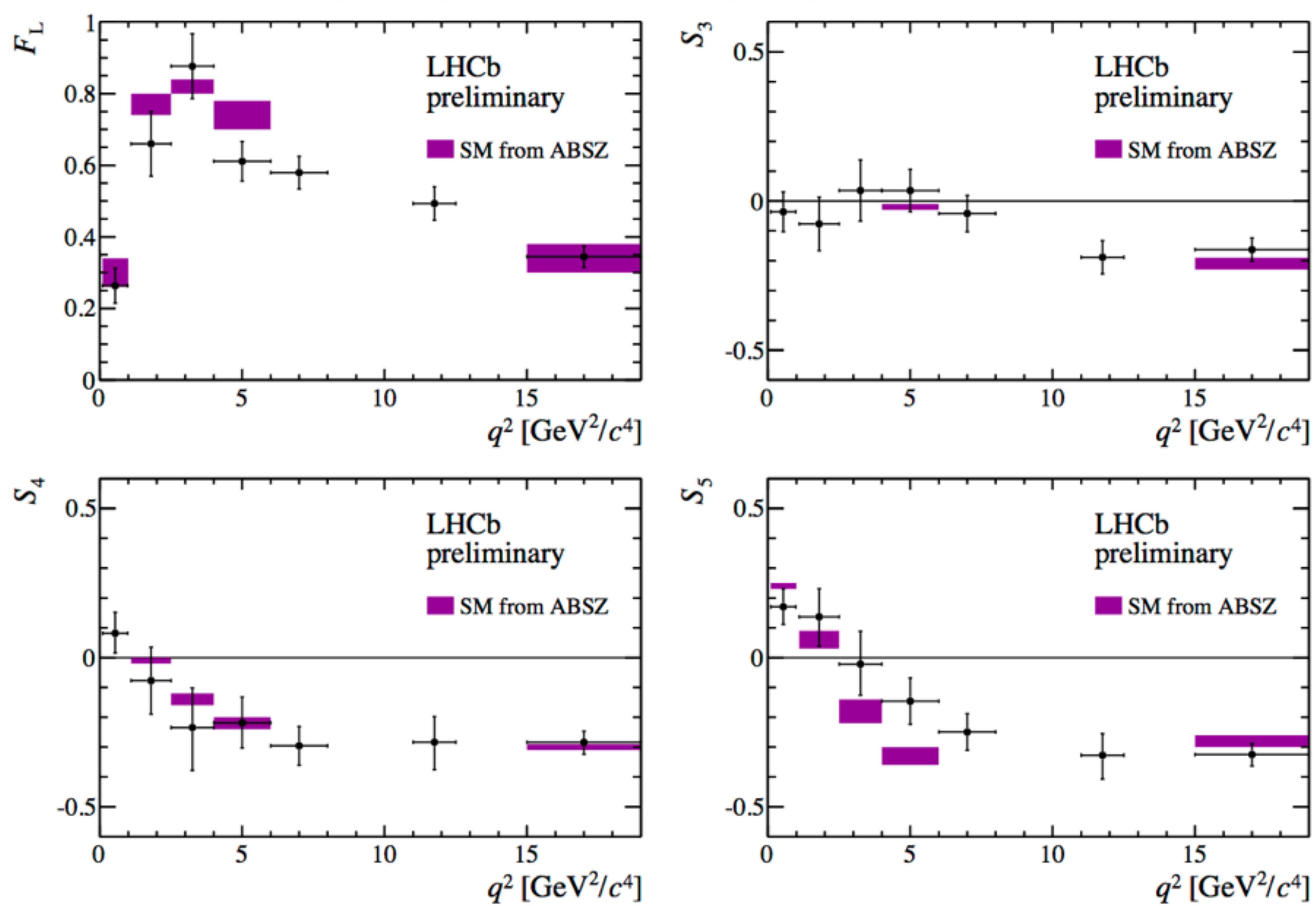
- These formulas hold at leading power and receive O(α_s) corrections (that are included in the numerics)

Exclusive: LHCb Results



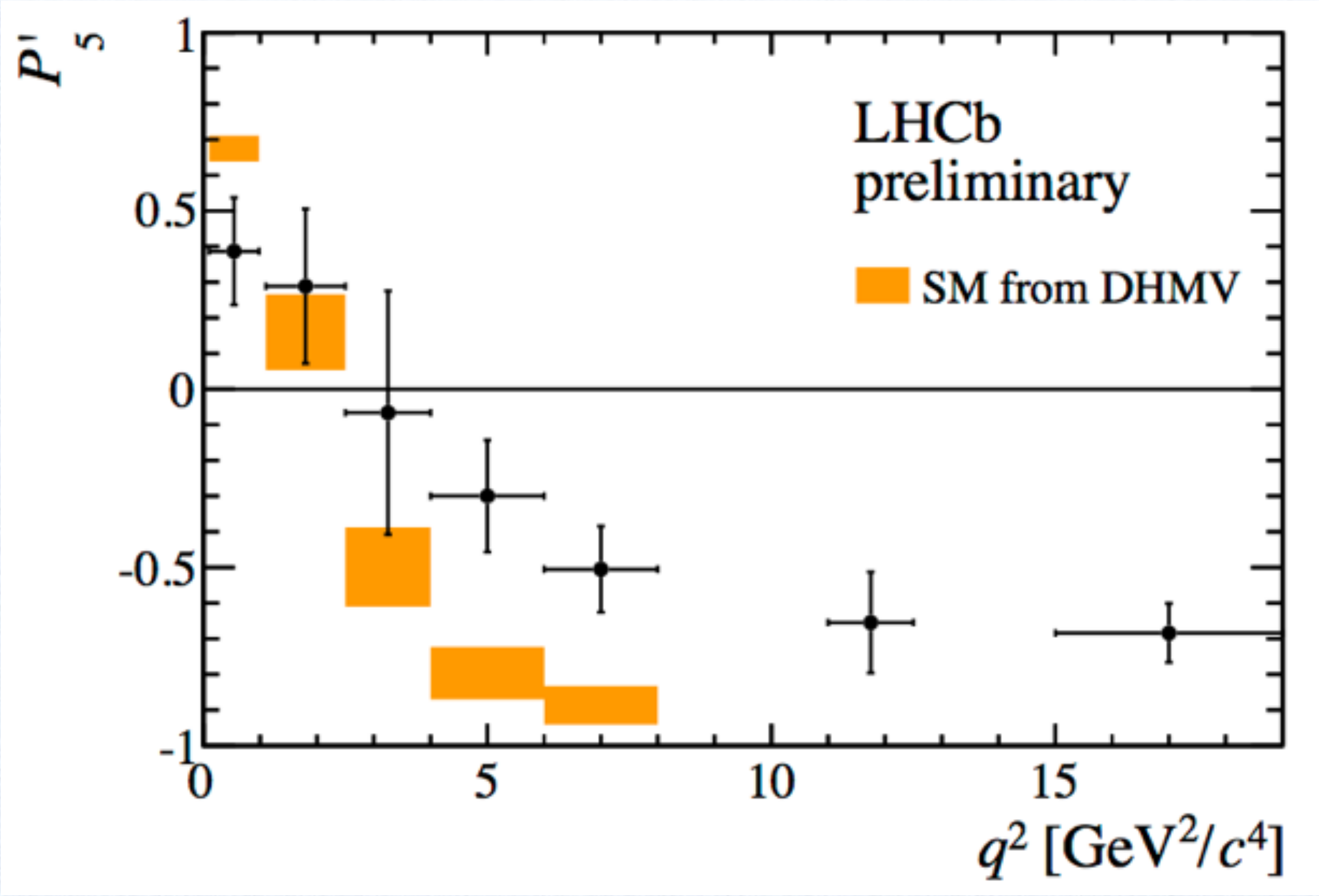


Exclusive: LHCb Results



Exclusive: LHCb Results

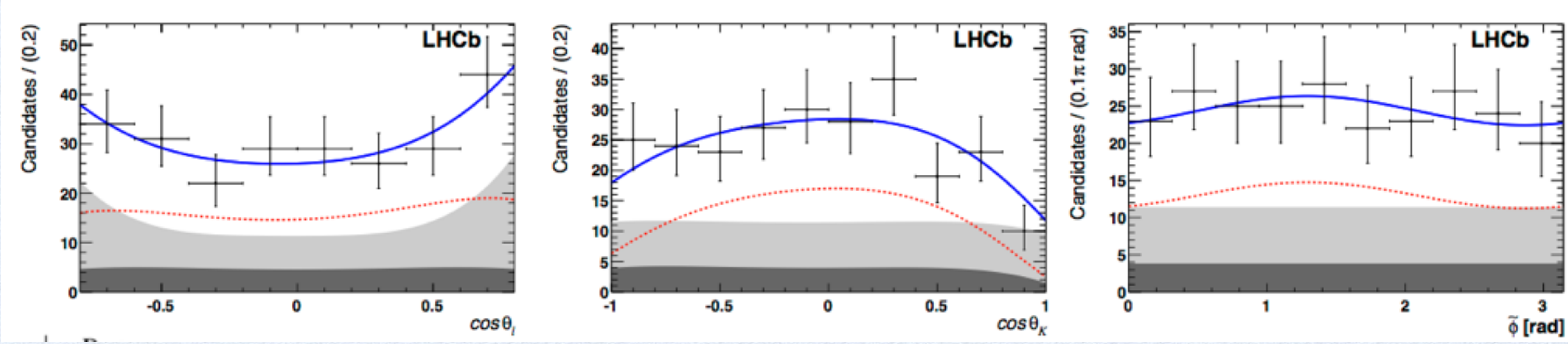
- In the elusive P_5' distribution a 3.7 sigma excess is observed



