QED Corrections to Hadronic Processes in Lattice QCD

Chris Sachrajda

School of Physics and Astronomy
University of Southampton
Southampton SO17 1BJ
UK

Lattice Gauge Theory for the LHC and Beyond

KITP Santa Barbara
August 3 - 25 September 2015

UNIVERSITY OF Southampton
School of Physics and Astronomy

1. Introduction - Mission Statement

- An important goal for the majority of our community is to find signatures of *new physics* and to try to unravel the underlying theoretical structure.
- Precision Flavour physics is a key tool, complementary to the large $p_\perp$ searches at the LHC, in this endeavour.
  - If the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to understand the underlying framework.
  - The discovery potential of precision flavour physics should also not be underestimated.
  - Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.

Personal Digression

- My interest in lattice QCD was inspired at an ITP meeting on *Nuclear Chromodynamics* organised by S.Brodsy and E.Moniz in 1985 at which Steve Gottlieb gave a talk:

LATTICE GAUGE THEORY AND PERTURBATIVE QCD MEET EXPERIMENT

*Steven Gottlieb**

Department of Physics
University of California, San Diego
La Jolla, California 92093

Andreas S. Kronfeld†

Newman Laboratory of Nuclear Studies
Cornell University
Ithaca, New York 14853

ABSTRACT

We use the results of a Monte Carlo simulation of quantum chromodynamics in the quenched approximation to compute the first two moments of the π and ρ distribution amplitude. Our results turn out to be surprisingly large, and we discuss some of the phenomenological implications of this turn of events. In addition to the distribution amplitude itself, we predict the pion form factor and the cross section for $\gamma\gamma \to \pi^0\pi^0$, and we compare the former with experiment. We also contrast lattice gauge theory predictions to those of QCD sum rules.

PACS numbers: 11.15.Ha, 12.35.Eq, 13.40.En, 13.60.Hb

- My talk was entitled *Status of Perturbative Calculations in QCD.*

N.Carrasco, V.Lubicz, G.Martinelli, CTS, N.Tantalo, C.Tarantino, M.Testa, (arXiv:1502.00257)
Current numerical studies also with F.Sanfilippo and S.Simula

- Electromagnetic corrections to hadronic masses are now being calculated. For a review see A.Portelli at Lattice 2014. arXiv:1505.07057
- The results of (some) weak matrix elements obtained from lattice QCD are now being quoted with $\lesssim O(1\%)$ precision e.g. FLAG Collaboration, arXiv:1310.8555

f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}
130.2(1.4)	156.3(0.8)	209.2(3.3)	248.6(2.7)	190.5(4.2)	227.7(4.5)

(results given in MeV)

- We therefore need to start considering electromagnetic (and other isospin breaking) effects if we are to use these results to extract CKM matrix elements at a similar precision.
- For illustration, we consider f_π but the discussion is general. We do not use ChPT.
  For a ChPT based discussion of f_π, see J.Gasser & G.R.S.Zarnauskas, arXiv:1008.3479

- At $O(\alpha^0)$

$$\Gamma(\pi^+ \to \mu^+\nu_\mu) = \frac{G_F^2\,|V_{ud}|^2 f_\pi^2}{8\pi}\, m_\pi\, m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 .$$

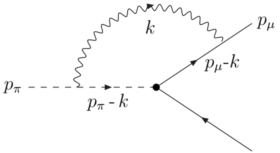
- At $O(\alpha)$ infrared divergences are present and we have to consider

$$\begin{aligned}\Gamma(\pi^+ \to \mu^+\nu_\mu(\gamma)) &= \Gamma(\pi^+ \to \mu^+\nu_\mu) + \Gamma(\pi^+ \to \mu^+\nu_\mu\gamma) \\ &\equiv \Gamma_0 + \Gamma_1\,,\end{aligned}$$

  where the suffix denotes the number of photons in the final state.
- Each of the two terms on the rhs is infrared divergent, the divergences cancel in the sum.
- The cancelation of infrared divergences between contributions with virtual and real photons is an old and well understood issue.

  F.Bloch and A.Nordsieck, PR 52 (1937) 54
- The question for our community is how best to combine this understanding with lattice calculations of non-perturbative hadronic effects.
- This is a generic problem if em corrections are to be included in the evaluation of a decay process.