Exclusive: LHCb Results

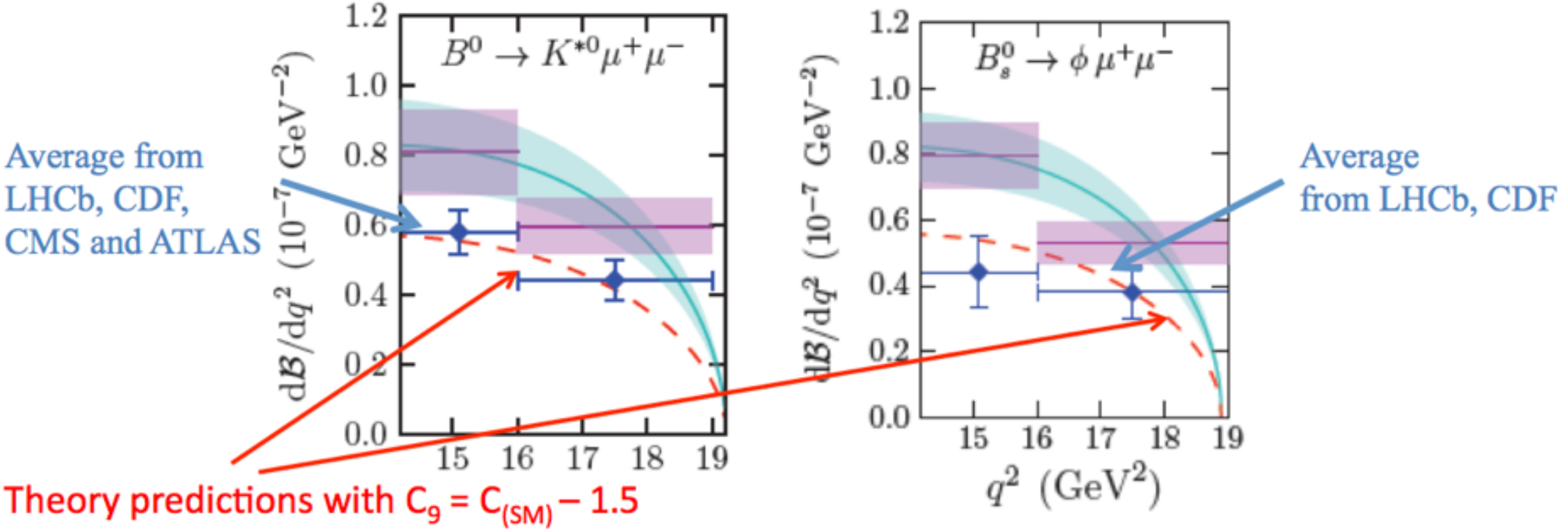
- Angular distributions in B→K*ll

Observable	Measurement	SM prediction†
$F_{\rm L}$	$+0.16 \pm 0.06 \pm 0.03$	$+0.10^{+0.11}_{-0.05}$
$A_{\rm T}^{(2)}$	$-0.23 \pm 0.23 \pm 0.05$	$0.03^{+0.05}_{-0.04}$
$A_{\rm T}^{\rm Re}$	$+0.10 \pm 0.18 \pm 0.05$	$-0.15^{+0.04}_{-0.03}$
$A_{\rm T}^{\rm Im}$	$+0.14 \pm 0.22 \pm 0.05$	$(-0.2^{+1.2}_{-1.2}) \times 10^{-4}$



Exclusive: LHCb Results

- Branching ratio at high-q^2



LHCb: JHEP 06 (2014) 133, JHEP 08 (2013)131, JHEP 07 (2013) 084
CDF: Public note 10894, CMS: arXiv: 1308.3409 ATLAS: ATLAS-CONF-2013-038

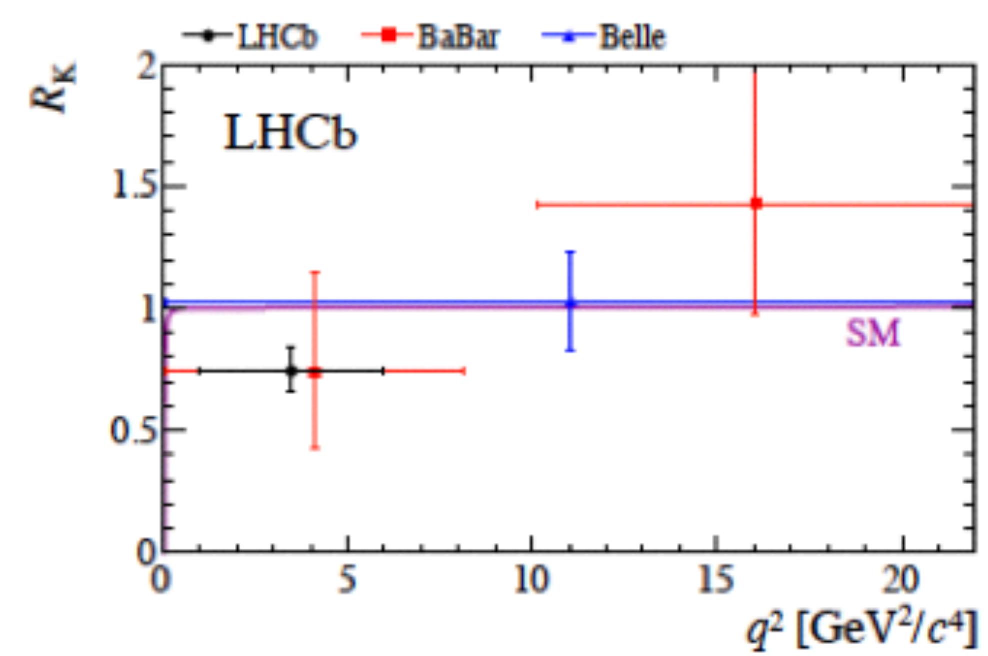
At high-q^2 the only sensible comparison is between rates integrated over a large enough range

Exclusive: LHCb Results

- Evidence for violation of lepton flavor universality?

$$R_k = BR(B^+ \to K^+\mu^+\mu^-)/BR(B^+ \to K^+e^+e^-)$$

- Experimentally the ratio is fairly clean (stat dominated)