Infrared Divergence - Example



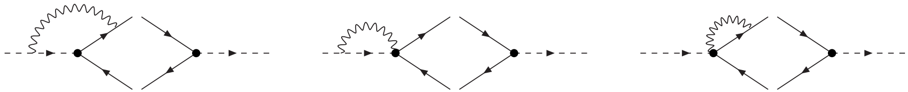
$$
\begin{aligned}
I \quad &\sim \quad \int_{\text{small } k} d^4k \, \frac{1}{(k^2+i\epsilon)((p_\mu-k)^2-m_\mu^2+i\epsilon)((p_\pi-k)^2-m_\pi^2+i\epsilon)} \\
&\sim \quad \int_{\text{small } k} d^4k \, \frac{1}{k^2(-2p_\mu\cdot k)(-2p_\pi\cdot k)} \\
&\sim \quad \int_{\text{small } k} d^4k \, \frac{1}{k^4} \qquad \Rightarrow \text{infrared divergence}\,.
\end{aligned}
$$

- This leads to a contribution to Γ_0 of

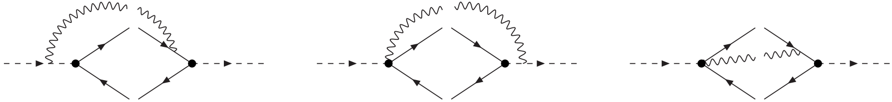
$$
\Gamma_0^{\pi\mu} = \Gamma_0^{\text{tree}} \, \frac{\alpha}{4\pi} \left(\frac{2(1+r_\mu^2)}{1-r_\mu^2} \log r_\mu^2 \log\left(\frac{m_\pi^2}{m_\gamma^2}\right) + \cdots \right) \,,
$$

where the photon mass, m_γ, is introduced to regulate the infrared divergences and $r_\mu = m_\mu/m_\pi$.

Infrared Divergence - Example (Cont)



$$\Gamma_0^{\pi\mu} = \Gamma_0^{\rm tree}\,\frac{\alpha}{4\pi}\left(\frac{2(1+r_\mu^2)}{1-r_\mu^2}\log r_\mu^2\,\log\left(\frac{m_\pi^2}{m_\gamma^2}\right)+\cdots\right)\,,$$



(c) (d) (e)

$$\Gamma_1^{\pi\mu} = \Gamma_0^{\rm tree}\,\frac{\alpha}{4\pi}\left(-\frac{2(1+r_\mu^2)}{1-r_\mu^2}\log r_\mu^2\,\log\left(\frac{4\Delta E^2}{m_\gamma^2}\right)+\cdots\right)\,.$$

- In principle, particularly as techniques and resources improve in the future, it may be better to compute Γ_1 nonperturbatively over a larger range of photon energies.
- Initially at least, we do not propose to compute Γ_1 nonperturbatively. Rather we consider only photons which are sufficiently soft for the point-like (pt) approximation to be valid.
  - A cut-off ΔE of $O(10\text{-}20\,\text{MeV})$ appears to be appropriate both experimentally and theoretically.

F.Ambrosino et al., KLOE collaboration, hep-ex/0509045; arXiv:0907.3594

A.Ceccucci, private communication

- We now write

$$\Gamma_0 + \Gamma_1(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\text{pt}}) + \lim_{V\to\infty}(\Gamma_0^{\text{pt}} + \Gamma_1(\Delta E))\,.$$

  - The second term on the rhs can be calculated in perturbation theory. It is infrared convergent, but does contain a term proportional to $\log\Delta E$.
  - The first term is also free of infrared divergences.
  - Γ_0 is calculated nonperturbatively and Γ_0^{pt} in perturbation theory.
  - There is no zero-mode contribution in $\Gamma_0 - \Gamma_0^{\text{pt}}$.

$$\Gamma_0 + \Gamma_1(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\rm pt}) + \lim_{V\to\infty}(\Gamma_0^{\rm pt} + \Gamma_1(\Delta E))\,.$$

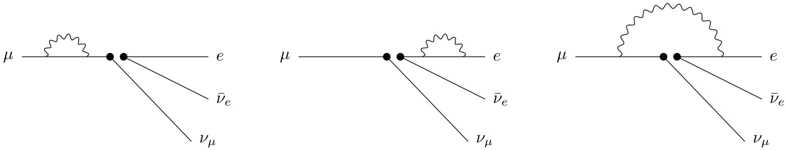
2. What is G_F at $O(\alpha)$?

1. The results for the widths are expressed in terms of G_F, the Fermi constant ($G_F = 1.16632(2) \times 10^{-5}\,\mathrm{GeV}^{-2}$). This is obtained from the muon lifetime:

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1 - \frac{8m_e^2}{m_\mu^2}\right]\left[1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right].$$

S.M.Berman, PR 112 (1958) 267; T.Kinoshita and A.Sirlin, PR 113 (1959) 1652

- This expression can be viewed as the definition of G_F. Many EW corrections are absorbed into the definition of G_F; the explicit $O(\alpha)$ corrections come from the following diagrams in the effective theory:



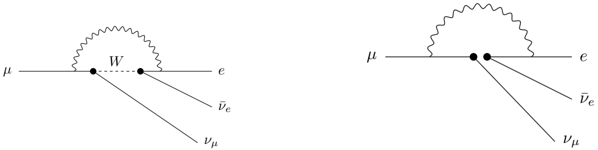
  together with the diagrams with a real photon.