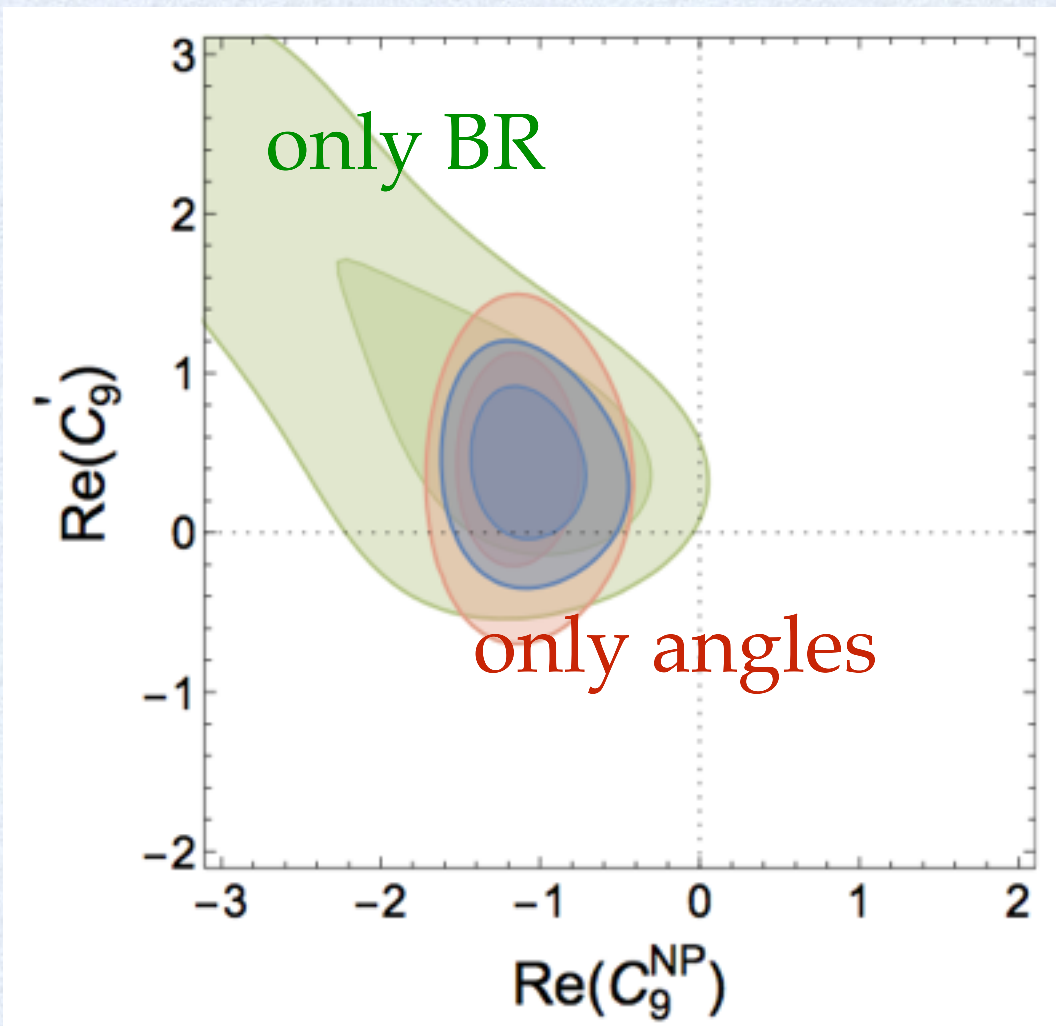
LHCb, PRL 113 (2014) 151601
Belle, PRL 103 (2009) 171801
Babar, PRD 86 (2012) 032012

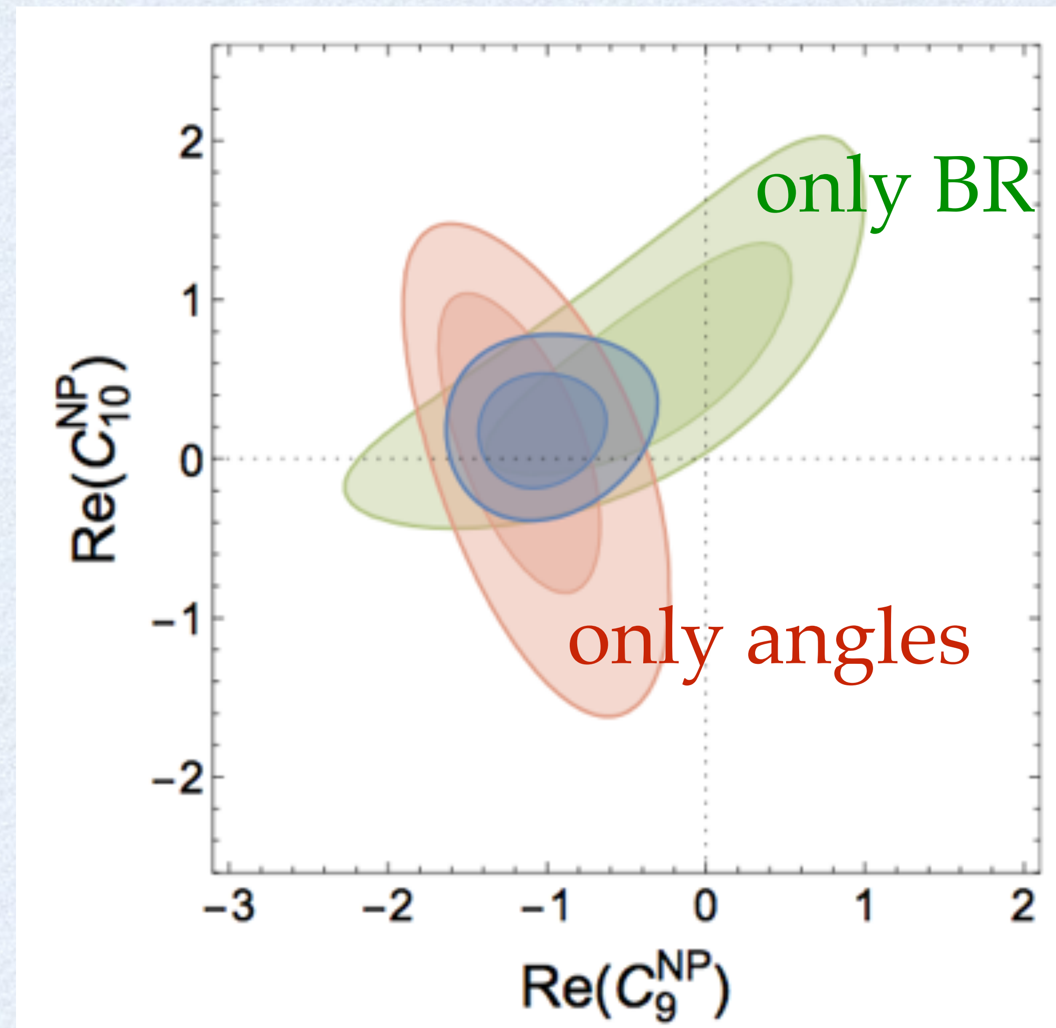
$$R_K = 0.745^{+0.090}_{-0.074}\,(\text{stat})\,^{+0.036}_{-0.036}\,(\text{syst})$$

$$R_K\,(\text{SM}) = 1.0003 \pm 0.0001$$

Wilson coefficients fits

- Deviations in P_5' seem to favor a negative shift in C_9 and a smaller positive contribution to C_9'