- These diagrams are evaluated in the W-regularisation in which the photon propagator is modified by: A.Sirlin, PRD 22 (1980) 971

$$\frac{1}{k^2} \to \frac{M_W^2}{M_W^2 - k^2}\,\frac{1}{k^2}. \qquad \left(\frac{1}{k^2} = \frac{1}{k^2 - M_W^2} + \frac{M_W^2}{M_W^2 - k^2}\,\frac{1}{k^2}\right)$$

W-regularization (cont)

- The $\gamma - W$ box diagram:



As an example providing some evidence & intuition that the W-regularization is useful consider the $\gamma - W$ box diagram.

- In the standard model (left-hand diagram) it contains both the γ and W propagators.
- In the effective theory this is preserved with the W-regularization where the photon propagator is proportional to

$$\frac{1}{k^2}\,\frac{1}{k^2 - M_W^2}$$

and the two diagrams are equal up to terms of $O(q^2/M_W^2)$, where q is the momentum of the e and ν_e.

3. Proposed calculation of $\Gamma_0 - \Gamma_0^{\text{pt}}$

- Most (but not all) of the EW corrections which are absorbed in G_F are common to other processes (including pion decay) $\Rightarrow$ factor in the amplitude of $(1 + 3\alpha/4\pi(1+2\bar{Q})\log M_Z/M_W)$, where $\bar{Q} = \frac{1}{2}(Q_u + Q_d) = 1/6$.

  A.Sirlin, NP B196 (1982) 83; E.Braaten & C.S.Li, PRD 42 (1990) 3888

- We therefore need to calculate the pion-decay diagrams in the effective theory with

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud}^* \left(1 + \frac{\alpha}{\pi}\log\frac{M_Z}{M_W}\right)(\bar{d}_L\gamma^\mu u_L)(\bar{\nu}_{\ell,L}\gamma_\mu \ell_L)$$

  in the W-regularization.

- Thus for example, with the Wilson action for both the gluons and fermions:

$$\begin{aligned} O_1^{\text{W-reg}} &= \left(1 + \frac{\alpha}{4\pi}\left(2\log a^2M_W^2 - 15.539\right)\right) O_1^{\text{bare}} + \frac{\alpha}{4\pi}\left(0.536\, O_2^{\text{bare}}\right. \\ &\qquad \left. +1.607\, O_3^{\text{bare}} - 3.214\, O_4^{\text{bare}} - 0.804\, O_5^{\text{bare}}\right), \end{aligned}$$

where

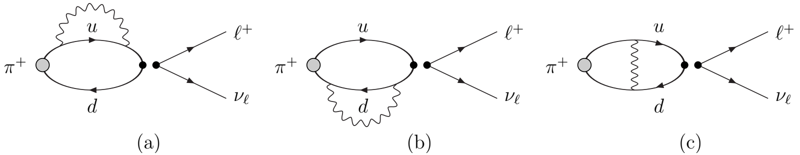
$$O_1 = (\bar{d}\gamma^\mu(1-\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell) \qquad O_2 = (\bar{d}\gamma^\mu(1+\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell)$$

$$O_3 = (\bar{d}(1-\gamma^5)u)\,(\bar{\nu}_\ell(1+\gamma^5)\ell) \qquad O_4 = (\bar{d}(1+\gamma^5)u)\,(\bar{\nu}_\ell(1+\gamma^5)\ell)$$

$$O_5 = (\bar{d}\sigma^{\mu\nu}(1+\gamma^5)u)\,(\bar{\nu}_\ell\sigma_{\mu\nu}(1+\gamma^5)\ell)\,.$$

Proposed calculation of $\Gamma_0 - \Gamma_0^{\mathbf{pt}}$ (Cont)

Consider now the evaluation of Γ_0.



- The correlation function for this set of diagrams is of the form:

$$C_1(t) = -\frac{1}{2}\int d^3\vec{x}\, d^4x_1\, d^4x_2\, \langle 0|T\Big\{J_W^\nu(0)\, j_\mu(x_1) j_\mu(x_2)\phi^\dagger(\vec{x},-t)\Big\}\,|\,0\rangle\, \Delta(x_1,x_2)\,,$$

  where $j_\mu(x) = \sum_f Q_f \bar{f}(x)\gamma_\mu f(x)$, J_W is the weak current, ϕ is an interpolating operator for the pion and Δ is the photon propagator.

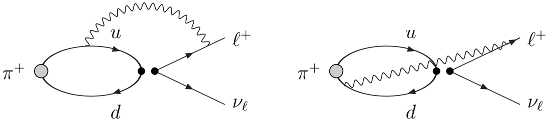
- Combining C_1 with the lowest order correlator:

$$C_0(t) + C_1(t) \simeq \frac{e^{-m_\pi t}}{2m_\pi} Z^\phi\, \langle\, 0\,|J_W^\nu(0)\,|\,\pi^+\rangle\,,$$

  where now $O(\alpha)$ terms are included.

- $e^{-m_\pi t} \simeq e^{-m_\pi^0 t}\,(1-\delta m_\pi\, t)$ and Z^ϕ is obtained from the two-point function.

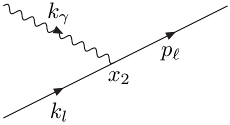
Proposed calculation of $\Gamma_0 - \Gamma_0^{\rm pt}$ (Cont)



$$\bar{C}_1(t)_{\alpha\beta} = -\int d^3\vec{x}\, d^4x_1\, d^4x_2\, \langle 0|T\{J_W^\nu(0)\,j_\mu(x_1)\,\phi^\dagger(\vec{x},-t)\}|\,0\rangle\, \Delta(x_1,x_2)$$
$$\times\left(\gamma_\nu(1-\gamma^5)S(0,x_2)\gamma_\mu\right)_{\alpha\beta}\, e^{E_\mu t_2}\, e^{-i\vec{p}_\mu\cdot\vec{x}_2}$$
$$\simeq Z_0^\phi\, \frac{e^{-m_\pi^0 t}}{2m_\pi^0}\, (\bar{M}_1)_{\alpha\beta}$$

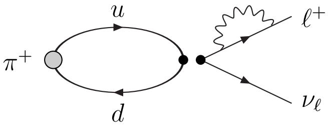
- Corresponding contribution to the amplitude is $\bar{u}_\alpha(p_{\nu_\mu})(\bar{M}_1)_{\alpha\beta}v_\beta(p_\mu)$.
- Diagrams (e) and (f) are not simply generalisations of the evaluation of f_π.
- We have to be able to isolate the finite-volume ground state (pion).
- The Minkowski $\leftrightarrow$ Euclidean continuation can be performed (the time integrations are convergent).

Convergence of the t_2 integration



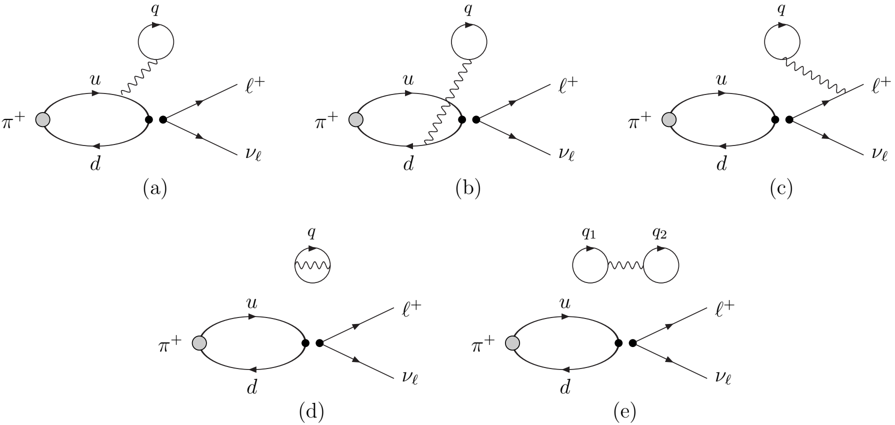
- For every term in the $\vec{k}_\gamma$ integration, $\omega_\gamma + \omega_l > E_l$ so the t_2 behaviour, $\exp[-(\omega_k + \omega_l - E_l)t_2]$ is convergent.

- The lepton's wave function renormalisation cancels in the difference $\Gamma_0 - \Gamma_0^{\rm pt}$.



(d)

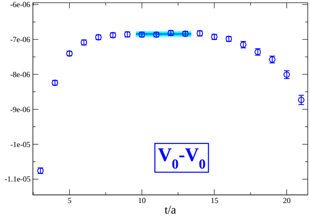
There are also disconnected diagrams to be evaluated.

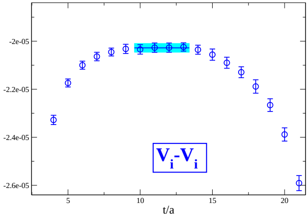


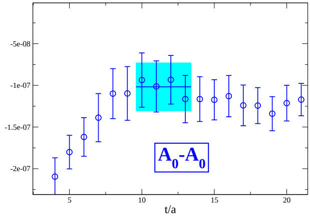
Preliminary Results for “Crossed" Diagrams

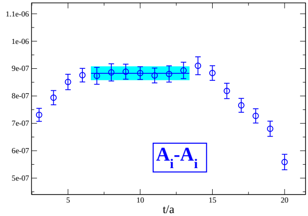
Twisted-mass study, $24^3 \times 48$ lattice with $a = 0.086\,$fm, $m_\pi \simeq 475\,$MeV, 240 configs with 3 stochastic sources per configuration.