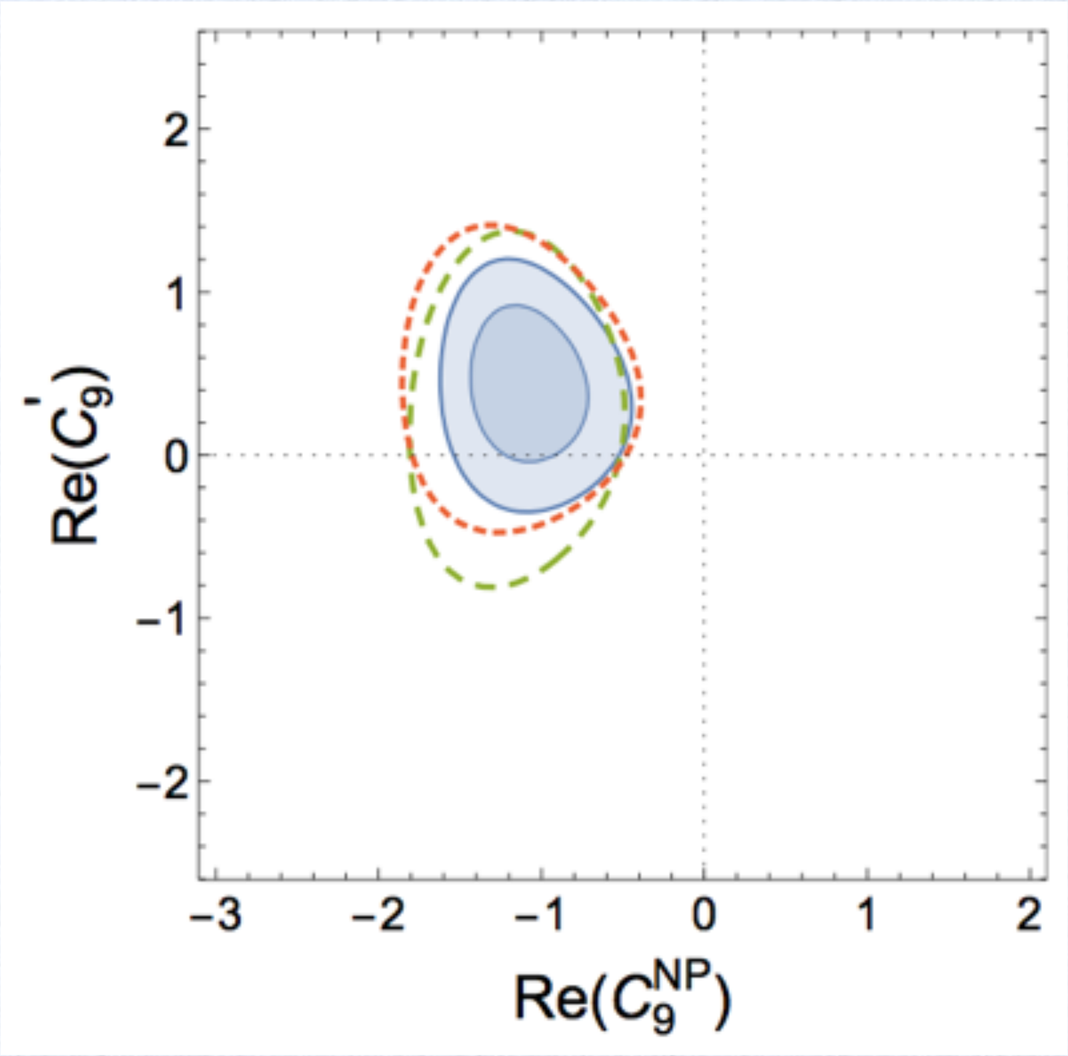


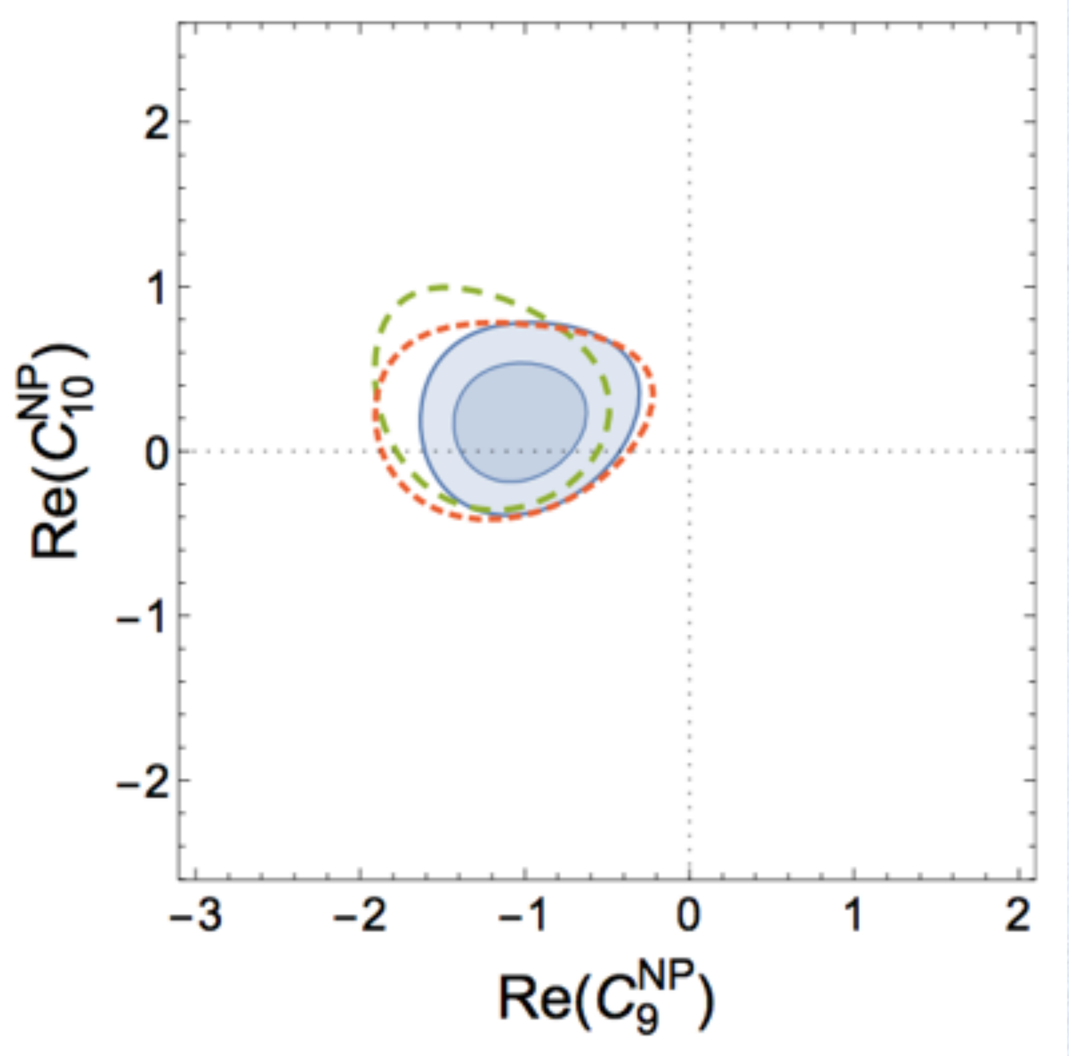
[Altmannshofer, Straub 1411.3161]

- BR data is compatible with the SM

Wilson coefficients fits

- Deviations in P_5' seem to favor a negative shift in C_9 and a smaller positive contribution to C_9'





[Altmannshofer, Straub 1411.3161]

- Dashed contours are obtained doubling some theory uncertainties (form factors, non-form factors)

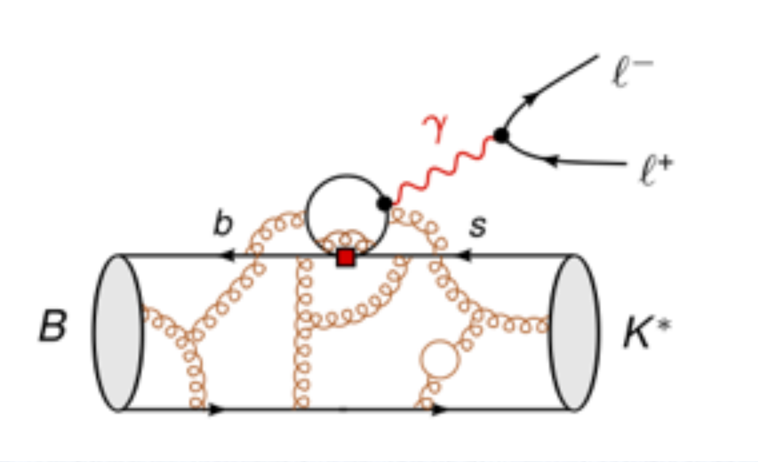
Fit Results

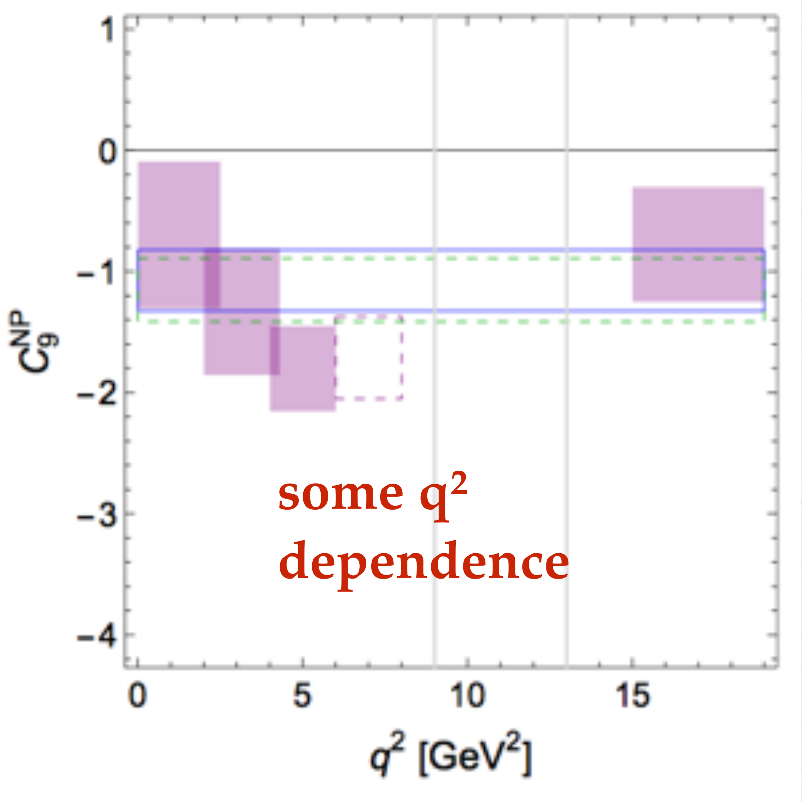
Coeff.	best fit	1σ	2σ	$\sqrt{\chi^2_{\text{b.f.}} - \chi^2_{\text{SM}}}$	p [%]
C_7^{NP}	-0.04	$[-0.07, -0.02]$	$[-0.10, 0.01]$	1.52	1.1
C_7'	0.00	$[-0.05, 0.06]$	$[-0.11, 0.11]$	0.05	0.8
C_9^{NP}	-1.12	$[-1.34, -0.88]$	$[-1.55, -0.63]$	4.33	10.6
C_9'	-0.04	$[-0.26, 0.18]$	$[-0.49, 0.40]$	0.18	0.8
C_{10}^{NP}	0.65	$[0.40, 0.91]$	$[0.17, 1.19]$	2.75	2.5
C_{10}'	-0.01	$[-0.19, 0.16]$	$[-0.36, 0.33]$	0.09	0.8
$C_9^{\text{NP}} = C_{10}^{\text{NP}}$	-0.20	$[-0.41, 0.05]$	$[-0.60, 0.33]$	0.82	0.8
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$	-0.57	$[-0.73, -0.41]$	$[-0.90, -0.27]$	3.88	6.8
$C_9' = C_{10}'$	-0.08	$[-0.33, 0.17]$	$[-0.58, 0.41]$	0.32	0.8
$C_9' = -C_{10}'$	-0.00	$[-0.11, 0.10]$	$[-0.22, 0.20]$	0.03	0.8