together with F.Sanfilippo and S.Simula

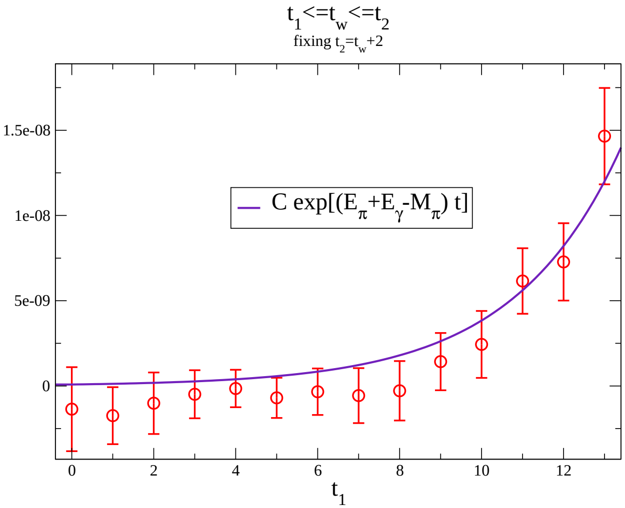






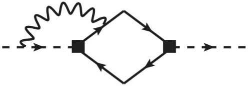


UNIVERSITY OF Southampton School of Physics and Astronomy



f_0^{pt} *The nasty diagram*

sum vs integral under study



$$\left(\frac{2\pi}{L}\right)^3 \sum_{\vec q} \frac{1}{\pi}$$

$$\Bigg(\frac{1}{m_W{}^2|\vec q\cdot\vec q|^{1/2}} - \frac{2m^2}{(m^2-m_\mu{}^2)m_W{}^2|\vec q\cdot\vec q|^{1/2}} + \frac{2m_\mu{}^2}{(m^2-m_\mu{}^2)m_W{}^2|\vec q\cdot\vec q|^{1/2}} + \frac{4|\vec q\cdot\vec q|^{1/2}}{m_W{}^4} - \frac{4m^2|\vec q\cdot\vec q|^{1/2}}{(m^2-m_\mu{}^2)m_W{}^4} +$$

$$\frac{4m_\mu{}^2|\vec q\cdot\vec q|^{1/2}}{(m^2-m_\mu{}^2)m_W{}^4} + \frac{m^2\sqrt{\frac{1}{m^2+\vec q\cdot\vec q}}}{2(m^2-m_\mu{}^2)\vec q\cdot\vec q} + \frac{m_\mu{}^2\sqrt{\frac{1}{m^2+\vec q\cdot\vec q}}}{2(m^2-m_\mu{}^2)\vec q\cdot\vec q} - \frac{m_\mu{}^2\sqrt{\frac{1}{m_\mu{}^2+\vec q\cdot\vec q}}}{(m^2-m_\mu{}^2)\vec q\cdot\vec q} - \frac{3\sqrt{\frac{1}{m_W{}^2+\vec q\cdot\vec q}}}{m_W{}^2} -$$

$$\frac{4\vec q\cdot\vec q\sqrt{\frac{1}{m_W{}^2+\vec q\cdot\vec q}}}{m_W{}^4} + \frac{4m^2\sqrt{m_W{}^2+\vec q\cdot\vec q}}{(m^2-m_\mu{}^2)m_W{}^4} - \frac{4m_\mu{}^2\sqrt{m_W{}^2+\vec q\cdot\vec q}}{(m^2-m_\mu{}^2)m_W{}^4} + \frac{m^2\left(\frac{1}{m_\gamma{}^2+\vec q\cdot\vec q}\right)^{3/2}\mathrm{Log}[r_\mu{}^2]}{2(m^2-m_\mu{}^2)} +$$

$$\frac{m_\mu{}^2\left(\frac{1}{m_\gamma{}^2+\vec q\cdot\vec q}\right)^{3/2}\mathrm{Log}[r_\mu{}^2]}{2(m^2-m_\mu{}^2)} - \frac{m^2\mathrm{Log}\left[\frac{1+|\vec q\cdot\vec q|^{1/2}\sqrt{\frac{1}{m^2+\vec q\cdot\vec q}}}{1-|\vec q\cdot\vec q|^{1/2}\sqrt{\frac{1}{m^2+\vec q\cdot\vec q}}}\right]}{2(m^2-m_\mu{}^2)\left(|\vec q\cdot\vec q|^{1/2}\right)^3} - \frac{m_\mu{}^2\mathrm{Log}\left[\frac{1+|\vec q\cdot\vec q|^{1/2}\sqrt{\frac{1}{m^2+\vec q\cdot\vec q}}}{1-|\vec q\cdot\vec q|^{1/2}\sqrt{\frac{1}{m^2+\vec q\cdot\vec q}}}\right]}{2(m^2-m_\mu{}^2)\left(|\vec q\cdot\vec q|^{1/2}\right)^3} +$$

$$\frac{m^2\mathrm{Log}\left[\frac{1+|\vec q\cdot\vec q|^{1/2}\sqrt{\frac{1}{m_\mu{}^2+\vec q\cdot\vec q}}}{1-|\vec q\cdot\vec q|^{1/2}\sqrt{\frac{1}{m_\mu{}^2+\vec q\cdot\vec q}}}\right]}{2(m^2-m_\mu{}^2)\left(|\vec q\cdot\vec q|^{1/2}\right)^3} + \frac{m_\mu{}^2\mathrm{Log}\left[\frac{1+|\vec q\cdot\vec q|^{1/2}\sqrt{\frac{1}{m_\mu{}^2+\vec q\cdot\vec q}}}{1-|\vec q\cdot\vec q|^{1/2}\sqrt{\frac{1}{m_\mu{}^2+\vec q\cdot\vec q}}}\right]}{2(m^2-m_\mu{}^2)\left(|\vec q\cdot\vec q|^{1/2}\right)^3}\Bigg)$$

slide from G.Martinelli, Benasque 2015

4. Calculation of $\Gamma^{\rm pt}=\Gamma_0^{\rm pt}+\Gamma_1^{\rm pt}$

- The total width, $\Gamma^{\rm pt}$ was calculated in 1958/9 using a Pauli-Villars regulator for the UV divergences and m_γ for the infrared divergences.

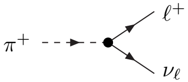
  S.Berman, PR **112** (1958) 267, T.Kinoshita, PRL **2** (1959) 477

  - This is a useful check on our perturbative calculation.

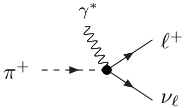
- In the perturbative calculation we use the following Lagrangian for the interaction of a point-like pion with the leptons:

$$\mathcal{L}_{\pi\text{-}\mu\text{-}\nu_\mu} \;=\; i\,G_F f_\pi V_{ud}^*\;\{(\partial_\mu - ieA_\mu)\pi\}\left\{\bar\psi_{\nu_\mu}\frac{1+\gamma_5}{2}\gamma^\mu\psi_\mu\right\}+\text{H.C.}\,.$$

The corresponding Feynman rules are:



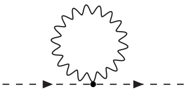
$$= -iG_F f_\pi V_{ud}^*\,p_\pi^\mu\,\tfrac{1+\gamma^5}{2}\gamma_\mu$$

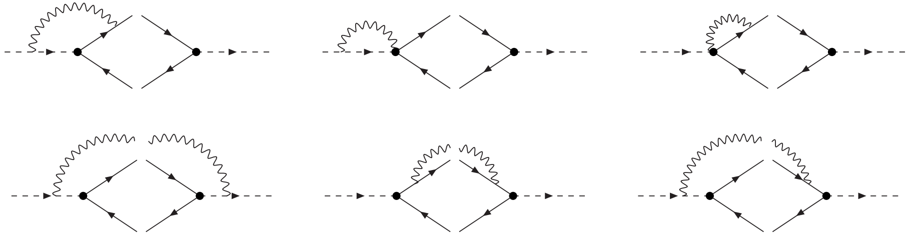


$$= ie\,G_F f_\pi V_{ud}^*\,g^{\mu\nu}\,\tfrac{1+\gamma^5}{2}\gamma_\mu$$

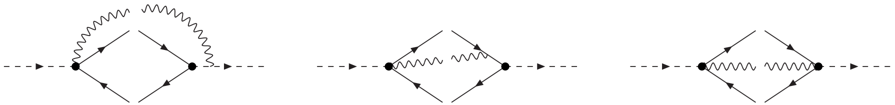
Diagrams to be evaluated

and





(a) (b) (c)



(d) (e) (f)

4. Calculation of $\Gamma^{\rm pt} = \Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}$ (cont)

- We find, for $E_\gamma < \Delta E$

$$\begin{aligned}\Gamma^{\rm pt}(\Delta E) \;=\; & \Gamma_0^{\rm tree} \times \Bigg(1 + \frac{\alpha}{4\pi}\Bigg\{3\log\left(\frac{m_\pi^2}{M_W^2}\right) + \log\left(r_\ell^2\right) - 4\log(r_E^2) + \frac{2-10r_\ell^2}{1-r_\ell^2}\log(r_\ell^2) \\ & -2\frac{1+r_\ell^2}{1-r_\ell^2}\log(r_E^2)\log(r_\ell^2) - 4\frac{1+r_\ell^2}{1-r_\ell^2}\,\mathrm{Li}_2(1-r_\ell^2) - 3 \\ & +\Big[\frac{3+r_E^2-6r_\ell^2+4r_E(-1+r_\ell^2)}{(1-r_\ell^2)^2}\log(1-r_E) + \frac{r_E(4-r_E-4r_\ell^2)}{(1-r_\ell^2)^2}\log(r_\ell^2) \\ & -\frac{r_E(-22+3r_E+28r_\ell^2)}{2(1-r_\ell^2)^2} - 4\frac{1+r_\ell^2}{1-r_\ell^2}\,\mathrm{Li}_2(r_E)\Big]\Bigg\}\Bigg)\,,\end{aligned}$$

where $r_E = 2\Delta E/m_\pi$ and $r_\ell = m_\ell/m_\pi$.