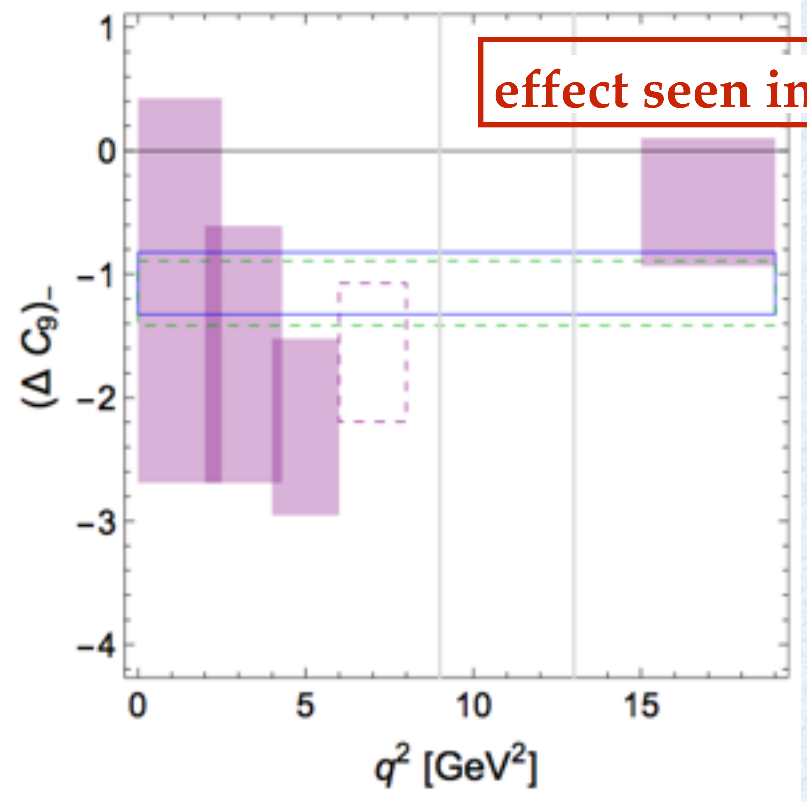
[Altmannshofer, Straub 1411.3161]

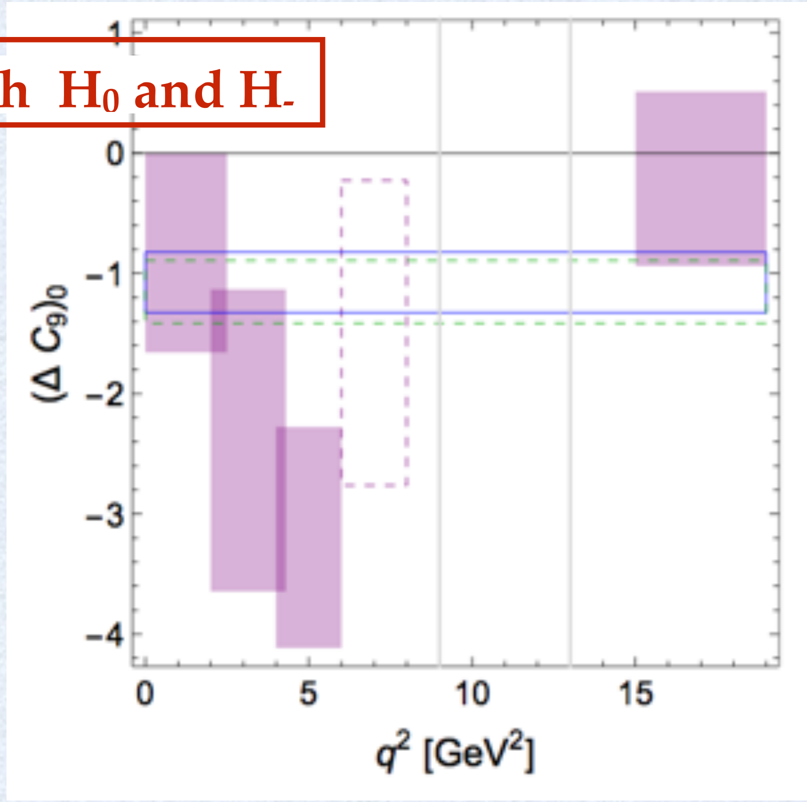
Charmonium troubles?

- Charm loops are included in C_9^{eff} using LCSR [Mannel et al]
- Issues in the calculation of charm effects could mimic NP in C_9 but effects should be:
  - q^2 dependent
  - lepton flavor universal
- What about resonant effects (tail of the J/ψ)?







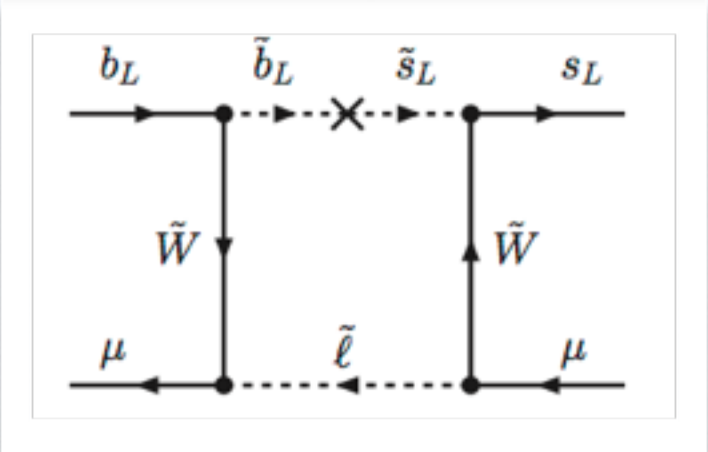


- In David [Straub]'s words: *"interesting hint or cruel coincidence?"*

[Altmannshofer, Straub 1411.3161]

NP interpretation

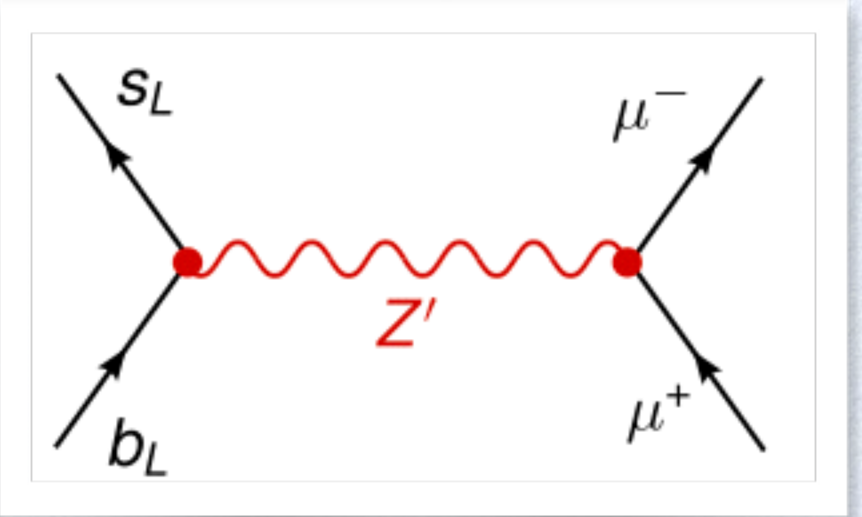
- The deviations in P_5' and R_K are **difficult to embed in NP models**
- Large contributions to C_9 or C_9' cannot be obtained in any minimal flavor violating MSSM and require additional flavor changing couplings (e.g. mass insertions in the 2-3 sector):