- We believe that this is a new result.

4. Calculation of $\Gamma^{\rm pt} = \Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}$ (cont)

- The total rate is readily computed by setting r_E to its maximum value, namely $r_E = 1 - r_\ell^2$, giving

$$\Gamma^{\rm pt} = \Gamma_0^{\rm tree} \times \left\{ 1 + \frac{\alpha}{4\pi} \left(3\log\left(\frac{m_\pi^2}{M_W^2}\right) - 8\log(1-r_\ell^2) - \frac{3r_\ell^4}{(1-r_\ell^2)^2}\log(r_\ell^2) \right.\right.$$
$$\left.\left. -8\frac{1+r_\ell^2}{1-r_\ell^2}\,\mathrm{Li}_2(1-r_\ell^2) + \frac{13-19r_\ell^2}{2(1-r_\ell^2)} + \frac{6-14r_\ell^2-4(1+r_\ell^2)\log(1-r_\ell^2)}{1-r_\ell^2}\log(r_\ell^2)\right)\right\}.$$

- This result agrees with the well known results in literature providing an important check of our calculation.

- It is of course possible instead to impose a cut-off on the energy of the final-state lepton, requiring it to be close to its maximum value $E_\ell^{\rm max}=\frac{m_\pi}{2}(1+r_\ell^2)$.
- We also give, up to $O(\Delta E_\ell)$, the distribution for $\Gamma^{\rm pt}(\Delta E_\ell)$ defined as

$$\Gamma^{\rm pt}(\Delta E_\ell)=\int_{E_\ell^{max}-\Delta E_\ell}^{E_\ell^{max}} dE_\ell^\prime\,\frac{d\Gamma^{\rm pt}}{dE_\ell^\prime}\,,$$

where $0\le\Delta E_\ell\le(m_\pi-m_\ell)^2/(2m_\pi)$;

$$\Gamma^{\rm pt}(\Delta E_\ell)=\Gamma_0^{\rm tree}\times\Bigg\{1+\frac{\alpha}{4\pi}\Bigg[3\log\left(\frac{m_\pi^2}{M_W^2}\right)+8\log\left(1-r_\ell^2\right)-7$$
$$+\log\left(r_\ell^2\right)\frac{3-7r_\ell^2+8\Delta E_\ell+4\left(1+r_\ell^2\right)\log\left(1-r_\ell^2\right)}{1-r_\ell^2}$$
$$+\log\left(2\Delta E_\ell\right)\left(-8-4\frac{1+r_\ell^2}{1-r_\ell^2}\log\left(r_\ell^2\right)\right)\Bigg]\Bigg\}\,.$$

- Summary: The perturbative calculation of $\Gamma_0^{\rm pt}+\Gamma_1(\Delta E)$ is done.

5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$

- For sufficiently small ΔE the structure dependent contributions to Γ_1 can be neglected.
- How big might they be for experimentally accessible values of ΔE?
  To estimate this for f_π and f_K we use Chiral Perturbation Theory.

J.Bijnens, G.Ecker and J.Gasser, hep-ph/9209261,
J.Bijnens, G.Colangelo, G.Ecker and J.Gasser, hep-ph/9411311.
V. Cirigliano and I. Rosell, arXiv:0707.3439 [hep-ph]],
L. Ametller, J. Bijnens, A. Bramon and F. Cornet, hep-ph/9302219.

- We define

$$R_1^A(\Delta E) = \frac{\Gamma_1^A(\Delta E)}{\Gamma_0^{\alpha,\text{pt}} + \Gamma_1^{\text{pt}}(\Delta E)}\ , \qquad A = \{\text{SD,INT}\}\ ,$$

where SD and INT refer to the structure dependent and interference (between SD and pt) contributions respectively.

  - Note that the notation I am using here differs from that in the paper.