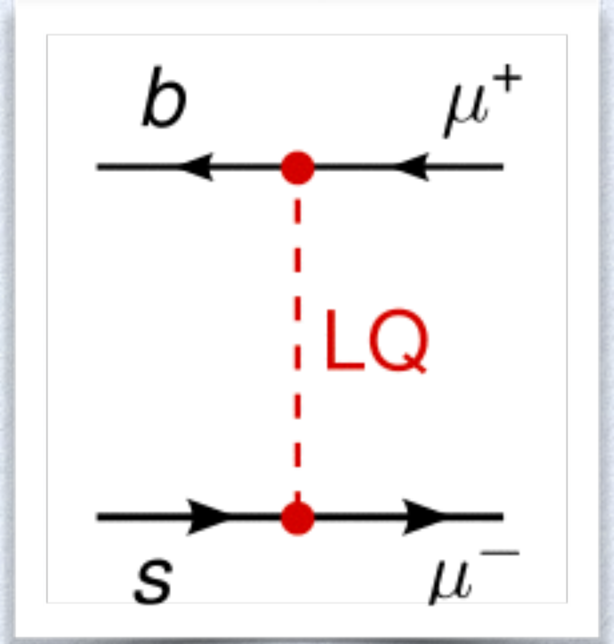
Z penguins can contribute to C10 but not to C9 because the Z current is mostly axial:

$$J_\mu^Z \propto (4s_W^2 - 1)\bar{\ell}\gamma_\mu\ell + \bar{\ell}\gamma_\mu\gamma_5\ell$$

- FC Z′ models:

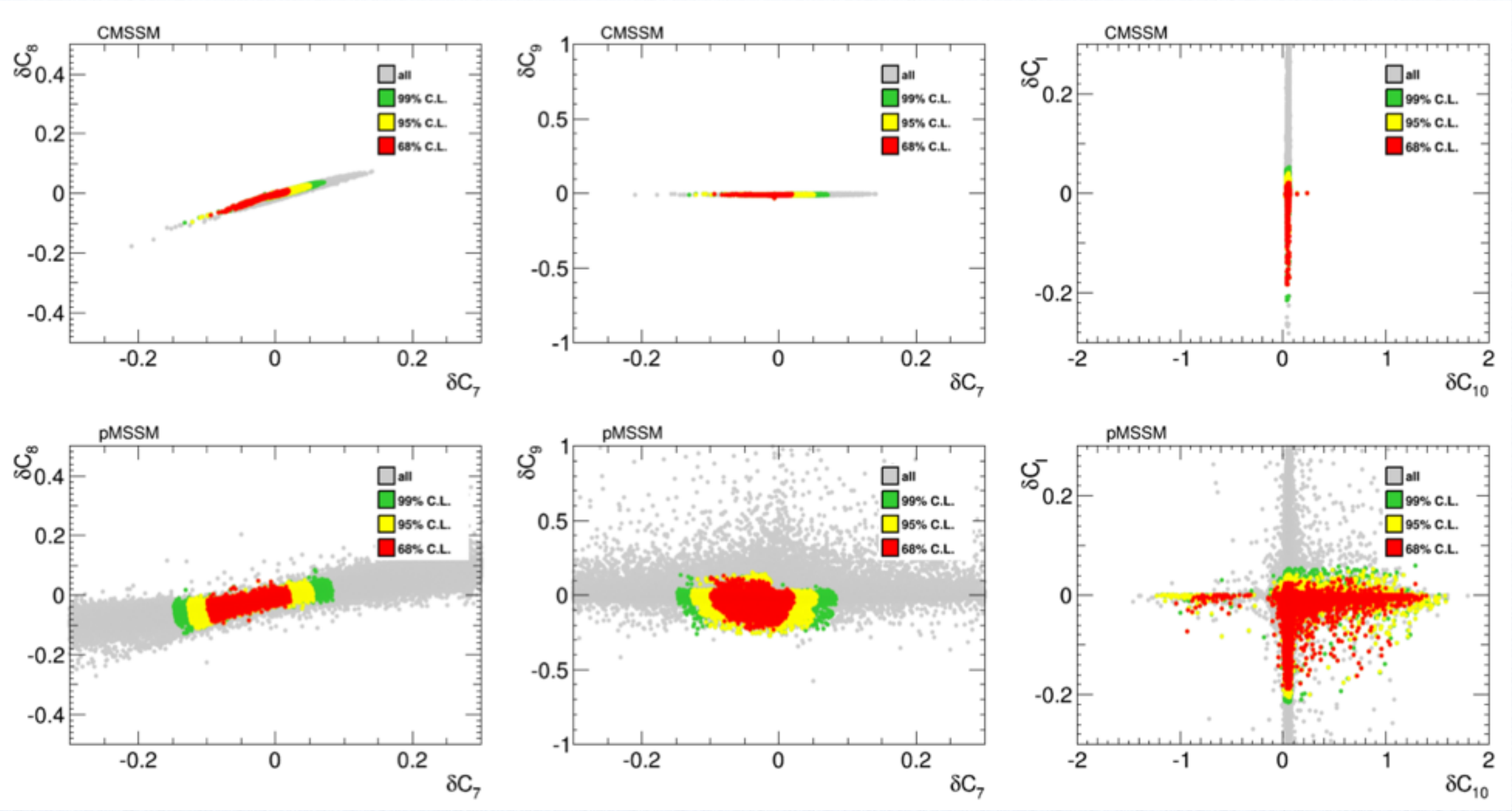


- Leptoquarks:



NP interpretation

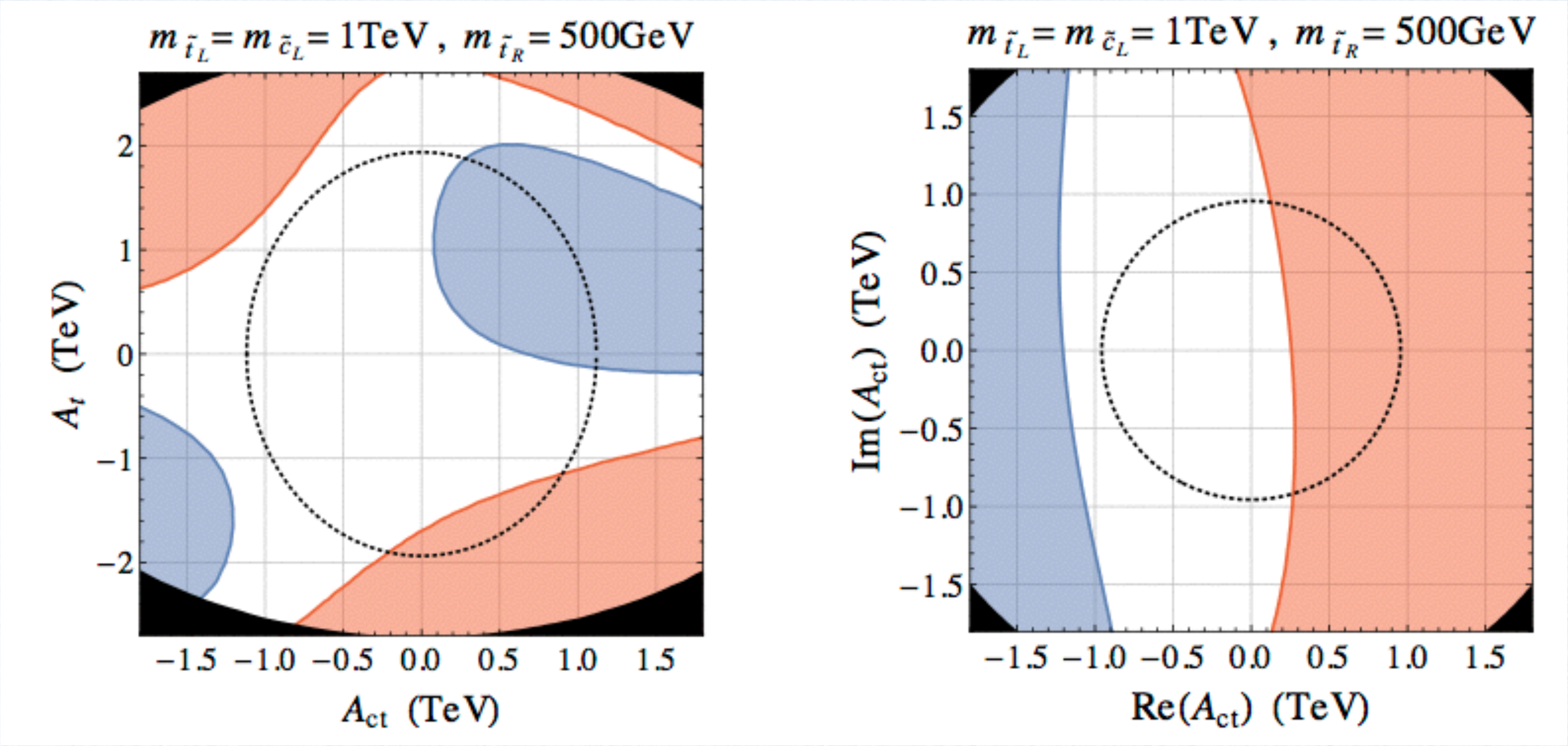
- No large effects on C_9 and C_9' are seen in the pMSSM