UNIVERSITY OF Southampton
School of Physics and Astronomy

- Start with a decomposition in terms of Lorenz invariant form factors of the hadronic matrix element

$$H^{\mu\nu}(k,p_\pi) = \int d^4x\, e^{ikx}\, T\langle 0|j^\mu(x)J_W^\nu(0)|\pi(p_\pi)\rangle$$

and separate the contribution corresponding to the approximation of a point-like pion $H_{\rm pt}^{\mu\nu}$, from the structure dependent part $H_{\rm SD}^{\mu\nu}$,

$$H^{\mu\nu} = H_{\rm SD}^{\mu\nu} + H_{\rm pt}^{\mu\nu}\,.$$

- $H_{\rm pt}^{\mu\nu}$ is simply given by

$$H_{\rm pt}^{\mu\nu} = f_\pi\left[g^{\mu\nu} - \frac{(2p_\pi-k)^\mu(p_\pi-k)^\nu}{(p_\pi-k)^2-m_\pi^2}\right]\,.$$

- The structure dependent component can be parametrised by four independent invariant form factors which we define as

$$H_{\rm SD}^{\mu\nu} = H_1\left[k^2g^{\mu\nu}-k^\mu k^\nu\right] + H_2\left\{\left[(k\cdot p_\pi-k^2)k^\mu - k^2(p_\pi-k)^\mu\right](p_\pi-k)^\nu\right\}$$
$$-i\frac{F_V}{m_\pi}\epsilon^{\mu\nu\alpha\beta}k_\alpha p_{\pi\,\beta} + \frac{F_A}{m_\pi}\left[(k\cdot p_\pi-k^2)g^{\mu\nu}-(p_\pi-k)^\mu k^\nu\right]\,.$$

5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$ (cont)

- For the decay into a real photon, only F_V and F_A contribute.
- At $O(p^4)$ in chiral perturbation theory,

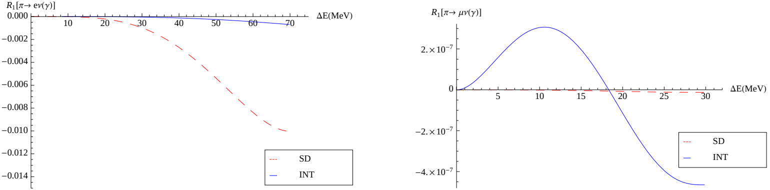
$$F_V = \frac{m_P}{4\pi^2 f_\pi} \quad \text{and} \quad F_A = \frac{8m_P}{f_\pi}\,(L_9^r + L_{10}^r)\,,$$

  where $P = \pi$ or K and L_9^r, L_{10}^r are Gasser-Leutwyler coefficients.
- The numerical values of these constants have been taken from the review by M.Bychkov and G.D'Ambrosio in the PDG. F_V and F_A are 0.0254 and 0.0119 for the pion and 0.096 and 0.042 for the Kaon (for the pion these values of the form factors, obtained from direct measurements, can be found in the supplement to the PDG.)

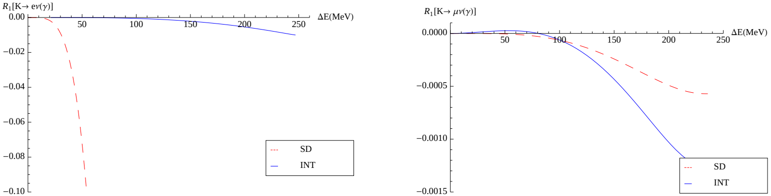
5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$ (cont)

UNIVERSITY OF Southampton
School of Physics and Astronomy

Pion



Kaon



5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$ (cont)

- For heavy-light mesons we don't have such ChPT calculations.
- For the B-meson in particular we have another small scale $< \Lambda_{\mathrm{QCD}}$, $m_{B^*} - m_B \simeq 45$ MeV so that we may expect that we will have to go to smaller ΔE in order to be able to neglect SD effects.
- Calculations based on the extreme approximation of single pole dominance suggest that this is likely to be the case.

  D. Becirevic, B. Haas and E. Kou, arXiv:0907.1845 [hep-ph]
- To be investigated further!

6. Summary and Conclusions

- Lattice calculations of some physical quantities are approaching $O(1\%)$ precision $\Rightarrow$ we need to include isospin-breaking effects, including electromagnetic effects, to make the tests of the SM even more stringent.
- For decay widths and scattering cross sections including em effects introduces infrared divergences.
- We propose a method for dealing with these divergences, illustrating the procedure by a detailed study of the leptonic (and semileptonic) decays of pseudoscalar mesons.
- Although challenging, the method is within reach of present simulations and we will now implement the procedure in an actual numerical computation.
  - Power-like FV corrections, $O(1/(L\Lambda_{\text{QCD}})^n)$, to be evaluated.
  - $O(\alpha\alpha_s)$ matching factors to be studied.
- In the future one can envisage relaxing the condition $\Delta E \ll \Lambda_{\text{QCD}}$, including the emission of real photons with energies which do resolve the structure of the initial hadron. Such calculations can be performed in Euclidean space under the same conditions as above, i.e. providing that there is a mass gap.
  - The natural extension of the present proposal is to subtract and add $\Gamma_1^{\text{pt}}(\Delta E)$ to determine $\Gamma_1(\Delta E) - \Gamma_1^{\text{pt}}(\Delta E)$, so that our calculation of $\Gamma_0^{\text{pt}} + \Gamma_1^{\text{pt}}(\Delta E)$ will still be useful.