[Hurth, Mahmoudi 1411.2786]

NP INTERPRETATION

- Example: MSSM with mass insertions in the 2-3 sector (A_{ct}):



[Altmannshofer, Straub 1411.3161]

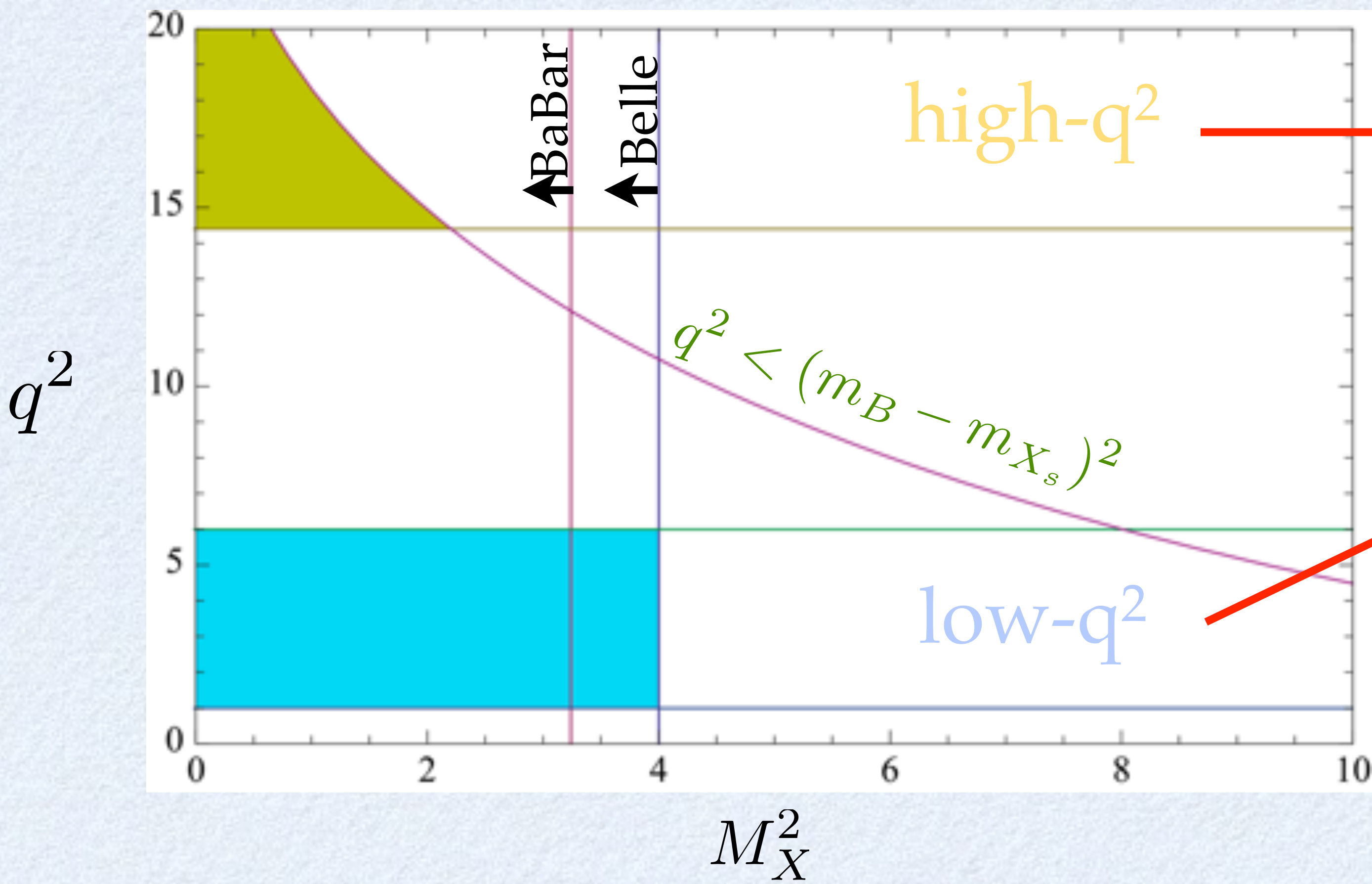
- Outside of the dashed circles: color/charge breaking minima
- Blue region: agreement with LHCb is "improved by more than one sigma"

INPUTS FOR B→SLL

$\alpha_s(M_z) = 0.1184 \pm 0.0007$	$m_e = 0.51099892$ MeV
$\alpha_e(M_z) = 1/127.918$	$m_\mu = 105.658369$ MeV
$s_W^2 \equiv \sin^2\theta_W = 0.2312$	$m_\tau = 1.77699$ GeV
$\lvert V_{ts}^* V_{tb}/V_{cb}\rvert^2 = 0.9621 \pm 0.0027$ [85]	$m_c(m_c) = (1.275 \pm 0.025)$ GeV
$\lvert V_{ts}^* V_{tb}/V_{ub}\rvert^2 = 130.5 \pm 11.6$ [85]	$m_b^{1S} = (4.691 \pm 0.037)$ GeV [86, 87]
$BR(B \to X_c e\bar{\nu})_{\text{exp}} = 0.1051 \pm 0.0013$ [86]	$m_{t,\text{pole}} = (173.5 \pm 1.0)$ GeV
$M_Z = 91.1876$ GeV	$m_B = 5.2794$ GeV
$M_W = 80.385$ GeV	$C = 0.574 \pm 0.019$ [71]
$\mu_b = 5^{+5}_{-2.5}$ GeV	$\mu_0 = 120^{+120}_{-60}$ GeV
$\lambda_2^{\text{eff}} = (0.12 \pm 0.02)$ GeV^2	$\rho_1 = (0.06 \pm 0.06)$ GeV^3 [88]
$\lambda_1^{\text{eff}} = (-0.362 \pm 0.067)$ GeV^2 [86, 87]	$f_u^0 + f_s = (0 \pm 0.2)$ GeV^3 [52]
$f_u^0 - f_s = (0 \pm 0.04)$ GeV^3 [52]	$f_u^\pm = (0 \pm 0.4)$ GeV^3 [52]

X_s cut

- MX cuts required to suppress the $b \to c\, l^- \nu \to s\, l^- l^+ \nu\, \nu$ background



unaffected

parton level at LO:

$M_{Xs} = m_s$

bremsstrahlung:

$m_s < M_{Xs} < m_b$

non-perturbative effects:

phase space (M_B-$m_b = \Lambda$)

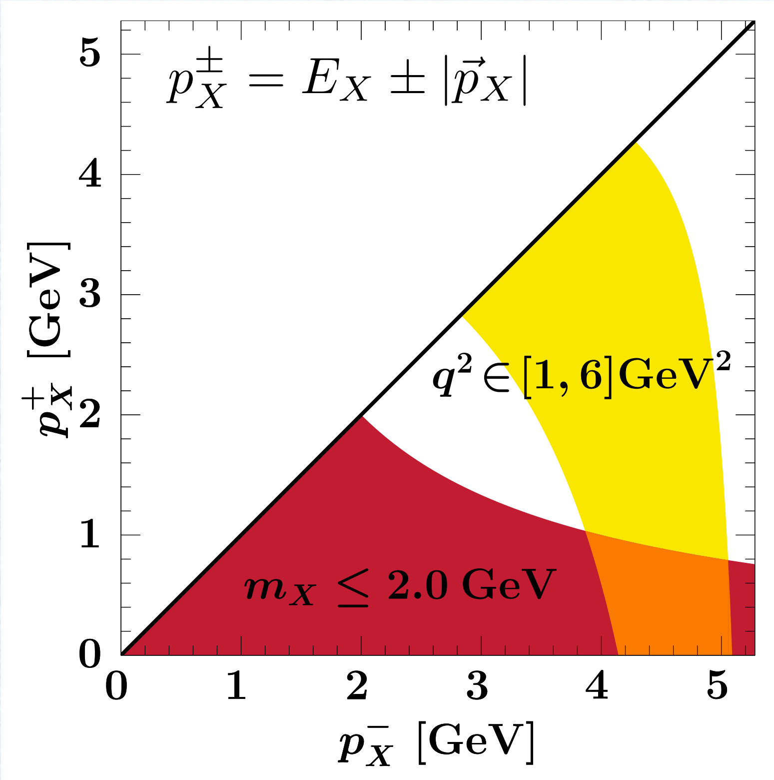
Fermi motion [Ali, Hiller]

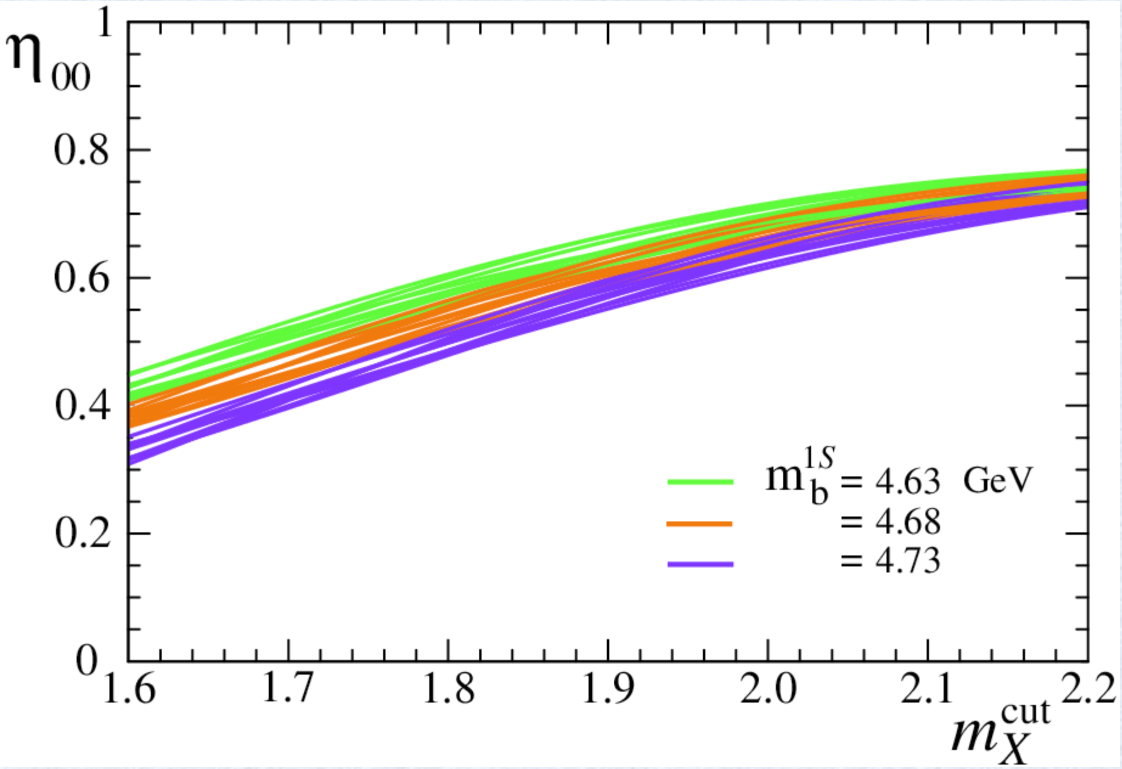
- Correction factor added in experimental results

- Framework: Fermi motion, SCET

Xs CUT

- New idea: use SCET to describe the X_s system





$$p_X^+ \ll p_X^- \Longrightarrow m_X^2 \ll E_X^2$$

X_S is a hard-collinear mode:

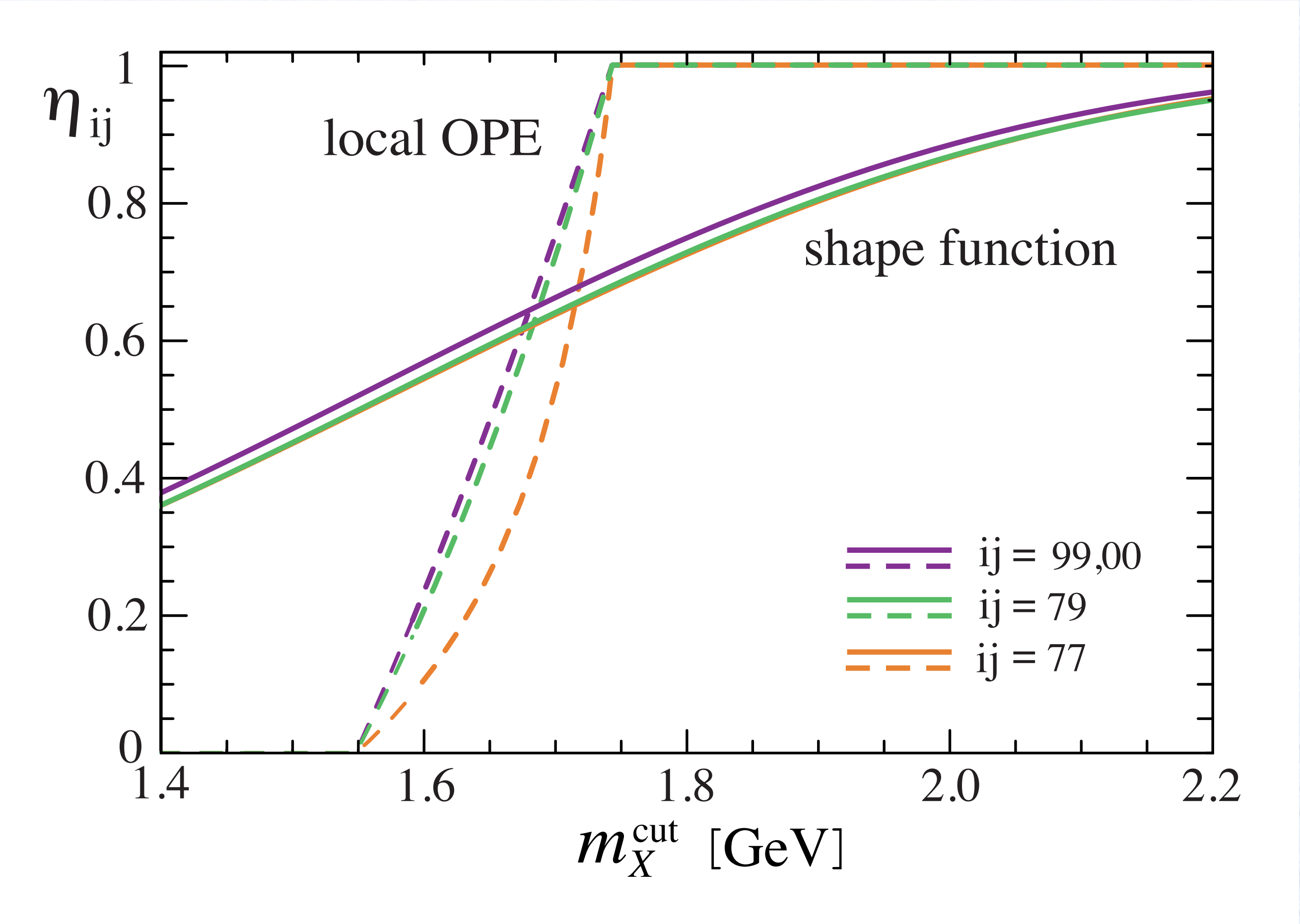
$$\Lambda^2 \ll p_{X_s}^2 \sim \Lambda m_b \ll m_b^2$$

$$\eta_{ij} = \frac{\displaystyle\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} dq^2 \int_0^{m_X^{\mathrm{cut}}} dm_X^2 \frac{d\Gamma_{ij}}{dq^2\, dm_X^2}}{\displaystyle\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} dq^2 \frac{d\Gamma_{ij}}{dq^2}}$$

ij: C_9^2 and C_{10}^2, $C_7 C_9$, C_7^2

X_s cut

- At leading power and at order α_s, these corrections are a universal multiplicative factor:



- Reduce non-perturbative effects by considering: [Lee, Ligeti, Stewart, Tackmann]

$\Gamma^{cut}(B \to X_s \ell^+ \ell^-)/\Gamma^{cut}(B \to X_u \ell \bar{\nu})$ *[same M_X cut]*